La matrice K - Ovvero: un approccio analitico ed unitario allo studio delle risonanze

Marco Scodeggio (INFN - Ferrara)

mscodegg@fe.infn.it

BESIII Italia
Torino - Aprile 2025

Outline

- Resonances and how to treat them
- Coupled-channel analysis of vector charmonia at BESIII
- Summary…

DISCLAIMER

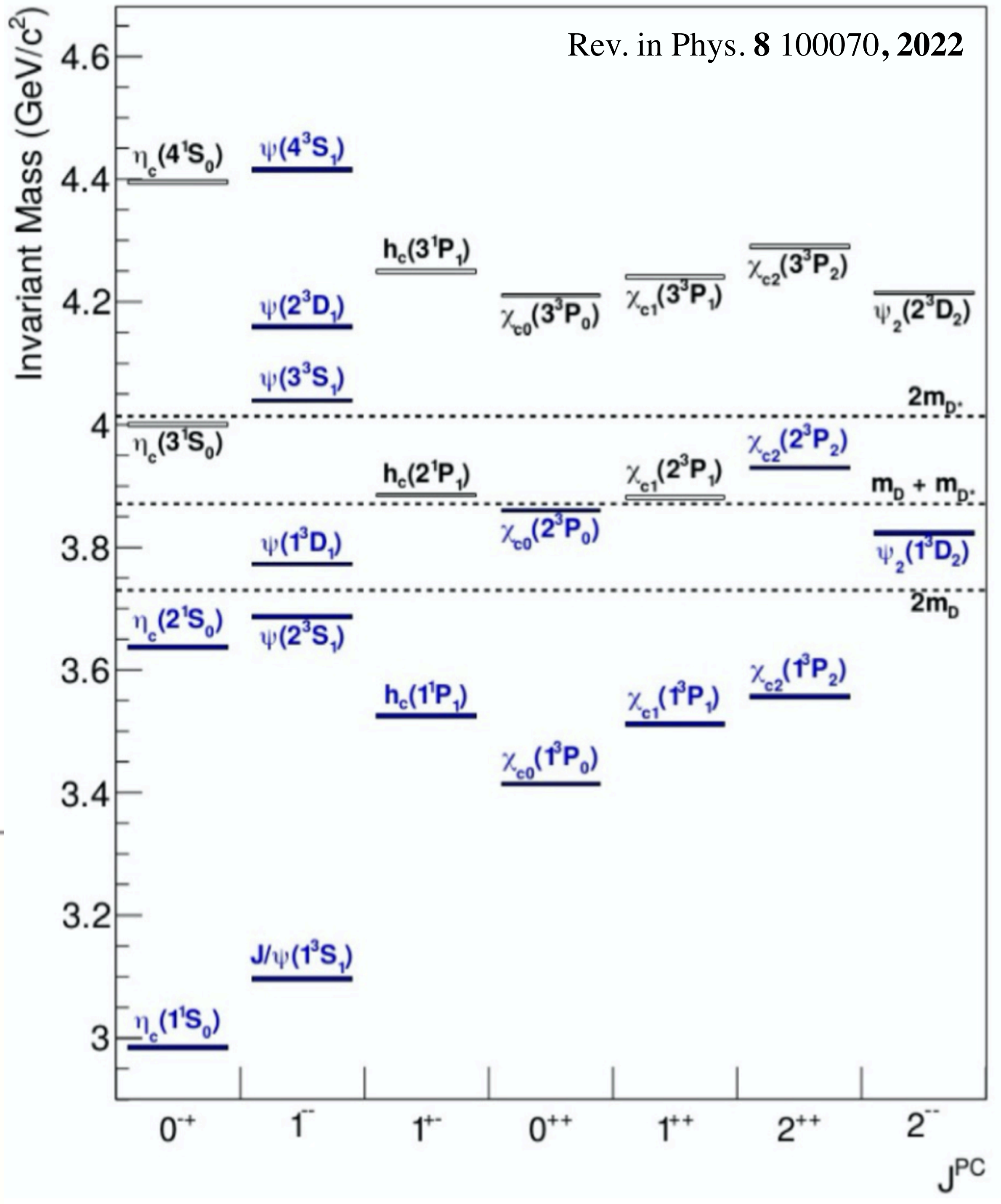
This presentation is kind of a work-in-progress which also involves me trying to understand this subject

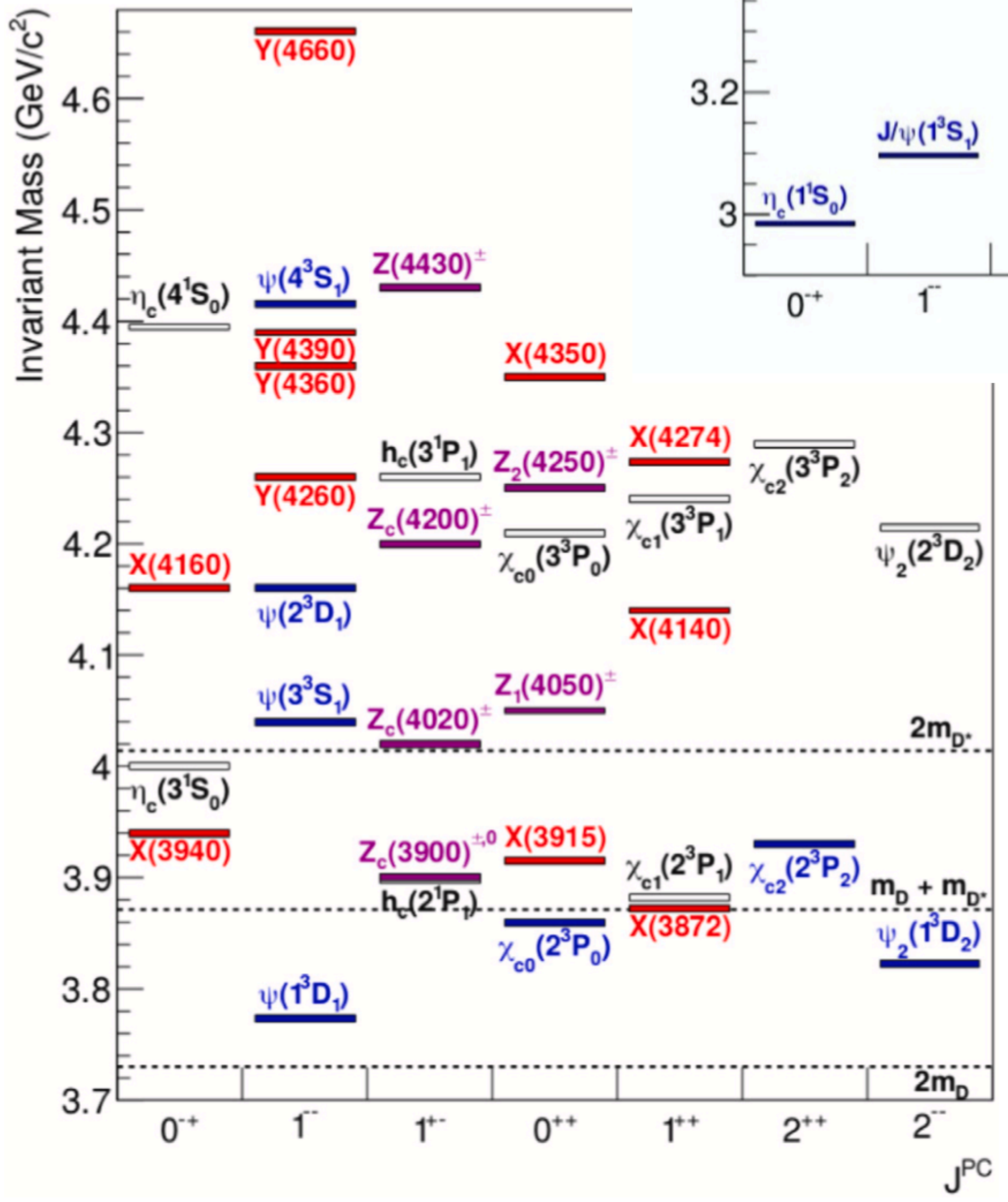
[Resonances (rev.) in PRD **110**, 030001 (2024)]

Resonances and how to treat them

Appear as poles of the S-matrix in the complex plane on unphysical sheets

Defined by pole locations and residues



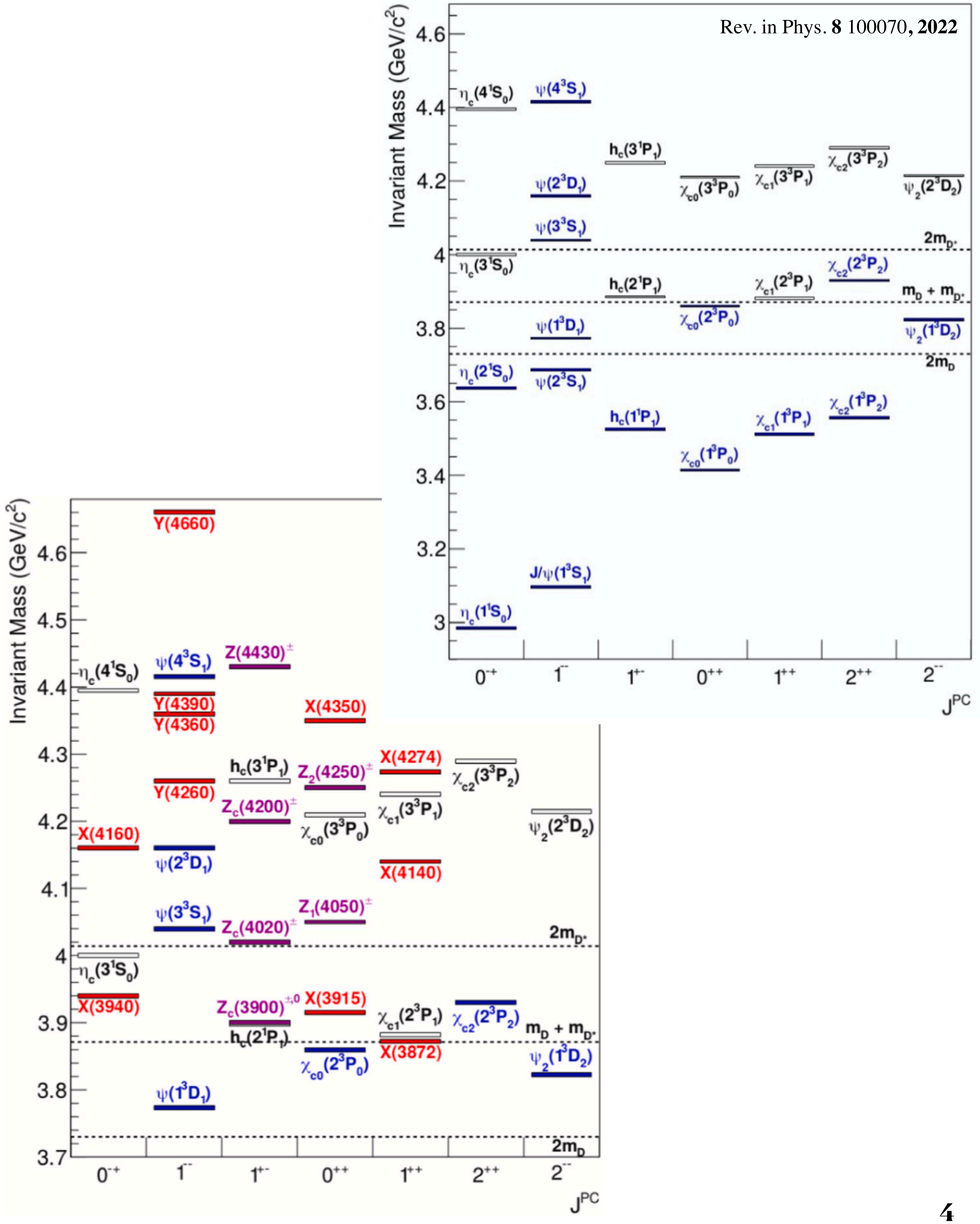


Resonances and how to treat them

Appear as poles of the S-matrix in the complex plane on unphysical sheets

Defined by pole locations and residues

A resonance appears as a peak in the total cross section
If it's narrow and there are no relevant thresholds or other resonances nearby one can employ a Breit–Wigner parameterization



Resonances and how to treat them

Appear as poles of the S-matrix in the complex plane on unphysical sheets

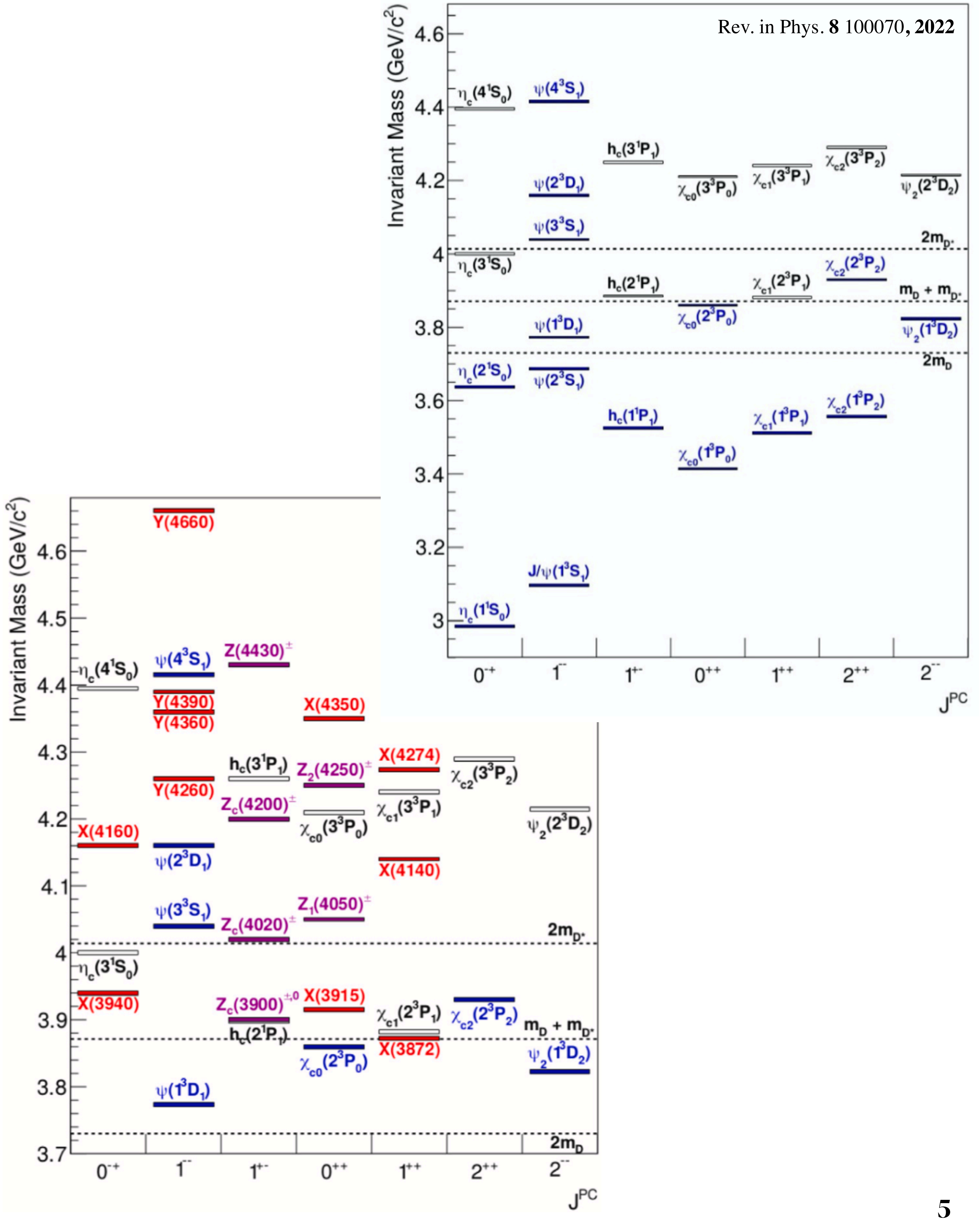
Defined by pole locations and residues

A resonance appears as a peak in the total cross section
If it’s narrow and there are no relevant thresholds or other resonances nearby one can employ a Breit–Wigner parameterisation

For broad resonances there is no direct relation between pole location and the total width/lifetime

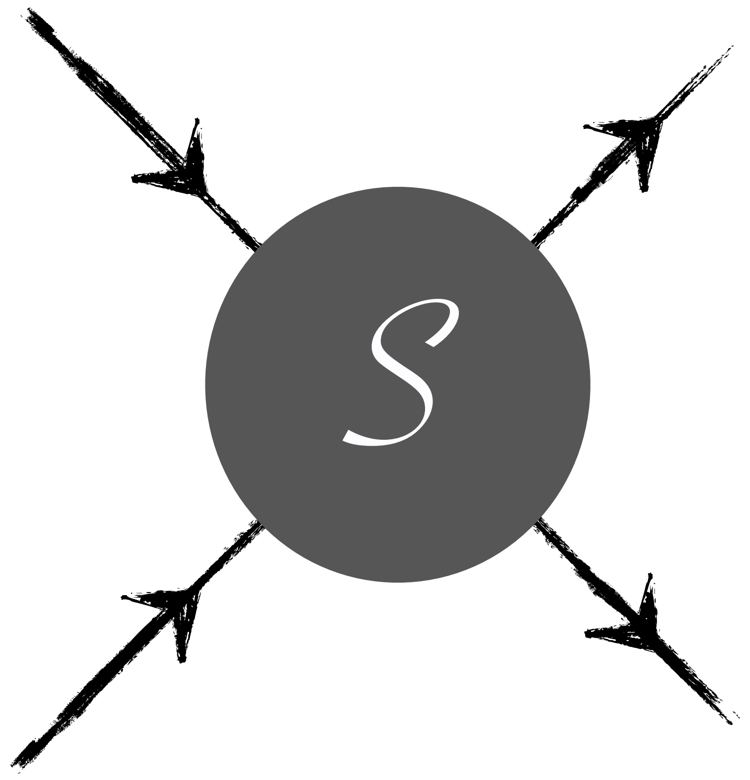
Unitarity and analyticity call for the use of more refined tools

“Entry level” → K-matrix approach



Resonances and how to treat them

Appear as poles of the **S-matrix** in the complex plane on unphysical sheets

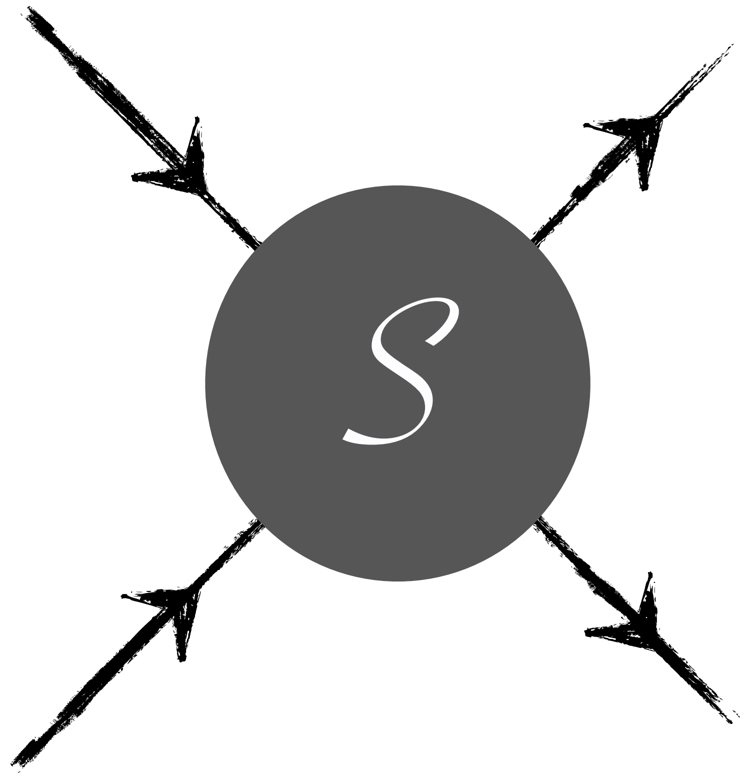


Resonances and how to treat them

Appear as poles of the **S-matrix** in the complex plane on unphysical sheets

Unitary operator that connect asymptotic in and out states undergoing a scattering process

A Feynman diagram represents one term in the Wick's expansion of the perturbative S-matrix

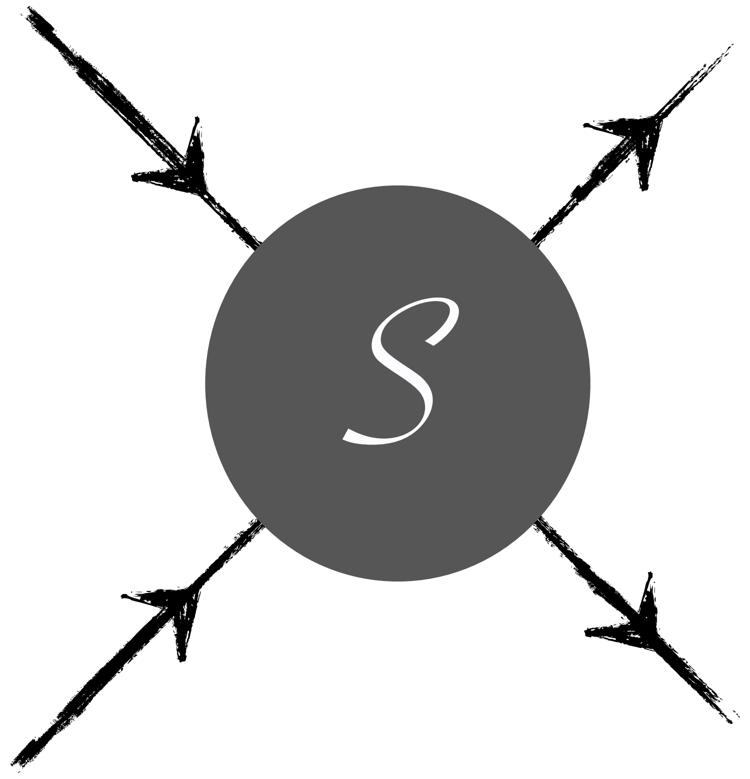


Resonances and how to treat them

Appear as poles of the **S-matrix** in the complex plane on unphysical sheets

Unitary operator that connect asymptotic in and out states undergoing a scattering process

A Feynman diagram represents one term in the Wick's expansion of the perturbative S-matrix



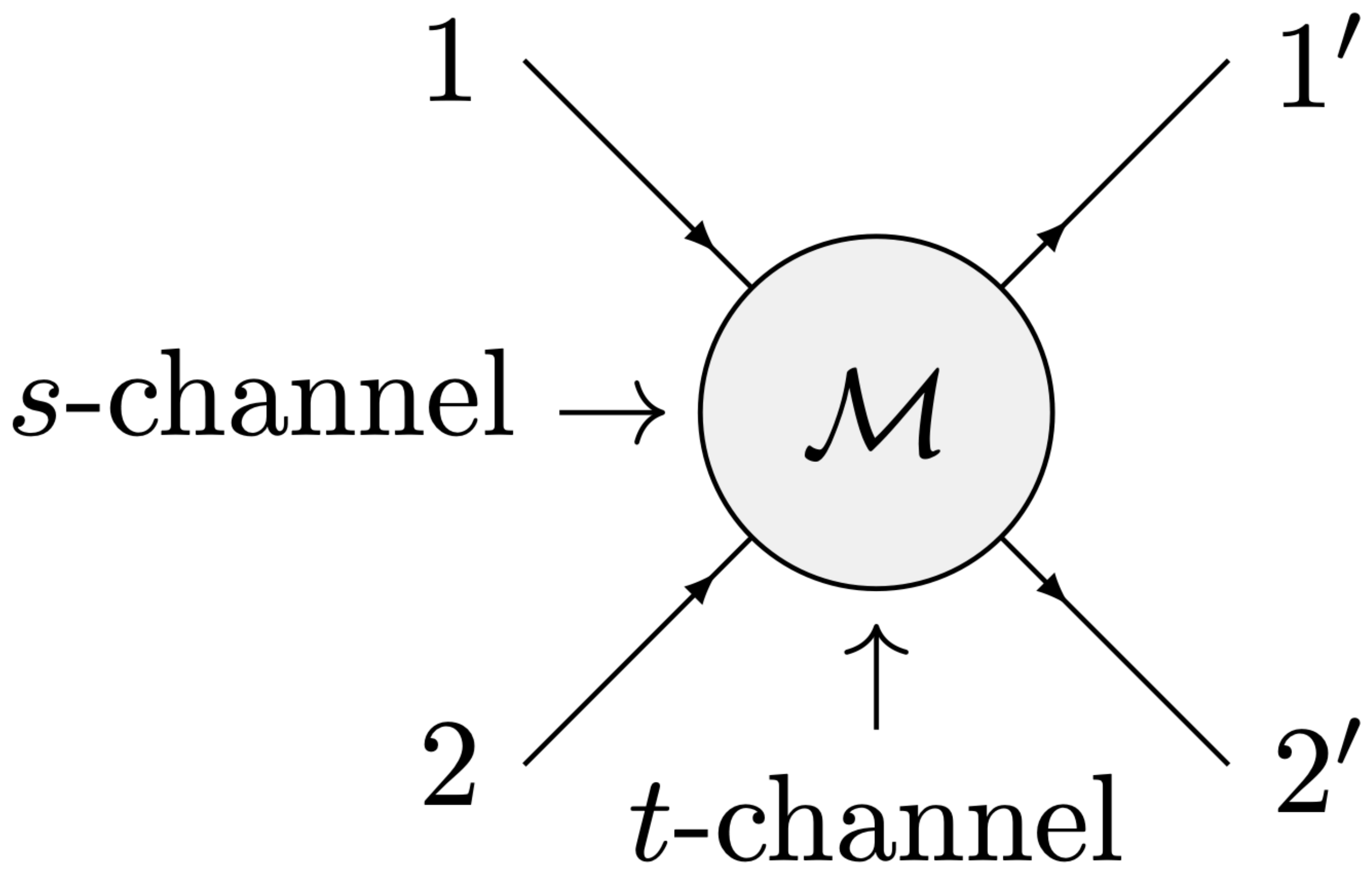
Scattering amplitude ($\mathscr{M}$) is defined as the interacting part of the S-matrix

$$i(2\pi)^4\delta^4(p_1+p_2-p_{1'}-p_{2'})\,\mathcal{M}(p_1,p_2;p_{1'},p_{2'})_{ba}$$
$$= {}_{\text{out}}\langle p_{1'}p_{2'},b|\,\mathcal{S}-1\,|p_1p_2,a\rangle_{\text{in}}$$

Resonances and how to treat them

Scattering amplitude ($\mathscr{M}$) is defined as the interacting part of the S-matrix

$$i(2\pi)^4\delta^4(p_1 + p_2 - p_{1'} - p_{2'})\,\mathcal{M}(p_1, p_2; p_{1'}, p_{2'})_{ba}$$
$$= {}_{\text{out}}\langle p_{1'}p_{2'}, b|\, \mathcal{S} - 1\, |p_1 p_2, a\rangle_{\text{in}}$$



Resonances and how to treat them

Scattering amplitude ($\mathscr{M}$) is defined as the interacting part of the S-matrix

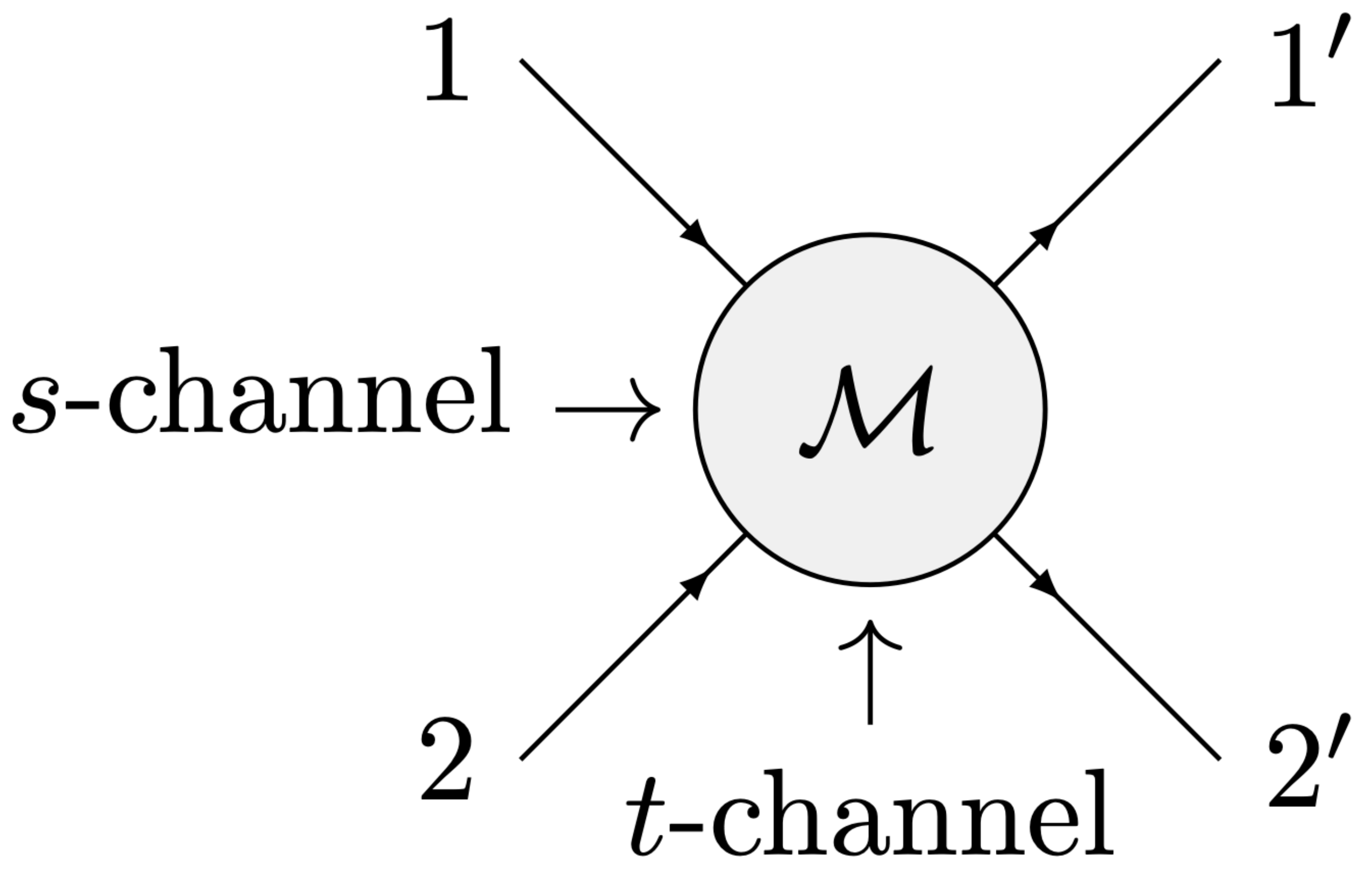
Mandelstam variables

$s = (p_1 + p_2)^2$, $t = (p_1 - p_1')^2$, $u = (p_1 - p_2')^2$

with $s + t + u = m_1^2 + m_2^2 + m_{1'}^2 + m_{2'}^2$

So, we can state $\mathscr{M} = \mathscr{M}(s, t)$

$$i(2\pi)^4\delta^4(p_1 + p_2 - p_{1'} - p_{2'})\,\mathcal{M}(p_1, p_2; p_{1'}, p_{2'})_{ba} = {}_{\text{out}}\langle p_{1'}p_{2'}, b|\,\mathcal{S} - 1\,|p_1p_2, a\rangle_{\text{in}}$$



Resonances and how to treat them

Scattering amplitude ($\mathscr{M}$) is defined as the interacting part of the S-matrix

Mandelstam variables

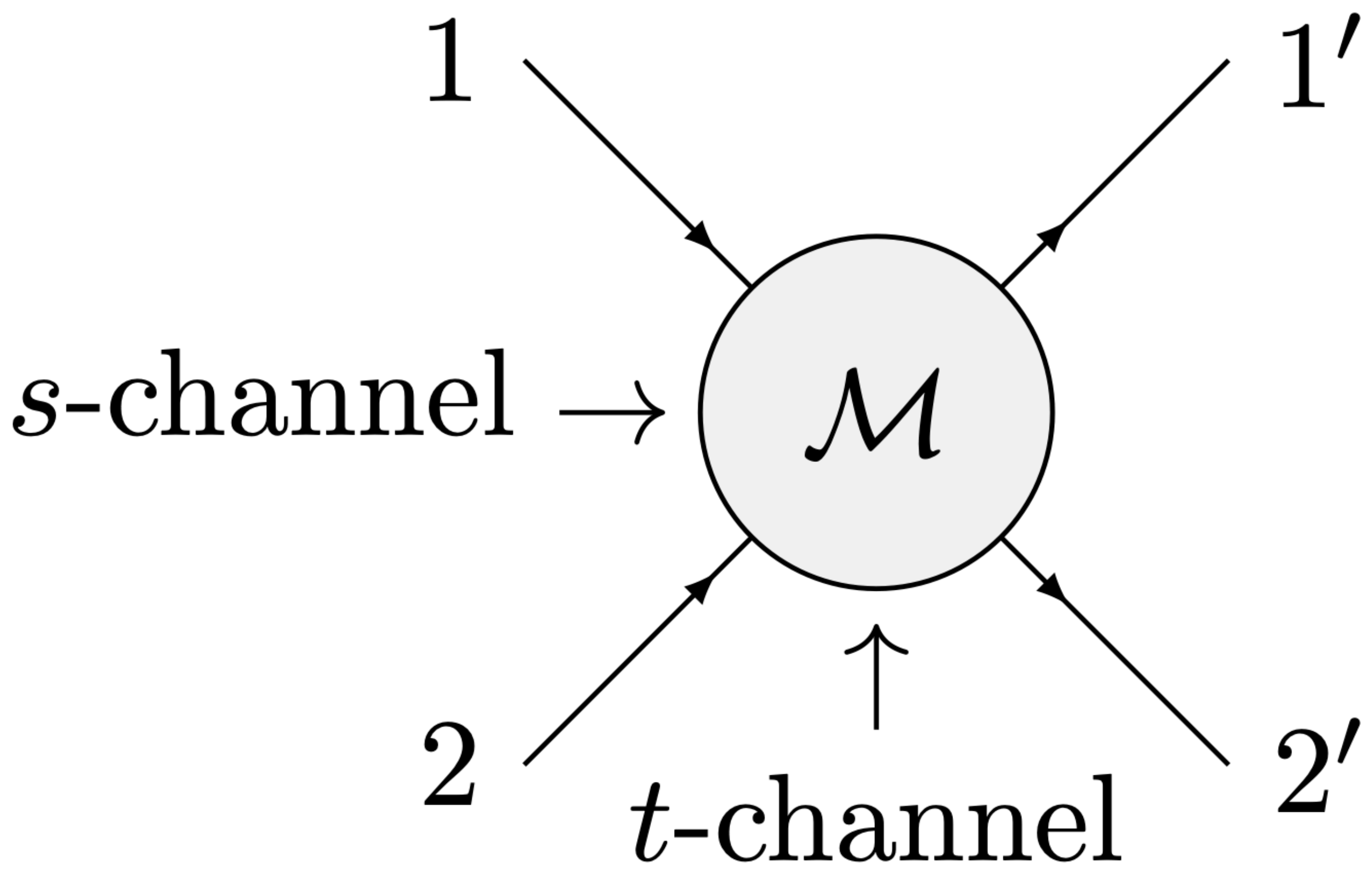
s = $(p_1 + p_2)^2$, t = $(p_1 - p_1')^2$, u = $(p_1 - p_2')^2$

with s + t + u = $m_1^2 + m_2^2 + m_{1'}^2 + m_{2'}^2$

So, we can state $\mathscr{M} = \mathscr{M}(s, t)$

$$i(2\pi)^4\delta^4(p_1 + p_2 - p_{1'} - p_{2'})\,\mathcal{M}(p_1, p_2; p_{1'}, p_{2'})_{ba} \\ = {}_{\text{out}}\langle p_{1'}p_{2'}, b|\,\mathcal{S} - 1\,|p_1p_2, a\rangle_{\text{in}}$$

$\mathscr{M}$(s, t) is a multivalued function due to the complex branch points associated with the Mandelstam variables



A branch point emerges when a new channel becomes accessible [s > $s_{thr,\,a} = (m_{1,a} + m_{2,a})^2$]

Resonances and how to treat them

Scattering amplitude ($\mathscr{M}$) is defined as the interacting part of the S-matrix

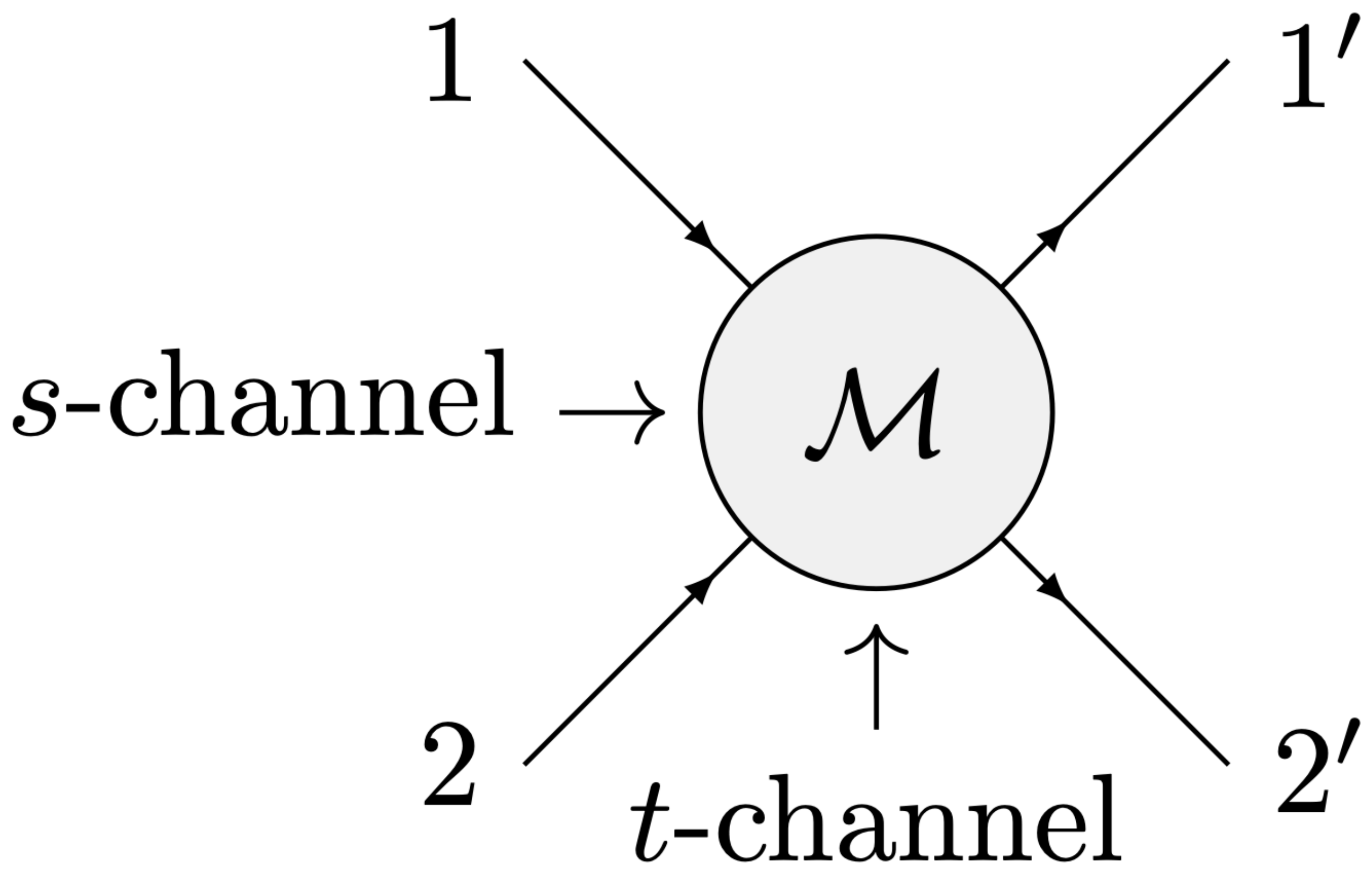
Mandelstam variables

$s = (p_1 + p_2)^2$, $t = (p_1 - p_1')^2$, $u = (p_1 - p_2')^2$

with $s + t + u = m_1^2 + m_2^2 + m_{1'}^2 + m_{2'}^2$

So, we can state $\mathscr{M} = \mathscr{M}(s, t)$

$$i(2\pi)^4\delta^4(p_1 + p_2 - p_{1'} - p_{2'})\,\mathcal{M}(p_1, p_2; p_{1'}, p_{2'})_{ba}$$
$$= {}_{\text{out}}\langle p_{1'}p_{2'}, b|\,\mathcal{S} - 1\,|p_1 p_2, a\rangle_{\text{in}}$$



$\mathscr{M}(s, t)$ is a multivalued function due to the complex branch points associated with the Mandelstam variables

A branch point emerges when a new channel becomes accessible [$s > s_{thr,\,a} = (m_{1,a} + m_{2,a})^2$]

A two-particle threshold introduces a square-root singularity
NB Odd number of particles produce logarithmic singularity

Resonances and how to treat them

$\mathscr{M}(s, t)$ is a multivalued function due to the complex branch points associated with the Mandelstam variables

A branch point emerges when a new channel becomes accessible [$s > s_{thr,\,a} = (m_{1,a} + m_{2,a})^2$]

A two-particle threshold introduces a square-root singularity
NB Odd number of particles produce logarithmic singularity

Resonances and how to treat them

$\mathscr{M}(s, t)$ is a multivalued function due to the complex branch points associated with the Mandelstam variables

A branch point emerges when a new channel becomes accessible [$s > s_{thr,\,a} = (m_{1,a} + m_{2,a})^2$]

A two-particle threshold introduces a square-root singularity
NB Odd number of particles produce logarithmic singularity

In the complex *s-plane*, these two solutions are represented as separate layers (Riemann sheets/surfaces)

NB Logarithmic singularity means ∞ sheets

Resonances and how to treat them

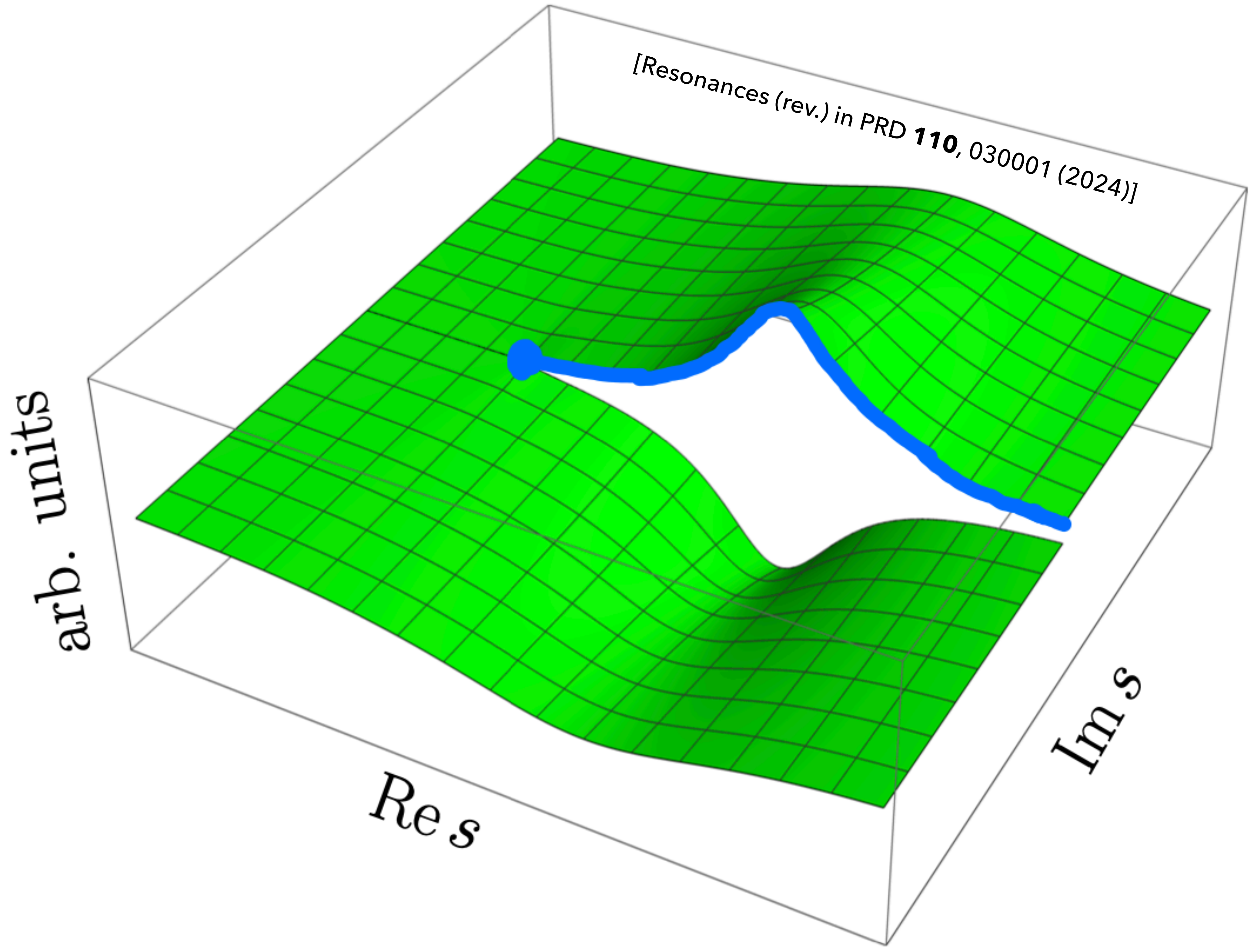
$\mathscr{M}(s, t)$ is a multivalued function due to the complex branch points associated with the Mandelstam variables

A branch point emerges when a new channel becomes accessible [$s > s_{thr,\,a} = (m_{1,a} + m_{2,a})^2$]

A two-particle threshold introduces a square-root singularity
NB Odd number of particles produce logarithmic singularity

In the complex *s-plane*, these two solutions are represented as separate layers (Riemann sheets/surfaces)

NB Logarithmic singularity means ∞ sheets



Resonances and how to treat them

$\mathscr{M}$(s, t) is a multivalued function due to the complex branch points associated with the Mandelstam variables

A branch point emerges when a new channel becomes accessible [$s > s_{thr,\, a} = (m_{1,a} + m_{2,a})^2$]

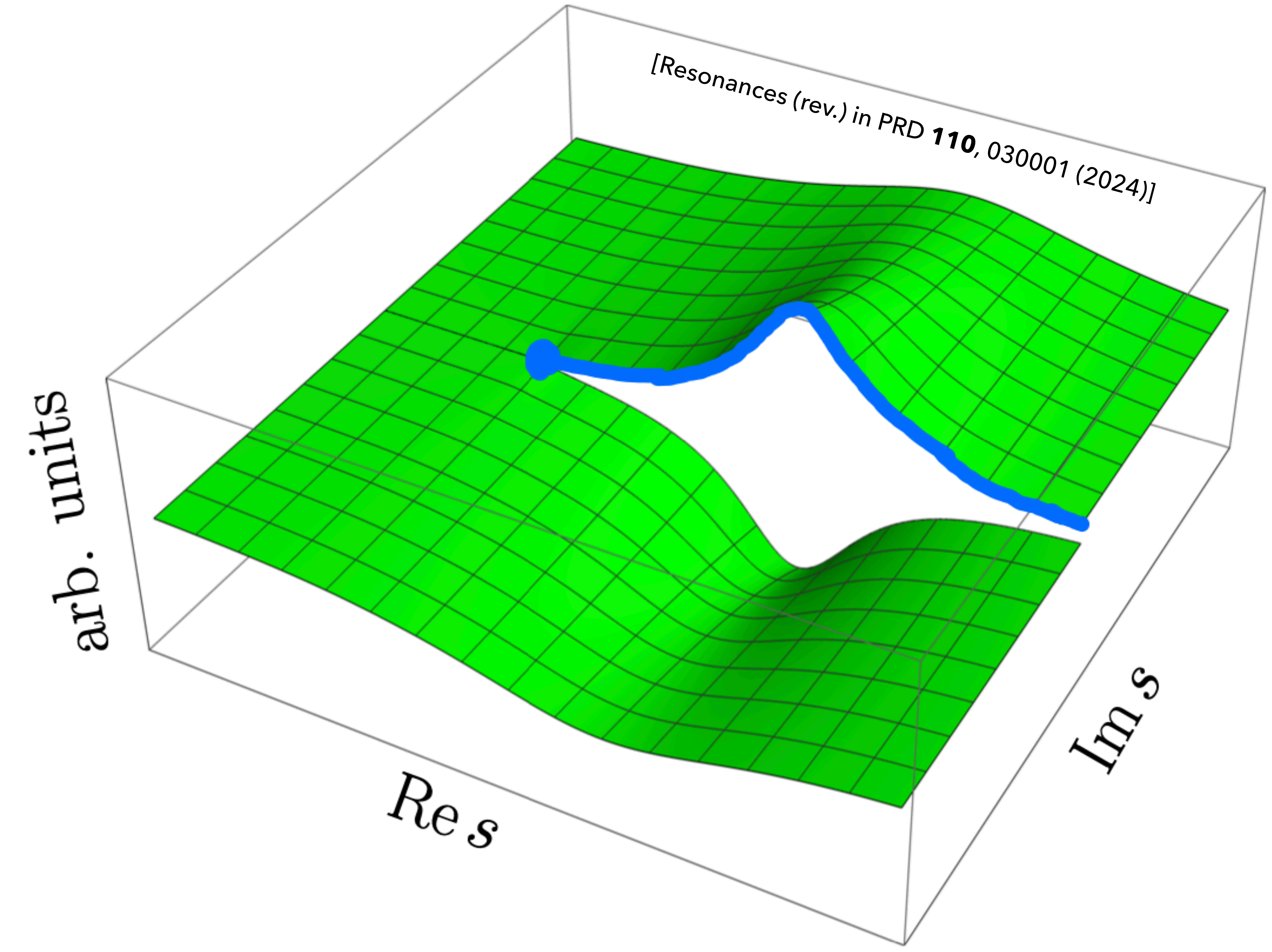
A two-particle threshold introduces a square-root singularity
NB Odd number of particles produce logarithmic singularity

In the complex *s-plane*, these two solutions are represented as separate layers (Riemann sheets/surfaces)

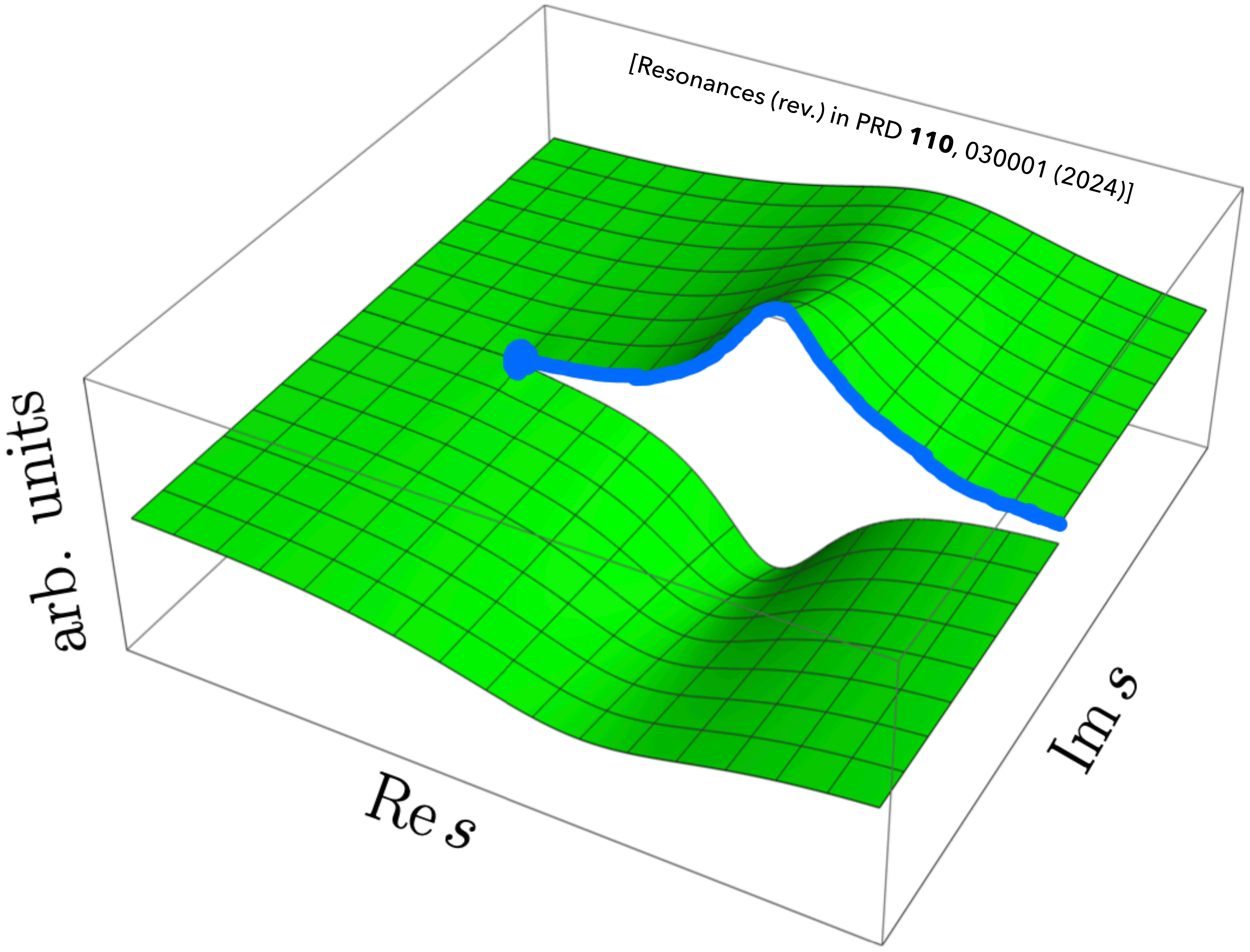
NB Logarithmic singularity means ∞ sheets

Branch points come with branch cuts from the s_{thr} to infinity (*right-hand cuts*)

s-plane directly accessible from the physical region is the so-called *first/physical sheet*



Resonances and how to treat them



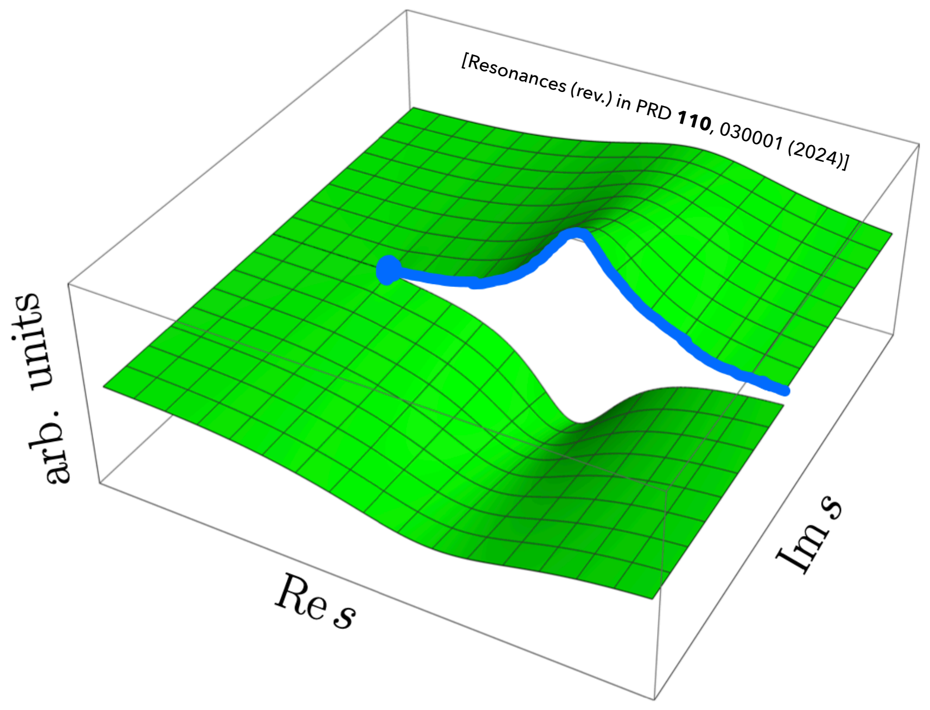
s-plane directly accessible from the physical region
is the so-called *first/physical sheet*

Resonances and how to treat them

s-plane directly accessible from the physical region is the so-called *first/physical sheet*

<u>Physical sheet is free of singularities</u>
(analicity ↔ causality) off the real axis

The unphysical sheets may contain resonance poles and branch points



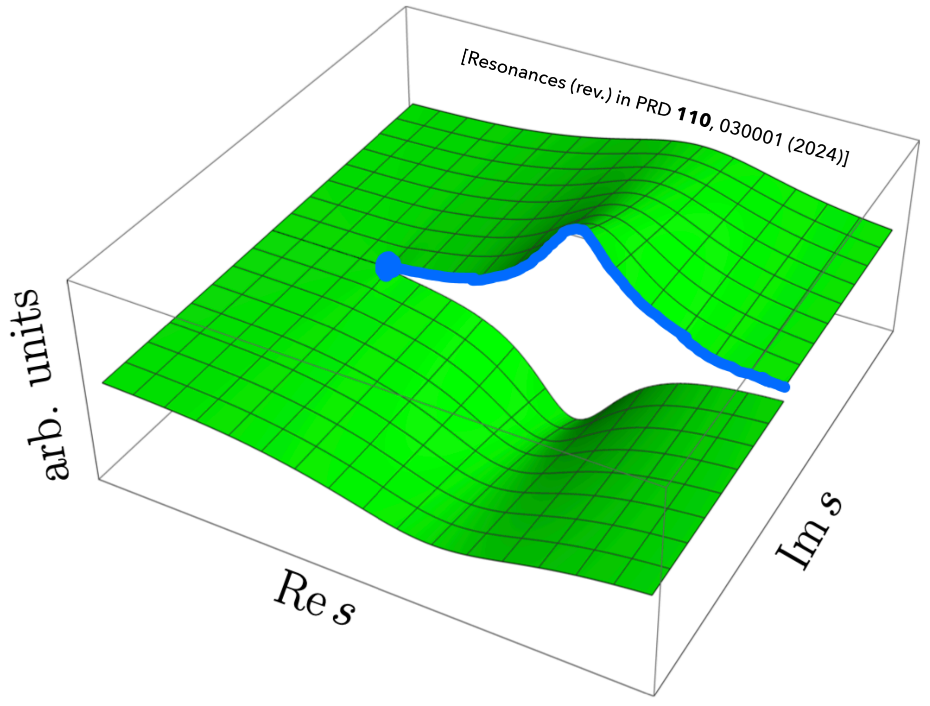
Resonances and how to treat them

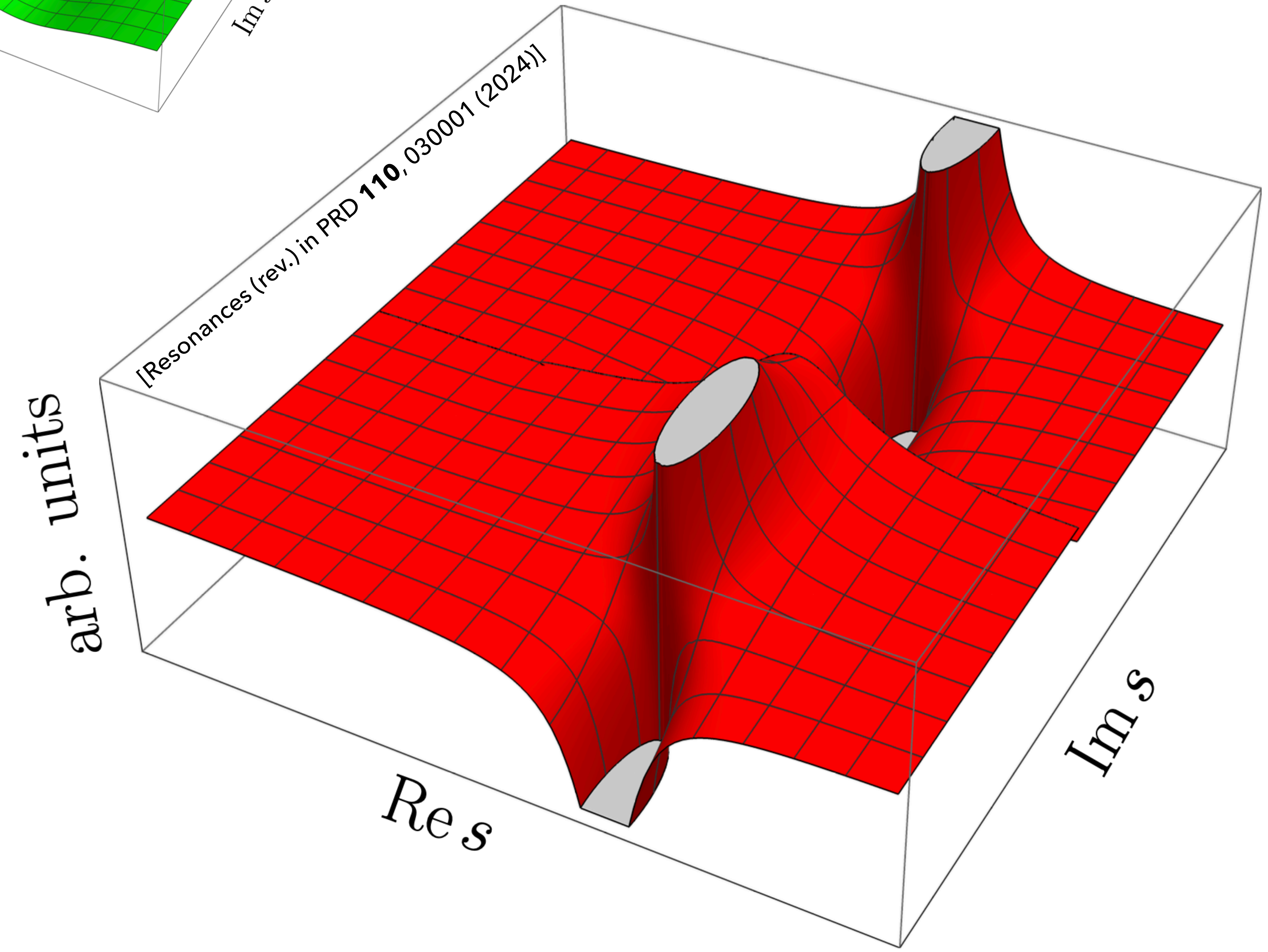
s-plane directly accessible from the physical region
is the so-called *first/physical sheet*

<u>Physical sheet is free of singularities</u>
(analicity ↔ causality) off the real axis

The unphysical sheets may contain resonance poles
and branch points

Resonance poles emerge inside the s-plane of
the unphysical sheets





Resonances and how to treat them

s-plane directly accessible from the physical region is the so-called *first/physical sheet*

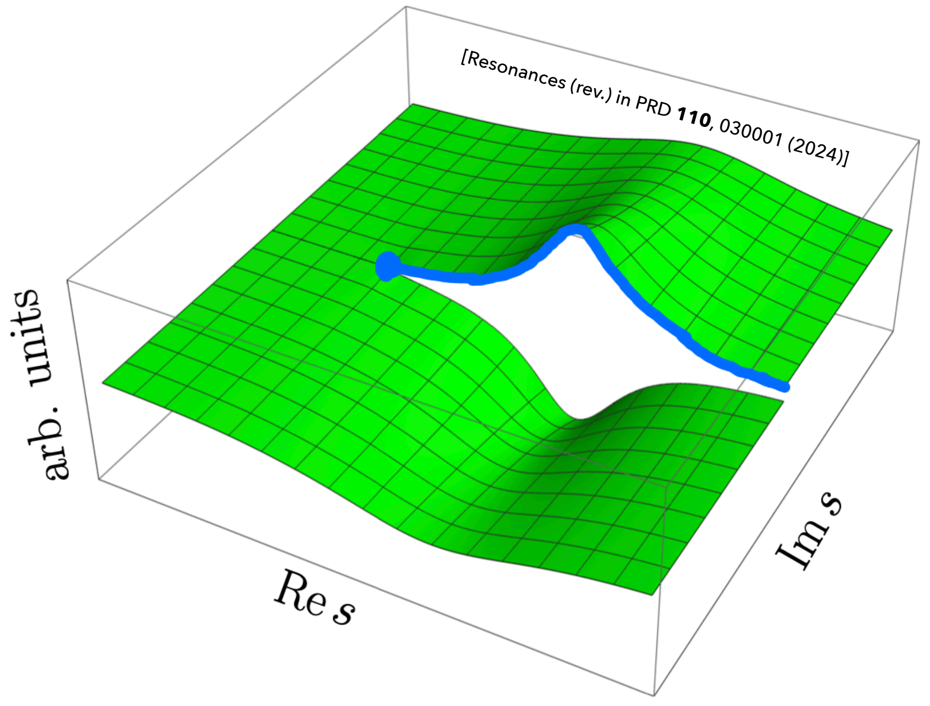
<ins>Physical sheet is free of singularities</ins>
(analicity ↔ causality) off the real axis

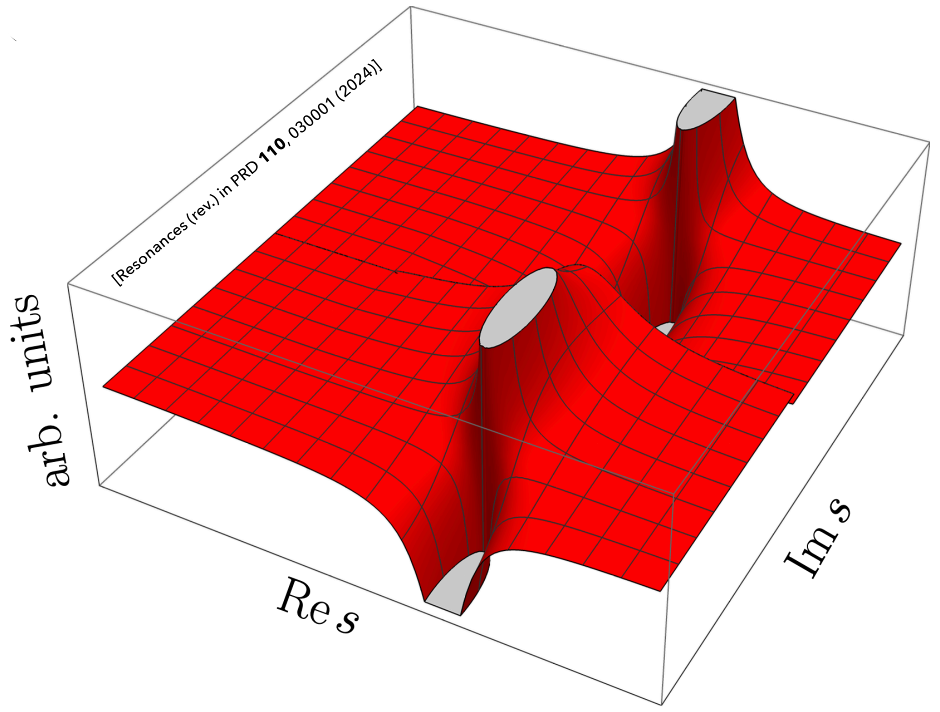
The unphysical sheets may contain resonance poles and branch points

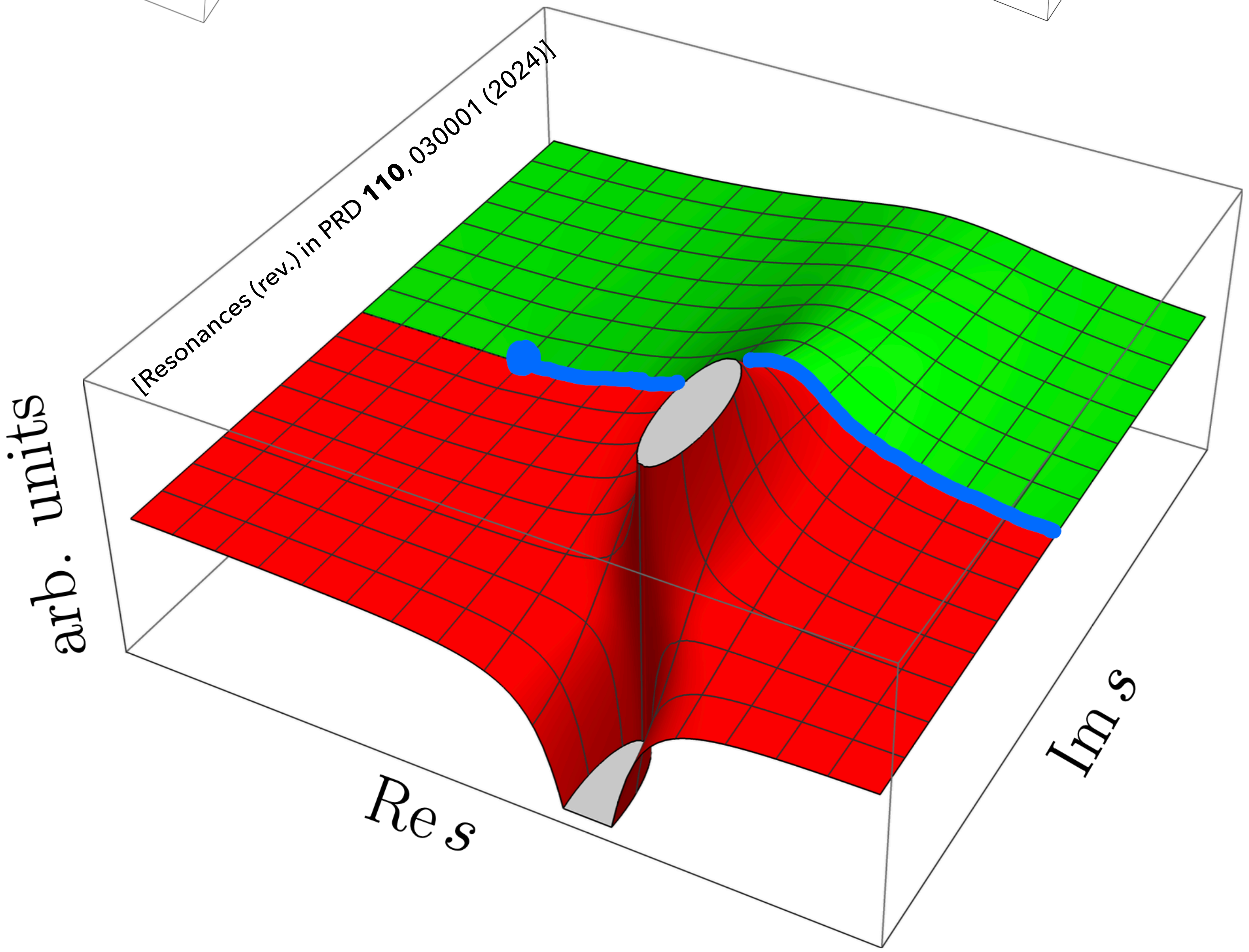
Resonance poles emerge inside the s-plane of the unphysical sheets

Poles that are on the unphysical sheet nearest to the physical region exert the most significant influence on experimental observables

NB Analyticity ↔ ∀ pole @s_p ⇒ a pole @s^*_p





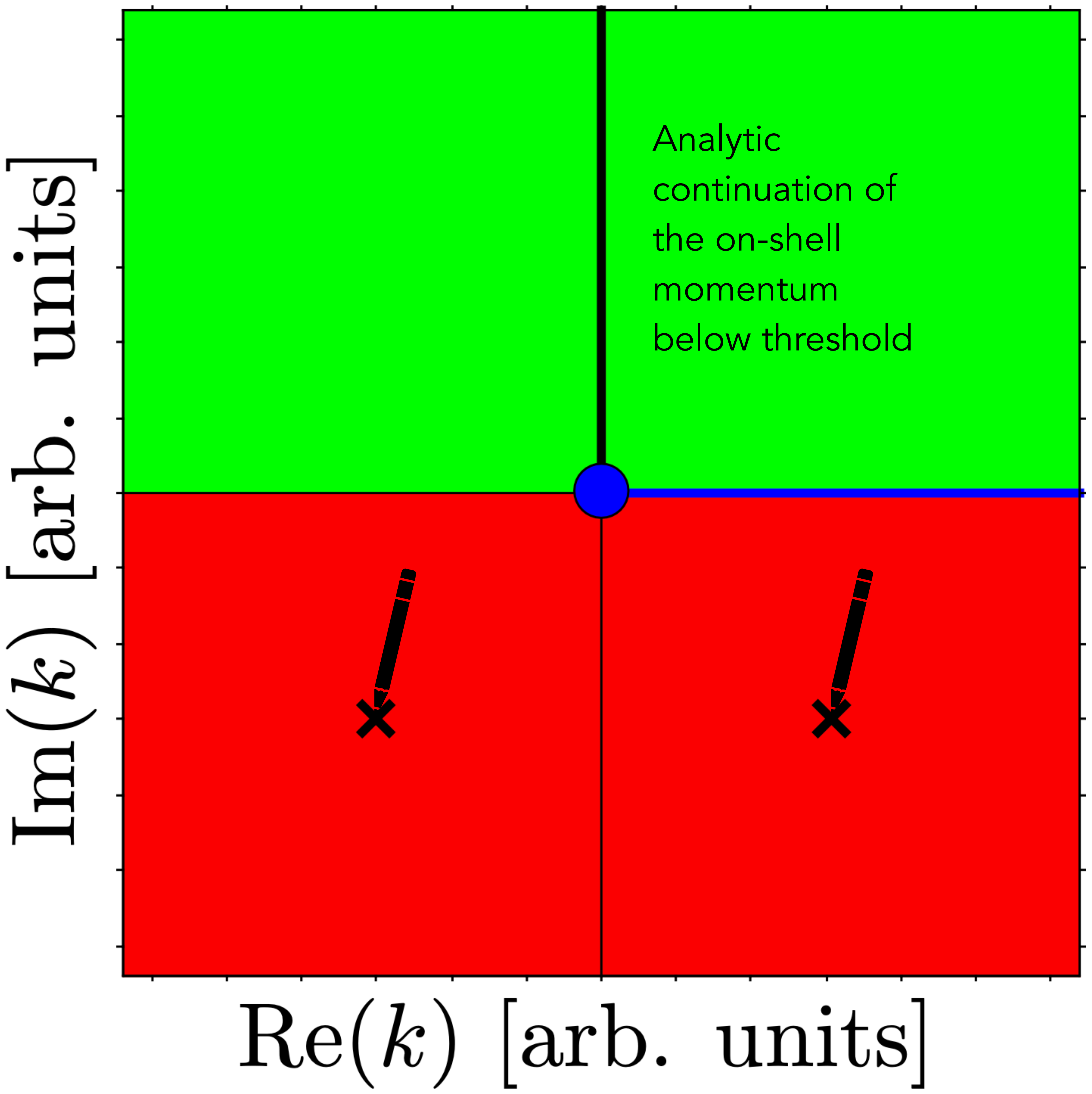


Resonances and how to treat them

Alternatively the k-plane (k being the relative momentum of the two particles in their $\sqrt{s}$ frame) can be used to picture resonances, poles, branch points, etc.

In this representation it's clearer that only one resonance pole drives the dynamics on the physical axis (at threshold, k = 0, both poles contribute)

[Resonances (rev.) in PRD **110**, 030001 (2024)]

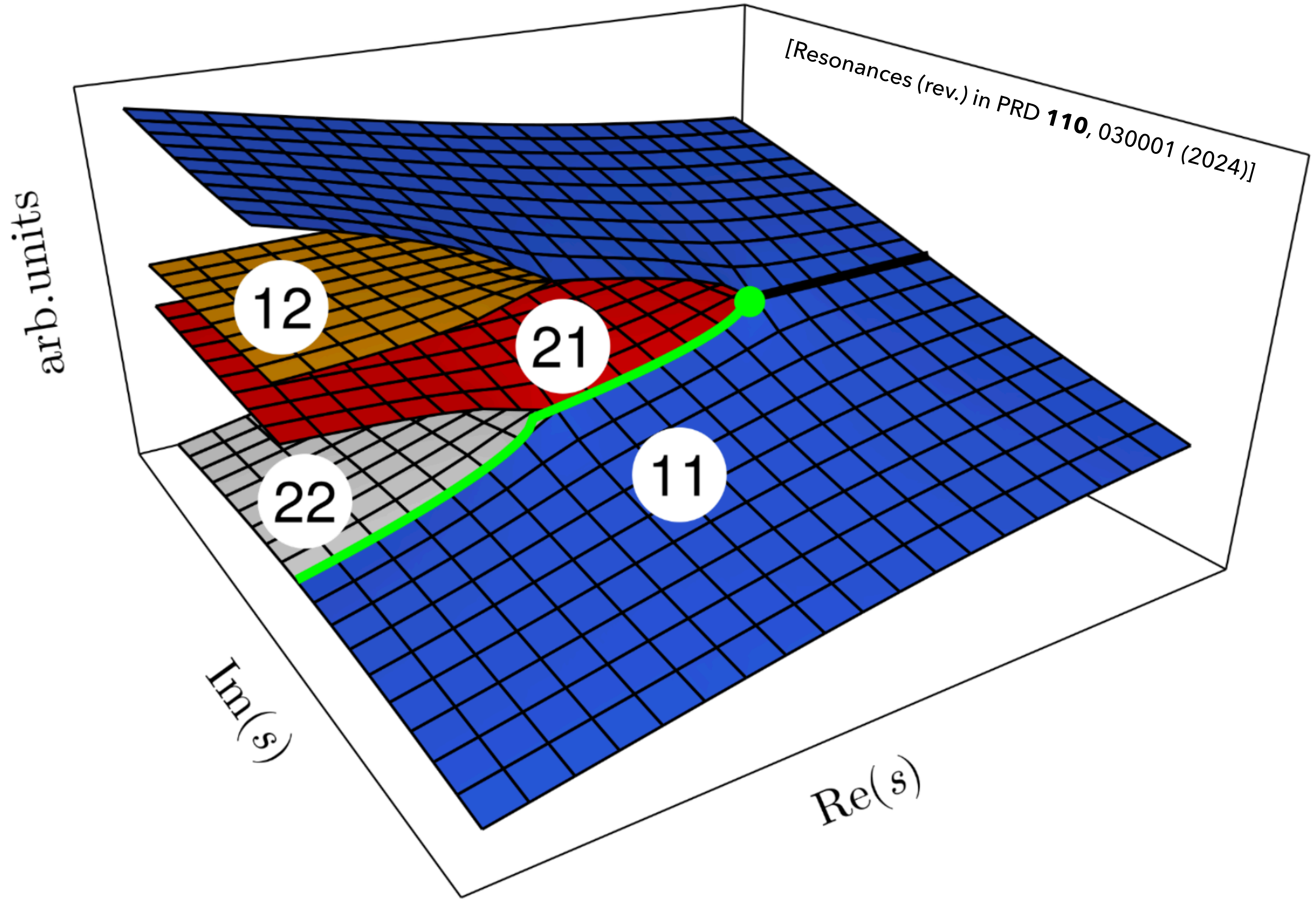


Resonances and how to treat them

What happens if the have two relevant channels…?

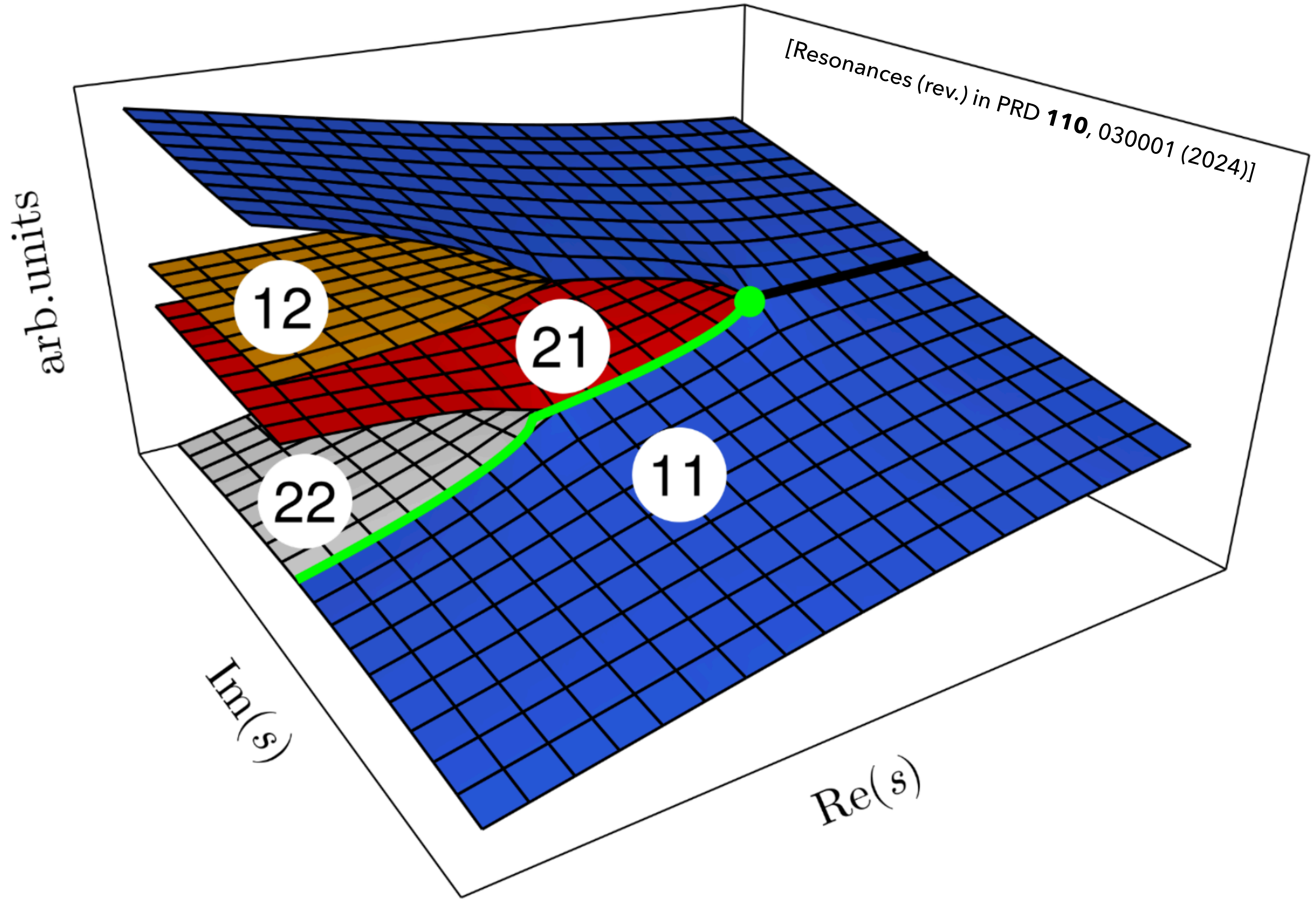
Resonances and how to treat them

What happens if the have two relevant channels…?



Resonances and how to treat them

What happens if the have two relevant channels…?

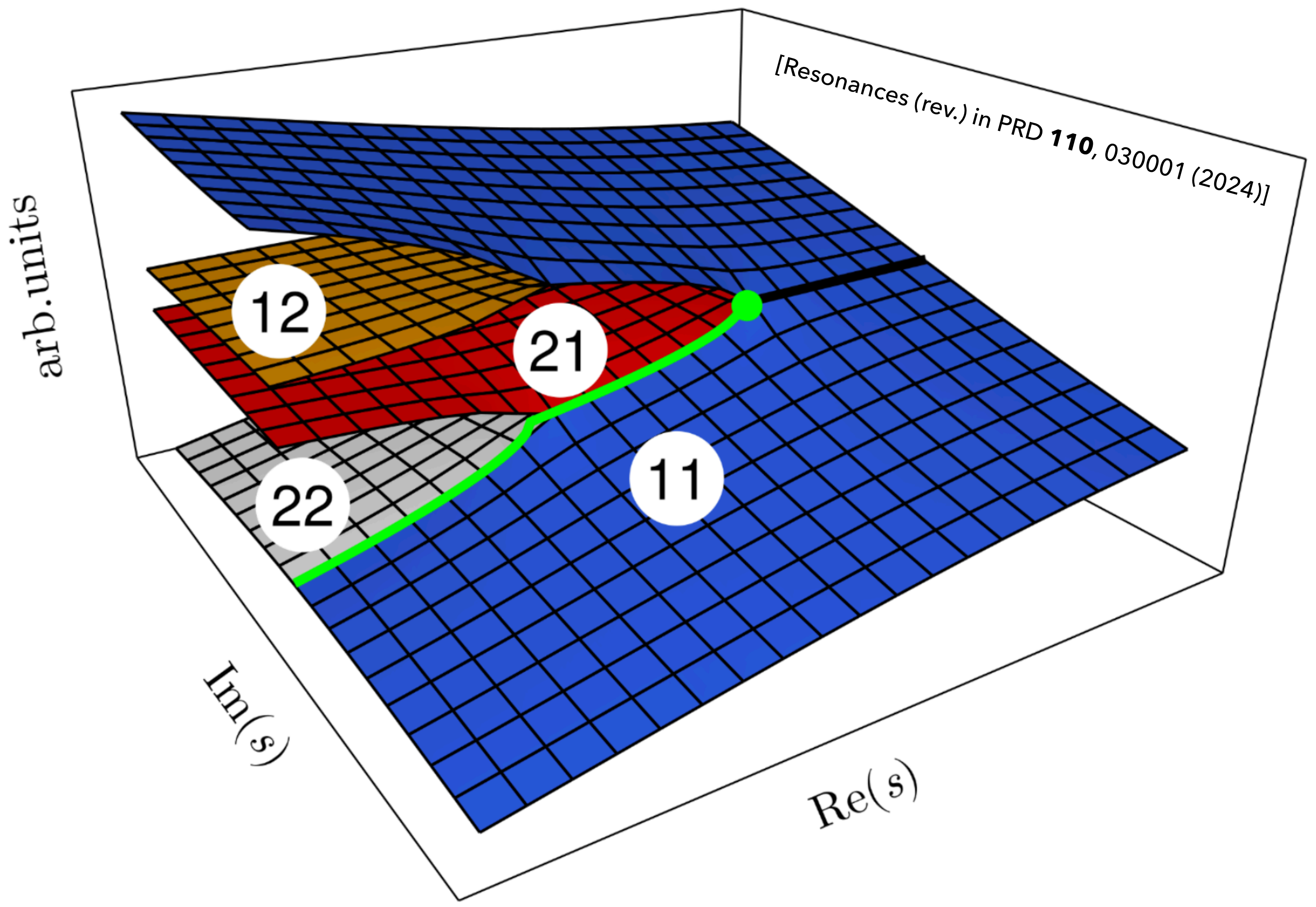


@[$s_{thr,\,11}$, $s_{thr,\,22}$] sheet-(21) is the one that smoothly connects to the physical sheet-(11), while @[$s_{thr,\,22}$, ∞] sheet-(22) assumes this role

Hence, any pole on sheet-(21) that lies above $s_{thr,\,22}$ will manifest in the data as a cusp

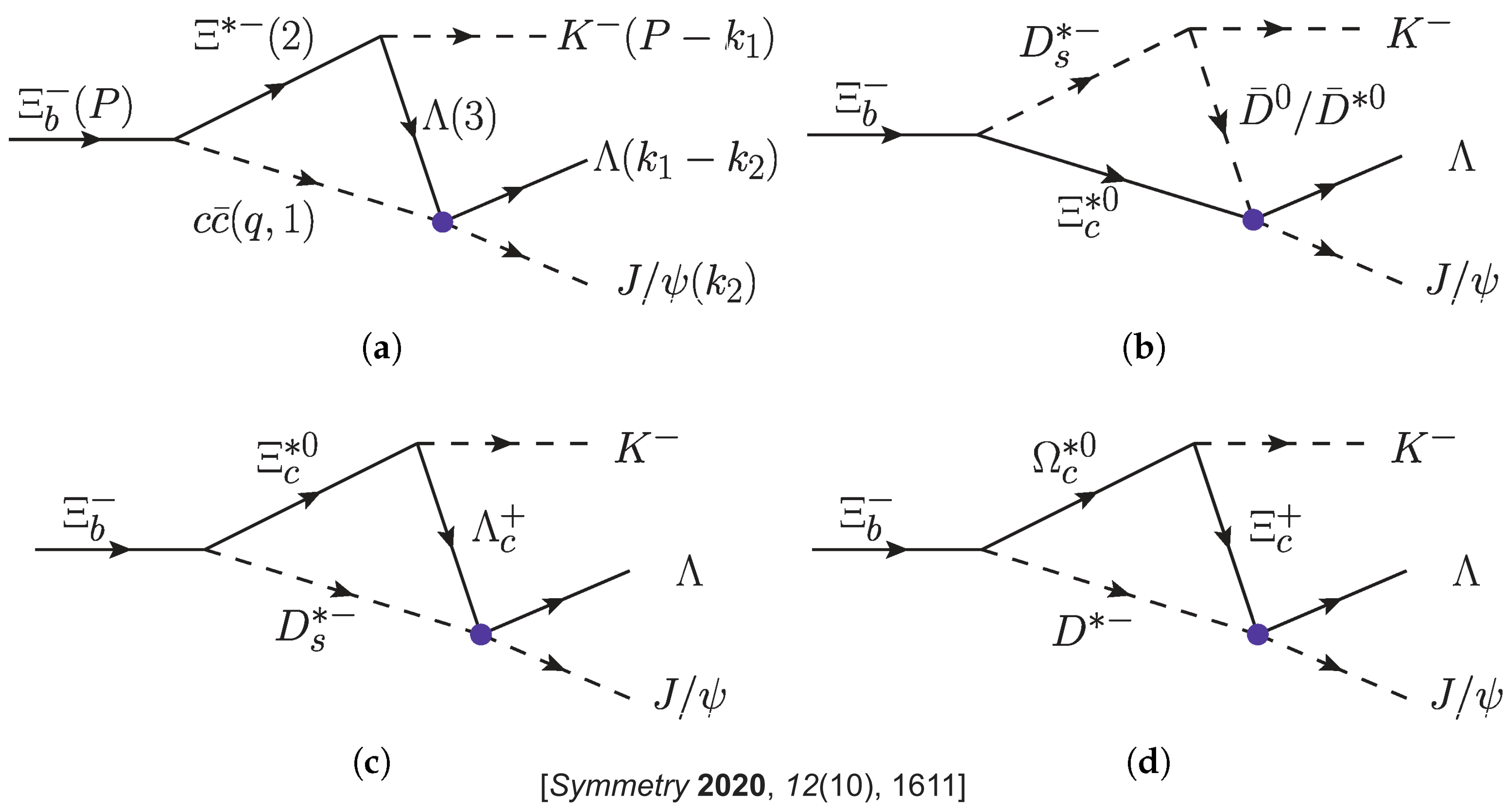
Sheet-(12) is remote for almost all energies

Resonances and how to treat them



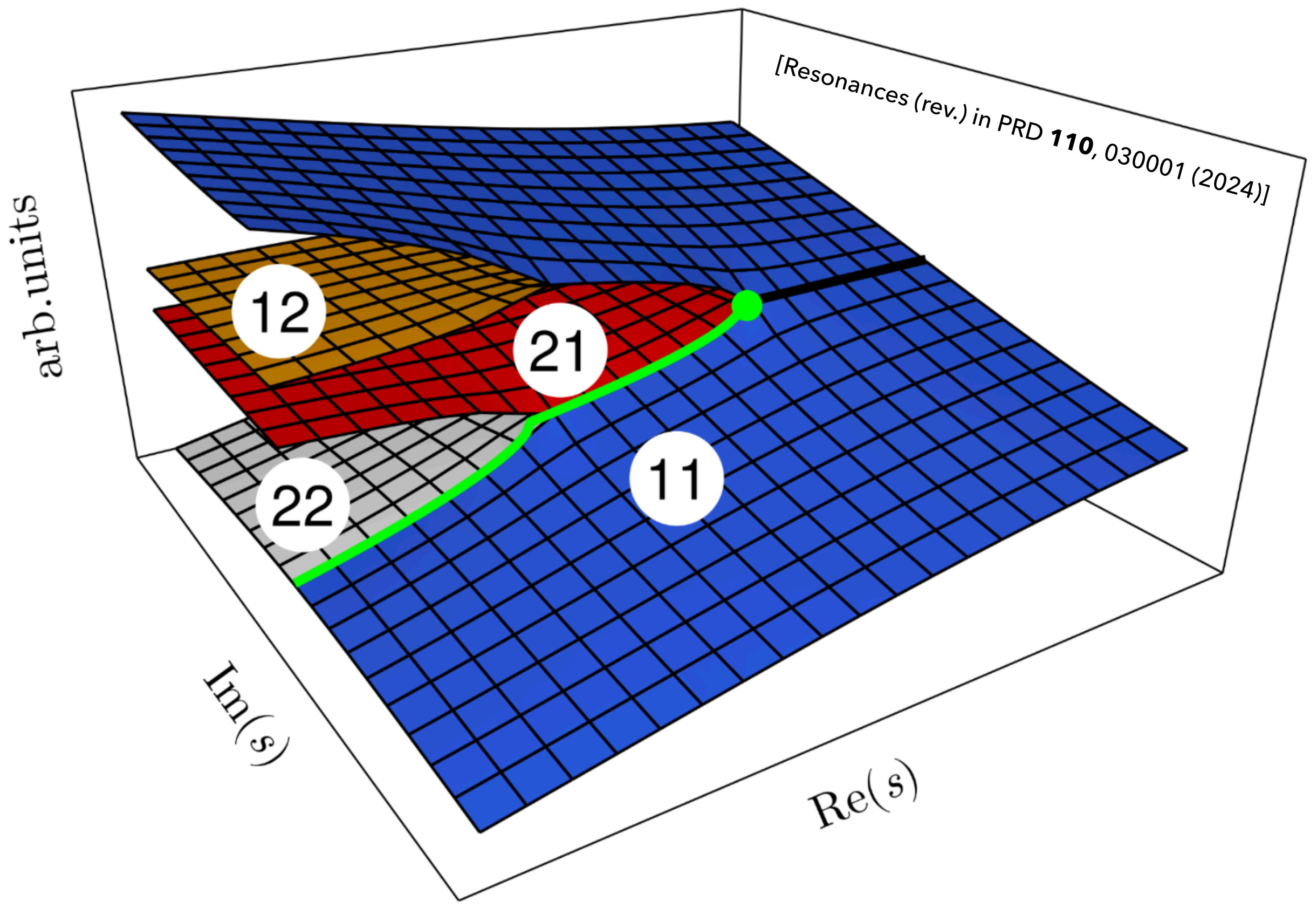
Singularities determine the visible structures in observables , so much so not every *bump* is indicative of a resonance
[Phys. Rev. **166**, 1719 (1968)]

Triangle Singularities, in particular, can either mimic resonance signals or significantly alter resonance signals



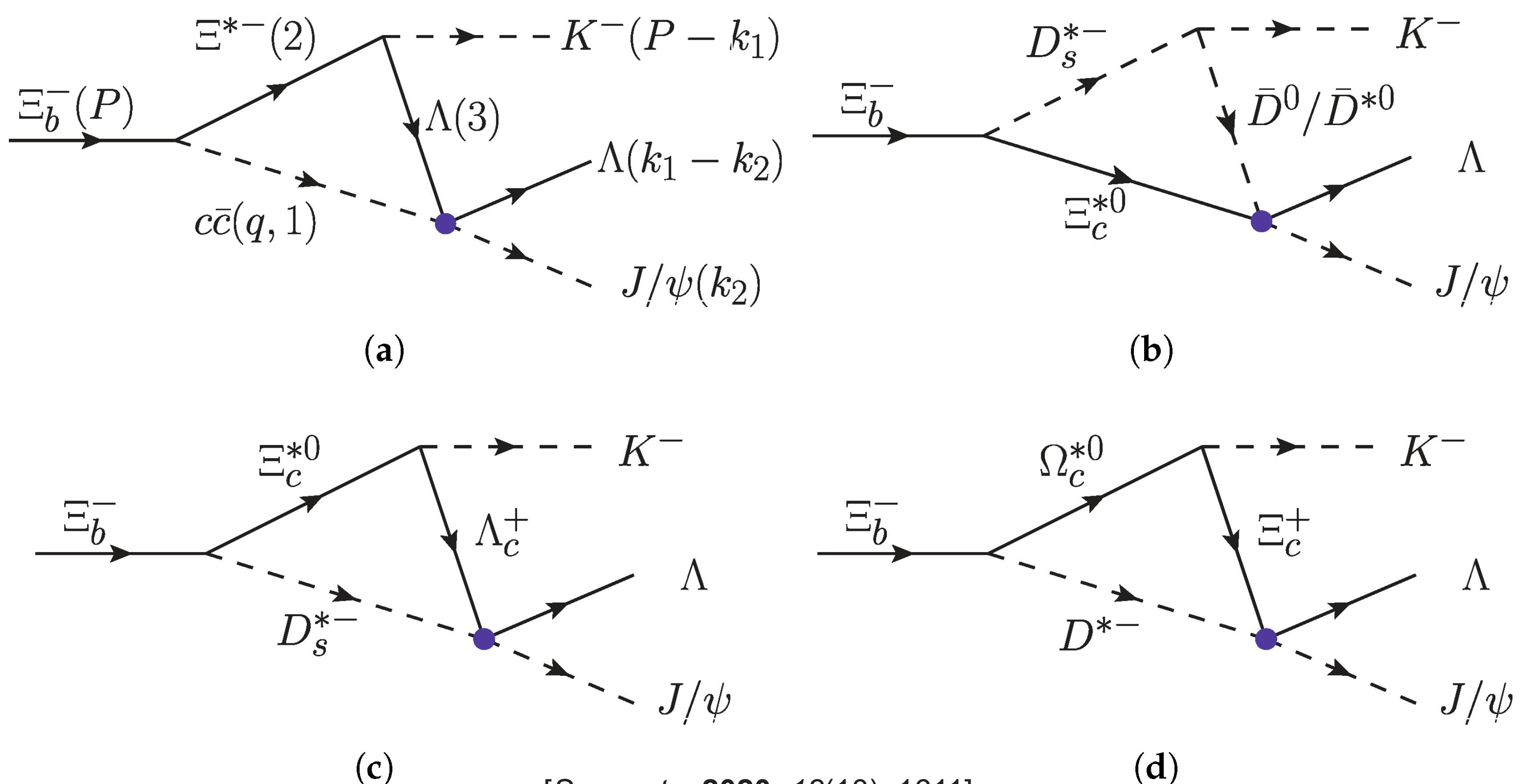
[*Symmetry* **2020**, *12*(10), 1611]

Resonances and how to treat them



Singularities determine the visible structures in observables , so much so not every *bump* is indicative of a resonance [Phys. Rev. **166**, 1719 (1968)]

Triangle Singularities, in particular, can either mimic resonance signals or significantly alter resonance signals



[*Symmetry* **2020**, *12*(10), 1611]

Also! Not all resonances produce a bump across all observables Roper resonance [Phys. Rev. Lett. **12** , 340 (1964)] or $f_0(500)$ hadron

Resonances and how to treat them

A resonance is defined by its *pole position* (s_R) and by the strength parameters of its couplings to decay channels evaluated at the pole (*pole residues*, $\mathscr{R}$)

$$\sqrt{s_{\mathrm{R}}} = M_{\mathrm{R}} - i\frac{\Gamma_{\mathrm{R}}}{2}$$

NB For states with the relevant thresholds are situated below the resonance location, the lifetime is $\tau_R = 1/\Gamma_R$

Resonances and how to treat them

A resonance is defined by its *pole position* (s_R) and by the strength parameters of its couplings to decay channels evaluated at the pole (*pole residues*, $\mathscr{R}$)

$$\sqrt{s_{\mathrm{R}}} = M_{\mathrm{R}} - i\frac{\Gamma_{\mathrm{R}}}{2}$$

NB For states with the relevant thresholds are situated below the resonance location, the lifetime is $\tau_R = 1/\Gamma_R$

$$\lim_{s \to s_{\mathrm{R}}} (s - s_{\mathrm{R}})\mathcal{M}_{ba} = -\mathcal{R}_{ba} \qquad \mathcal{R}_{ba} = -\frac{1}{2\pi i}\oint ds \mathcal{M}_{ba}$$

Pole couplings ⇒ $\tilde{g}_a = \mathcal{R}_{ba}/\sqrt{\mathcal{R}_{bb}}$

Transition strength of R to *a*-channel

Resonances and how to treat them

A resonance is defined by its *pole position* (s_R) and by the strength parameters of its couplings to decay channels evaluated at the pole (*pole residues*, $\mathscr{R}$)

$$\sqrt{s_{\mathrm{R}}} = M_{\mathrm{R}} - i\frac{\Gamma_{\mathrm{R}}}{2}$$

NB For states with the relevant thresholds are situated below the resonance location, the lifetime is $\tau_R = 1/\Gamma_R$

$$\lim_{s \to s_{\mathrm{R}}} (s - s_{\mathrm{R}})\mathcal{M}_{ba} = -\mathcal{R}_{ba} \qquad \mathcal{R}_{ba} = -\frac{1}{2\pi i}\oint ds\mathcal{M}_{ba}$$

Pole couplings ⇒ $\tilde{g}_a = \mathcal{R}_{ba}/\sqrt{\mathcal{R}_{bb}}$

Transition strength of R to *a*-channel

Typical branching fraction ⇒ $\mathcal{B}_a = \dfrac{N_a}{N_{Tot}}$

Resonances and how to treat them

A resonance is defined by its *pole position* (s_R) and by the strength parameters of its couplings to decay channels evaluated at the pole (*pole residues*, $\mathscr{R}$)

$$\sqrt{s_{\mathrm{R}}} = M_{\mathrm{R}} - i\frac{\Gamma_{\mathrm{R}}}{2}$$

NB For states with the relevant thresholds are situated below the resonance location, the lifetime is $\tau_R = 1/\Gamma_R$

$$\lim_{s \to s_{\mathrm{R}}} (s - s_{\mathrm{R}})\mathcal{M}_{ba} = -\mathcal{R}_{ba} \qquad \mathcal{R}_{ba} = -\frac{1}{2\pi i}\oint ds\mathcal{M}_{ba}$$

Pole couplings ⇒ $\tilde{g}_a = \mathcal{R}_{ba}/\sqrt{\mathcal{R}_{bb}}$

Transition strength of R to *a*-channel

Typical branching fraction ⇒ $\mathcal{B}_a = \dfrac{N_a}{N_{Tot}}$

For broad (overlapping) resonances (*e.g.*, $f_0(500)$) ⇒ $\mathcal{B}_a = \dfrac{|\tilde{g}_a|}{M_{\mathrm{R}}\Gamma_{\mathrm{R}}}\rho_a(M_{\mathrm{R}}^2)$

$$\rho_c(s) = \frac{(2\pi)^4}{2}\int \mathrm{d}\Phi_2 = \frac{1}{16\pi}\frac{2|\vec{q}_c|}{\sqrt{s}}$$

q_a break-up momentum

$$q_a = \frac{\lambda^{1/2}(s, m_{1,a}^2, m_{2,a}^2)}{2\sqrt{s}}$$

$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$
is the Källén function

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

$$\mathcal{N}_a(s) = \alpha\, g_a\, n_a(s)$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s)$$

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

$$\mathcal{N}_a(s) = \alpha\, g_a\, n_a(s)$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s)$$

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)$$

$1/q_0 = [1, 5]\ \mathrm{GeV}^{-1}$

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

$$\mathcal{N}_a(s) = \alpha\, g_a\, n_a(s)$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s)$$

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)$$

$1/q_0 = [1, 5]\ \mathrm{GeV}^{-1}$

$F_{la}(q_a, q_0)$ is a phenomenological form factor (typically the Blatt-Weisskopf ones)
[Theoretical nuclear physics]

⇒

Presence of these $F_{la}(q_a, q_0)$ is a requirement from positivity, which demands that the dressed propagator is not allowed to drop faster than 1/s

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

$$\mathcal{N}_a(s) = \alpha\, g_a\, n_a(s)$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s)$$

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)$$

$1/q_0 = [1, 5]\ \mathrm{GeV}^{-1}$

$F_{la}(q_a, q_0)$ is a phenomenological form factor
(typically the Blatt-Weisskopf ones)
[Theoretical nuclear physics]

CAVEAT

(i) Decay channel's threshold must be below M_{BW}, its evaluation above requires analytic continuation. Flatté parameterisation refers to the S-wave channels amplitude near a threshold of a heavier channel

(ii) The parameters differ from the pole ones (they align if the Γ_{BW} is small)

(iii) It is an accurate representation of resonance phenomena in $\Gamma/\Delta \to 0$ (Δ being the distance to the closest unaccounted singularity)

(iv) If, in the same partial wave, there is another resonance that significantly couples to the same channel, it is inappropriate. Breaching unitarity and adding bias to the inferred resonance

Resonances and how to treat them

Relativistic Breit–Wigner parameterisation (a dressed propagator for an isolated resonance)

$$\mathcal{A}_a(s) = \frac{\mathcal{N}_a(s)}{M_{\mathrm{BW}}^2 - s - iM_{\mathrm{BW}}\Gamma(s)}$$

$\Gamma_{BW} = \Gamma(M^2_{BW})$
being the Breit–Wigner width

$$\mathcal{N}_a(s) = \alpha\, g_a\, n_a(s)$$

$$\Gamma(s) = \frac{1}{M_{\mathrm{BW}}} \sum_b g_b^2 \rho_b(s) n_b^2(s)$$

$$n_a = (q_a/q_0)^{l_a} F_{l_a}(q_a/q_0)$$

$1/q_0 = [1, 5]\ \mathrm{GeV}^{-1}$

$F_{la}(q_a, q_0)$ is a phenomenological form factor
(typically the Blatt-Weisskopf ones)
[Theoretical nuclear physics]

CAVEAT

(i) Decay channel's threshold must be below M_{BW}, its evaluation above requires analytic continuation. Flatté parameterisation refers to the S-wave channels amplitude near a threshold of a heavier channel

(ii) The parameters differ from the pole ones
(they align if the Γ_{BW} is small)

(iii)

Situation more complex that this…

(iv) If, … cou…

The ρ-meson pole is well described by $\mathscr{BW}$…
because the ω-pole isospin breaking effect is insignificant

Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

$$n_b \, \mathcal{M}^{-1}_{ba} \, n_a = \mathcal{K}^{-1}_{ba} - i\delta_{ba}\rho_a n_a^2 \quad \leftrightarrow \quad \mathcal{M} = n[1 - \mathcal{K}\, i\rho\, n^2]^{-1}\mathcal{K}\, n$$

K_{ba} represents a real function (such that $\mathscr{M}_{ba}$ remains unitary) and is subject to modelling

Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

$$n_b\,\mathcal{M}_{ba}^{-1}\,n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2 \quad \leftrightarrow \quad \mathcal{M} = n[1-\mathcal{K}\,i\rho\,n^2]^{-1}\mathcal{K}\,n$$

K_{ba} represents a real function (such that $\mathscr{M}_{ba}$ remains unitary)
and is subject to modelling

$$\mathcal{K}_{ba}(s) = \sum_{\mathrm{R}} \frac{g_b^{\mathrm{R}} g_a^{\mathrm{R}}}{m_{\mathrm{R}}^2 - s} + b_{ba}$$

m_R and g_{Ra} the bare mass and couplings
b_{ba} parameterises the non-pole components

Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

$$n_b\,\mathcal{M}_{ba}^{-1}\,n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2 \quad \leftrightarrow \quad \mathcal{M} = n[1-\mathcal{K}\,i\rho\,n^2]^{-1}\mathcal{K}\,n$$

K_{ba} represents a real function (such that $\mathscr{M}_{ba}$ remains unitary) and is subject to modelling

$$\mathcal{K}_{ba}(s) = \sum_{\mathrm{R}} \frac{g_b^{\mathrm{R}} g_a^{\mathrm{R}}}{m_{\mathrm{R}}^2 - s} + b_{ba}$$

m_R and g_{Ra} the bare mass and couplings
b_{ba} parameterises the non-pole components

Amplitudes exclude the left-hand cuts, but can be incorporated into the b_{ba}

Same functional structure can be used to parameterise the inverse K-matrix (the M-matrix [Phys. Rev. D **35**, 1633 (1987)])

The position of the resonance poles can be determined by examining the zeros of *det[1−Kiρn²]*

The function has a complex multi-sheet structure, but, again, the nearest unphysical sheet has the highest influence

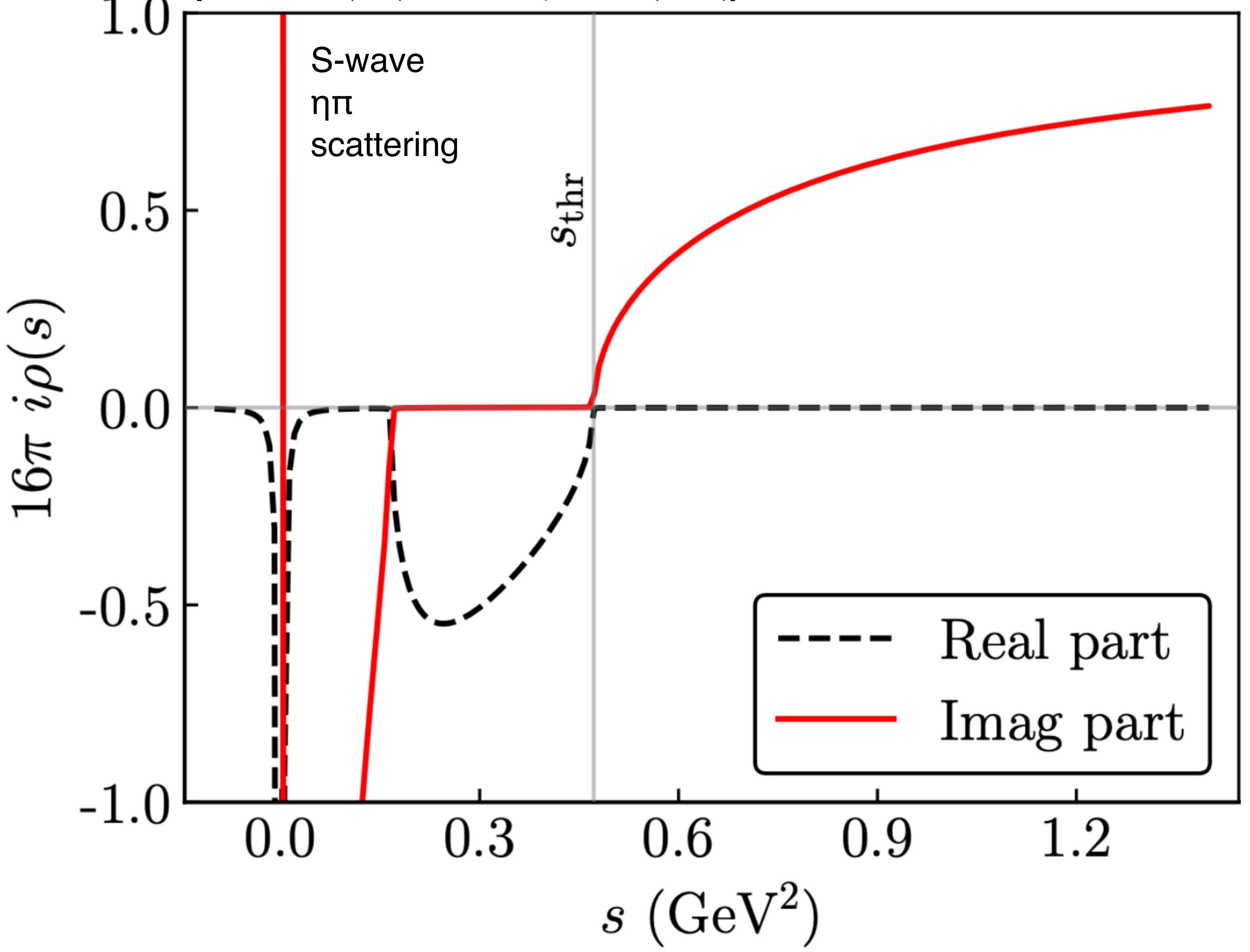
Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

$$n_b \mathcal{M}^{-1}_{ba} n_a = \mathcal{K}^{-1}_{ba} - i\delta_{ba}\rho_a n_a^2$$

A method to improve the analytic properties consists of $i\rho_a(s)n_a^2 \leftrightarrow \Sigma_a(s)$ (the Chew-Mandelstam function)

[Resonances (rev.) in PRD **110**, 030001 (2024)]



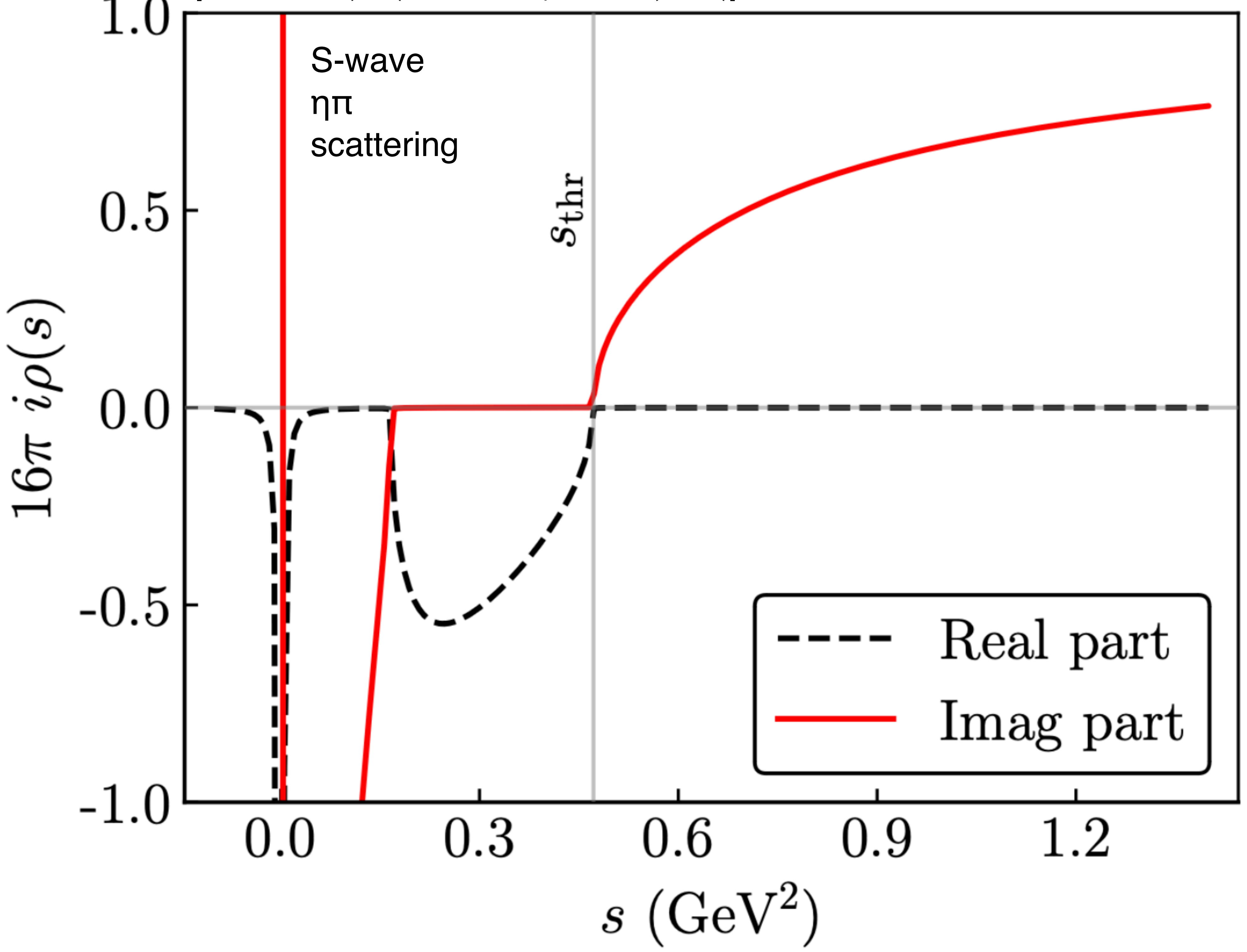
Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

$$n_b\,\mathcal{M}_{ba}^{-1}\,n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2$$

A method to improve the analytic properties consists of $i\rho_a(s)n_a^2 \leftrightarrow \Sigma_a(s)$ (the Chew-Mandelstam function)

[Resonances (rev.) in PRD **110**, 030001 (2024)]



Resonances and how to treat them

To ensures two-particle unitarity: K-matrix

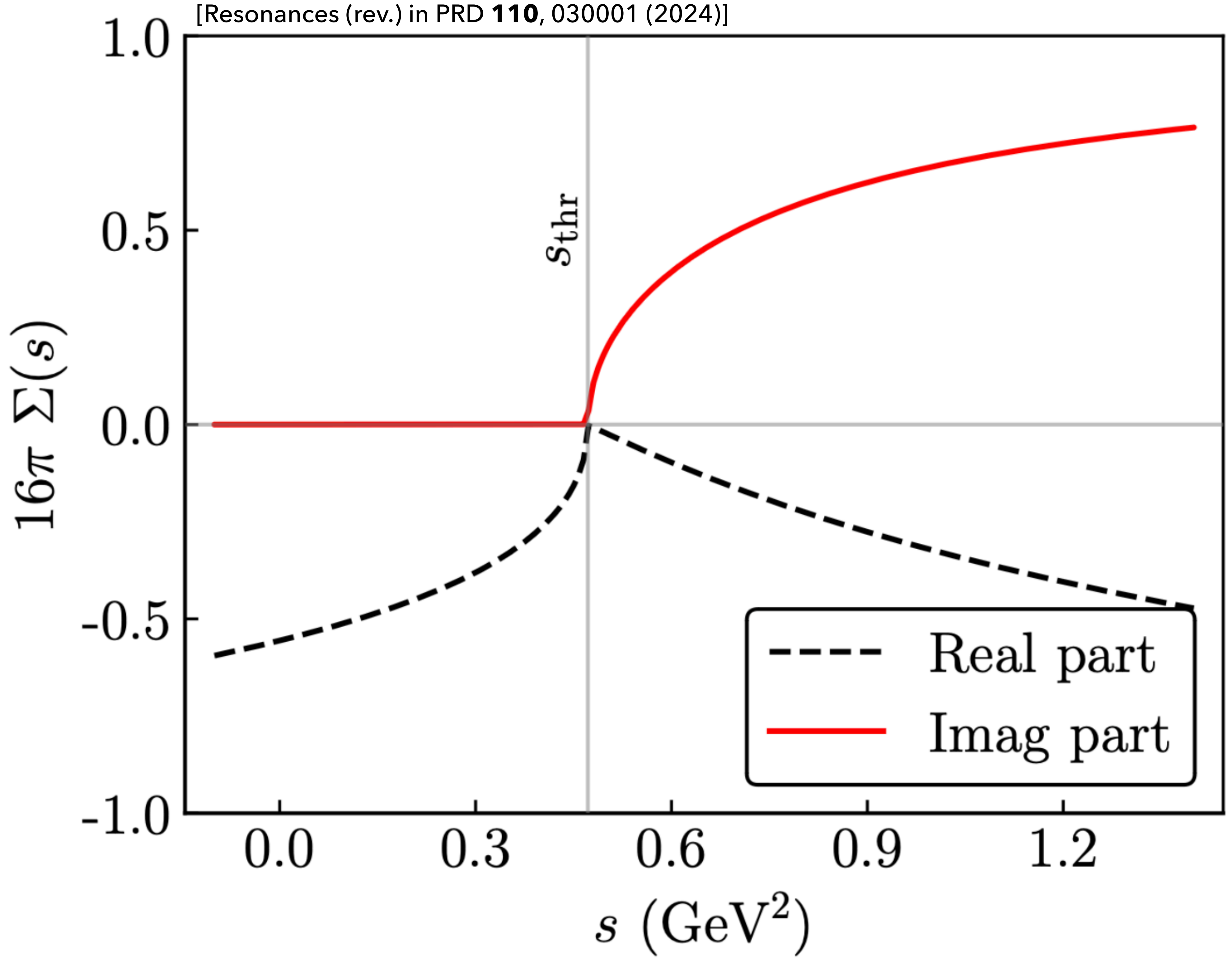
$$n_b\,\mathcal{M}^{-1}_{ba}\,n_a = \mathcal{K}^{-1}_{ba} - i\delta_{ba}\rho_a n_a^2$$

A method to improve the analytic properties consists of
$i\rho_a(s)n_a^2 \leftrightarrow \Sigma_a(s)$ (the Chew-Mandelstam function [Phys. Rev. D **16** (1977) 657])

$$\Sigma_a(s+i0) = \frac{s - s_{\mathrm{thr}_a}}{\pi}\int_{s_{\mathrm{thr}_a}}^{\infty} \frac{\rho_a(s')n_a^2(s')}{(s' - s_{\mathrm{thr}_a})(s' - s - i0)}\,\mathrm{d}s'$$

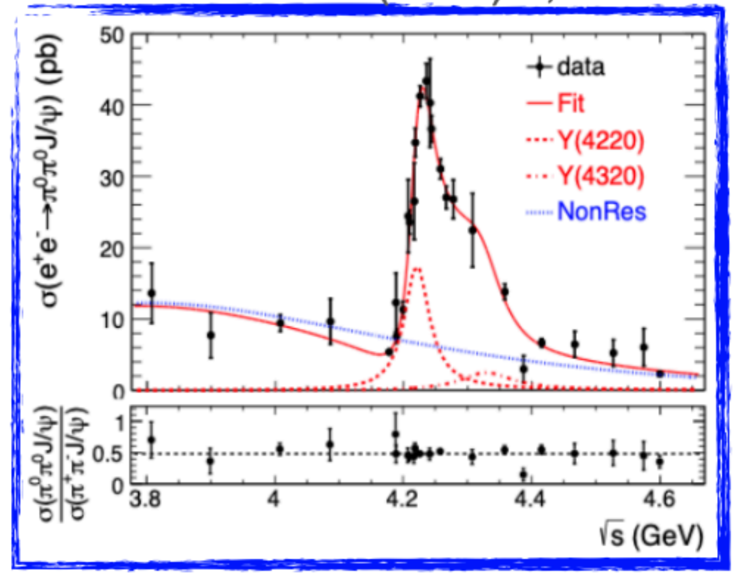
produces the imaginary part on the right-hand cut, maintaining analyticity below threshold

[Resonances (rev.) in PRD **110**, 030001 (2024)]

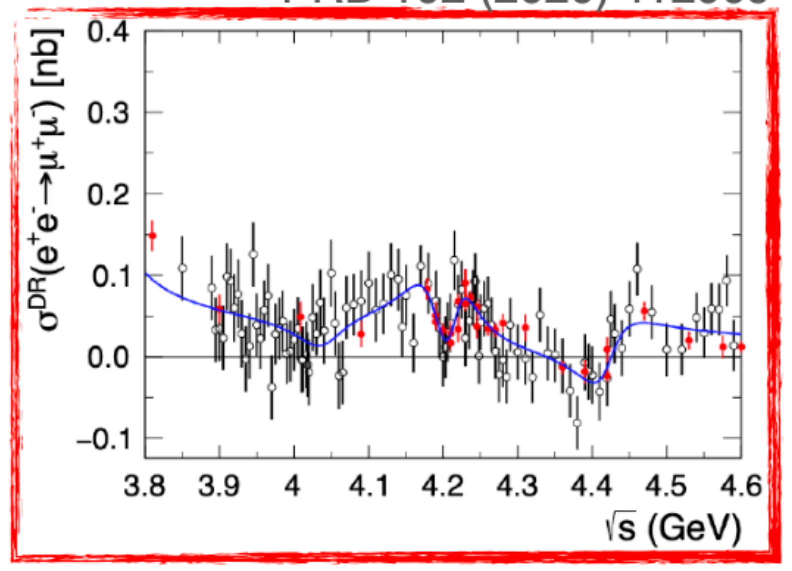


Coupled-channel analysis of vector charmonia at BESIII

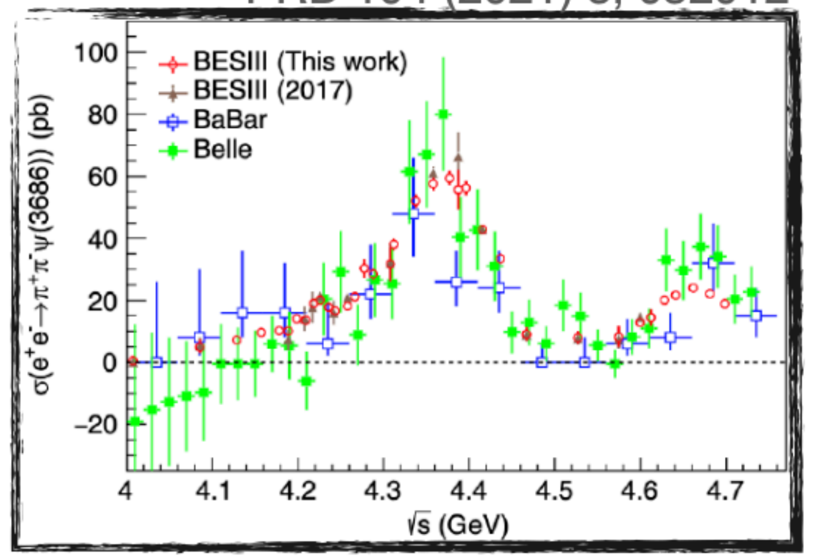
PRD 102 (2020) 1, 012009

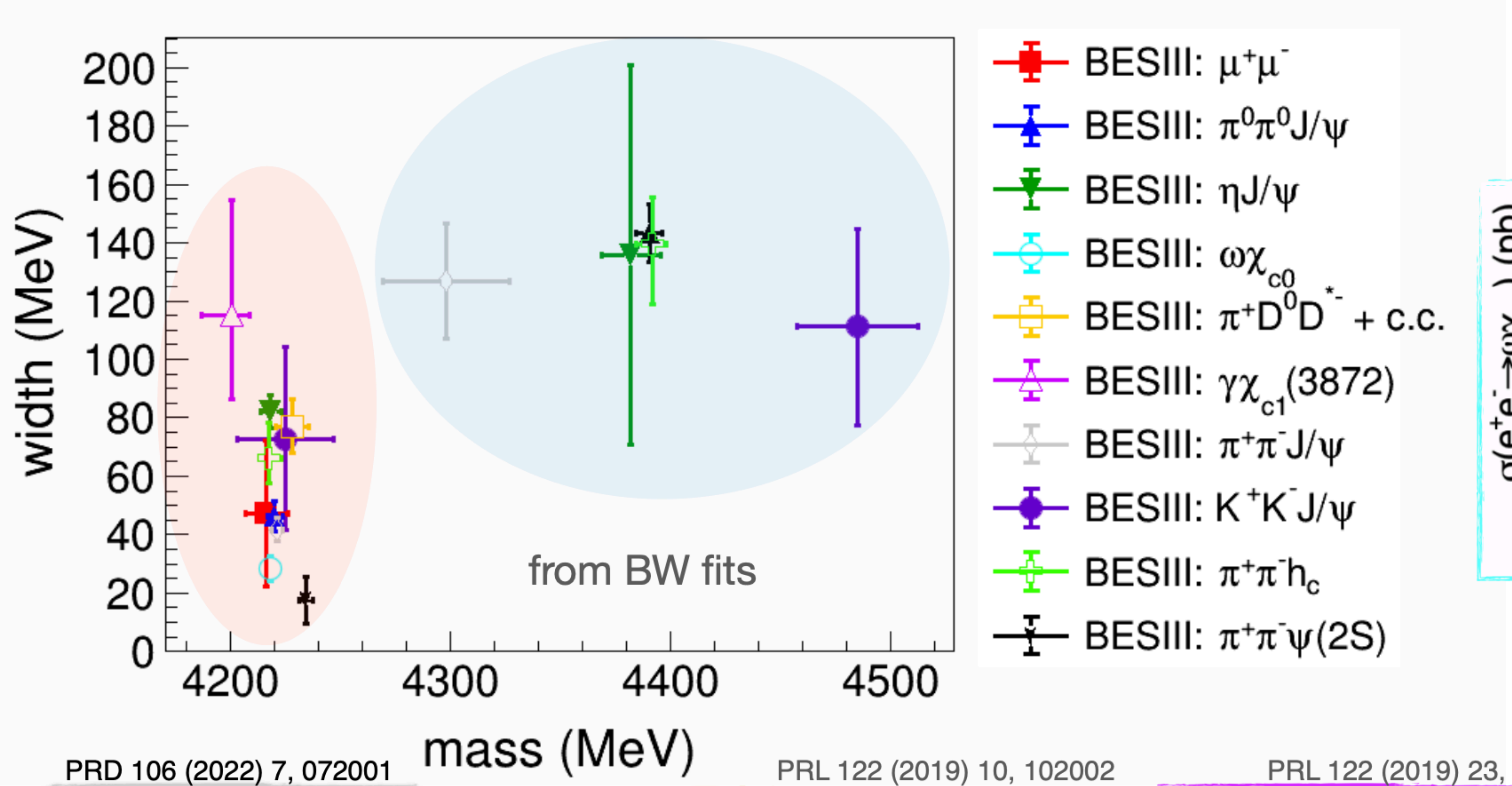


PRD 102 (2020) 112009

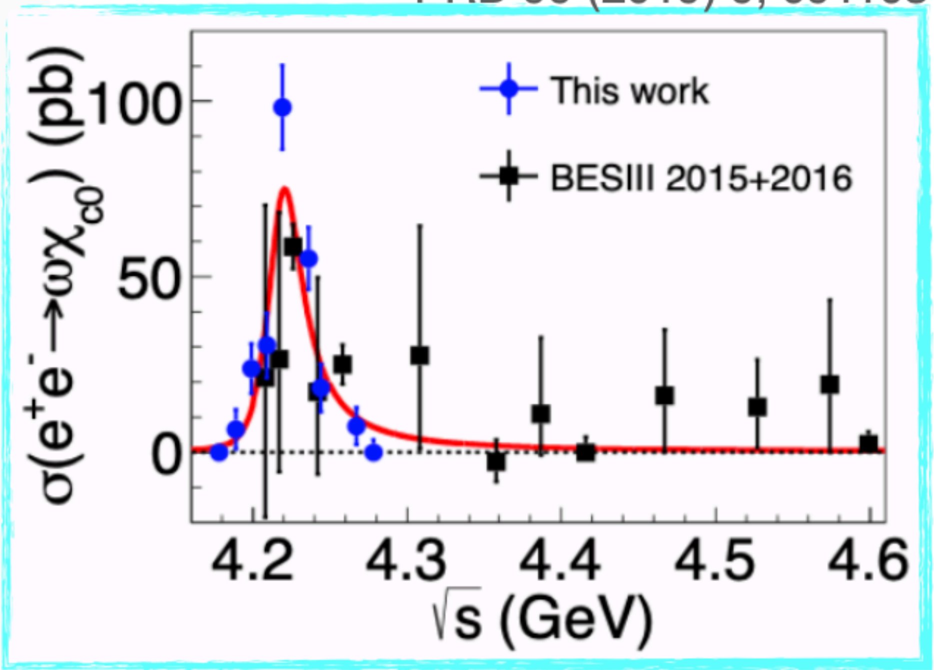


PRD 104 (2021) 5, 052012

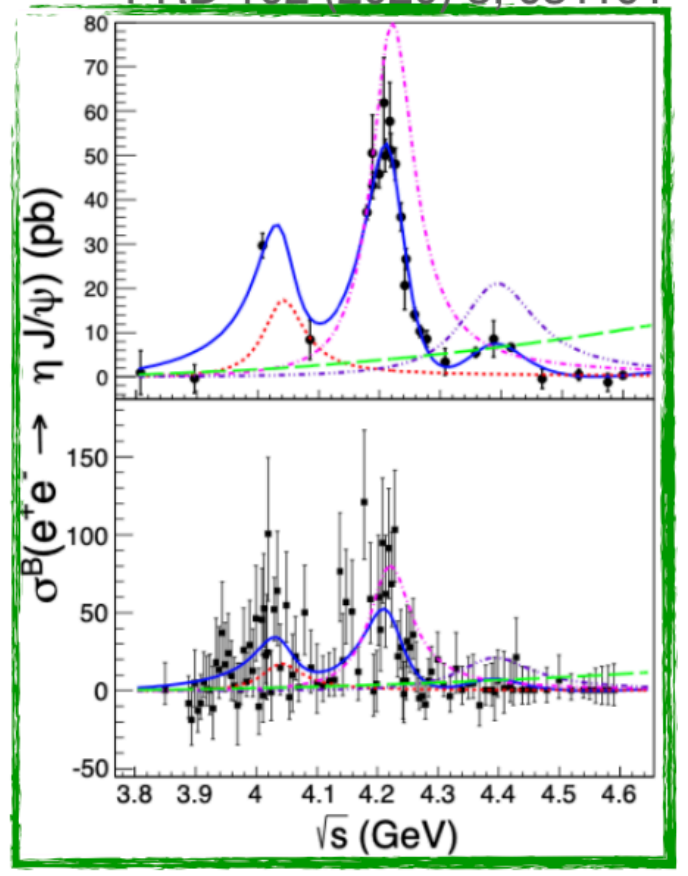




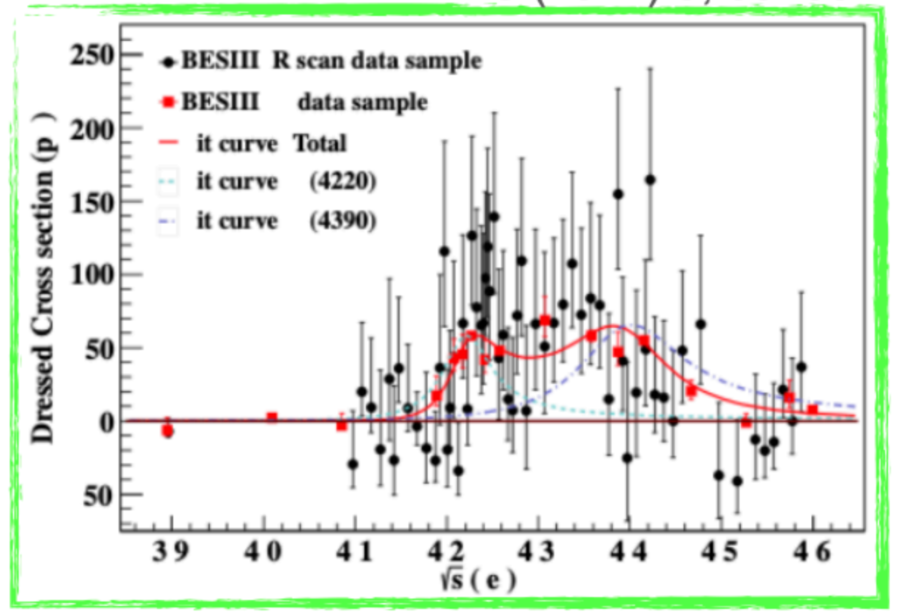
PRD 99 (2019) 9, 091103



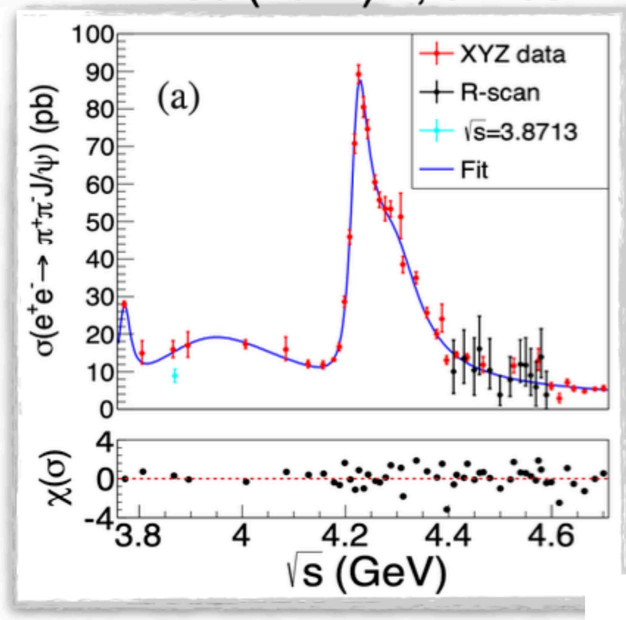
PRD 102 (2020) 3, 031101



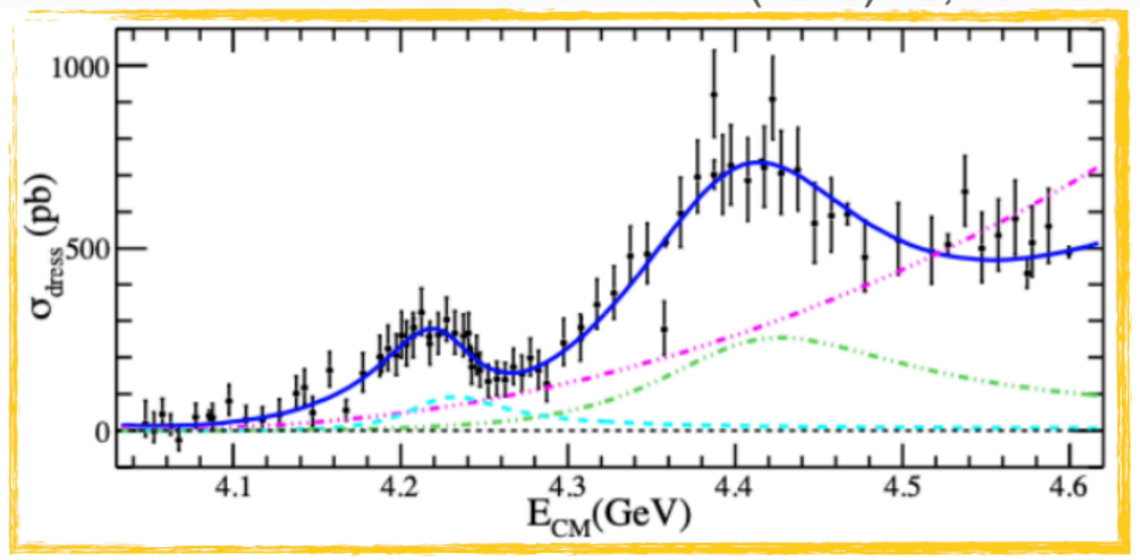
PRL 118 (2017) 9, 092002



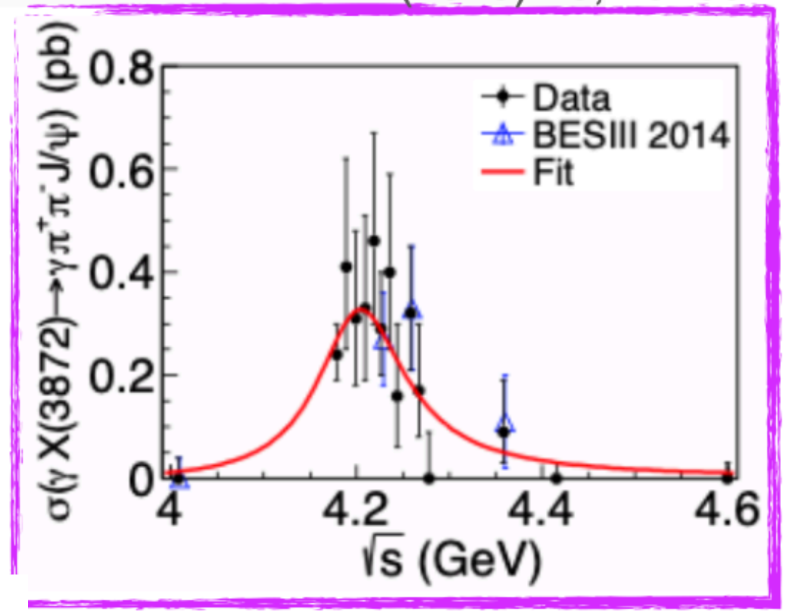
PRD 106 (2022) 7, 072001



PRL 122 (2019) 10, 102002

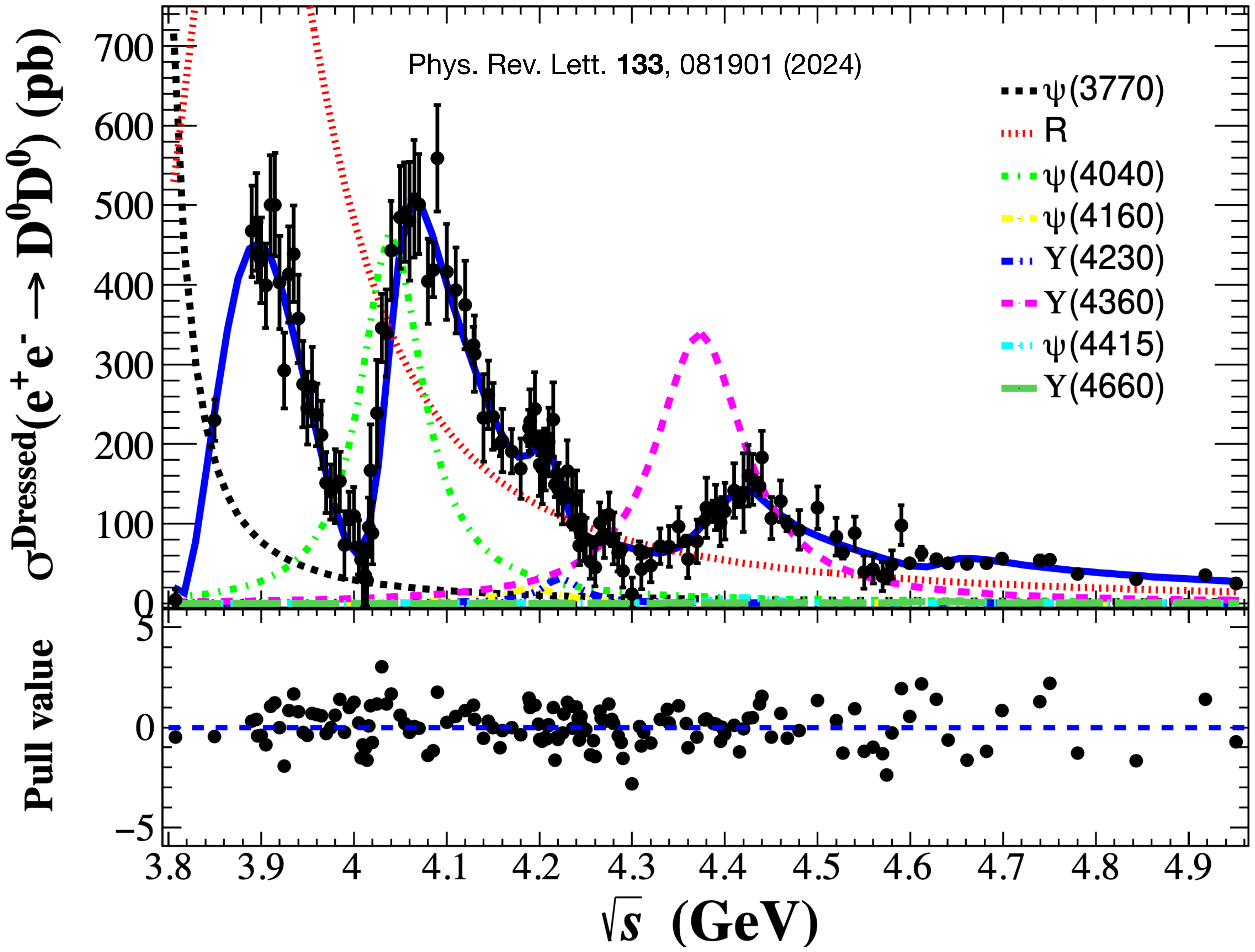


PRL 122 (2019) 23, 232002

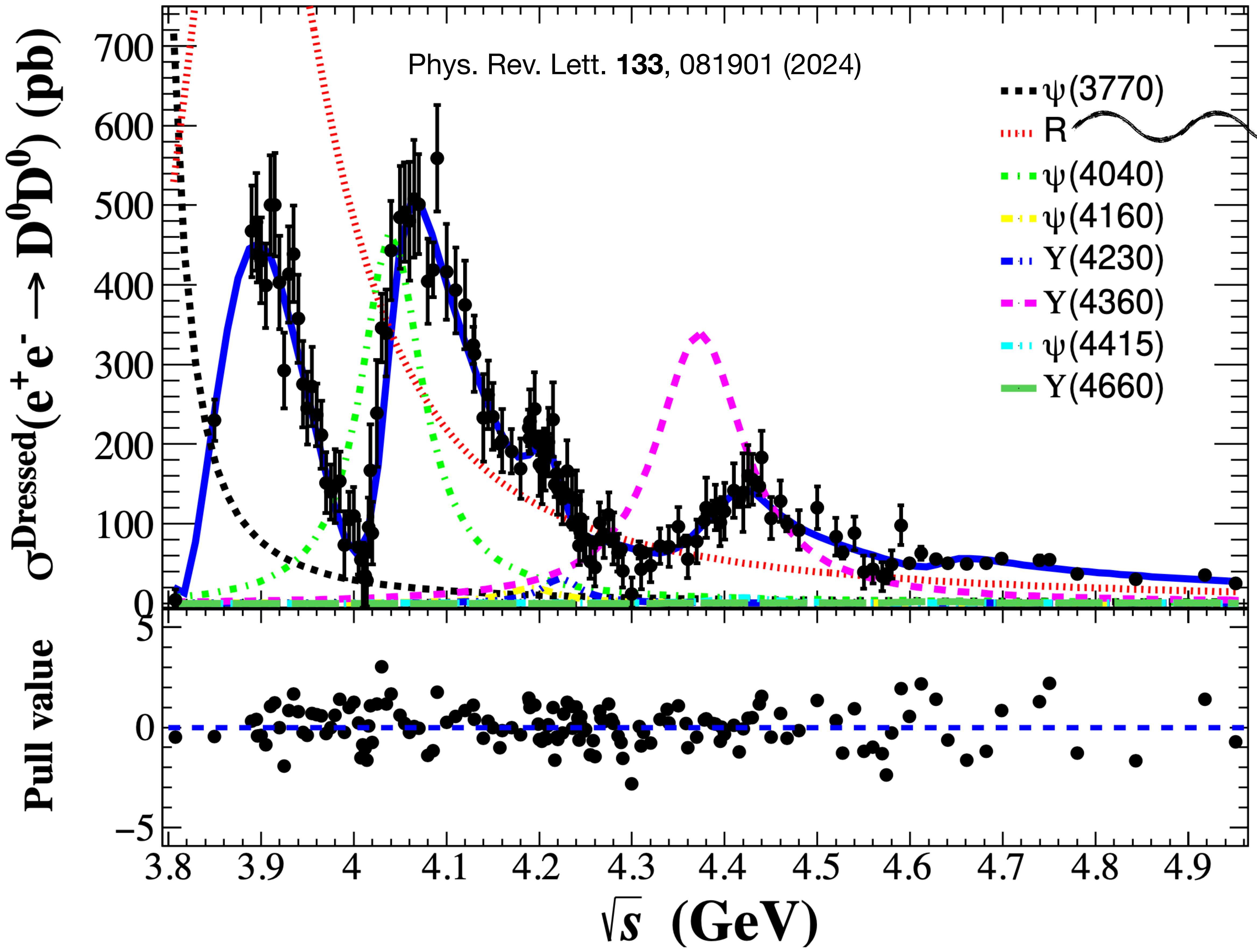


N. Huesken
Workshop on coupled channel analyses of charmonia

Coupled-channel analysis of vector charmonia at BESIII

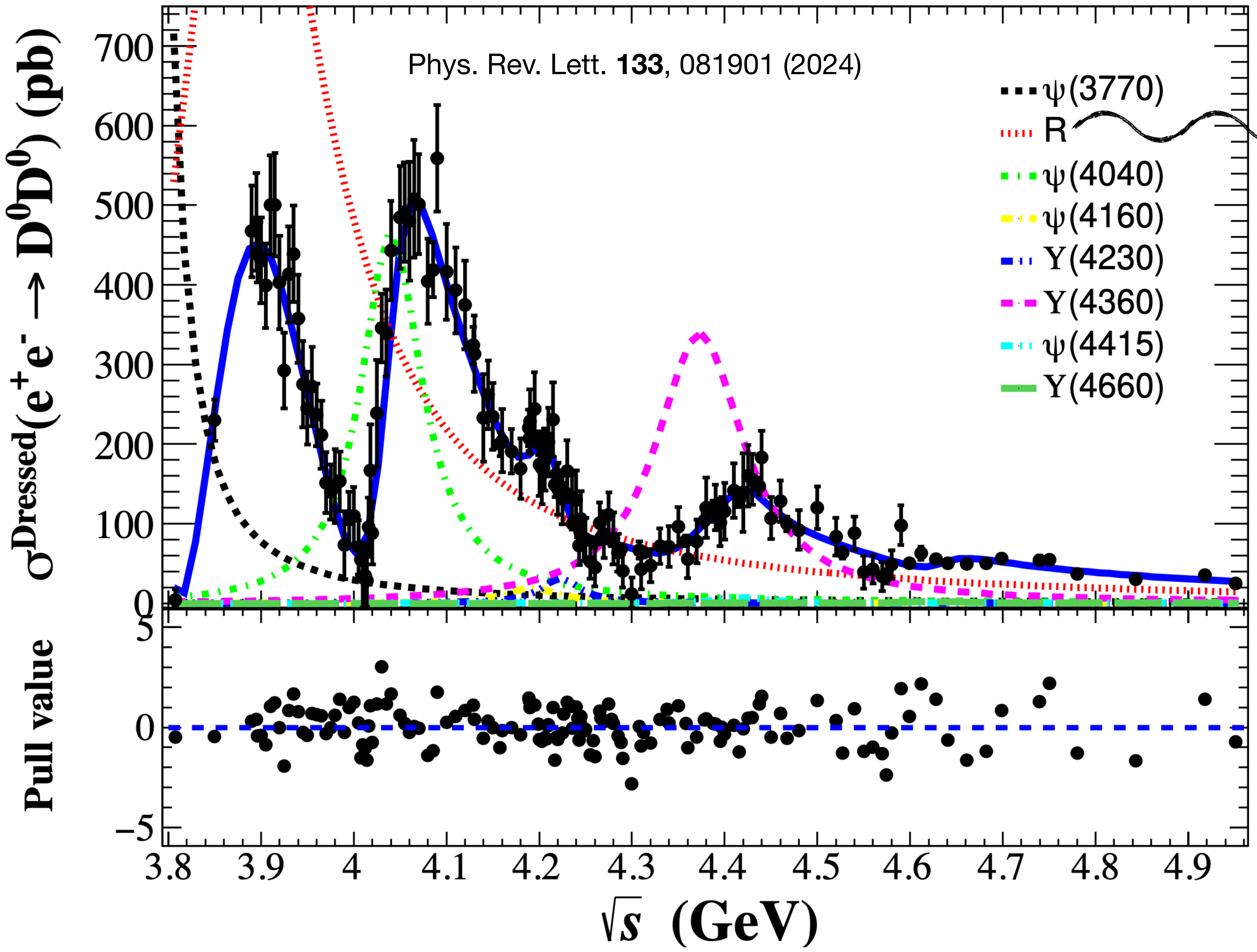


Coupled-channel analysis of vector charmonia at BESIII

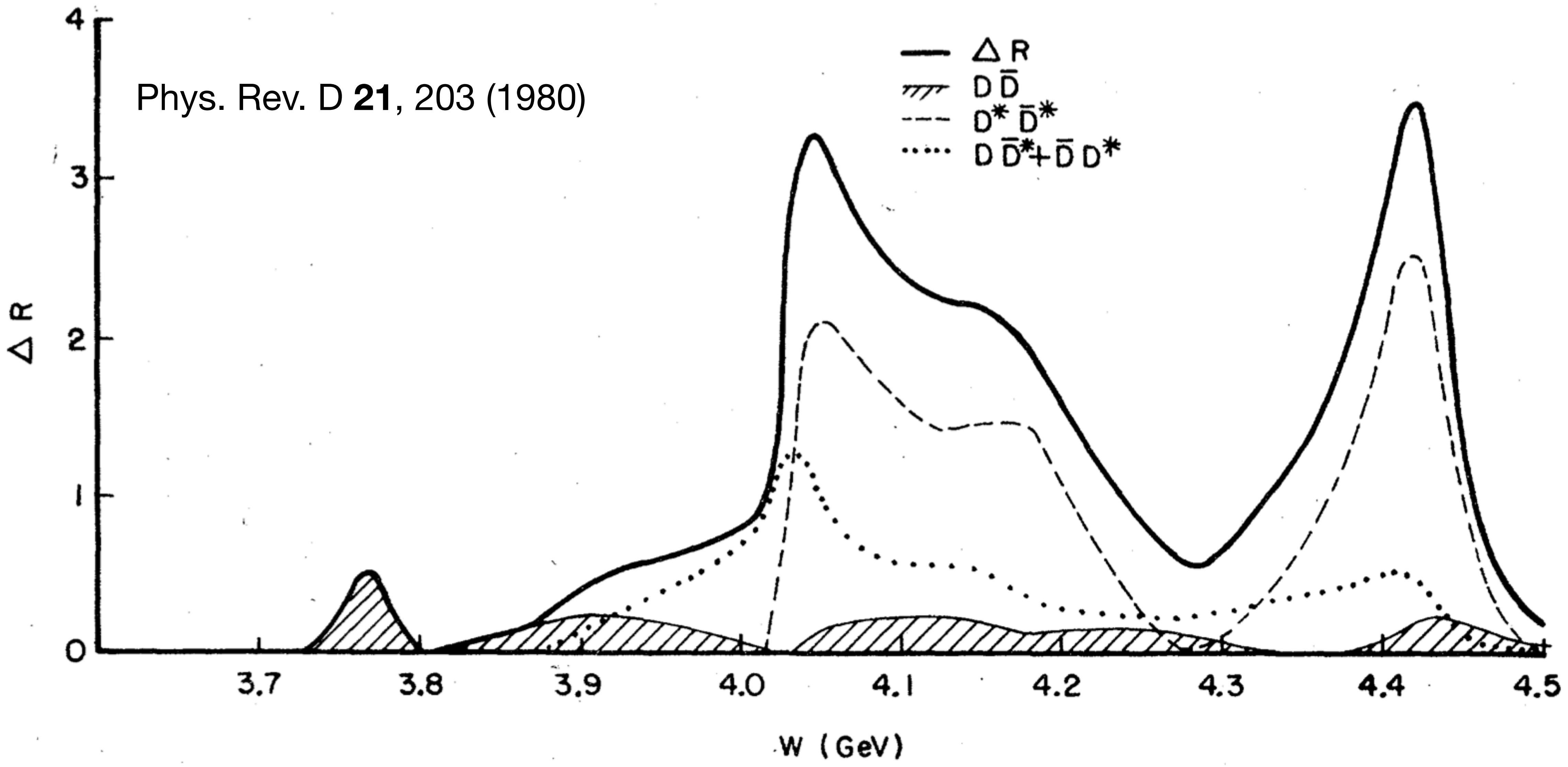


A new state?
The *G(3900)*…?

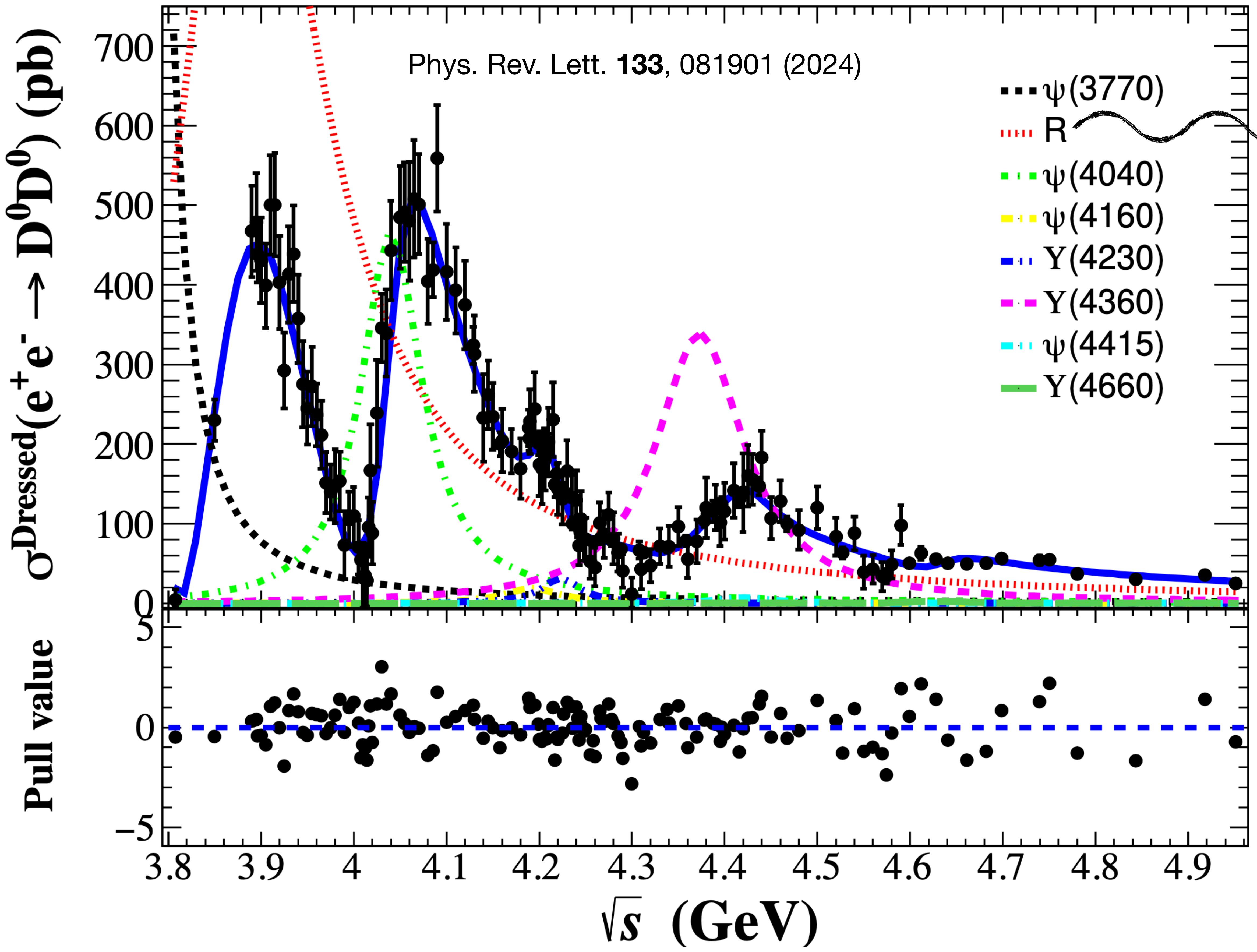
Coupled-channel analysis of vector charmonia at BESIII



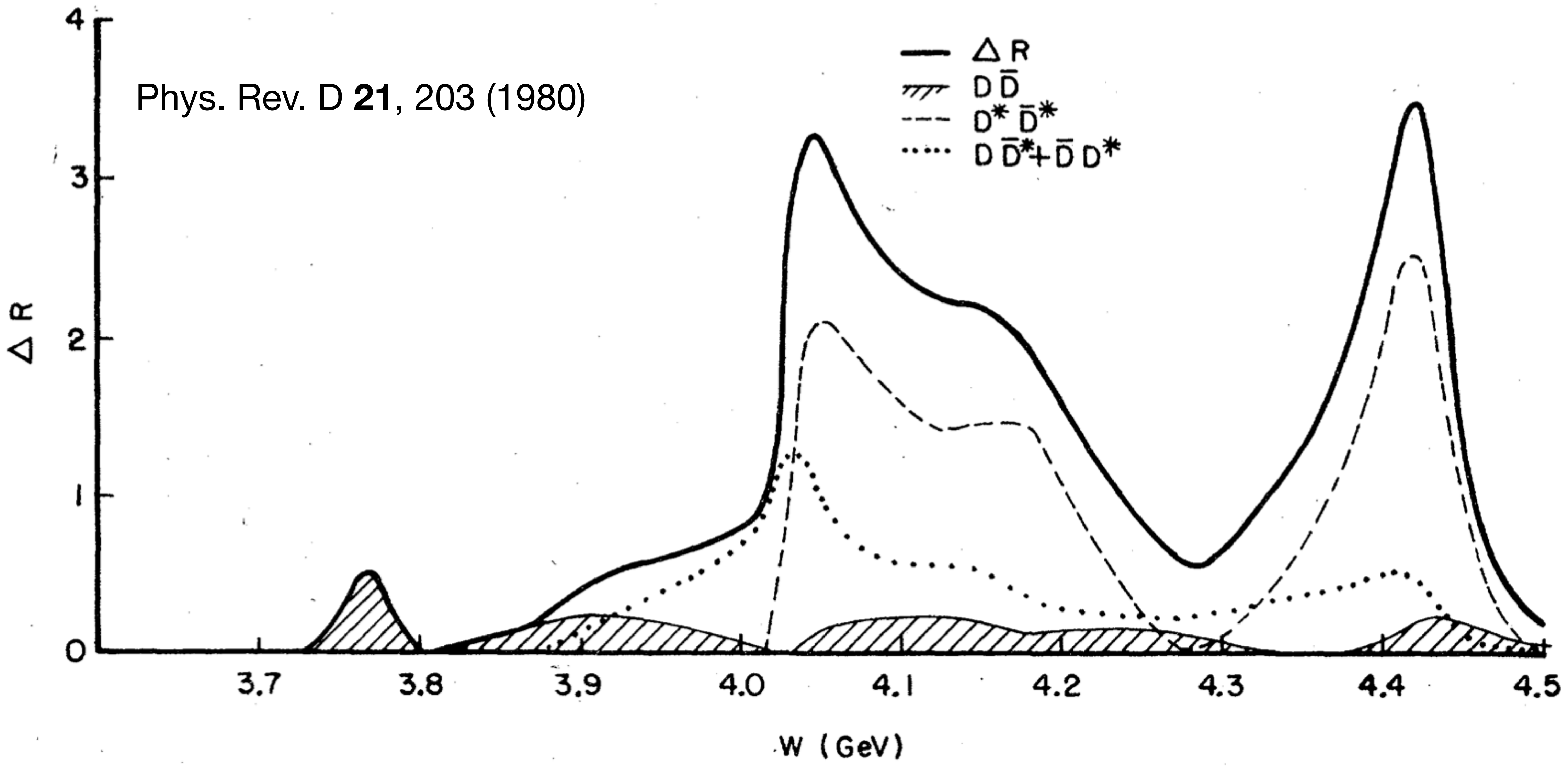
A new state?
The *G(3900)*…?



Coupled-channel analysis of vector charmonia at BESIII

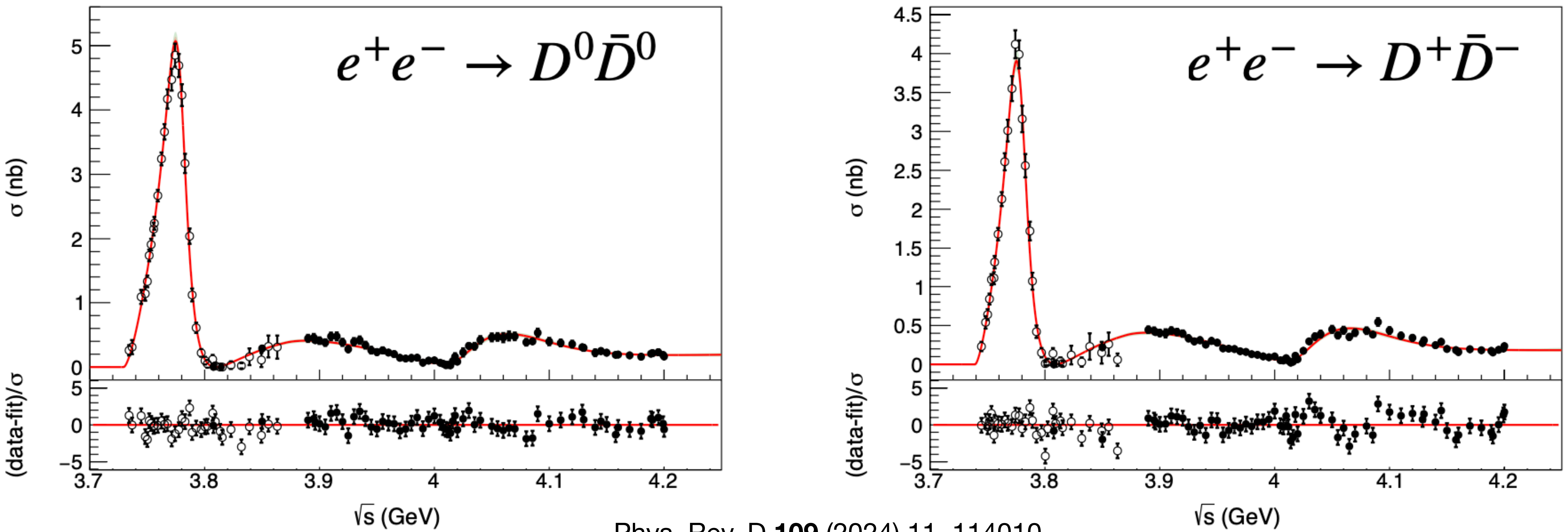


A new state?
The *G(3900)*…?

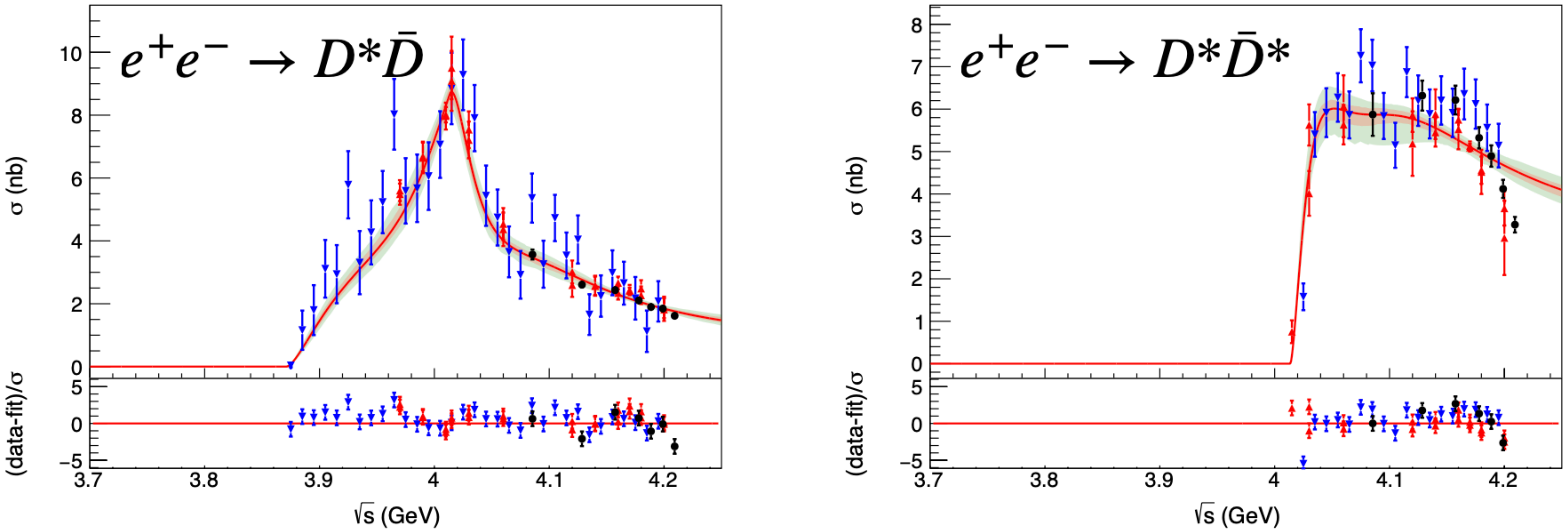


In our calculation there is some weak structure in the 3.9–4.0 GeV region. It does not arise from a $c\bar{c}$ resonance, but from the opening of the $D\bar{D}^* + D^*\bar{D}$ channel and a decrease in the $D\bar{D}$ channel due to a nearby zero in the 3S decay amplitude.

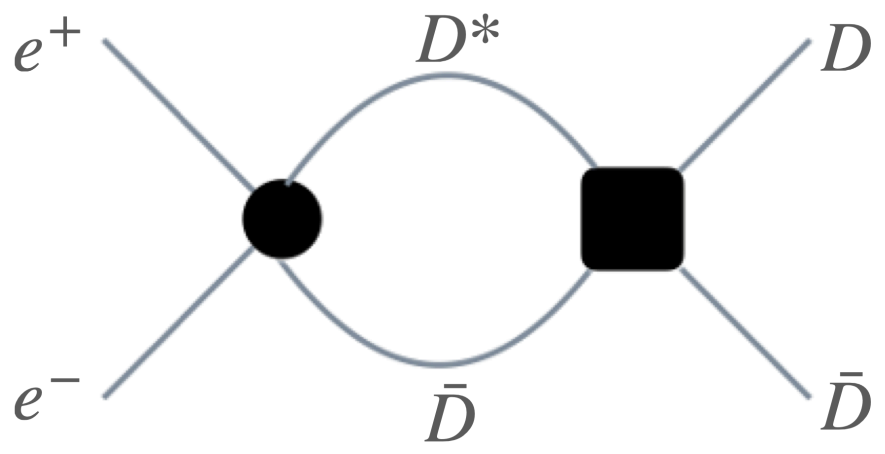
Coupled-channel analysis of vector charmonia at BESIII



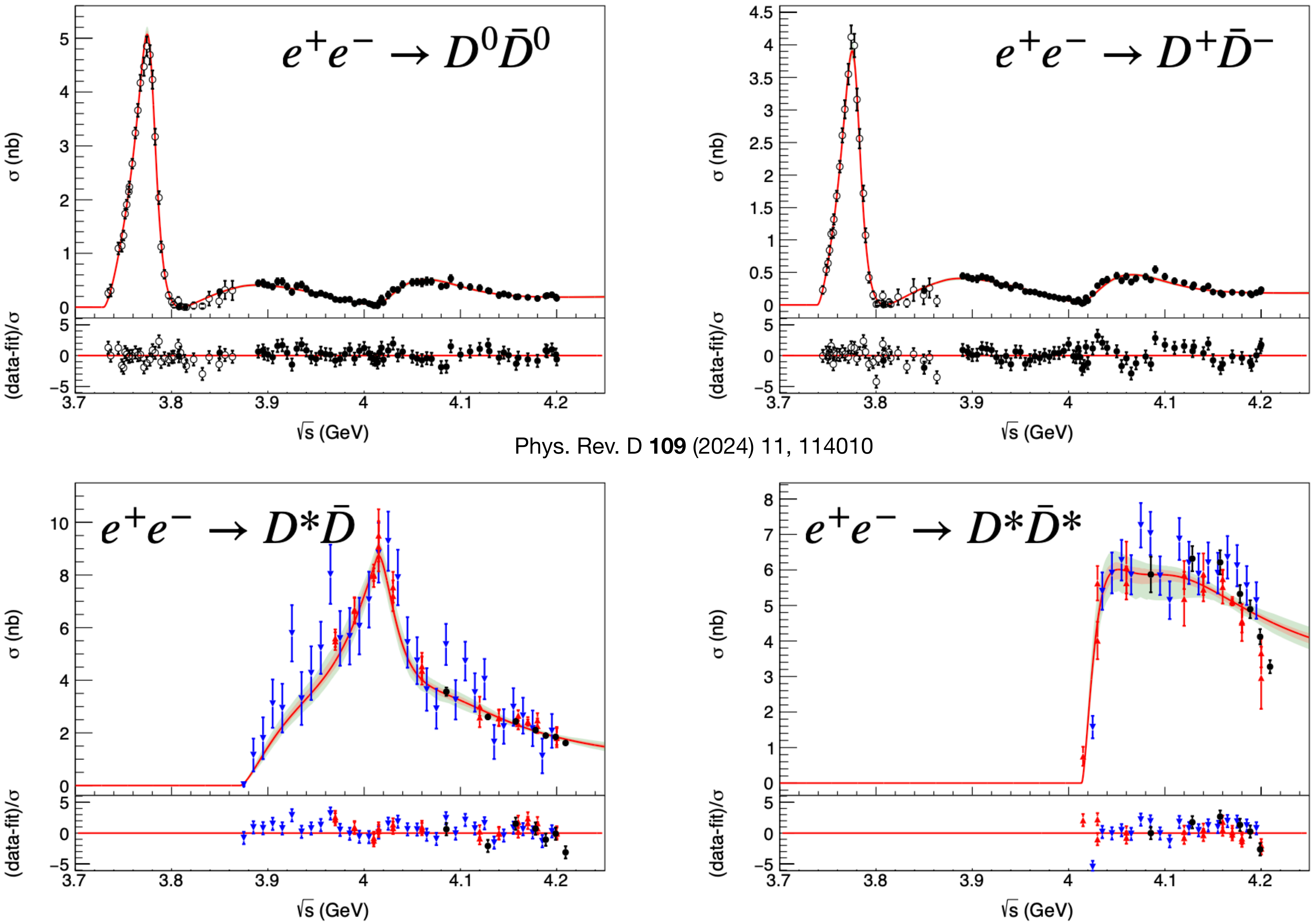
Phys. Rev. D **109** (2024) 11, 114010



The opening of other channels create the feature @3.9 GeV

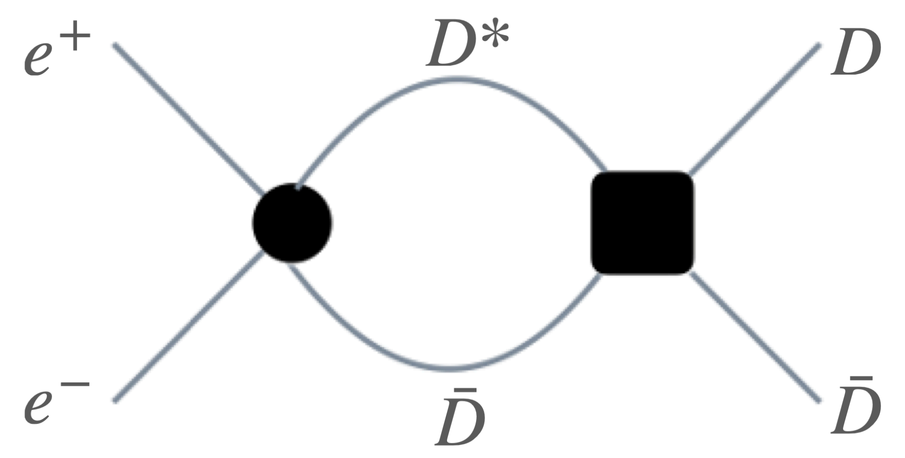


Coupled-channel analysis of vector charmonia at BESIII



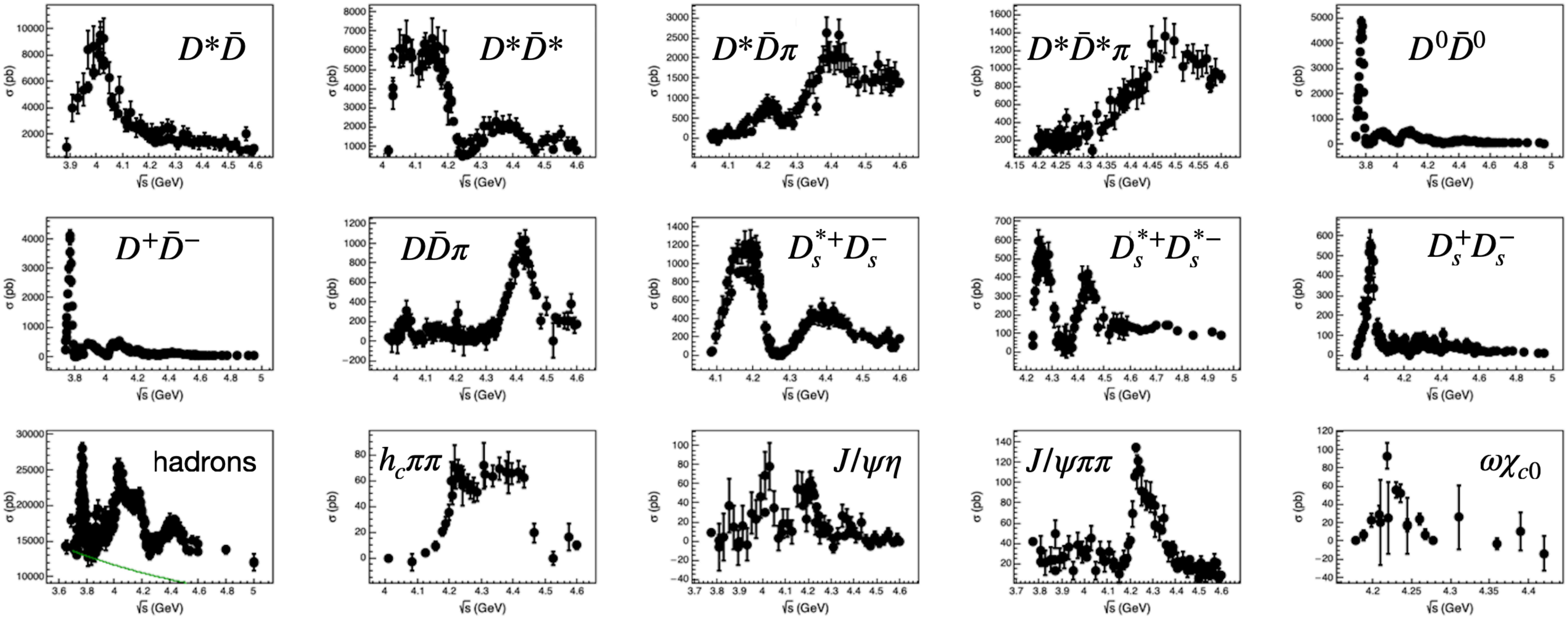
Phys. Rev. D **109** (2024) 11, 114010

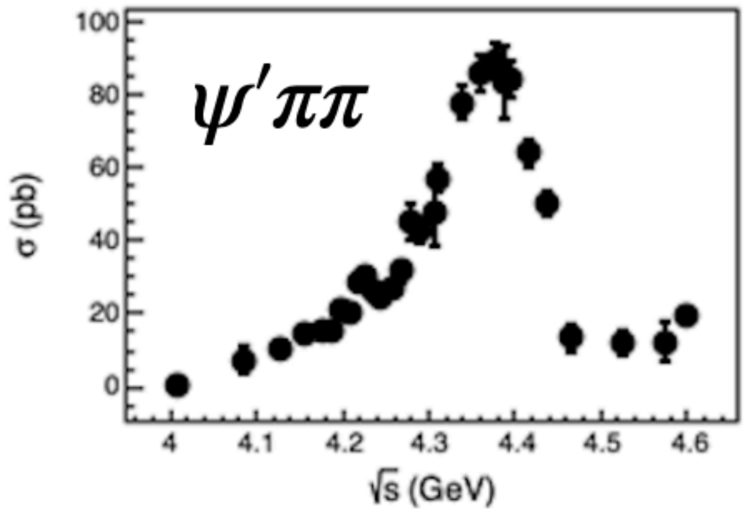
The opening of other channels create the feature @3.9 GeV



NB
Using a unified meson-exchange model
Phys. Rev. Lett. **133**, 241903 (2024) finds instead the G(3900) state

Coupled-channel analysis of vector charmonia at BESIII





15 exclusive channels + 1 inclusive (+ 1 dummy)
Data is from BaBar, Belle, BESIII, and CLEO
Some of it is not yet public or is from early analysis stages
4 resonances ($\psi(3770)$, $\psi(4040)$, $\psi(4230)$, and $\psi(4415)$)

Coupled-channel analysis of vector charmonia at BESIII

$$n_b\,\mathcal{M}_{ba}^{-1}\,n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2$$

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu}g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

Coupled-channel analysis of vector charmonia at BESIII

$$n_b \mathcal{M}^{-1}_{ba} n_a = \mathcal{K}^{-1}_{ba} - i\delta_{ba}\rho_a n_a^2$$

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

$$\mathcal{M} = (1 + KC)^{-1} K$$

$$C \equiv R - i\rho$$

$$g_{R:\mu}(s) = \hat{g}_{R:\mu} \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \exp[-k_\mu^2(s)/\beta^2]$$

$$f_{\mu,\nu} = \hat{f}_{\mu,\nu} \cdot \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \left(\frac{k_\nu(s)}{\beta}\right)^{\ell_\nu} \cdot \exp\left[-\left(\frac{k_\nu^2(s) + k_\mu^2(s)}{\beta^2}\right)\right].$$

Coupled-channel analysis of vector charmonia at BESIII

$$n_b \mathcal{M}^{-1}_{ba} n_a = \mathcal{K}^{-1}_{ba} - i\delta_{ba}\rho_a n_a^2$$

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

The bare masses m_R and couplings $g_{R:\mu}$ are free parameters in the fit

$$\mathcal{M} = (1 + KC)^{-1} K$$

$$C \equiv R - i\rho$$

$$g_{R:\mu}(s) = \hat{g}_{R:\mu} \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \exp[-k_\mu^2(s)/\beta^2]$$

$$f_{\mu,\nu} = \hat{f}_{\mu,\nu} \cdot \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \left(\frac{k_\nu(s)}{\beta}\right)^{\ell_\nu} \cdot \exp\left[-\left(\frac{k_\nu^2(s) + k_\mu^2(s)}{\beta^2}\right)\right].$$

Coupled-channel analysis of vector charmonia at BESIII

$$n_b \mathcal{M}_{ba}^{-1} n_a = \mathcal{K}_{ba}^{-1} - i\delta_{ba}\rho_a n_a^2$$

$$K_{\mu,\nu} = \sum_R \frac{g_{R:\mu} g_{R:\nu}}{m_R^2 - s} + f_{\mu,\nu}$$

The bare masses m_R and couplings $g_{R:\mu}$ are free parameters in the fit

$$\mathcal{M} = (1 + KC)^{-1} K$$

$$C \equiv R - i\rho$$

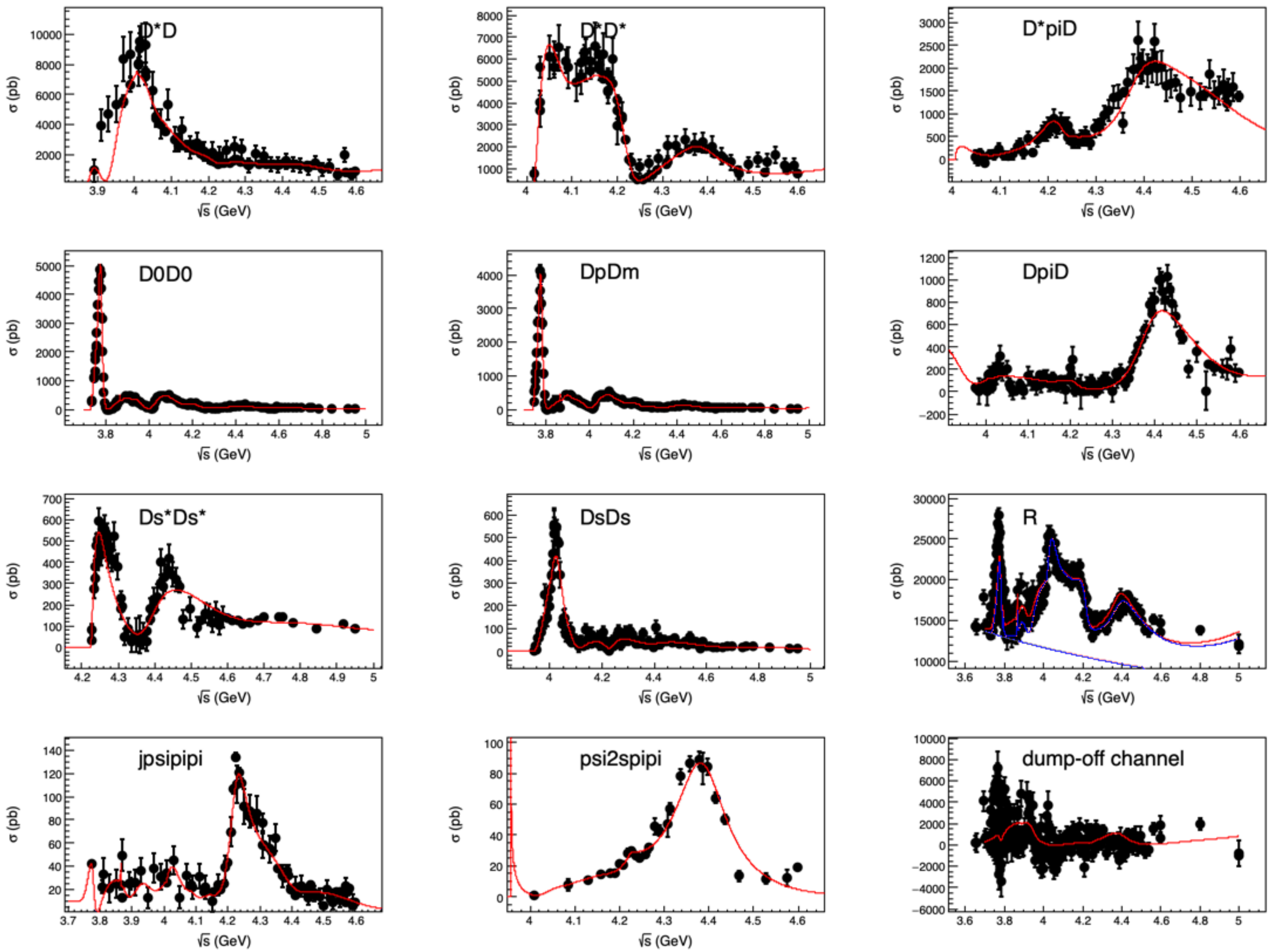
3-body channels are modelled based on two-body "mock" channels (the DDπ as subsequent D[Dπ] ones)

$$g_{R:\mu}(s) = \hat{g}_{R:\mu} \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \exp[-k_\mu^2(s)/\beta^2]$$

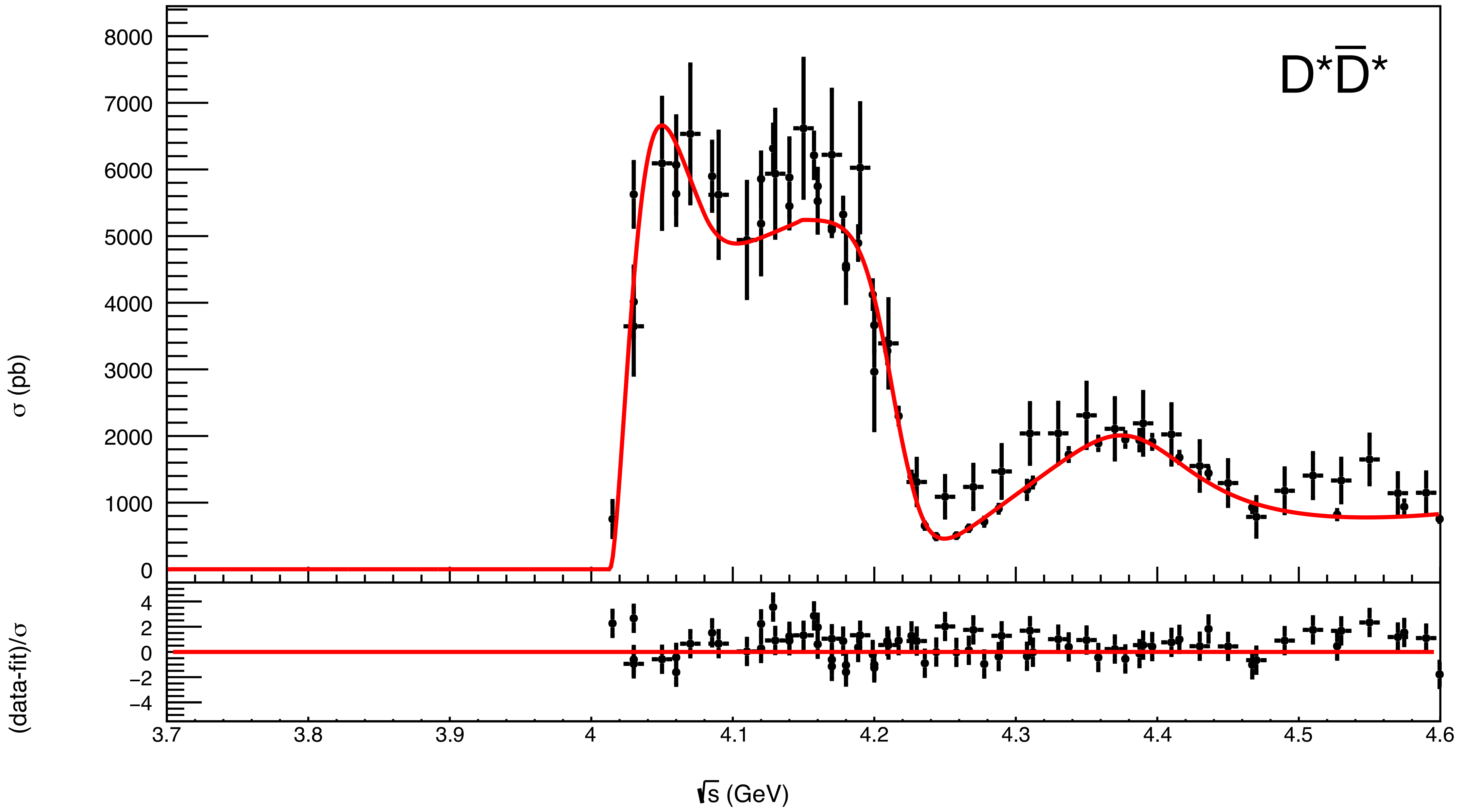
Justified by… 3-body channels cross-sections are 1 order smaller than open-charm 2-body ones

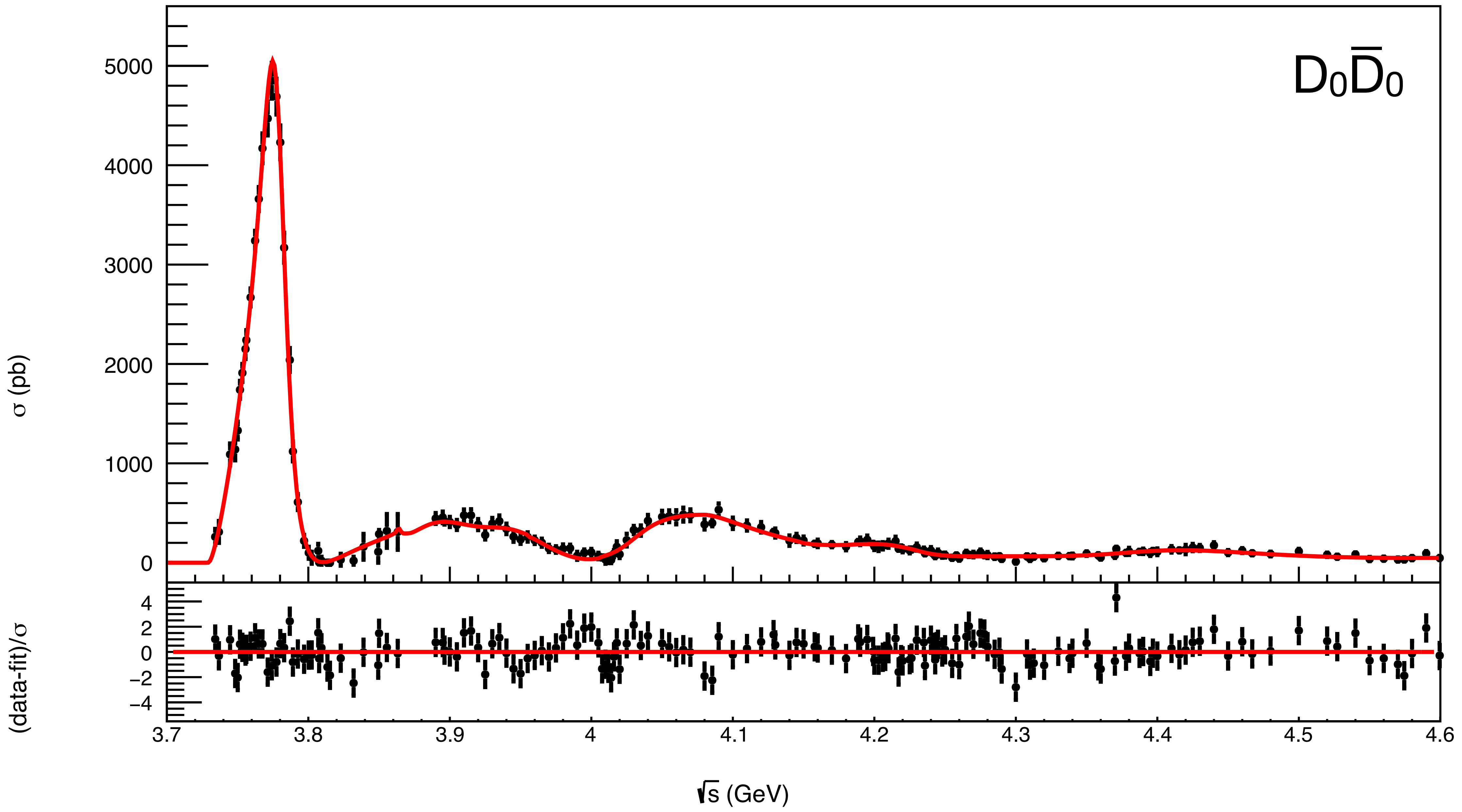
$$f_{\mu,\nu} = \hat{f}_{\mu,\nu} \cdot \left(\frac{k_\mu(s)}{\beta}\right)^{\ell_\mu} \cdot \left(\frac{k_\nu(s)}{\beta}\right)^{\ell_\nu} \cdot \exp\left[-\left(\frac{k_\nu^2(s) + k_\mu^2(s)}{\beta^2}\right)\right].$$

Coupled-channel analysis of vector charmonia at BESIII



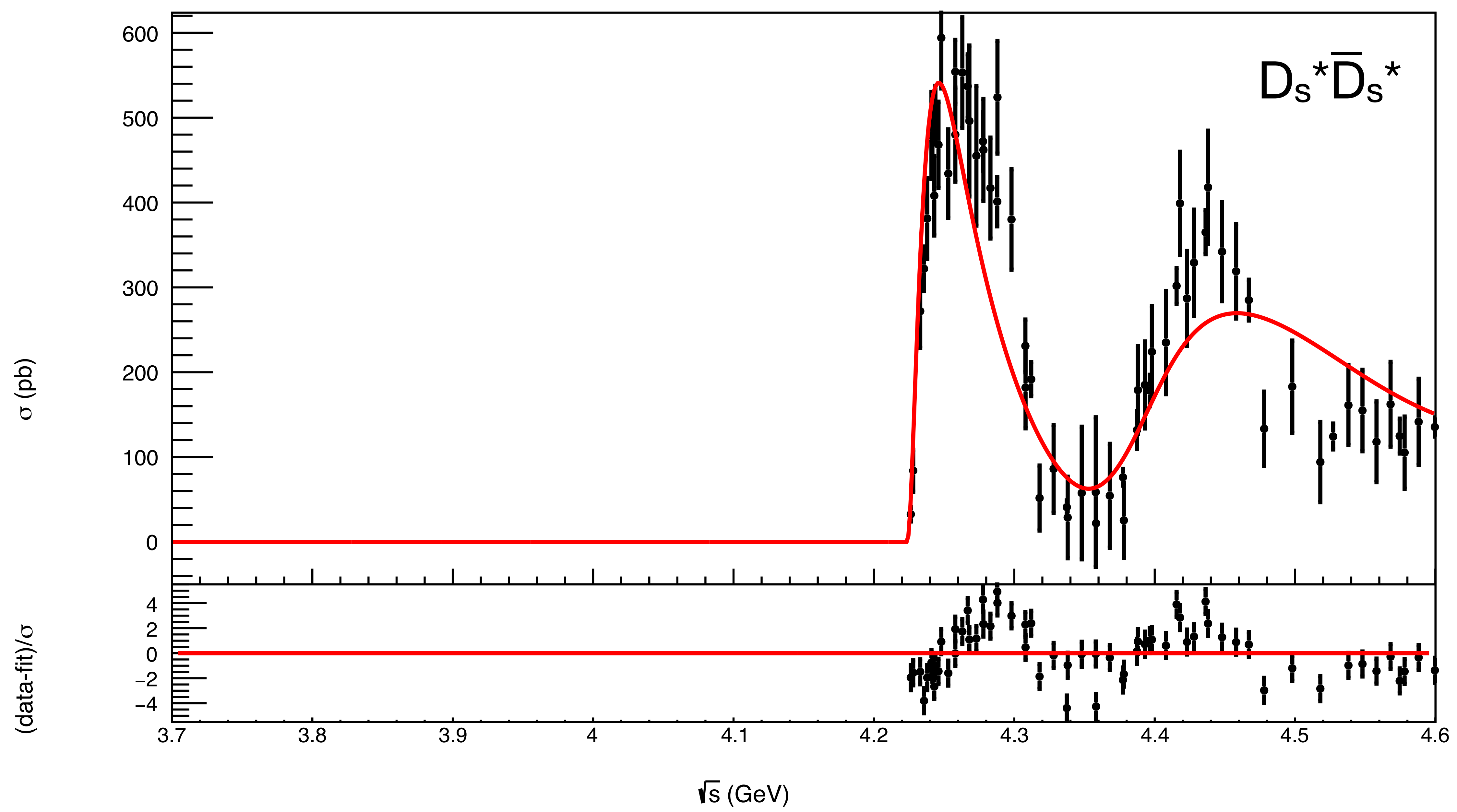
Coupled-channel analysis of vector charmonia at BESIII

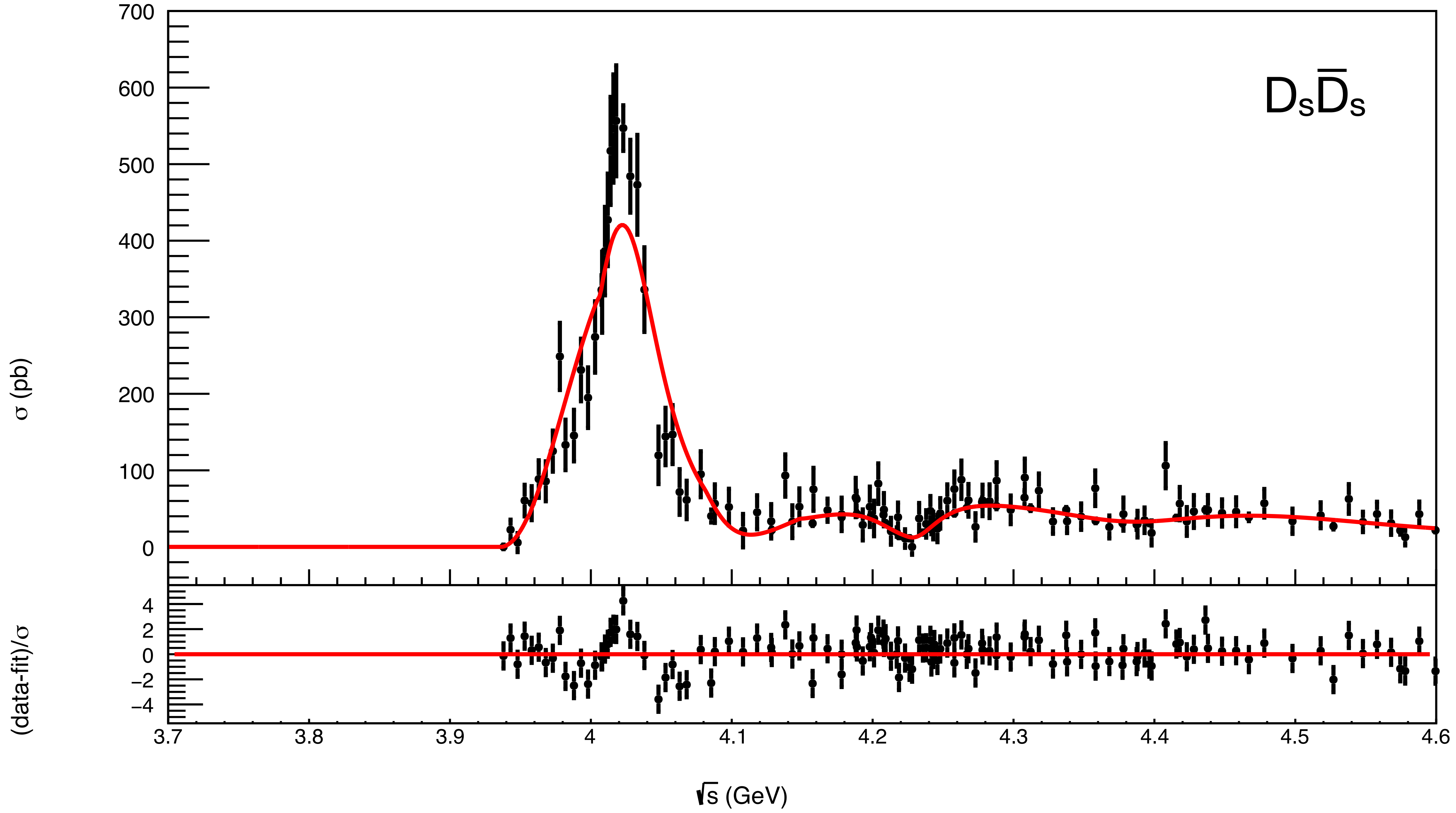




Cross-sections raise quickly close to channel's threshold
$D\bar{D}$ shows a structure near $D^*\bar{D}$ threshold

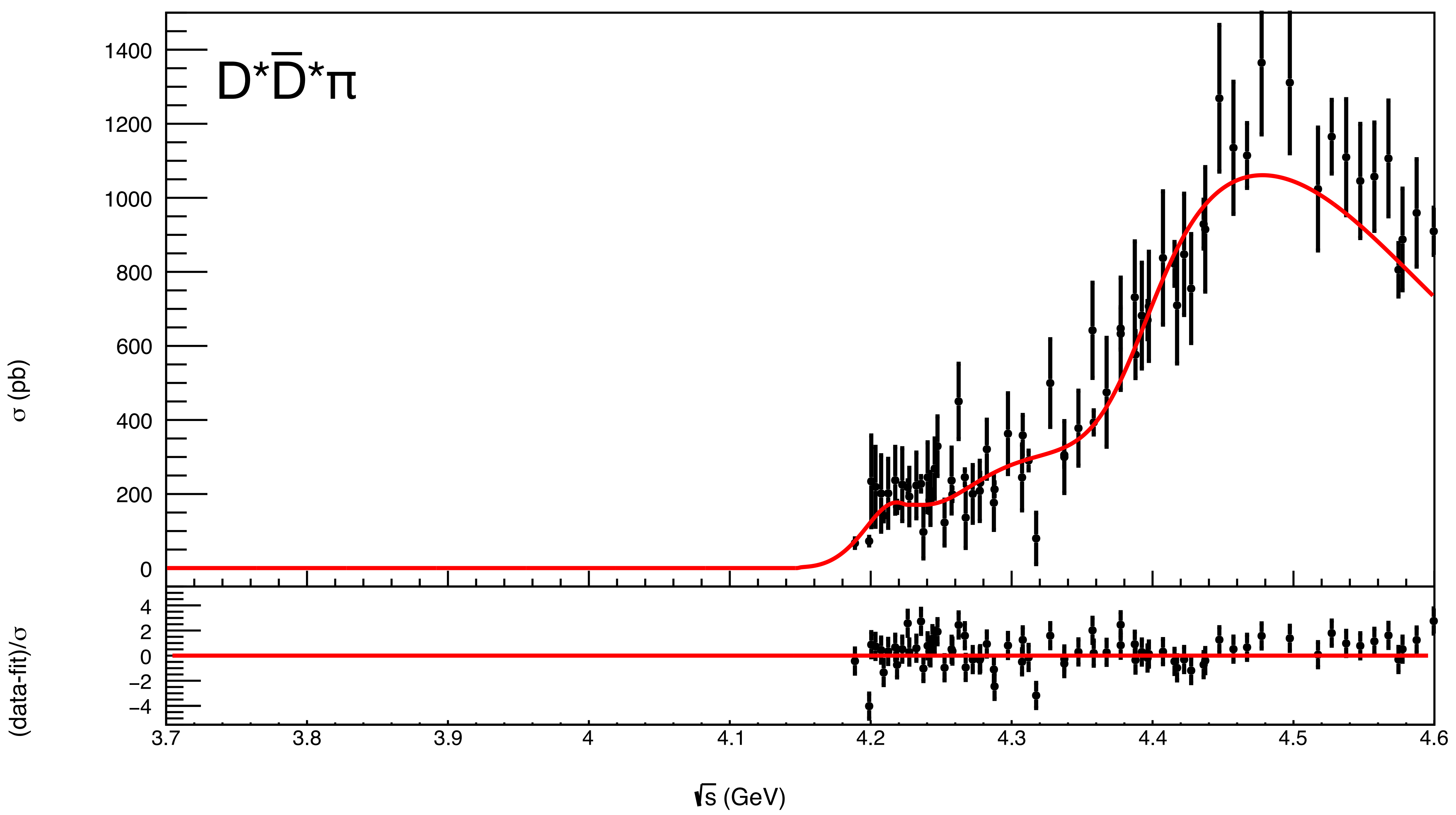
Coupled-channel analysis of vector charmonia at BESIII

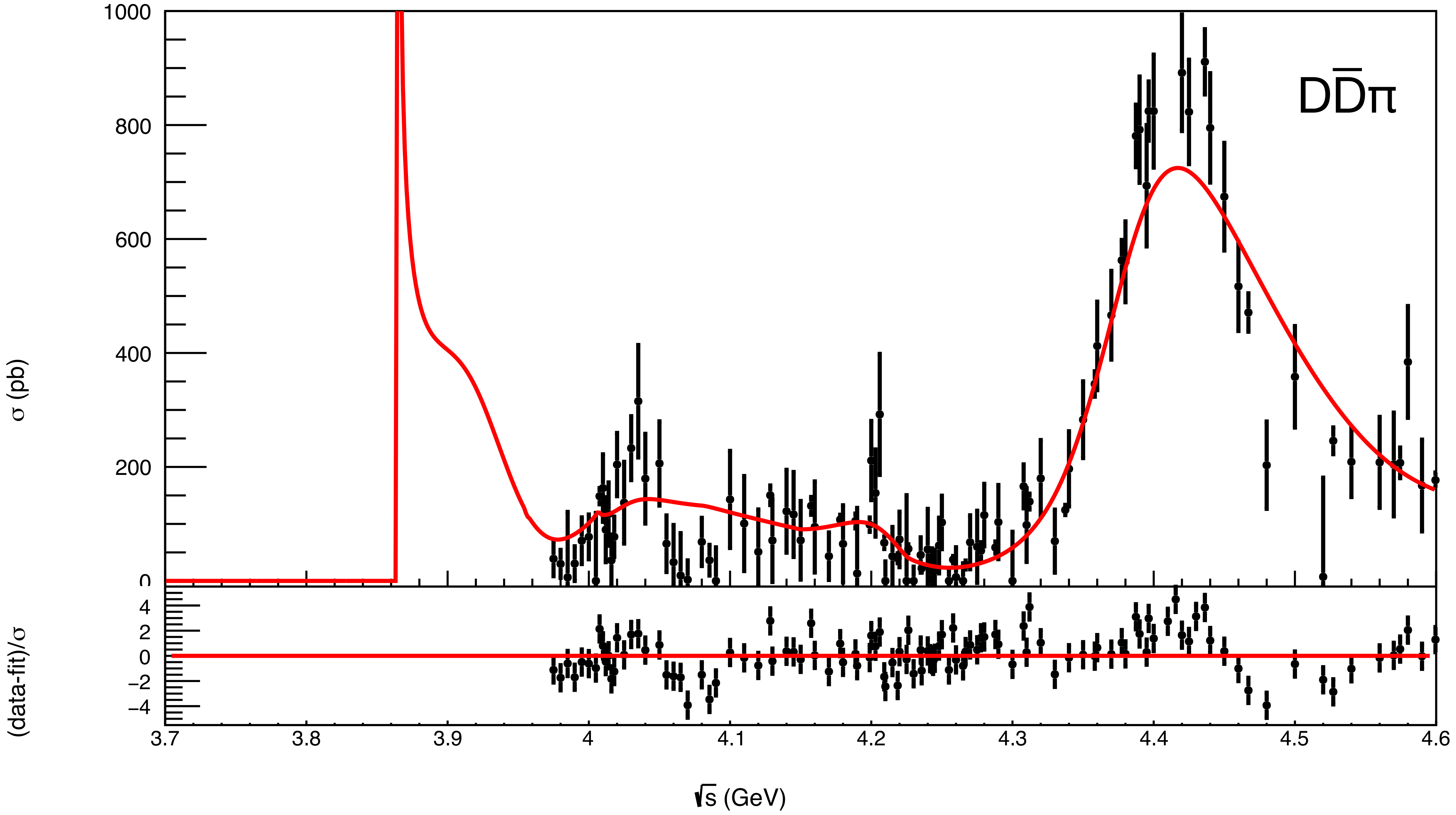




Cross-sections raise quickly close to channel's threshold
Fit must be improved, but it captures the trend

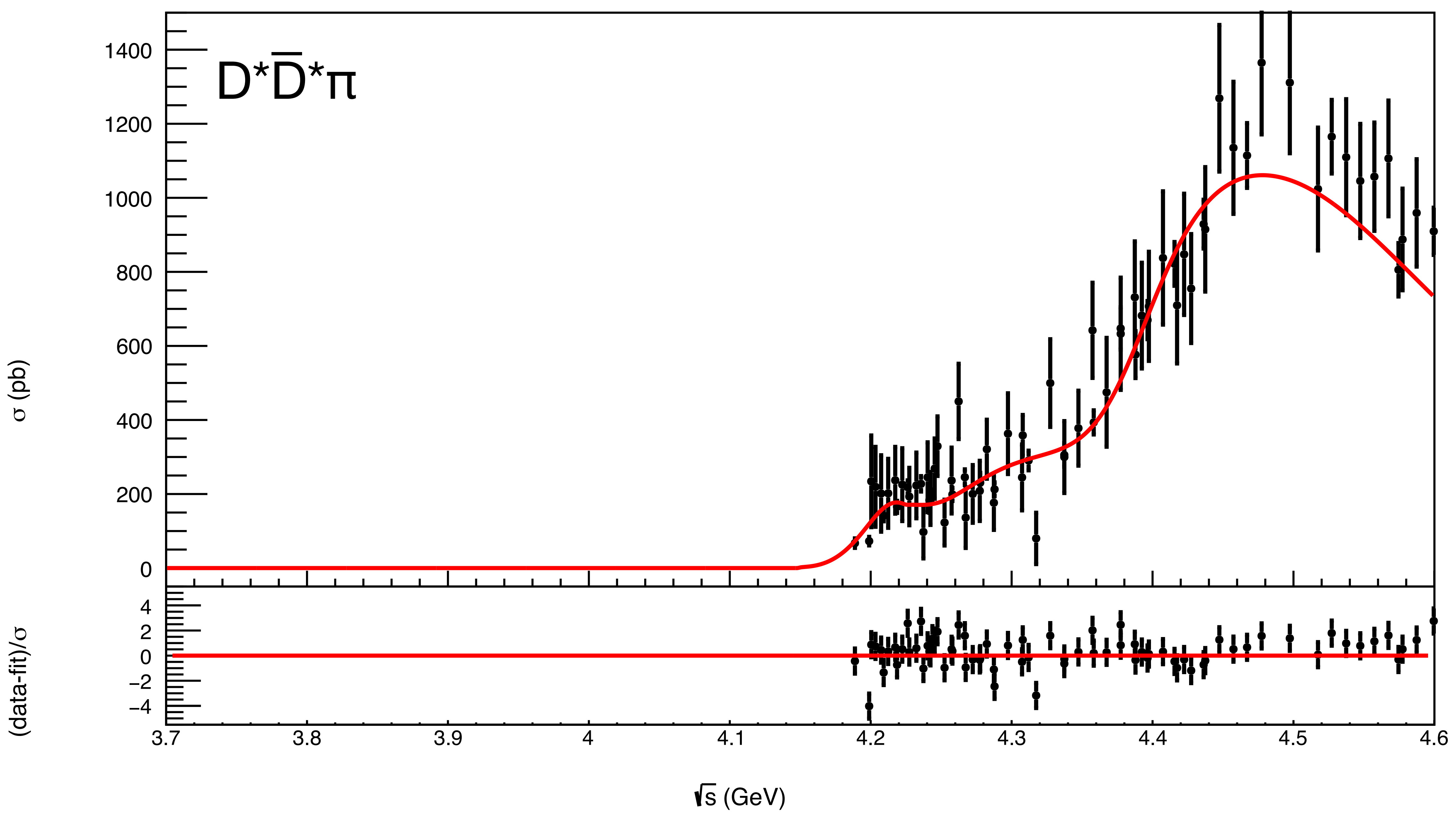
Coupled-channel analysis of vector charmonia at BESIII

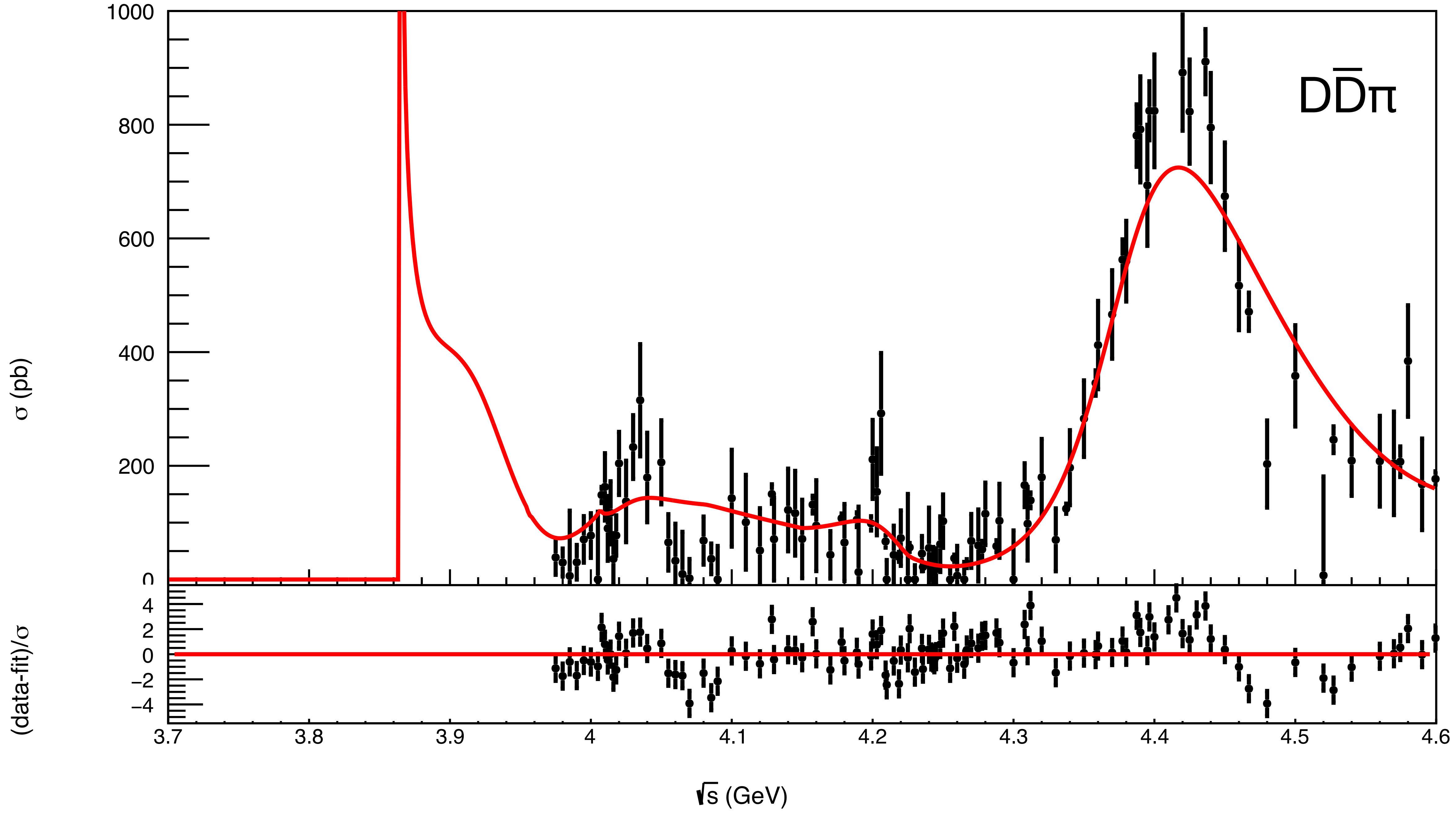




Cross-sections raise slowly
Fit must be improved, there are artefacts (some are unavoidable and depend on the minima)
$\psi(4415)$ seems to be an important driver for these cross-sections

Coupled-channel analysis of vector charmonia at BESIII



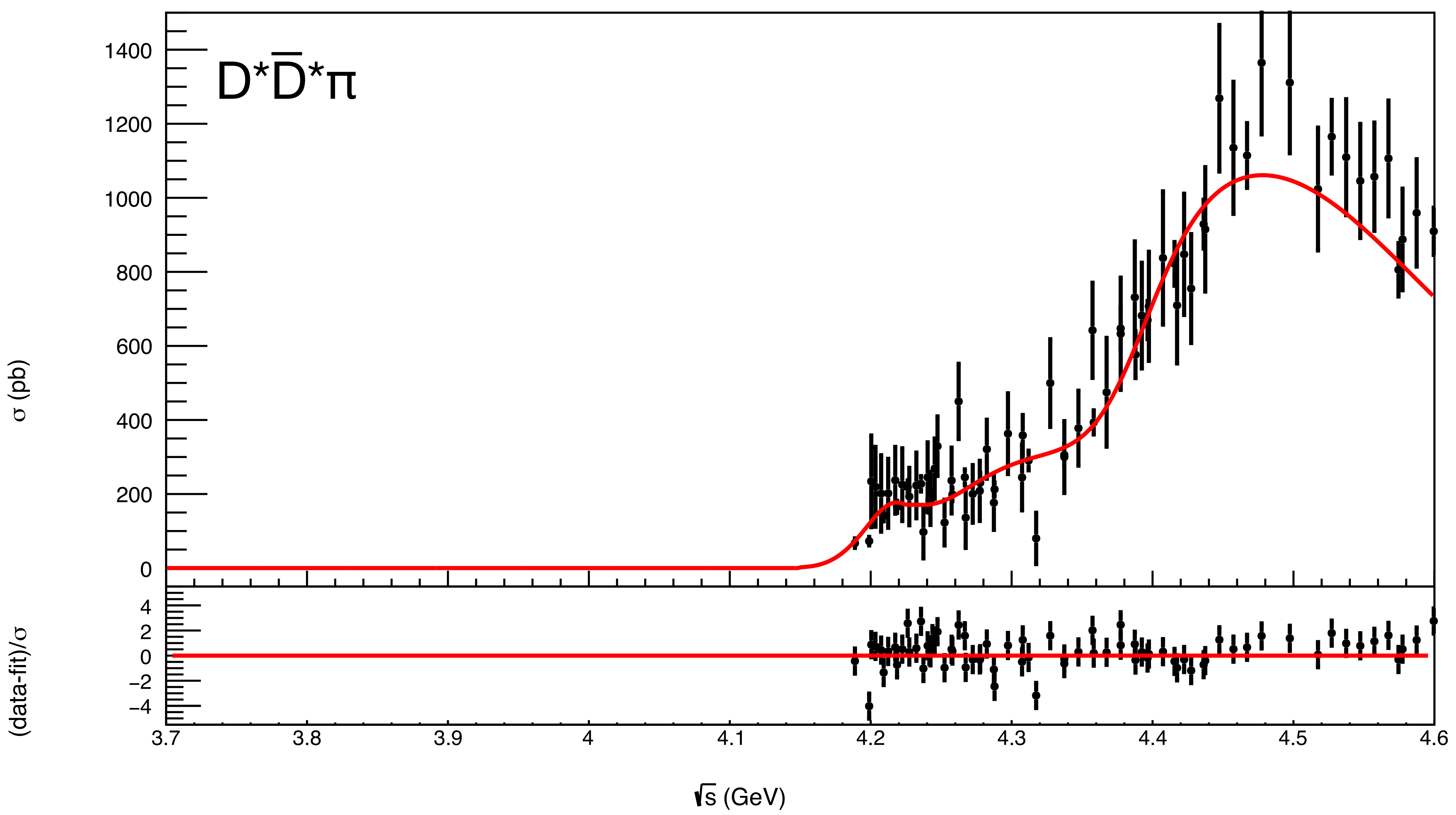


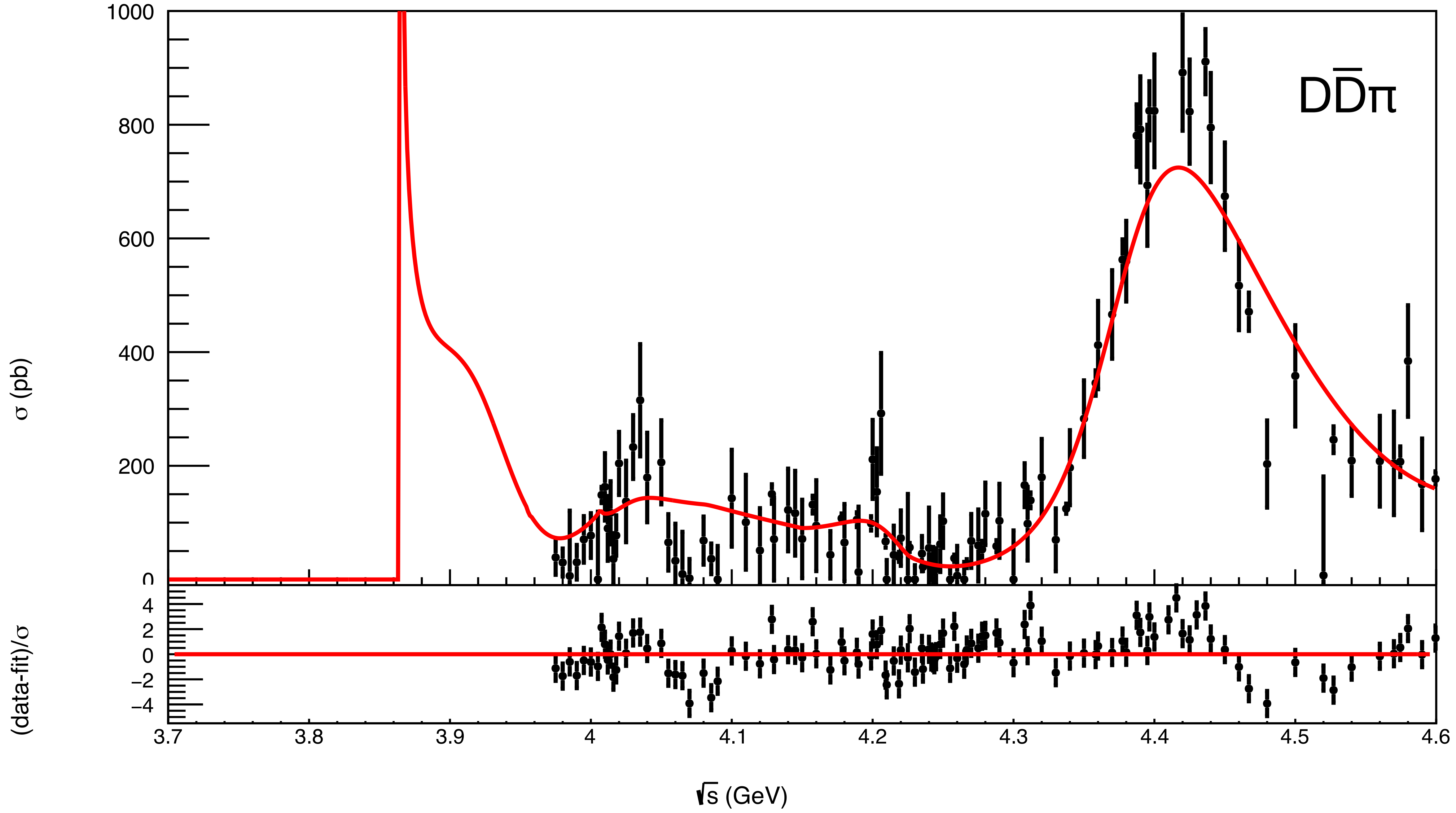
Cross-sections raise slowly

Fit must be improved, there are artefacts (some are unavoidable and depend on the minima)

$\psi(4415)$ seems to be an important driver for these cross-sections

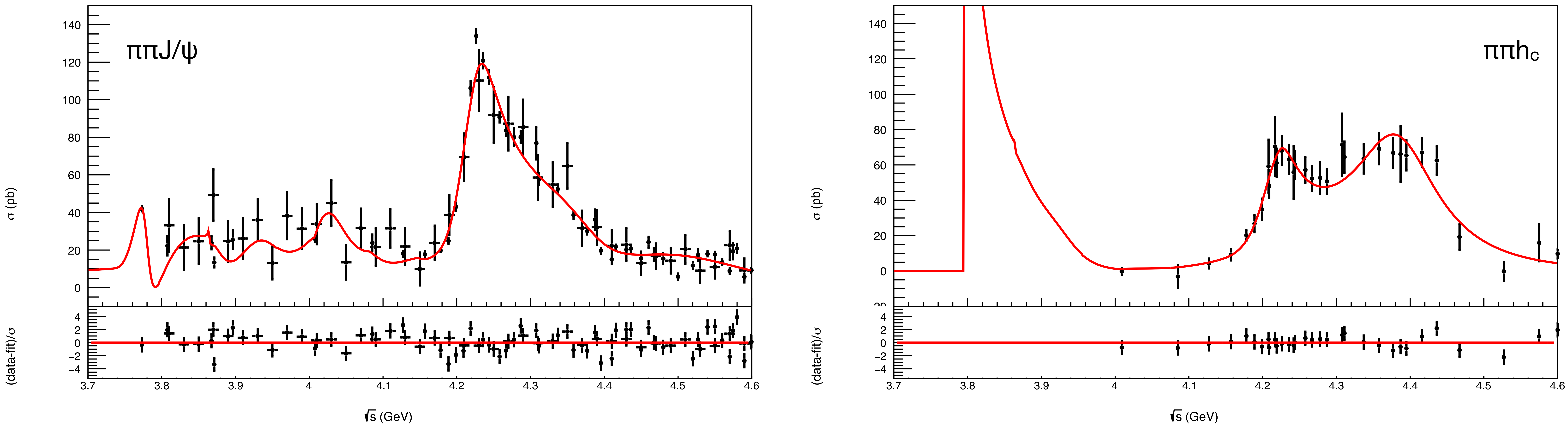
Coupled-channel analysis of vector charmonia at BESIII





Cross-sections raise slowly
Fit must be improved, there are artefacts (some are unavoidable and depend on the minima)
$\psi(4415)$ seems to be an important driver for these cross-sections

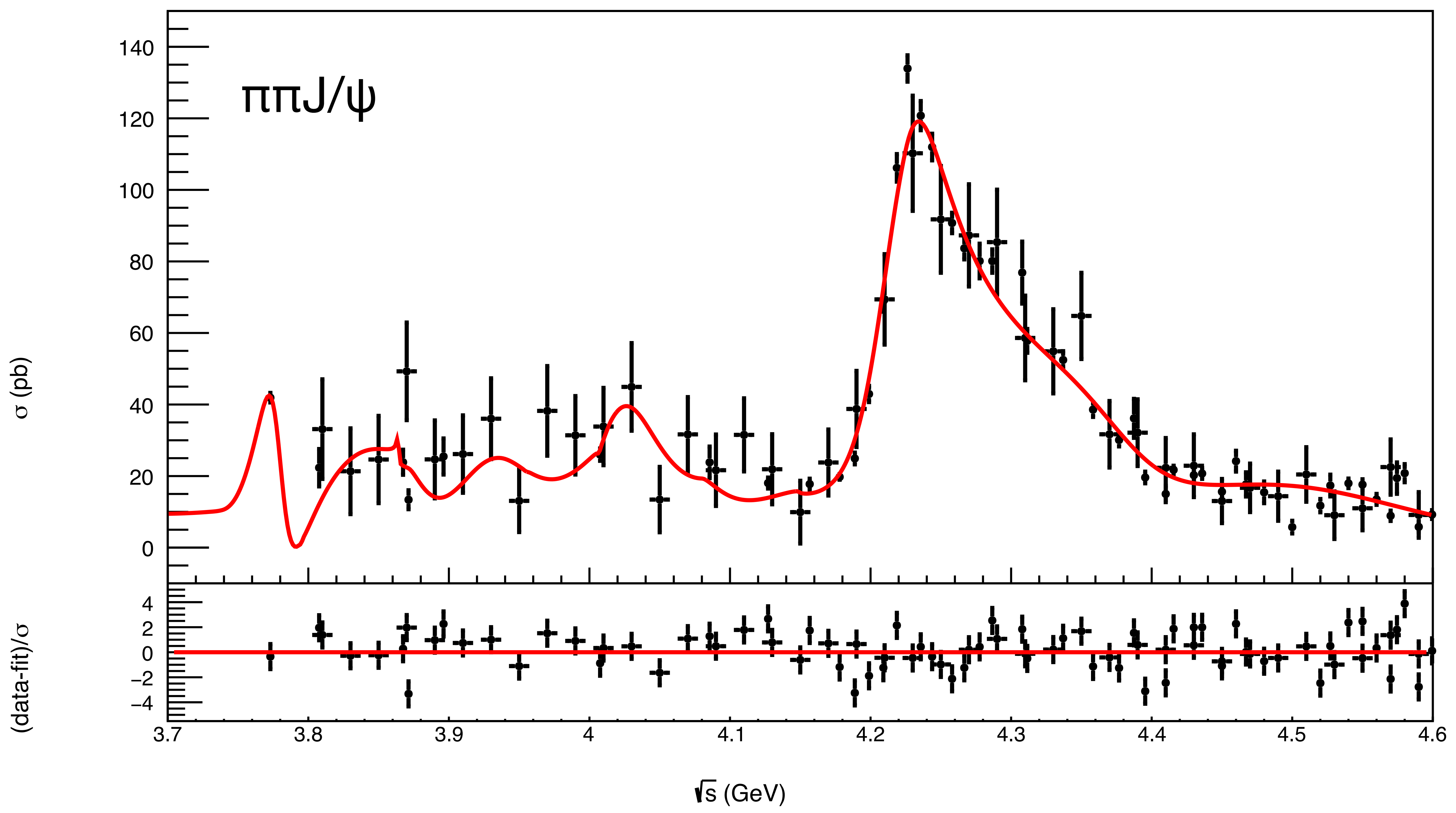
Coupled-channel analysis of vector charmonia at BESIII

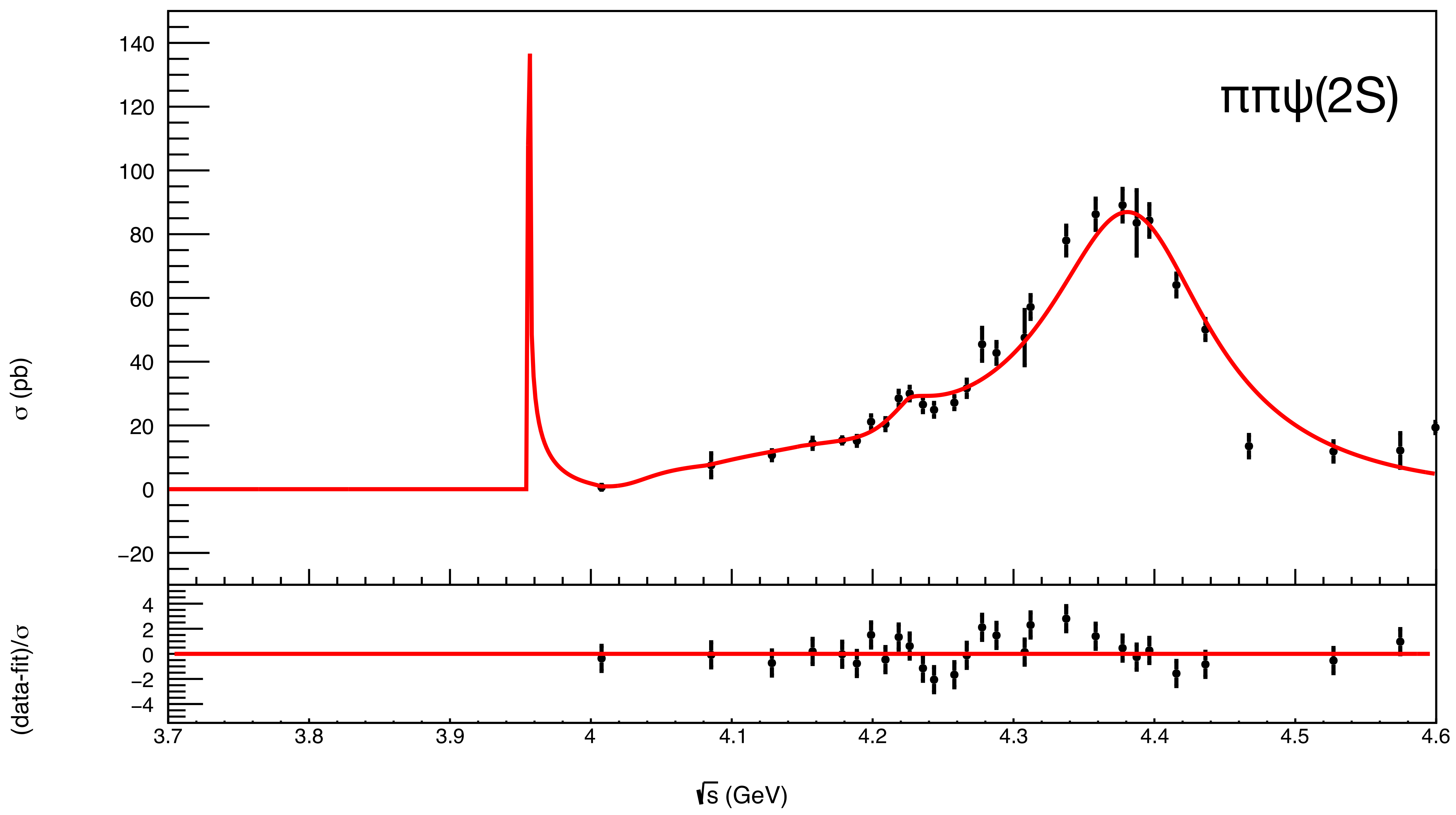


ψ(4230)'s peak and shoulder are described well

Also here, there are artefacts (some are unavoidable and depend on the minima)

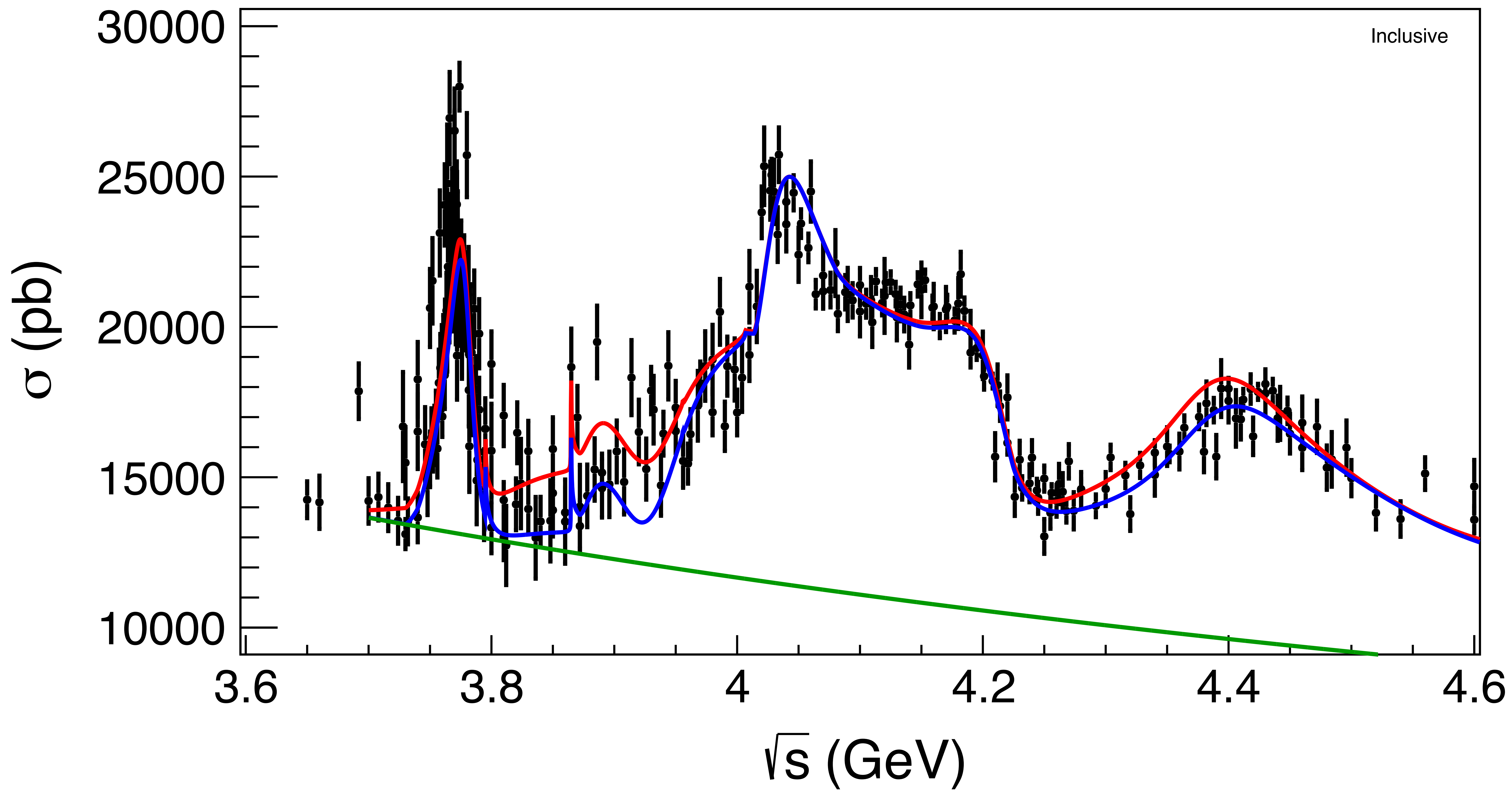
Coupled-channel analysis of vector charmonia at BESIII





ψ(4230)'s peak and shoulder are described well
Also here, there are artefacts (some are unavoidable and depend on the minima)

Coupled-channel analysis of vector charmonia at BESIII



The total cross-section might profit from better exclusive fits,
but all the structures are properly modelled

Summary… did we understand anything?

Using the “old” adagio one can say…

Not every resonance is a peak, and not all peaks are resonances

The **Breit-Wigner** approach is still a **powerful tool**, but for the case of an **isolated** ($\Gamma/\Delta \to 0$) **resonance**

Many coupled-channel analysis tools were not presented

K-Matrix applied to BESIII (**charmonia**) data can be a **powerful tool** to remove “dummy” resonances

Finally, the **K-Matrix project** I’m a part of is currently ongoing, trying to fit the open/hidden/inclusive charmonia cross-sections in the [3.7, 4.6] GeV range

Thank you for the attention!