Theory overview of charged Lepton Flavour Violation

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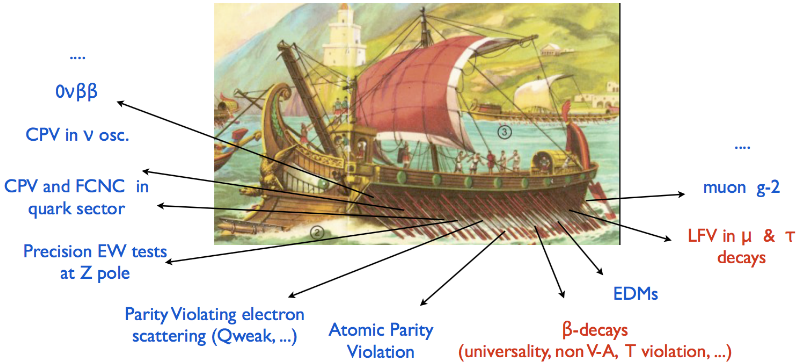
CLFV Searches with the MU2E Experiment
9-13 June 2025 INFN - LNF

Plan of the talk

1. **Strategies to look for New Physics at low-energy**
2. **Current status of LFV**
3. **EDMs, g-2 and LFV interrelationship**
4. **LFV @ FCC-ee & Muon Collider (MuC)**
5. **Conclusions and future prospects**

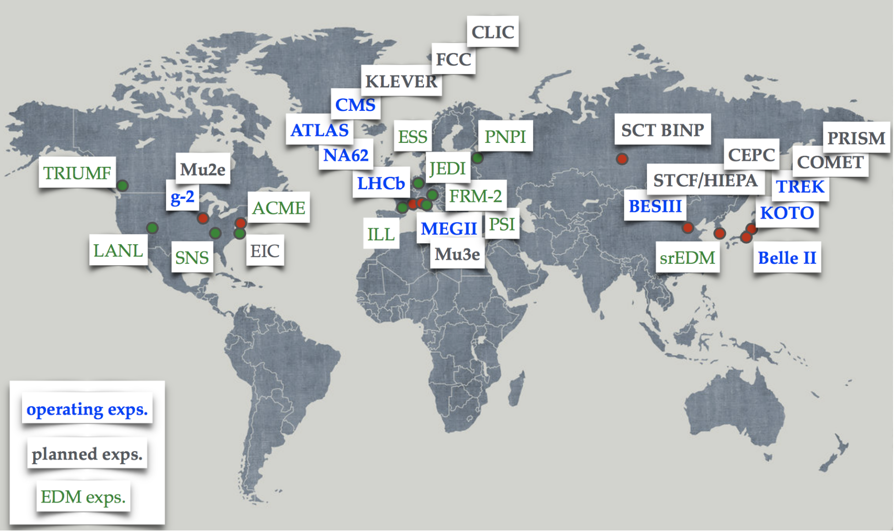
Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
- **Processes predicted with high precision in the SM**



High-intensity frontier: A collective effort to determine the NP dynamics

Experimental status



Experimental bounds

Process	Present	Experiment	Future	Experiment
$\mu \to e\gamma$	1.5×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \to 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
μ^- Au $\to e^-$ Au	7.0×10^{-13}	SINDRUM II	?	
μ^- Ti $\to e^-$ Ti	4.3×10^{-12}	SINDRUM II	?	
μ^- Al $\to e^-$ Al	–		$\approx 10^{-16}$	COMET, MU2e
$\tau \to e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \to 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\mathrm{e\ cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\mathrm{e\ cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

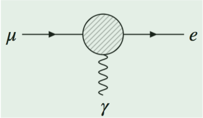
Table: Present and future experimental sensitivities for relevant low-energy observables.

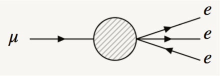
- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

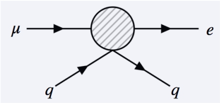
Experimental bounds on $\mu \to e$ processes

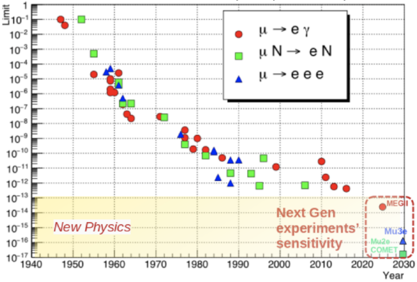
$$\mu \to e\gamma, \; \mu \to 3e, \; \mu A \to eA$$

- The possibility of extremely intense muon beams make these the **golden channels for LFV**
- Experimental sensitivities are expected to be **improved by up to five orders of magnitude**









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Charged LFV in the SM

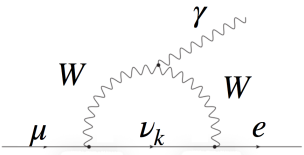
- GIM mechanism very effective in LFV transitions
- amplitude proportional to $A(\mu \to e\gamma) \propto m_\nu^2$

Very small !!!

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3}\frac{U_{\mu k}U^*_{ek}m^2_{\nu_k}}{M^2_W}\right|^2 .$$

$$\mathrm{BR}(\mu \to e\gamma) = 10^{-55} \div 10^{-54}$$

- similar suppressions for $\mu \to 3e, \tau \to 3\mu, \mu \to e, \ldots$

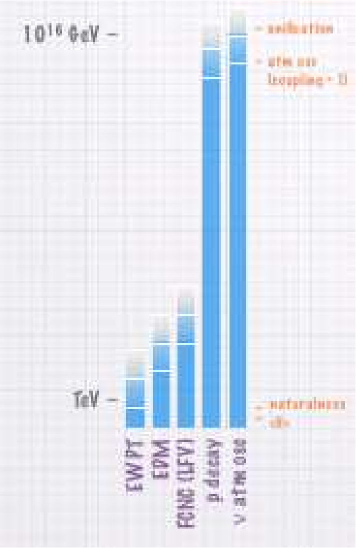


Why flavor violation is visible in neutrino oscillation while it's not in charged LFV?
The uncertainty principle sets the oscillation time for $\mu \to e\gamma$ to be $t \sim h/M_W$!

Message: Any evidence for LFV would be an unambiguous signal of NP!

Why do we need New Physics (NP)?

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}\ \text{GeV}$
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \text{GeV}$
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \text{GeV}$
- **Dark Matter (WIMP)** $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$



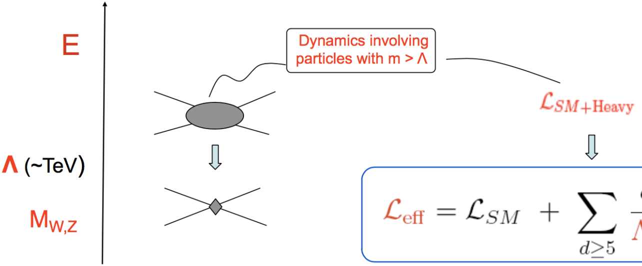
SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}}\, O_{ij}^{(d)}$$

- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_\nu^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi\phi,$
- $\mathcal{L}_{\text{eff}}^{d=6}$ generates FCNC operators

EFT approach to NP

- **Dynamics below the scale Λ [$\sim$ mass of new particles] is described by $L_{\rm eff}$**



$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \sum_{d\geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

  - $L_{\rm eff}$ is built out of relevant low-energy degrees of freedom (SM fields)
  - $L_{\rm eff}$ respects the SM gauge symmetries $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
  - $L_{\rm eff}$ is organized in inverse powers of Λ (amplitudes suppressed by powers of E/Λ)

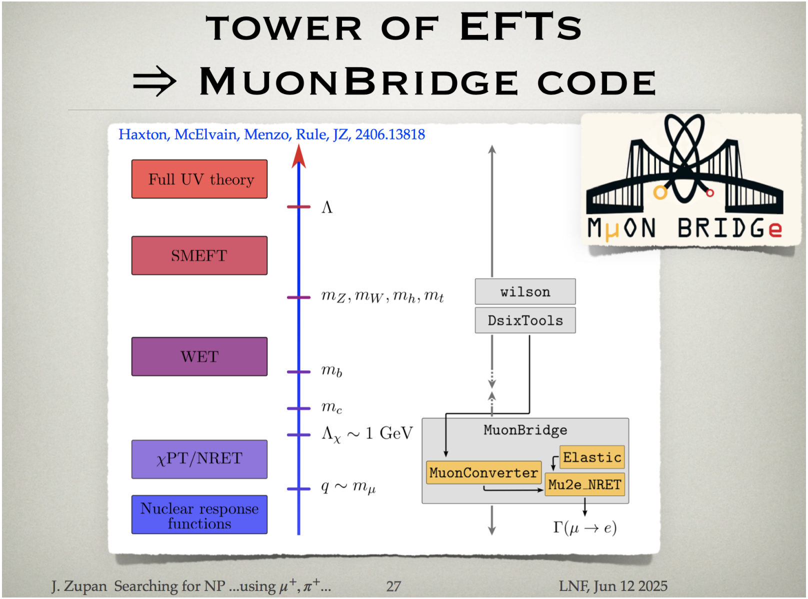
- **Experiments at the precision frontier probe energy scale Λ and symmetries of the new interactions (coeff. & structure of $\hat{O}_n^{(d)}$)**

EFT approach to LFV

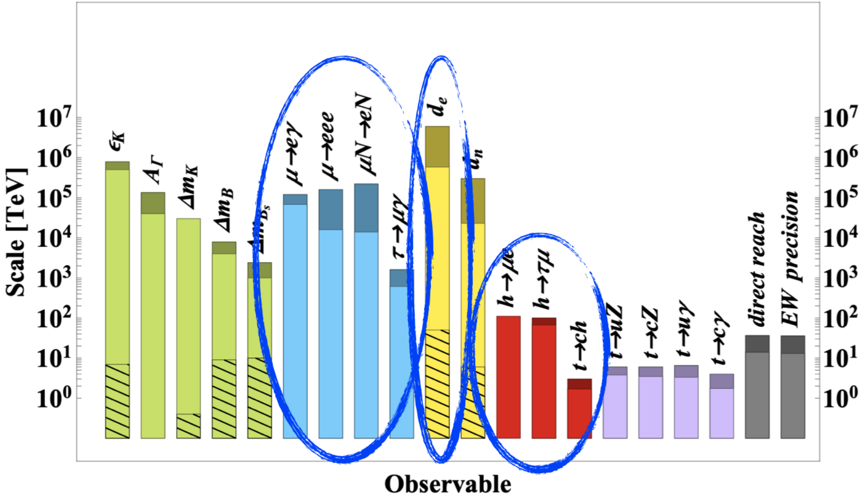
4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

$\mu \to e\gamma$ $\qquad$ $\mu \to 3e$ $\qquad$ $\mu \to e$

MuonBridge: a $\mu \to e$ conversion in Nuclei code



Bounds on the NP scale



[Physics Briefing Book, 1910.11775]

Correlating LFV signals

- **LFV** operators **@ dim-6**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\textbf{LFV}}}\,\mathcal{O}^{\textbf{dim}-\textbf{6}} + \ldots .$$

$$\mathcal{O}^{\rm dim-6} \ni \;\; \bar{\mu}_R\,\sigma^{\mu\nu}\,H\,e_L\,F_{\mu\nu}\,, \;\; (\bar{\mu}_L\gamma^\mu e_L)\left(\bar{f}_L\gamma^\mu f_L\right)\,, \;\; (\bar{\mu}_R e_L)\left(\bar{f}_R f_L\right)\,, \; f = e, u, d$$

- $\ell \to \ell'\gamma$ probe ONLY the dipole-operator (at tree level)
- $\ell_i \to \ell_j\bar{\ell}_k\ell_k$ and $\mu \to e$ in Nuclei probe dipole and 4-fermion operators
- When the dipole-operator is dominant:

$$\mathrm{BR}(\ell_i \to \ell_j\ell_k\bar{\ell}_k) \approx \alpha \times \mathrm{BR}(\ell_i \to \ell_j\gamma)$$
$$\mathrm{CR}(\mu \to e \text{ in N}) \approx \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

$$\frac{\mathrm{BR}(\mu \to \mathbf{3e})}{\mathbf{3 \times 10^{-15}}} \approx \frac{\mathrm{BR}(\mu \to \mathbf{e}\gamma)}{\mathbf{5 \times 10^{-13}}} \approx \frac{\mathrm{CR}(\mu \to \mathbf{e} \text{ in N})}{\mathbf{3 \times 10^{-15}}}$$

- **Ratios like $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ probe the NP flavor structure**
- **Ratios like $Br(\mu \to e\gamma)/Br(\mu \to eee)$ probe the NP operator at work**

SMEFT parameters [Sanchez @ this workshop]

Flavor structure in SMEFT

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Dimension 6: [Grzadkowski, Iskrzynski, Misiak, Rosiek]

- Single generation: 59 parameters
- Three generations: 2499 parameters

Proliferation arises from flavor: What if we assume flavor? [Greljo, Palavrić, Thomsen]

SMEFT $\mathcal{O}(1)$ terms (dim-6, $\Delta B = 0$)		Lepton sector																	
		MFV_L		$\mathrm{U}(3)_V$		$\mathrm{U}(2)^2\times\mathrm{U}(1)^2$		$\mathrm{U}(2)^2$		$\mathrm{U}(2)_V$		$\mathrm{U}(1)^6$		$\mathrm{U}(1)^3$		No symm.			
Quark sector	MFV_Q	41	6	45	9	59	6	62	9	67	13	81	6	93	18	207	132		
	$\mathrm{U}(2)^2\times\mathrm{U}(3)_d$	72	10	78	15	95	10	100	15	107	21	122	10	140	28	281	169		
	$\mathrm{U}(2)^3\times\mathrm{U}(1)_{d_3}$	86	10	92	15	111	10	116	12	123	21	140	10	158	28	305	175		
	$\mathrm{U}(2)^3$	93	17	100	23	118	17	124	23	132	30	147	17	168	38	321	191		
	No symmetry	703	570	734	600	756	591	786	621	818	652	813	612	906	705	1350	1149		

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The relevant SMEFT operators drastically reduce adding more flavor structures!

SMEFT parameters [Sanchez @ this workshop]

Granada Dictionary

[de Blas, Criado, Pérez-Victoria, Santiago]

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- All possible renormalizable heavy extensions in the leptonic sector
- Their matching to the SMEFT is automatized at tree-level and one-loop
- SMEFT directions: Classification with a flavor assumption

UV Field	$-\mathcal{L}_{\text{UV}}^{(4)} \supset$
$\mathcal{S}_1 \sim (\mathbf{1},\mathbf{1})_1$	$[y_{\mathcal{S}_1}]_{rij}\mathcal{S}_{1r}^\dagger \bar{\ell}_i i\sigma_2 \ell_j^c$
$\mathcal{S}_2 \sim (\mathbf{1},\mathbf{1})_2$	$[y_{\mathcal{S}_2}]_{rij}\mathcal{S}_{2r}^\dagger \bar{e}_i e_j^c$
$\varphi \sim (\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$[y_\varphi]_{rij}\varphi_r \bar{\ell}_i e_j$
$\Xi_1 \sim (\mathbf{1},\mathbf{3})_1$	$[y_{\Xi_1}]_{rij}\Xi_{1r}^{a\dagger}\bar{\ell}_i\sigma^a i\sigma_2\ell_j^c$
$N \sim (\mathbf{1},\mathbf{1})_0$	$[\lambda_N]_{ri}\bar{N}_{R,r}\tilde{\phi}^\dagger\ell_i$
$E \sim (\mathbf{1},\mathbf{1})_{-1}$	$[\lambda_E]_{ri}\bar{E}_{R,r}\phi^\dagger\ell_i$
$\Delta_1 \sim (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$[\lambda_{\Delta_1}]_{ri}\bar{\Delta}_{1L,r}\phi e_i$
$\Delta_3 \sim (\mathbf{1},\mathbf{2})_{-\frac{3}{2}}$	$[\lambda_{\Delta_3}]_{ri}\bar{\Delta}_{3L,r}\tilde{\phi} e_i$
$\Sigma \sim (\mathbf{1},\mathbf{3})_0$	$\frac{1}{2}[\lambda_\Sigma]_{ri}\bar{\Sigma}_{R,r}^a\tilde{\phi}^\dagger\sigma^a\ell_i$
$\Sigma_1 \sim (\mathbf{1},\mathbf{3})_{-1}$	$\frac{1}{2}[\lambda_{\Sigma_1}]_{ri}\bar{\Sigma}_{1R,r}^a\phi^\dagger\sigma^a\ell_i$
$\mathcal{B} \sim (\mathbf{1},\mathbf{1})_0$	$[g_{\mathcal{B}}^\ell]_{rij}\mathcal{B}_r^\mu\bar{\ell}_i\gamma_\mu\ell_j + [g_{\mathcal{B}}^e]_{rij}\mathcal{B}_r^\mu\bar{e}_i\gamma_\mu e_j$
$\mathcal{W} \sim (\mathbf{1},\mathbf{3})_0$	$\frac{1}{2}[g_{\mathcal{W}}^\ell]_{rij}\mathcal{W}_r^{\mu\,a}\bar{\ell}_i\sigma^a\gamma_\mu\ell_j$
$\mathcal{L}_3 \sim (\mathbf{1},\mathbf{2})_{-\frac{3}{2}}$	$[g_{\mathcal{L}_3}]_{rij}\mathcal{L}_{3r}^{\mu\dagger}\bar{e}_i^c\gamma_\mu\ell_j$

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SMEFT parameters [Sanchez @ this workshop]

- Consider the UV interaction

$$[y_{\mathcal{S}_1}]_{rij}\mathcal{S}^\dagger_{1r}\bar{\ell}_i i\sigma_2 \ell^c_j$$

- Extract the flavor tensor

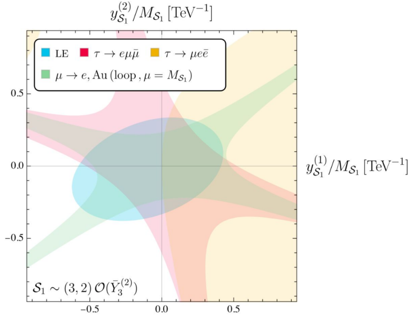
$$[y_{\mathcal{S}_1}]_{rij} = \frac{y^{(1)}_{\mathcal{S}_1}}{2}\Big[\delta_{r1}(\delta_{i2}\delta_{j3}-\delta_{i3}\delta_{j2})+\delta_{r2}(\delta_{i3}\delta_{j1}-\delta_{i1}\delta_{j3})+\delta_{r3}(\delta_{i1}\delta_{j2}-\delta_{i2}\delta_{j1})\Big],$$

- Perform the matching to the SMEFT

$$\frac{1}{M^2_{\mathcal{S}_1}}[y_{\mathcal{S}_1}]^*_{rjl}[y_{\mathcal{S}_1}]_{rik}[\mathcal{O}_{\ell\ell}]_{ijkl}$$

Flavor tensor with various different couplings

- Combined fit with correlations
- Projected two-dimensional likelihoods
- Blind lines



Polarising the muon to distinguish operators

Kuno, Okada hep-ph/9909265

$$\frac{dB(\mu\to e\gamma)}{d(\cos\theta_e)} = 192\pi^2\left(\frac{v}{\Lambda}\right)^4\left[\left|C_{D,R}\right|^2\left(1-P_\mu\cos\theta_e\right)+\left|C_{D,L}\right|^2\left(1+P_\mu\cos\theta_e\right)\right]$$

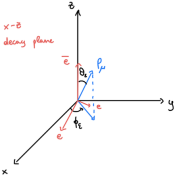
Muon polarization vector

Angle between e momentum and $\vec{P}_\mu$

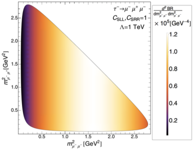
Petcov, Bolton 2204.03468

$$\frac{dB_{\mu\to 3e}}{dx_1 dx_2 d\Omega_e} = \frac{3}{2\pi}\left[C_1(x_1,x_2)+C_2(x_1,x_2)\,P_\mu\cos\theta_e+C_3(x_1,x_2)\,P_\mu\sin\theta_e\cos\phi_e+C_4(x_1,x_2)\,P_\mu\sin\theta_e\sin\phi_e\right]$$

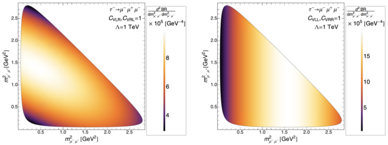
Can distinguish $C_{V,LX}, C_{V,LX}, C_{S,R}$ from $C_{V,RX}, C_{V,RX}, C_{S,L}$ but not scalars from vectors
CP asymmetries are also measureable (phase between dipoles and vectors)



- Dalitz plots for the three body also possible to distinguish operators (vector vs scalar)

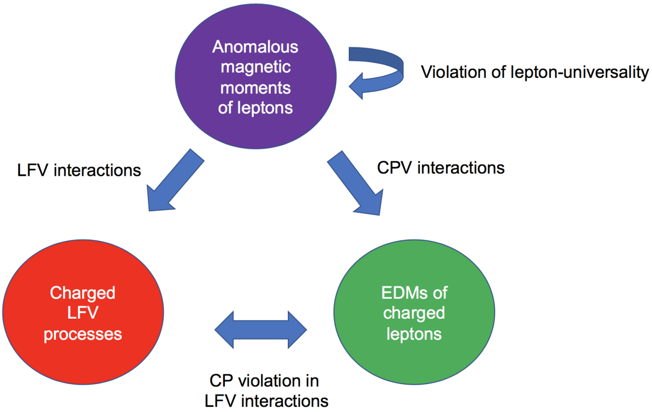


Scalars



Vectors

Probing NP in the leptonic sector



On leptonic dipoles: $\ell \to \ell' \gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R\right) F^{\mu\nu} \qquad \ell, \ell' = e, \mu, \tau \,,$$

  - **Branching ratios of $\ell \to \ell' \gamma$**

$$\frac{\mathrm{BR}(\ell \to \ell' \gamma)}{\mathrm{BR}(\ell \to \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3 \alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2 m_\ell^2 \,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell \,\mathrm{Im}(A_{\ell\ell}) \,.$$

  - **"Naive scaling":** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}} \,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j \gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}} \right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}} \right)^2$$

- **EDMs vs.** $(g-2)_\mu$

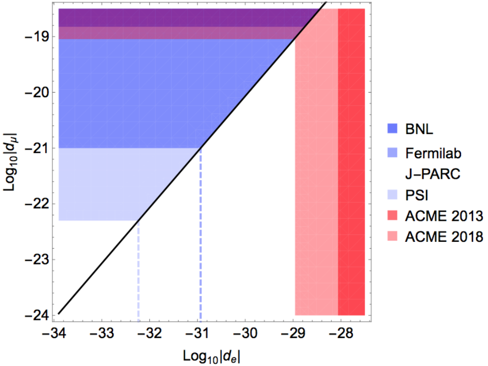
$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}} \right) e\,\mathrm{cm}\,,$$

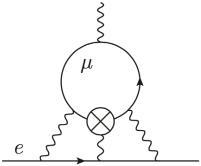
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22}\, \phi_\mu^{CPV}\; e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM





$$d_\mu \lesssim 10^{-21}\, e\,\mathrm{cm} \left(\frac{d_e}{10^{-31}\, e\,\mathrm{cm}} \right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, '18]

$$d_\mu \quad \simeq \quad \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22}\, \phi_\mu^{CPV} \quad e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, '12]

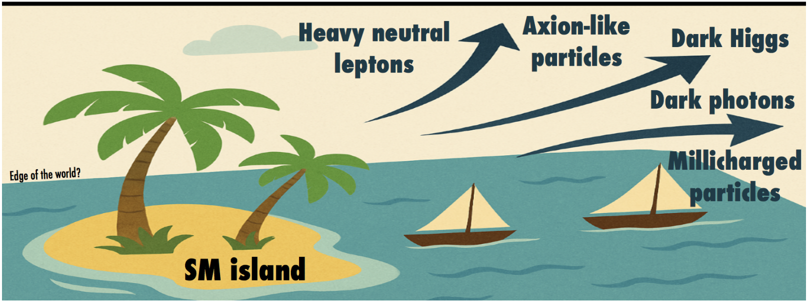
Many Options, Much Fun?

How do I make sense of the landscape of singlet options?

EFT: $$\mathcal{L}_{\rm portal} \supset \frac{c_n}{\Lambda^{\Delta_{\rm dark}+\Delta_{\rm SM}-4}} \mathcal{O}_{\rm dark}\mathcal{O}_{\rm SM}$$

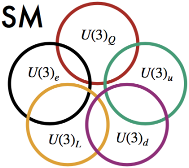
Any gauge invariant combination of SM fields

Lower dimensional portals $\mathcal{O}_{\rm dark} = \{\underbrace{\phi}_{\text{Scalar}}, \underbrace{N}_{\text{Fermion}}, \underbrace{V_\mu}_{\text{Vector}}, \dots\}$ $\longrightarrow$ **Largest rates**



Flavoured Axion-like Portal

Accidental symmetries of the SM might be broken by light new particles feebly coupled to the SM



Example: Flavour dependent Peccei-Quinn charges

[Calibbi et al 1612.08040]
[Ema et al 1612.05492]

Flavour conserving + **Flavour violating**

$$\mathscr{L} \supset \sum_i \frac{\partial_\mu a}{2f_a} \bar{f}_i C^A_{ii} \gamma_\mu \gamma_5 f_i + \sum_{i \neq j} \frac{\partial_\mu a}{2f_a} \bar{f}_i \gamma^\mu \left(C^V_{ij} + C^A_{ij} \gamma_5 \right) f_j$$

[Feng et al hep-ph/9709411]

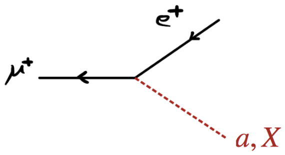
⚠ Hierarchy between **flavour-conserving** and **flavour-violating** depends on the UV theory

Flavour anarchy: $C^{A,V}_{ij}(\Lambda_{\mathrm{UV}}) \sim \mathcal{O}(1)$

Minimal favour violation: $C^{A,V}_{ij}(\Lambda_{\mathrm{UV}}) = 0$

What New Physics?

Lepton-flavour Violating

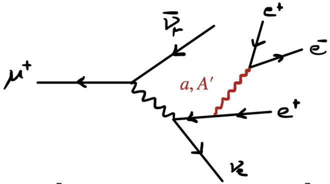


$$\mathcal{L}_{\mathrm{FV-a}} = \frac{m_\mu}{2f_a}\frac{1}{|C_{e\mu}|}a\bar{\mu}(C^V_{e\mu} + C^A_{e\mu}\gamma_5)e$$

Bump hunt in electron momentum p_e

[Bayes et al (TWIST Collaboration) 1411.1770]
[Perrevoort et al (Mu3e Collaboration) 1812.00741]

Lepton-flavour Conserving



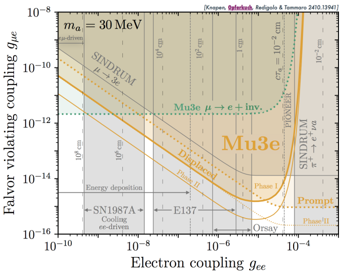
$$\mathcal{L}_0 \supset X\left[\bar{\mu}(g^\mu_S + g^\mu_P\gamma_5)\mu + \bar{e}(g^e_S + g^e_P\gamma_5)e\right]$$

$$\mathcal{L}_1 \supset X^\alpha\left[\bar{\mu}\gamma_\alpha(g^\mu_V + g^\mu_A\gamma_5)\mu + \bar{e}\gamma_\alpha(g^e_V + g^e_A\gamma_5)e\right]$$

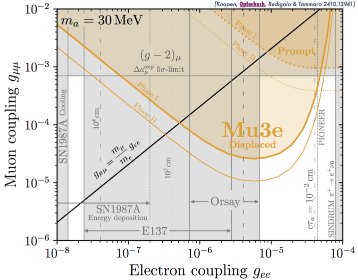
Bump hunt in e^+e^--pair invariant mass

[Echenard et al 1411.1770]
[Perrevoort et al (Mu3e Collaboration) 1812.00741]

Lepton Flavour Violating ALP



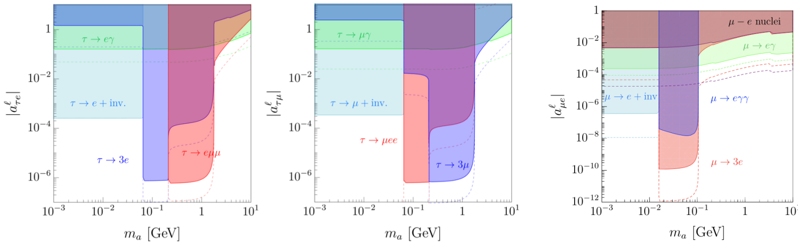
Lepton Flavour Conserving ALP



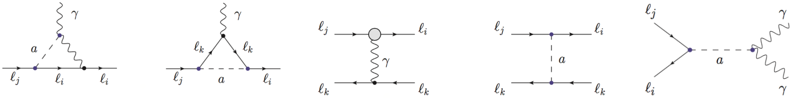
[See also Jure Zupan's talk @ this workshop]

LFV from light NP: axionlike particles (ALP)

$$\mathcal{L}_{\rm eff}^{d\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2 a^2}{2}$$
$$+ e^2\, c_{\gamma\gamma}\frac{a}{\Lambda}F_{\mu\nu}\widetilde{F}^{\mu\nu} + g_s^2\, c_{gg}\frac{a}{\Lambda}G_{\mu\nu}\widetilde{G}^{\mu\nu} - \frac{\partial_\mu a}{\Lambda}\sum_{f,i,j}\bar{f}_i\gamma^\mu(v_{ij}^f - a_{ij}^f\gamma_5)f_j$$



[Cornella, P.P. & Sumensari, '19.]



$$\mathcal{B}(\ell_i \to \ell_j\gamma\gamma) \approx \mathcal{B}(\ell_i \to \ell_j a) \times \mathcal{B}(a \to \gamma\gamma) \quad \Rightarrow \quad \mathrm{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k) \not\approx \alpha \times \mathrm{BR}(\ell_i \to \ell_j\gamma)$$

Message: correlations among LFV signals discriminate heavy vs. light NP!

Low-energy vs high-energy frontiers

Complementary (*not* ALTERNATIVE!) approach →
HIGH-PRECISION SMALL/MID-SCALE EXPS.

Low-energy high-precision exps. can exploit :

- many recent ***advances in experimental techniques and technologies*** + **(experimental as well as theoretical)** ***synergies*** with adjacent areas of particle physics (atomic, molecular, optical, nuclear, particle physics)
- the relevant impact of ***quantum mechanical virtual effects*** on physical phenomena → access to the exploration of BSM new physics areas (large energy scales, very feebly coupled new particles, hidden sectors, etc.) difficult to be probed by traditional HE particle physics

SYNERGY between small/mid-scale & large-scale experiments → casting a wider and tighter net for possible effects of BSM physics

Community Planning Exercise: Snowmass 2021 Blum, Winter et al. arXiv:2209.08041v2 ⇒ **2023 P5 (Particle Physics Project Prioritization Panel) Report**

[Masiero @ this workshop]

Low-energy vs high-energy LFV

Purely leptonic contact interations:

- $e^+e^- \to \mu\tau$,
- $\mu^+\mu^- \to \mu e$,
- $\mu^- \to eee$, … .

4-leptons operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$

Dipole operators	
Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$

Loop generated dipoles:

- $\mu^- \to e\gamma$,
- $\tau^- \to e\gamma$,
- $\tau^- \to \mu\gamma$.

2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$

Semi-leptonic operators:

- $\mu N \to e N$,
- $pp \to l_i l_k$,
- $B \to \pi e\mu$,
- $B \to K\mu\tau$, … .

Lepton-Higgs Interactions:

- $h \to \tau\mu$,
- $Z \to \tau\mu$,
- $e^+e^- \to e\tau$,
- $\tau^- \to \mu ee$.

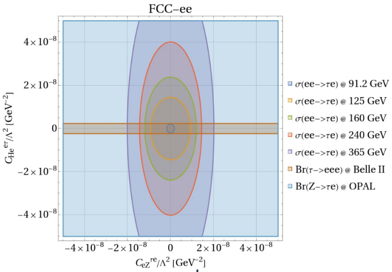
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

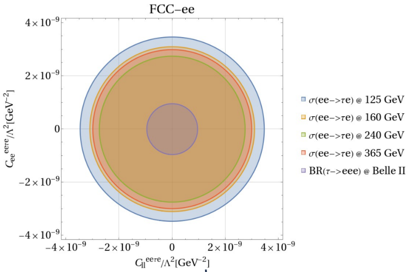
Energy dependence of LFV signals

$$\Gamma(\tau \to 3e) \sim \frac{m_\tau^5}{\Lambda^4} \qquad \Gamma(Z \to \tau e) \sim \frac{m_Z^5}{\Lambda^4} \qquad \sigma(ee \to \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

- # of Z events @ FCC-ee: $N_Z(e^+e^- \to Z) \sim 10^{11}$
- # of Z events @ MuC: $N_Z(\mu^+\mu^- \to \bar{\nu}\nu Z) \sim 10^{11}\left(\frac{\sqrt{s}}{30\ TeV}\right)^4$

LFV @ FCC-ee





$(\bar{L}_L \sigma^{\mu\nu} e_R)\Phi Z_{\mu\nu}$
vs
$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R \gamma^\mu e_R)$

FCC-ee constraints on 2-dimensional parameter space:

- LFV Z couplings better probed by FCC-ee than BelleII,
- LFV 4f operators better probed by Belle II than FCC-ee.

$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$
vs
$(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$

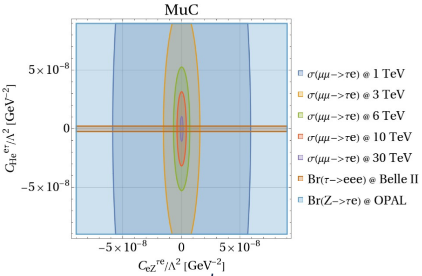
Energy dependence of LFV signals

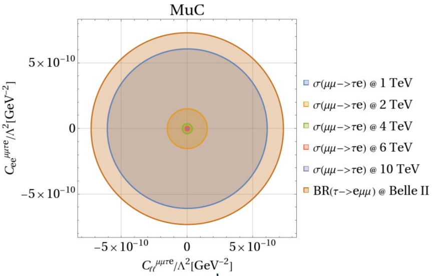
$$\Gamma(\tau \to 3e) \sim \frac{m_\tau^5}{\Lambda^4} \qquad \Gamma(Z \to \tau e) \sim \frac{m_Z^5}{\Lambda^4} \qquad \sigma(ee \to \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

Message: The high luminosity FCC-ee is competitive with Belle II to probe LFV

[Bartocci, Pagani, PP & Scantamburlo, to appear. See also Calibbi et al., '21 & Altmannshofer et al. '23]

LFV @ Muon Collider





$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi Z_{\mu\nu}$
vs
$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$

MuC constraints on 2-dimensional parameter space:

- LFV Z couplings probed by both FCC-ee and BelleII,
- LFV 4f operators better probed by MuC than Belle II.

$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$
vs
$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$

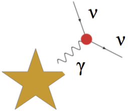
Energy dependence of LFV signals

$$\Gamma(\tau\to 3e)\sim\frac{m_\tau^5}{\Lambda^4}\qquad \Gamma(Z\to\tau e)\sim\frac{m_Z^5}{\Lambda^4}\qquad \sigma(\mu\mu\to\tau e)\sim\frac{E_{CM}^2}{\Lambda^4}$$

Message: Energy helps accuracy!

[Bartocci, Pagani, PP & Scantamburlo, to appear.]

Probes of Neutrino Magnetic Moments



- **Astrophysical and cosmological observations**

Astrophysical bounds from stellar energy loss measurements

$$\mu_\nu \lesssim 2 \times 10^{-12}\, \mu_B$$

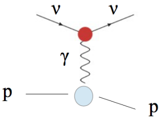
From the red giant branch of modular clusters

[Capozzi, Raffelt, PRD 102 (2020) 8, 083007]

Bounds from CMB and BBN measurements of the effective neutrino number

$$\mu_\nu \lesssim 3 \times 10^{-12}\, \mu_B$$

[Li, Xu, *JHEP* 02 (2023) 085]



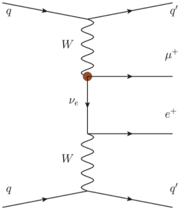
- **Low energy (solar) neutrino scattering experiments**

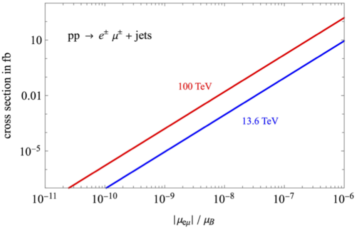
$$\mu_\nu \lesssim 6.3 \times 10^{-12}\, \mu_B$$ XENONnT Phys.Rev.Lett. 129 (2022) 16, 161805

$$\mu_\nu \lesssim 6.2 \times 10^{-12}\, \mu_B$$ LUX-ZEPLIN Phys.Rev.Lett. 131 (2023) 4, 041002

(Future) Collider Probes: Hadron Colliders

$$C^W_{\alpha\beta}(\Lambda_{NP}) = -\frac{[\mu]_{\alpha\beta}}{2v^2 e}$$





Background evaluated from a recast of the ATLAS search [arXiv:2403.15016 [hep-ex]].

"Search for heavy Majorana neutrinos in $e^{\pm}e^{\pm}$ and $e^{\pm}\mu^{\pm}$ final states via WW scattering in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector"

2σ excl. sensitivity   In the most optimistic case of a **negligible background**

HL-LHC	$\lvert\mu_{e\mu}\rvert < 6.1 \cdot 10^{-8}\,\mu_B$	$\{300\,\text{fb}^{-1}\}$,	$\lvert\mu_{e\mu}\rvert < 1.9 \cdot 10^{-8}\,\mu_B$	$\{3\,\text{ab}^{-1}\}$
FCC-hh	$\lvert\mu_{e\mu}\rvert < 3.4 \cdot 10^{-9}\,\mu_B$	$\{3\,\text{ab}^{-1}\}$,	$\lvert\mu_{e\mu}\rvert < 1.0 \cdot 10^{-9}\,\mu_B$	$\{30\,\text{ab}^{-1}\}$

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Large electron-muon transition moments of the order of $10^{-12}\mu_B$ for Majorana neutrinos are accessible experimentally at a high-energy Muon Collider.

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - We can expect any deviation from the SM expectations below the current bounds.
  - LFV processes and leptonic EDMs do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
  - FCC-ee and a high-energy Muon Collider are complementary with each other and also with Belle II and LHCb to probe LFV.

Message: an exciting program is in progress at the Intensity Frontier which would greatly benefit from the next high-energy frontier program (FCC & MuC)!