OPTIMAL CONTROL METHODS FOR NUCLEAR PHYSICS

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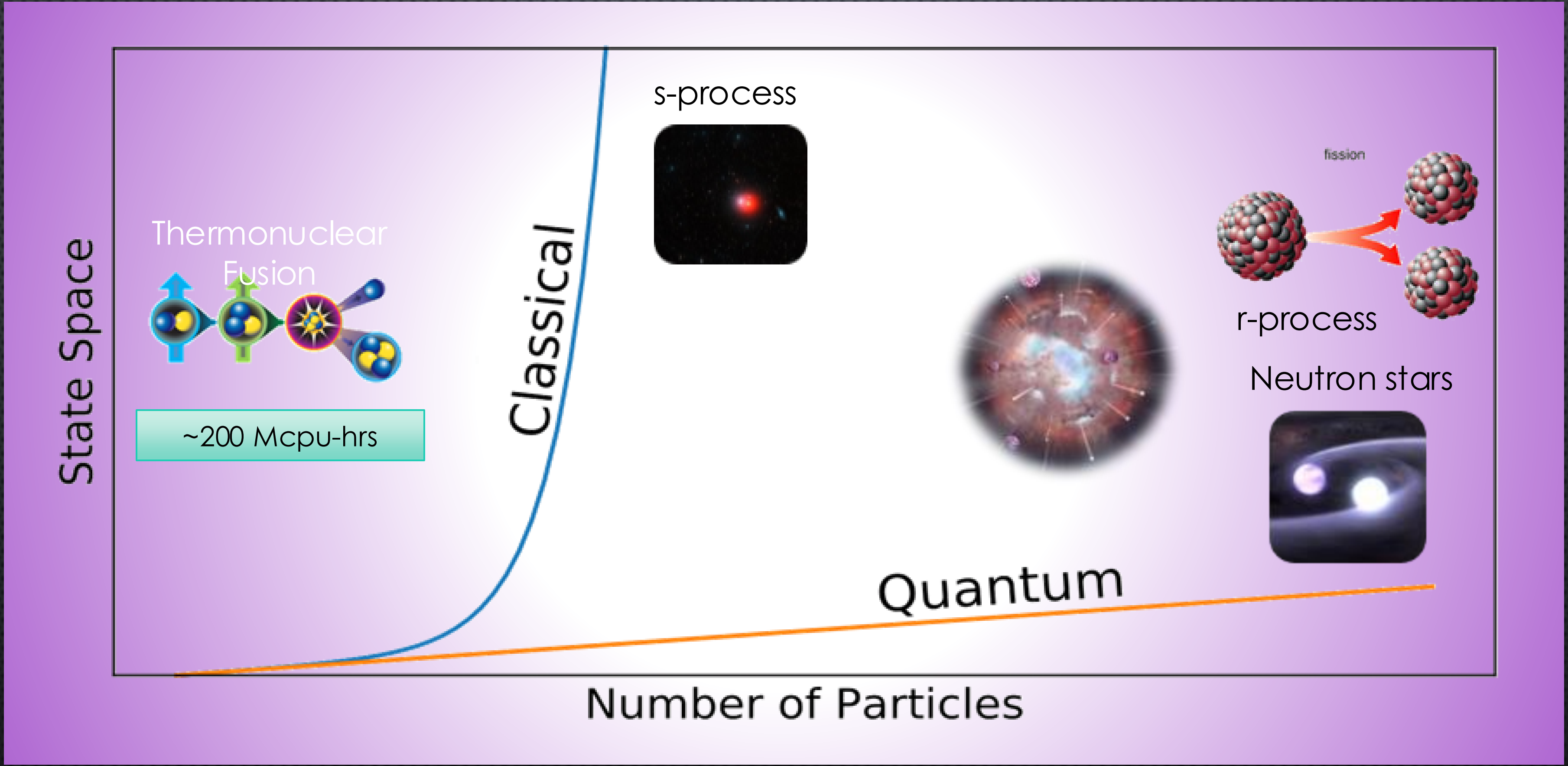
SIMULATING REACTIONS

The concept is somewhat simple. A reaction is just a time dependent process. A quantum computer is a tool that implements the time evolution of a quantum system so It should be relatively "easy" to describe a process just as it occurs, without approximations, provided that:

1) We actually know the Hamiltonian of the system (and for nuclear physics this is already not trivial, unless you do full fledged QCD…)
2) We can implement computations:
   a) In a reasonably large model Hilbert space (we don't really need it to be infinite, right?)
   b) Such that noise doesn't kill the signal over the time scale relevant for the process
3) We know how to tame dissipation (since nature gave us quasi elastic and unelastic processes…)

And yes, at this point one would just turn around and give up, but if it is not difficult, it is not fun!

THE DREAM…..


State Space
Thermonuclear
Fusion
~200 Mcpu-hrs
Classical
s-process
fission
r-process
Neutron stars
Quantum
Number of Particles

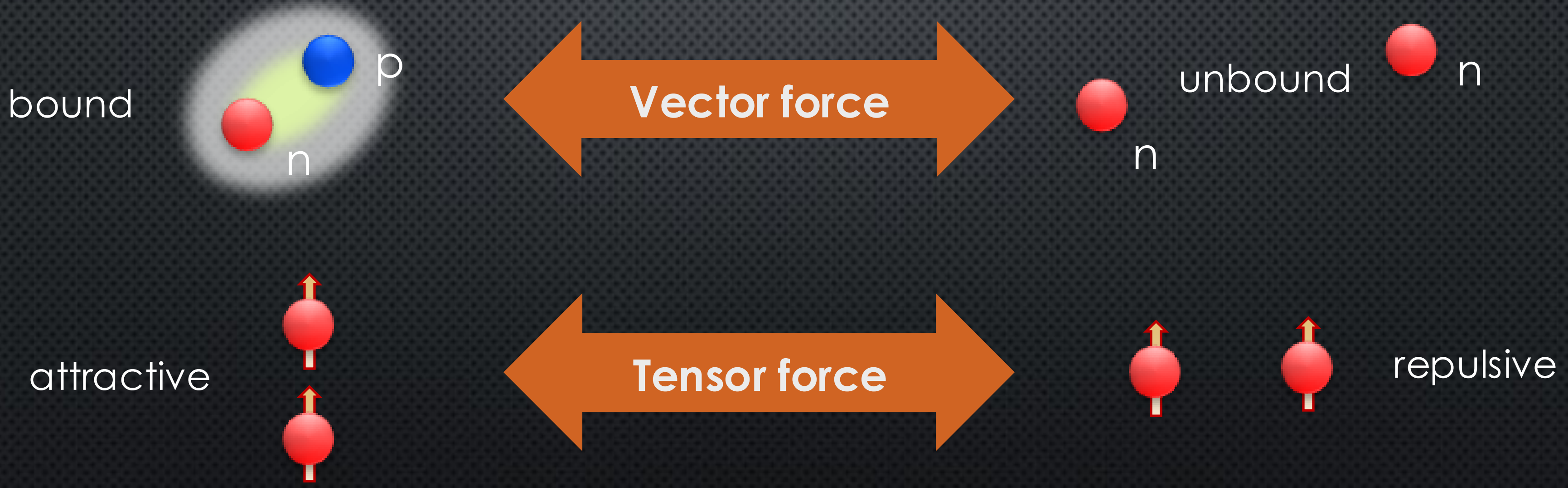
REALITY: A SIMPLE, YET NON-TRIVIAL, NUCLEAR PHYSICS PROBLEM…

Describe how two neutrons evolve in time under the effect of their mutual interaction

- Interaction at leading-order (LO) of chiral effective field theory (spin tensor!)
- Implementation of real-time evolution of the system of 2 neutrons
- Realistic device-level simulations gauged on LLNL’s QPU without/with measured noise
- Measurements/calculations on the LLNL testbed
- A simple extension on a digital machine (AQT testbed @ LBL) including some kind of evolution of the coordinates

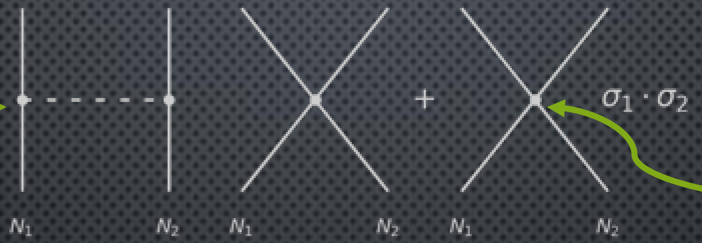
THE STRONG NUCLEAR INTERACTION HAS A NON-TRIVIAL DEPENDENCE ON THE QUANTUM STATE OF THE NUCLEONS

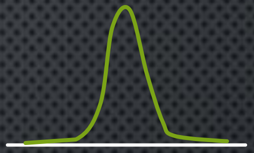
- One of the main features of the nucleon-nucleon interaction is its spin/isospin dependence. This dependence accounts for some very basic phenomenology.



NUCLEON NUCLEON INTERACTION (LO)

One-pion exchange (OPE)





Regularized contact (cutoff in momentum)

$$H_{\text{int}}^{\text{LO}} = V_{\text{OPE}}\left[1 - \delta_{R_0}(\vec{r})\right] + \left[C_0 + C_1\, \vec{\sigma}^1 \cdot \vec{\sigma}^2\right] \delta_{R_0}(\vec{r})$$

regulator function

$$V_{\text{OPE}} = \frac{f_\pi^2 m_\pi}{12\pi}\left[T_\pi(r)\, S_{12} - \left(Y_\pi(r) - \frac{4\pi}{m_\pi^3}\,\delta(\vec{r})\right)\vec{\sigma}^1 \cdot \vec{\sigma}^2\right]\vec{\tau}^1 \cdot \vec{\tau}^2$$

Spin independent

Spin dependent

TIME PROPAGATION

Formal solution of the time-dependent Schroedinger equation:

$$\exp\left(-\frac{i}{\hbar}\hat{H}t\right) = \exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}+\hat{V}_{SD}\right)t\right]$$

- V_{SI}: **SPIN-INDEPENDENT** part of the interaction
- V_{SD}: **SPIN-DEPENDENT** part of the interaction

In the short-time limit, we can separate the terms depending on V_{SI} and V_{SD}:

$$\exp\left[-\frac{i}{\hbar}\left(\hat{T}+\hat{V}_{SI}\right)\delta t\right]\exp\left[-\frac{i}{\hbar}\hat{V}_{SD}\delta t\right] + o(\delta t^2)$$

TIME PROPAGATION

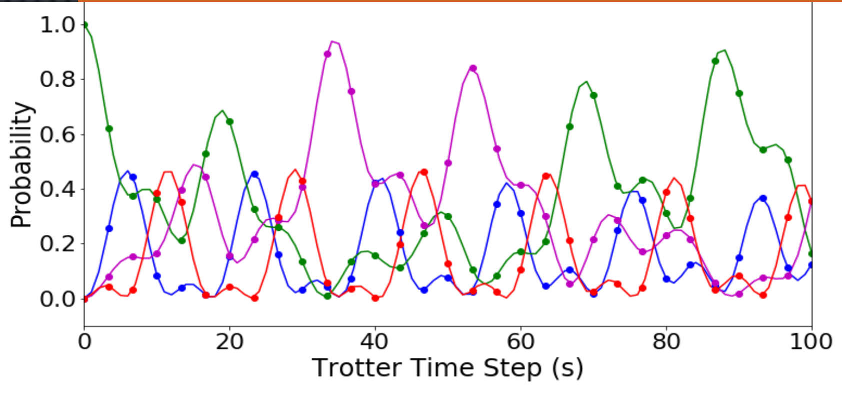
First application: "frozen" nucleons → Spin/isospin Hamiltonian only

$$\exp\left[-\frac{i}{\hbar}\hat{V}_{SD}\delta t\right] = \exp\left[-\frac{i}{\hbar}\left(\sum_{i,j=1}^{A}\sum_{\alpha,\beta=x,y,z}\sigma_{i\alpha}A(r_{ij})_{ij;\alpha\beta}\sigma_{j\beta}\right)\delta t\right]$$

coordinates appear as "parameters"

Computer simulation of the actual device

Occupation probability



Lines: analytic results

Circles: synthetic data: probability of measuring state $|\alpha\rangle$ after a time t (obtained by repeating the process over and over)

The time evolution with a tensor Hamiltonian introduces spin-parallel components

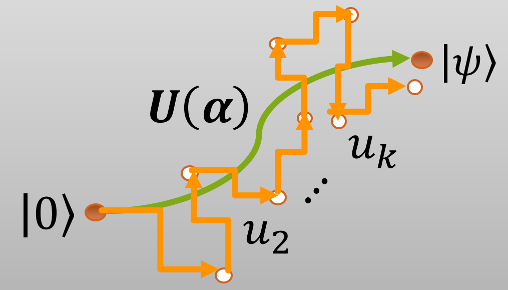
$|0\rangle = |\uparrow\uparrow\rangle$ $|1\rangle = |\uparrow\downarrow\rangle$ $|2\rangle = |\downarrow\uparrow\rangle$ $|3\rangle = |\downarrow\downarrow\rangle$

"DIGITAL" QUANTUM COMPUTING

Useful theorem (Solovay-Kitaev)

Given an Hilbert space $\mathscr{H}$, any unitary transformation $U \in U(\mathscr{H})$ can be approximated with arbitrary precision δ by a circuit of size *poly(1/ δ)* over the standard gates basis, possibly using ancillary qubits (i.e there exists a

This guarantees that we can approximate for instance an arbitrary time evolution under a Hamiltonian H with a circuit depth that will increase at most a polinomially with the required accuracy.



$$U(\alpha, t) = u_1(\alpha, \Delta t) u_2(\alpha, \Delta_t) \ldots$$

THIS THEOREM IS AT THE BASIS OF UNIVERSAL QC SCHEME

WARNING: other problems like decoherence (thermal effects) and dissipation

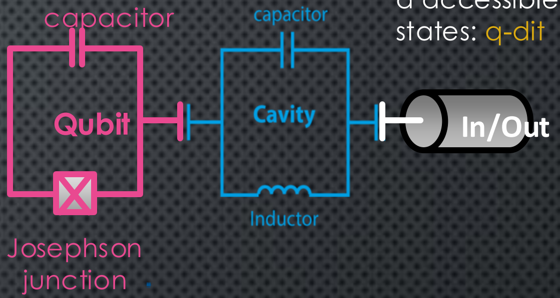
"CONTROL-CENTRIC" APPROACH TO QUANTUM COMPUTATION

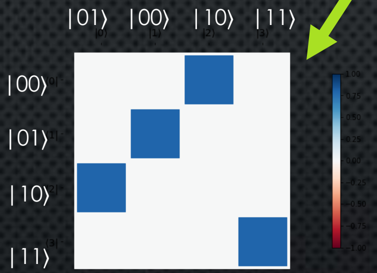
Given some control parameterization $f(t,\alpha) = \sum_k \alpha_k \phi_k(t)$

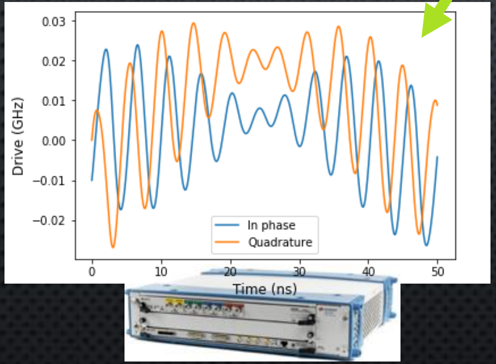
all unitary transformations in time T can be expressed as

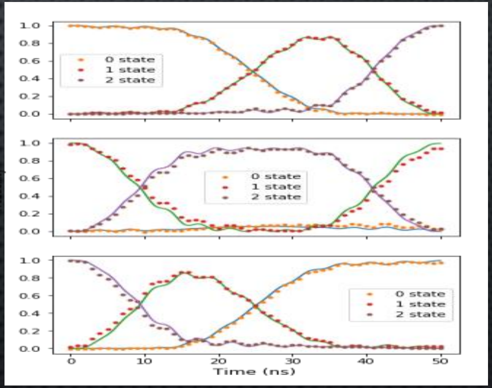
$$U(t,\alpha) = \mathcal{T} e^{-i\int_0^t dt' [\hat{H}_0 + f(t',\alpha)\hat{H}_c]}$$

Device with d accessible states: q-dit





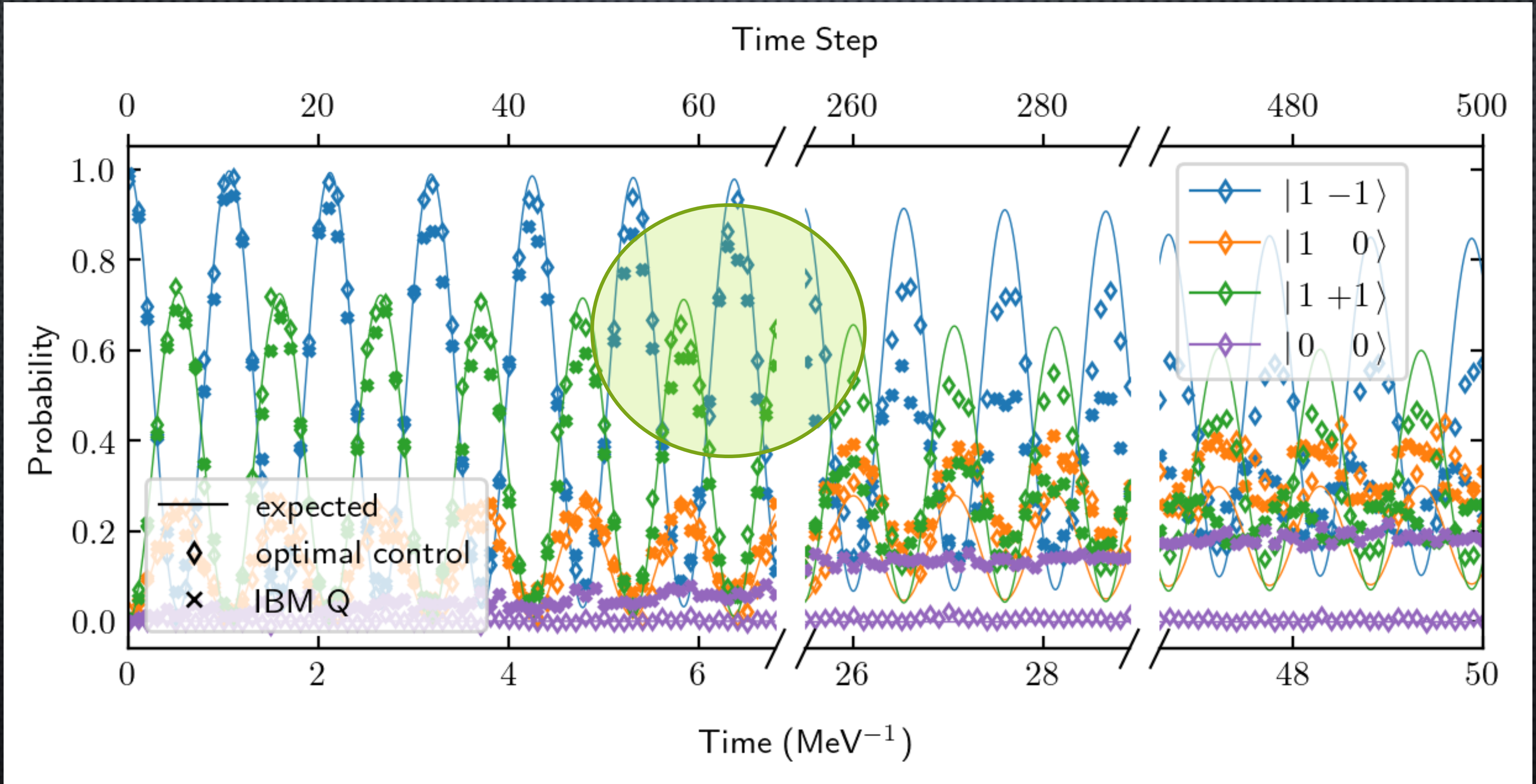




See e.g. S. Schirmer, "Hamiltonian Engineering for Quantum Systems." , Proceedings of the 7th International Conference on Cooperative Control and Optimization (2007)

Optimization methods, machine learning (Q@Tn ML-QFORGE project (Jonathan Dubois, Kyle Wendt (LLNL), Simone Taioli, Paolo Trevisanutto (ECT*-LISC), FP, Piero Luchi, (UNITN)) – PRA 101, 062307 (2020) – INFN Quart&T Project (UNIMIB, FBK, TIFPA, INFN-LNF and others, 2024)

RESULTS ON LLNL TEST BED SIMULATION



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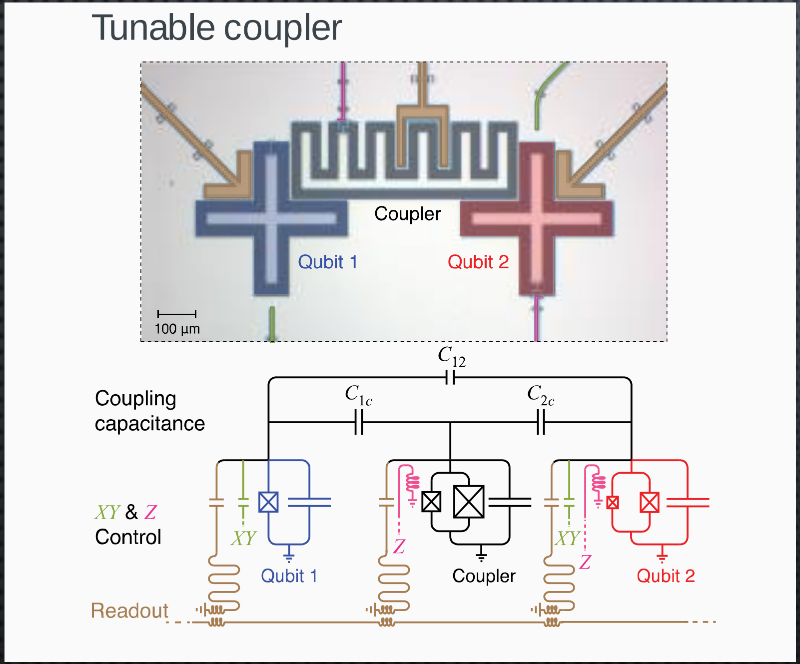
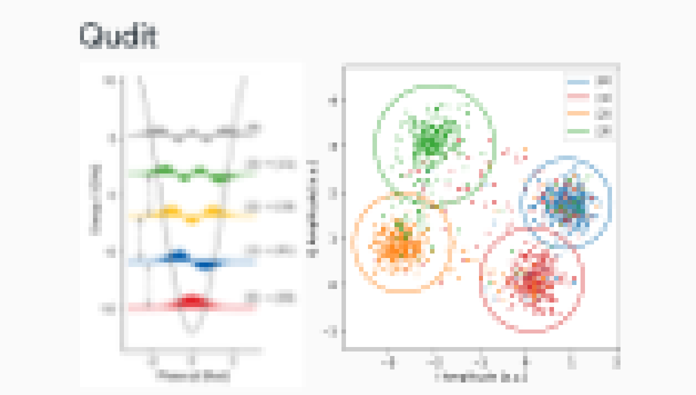




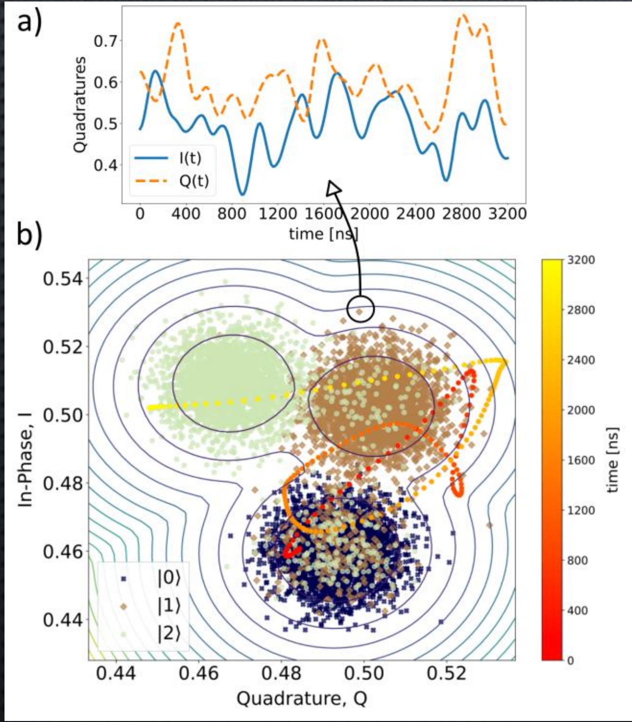
Implementation of qudits with all-to-all connectivity.

Tunable coupling: possibility of controlling The coupling between qubits (change the machine Hamiltonian)

CO-DESIGN

IMPROVING READOUT FORM CAVITIES WITH AUTOENCODERS

Autoencoders can be used as a way to reduce misclassifications when reading out the state of a cavity.

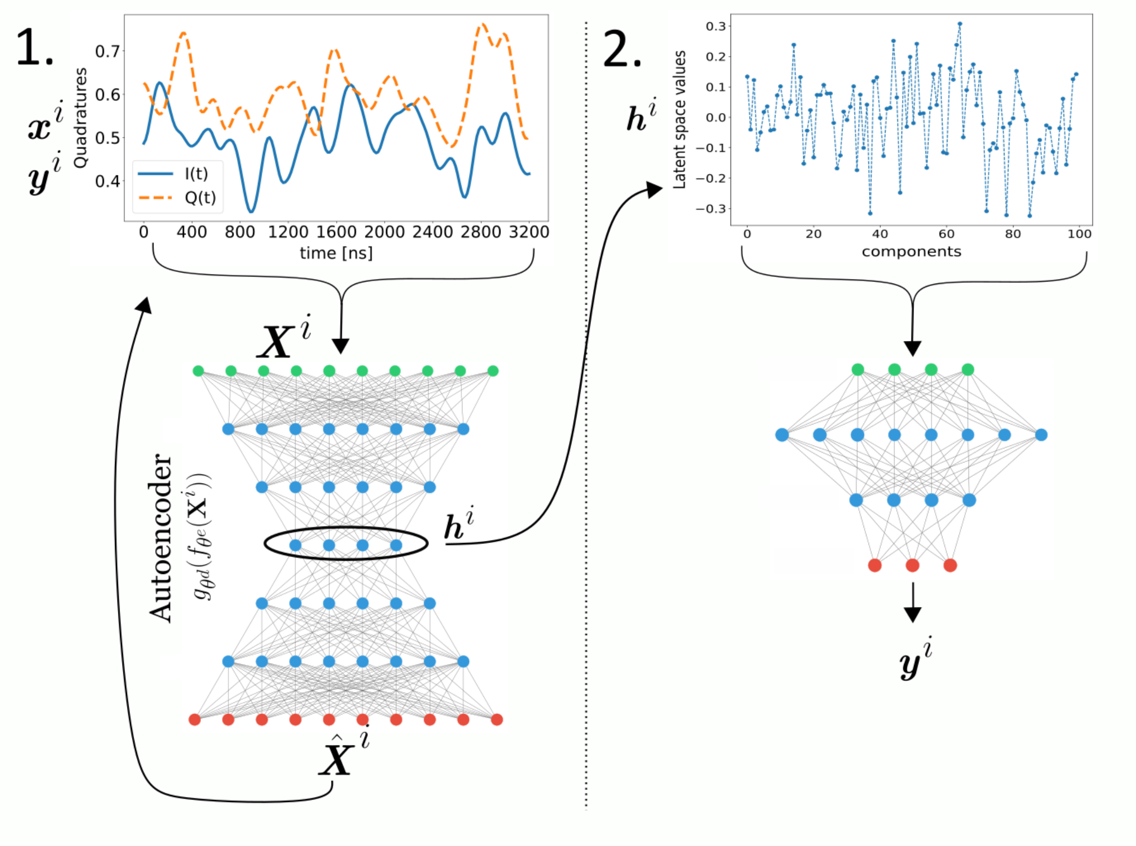


Demodulated signal [I(t): in-phase Q(t): quadrature]. The average of the signals identifies a point in the panel below.

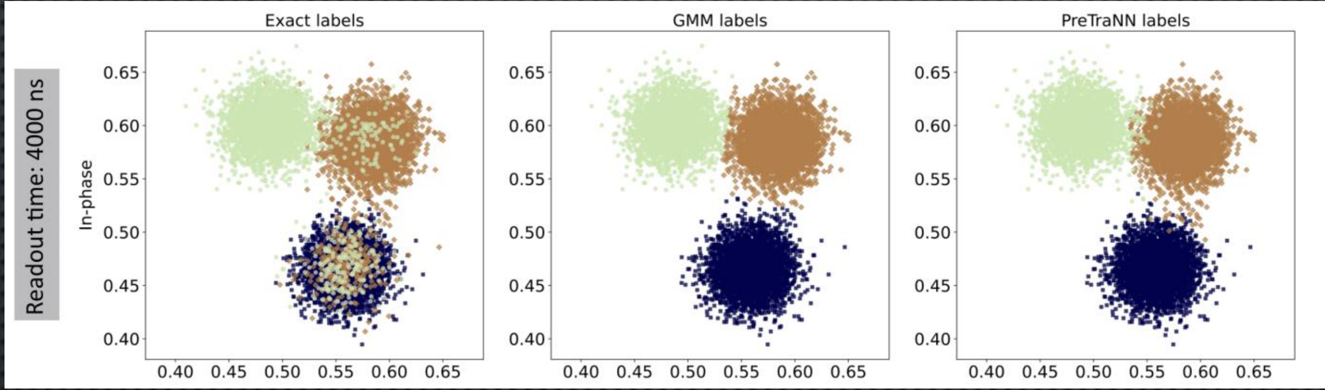
Readout points from many shots of a given experiment.

- Notice the evident effects of the decay of the state along the measurement and the consequent possible misreadings

IMPROVING READOUT FORM CAVITIES WITH AUTOENCODERS



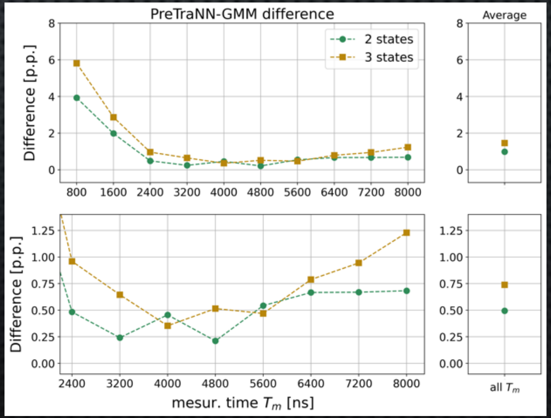
IMPROVING READOUT FORM CAVITIES WITH AUTOENCODERS



	Average Class. Accuracy
GMM	93.2 %
PreTraNN	93.3 %

Different schemes have been tested and compared:

- PreTraNN: Autoencoder pre-training
- GMM: Gaussian Mixture Model



CONCLUSIONS

- Optimal control techniques can definitely help to realize proofs of concept for quantum algorithms that need to be robust over time (as for the case of scattering studies)

- A path for the study of nuclear reactions with realistic interactions is visible. However, there are still many open problems (like a more accurate implementation of space propagators with a reasonable scaling) which still need to be addressed.

- Digital QC requires a more elaborated circuit optimization, but some interesting demonstration might not be too far.

- In both cases, it is very important to have access to low level optimization -> work in strict cooperation with the experimental groups!

THANK YOU!