

Tunable Couplers and Their Applications in Analog Quantum Simulations

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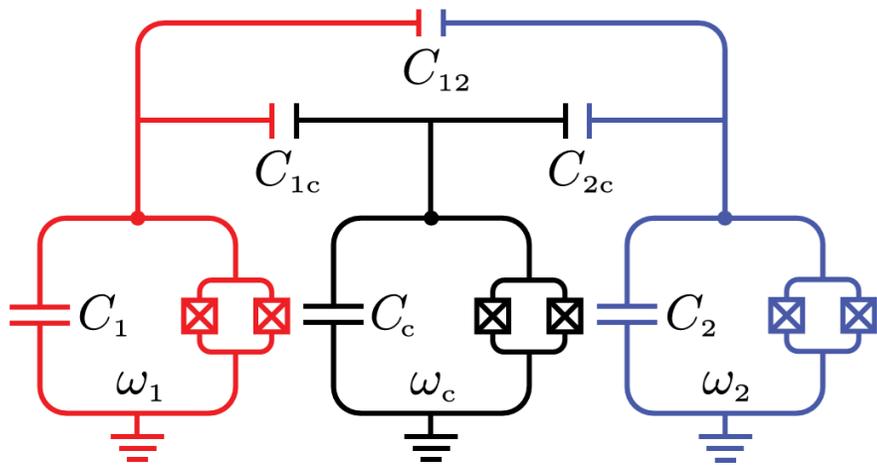
Tunable couplers

Device capable of parametrically control the coupling between multiple qubits. How:

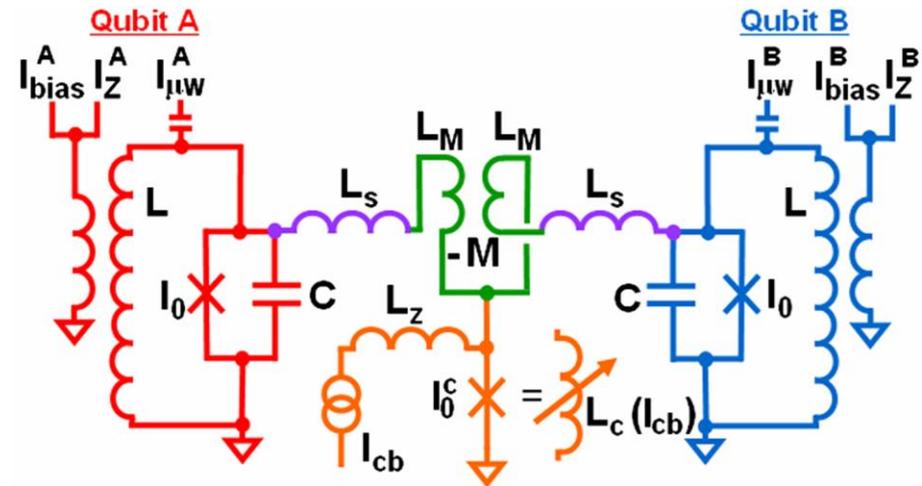
- Tuning g : $g \rightarrow g(\Phi(t))$
- Frequency of the coupler: $\omega_{coupler} \rightarrow \omega_{coupler}(\Phi(t))$

Advantages:

- The possibility to define a working point «**idling**» where the coupling is completely turn off.
- Activate the interaction only between desired qubits.



Typical transmon-based coupler. The frequency of the transmon is used to modulate the interaction.



More complicated design between phase qubits where the coupling strength g is modulated by the current I_{cb} .

Tunable coupling implementation

- The most popular way is to reach the tunability **with a flux-sensitive device (SQUID)** that allow us to modulate the interaction.

Two different way of usage:

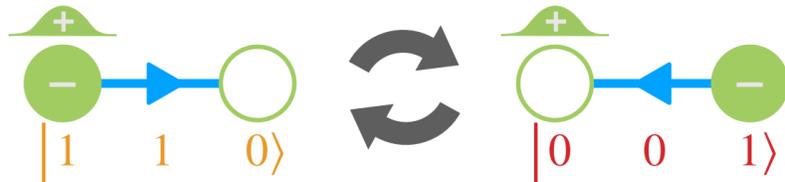
- Using a **DC signal** to move the coupler frequency:

$$\omega_c \sim \omega_1, \omega_2 \rightarrow \text{TURN ON}$$

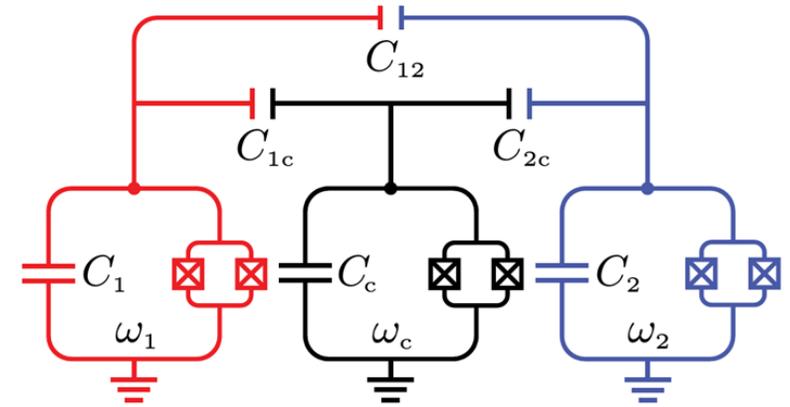
$$\omega_c \neq \omega_1, \omega_2 \rightarrow \text{TURN OFF}$$

- Using an **AC signal** to activate specific interactions between coupled elements:

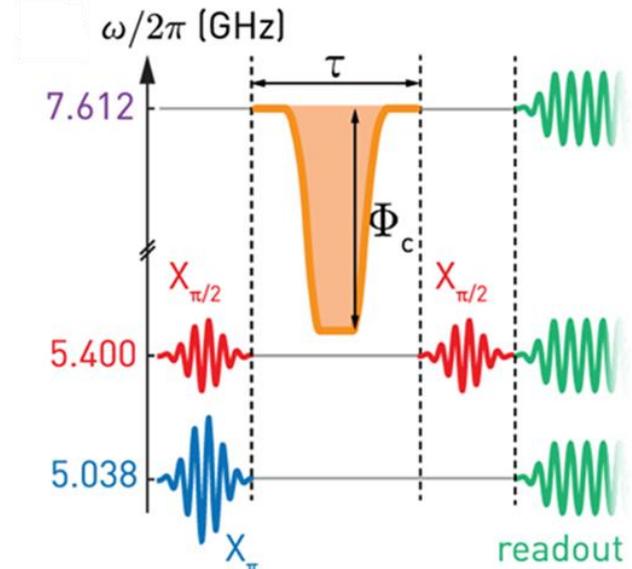
$$\omega_p \approx \omega_{|i\rangle \rightarrow |f\rangle} \rightarrow \text{TURN ON Rabi oscillation between } |i\rangle \rightarrow |f\rangle$$



[Preprint, 2025]



[Phys. Rev. Applied **10**, 2018]



[Phys. Rev. Lett. **125**, 2020]

Example of architecture

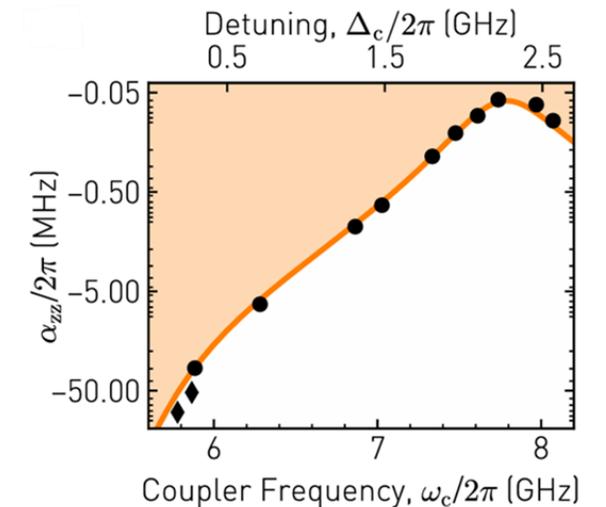
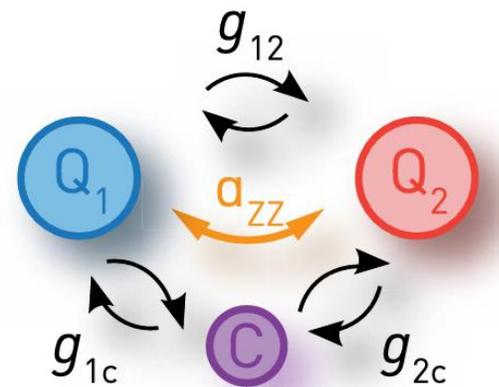
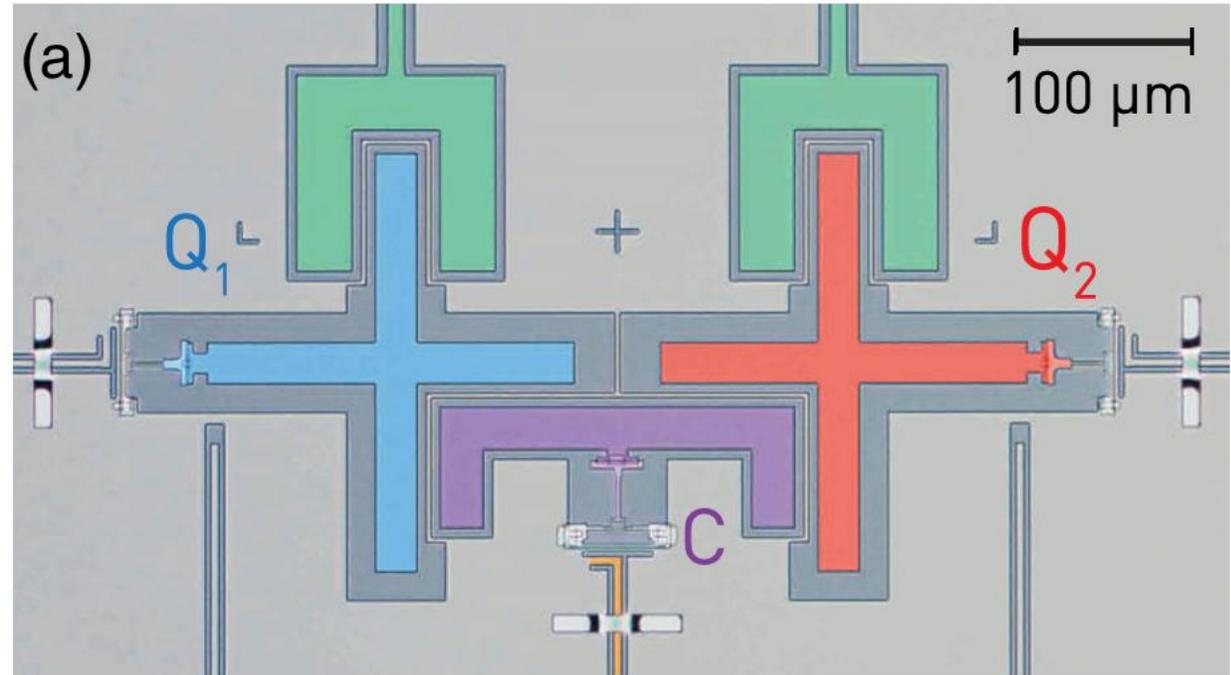
Implementation of Conditional Phase Gates Based on Tunable ZZ Interactions

- **All the three elements are tunable** and the qubits interact both directly and through the coupler;
- The interested interaction α_{ZZ} arises from the two coupling channels and the hybridization of the three local modes;
- The interaction is turned on **using gaussian pulses** on the DC SQUID to bring the coupler frequency close to the qubit frequencies;

$$\mathcal{H}_{\text{eff}}/\hbar = \frac{1}{2} \sum_{i=1,2} \left(\omega_i + \frac{\alpha_{ZZ}}{2} \right) \sigma_z^{(i)} + \frac{\alpha_{ZZ}}{4} \sigma_z^{(1)} \sigma_z^{(2)} + J \left(\sigma_+^{(1)} \sigma_-^{(2)} + \sigma_-^{(1)} \sigma_+^{(2)} \right)$$

Effective Hamiltonian. Valid in the dispersive regime $\Delta_{ic} > g_{ic}$. The residual transversal coupling J is small compared to α_{ZZ} .

[\[Phys. Rev. Lett. 125, 2020\]](#)



Existing multi-qubits 2D architectures

- Demonstration of Tunable Three-Body Interactions between Superconducting Qubits (2022 *MIT*)

[\[https://doi.org/10.1103/PhysRevLett.129.220501\]](https://doi.org/10.1103/PhysRevLett.129.220501)

- Native Three-Body Interactions in a Superconducting Lattice Gauge Quantum Simulator (2025)

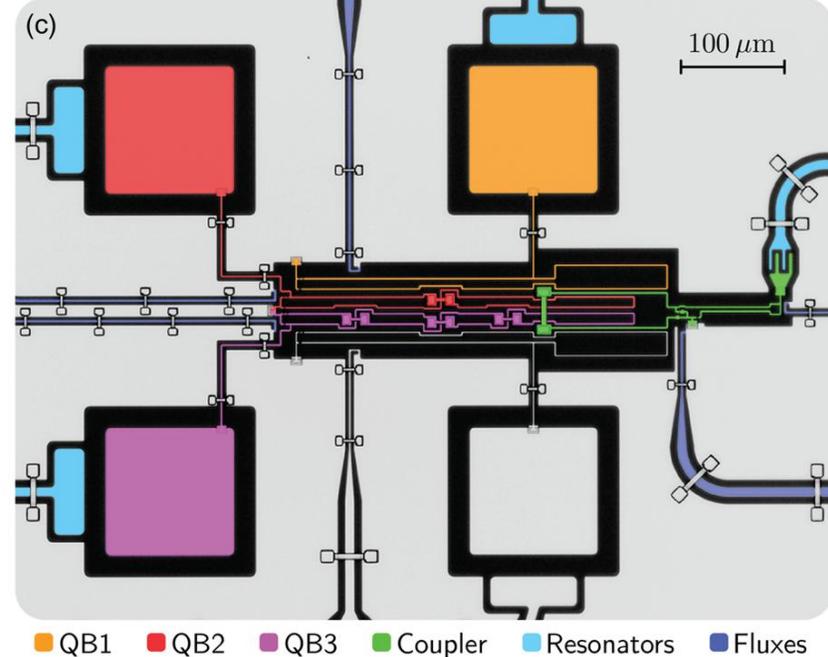
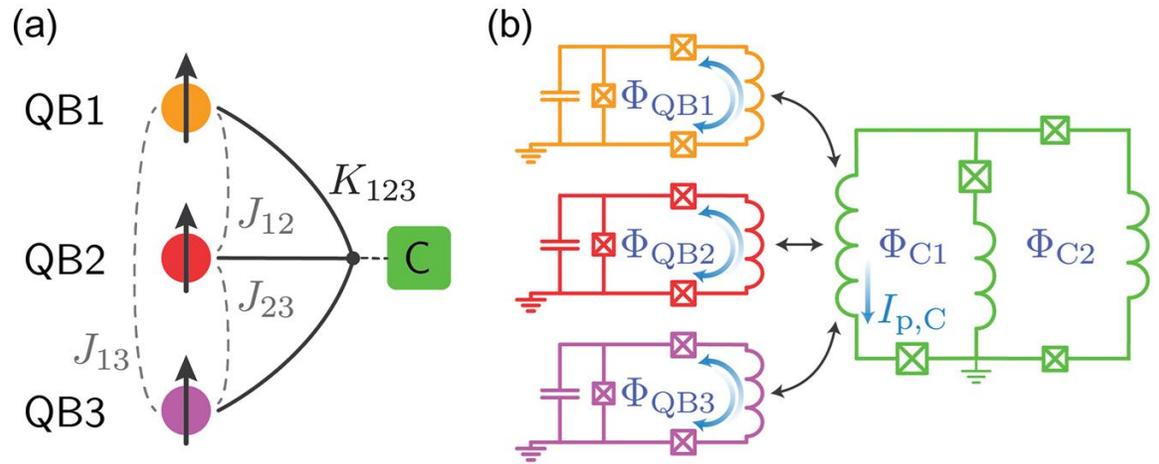
[\[https://doi.org/10.48550/arXiv.2501.13383\]](https://doi.org/10.48550/arXiv.2501.13383)

Demonstration of Tunable Three-Body Interactions between Superconducting Qubits [Phys. Rev. Lett. 129, 2022]

- GOAL: simulate, realize and characterize a **three-qubits tunable coupled device**;
- The qubits are flux qubits.
- The three body interaction rise entirely from the non linear coupling of the coupler. It arises from interactions between the coupler excited state and higher excited states of the qubit system;
- Hamiltonian parameters estimated numerically [<https://arxiv.org/abs/2010.14929>];

$$H/\hbar = - \sum_{i=1}^3 \frac{\omega_i}{2} \hat{Z}_i + \sum_{\substack{i,j=1 \\ i < j}}^3 J_{ij} \hat{Z}_i \hat{Z}_j + K_{123} \hat{Z}_1 \hat{Z}_2 \hat{Z}_3$$

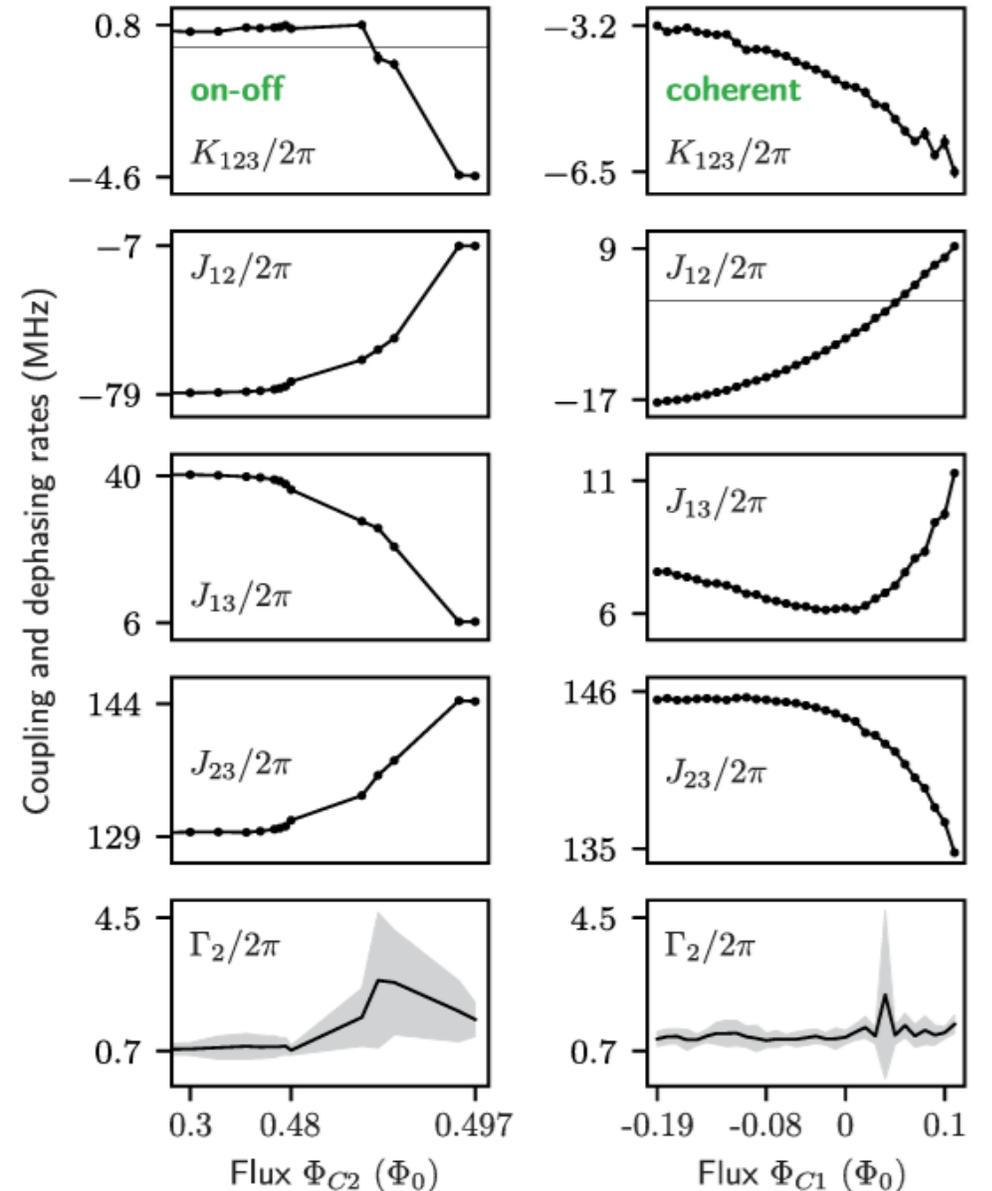
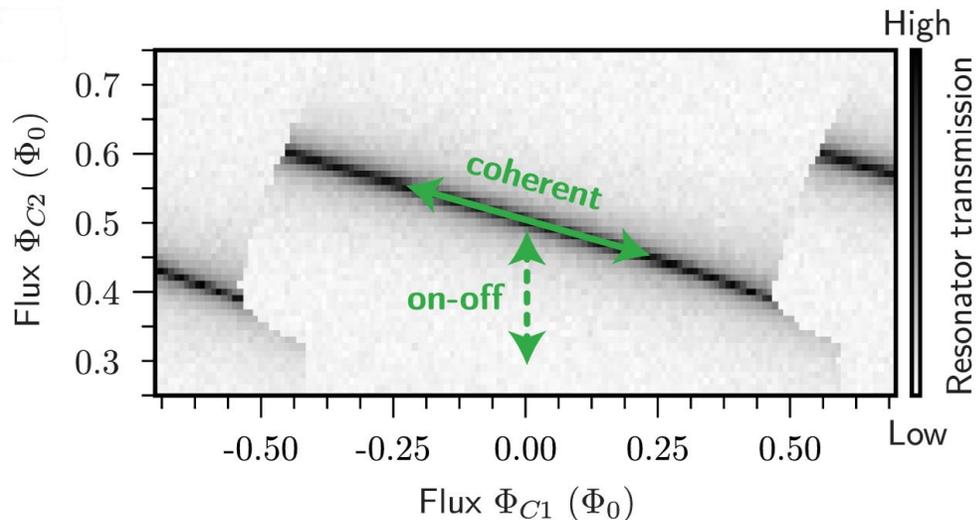
Hamiltonian of the system. K_{123} is the three-qubit interaction. J_{ij} are the two local interaction terms including capacitively, inductively and spurious coupler-mediate interactions.



Demonstration of Tunable Three-Body Interactions between Superconducting Qubits

The interaction between the qubits can be **modulated changing the two fluxes of the tunable coupler**. The modulation is done along two direction in the parameter space:

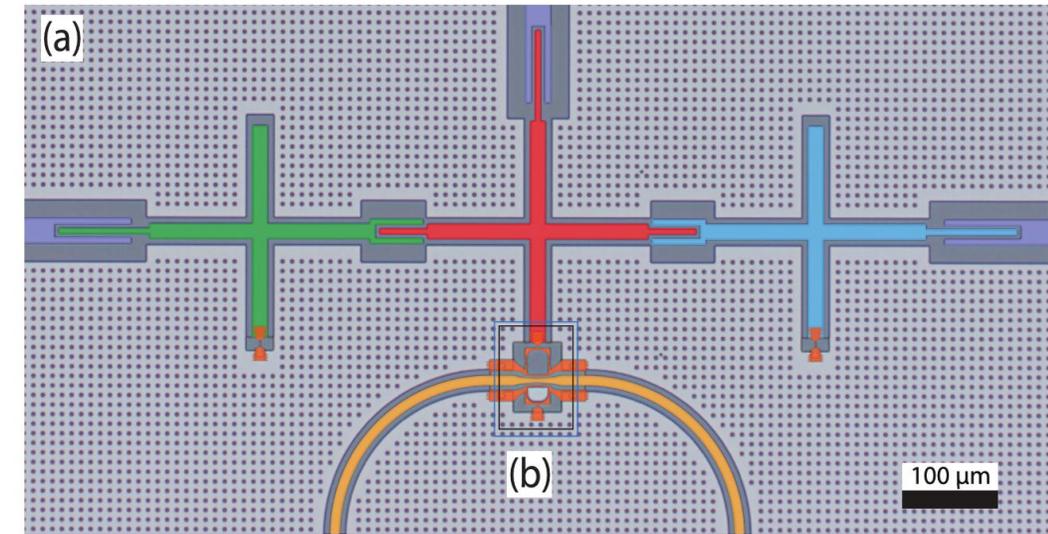
1. **On-off** direction. Quick turn off and on of the coupling. Suitable for annealing protocol.
2. **Coherent** direction. Mantain coherence between qubits. Suitable for analog simulations or for digital gates.



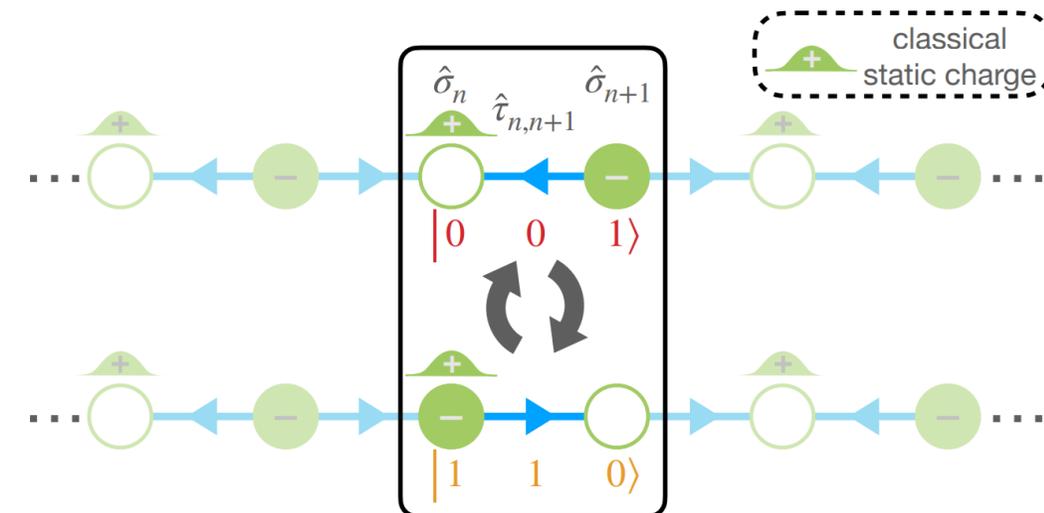
- GOAL: realization of three body interaction for **analog quantum simulation of U(1) lattice gauge theories (LGTs)**;
- Quantum Monte Carlo simulations are not efficient for investigating LGTs in nonperturbative regimes. Quantum simulations have emerged as a promising tool;
- The device is constituted by 2 fixed frequency transmons (data qubits) and a flux dependant transmon (coupler);
- A three-body quantum interaction is implemented using two qubits as 'matter fields' and a coupling qubit as the 'gauge field.' **This interaction is engineered to intrinsically preserve gauge symmetry;**

$$\hat{\mathcal{H}}_m = \frac{\mu}{2} \sum_{n=1}^L (-1)^n \hat{\sigma}_n^z - J \sum_{n=1}^{L-1} (\hat{\sigma}_n^+ \hat{\tau}_{n,n+1}^+ \hat{\sigma}_{n+1}^- + \text{h.c.})$$

LTG Hamiltonian to simulate



■ Qubit 1 ■ Qubit 2 ■ Qubit 3 ■ Resonators ■ AC flux

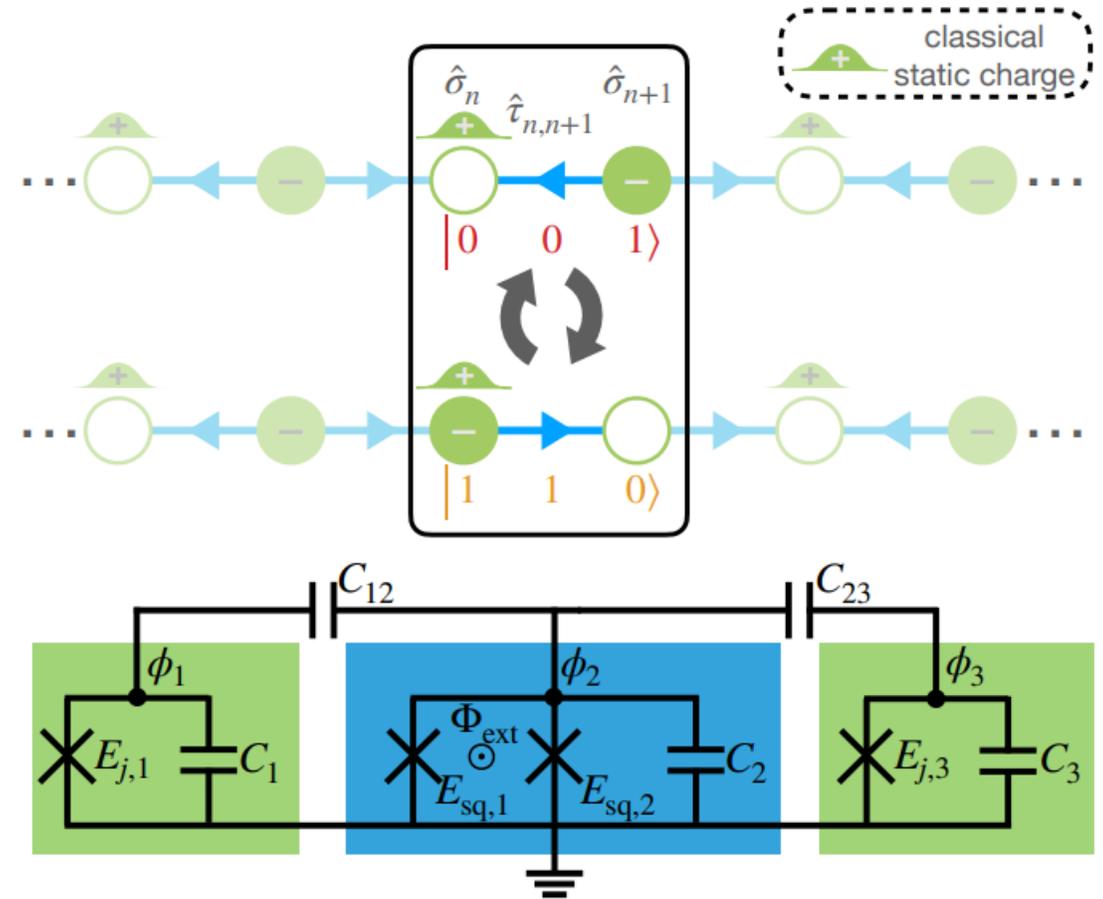


- The **three body interaction** rise from the **asymmetric SQUID** Hamiltonian which is dependant from the transmon states;
- Driving the coupler frequency with an AC pulse allow us to realize SWAP interactions between selected system eigenstates;
- $\omega_p \approx \omega_{001 \rightarrow 110} = \omega_1 + \omega_2 + \chi_{12} - \omega_3$ activate the 3 qubits interaction between $|001\rangle$ and $|110\rangle$ which preserves the Gauss's Law (electromagnetis U(1) symmetry);

$$\hat{H}_{\text{sq}} = -E_J \cos\left(\frac{\pi \hat{\Phi}_{\text{ext}}}{\Phi_0}\right) \cos \hat{\phi}_2 - \delta E_J \sin\left(\frac{\pi \hat{\Phi}_{\text{ext}}}{\Phi_0}\right) \sin \hat{\phi}_2$$

$$\begin{aligned} \hat{\Phi}_{\text{ext}} &= \alpha_p(t) = A_p \cos(\omega_p t + \varphi) \\ \hat{\phi}_2 &\sim (\hat{b}_j^\dagger + \hat{b}_j) \end{aligned}$$

$$\hat{H}_{\text{sq}} = - \sum_k g_k(A_p) \left[\sum_{n=1}^3 (\lambda_n \hat{\sigma}_n^+ + \lambda_n^* \hat{\sigma}_n^-) \right]^k \xrightarrow[\text{RWA}]{\alpha_p(t) = A_p \cos(\omega_{001 \rightarrow 110} t + \varphi)} \hat{H}_{\text{int}} \approx -J(A_p) \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3 + \text{h.c.}$$



Project flow

Idea: a tunable coupler for QUART&T project, how many qubits, type of qubit, type of gate, type of application

Hamiltonian estimation: analytical approximated solutions of H , numerical solutions, ideal circuit design

Physical circuit design: design on Qiskit Metal, parameters optimization doing multiple simulations

Fabrication

Characterization

Thank you for the attention

A. Cattaneo, R. Moretti, P. Campana, R. Carobene, M. Gobbo, M. Borghesi, D. Labranca, H. A. Corti, M. Faverzani, E. Ferri, S. Gamba, A. Nucciotti, L. Origo, A. Giachero

Backup slides

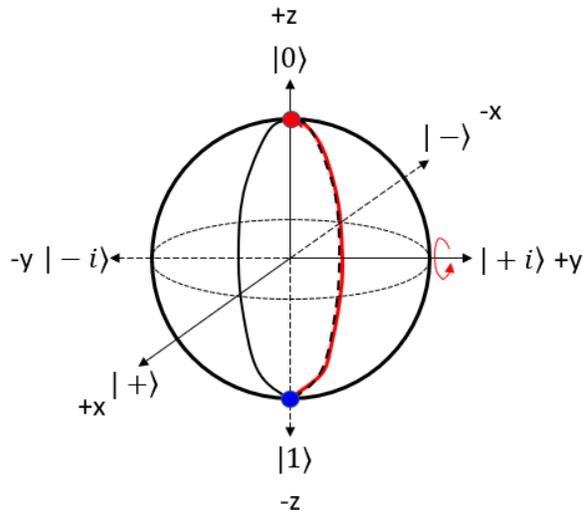
Type of couplings

Transversal (XX or YY) interaction

$$\hat{H} = -\frac{\hbar\omega_1}{2}\hat{\sigma}_{z1} - \frac{\hbar\omega_2}{2}\hat{\sigma}_{z2} + \hbar g(\hat{\sigma}^-\hat{\sigma}^+ + \hat{\sigma}^+\hat{\sigma}^-)$$

Reached when the **qubits are brought in resonance**.

- iSWAP gate \rightarrow tunable qubits (or fixed qubits with a tunable coupler)



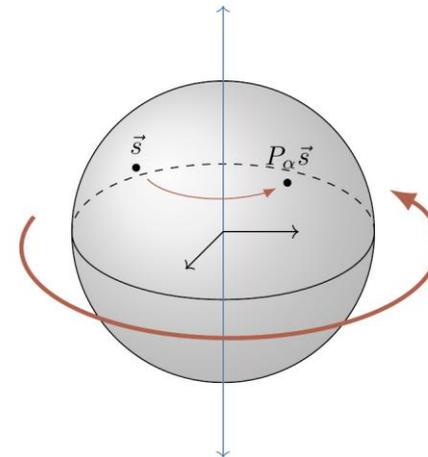
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Longitudinal (ZZ) interaction

$$\mathcal{H}_{\text{eff}}/\hbar = \frac{1}{2} \sum_{i=1,2} \left(\omega_i + \frac{\alpha_{ZZ}}{2} \right) \sigma_z^{(i)} + \frac{\alpha_{ZZ}}{4} \sigma_z^{(1)} \sigma_z^{(2)}$$

Reached when the **qubits frequencies are detuned by an anharmonicity**.

- CPHASE gate \rightarrow tunable qubits (or fixed qubits with a tunable coupler)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{bmatrix}$$

Which one is better depends on the application.

CR gate use a mixture of the two interactions: $\sigma_x \otimes \sigma_z \rightarrow$ fixed qubits.

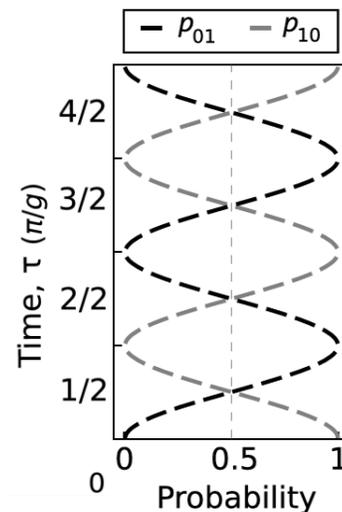
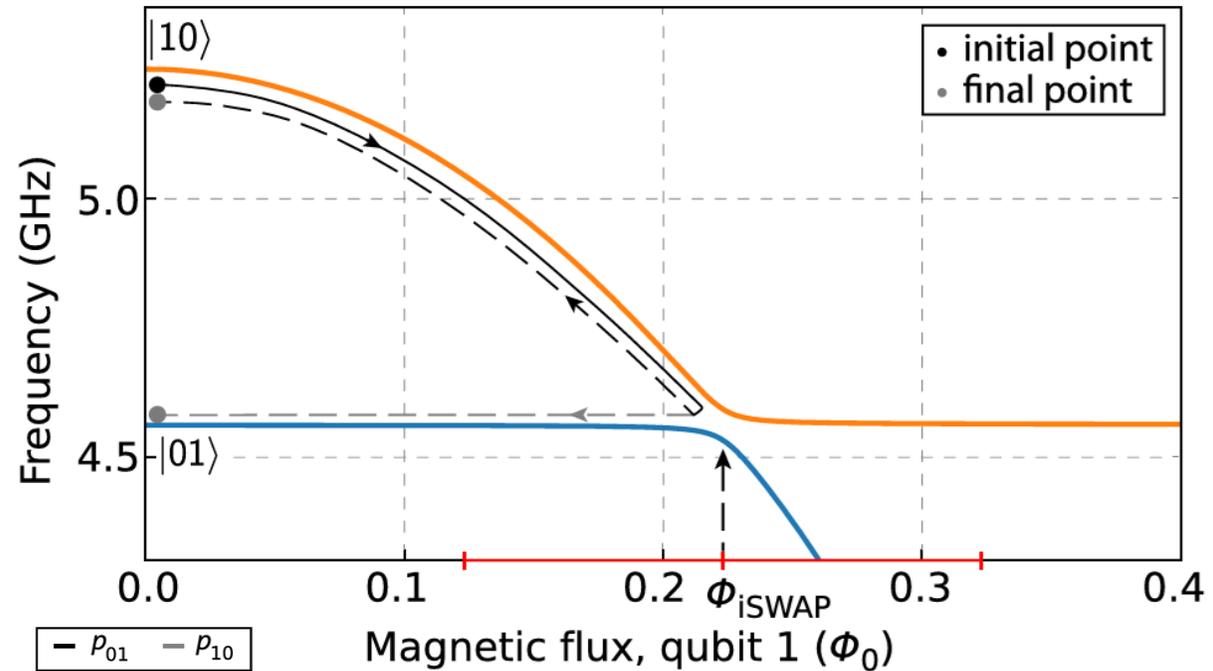
If the qubits are tunable all the three gates can be realized with the same architecture.

iSWAP gate

- Realized bringing the **two qubits in resonance** ($\omega_{Q1} = \omega_{Q2}$) for a determined period of time τ ;
- Necessity of having at least one tunable qubit;
- Achievable also with a fixed frequency coupler;

Tunable coupling advantages:

- A tunable coupler allows to **turn off completely the interaction** $\tilde{g}(\omega_c^{off}) = 0$;
- Allow to **cancel the unwanted ZZ coupling** adjusting the energy level $|020\rangle$;
- Allow **faster gates**;
- Specific control coupler pulses allows to reduce leakage through the coupler during the interaction = best working point;
- Achievable with fixed frequency qubits;

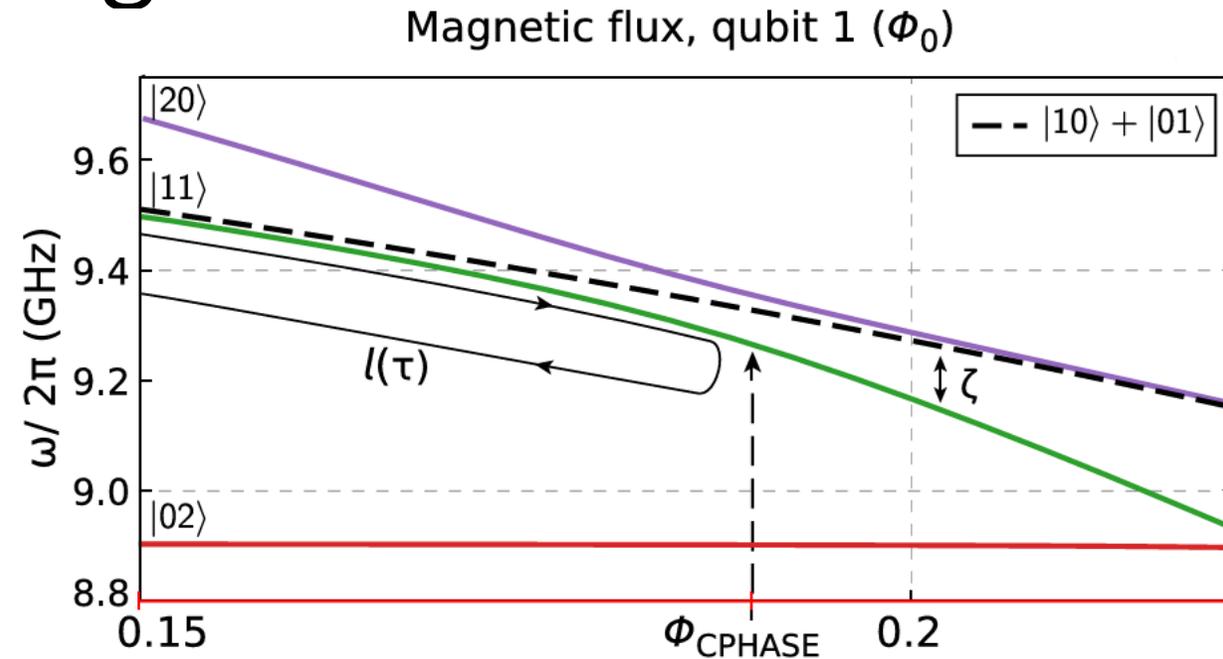


CPHASE gate

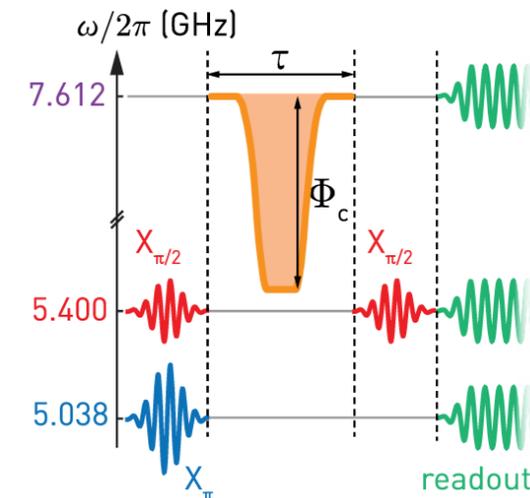
- Realized thanks to the frequency difference between $|11\rangle$ and single state excitations: $\zeta = \omega_{11} - (\omega_{10} + \omega_{01})$;
- Realized starting from the $|11\rangle$ state and **changing one qubit frequency moving adiabatically the flux through the path identified by $l(\tau)$** . ϕ_{CPHASE} so that $\omega_{q1} = \omega_{q2} - \alpha_{min}$; τ length of the process;
- Also compatible with fixed frequency qubits (if using a tunable coupler);
- Achievable also with a fixed frequency coupler;

Tunable coupling advantages:

- A tunable coupler allows to **turn off completely the interaction** $\tilde{g}(\omega_c^{off}) = 0$. Something impossible for fixed frequency coupler due to always on ZZ interaction;
- Using a tunable element should allow faster gate operations and optimal working point;



Magnetic flux, qubit 1 (Φ_0)



Analyzed architectures

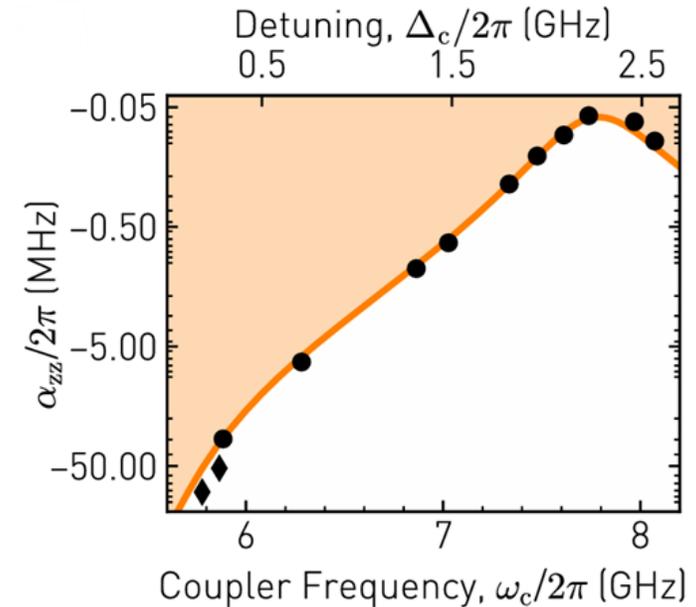
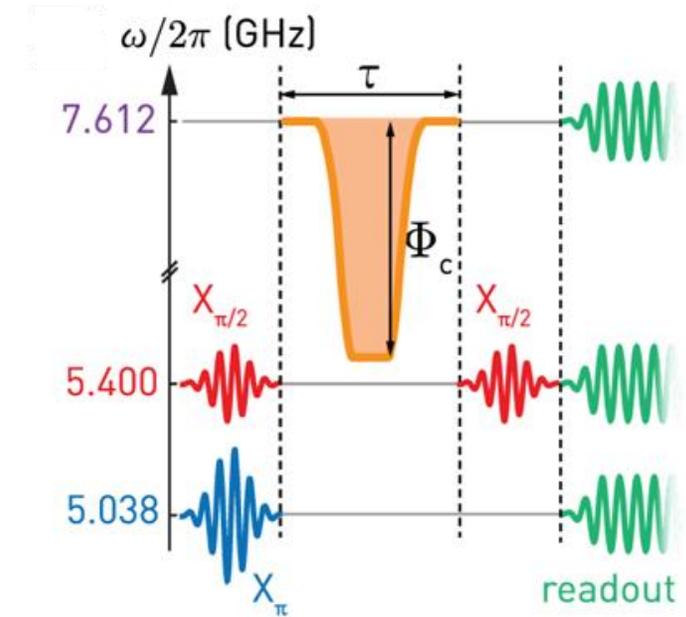
- Implementation of Conditional Phase Gates Based on Tunable ZZ Interactions (2020 ETH)
[\[https://doi.org/10.1103/PhysRevLett.125.240502\]](https://doi.org/10.1103/PhysRevLett.125.240502)
- Long-Distance Transmon Coupler with CZ-Gate Fidelity above 99.8% (2023 IQM)
[\[http://dx.doi.org/10.1103/PRXQuantum.4.010314\]](http://dx.doi.org/10.1103/PRXQuantum.4.010314)
- Tunable Coupling Scheme for Implementing High-Fidelity Two-Qubit Gates (2018 MIT)
[\[http://dx.doi.org/10.1103/PhysRevApplied.10.054062\]](http://dx.doi.org/10.1103/PhysRevApplied.10.054062)
- Realization of High-Fidelity CZ and ZZ-Free iSWAP Gates with a Tunable Coupler (2021 MIT)
[\[https://doi.org/10.1103/PhysRevX.11.021058\]](https://doi.org/10.1103/PhysRevX.11.021058)
- Modular tunable coupler for superconducting circuits (2023 MIT)
[\[https://doi.org/10.1103/PhysRevApplied.19.064043\]](https://doi.org/10.1103/PhysRevApplied.19.064043)

Implementation of Conditional Phase Gates Based on Tunable ZZ Interactions

ω_1 [GHz]	5.4
ω_2 [GHz]	5.038
ω_c [GHz]	7.612-5.500
g_{12} [MHz]	33
g_{1c} [MHz]	265
g_{2c} [MHz]	274
τ [ns]	38

$$\Delta\omega_q = 360 \text{ MHz} > g_{12}, g_{1c}, g_{2c}$$

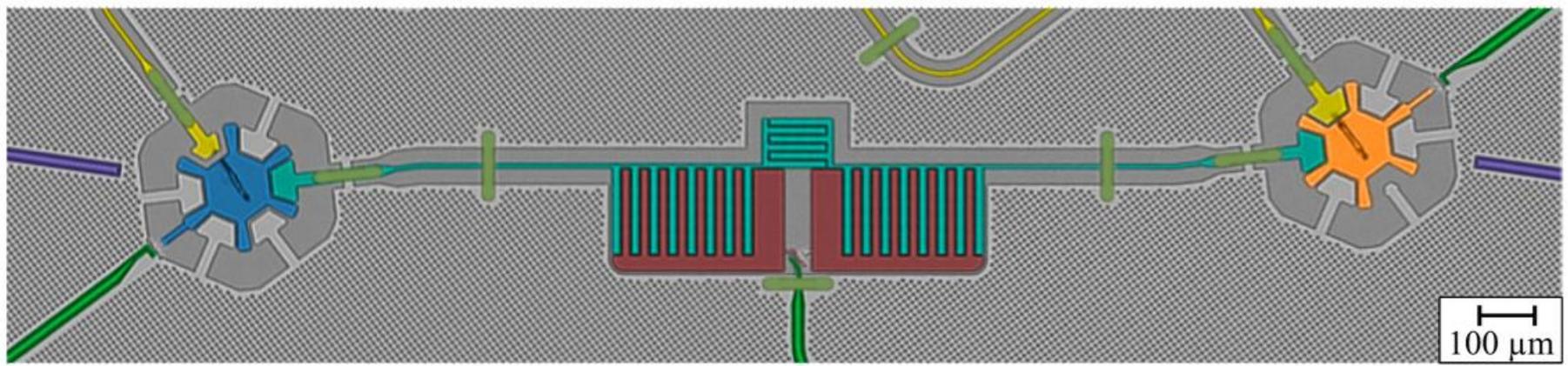
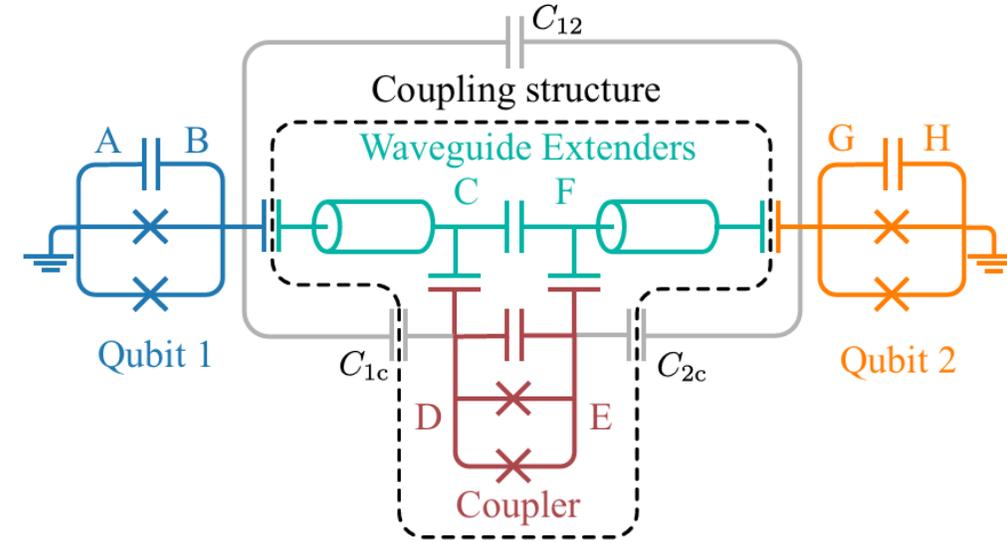
Squared flux pulses gaussian filtered to control the coupler, with average length $\tau = 60 \text{ ns}$;



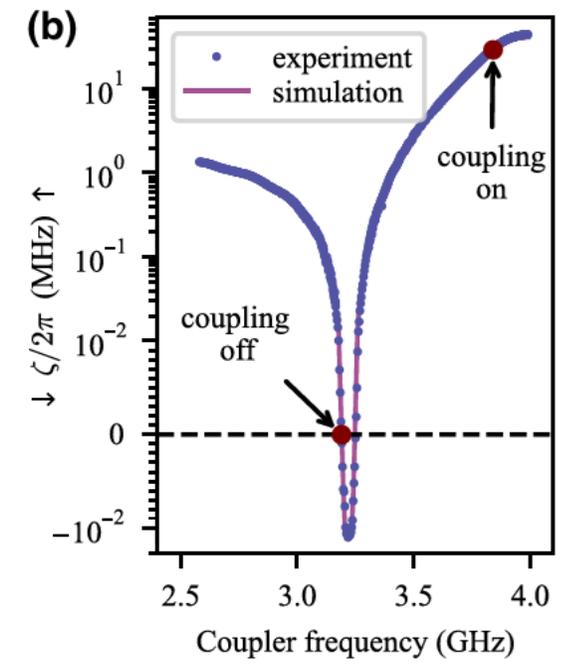
Long-Distance Transmon Coupler with CZ-Gate Fidelity above 99.8%

[PRX Quantum 4, 2023]

- Couplings mediated by two waveguides;
- The distance reduce **non-nearest-neighbor couplings, cross talk** between different drive lines (more effective for flip-chip) and allows the **usage of purcell filters**;
- All the elements are tunable;
- $\alpha_{ZZ}(\omega_c)$;
- $\mathcal{F}_{qpt} = 99.81\%$ for $\varphi_c^{target} = \pi$;
- Gate fidelity limited by qubits T_1 ;



Drive Lines Flux Lines Readout Resonators Air Bridges



CR gate

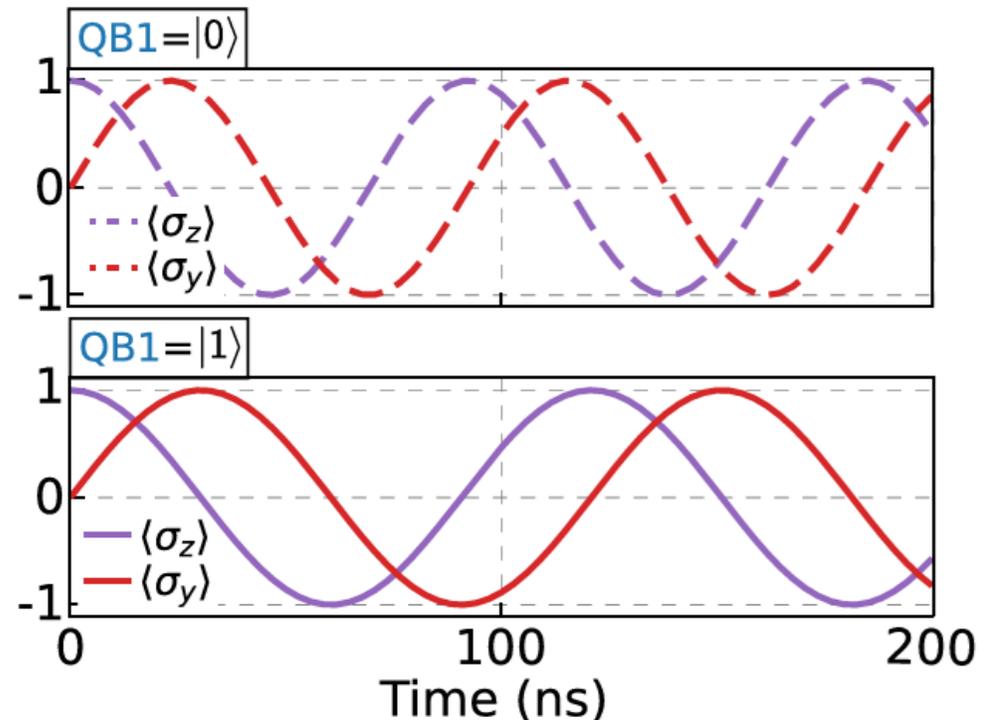
- Fixed frequency qubits;
- Drive of Q1 at the frequency of Q2. **Depending on Q1 state different Rabi Oscillation frequency.** Q2 gain a phase shift;
- The matrix representation want the angle θ which is directly proportional to the qubit driving (amplitude, time and shape);
- Work with detuned qubits ($\Delta \gg g$);

Tunable coupling advantages:

- The coupling could be **completely turned off**;
- Tunability of coupling element can be used to **reduce unwanted ZZ interaction**;

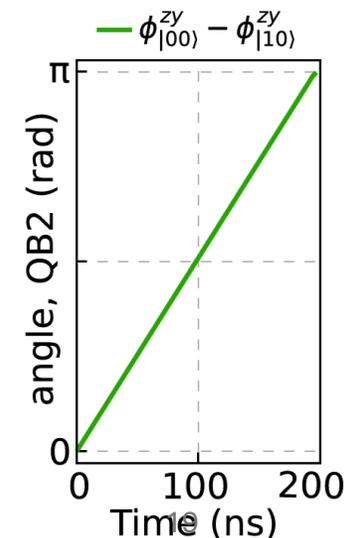
It is not much popular to implement CR gates on tunable couplers architectures.

$$\tau_{CR} \sim 10 \cdot \tau_{iSWAP} (\tau_{CPHASE})$$



$$\begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} & 0 & 0 \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \cos \frac{\theta}{2} & i \sin \frac{\theta}{2} \\ 0 & 0 & i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

XZ coupling matrix



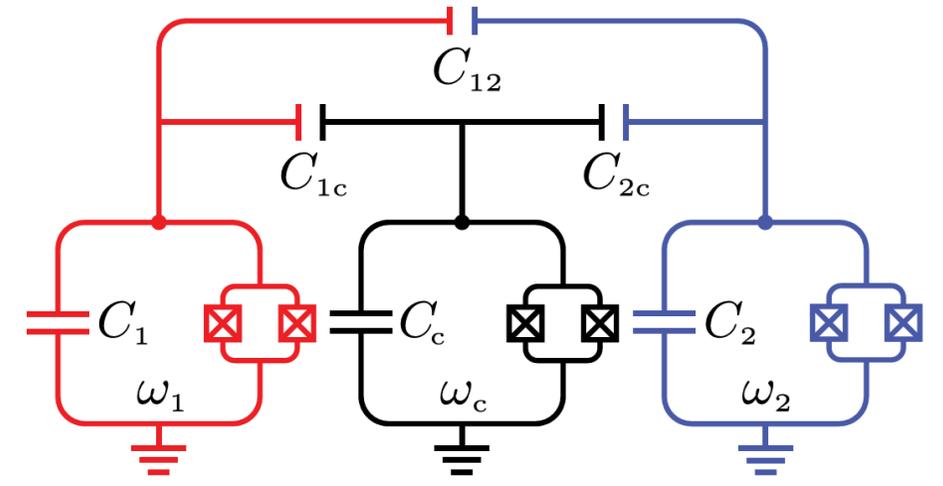
Comparison between long and short coupler

	short	long
g_{12} [MHz]	33	3.7
g_{1c} [MHz]	265	51.5
g_{2c} [MHz]	274	53.9
τ [ns]	38	33
fidelity	98.9	99.8

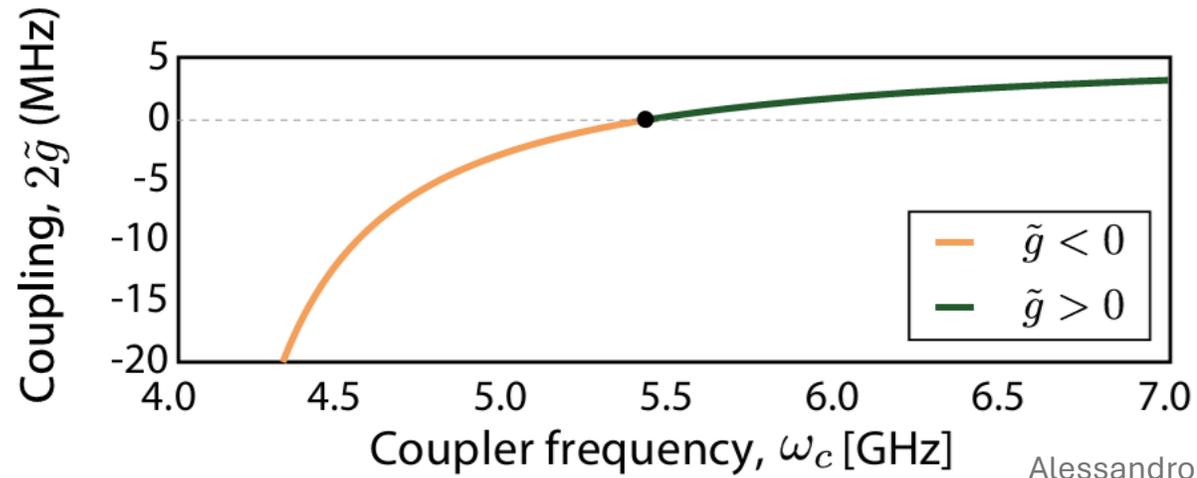
Tunable Coupling Scheme for Implementing High-Fidelity Two-Qubit Gates

[Phys. Rev. Applied **10**, 2018]

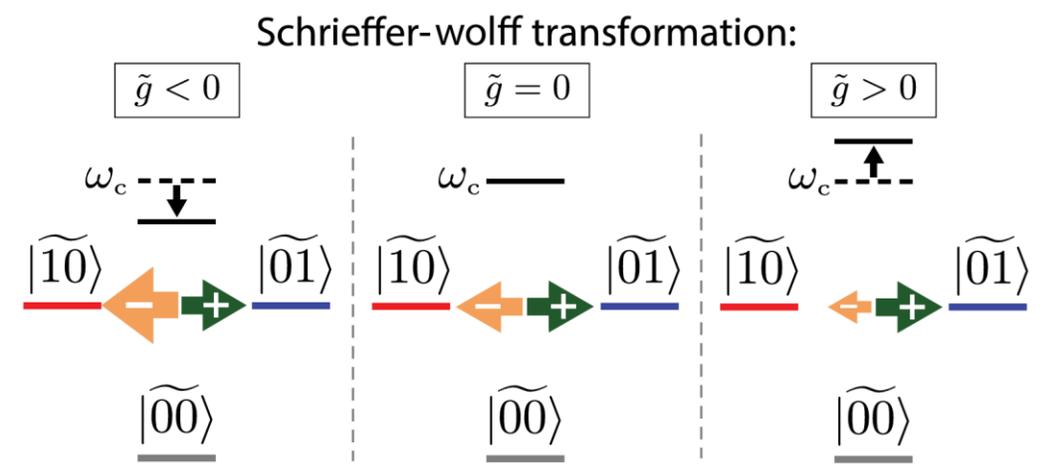
- Device realized only at the simulation level;
- Rewrite the three-body Hamiltonian through a SWT into a **two-body Hamiltonian with an effective coupling \tilde{g}** . The qubit frequencies are redefined: $\tilde{\omega}_j = \omega_j + g_j^2/\Delta_j$
- Traditional SWAP interaction. The total effective coupling: \tilde{g} is a function of ω_c and **can be always reduced to 0** for ω_c^{off} ;
- Simulate iSWAP with qubit with $T_1 = 10 \mu s$: $\tau = 35 ns$, $\mathcal{F} = 99.4\%$ (fidelity just from error gate);



$$\tilde{H} = \sum_{j=1,2} \frac{1}{2} \tilde{\omega}_j \sigma_j^z + \left[\frac{g_1 g_2}{\Delta} + g_{12} \right] (\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+)$$

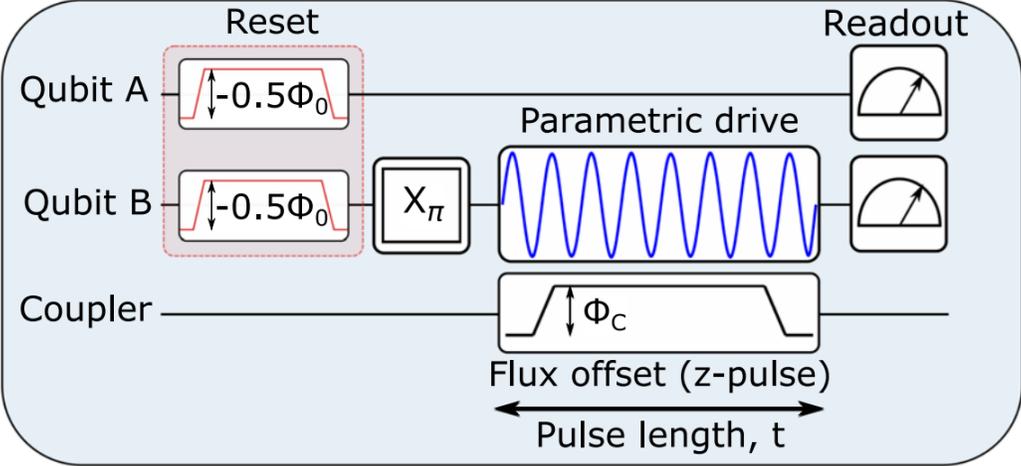


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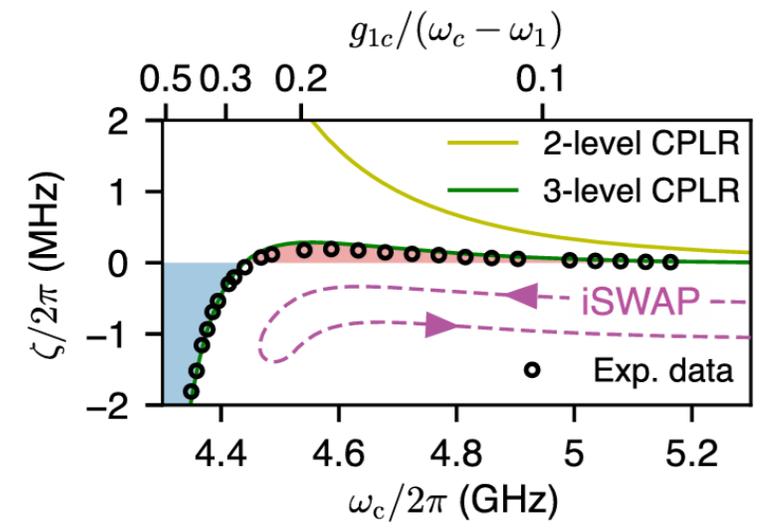
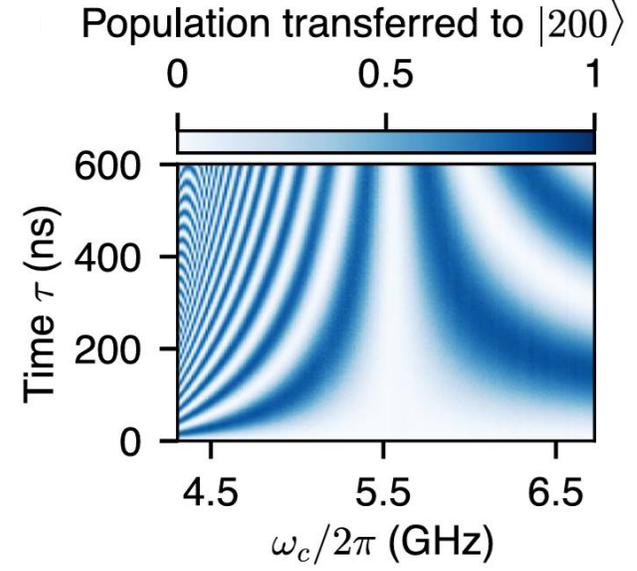
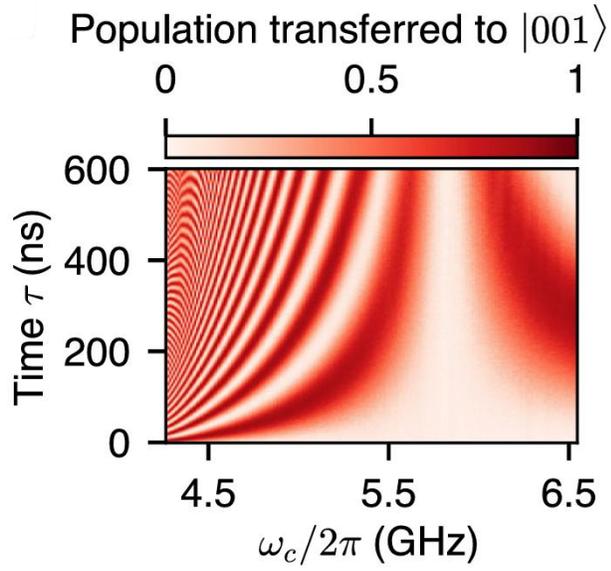
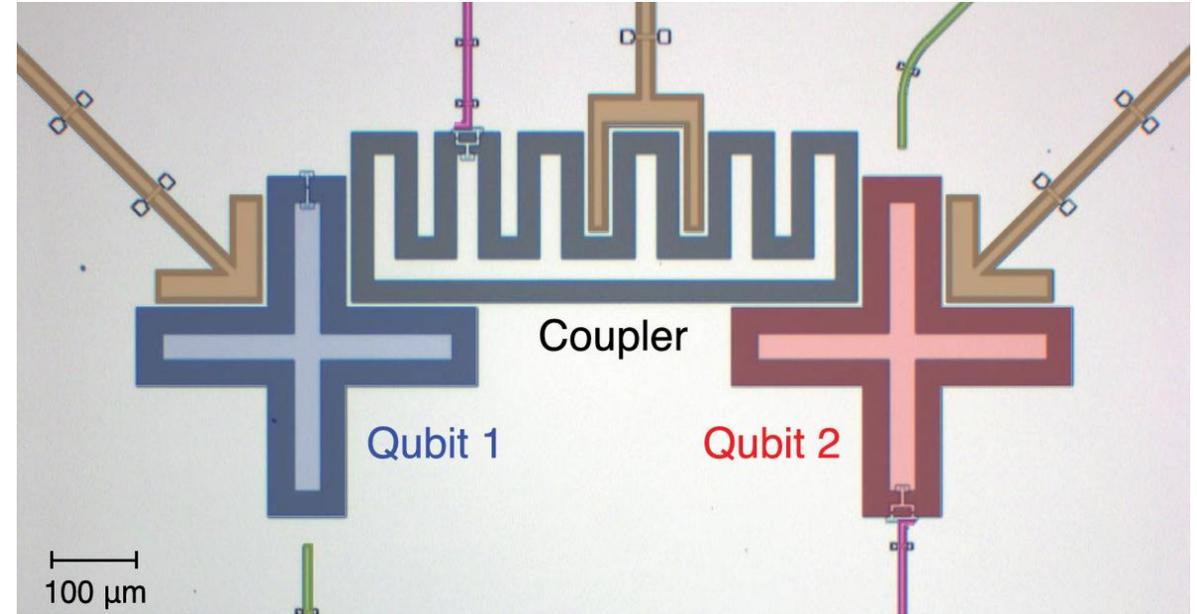
Realization of High-Fidelity CZ and ZZ-Free iSWAP Gates with a Tunable Coupler

ω_1 [GHz]	4
ω_2 [GHz]	4
ω_c [GHz]	5
τ [ns]	35



Realization of High-Fidelity CZ and ZZ-Free iSWAP Gates with a Tunable Coupler [Phys. Rev. X **11**, 2021]

- Q2 and C tunable;
- CZ: $\tau = 60 \text{ ns}$, $\mathcal{F} = 99.76\%$;
- iSWAP: $\tau = 30 \text{ ns}$, $\mathcal{F} = 99.97\%$;
- **Both gates can be turned off** and on biasing the coupler frequency;
- It is possible to define a range of frequencies ω_c at which the **unwanted ZZ coupling is completely suppressed**;

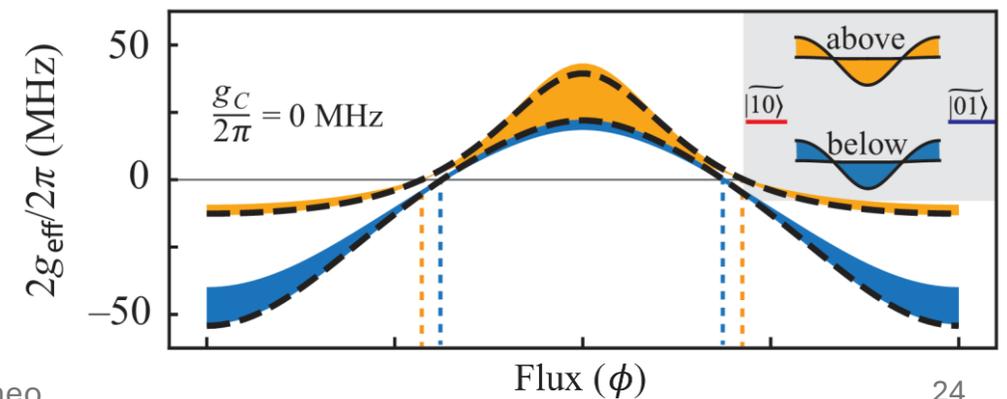
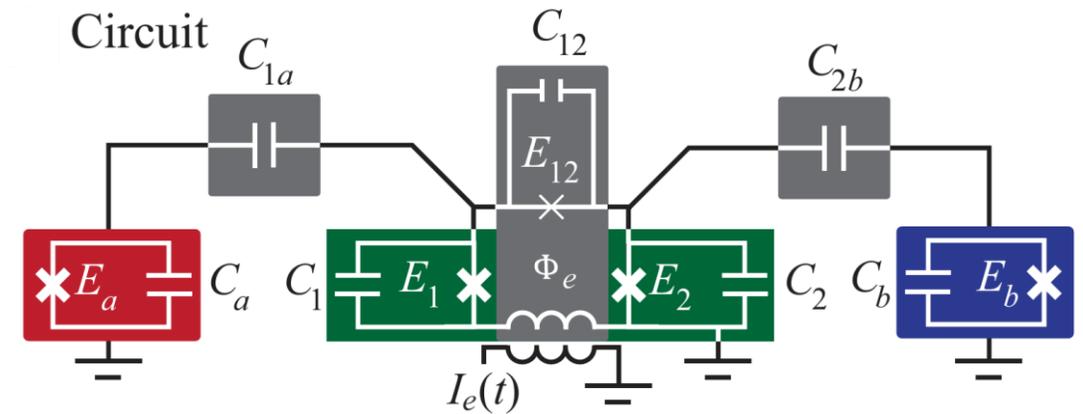
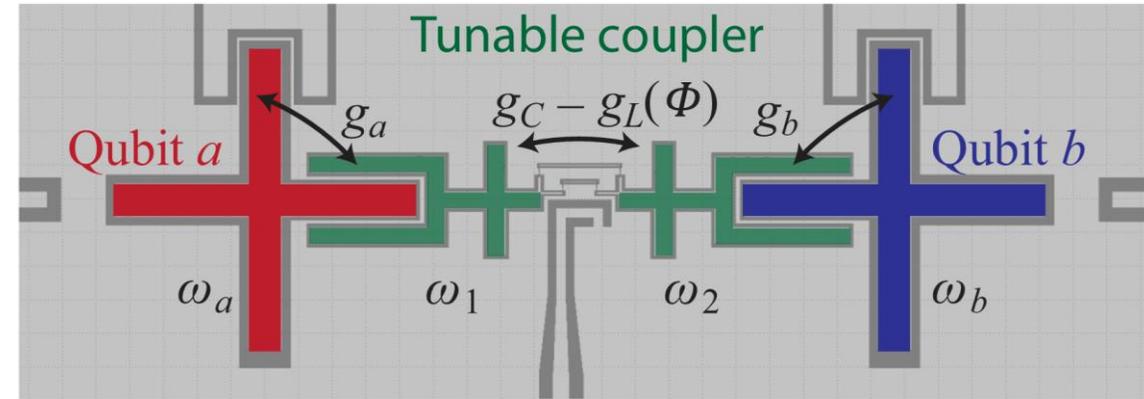


Original approaches: Modular Tunable Coupler for Superconducting Circuits [Phys. Rev. Applied 19, 2023]

- Coupling realized with a **double transmon coupler (DTC)** architecture: 2 transmon connected by a JJ in parallel to a capacitance;
- The connection has a capacitive nature g_c , and an inductive one $g_L(\Phi)$;
- An **effective couplig** g_{eff} between the qubits can be estimated with an analytical approach in the dispersive limit ($\frac{g_j}{|\omega_j - \omega_{\pm}|} < 0.1$);
- Traditional gates like **iSWAP, CZ and CR** can be realized bringing the qubits at the required frequencies and turning on the coupling for a short time through external flux;
- Drive in a «**parametric regime**»: varying g_L at a specific frequency allow to make in communication detuned qubits of order 80 MHz;

Big advantages:

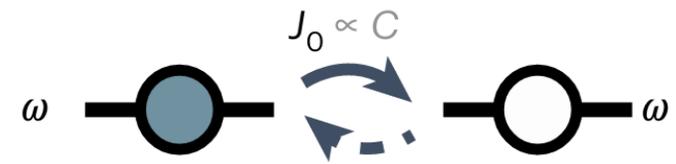
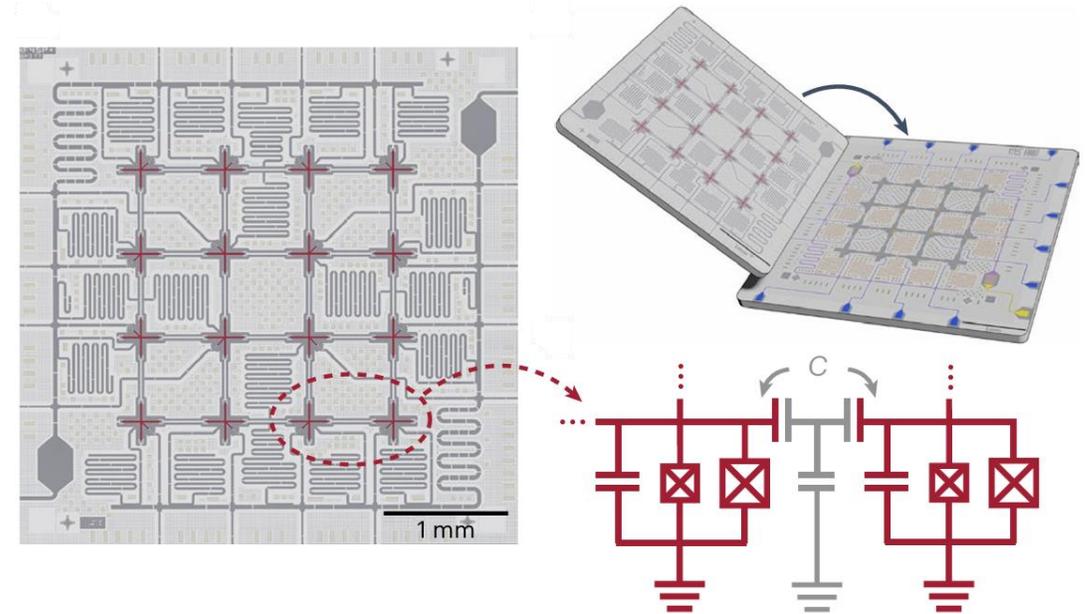
- possesses an **internally defined zero-coupling** state that is independent of the coupled data qubits or circuit resonators;
- The direct capacitive coupling between data qubits is turned off by the third JJ.



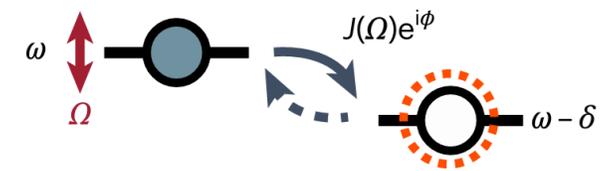
A synthetic magnetic vector potential in a 2D superconducting qubit array

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- GOAL: simulate the movement of an electron inside of a 2D lattice under external electromagnetic fields. Use the **Harper-Hofstadter (HH)** model to describe it. The Hamiltonian of the model has been simulated with the device;
- All the qubits are **capacitively coupled**. The interaction happens bringing two qubits in resonance;
- A non-zero phase of the modulation tone allow to break the spacial inversion symmetry to simulate the presence of **faraday induced electric fields**;
- Observed **Aharonov-Bohm interference patterns** in 8- and 12-site rings. Observed Aharonov-Bohm caging effect
- Tunable parametric coupling would enable easier and more efficient coupling in expense of hardware complexity.



Two qubits at resonance have a probability J_0 to interact exchanging an excitation.



Parametric coupling realized in this work depending on energy modulation amplitude Ω .

$$\hat{H}_{\text{BH}}/\hbar = \sum_i \left(\omega_i \hat{n}_i + \frac{U_i}{2} \hat{n}_i (\hat{n}_i - 1) \right) + \sum_{\langle i,j \rangle} J_0^{ij} \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger \right)$$

Bose-Hubbard model to simulate without magnetic field situations.

$$\hat{H}_{\text{HH}}/\hbar = \sum_{\langle i,j \rangle} J \left(e^{-i\phi_{ij}} \hat{a}_i^\dagger \hat{a}_j + e^{i\phi_{ij}} \hat{a}_i \hat{a}_j^\dagger \right)$$

Harper-Hofstadter model.

Demonstration of Tunable Three-Body Interactions between Superconducting Qubits

- To estimate experimentally the Hamiltonian parameters all the qubits fundamental frequencies need to be identified, as well as each transition up to a total of three excitations:

- Low energy spectroscopy;**
- Double photons excitations** on a single qubit found applying a Rabi spectroscopy between states $|1\rangle$ and $|2\rangle$;
- Use **Ramsey experiments** to identify multi-qubit excitations;

- The set of computational states is then used to fit a full circuit Hamiltonian model to the spectroscopy data;
- The model is valid around the flux insensitive point of the qubits, which corresponds to $\varepsilon = 0$;

