Axionless strong CP problem solution and the 2 Higgs doublet model

Gustavo Burdman
Tania Robens
Miguel Nebot
Barbara Mele

Stefano Morisi, University of Naples

In collaboration with E.Peinado and R.Ferro-Hernandez

Phys.Rev.D 111 (2025) 7, 073009 e-Print: 2407.18161 [hep-ph]

$$\mathcal{L}_{\rm QCD} \supset \frac{\theta}{32\pi^2} G^{\mu\nu}\tilde{G}_{\mu\nu}$$ Violate CP

$$\bar{\theta} = \theta + \mathrm{Arg\,Det}(M_u M_d)$$

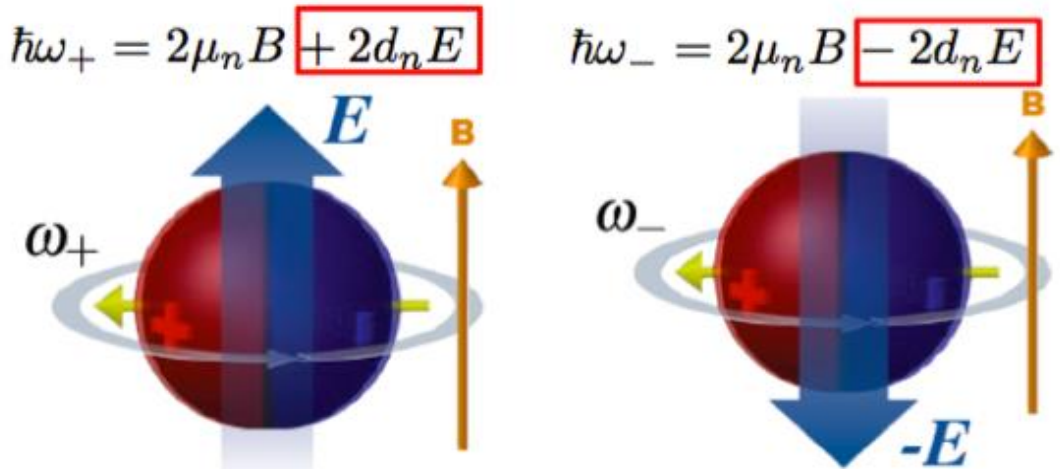
invariant under a quark chiral rotation and has physical meaning

neutron Electric Dipole Moment (nEDM)

$$H = -d_n \vec{E}\cdot\hat{S}$$

$$d_n = 2.4\,(1.0)\cdot 10^{-16}\,\bar{\theta}\ e\ \mathrm{cm}$$

Pospelov, Ritz, *Theta vacua, QCD sum rules, and the nEDM* *NPB573(2000)*



$$|d_n^{\rm exp}| < 3.0\cdot 10^{-26}\ e\ \mathrm{cm}$$

$$|\bar{\theta}| \lesssim 10^{-10}$$

$|\bar{\theta}| \lesssim 10^{-10}$ Why this is small is just a problem of naturalness

t'Hooft has convincingly argued that a physical parameter may only be very small if replacing it by zero increases the symmetry of the theory.

In this case the symmetry is CP, however in the Standard Model CP is explicitly broken in week interactions, thus θ can not be naturally small

Most popular solutions

- If $m_u = 0$, θ can be reabsorbed by means of chiral trasformation
- Peccei-Quinn solution, the Lagrangian possesses a spontaneously broken GLOBAL symmetry U(1) that is color anomaly, so θ=0 dynamically → Axion
- if CP invariance is softly or spontaneously broken, then θ=0 at tree-level and is a calculable quantity at loop level (Wilczek)

Wilczek, Problem of Strong P and T Invariance in the Presence of Instantons, PRL40(1978), 6000 citations

The main idea

Imposing CP at the lagrangian-level, then θ=0

Then CP invariance is softly or spontaneously broken

$M_{u,d}$ complex to fit CKM

$$\bar{\theta} = \theta + \mathrm{Arg\,Det}(M_u M_d)$$

$|\bar{\theta}| \lesssim 10^{-10}$

To be small or zero at tree level

$$\mathrm{Arg\,Det}(M_u M_d) = 0$$

Nelson-Barr

$$\mathcal{M} = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix}$$

Nelson, PLB136 (1984)
Barr, PRL53 (1984)

Left-Right models

$$SU(2)_L \times SU(2)_R$$

Assuming further Parity invariance, then $M_{u,d}$ are Hermitian

Mohapatra, Senjanovic, PLB79(1978)
Mohapatra, Pati,PRD11(1975)
Mohapatra, Senjanovic, PRD12(1975)

Special textures

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i}* & * \\ 0 & 0 & * \end{pmatrix}$$

Antusch,Holthausen,Schmidt,Spinrath,NPB(2013)

Modular Invariance

The Yukawa couplings Y depend on a super-field τ

$$Y(\tau) \to (c\tau + d)^{k_Y} Y(\tau)$$

$$\text{with weight } k_Y = k_{q_{Ri}} + k_{q_{Li}} + k_{H_q}$$

$$\sum_{i=1}^{3} (2k_{Q_i} + k_{u_{Ri}} + k_{d_{Ri}}) = 0$$

Feruglio, Strumia,Titov, JHEP(2023)

A MODEL OF SOFT CP VIOLATION

Howard Georgi*

Lyman Laboratory of Physics

Harvard University

Cambridge, Massachusetts 02138

Hadronic J. 1 (1978) 155 with 300 citation

This model has only 2 families
and
2 Higgs scalars (2HDM)

$\mathbb{Z}_2$: $\phi_1^+ - \phi_1$, $U_{1R}^+ - U_{1R}$, $D_{1R}^+ - D_{1R}$

all the other fields are even

$$\sum_{i=1}^{2} \{ \bar{U}_{iR} \phi_i^+ A_{ij} \psi_{jL} + \bar{D}_{iR} \tilde{\phi}_i^+ B_{ij} \psi_{jL} \} + h.c.$$

$$\psi_{1L} = \begin{pmatrix} U_{1L} \\ D_{1L} \end{pmatrix}, \quad \psi_{2L} = \begin{pmatrix} U_{2L} \\ D_{2L} \end{pmatrix},$$

CP imposed → all Yukawas are real

2HDM with CP and $\mathbb{Z}_2$ invariance, $\mathbb{Z}_2$ softly broken by μ_{12}

Nebot, Botella, Branco, Eur. Phys. J. C 79 (2019)

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2\Phi_1^\dagger\Phi_1 + \mu_{22}^2\Phi_2^\dagger\Phi_2 + \mu_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_5[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]\,.$$

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ e^{i\theta_1}v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ e^{i\theta_2}v_2/\sqrt{2} \end{pmatrix}$$

$$\frac{\partial V}{\partial\theta} = -v_1v_2\sin\theta(\mu_{12}^2 + 2\lambda_5v_1v_2\cos\theta) = 0,$$
$$\frac{\partial V}{\partial v_1} = \mu_{11}^2v_1 + \lambda_1v_1^3 + (\lambda_3+\lambda_4)v_1v_2^2 + v_2(\mu_{12}^2\cos\theta + \lambda_5v_1v_2\cos 2\theta) = 0,$$
$$\frac{\partial V}{\partial v_2} = \mu_{22}^2v_2 + \lambda_2v_2^3 + (\lambda_3+\lambda_4)v_1^2v_2 + v_1(\mu_{12}^2\cos\theta + \lambda_5v_1v_2\cos 2\theta) = 0.$$

$$\cos\theta = \frac{-\mu_{12}^2}{2\lambda_5v_1v_2}$$

$$\theta = \theta_2 - \theta_1$$

$$\text{Mu} \;:\; (\,\text{U1R}\;\;\text{U2R}\,)\begin{pmatrix} \text{A11 v1} & \text{A12 v1} \\ \text{A21 e}^{-\text{I}\theta}\,\text{v2} & \text{A22 e}^{-\text{I}\theta}\,\text{v2} \end{pmatrix}\begin{pmatrix} \text{U1L} \\ \text{U2L} \end{pmatrix}$$

$$\text{Md} \;:\; (\,\text{D1R}\;\;\text{D2R}\,)\begin{pmatrix} \text{B11 v1} & \text{B12 v1} \\ \text{B21 e}^{\text{I}\theta}\,\text{v2} & \text{B22 e}^{\text{I}\theta}\,\text{v2} \end{pmatrix}\begin{pmatrix} \text{D1L} \\ \text{D2L} \end{pmatrix}$$

$$\text{Mu.Md} = \begin{pmatrix} \text{A11 B11 v1}^2 + \text{A12 B21 e}^{\text{I}\theta}\,\text{v1 v2} & \text{A11 B12 v1}^2 + \text{A12 B22 e}^{\text{I}\theta}\,\text{v1 v2} \\ \text{A21 B11 e}^{-\text{I}\theta}\,\text{v1 v2} + \text{A22 B21 v2}^2 & \text{A21 B12 e}^{-\text{I}\theta}\,\text{v1 v2} + \text{A22 B22 v2}^2 \end{pmatrix}$$

$$\text{Det[Mu.Md]} = \text{A12 A21 B12 B21 v1}^2\,\text{v2}^2 - \text{A11 A22 B12 B21 v1}^2\,\text{v2}^2 - \text{A12 A21 B11 B22 v1}^2\,\text{v2}^2 + \text{A11 A22 B11 B22 v1}^2\,\text{v2}^2$$

is real

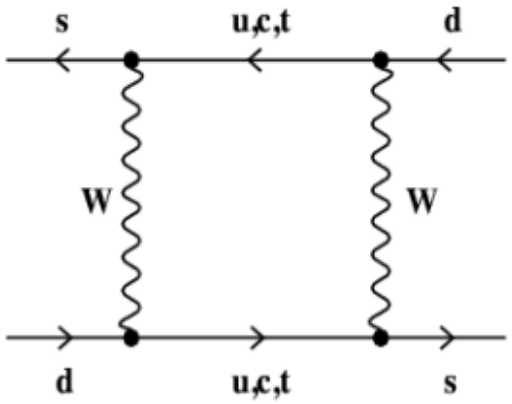
$$Det[M_u \cdot M_u^\dagger, M_d \cdot M_d^\dagger] = \text{e}^{-2\,\text{I}\theta}\,\text{v1}^2\,\text{v2}^2\left(-\text{A11}^2\,(\text{B11 B21} + \text{B12 B22})\,\text{e}^{2\,\text{I}\theta}\,\text{v1}^2 - \text{A12}^2\,(\text{B11 B21} + \text{B12 B22})\,\text{e}^{2\,\text{I}\theta}\,\text{v1}^2 + \left(\text{A21}^2 + \text{A22}^2\right)(\text{B11 B21} + \text{B12 B22})\,\text{v2}^2 + \text{A11 A21}\left(\text{B11}^2\,\text{v1}^2 + \text{B12}^2\,\text{v1}^2 - \left(\text{B21}^2 + \text{B22}^2\right)\text{e}^{2\,\text{I}\theta}\,\text{v2}^2\right) + \text{A12 A22}\left(\text{B11}^2\,\text{v1}^2 + \text{B12}^2\,\text{v1}^2 - \left(\text{B21}^2 + \text{B22}^2\right)\text{e}^{2\,\text{I}\theta}\,\text{v2}^2\right)\right)^2$$

is proportional to Jarlskog and is complex (CKM)

if CP invariance is softly or spontaneously broken, then $\bar{\theta}$ is a calculable quantity at loop level (Wilczek)

$$\mu_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$

$\mathbb{Z}_2$ breaking term



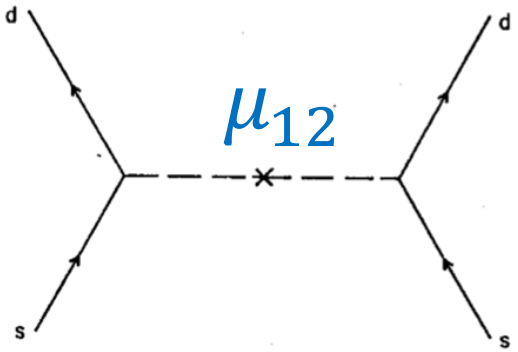
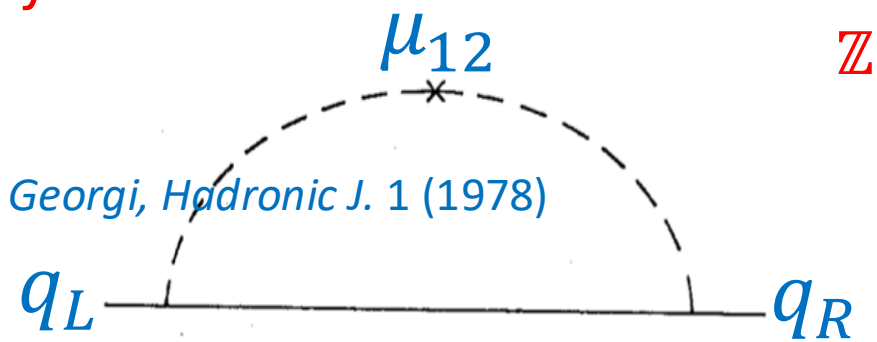


Figure 2

K-$\bar{K}$

$$\frac{\Delta}{m_H^2} \simeq 10^{-3}$$



Georgi, *Hadronic J.* 1 (1978)

$f = m_q \sqrt{G_f}$

Figure 3

$$f^2\Delta/m_H^2 \simeq 10^{-3} m_q^2 G_f \rightarrow \boxed{\bar{\theta} \simeq 10^{-8}}$$

The estimates are very rough. In fact, it is possible by choosing the parameters carefully to make the one-loop contribution to $\bar{\theta}$ vanish. But that is not the point. We know we can always eliminate $\bar{\theta}$ by choosing parameters. The question is what kind of suppression is expected for a random choice of parameters in the specified range. For our model, $\bar{\theta} \simeq 10^{-8}$ the answer.

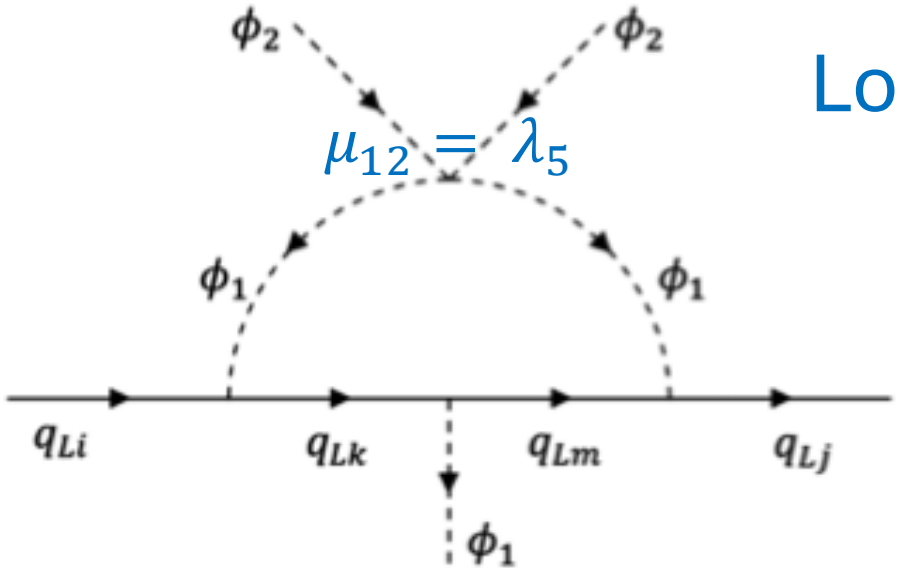
From 1978 to 2025

$$\mathcal{L}_Y = \sum_{i=1}^{2}\sum_{k=1}^{3} Y^u_{ik}\overline{Q_{iL}}U_{kR}\phi_1 + \sum_{i=1}^{3} Y^u_{3i}\overline{Q_{3L}}U_{iR}\phi_2$$
$$+ \sum_{i=1}^{2}\sum_{k=1}^{3} Y^d_{ik}\overline{Q_{iL}}D_{kR}\tilde{\phi}_1 + \sum_{i=1}^{3} Y^d_{3i}\overline{Q_{3L}}D_{iR}\tilde{\phi}_2 + \text{H.c.}$$

$$M_{u,d} = M^0_{u,d} + M^{(1)}_{u,d}$$

$$M^0_u = \frac{v_1}{\sqrt{2}}\begin{pmatrix} Y^u_{11} & Y^u_{12} & Y^u_{13} \\ Y^u_{21} & Y^u_{22} & Y^u_{23} \\ Y^u_{31}t_\beta e^{i\alpha} & Y^u_{32}t_\beta e^{i\alpha} & Y^u_{33}t_\beta e^{i\alpha} \end{pmatrix}, \qquad M^{(1)}_u \approx \begin{pmatrix} \delta^u_{11}e^{+2i\alpha} & \delta^u_{12}e^{+2i\alpha} & \delta^u_{13}e^{+2i\alpha} \\ \delta^u_{21}e^{+2i\alpha} & \delta^u_{22}e^{+2i\alpha} & \delta^u_{23}e^{+2i\alpha} \\ \delta^u_{31}t_\beta^{-1}e^{-i\alpha} & \delta^u_{32}t_\beta^{-1}e^{-i\alpha} & \delta^u_{33}t_\beta^{-1}e^{-i\alpha} \end{pmatrix},$$

$$M^0_d = \frac{v_1}{\sqrt{2}}\begin{pmatrix} Y^d_{11} & Y^d_{12} & Y^d_{13} \\ Y^d_{21} & Y^d_{22} & Y^d_{23} \\ Y^d_{31}t_\beta e^{-i\alpha} & Y^d_{32}t_\beta e^{-i\alpha} & Y^d_{33}t_\beta e^{-i\alpha} \end{pmatrix}, \qquad M^{(1)}_d \approx \begin{pmatrix} \delta^d_{11}e^{-2i\alpha} & \delta^d_{12}e^{-2i\alpha} & \delta^d_{13}e^{-2i\alpha} \\ \delta^d_{21}e^{-2i\alpha} & \delta^d_{22}e^{-2i\alpha} & \delta^d_{23}e^{-2i\alpha} \\ \delta^d_{31}t_\beta^{-1}e^{+i\alpha} & \delta^d_{32}t_\beta^{-1}e^{+i\alpha} & \delta^d_{33}t_\beta^{-1}e^{+i\alpha} \end{pmatrix},$$



Loop correction and CP violation

$$\delta_{ij}^{u,d} = \frac{\lambda_5 v_1^3 t_\beta^2}{16\pi^2 M^2} \sum_{k=1}^{3} \sum_{m=1}^{2} Y_{ik}^{u,d} Y_{mk}^{u,d*} Y_{mj}^{u,d},$$

$$\mathrm{Arg\,Det}(M_u M_d) \sim \sin\alpha \lambda_5 f_Y \frac{v_1^2 \tan\beta}{16\pi^2 M^2} \sim \lambda_5 f_Y 10^{-5}$$

λ_5 can not be too small because of limits on the CP-odd mass *A*:

$$\lambda_5 \gtrsim 0.02 \; and \; m_A \gtrsim 50 \; GeV; \qquad m_A^2 = 2\,\lambda_5 (v_1^2 + v_2^2)$$

Flavor Changing Neutral Currents (FCNC)

$$\mathcal{L}_{\text{eff}} = \sum_{i,j=d,s,b i\neq j} C^d_{ij}\bar{d}^i_L d^j_R h + \sum_{i,j=u,c,t i\neq j} C^u_{ij}\bar{u}^i_L u^j_R h,$$

Blankenburg, Ellis, Isidori, Flavour-Changing Decays of a 125 GeV Higgs-like Particle, PLB(2012)

ΔF=2 processes

Observables
Δm_K; ϵ_K
Δm_D; $\lvert q/p\rvert, \phi_D$
Δm_{B_d}; $S_{B_d\to\psi K}$
Δm_{B_s}

Observable	Bound 95%
$\lvert C_{sd}C^*_{ds}\rvert$	1.1×10^{-10}
$\lvert C^2_{ds}\rvert$	2.2×10^{-10}
$\lvert C^2_{sd}\rvert$	2.2×10^{-10}
$\mathrm{Im}(C_{sd}C^*_{ds})$	4.1×10^{-13}
$\mathrm{Im}(C^2_{ds})$	8×10^{-13}
$\mathrm{Im}(C^2_{sd})$	8×10^{-13}
$\lvert C_{cu}C^*_{uc}\rvert$	9×10^{-10}
$\lvert C^2_{uc}\rvert$	1.4×10^{-9}
$\lvert C^2_{cu}\rvert$	1.4×10^{-9}
$\mathrm{Im}(C_{cu}C^*_{uc})$	1.7×10^{-10}
$\mathrm{Im}(C^2_{uc})$	2.5×10^{-10}
$\mathrm{Im}(C^2_{cu})$	2.5×10^{-10}
$\lvert C_{bd}C^*_{db}\rvert$	9×10^{-9}
$\lvert C^2_{db}\rvert$	1×10^{-8}
$\lvert C^2_{bd}\rvert$	1×10^{-8}
$\mathrm{Im}(C_{bd}C^*_{db})$	2.7×10^{-9}
$\mathrm{Im}(C^2_{db})$	3×10^{-9}
$\mathrm{Im}(C^2_{bd})$	3×10^{-9}
$\lvert C_{bs}C^*_{sb}\rvert$	2×10^{-7}
$\lvert C^2_{sb}\rvert$	2.2×10^{-7}
$\lvert C^2_{bs}\rvert$	2.2×10^{-7}
$\mathrm{Im}(C_{bs}C^*_{sb})$	2×10^{-7}
$\mathrm{Im}(C^2_{sb})$	2.2×10^{-7}
$\mathrm{Im}(C^2_{bs})$	2.2×10^{-7}

Alves, Botella, Branco, Cornet-Gomez Nebot, EPJC(2017)

$$\Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

2HDM lead to controlled flavor-changing contributions

To determine whether the model is viable, we constructed a χ^2 which includes 6 quark masses, 3 CKM mixing angles, Jarlskog , 24 FCNC parameters, the strong CP parameters giving **35 constraints**
while the number of free the free parameters are 18 real Yukawas, the CP breaking phase, tanβ: **20 parameters**

Observable	Optimal point	Experimental error	Pull
$\hat{m}_u(M_W)$	1.3 MeV	0.4 MeV	0.08
$\hat{m}_d(M_W)$	2.7 MeV	0.4 MeV	0.04
$\hat{m}_s(M_W)$	54 MeV	10 MeV	0.57
$\hat{m}_c(M_W)$	0.631 GeV	0.009 GeV	0.05
$\hat{m}_b(M_W)$	2.883 GeV	0.008 GeV	0.04
$\hat{m}_t(M_W)$	173.1 GeV	0.6 GeV	0.04
$\lvert V_{ud}\rvert$	0.97435	0.00016	0.01
$\lvert V_{us}\rvert$	0.22501	0.00068	0.00
$\lvert V_{ub}\rvert$	0.003732	0.000090	0.19
$\lvert V_{cd}\rvert$	0.22487	0.00068	−0.02
$\lvert V_{cs}\rvert$	0.97349	0.00016	0.02
$\lvert V_{cb}\rvert$	0.04183	0.0008	−0.02
$\lvert V_{td}\rvert$	0.00858	0.00019	0.05
$\lvert V_{ts}\rvert$	0.041110	0.00077	−0.03
$\lvert V_{tb}\rvert$	0.999118	0.000034	0.02
J	0.000031	0.000001	0.17
$\bar{\theta}$	-8.3×10^{-10}	5×10^{-10}	−1.7

$(\mathcal{O}_{\rm exp} - \mathcal{O}_{\rm model})/\delta O_{\rm exp}$

λ_5=1 and M_H=1TeV

Model/bound >1 gives 2σ tension

Observable	Bound 95%	Model/bound
$\lvert C_{sd}C^*_{ds}\rvert$	1.1×10^{-10}	8×10^{-8}
$\lvert C^2_{ds}\rvert$	2.2×10^{-10}	8×10^{-7}
$\lvert C^2_{sd}\rvert$	2.2×10^{-10}	2×10^{-9}
$\mathrm{Im}(C_{sd}C^*_{ds})$	4.1×10^{-13}	3×10^{-14}
$\mathrm{Im}(C^2_{ds})$	8×10^{-13}	-1×10^{-14}
$\mathrm{Im}(C^2_{sd})$	8×10^{-13}	-2×10^{-15}
$\lvert C_{cu}C^*_{uc}\rvert$	9×10^{-10}	2×10^{-7}
$\lvert C^2_{uc}\rvert$	1.4×10^{-9}	8×10^{-5}
$\lvert C^2_{cu}\rvert$	1.4×10^{-9}	3×10^{-10}
$\mathrm{Im}(C_{cu}C^*_{uc})$	1.7×10^{-10}	2×10^{-11}
$\mathrm{Im}(C^2_{uc})$	2.5×10^{-10}	4×10^{-12}
$\mathrm{Im}(C^2_{cu})$	2.5×10^{-10}	-6×10^{-14}
$\lvert C_{bd}C^*_{db}\rvert$	9×10^{-9}	9×10^{-4}
$\lvert C^2_{db}\rvert$	1×10^{-8}	0.8
$\lvert C^2_{bd}\rvert$	1×10^{-8}	7×10^{-7}
$\mathrm{Im}(C_{bd}C^*_{db})$	2.7×10^{-9}	6×10^{-16}
$\mathrm{Im}(C^2_{db})$	3×10^{-9}	2×10^{-14}
$\mathrm{Im}(C^2_{bd})$	3×10^{-9}	1×10^{-18}
$\lvert C_{bs}C^*_{sb}\rvert$	2×10^{-7}	0.02
$\lvert C^2_{sb}\rvert$	2.2×10^{-7}	0.8
$\lvert C^2_{bs}\rvert$	2.2×10^{-7}	2×10^{-4}
$\mathrm{Im}(C_{bs}C^*_{sb})$	2×10^{-7}	-2×10^{-17}
$\mathrm{Im}(C^2_{sb})$	2.2×10^{-7}	1×10^{-15}
$\mathrm{Im}(C^2_{bs})$	2.2×10^{-7}	1×10^{-18}

Conclusions

The 2HDM can solve the strong CP problem without Axion quality problem*

Thank you for your attention

**be aware that there is fine-tuning since relatively small variations ~1% yield a drastic increase of the tension. For example, changing Y11 by 1% changes the χ^2 by 4 orders of magnitude.*
This is a typical situation in two Higgs doublet models.

Best fit point: parameters

Parameter	Value
$\tan\beta$	4.14177
Y^u_{11}	$+6.99737 \times 10^{-2}$
Y^u_{12}	-7.33074×10^{-2}
Y^u_{13}	-7.11714×10^{-2}
Y^u_{21}	-6.87086×10^{-2}
Y^u_{22}	$+7.22767 \times 10^{-2}$
Y^u_{23}	$+6.96334 \times 10^{-2}$
Y^u_{31}	$+5.63088 \times 10^{-1}$
Y^u_{32}	-6.72942×10^{-1}
Y^u_{33}	-5.25847×10^{-1}
Y^d_{11}	$+8.51142 \times 10^{-6}$
Y^d_{12}	-6.09872×10^{-4}
Y^d_{13}	$+3.37296 \times 10^{-4}$
Y^d_{21}	$+1.81040 \times 10^{-4}$
Y^d_{22}	$+1.05304 \times 10^{-3}$
Y^d_{23}	-4.03567×10^{-4}
Y^d_{31}	$+1.58555 \times 10^{-2}$
Y^d_{32}	$+6.25660 \times 10^{-3}$
Y^d_{33}	-2.00748×10^{-4}
α	-2.37994

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \theta\frac{g^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi$$

$$\psi' = e^{i\alpha\gamma_5/2}\psi, \qquad \bar{\psi}' = \bar{\psi}e^{i\alpha\gamma_5/2} \qquad \theta' \to \theta' - \alpha$$

Chiral trasformation $\qquad \theta \to \theta + \alpha$

$$\theta' + \theta \to (\theta' - \alpha) + (\theta + \alpha) = \theta' + \theta$$ unchange

If $m_u = 0$ then Arg(Det($M_u . M_d$)) = 0 and we can use a chiral trasformation to absorb θ