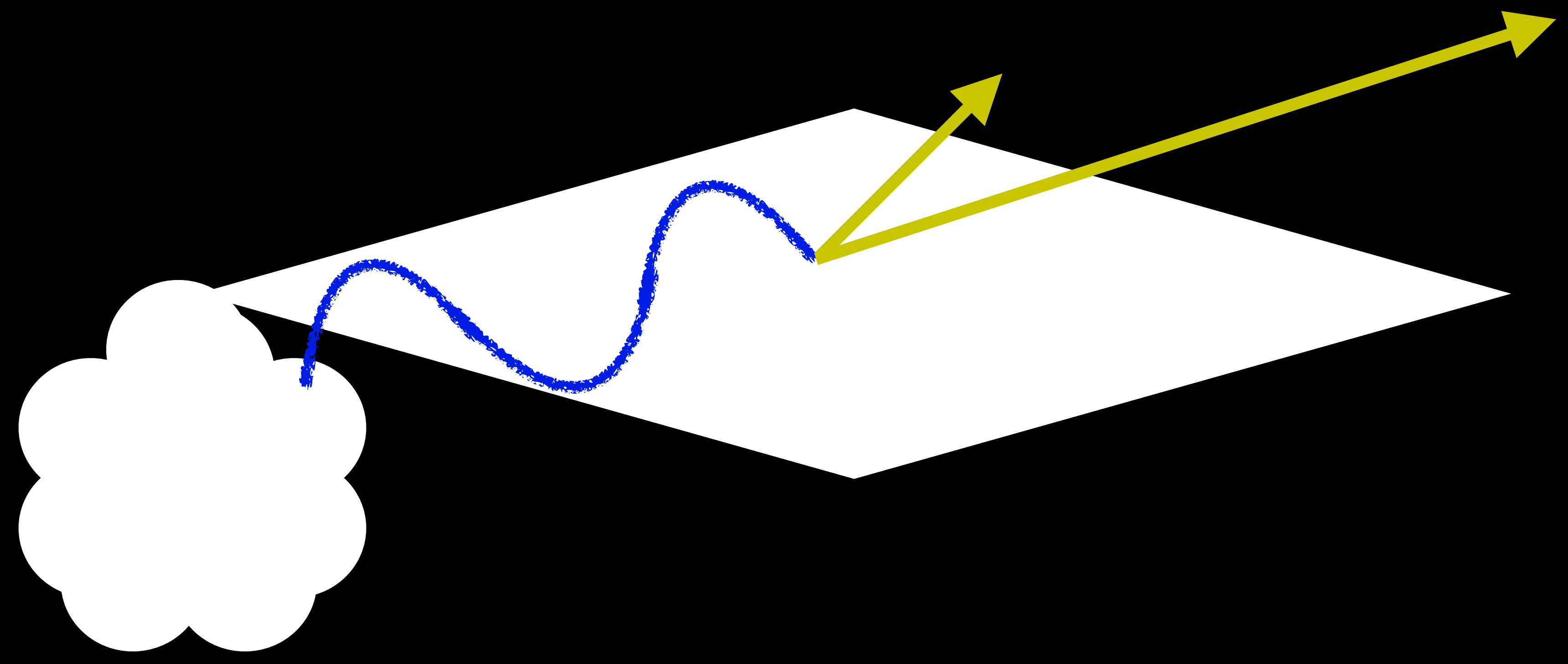
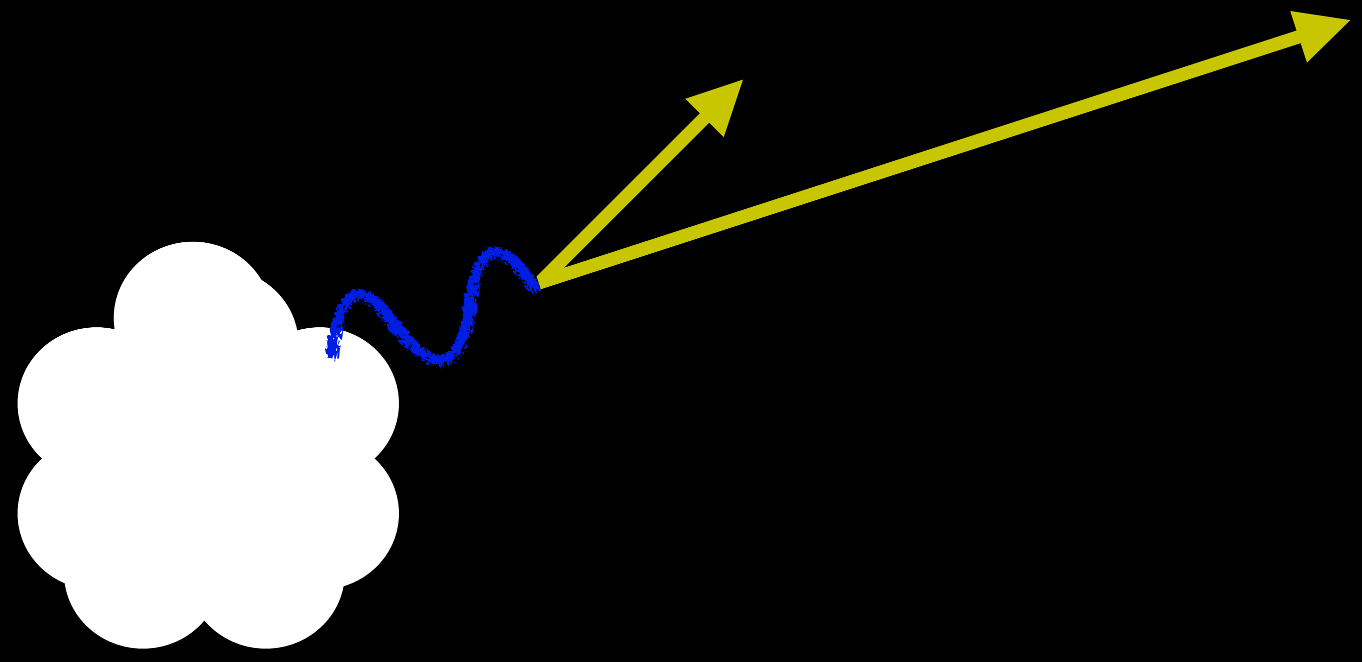
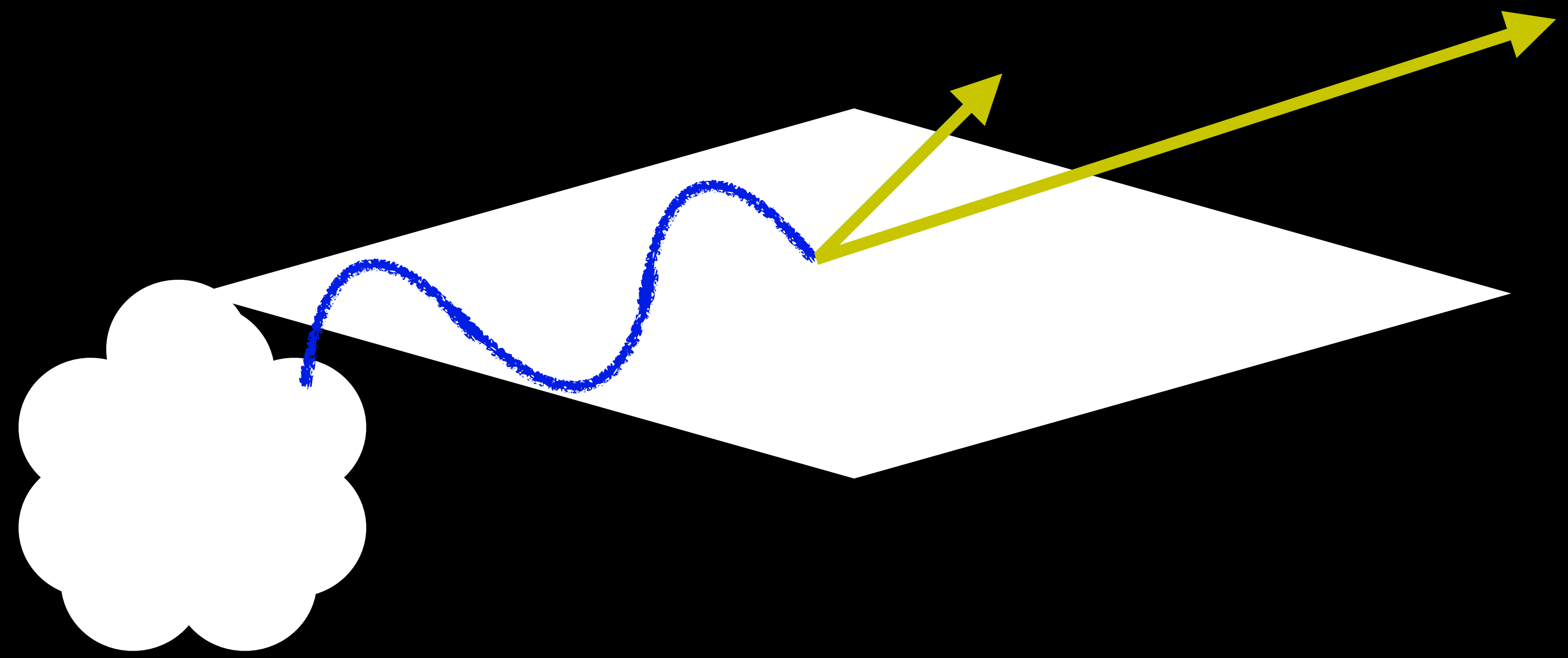


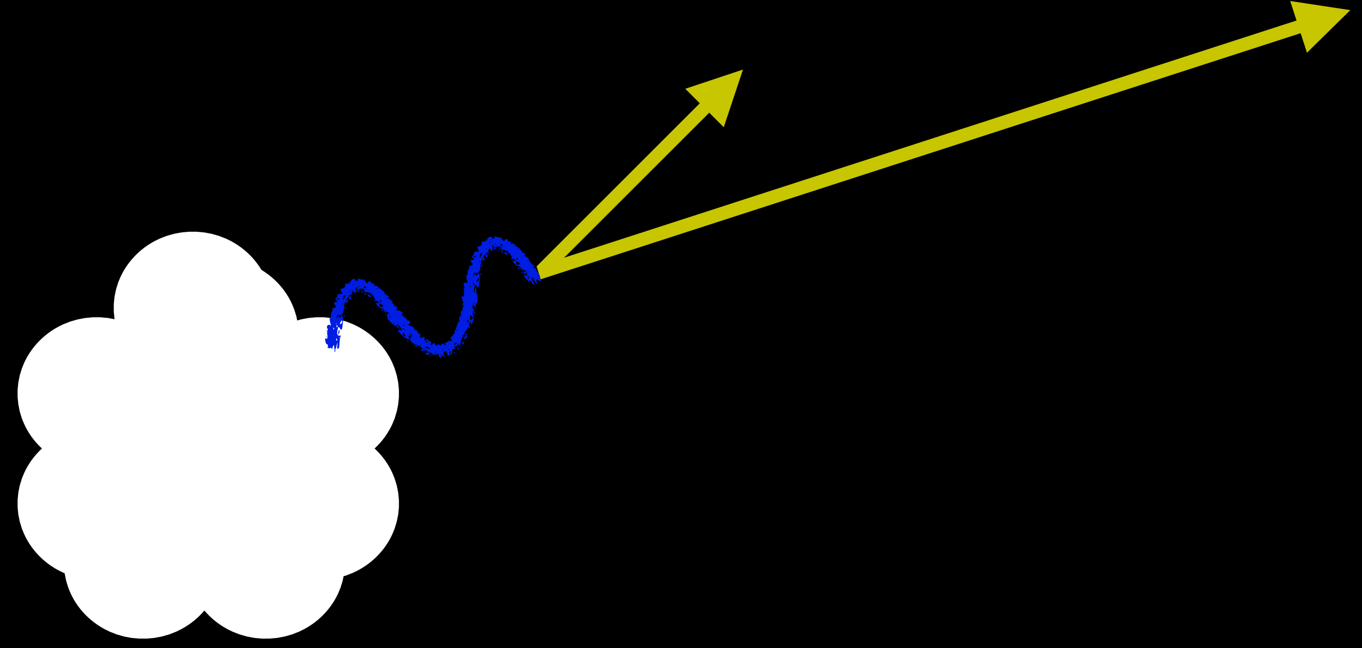


Radiative muon capture as a background at Mu2e

$$\mu^- \, \mathrm{Al} \to \nu_\mu \, \gamma^{(*)} \, \mathrm{Mg} \qquad \mu^- \, \mathrm{Al} \to e^+ \, \mathrm{Na}$$

Ryan Plestid | Friday June 13 2025 | CLFV Searches with the Mu2e Experiment | LNF Frascati, IT





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Ryan Plestid | Friday June 13 2025 | CLFV Searches with the Mu2e Experiment | LNF Frascati, IT

Background and Motivation

Motivation:

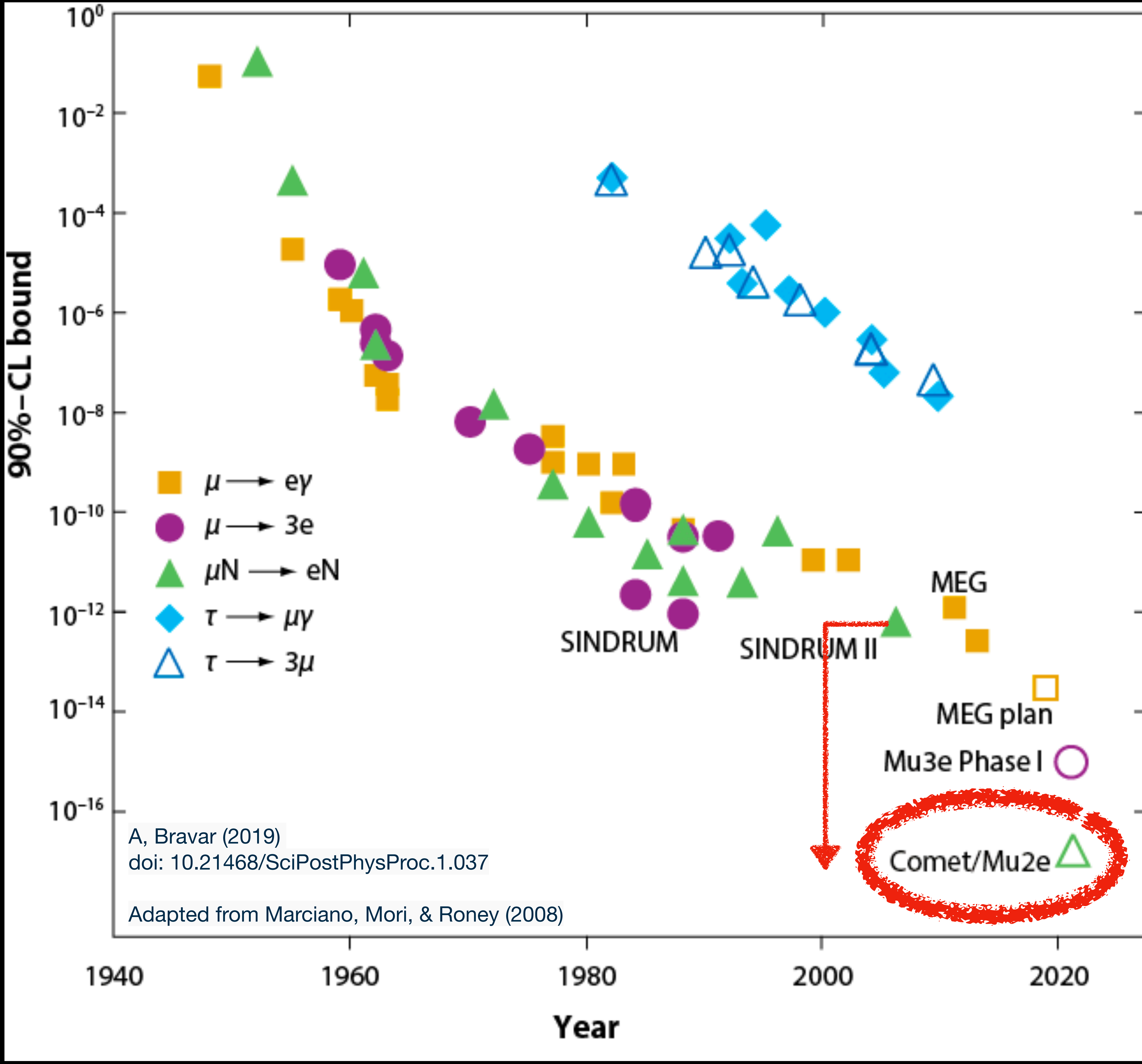
Smoking guns of BSM physics

$$\mu \to 3e$$

$$\mu \to e\gamma$$

$$\mu A \to eA$$

$$\mu^- A \to e^+ A^{2-}$$



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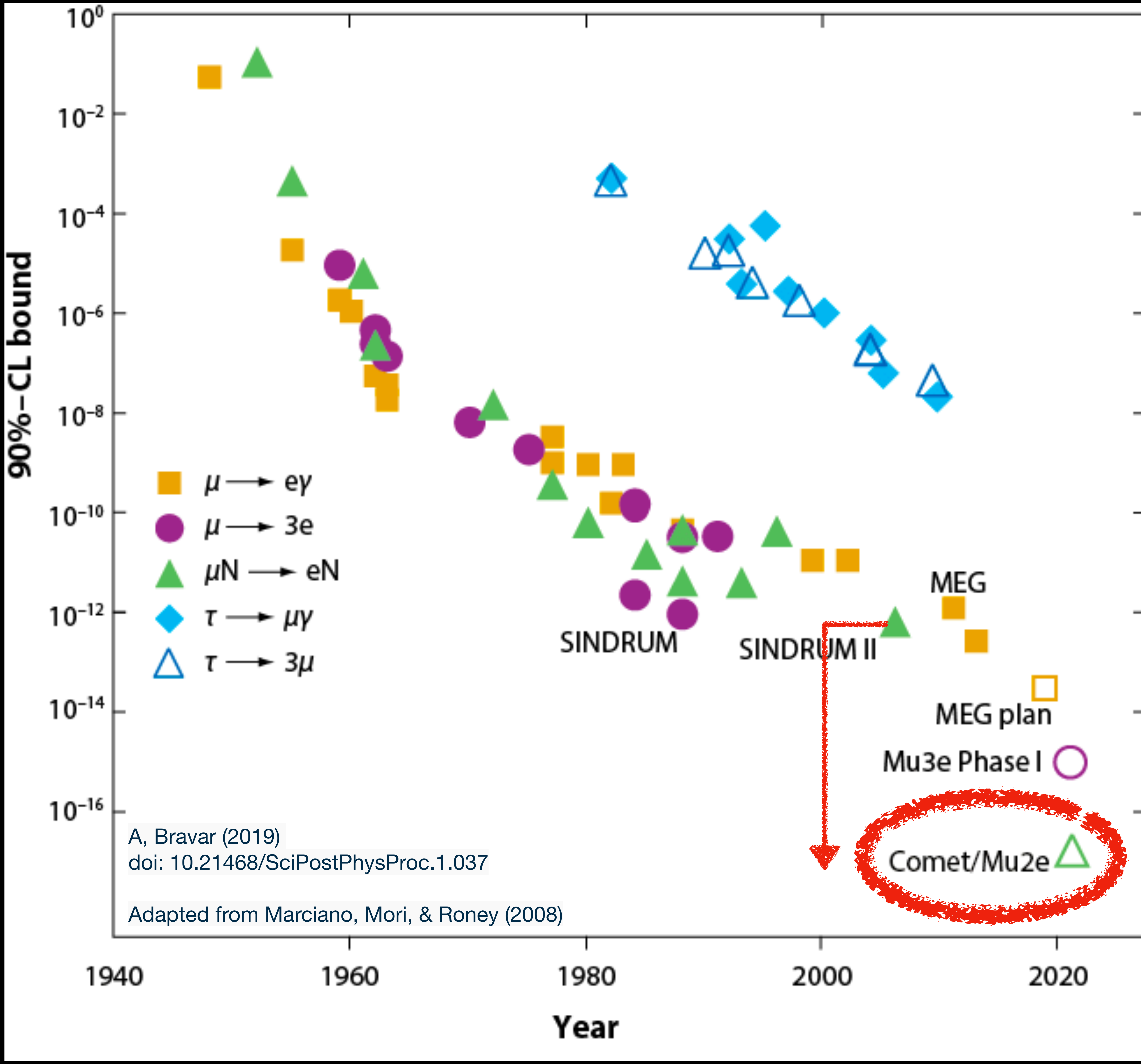

$$\mu A \to eA$$

$$\mu^- A \to e^+ A^{2-}$$

$$E_{e^-} = m_\mu - B_\mu - T_R \approx 105 \text{ MeV}$$

$$E_{e^-} = m_\mu - B_\mu - \Delta M_A - T_R \approx 92 \text{ MeV}$$

$$\mu A \to eA$$

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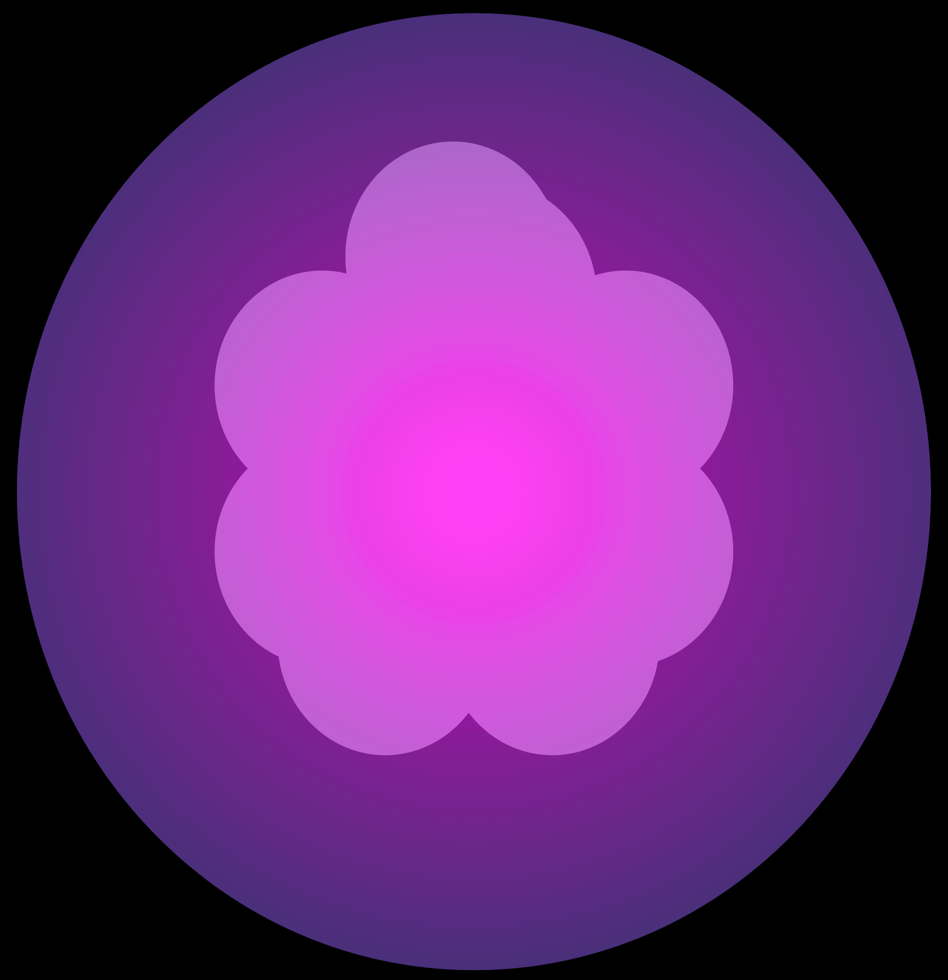

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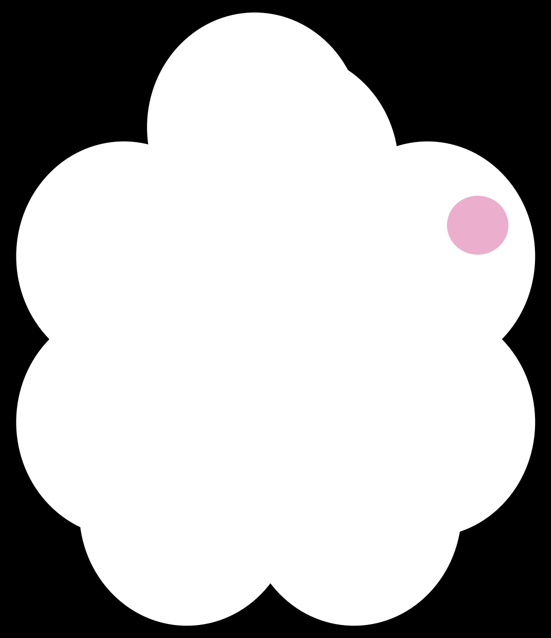

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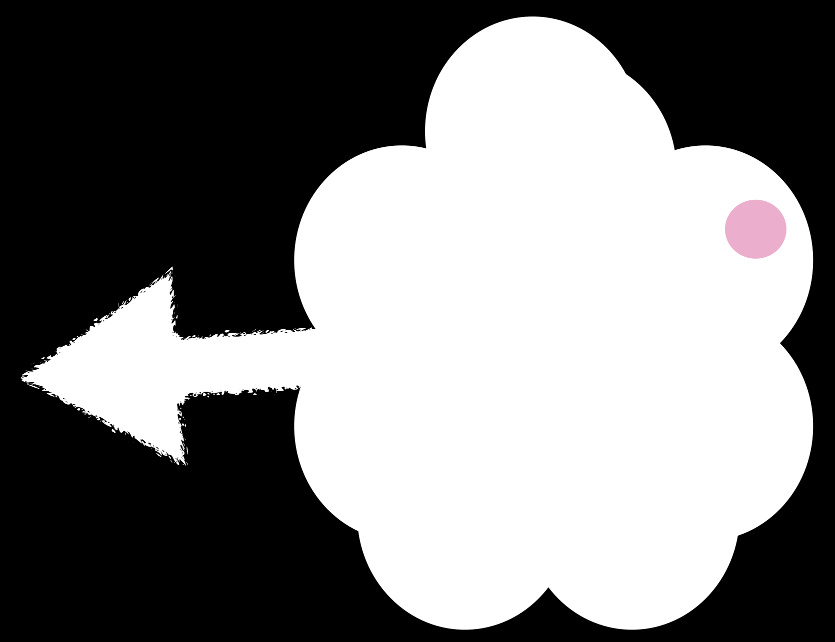

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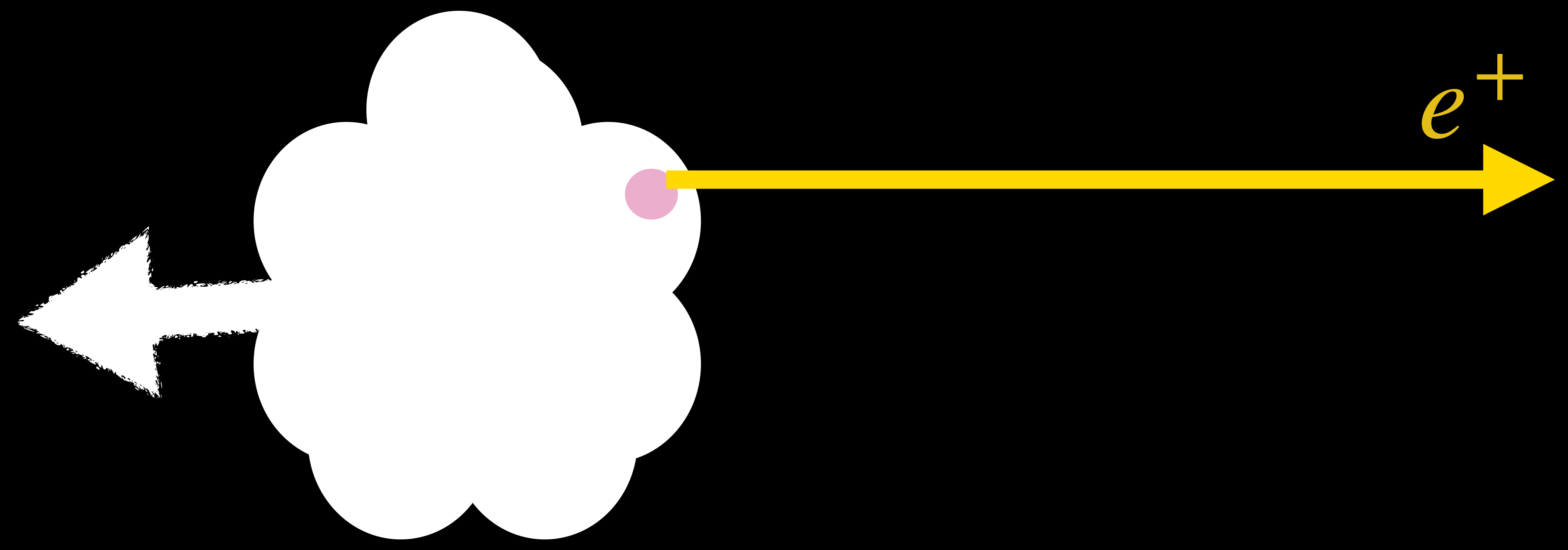

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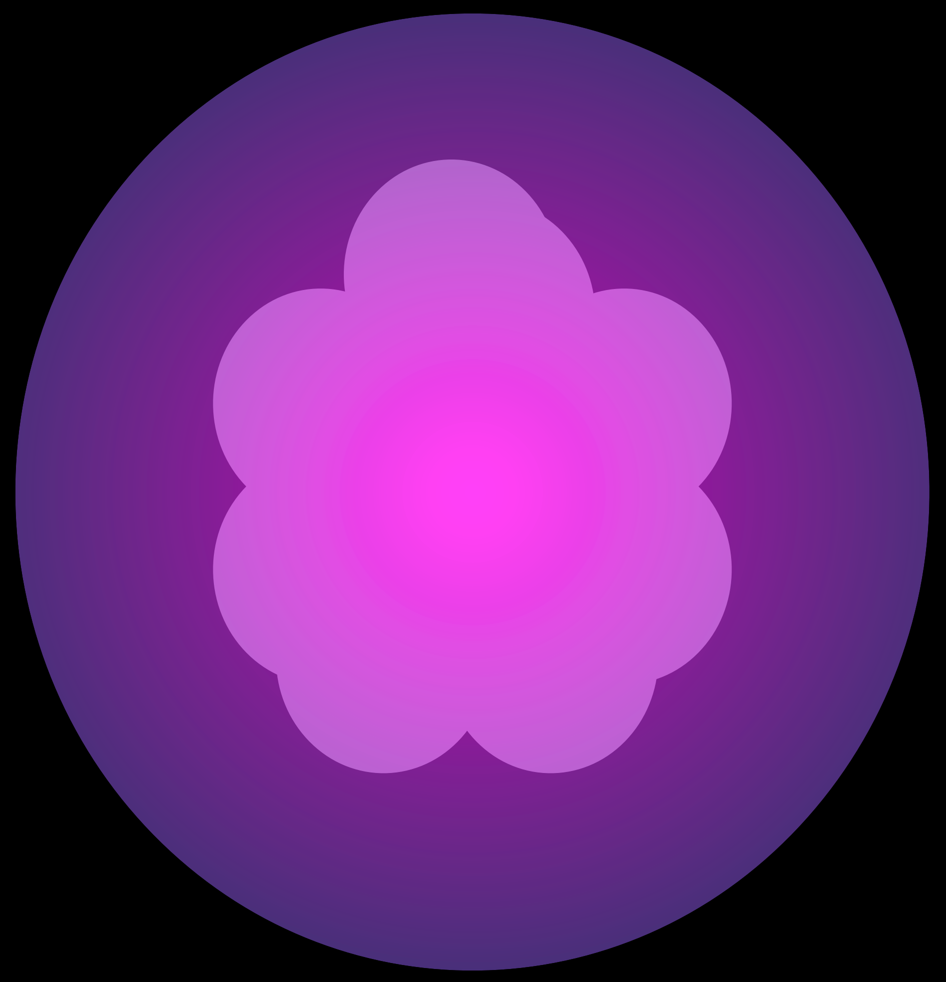

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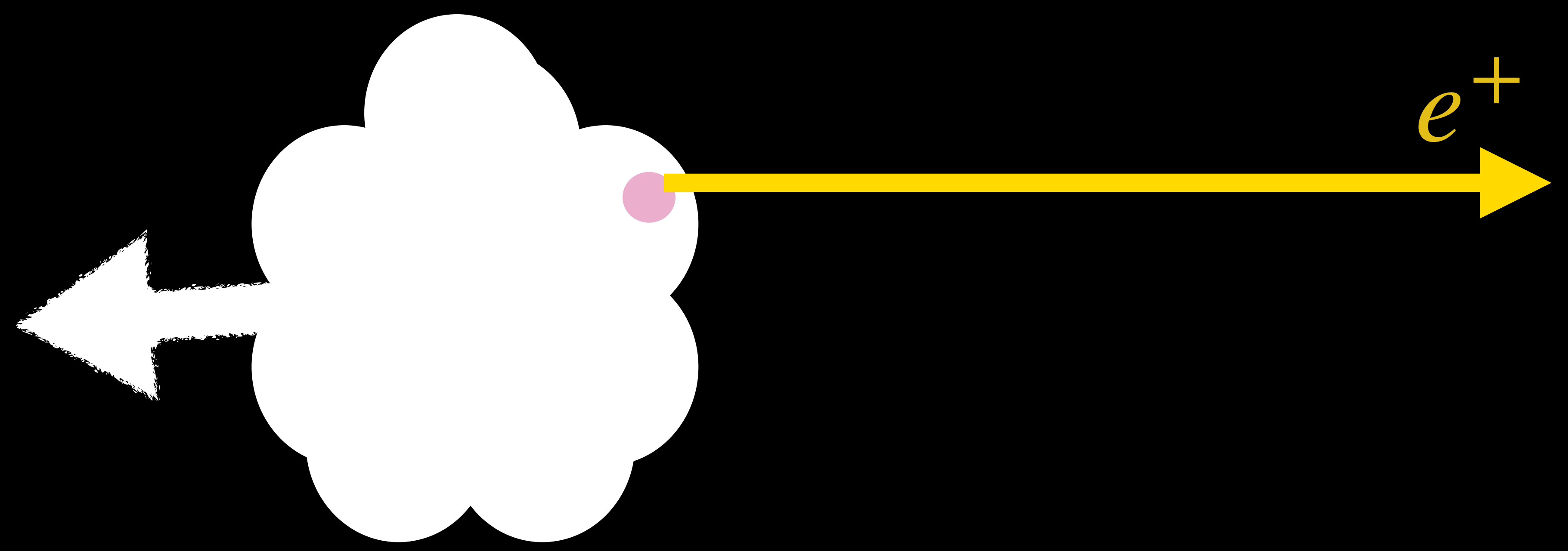

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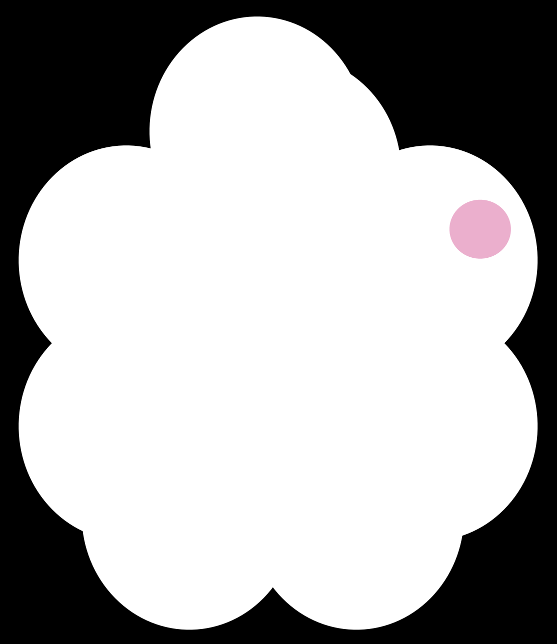

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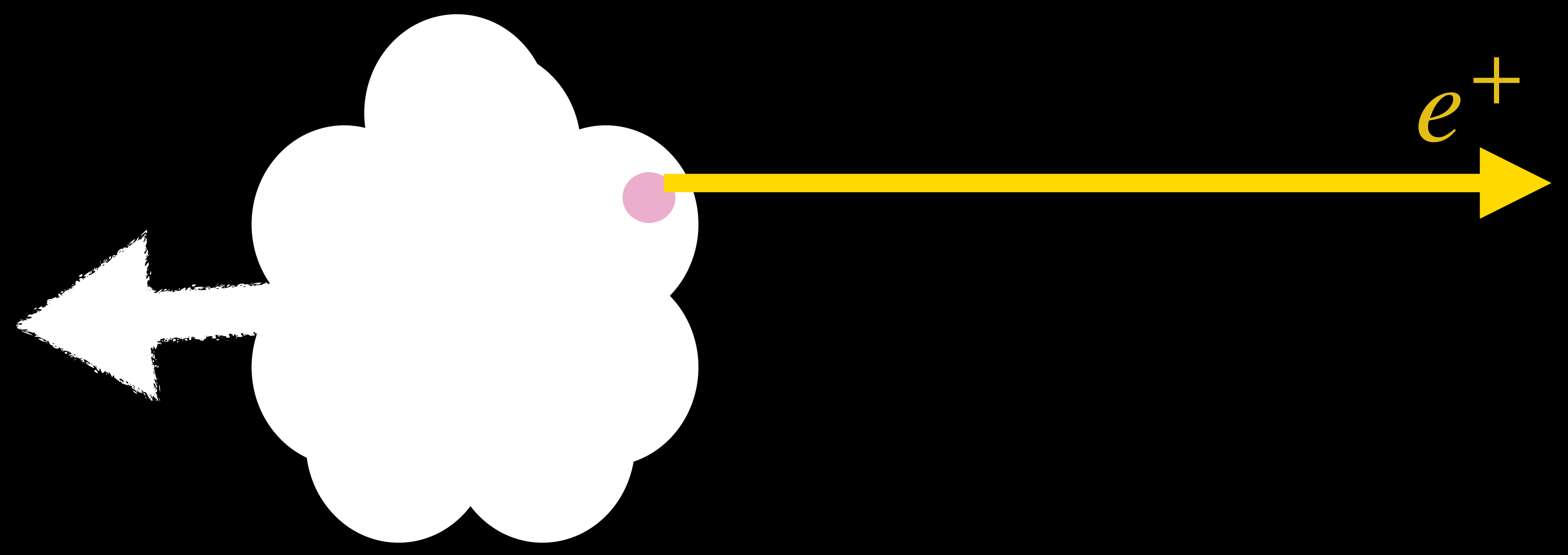

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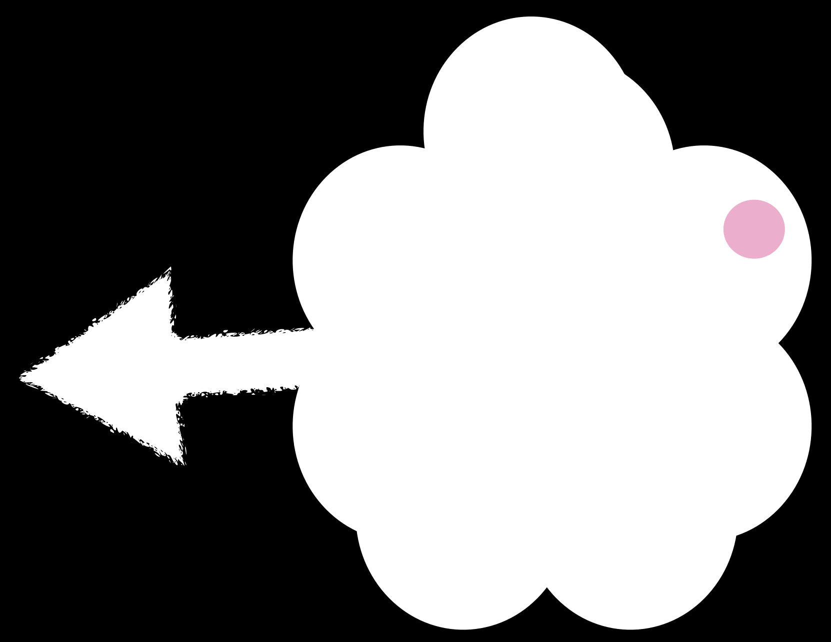

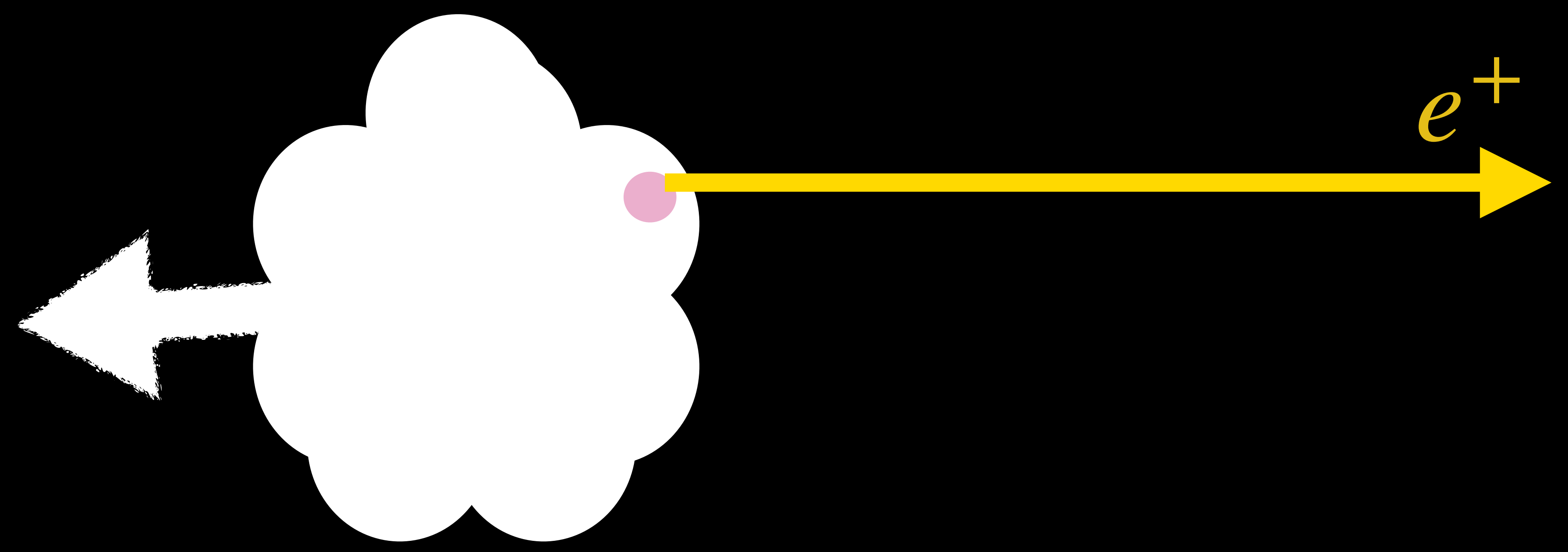

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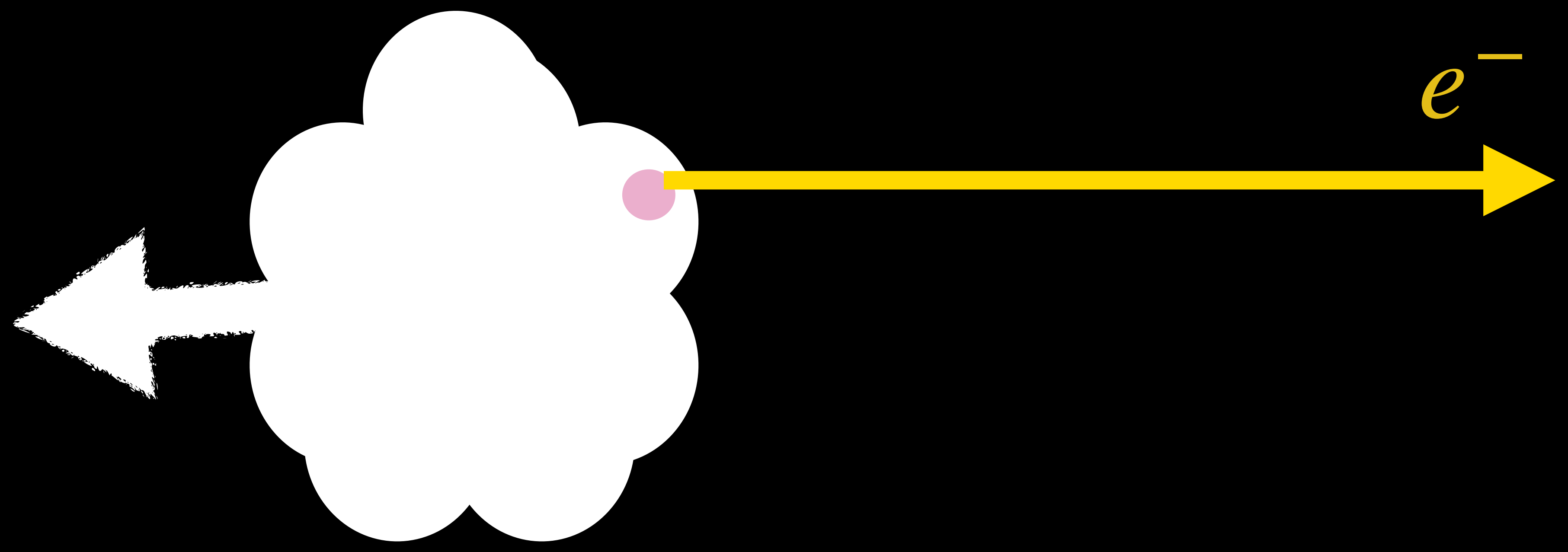

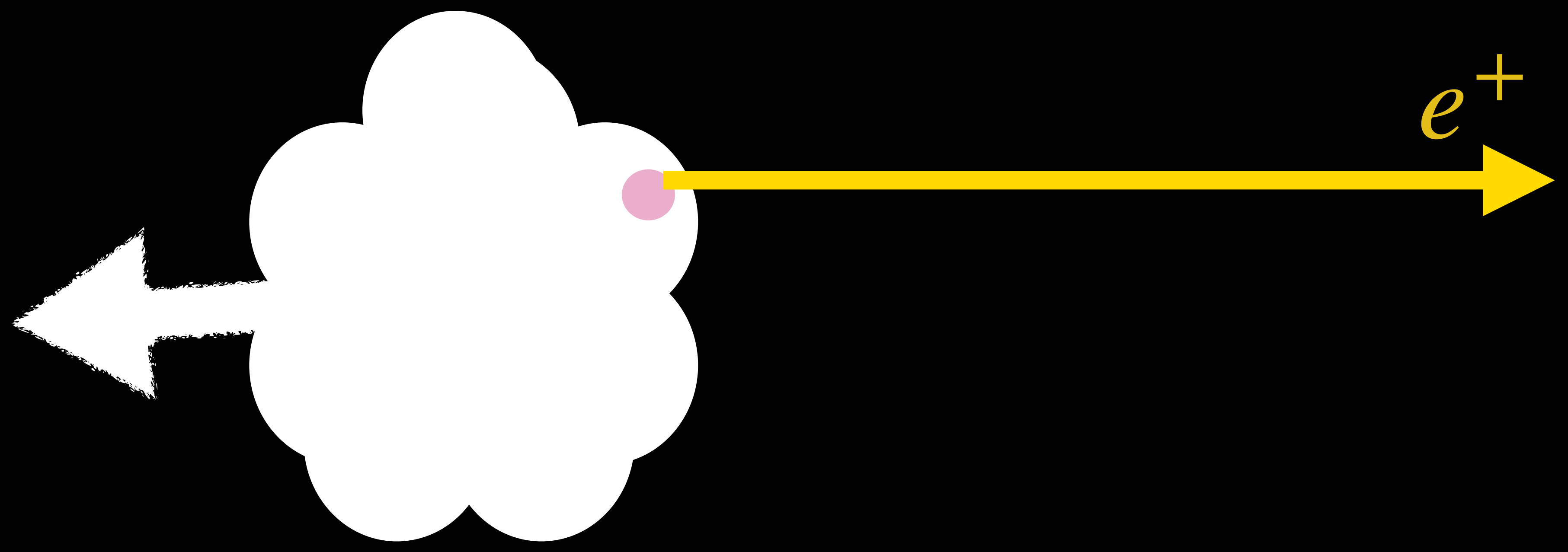

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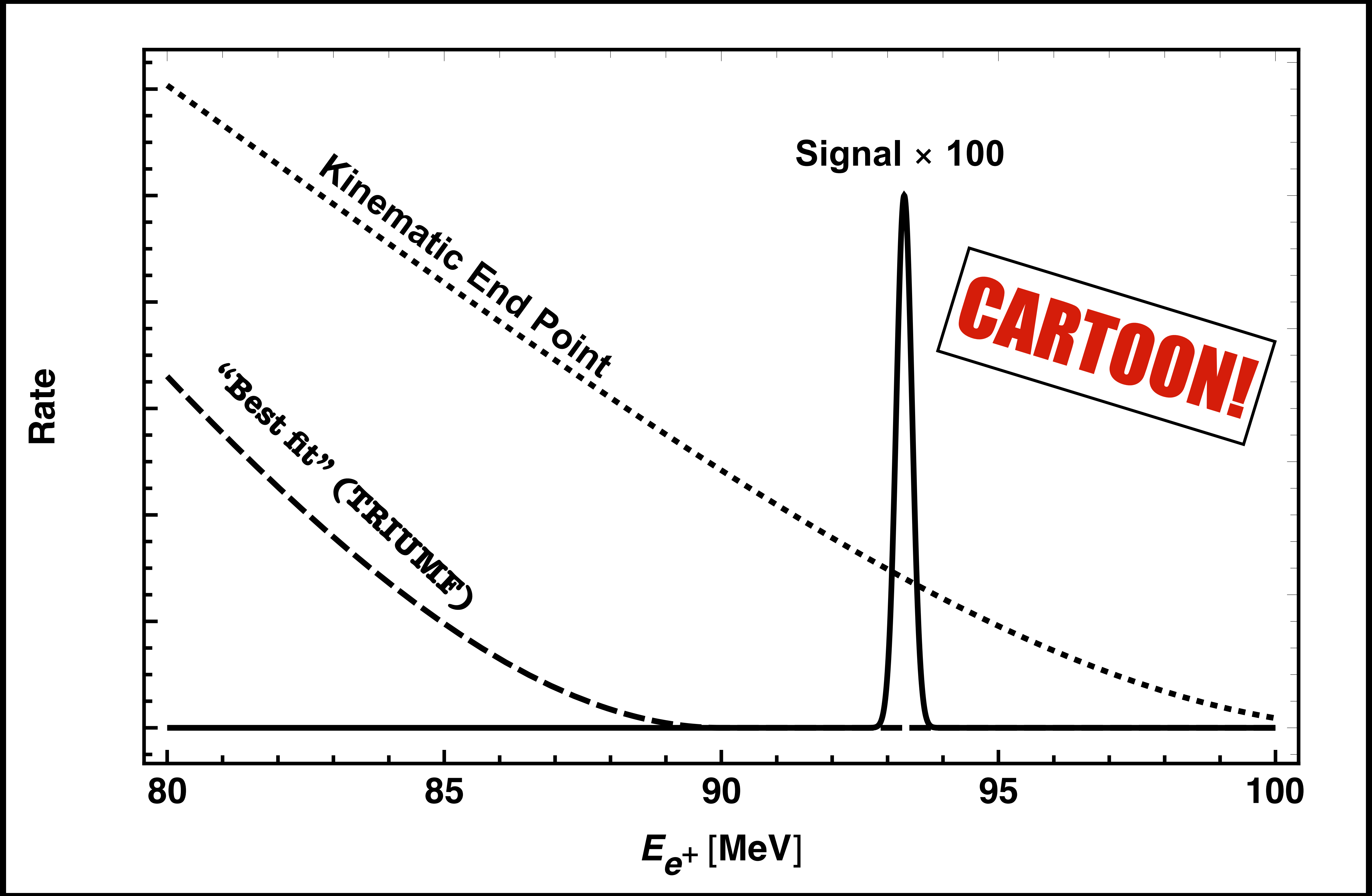
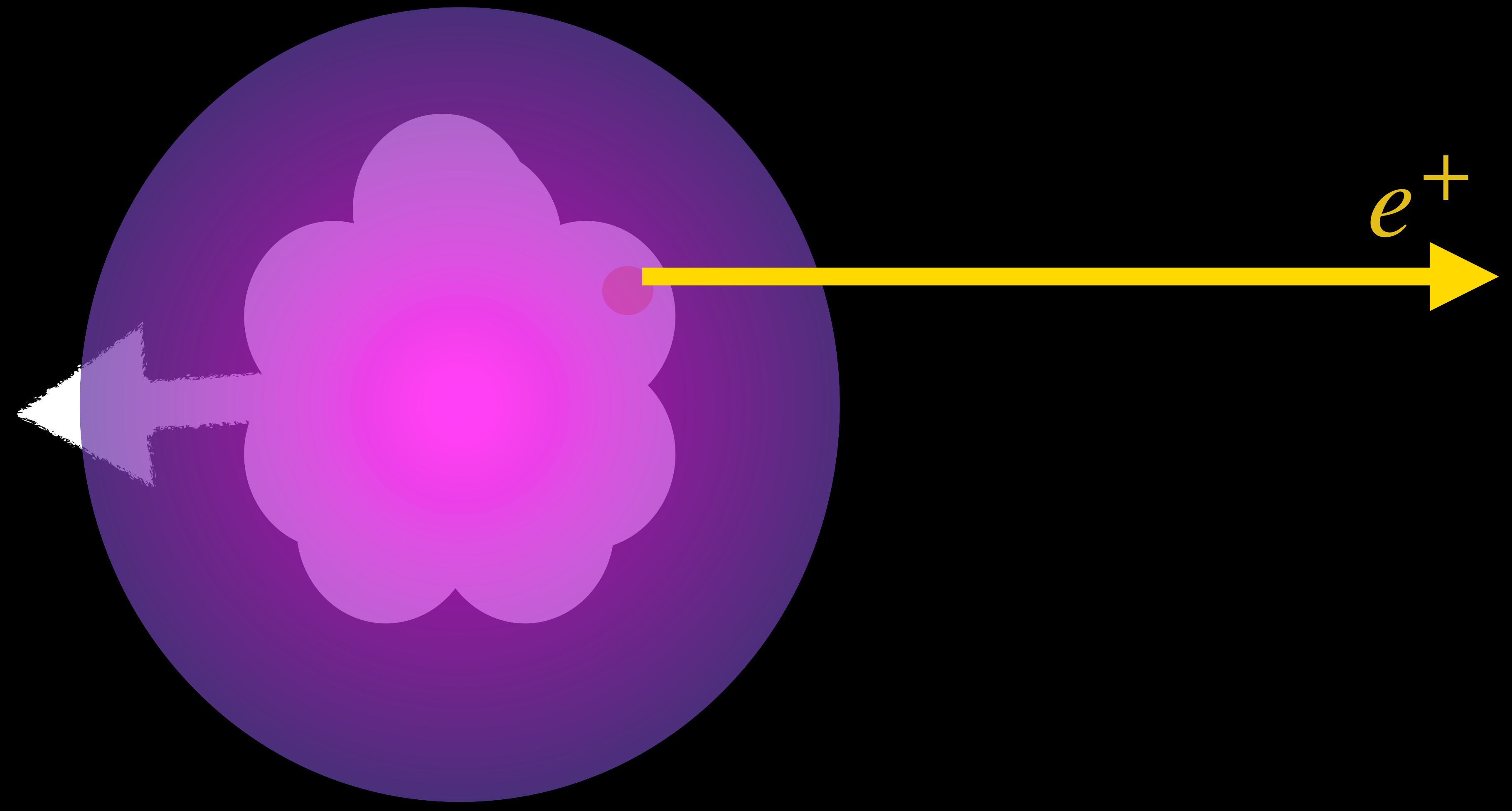
Signal: Positron from muon capture on ^{27}Al





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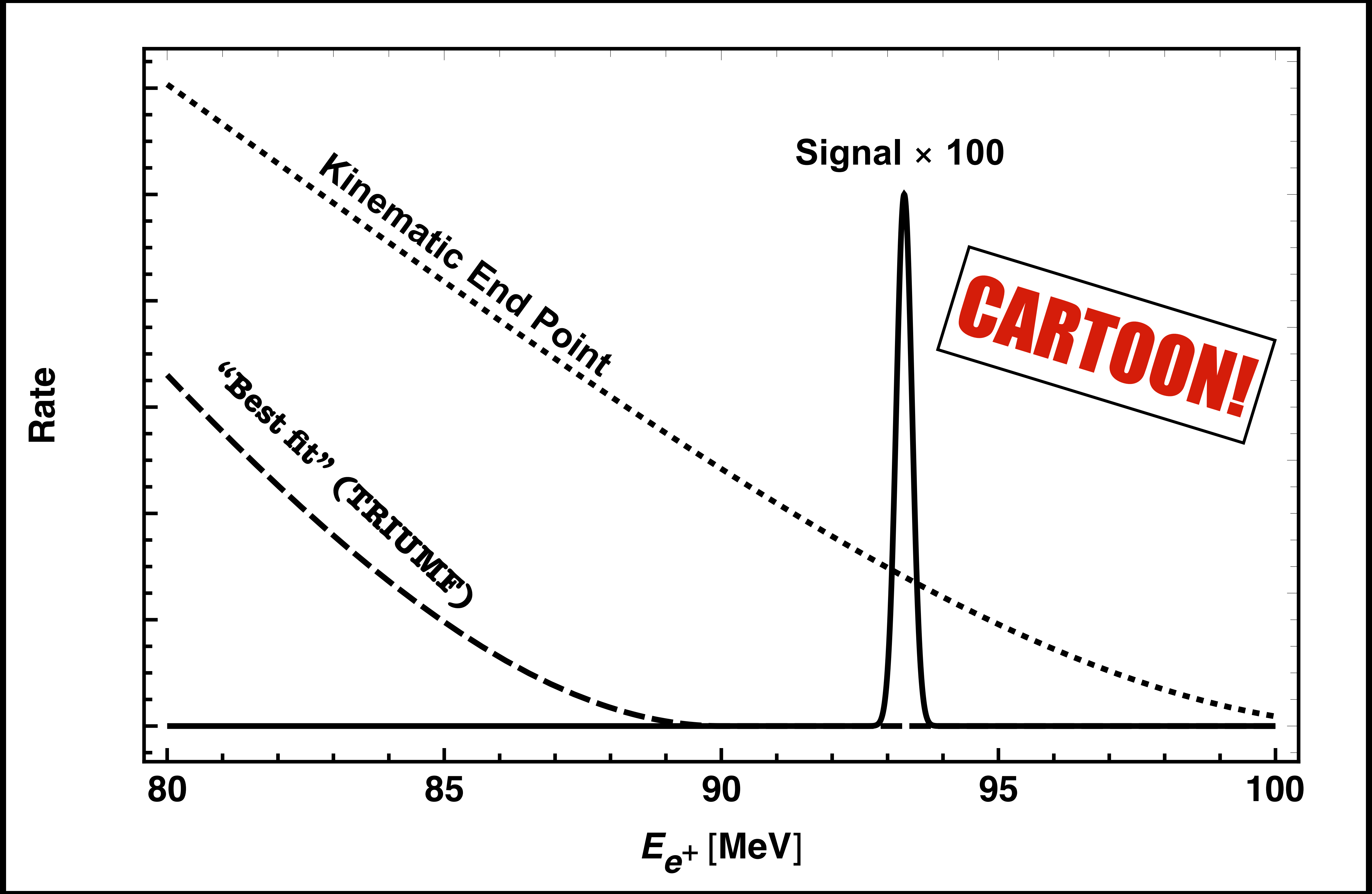
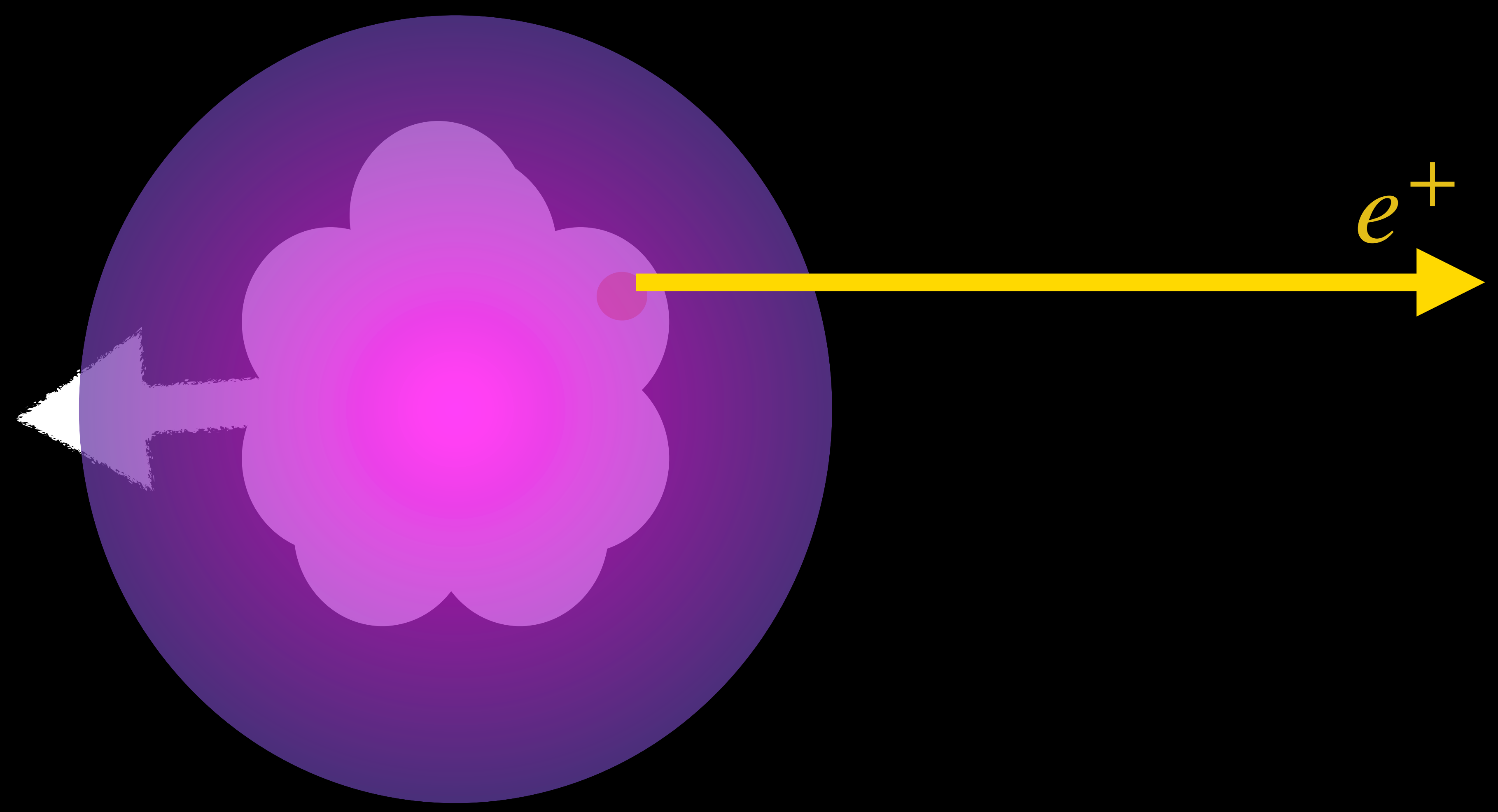
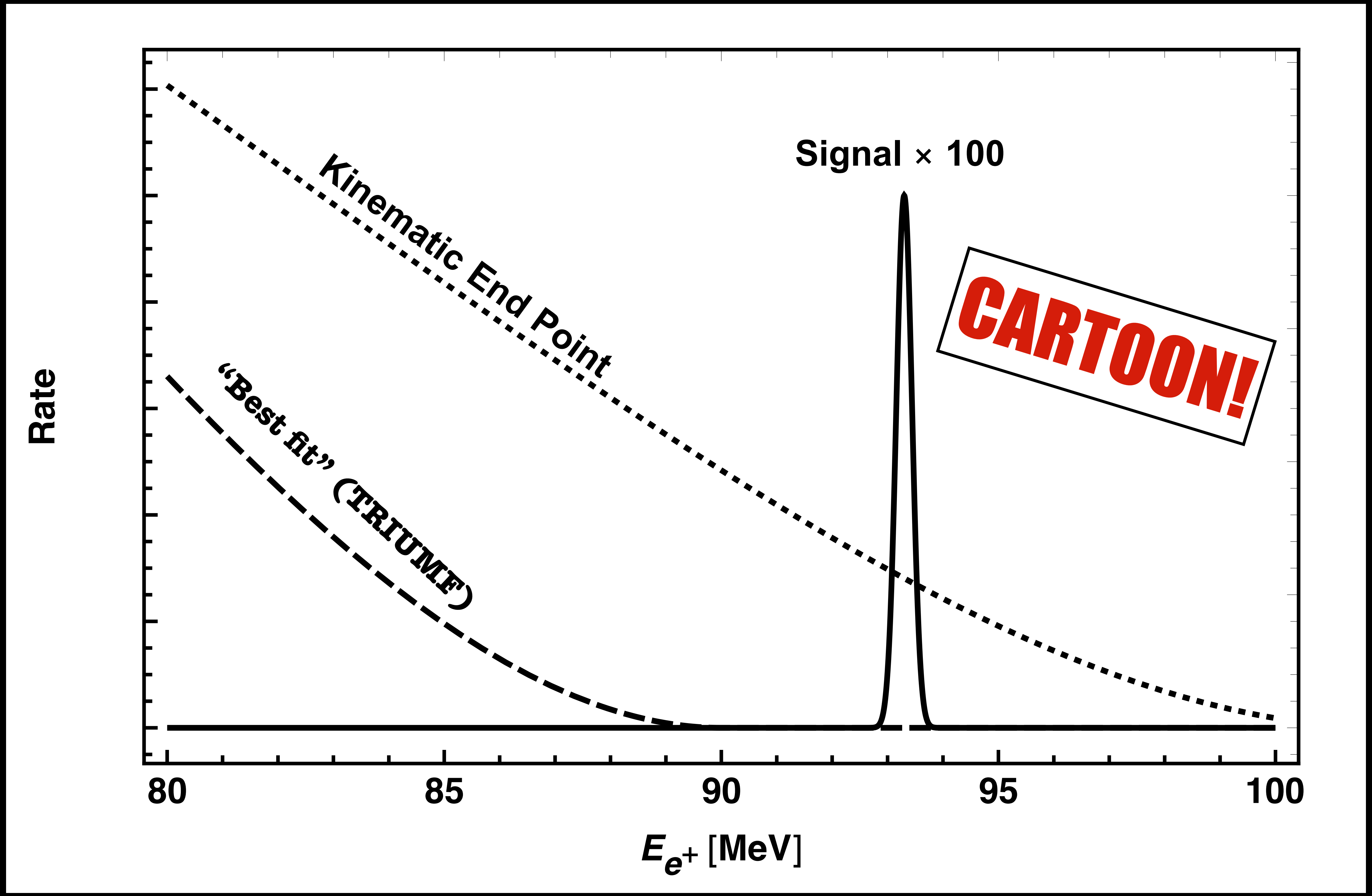
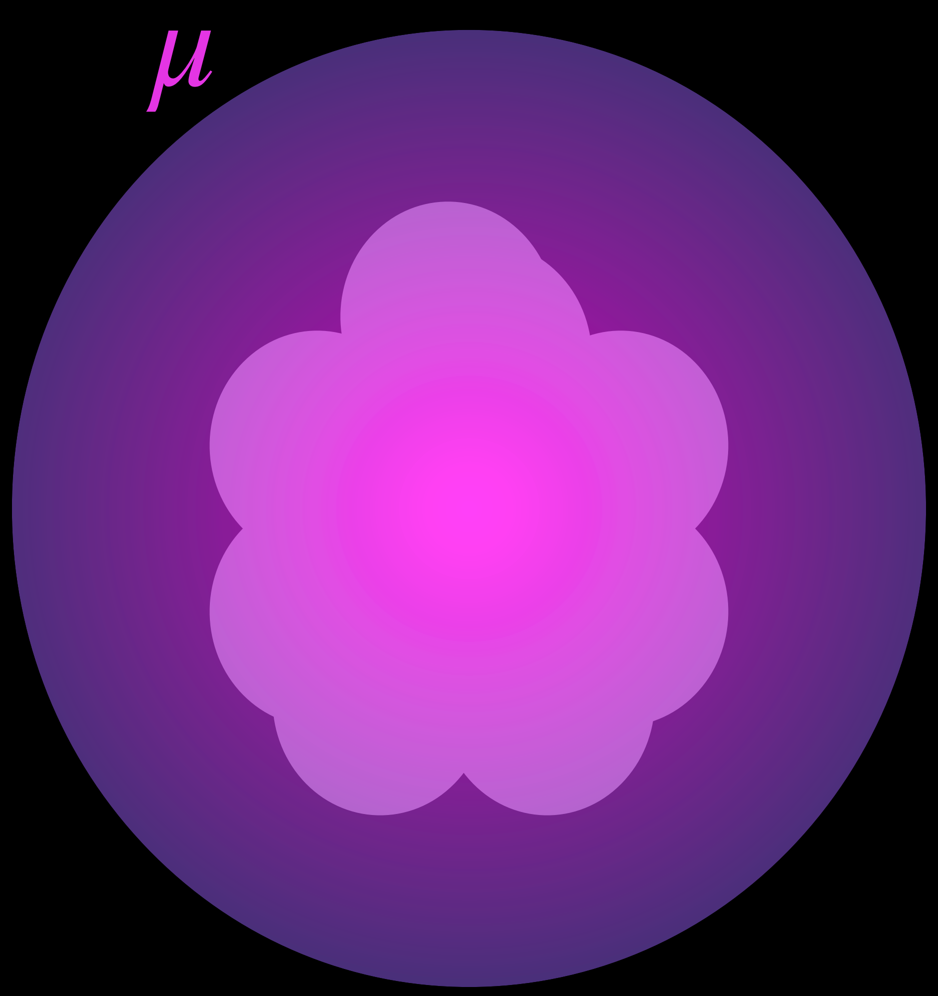




Background: Photon from RMC on ^{27}Al



$$\mu^- \ \mathrm{Al} \to \nu_\mu \, \gamma \ \mathrm{Mg}$$



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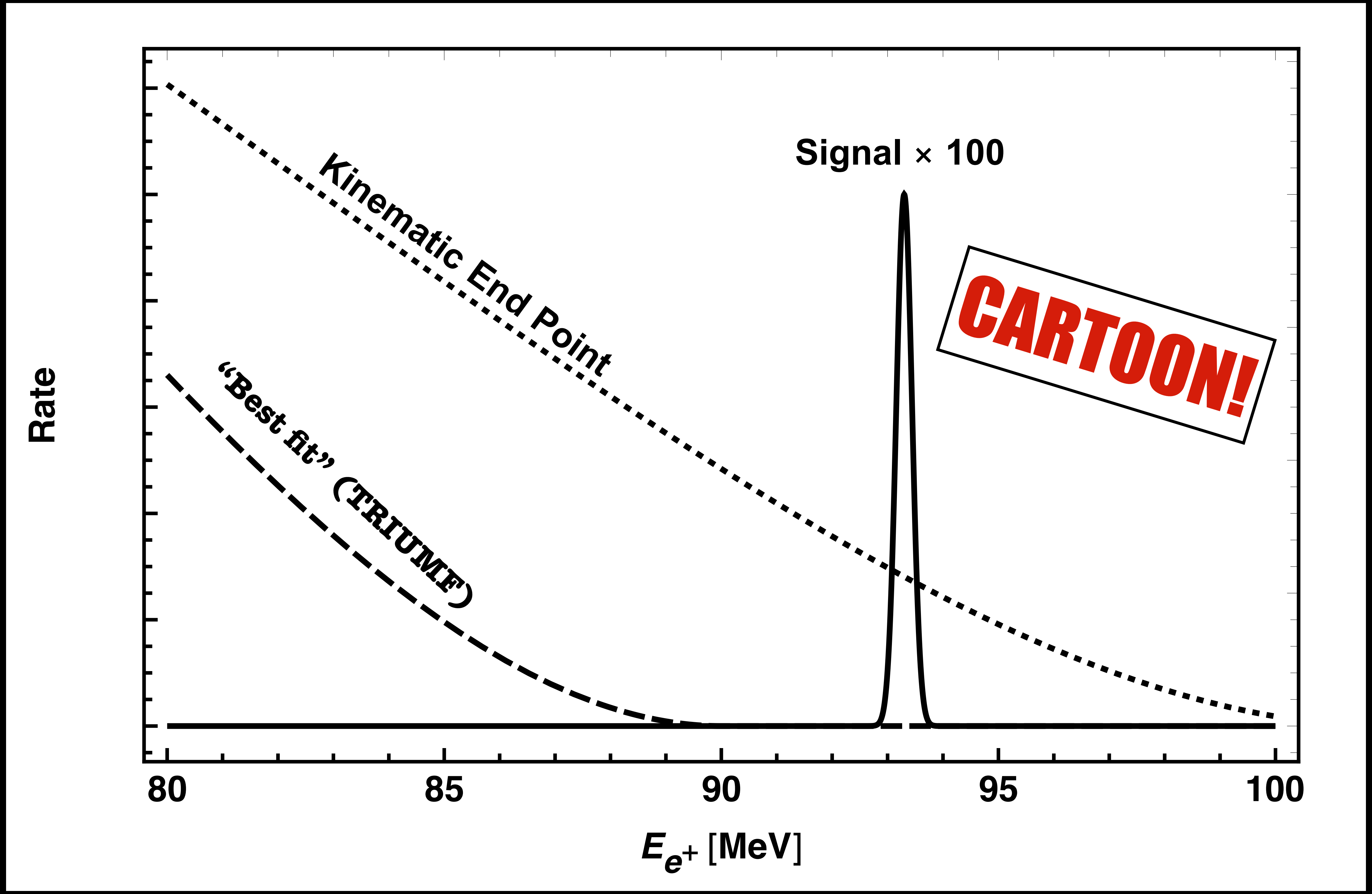

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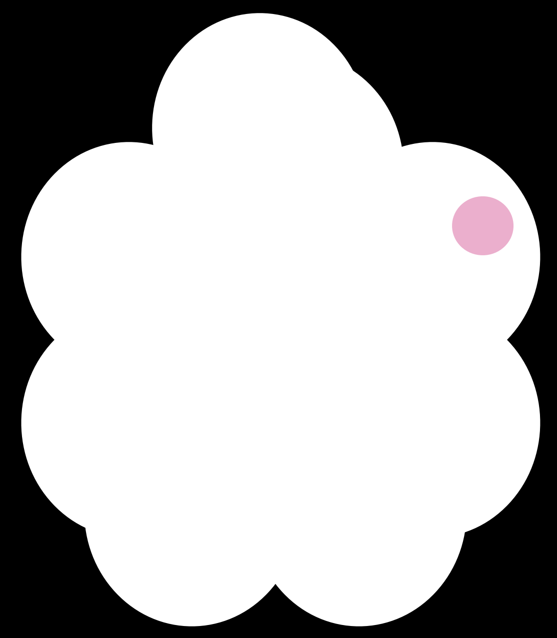

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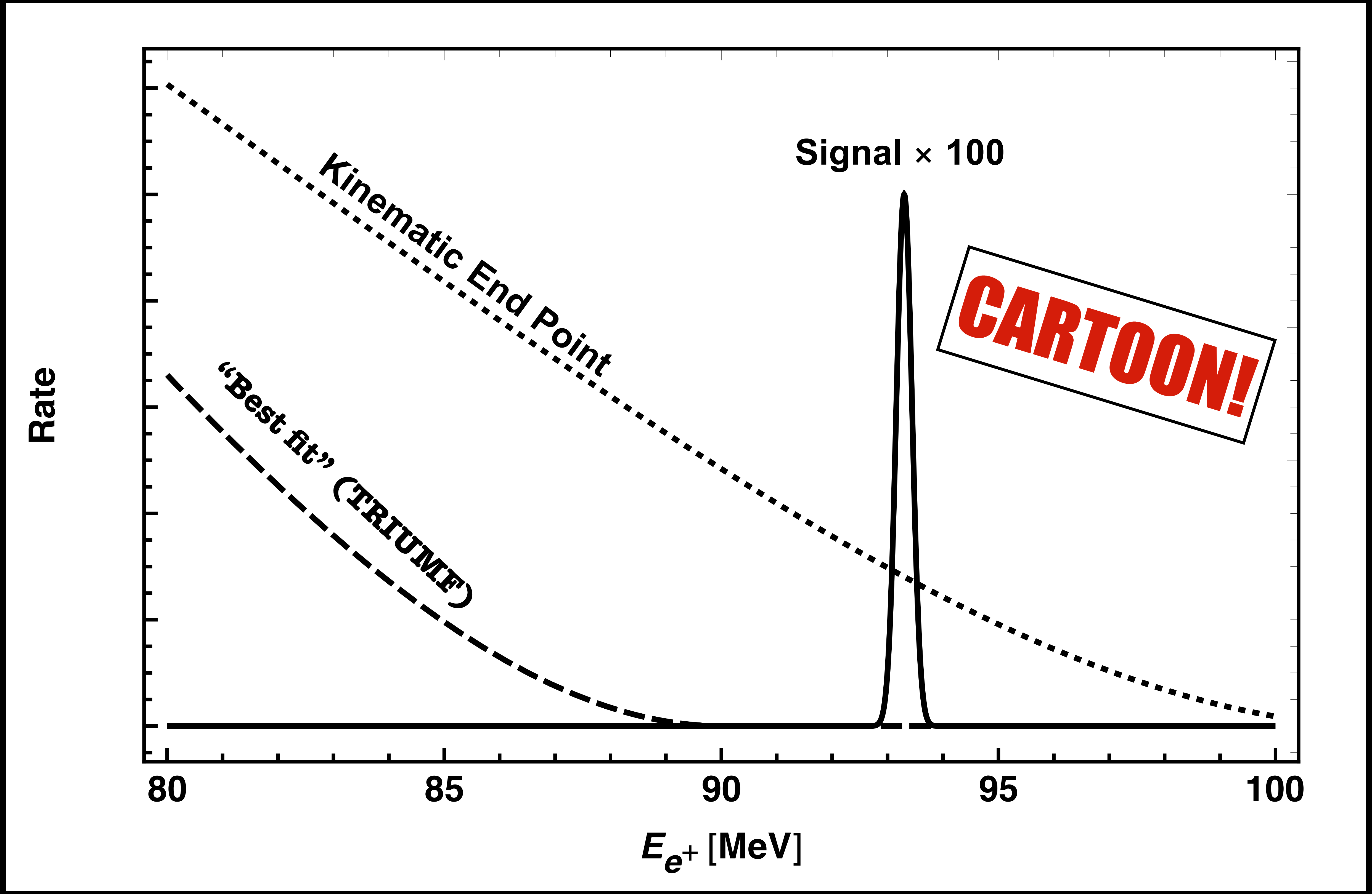

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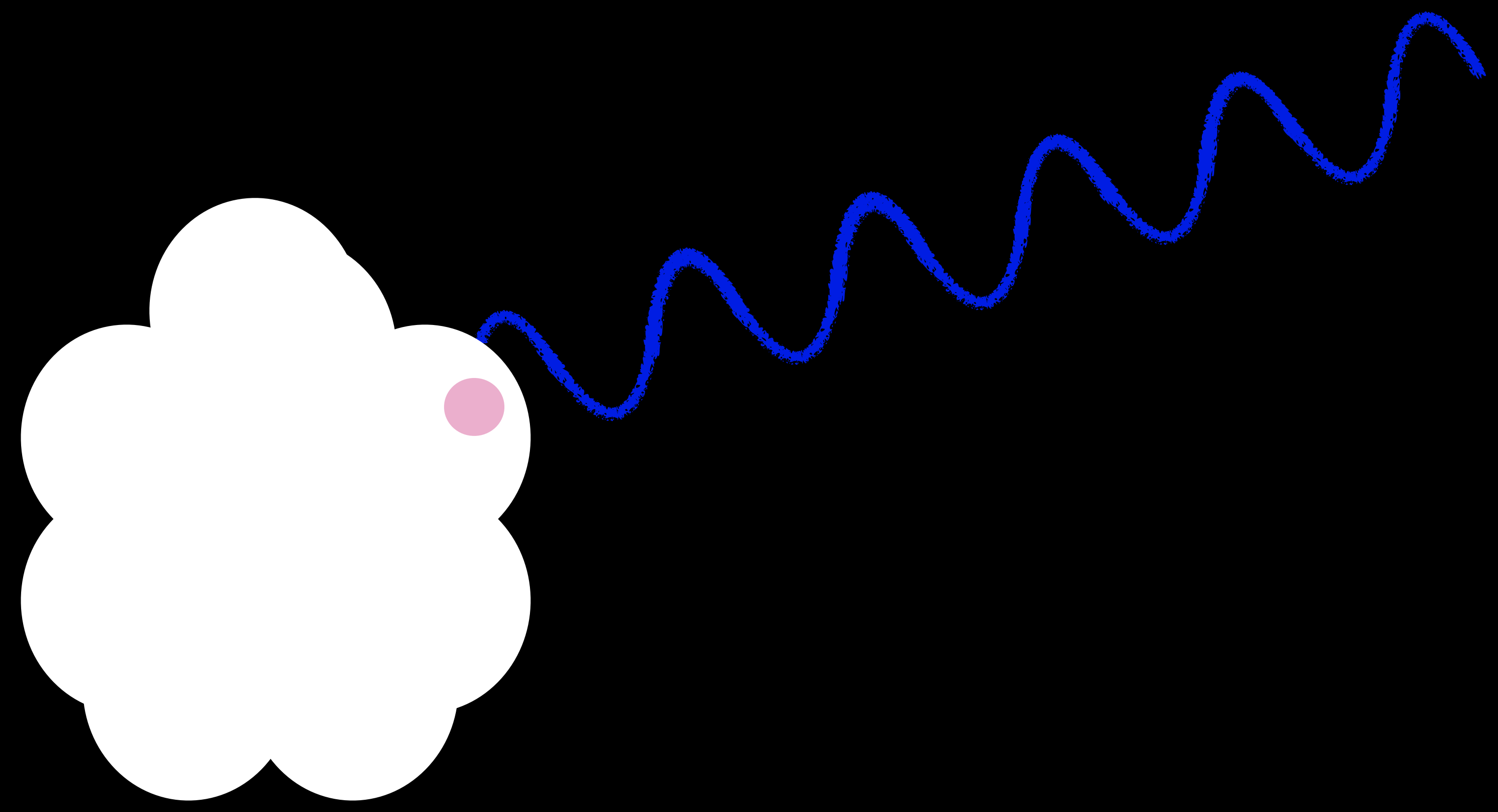

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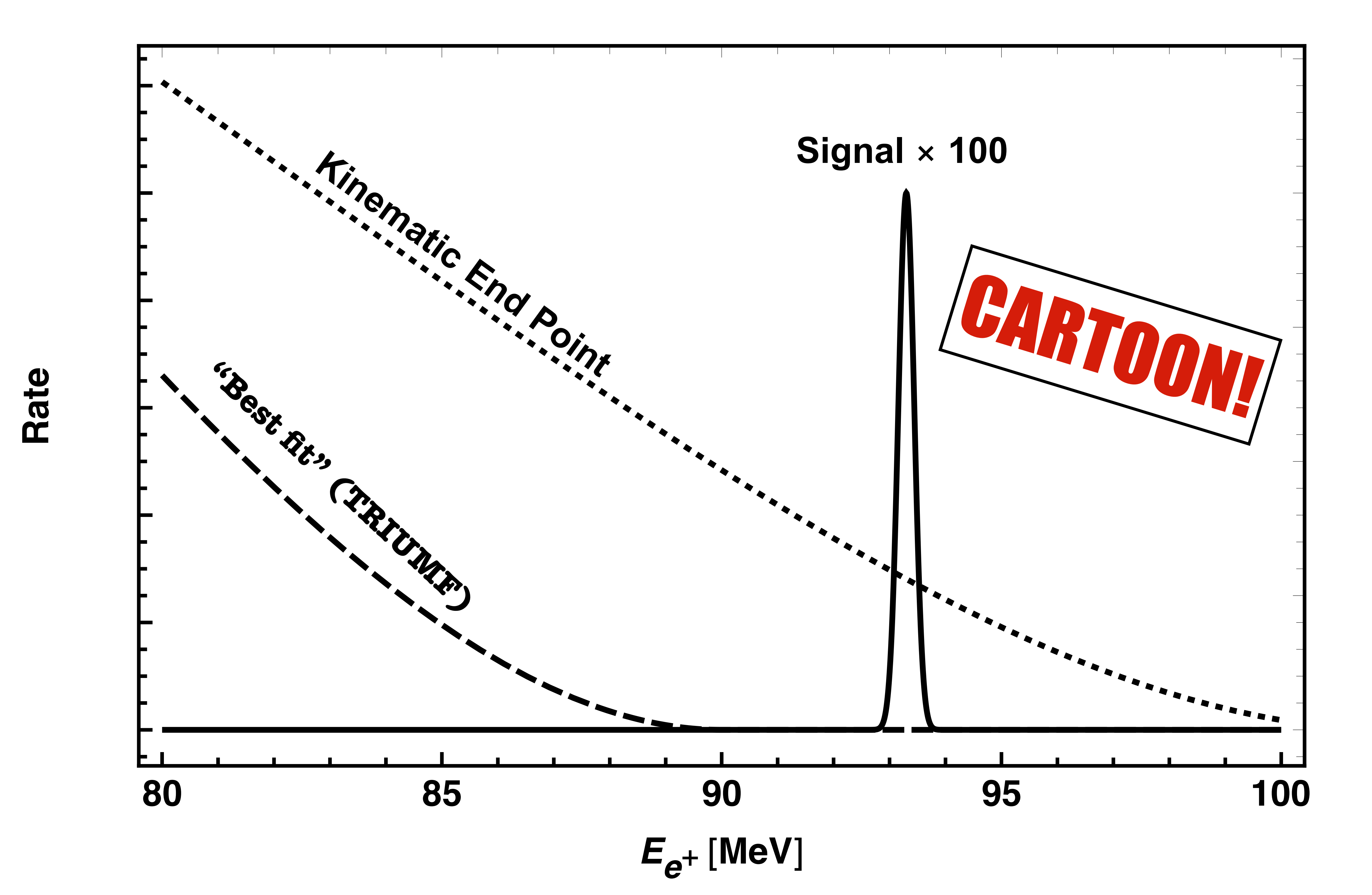

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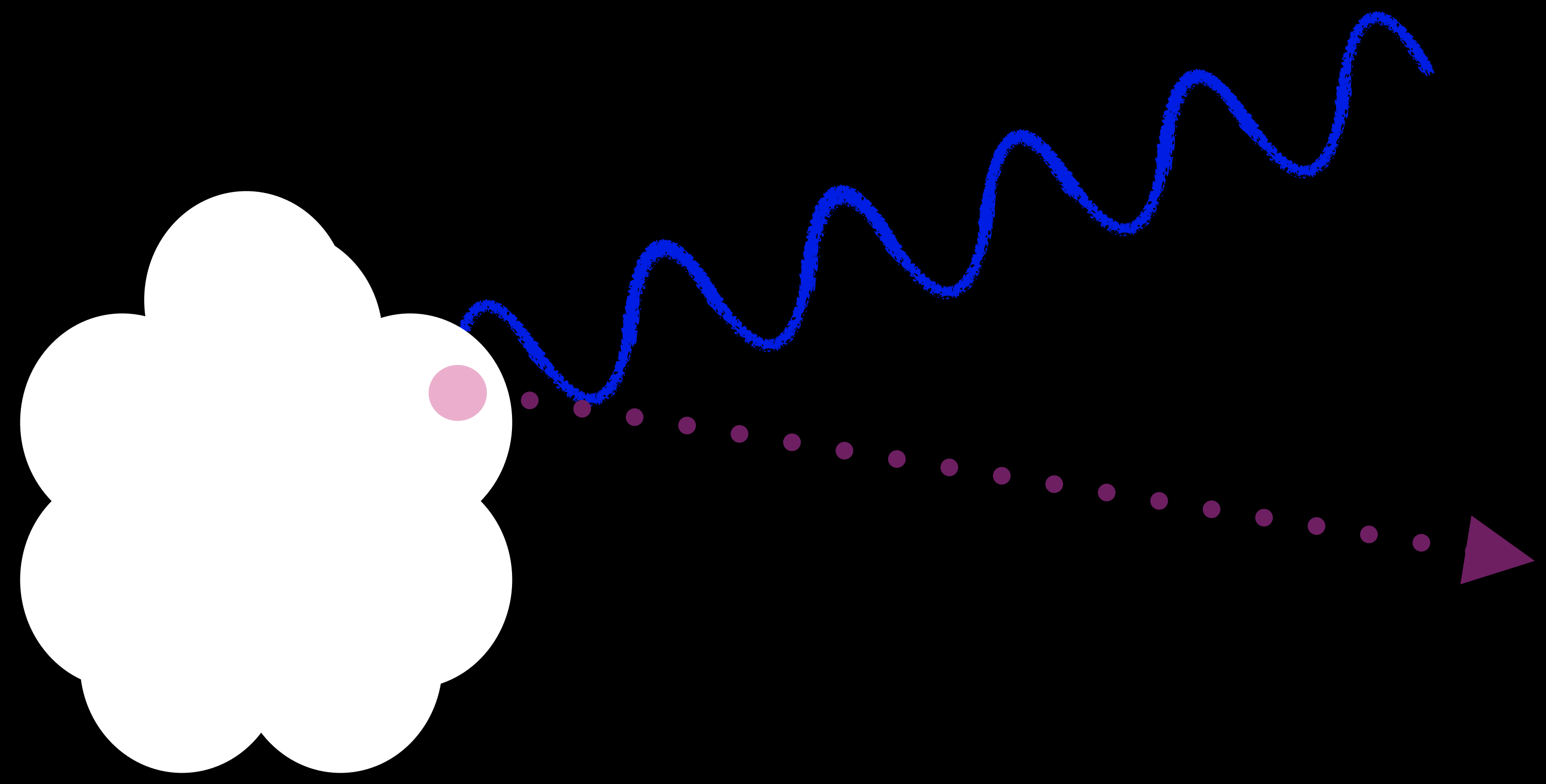

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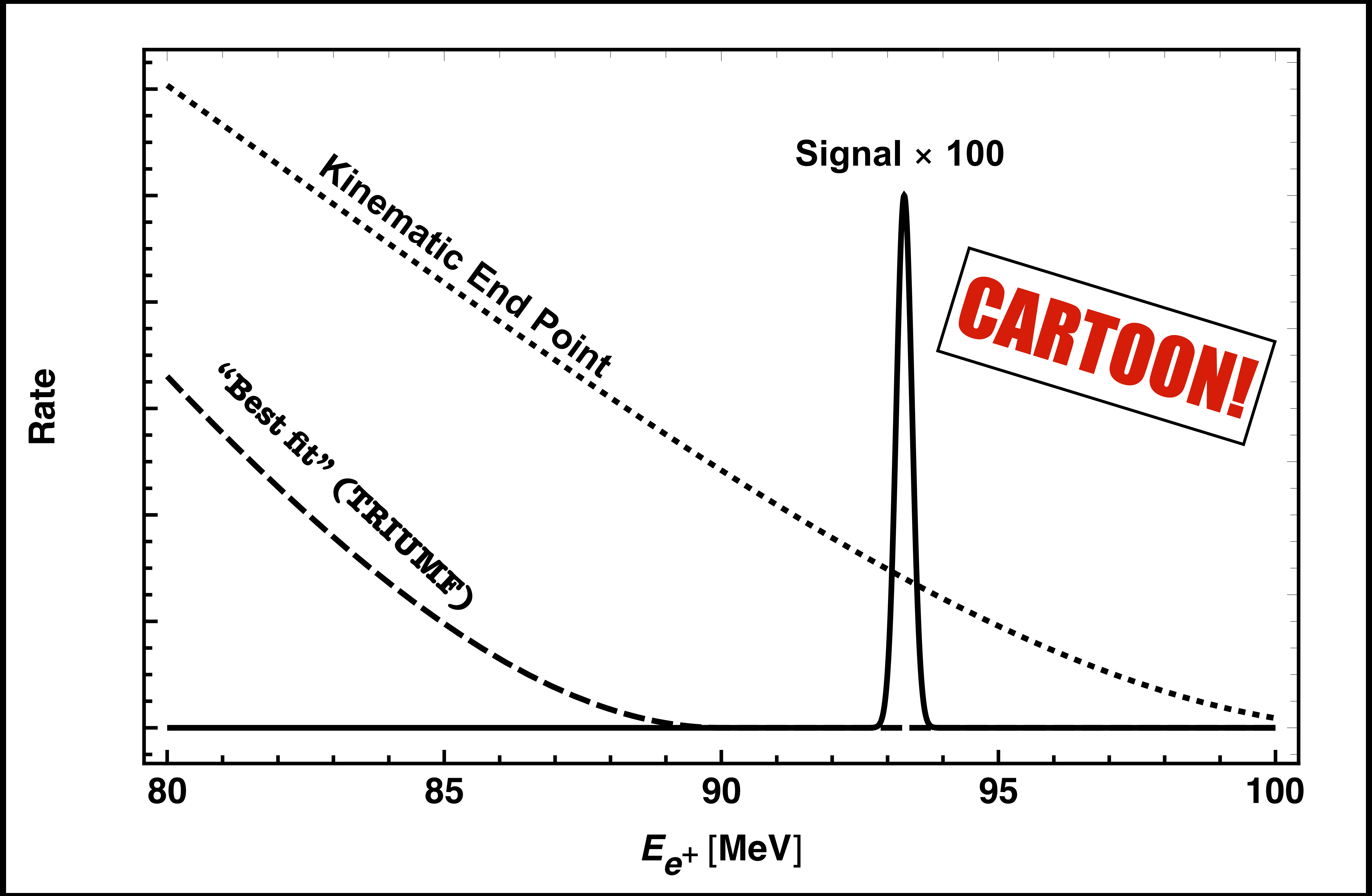

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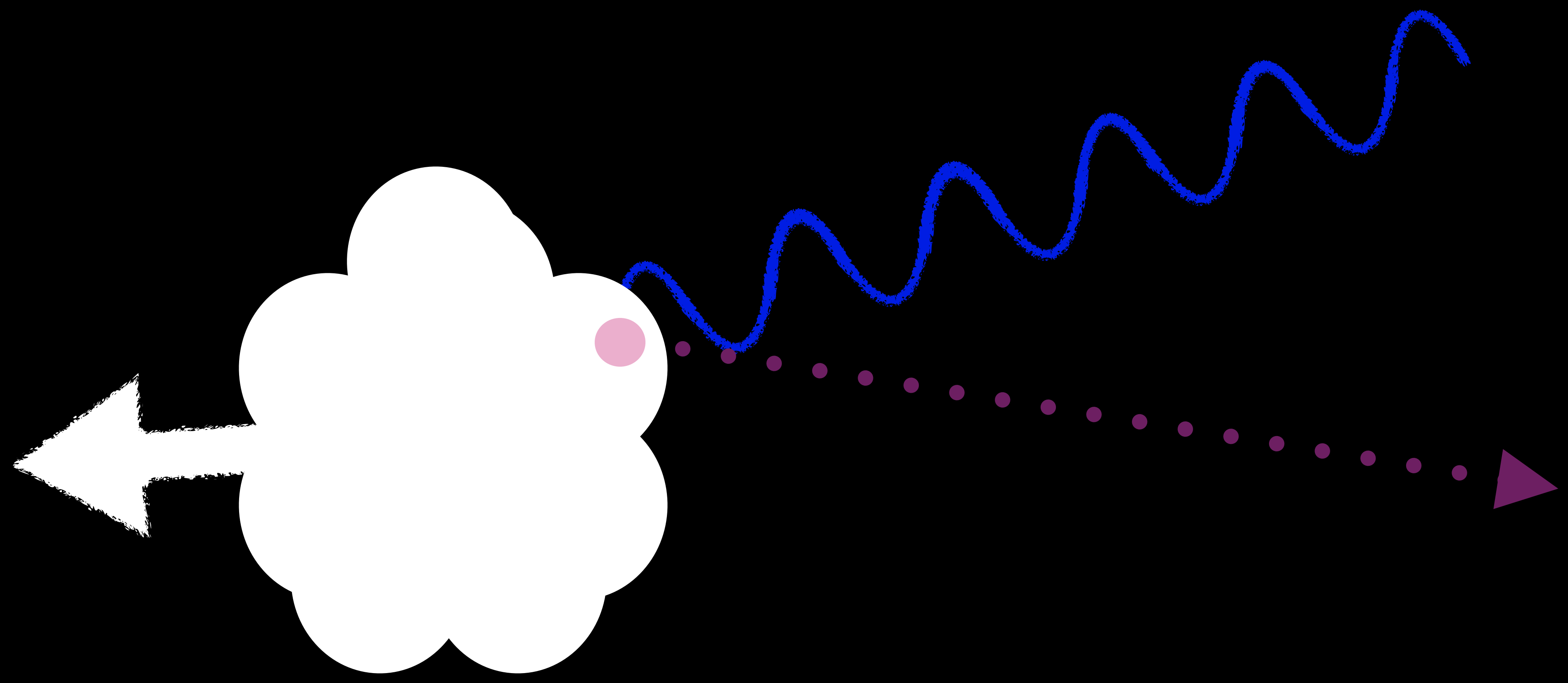
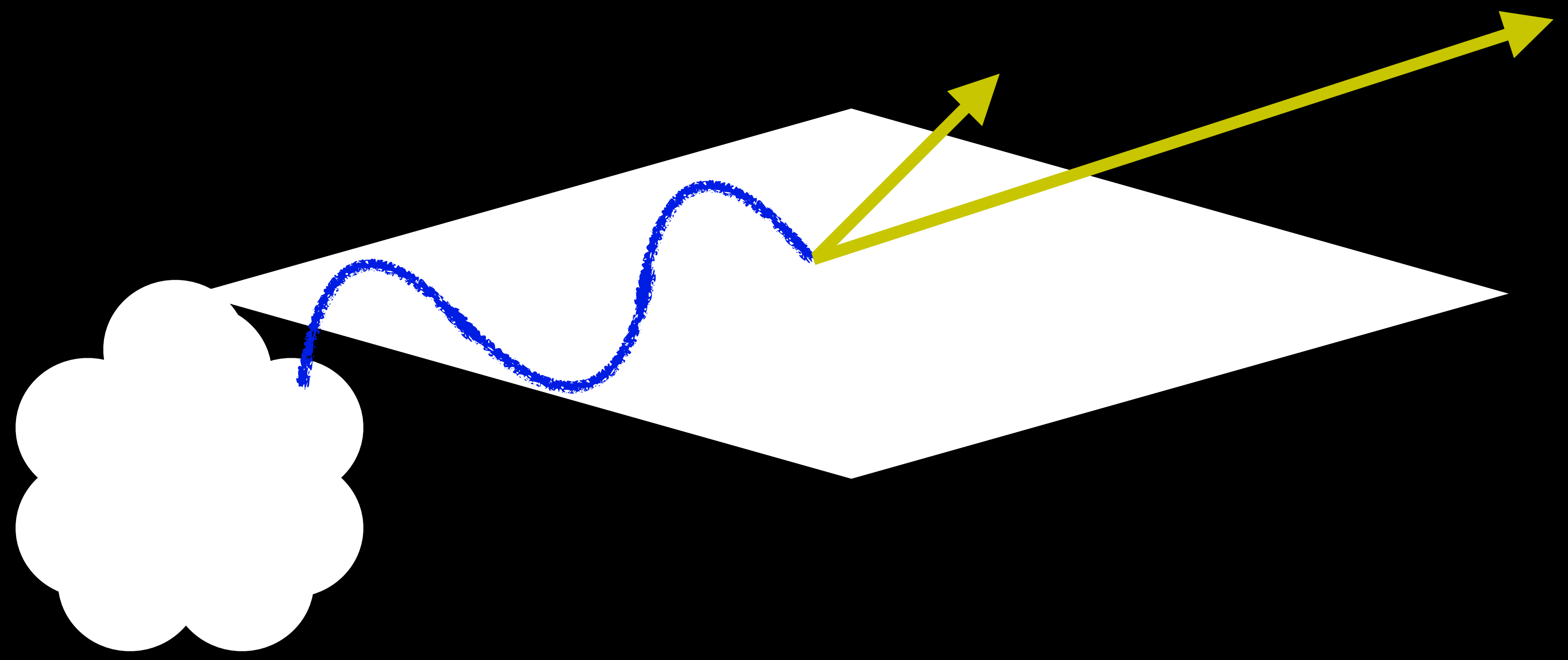


External

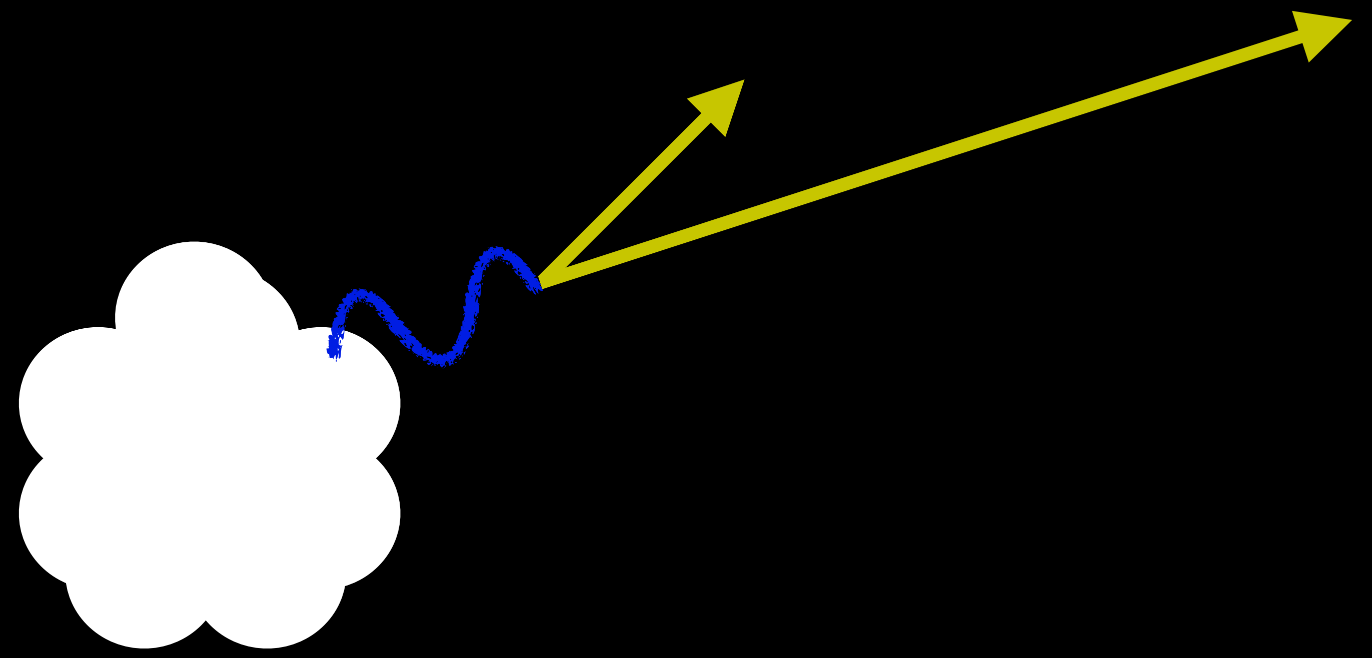


Compton scattering

In-medium pair production

Detector/target dependent

Internal

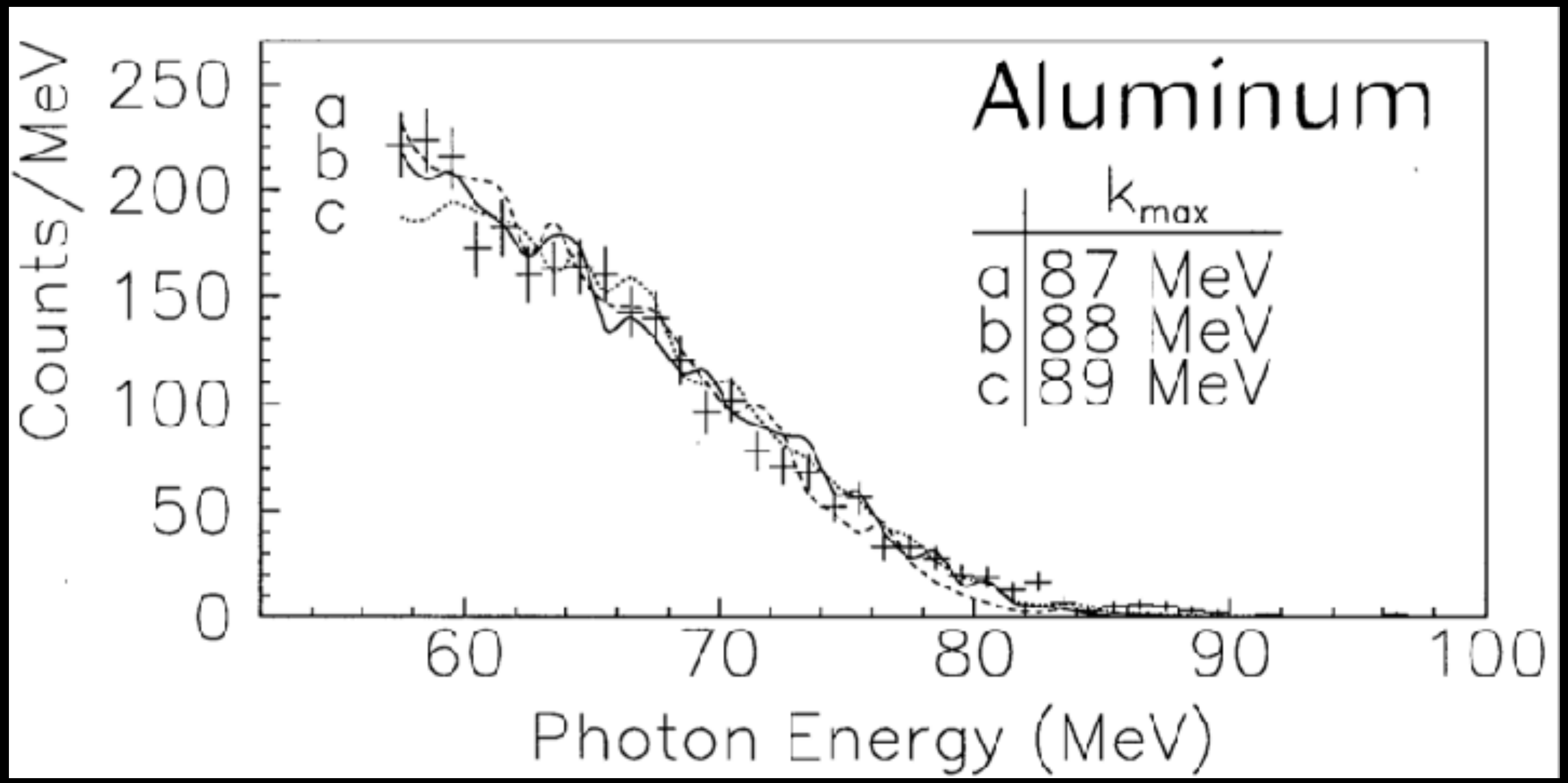


Virtual photon to $e^+ e^-$

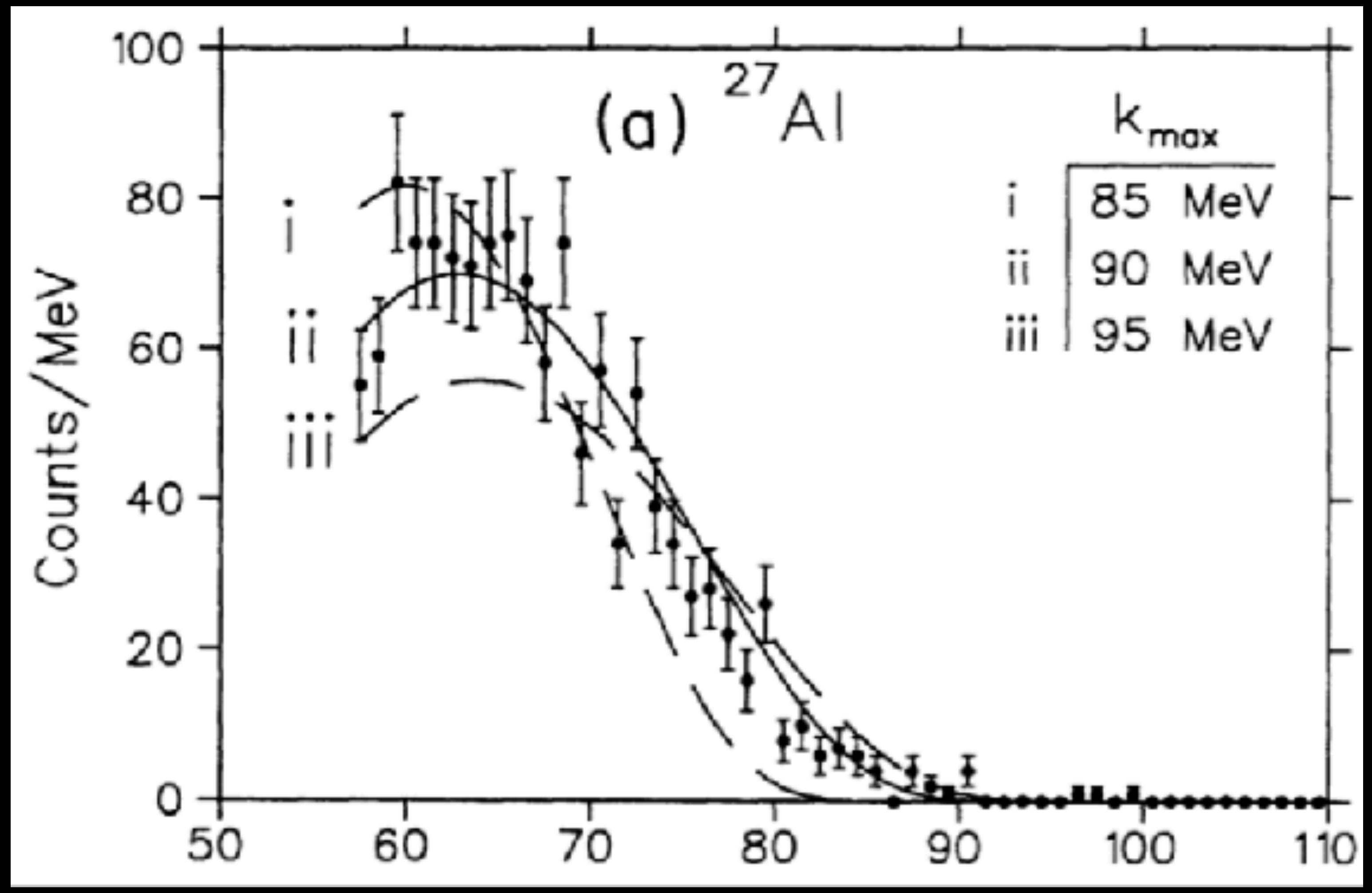
Detector independent

What do we know about RMC on Al ?

- 60% of muons capture on Al from 1s orbit.
- $\sim 10^{-5}$ RMC events per muon capture.
- $\sim 0.1 - 1\,\%$ probability for $\gamma \to e^+e^-$.
- Data from TRIUMF in mid-90's.



Bergbusch M.Sc. Thesis (1994)



Armstrong *et. al.* PRC **46:**3 1094-1106 (1993)

What do Mu2e/COMET ***need to know*** about RMC on Al ?

- Number of electrons & positrons in the high-energy tail of the spectrum.
- Estimates of both external and internal pair production rates.
- Data from TRIUMF in mid-90's ***is not good enough!***

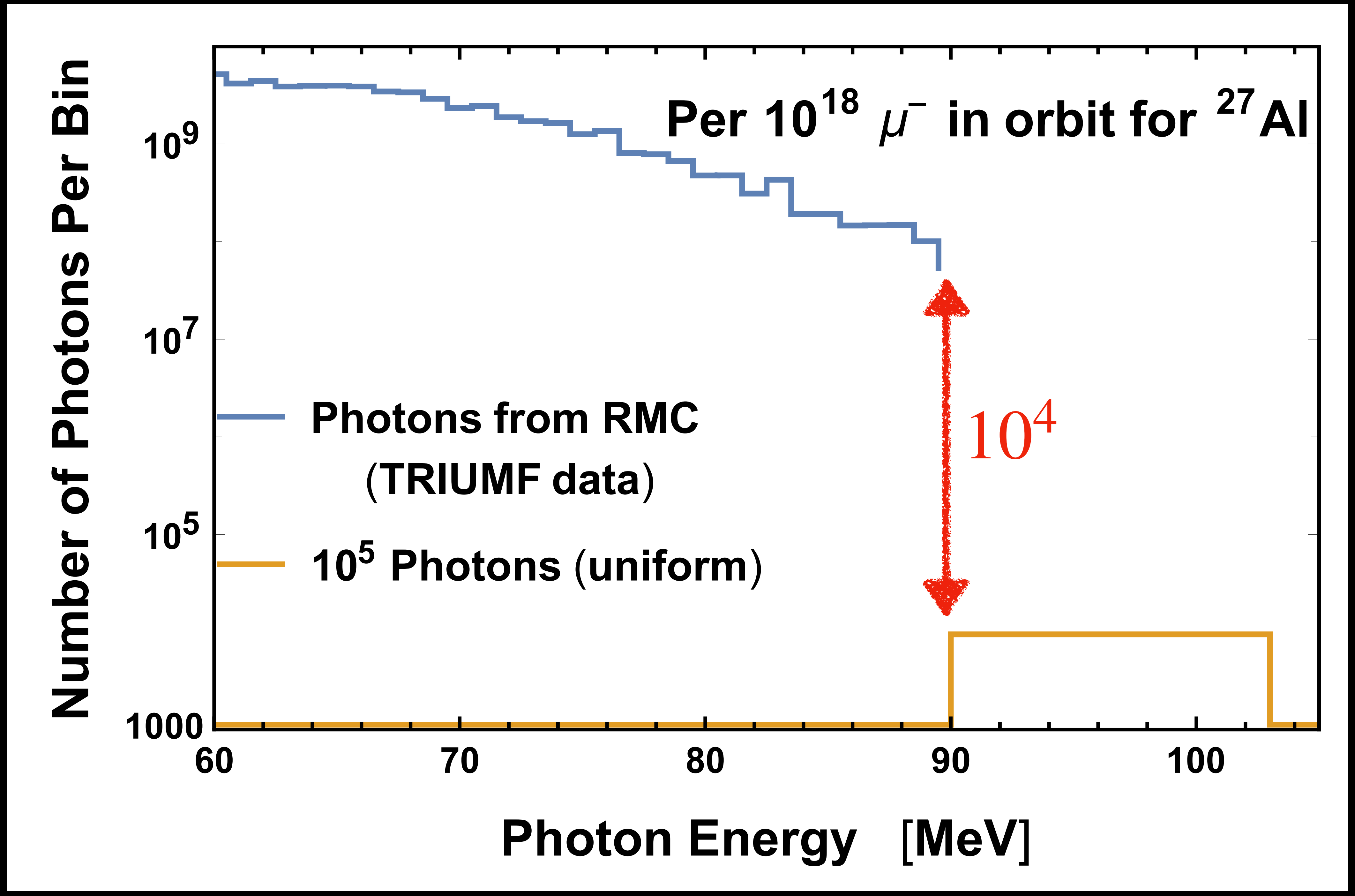
Quick Estimate

$$N_\gamma \sim 10^{13} \sim [10^{18}\ \text{muons}] \times 0.6 \times 10^{-5}$$

$$N_{e^\pm} \sim 10^{8} \sim N_\gamma \times 10^{-3} \times 10^{-2}$$

acceptance

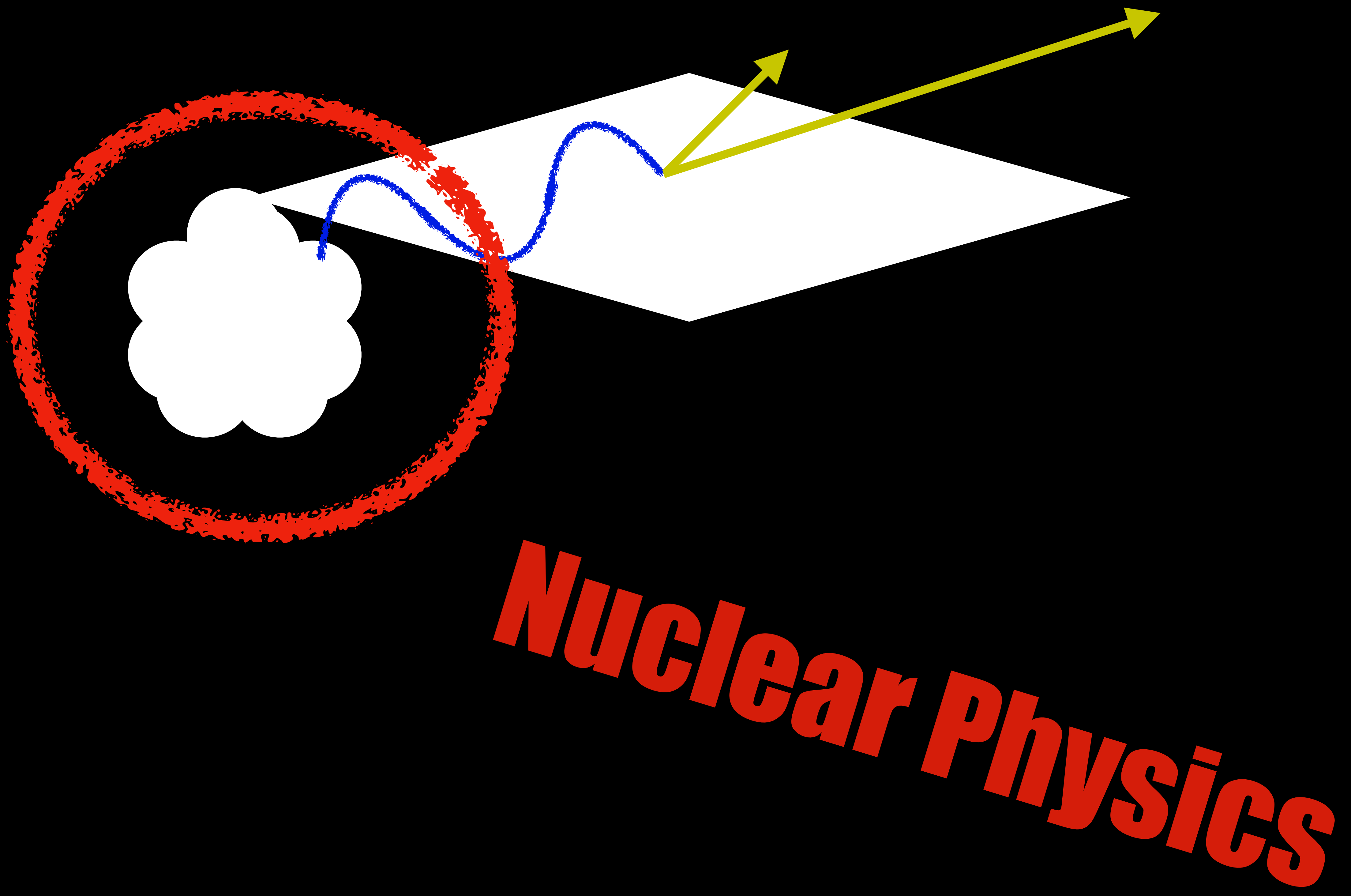
$$N_{\text{bkg}\ e^\pm} \sim f_{90\ \text{MeV}} \times N_{e^\pm}$$

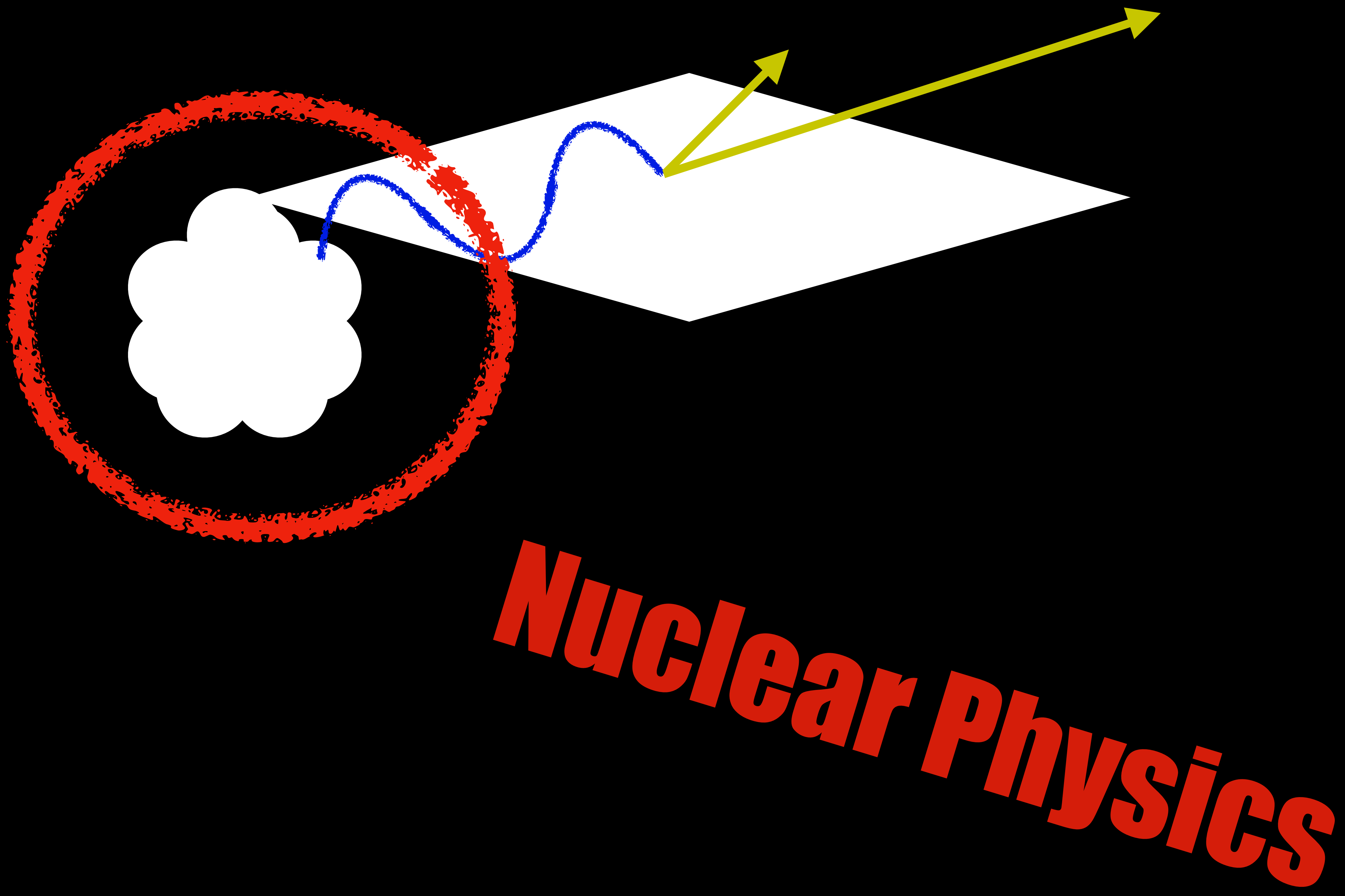


Internal conversion as external conversion

Summary:

Pair production via virtual photons can be related to the real-photon spectrum of RMC. Errors are controlled by calculable kinematic effects, the non-zero ``mass" of the virtual photon, and longitudinally polarized virtual photons. All of these corrections become small as you approach the end-point of the spectrum.


Nuclear Physics



Can we absorb all the nuclear physics into measurable quantities?

Yes, but only near the end point

This work revisits an old topic covered in:

PHYSICAL REVIEW VOLUME 98, NUMBER 5 JUNE 1, 1955

Internal Pair Production Associated with the Emission of High-Energy Gamma Rays

NORMAN M. KROLL, *Columbia University, New York, New York*

AND

WALTER WADA, *Naval Research Laboratories, Washington, D. C.*

(Received January 10, 1955)

The theory of inner pair production associated with the radiative capture of π^- mesons and with the decay of the π^0 meson is discussed. Appropriate distribution functions are derived and compared with recently obtained experimental results. The weak dependence of the theoretical predictions upon the details of meson theory is emphasized. The possible utility of the double conversion process, in which the π^0 meson decays into two electron-positron pairs, for the determination of the π^0 parity is also discussed.

We disagree with this paper in a couple places. If interested see Appendix C of our paper

arXiv:2010.09509

This ... n:

PHYSICAL REVIEW D **103,** 033002 (2021)

PHYS ... 1955

Int ... ays

High energy spectrum of internal positrons from radiative muon capture on nuclei

Ryan Plestid* and Richard J. Hill†

Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA and Theoretical Physics Department, Fermilab, Batavia, Illinois 60510, USA

(Received 19 October 2020; accepted 24 December 2020; published 15 February 2021)

The Mu2e and COMET collaborations will search for nucleus-catalyzed muon conversion to positrons ($\mu^- \to e^+$) as a signal of lepton number violation. A key background for this search is radiative muon capture where either (1) a real photon converts to an e^+e^- pair "externally" in surrounding material, or (2) a virtual photon mediates the production of an e^+e^- pair "internally." If the e^+ has an energy approaching the signal region then it can serve as an irreducible background. In this work we describe how the near end point internal positron spectrum can be related to the real photon spectrum from the same nucleus, which encodes all nontrivial nuclear physics.

arXiv:2010.09509

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PHYSICAL REVIEW D **103**, 033002 (2021)

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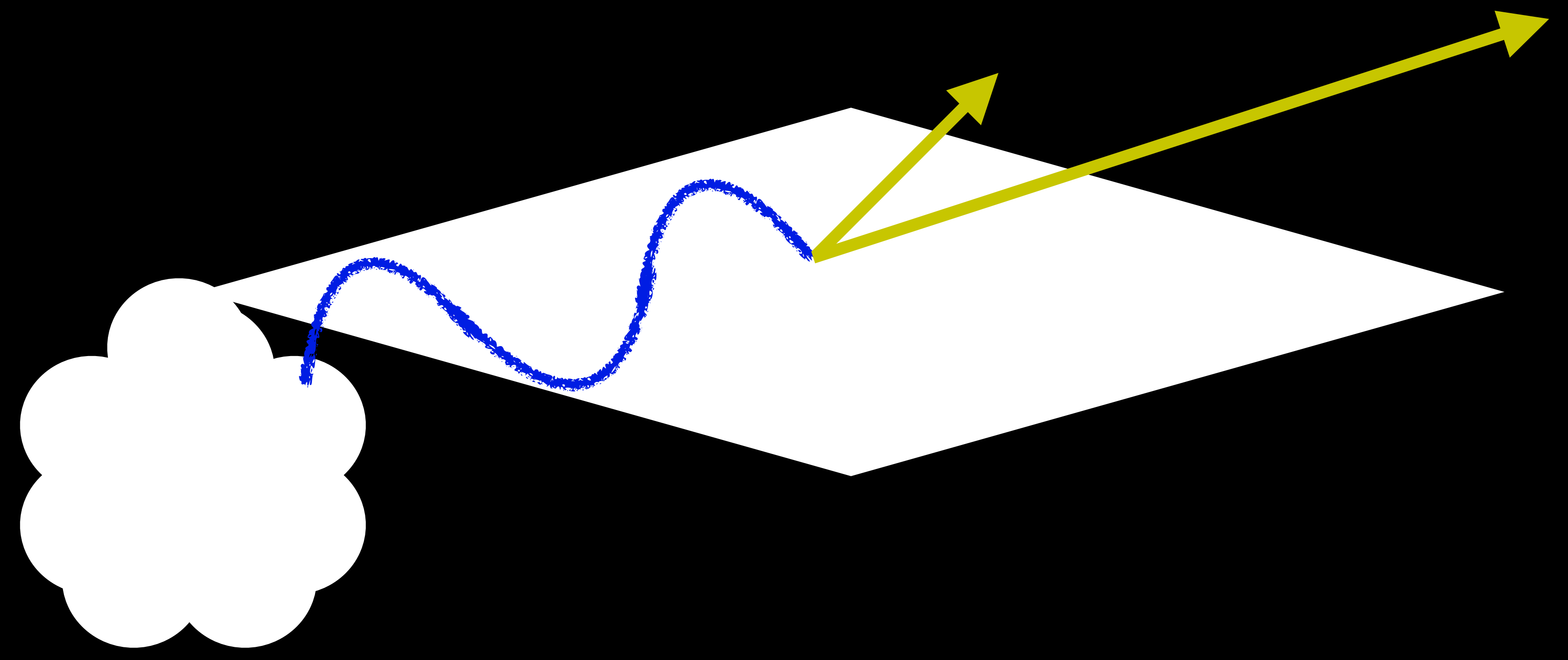
External conversion

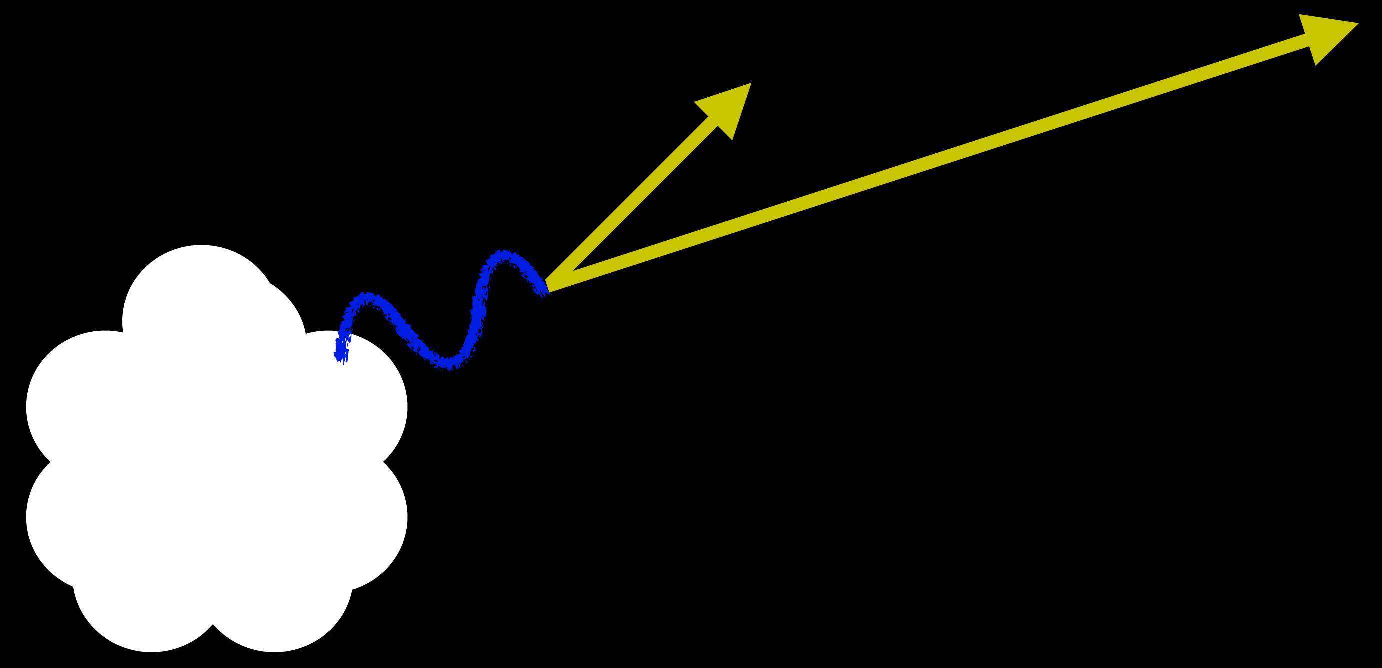


$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma P(E_+ | E_\gamma) \times \frac{\mathrm{d}\Gamma_{\mathrm{RMC}}}{\mathrm{d}E_\gamma}$$

Bethe – Heitler
+Geometry

$\langle |\mathscr{M}_0|^2 \rangle_{\mathrm{Nuclear}}$

Internal conversion



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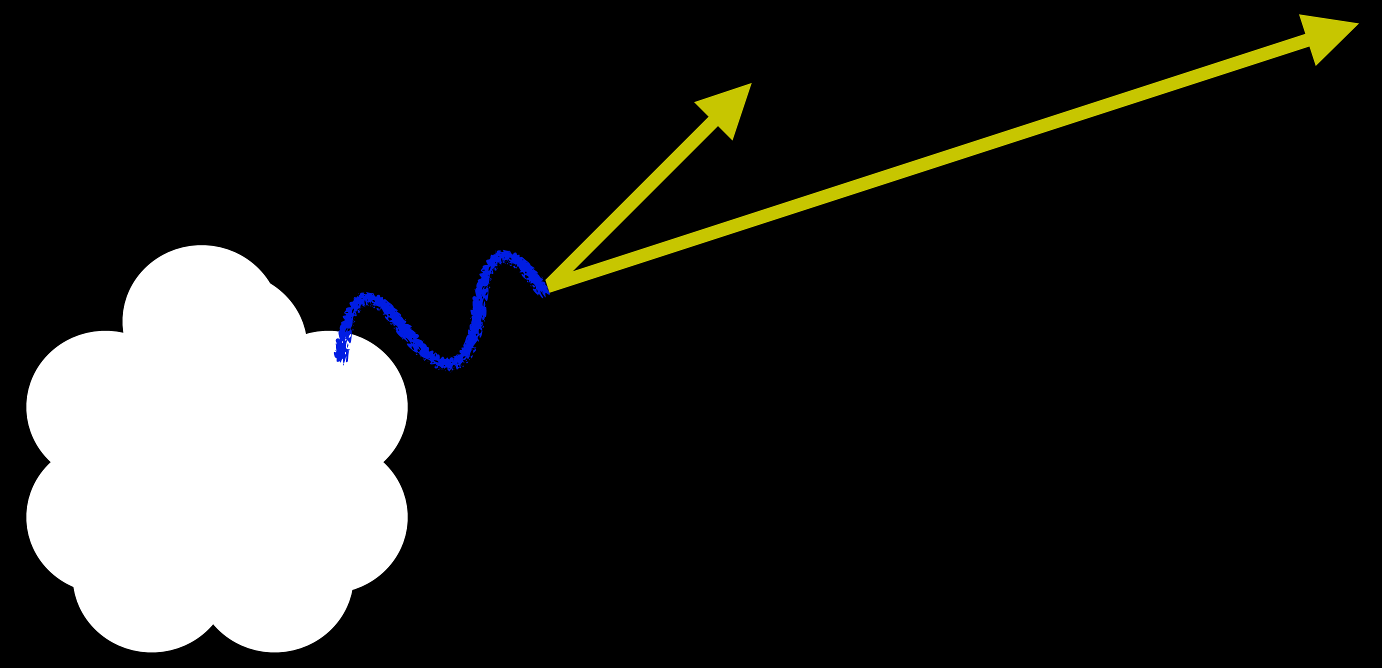
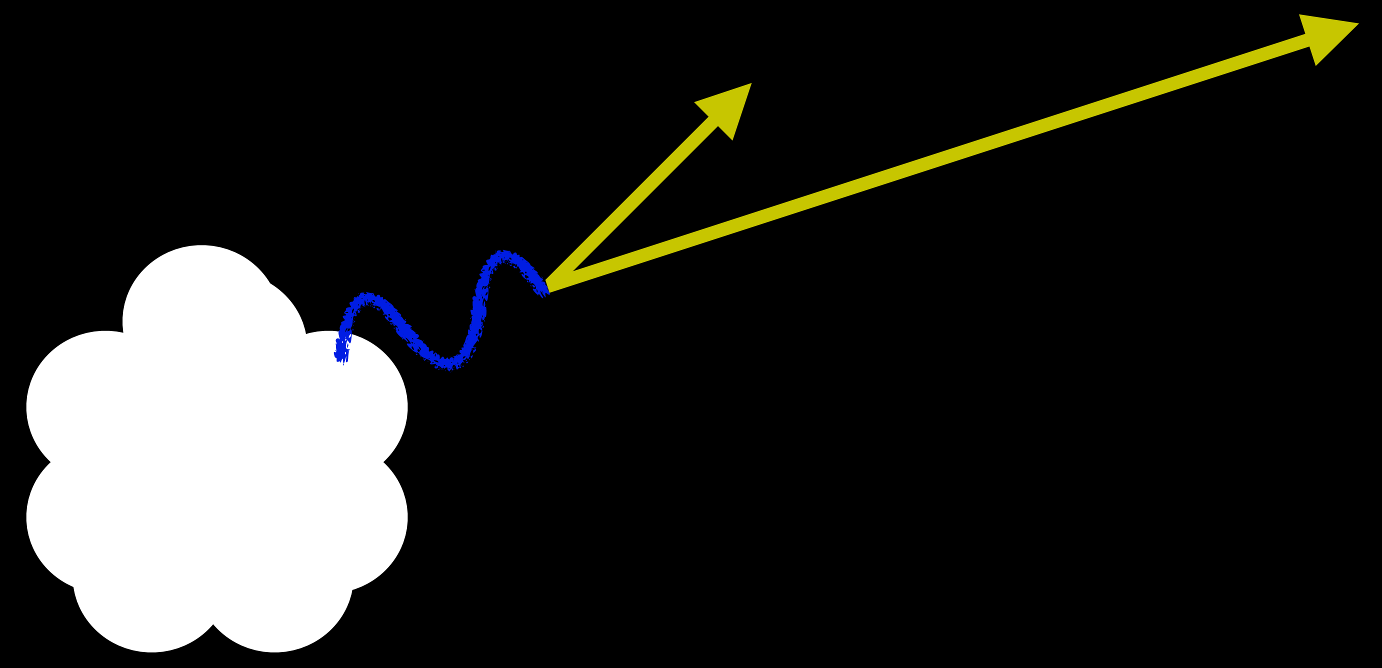


$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

Internal conversion



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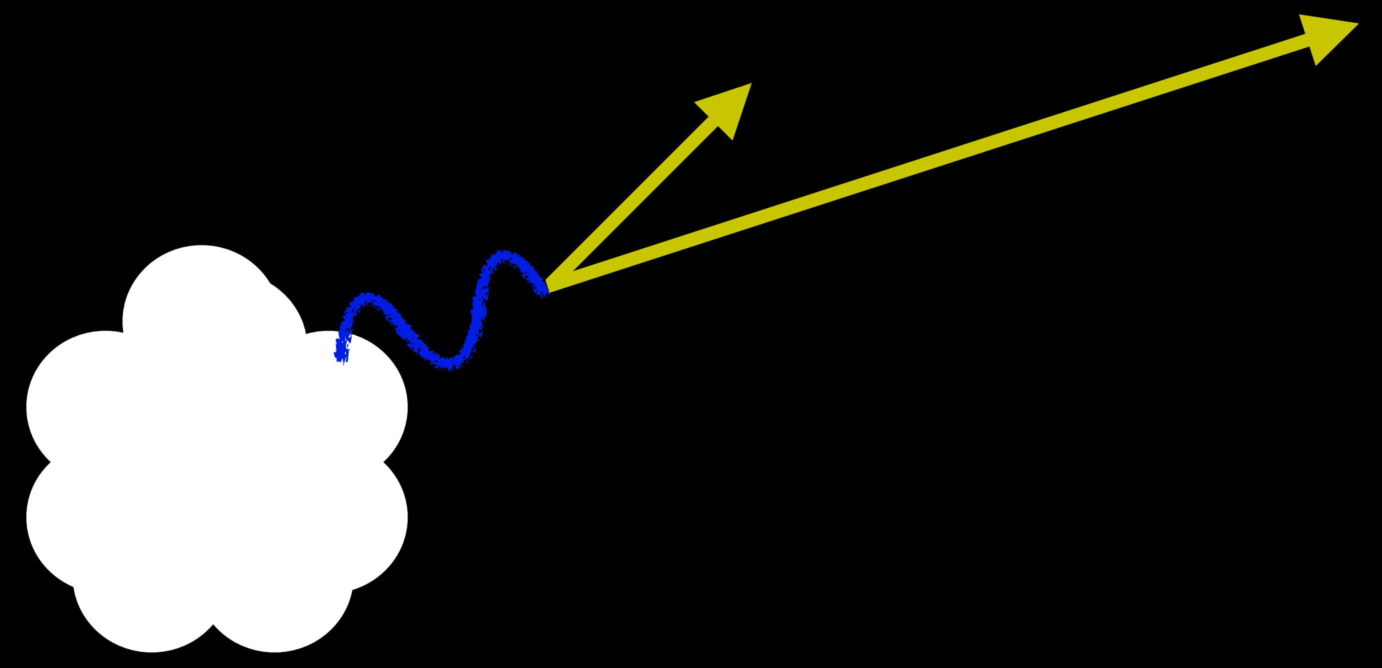
$$\langle |\mathscr{M}_*|^2 \rangle_{\text{Nuclear}}$$

Internal conversion

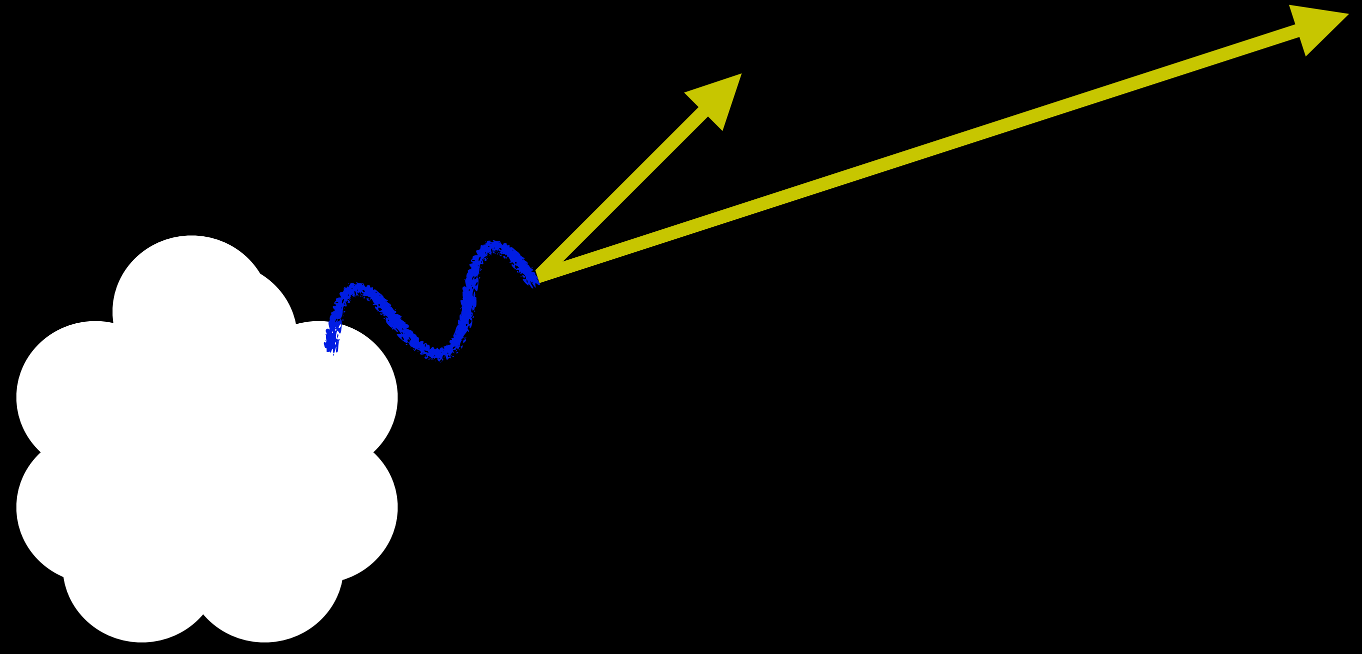


$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

Off-shell

$$\langle |\mathscr{M}_*|^2 \rangle_{\mathrm{Nuclear}}$$

Internal conversion

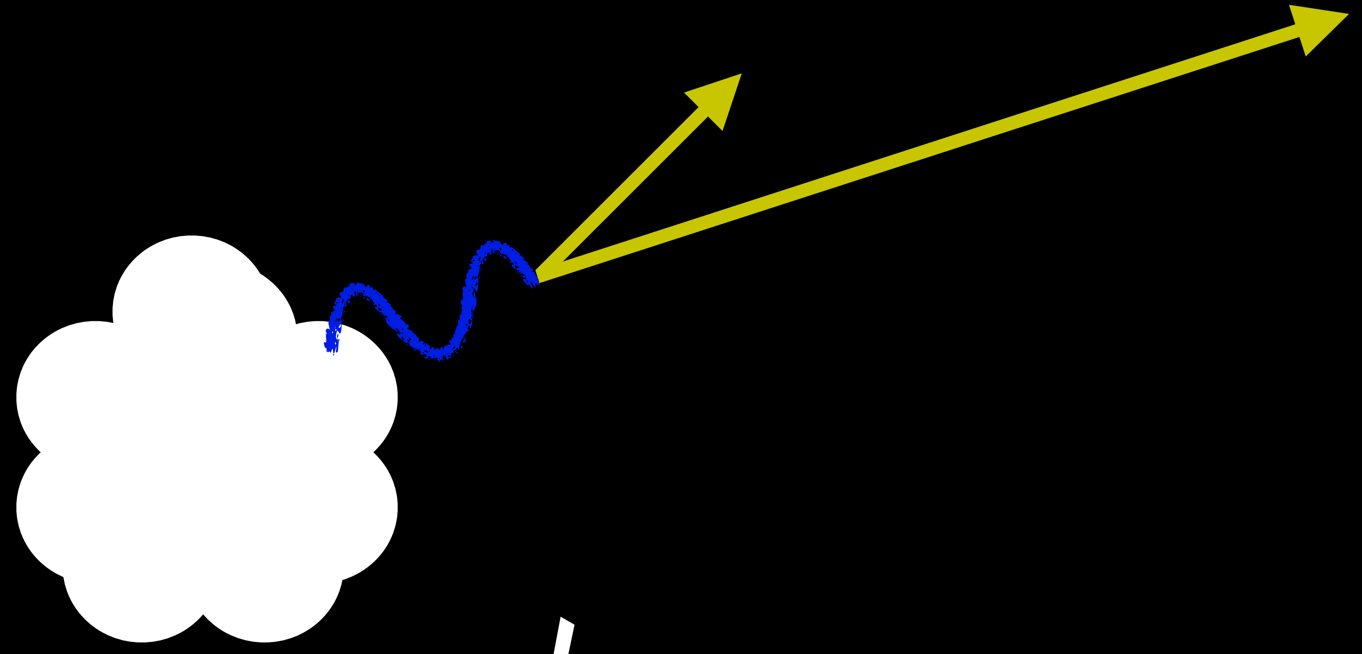


$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

4-body phase space

Off-shell $\langle |\mathscr{M}_*|^2 \rangle_{\mathrm{Nuclear}}$

Internal conversion



$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

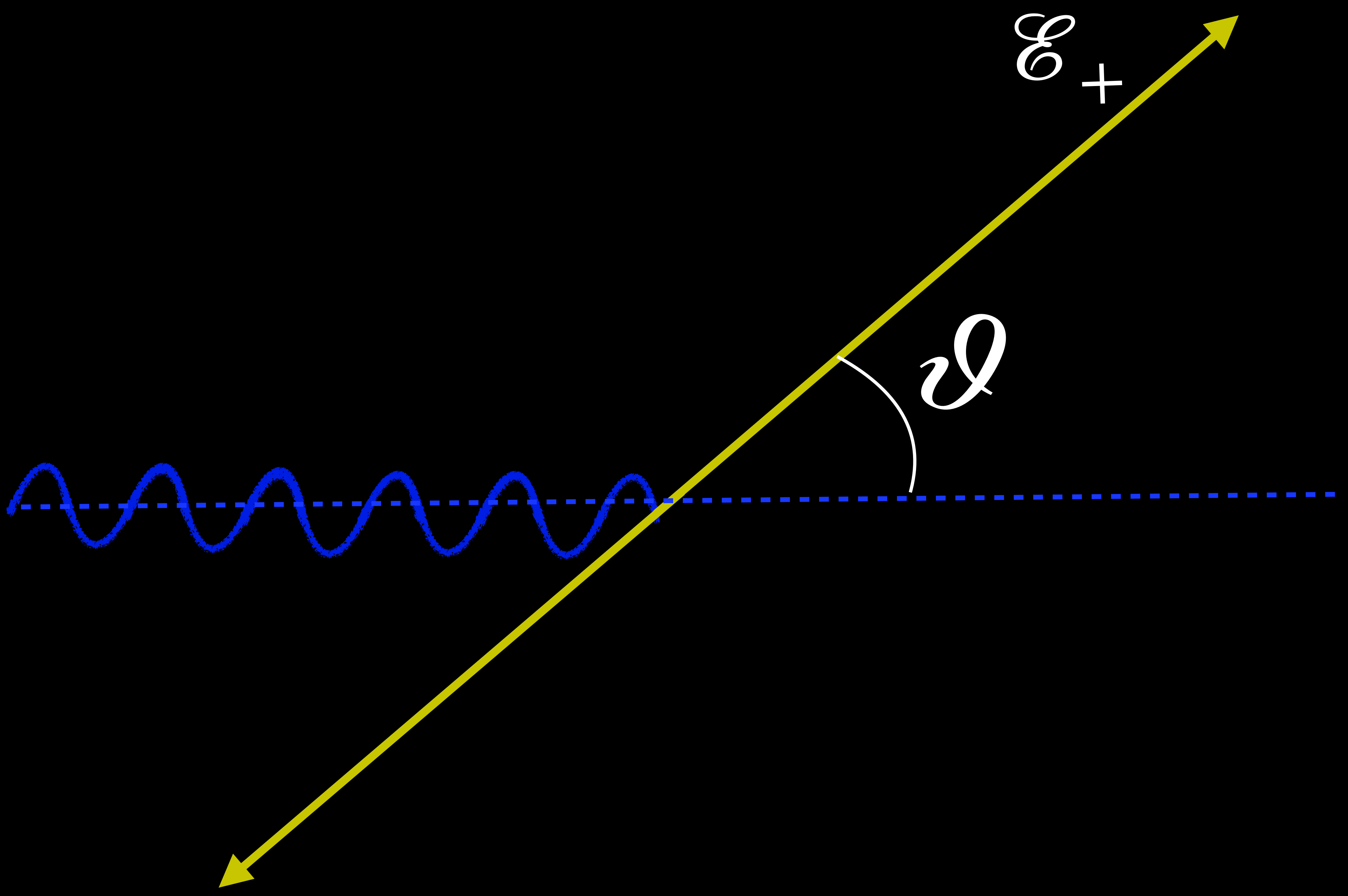
Virtual photon mass

Off-shell

$$\langle |\mathscr{M}_*|^2 \rangle_{\mathrm{Nuclear}}$$

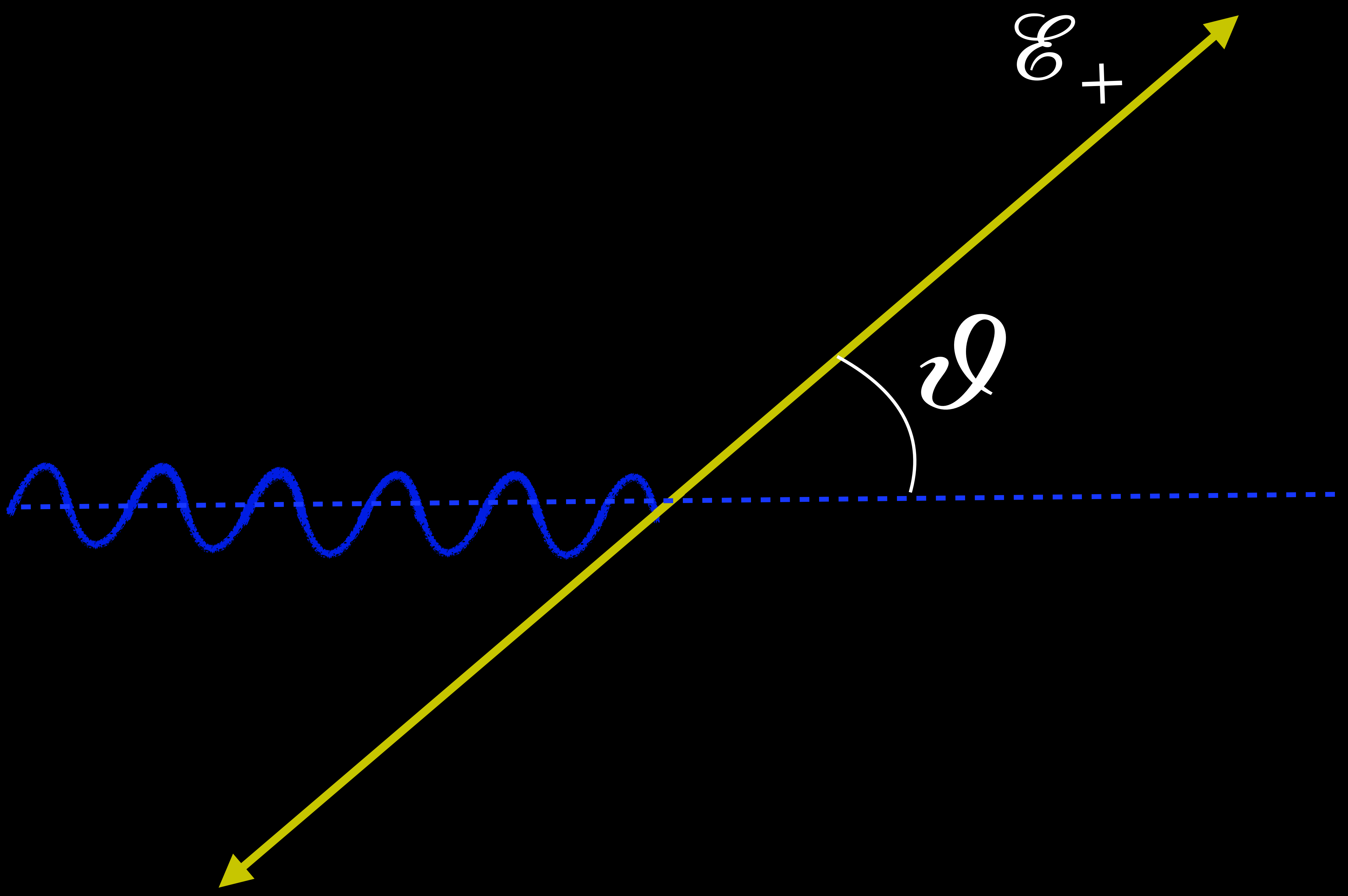
4-body phase space

Photon rest frame



Photon rest frame

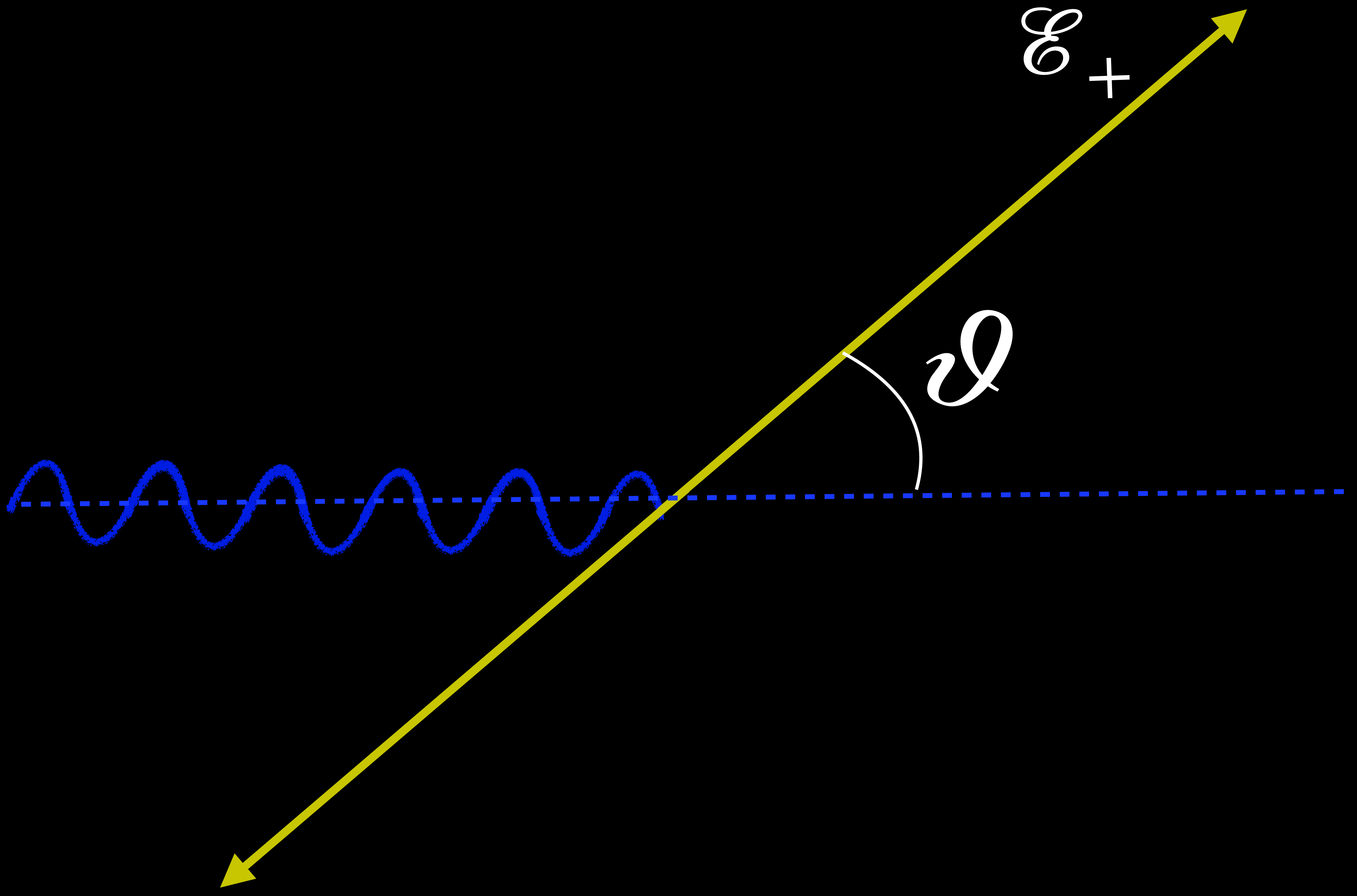
Boost



Photon rest frame

Boost

Lab frame



Photon rest frame

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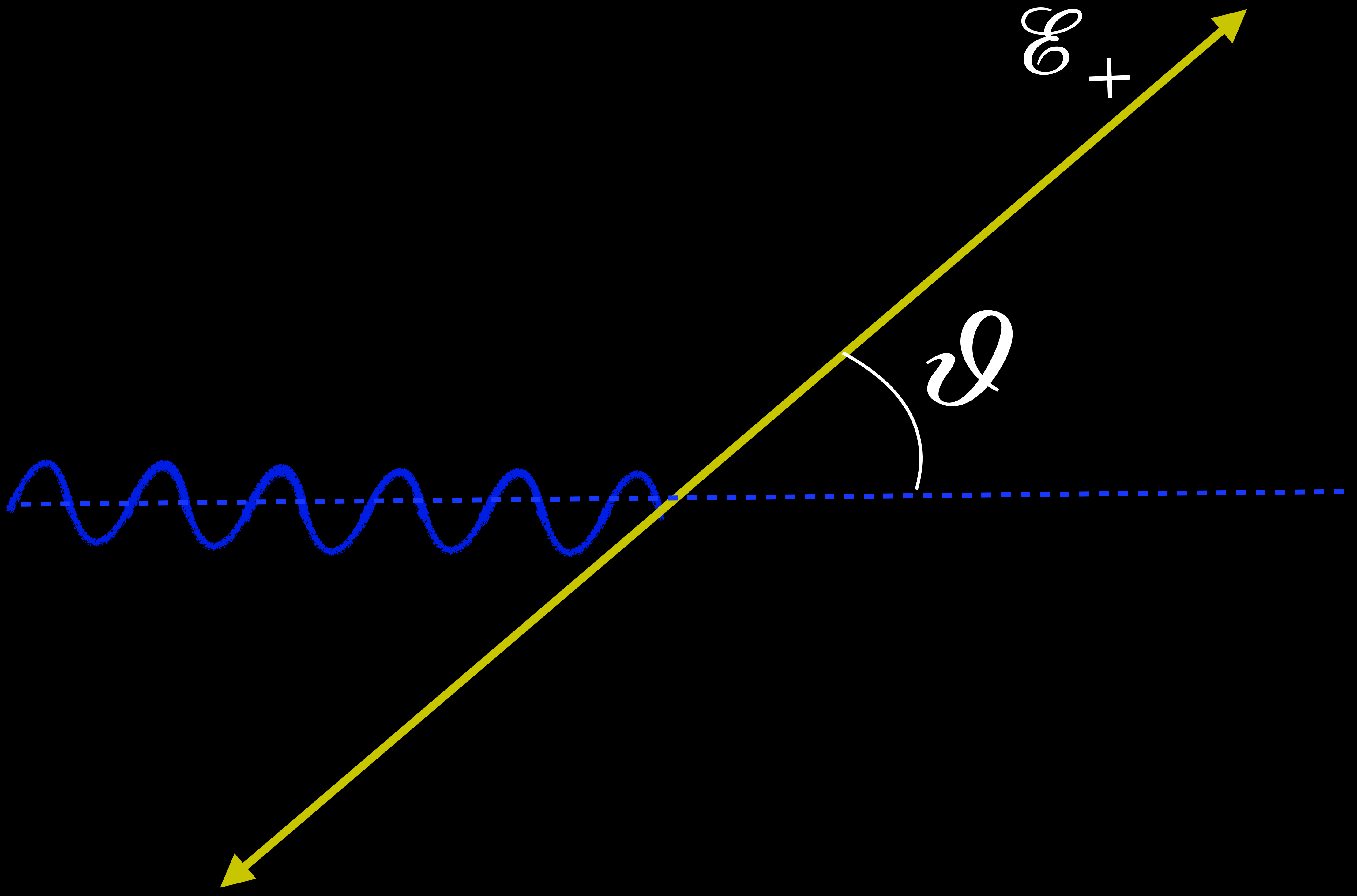

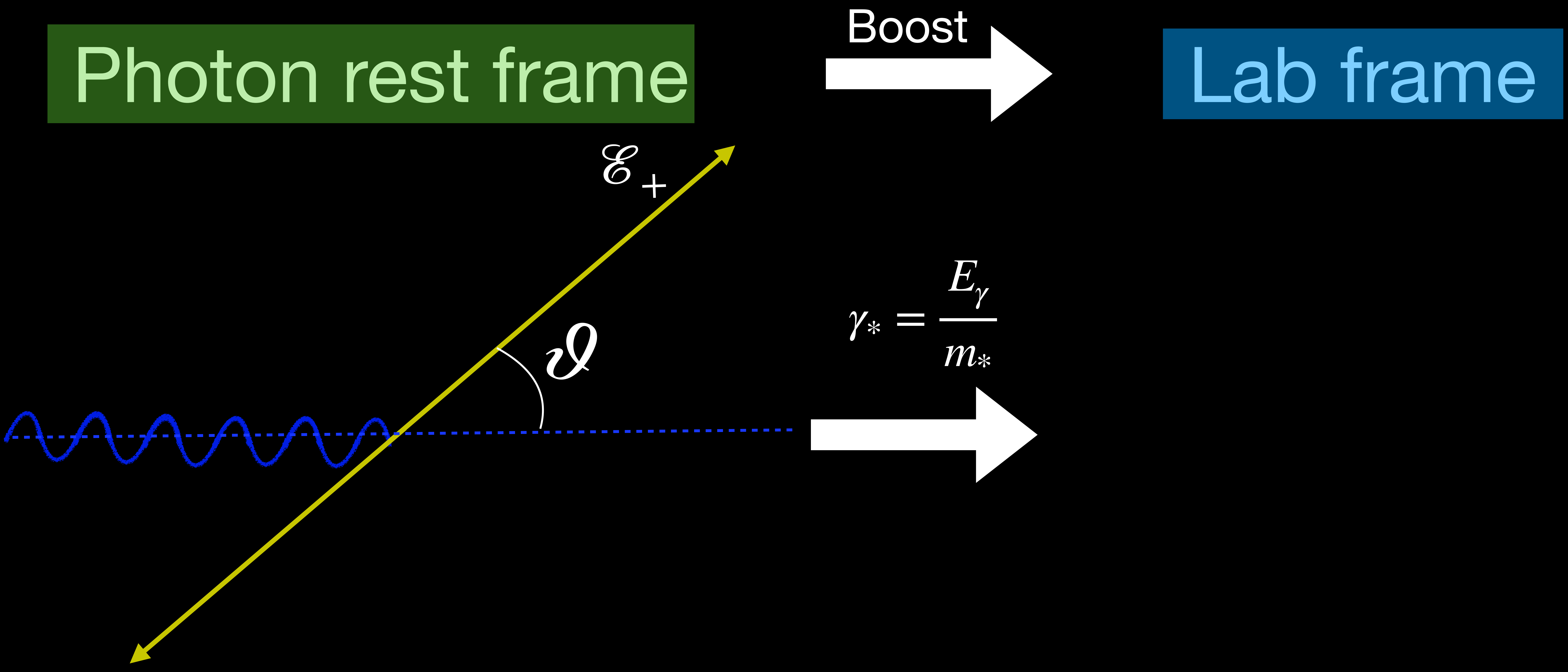
Photon rest frame
Boost
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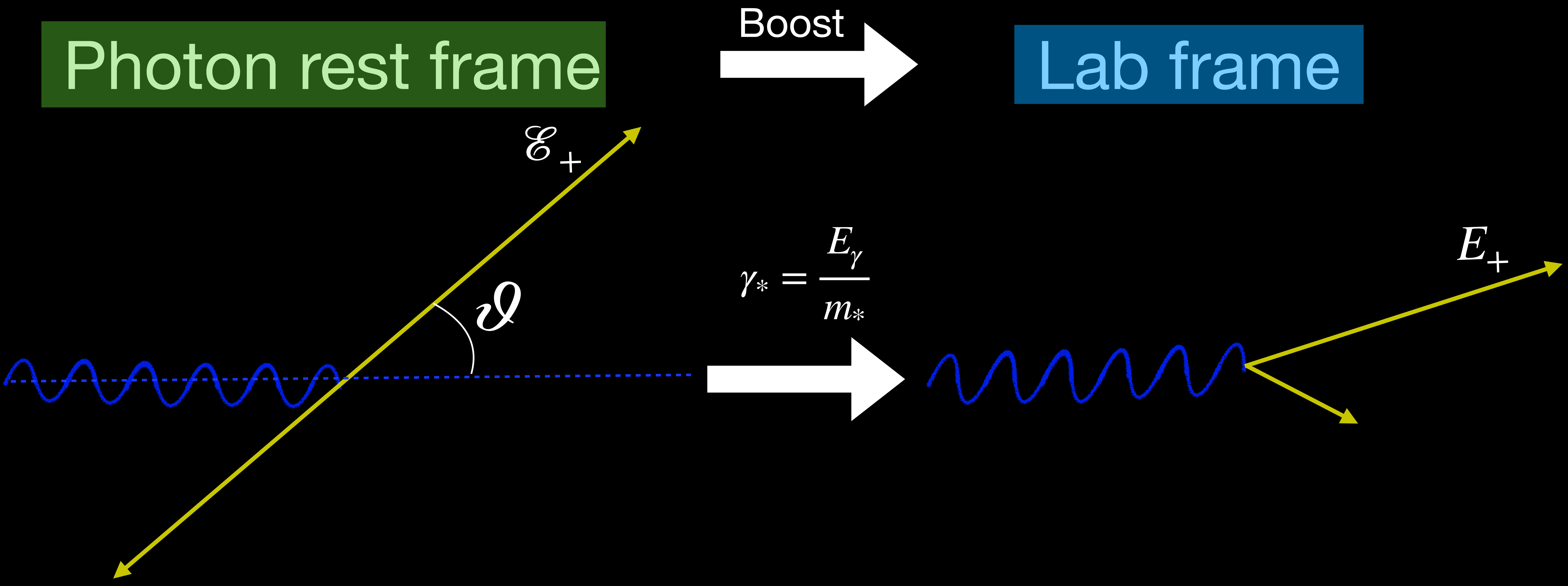
$\mathscr{E}_+$
ϑ
$\gamma_* = \frac{E_\gamma}{m_*}$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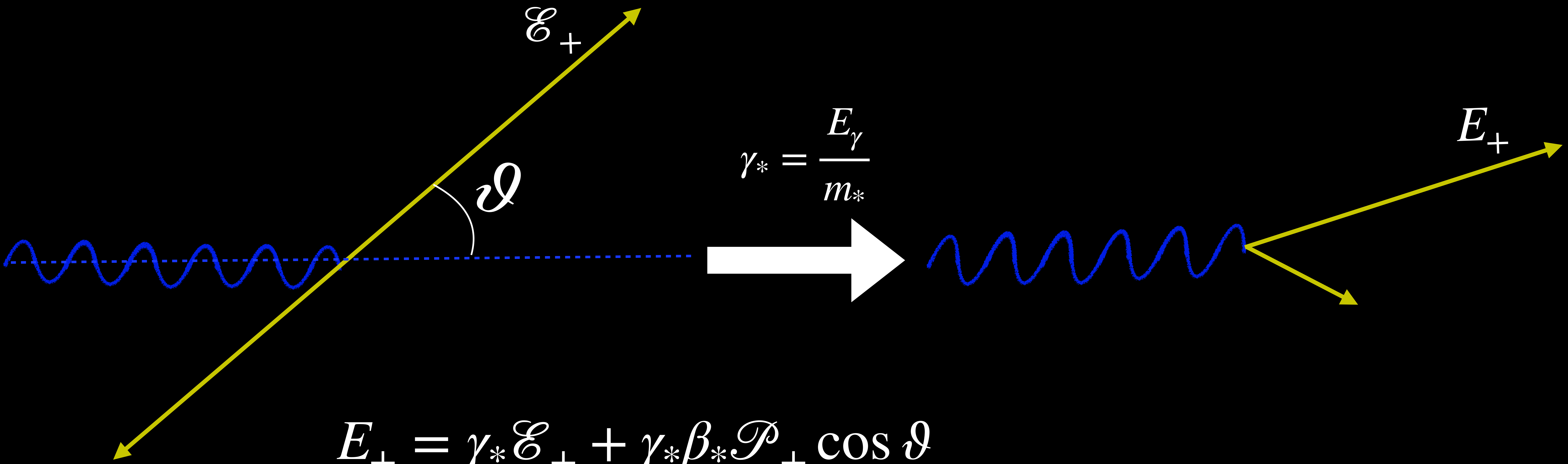
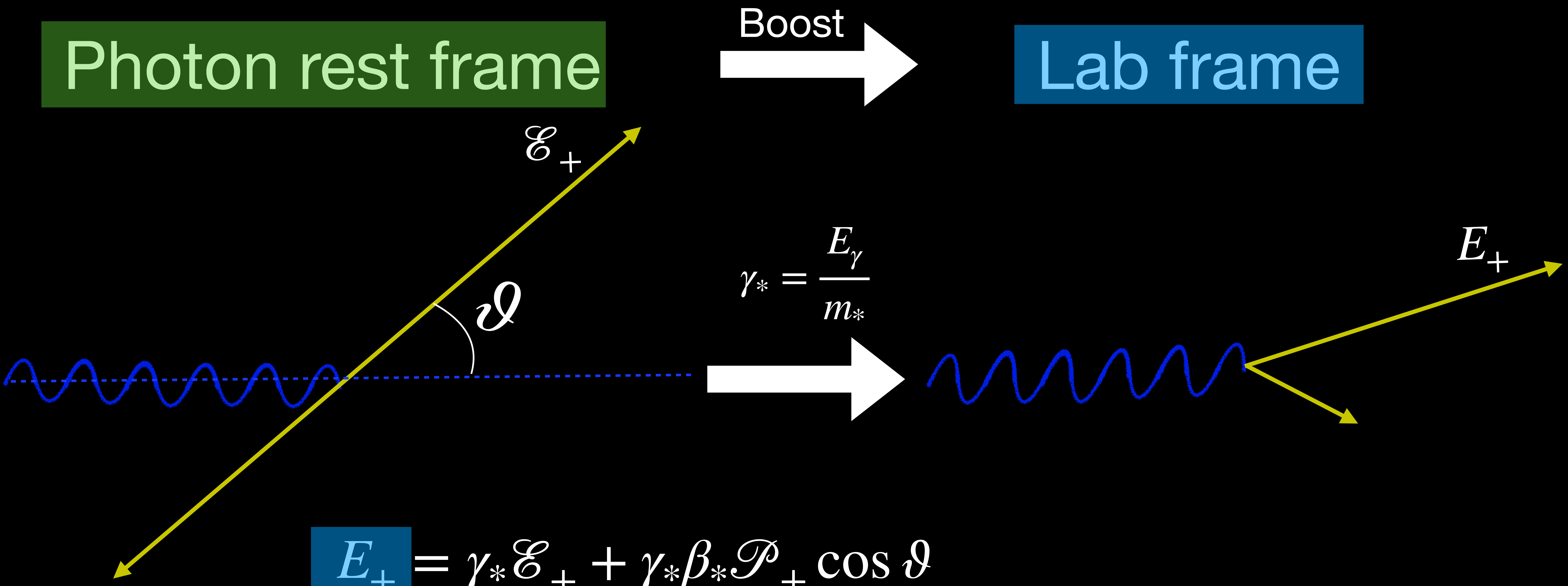


$$E_+ = \gamma_* \mathscr{E}_+ + \gamma_* \beta_* \mathscr{P}_+ \cos\vartheta$$



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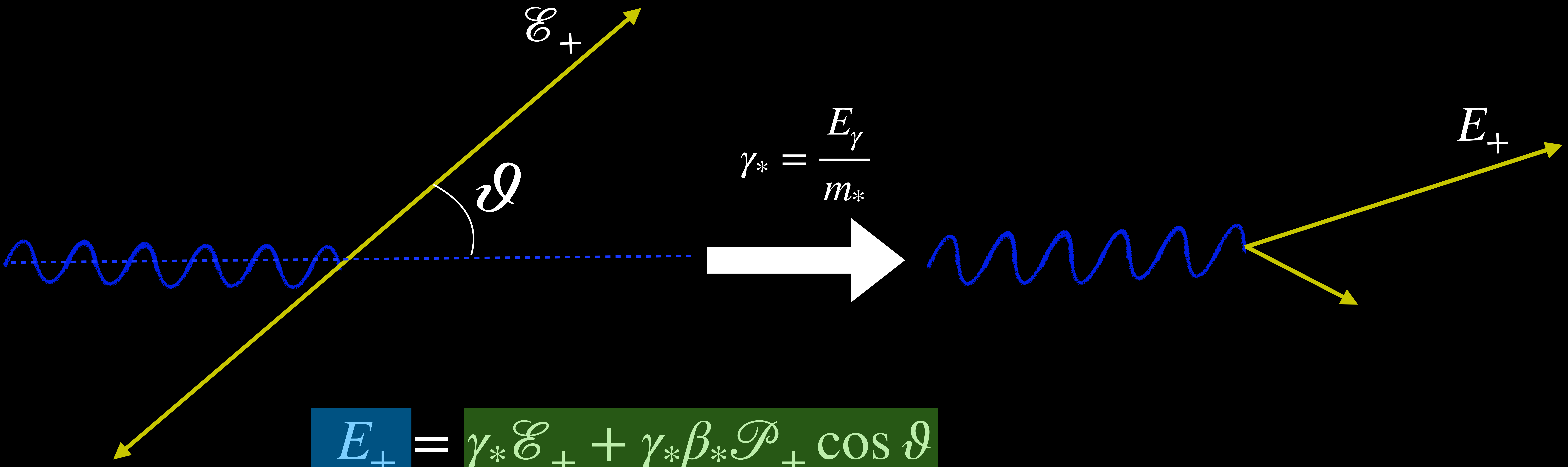
Photon rest frame → Boost → Lab frame



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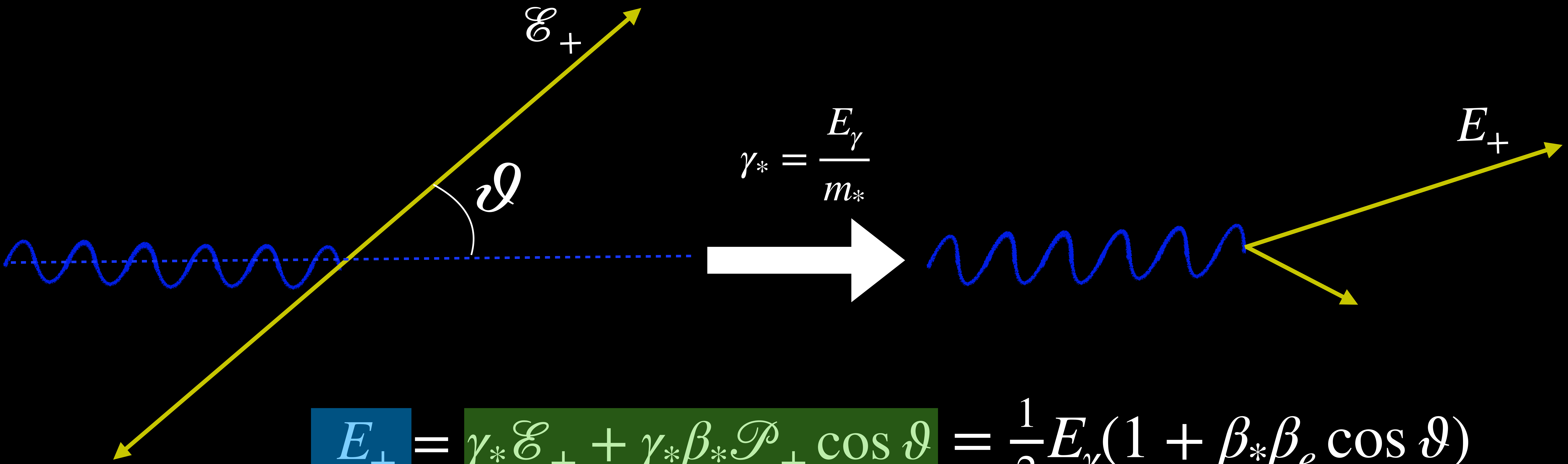

$$E_+ = \gamma_* \mathscr{E}_+ + \gamma_* \beta_* \mathscr{P}_+ \cos\vartheta = \frac{1}{2} E_\gamma (1 + \beta_* \beta_e \cos\vartheta)$$

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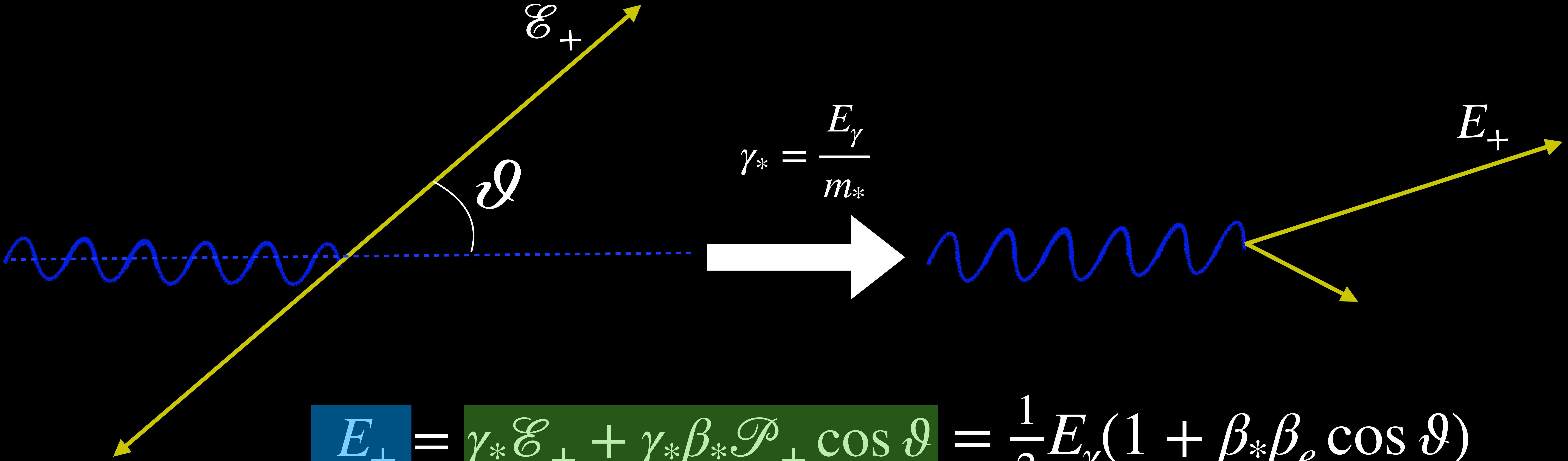

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Maximum energy $\Longrightarrow$ Small angle

$$E_+ = \frac{1}{2} E_\gamma (1 + \beta_* \beta_e \cos \vartheta)$$

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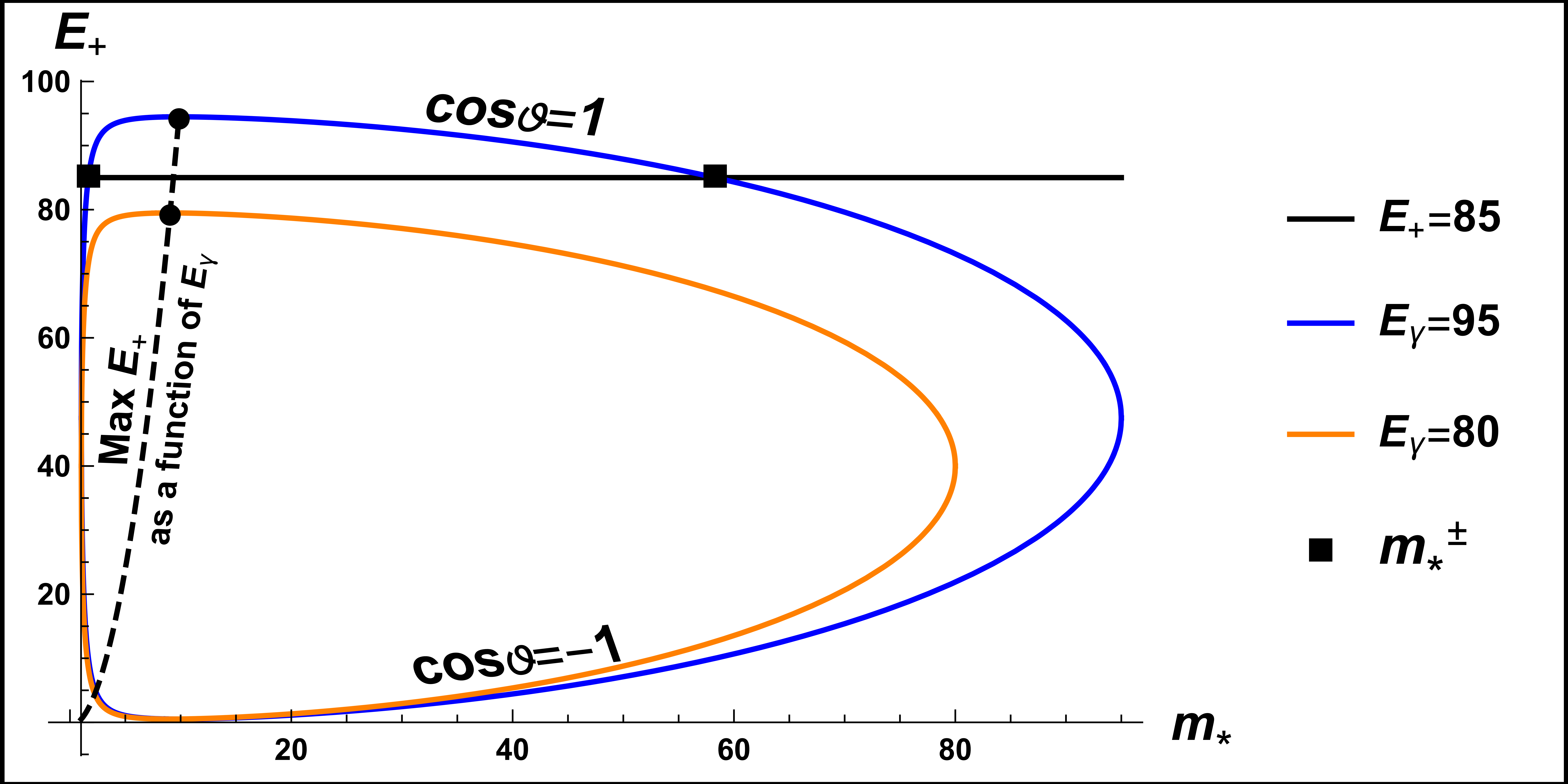

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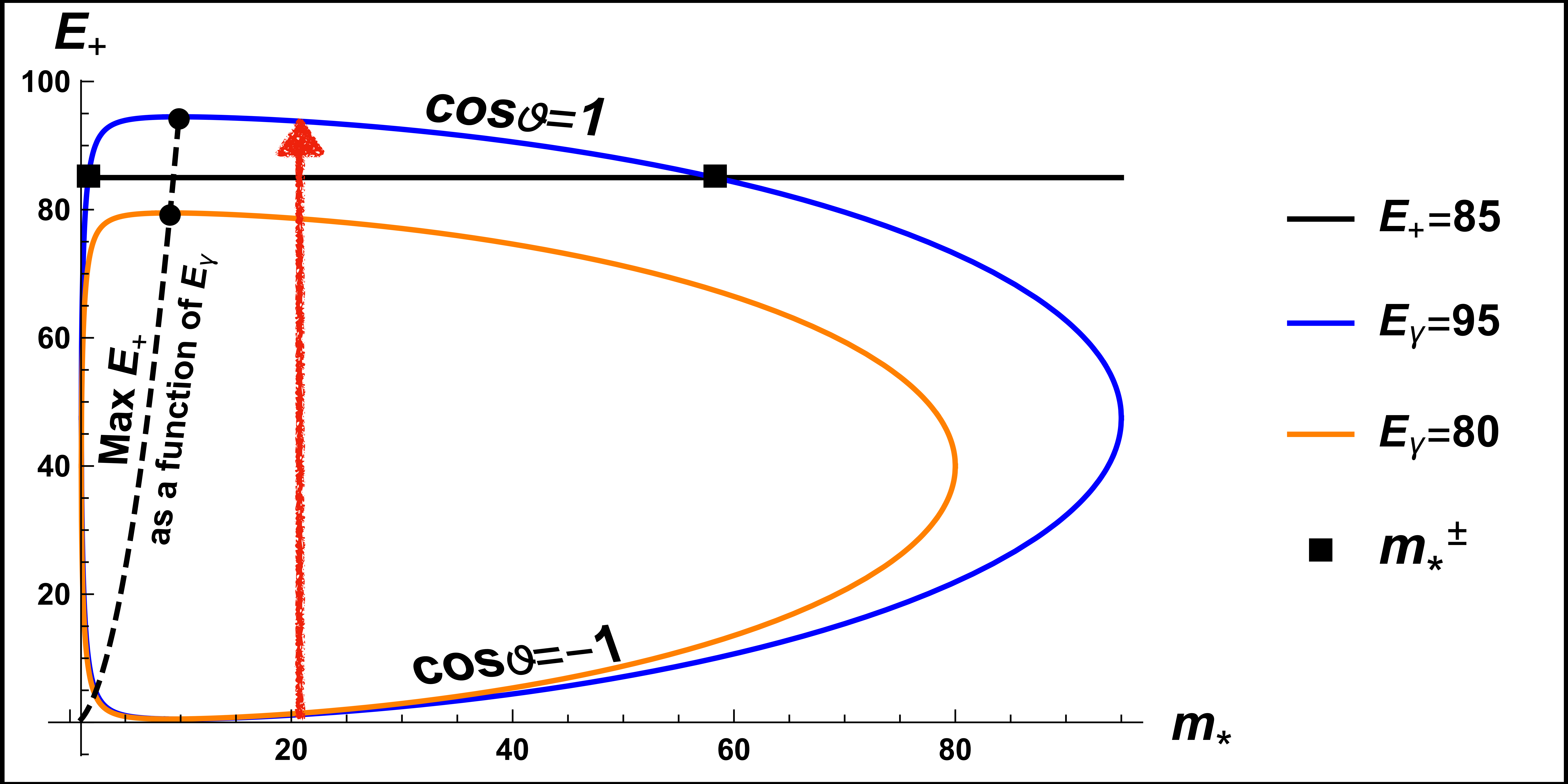

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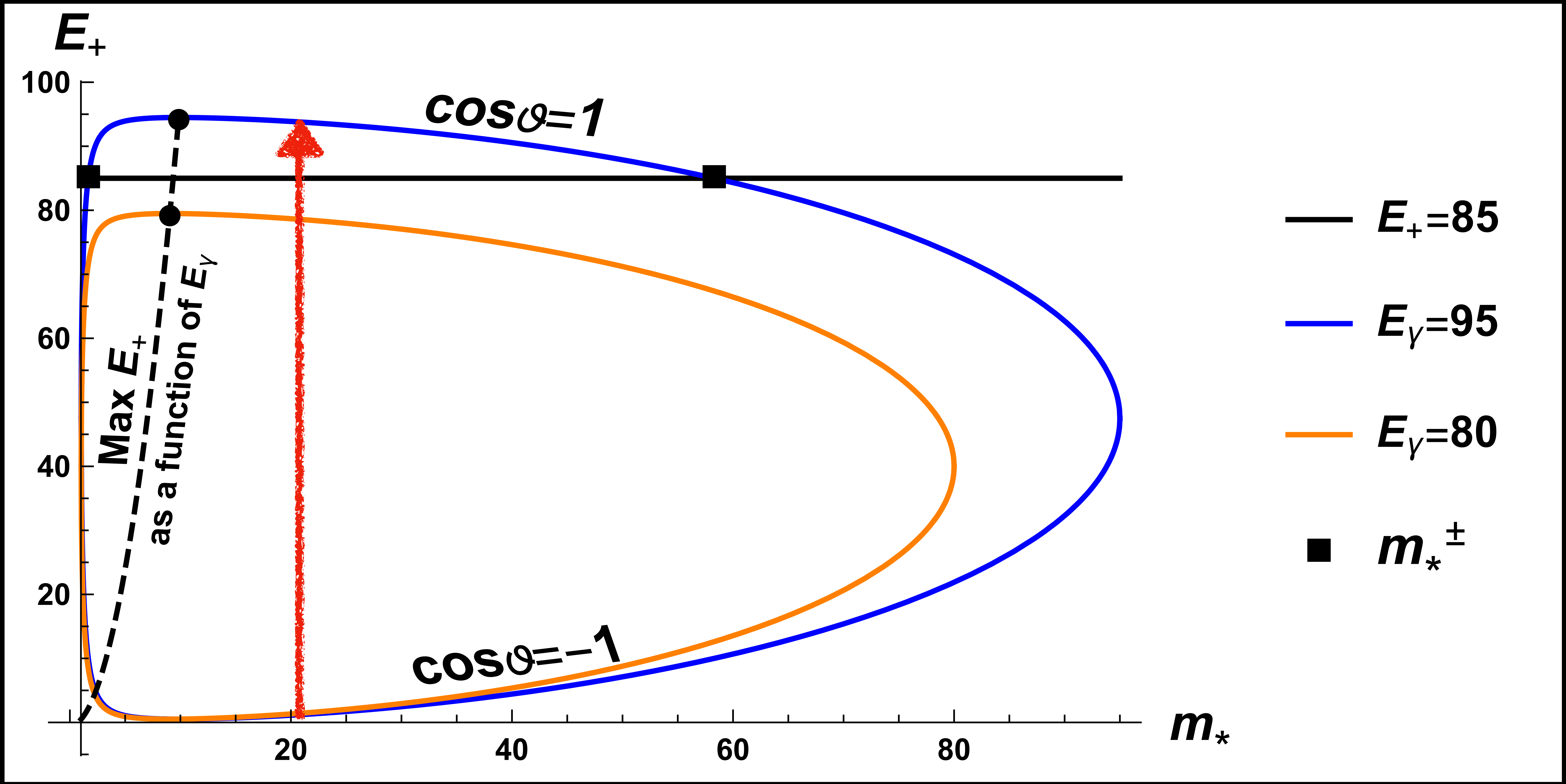

Expand around endpoint

$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

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"off-shell RMC" phase space

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"off-shell RMC" phase space **Virtual photon mass**

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"off-shell RMC" phase space — **Virtual photon mass** — **Electron phase space**

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"off-shell RMC" phase space

Virtual photon mass

Electron phase space

Expand around endpoint

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"off-shell RMC" phase space — Virtual photon mass — Electron phase space

Expand around endpoint

Do these integrals

$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

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NUCLEAR PHYSICS

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Expand around endpoint

Do these integrals

arXiv:2010.09509

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma}\; P_{\mathrm{int}}(E_+ \,|\, E_\gamma)$$

$$P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi) \approx \frac{\alpha}{2\pi E_\gamma} \int_{m_{*-}}^{m_*^+} \frac{\mathrm{d}m_*}{m_*} = \frac{\alpha}{\pi E_\gamma} \log\left[\frac{m_*^+}{m_*^-}\right]$$

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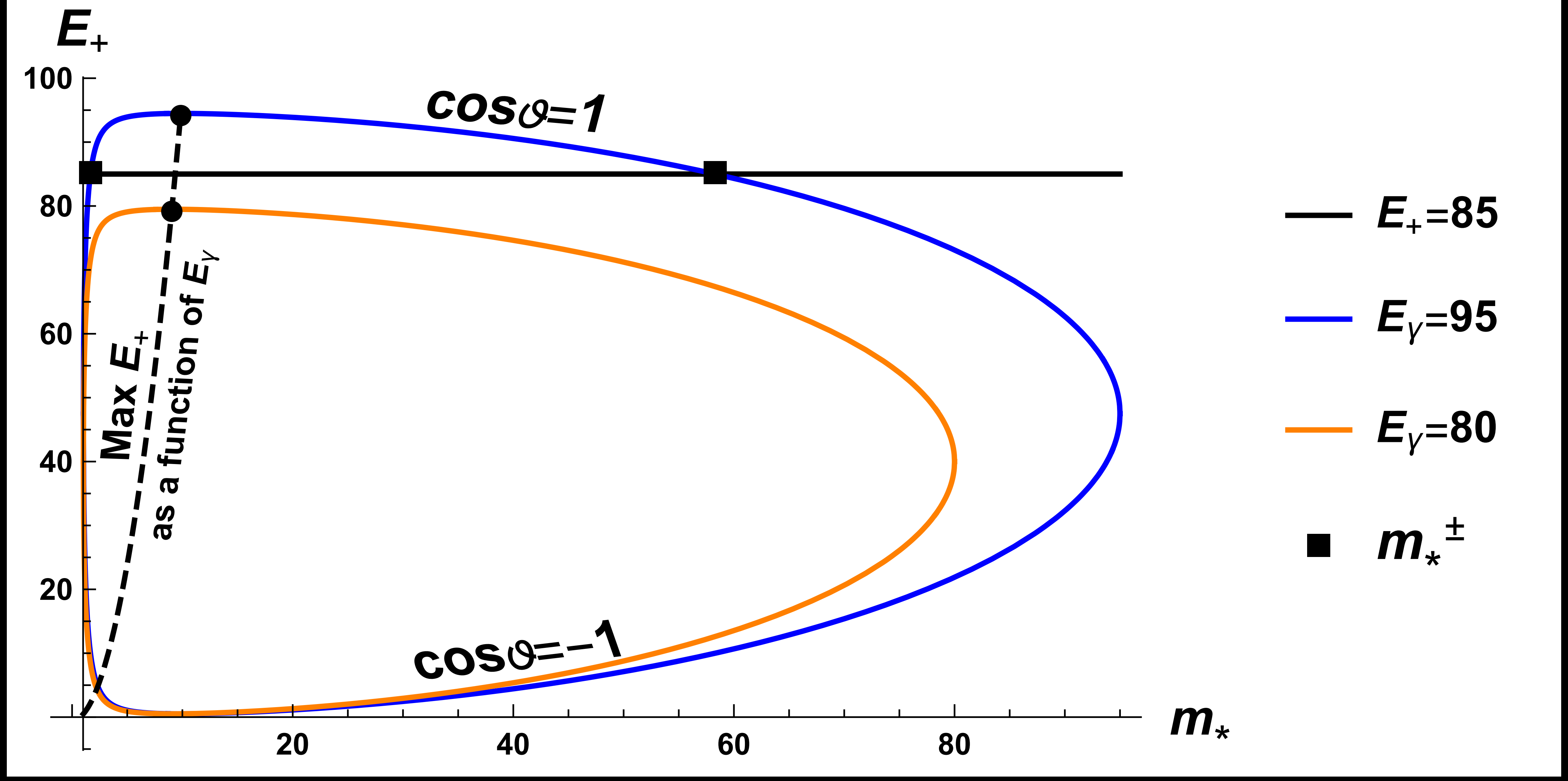
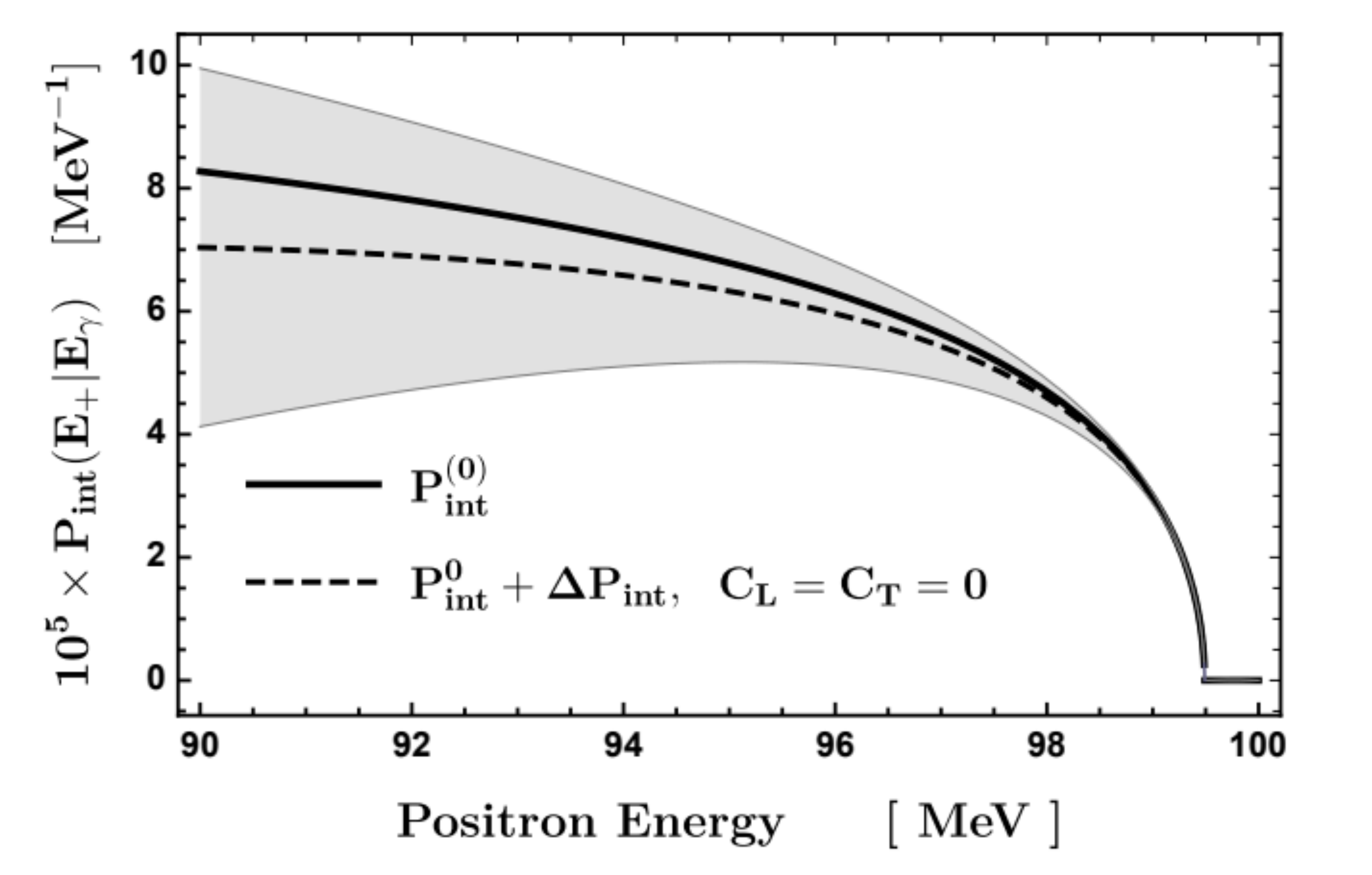
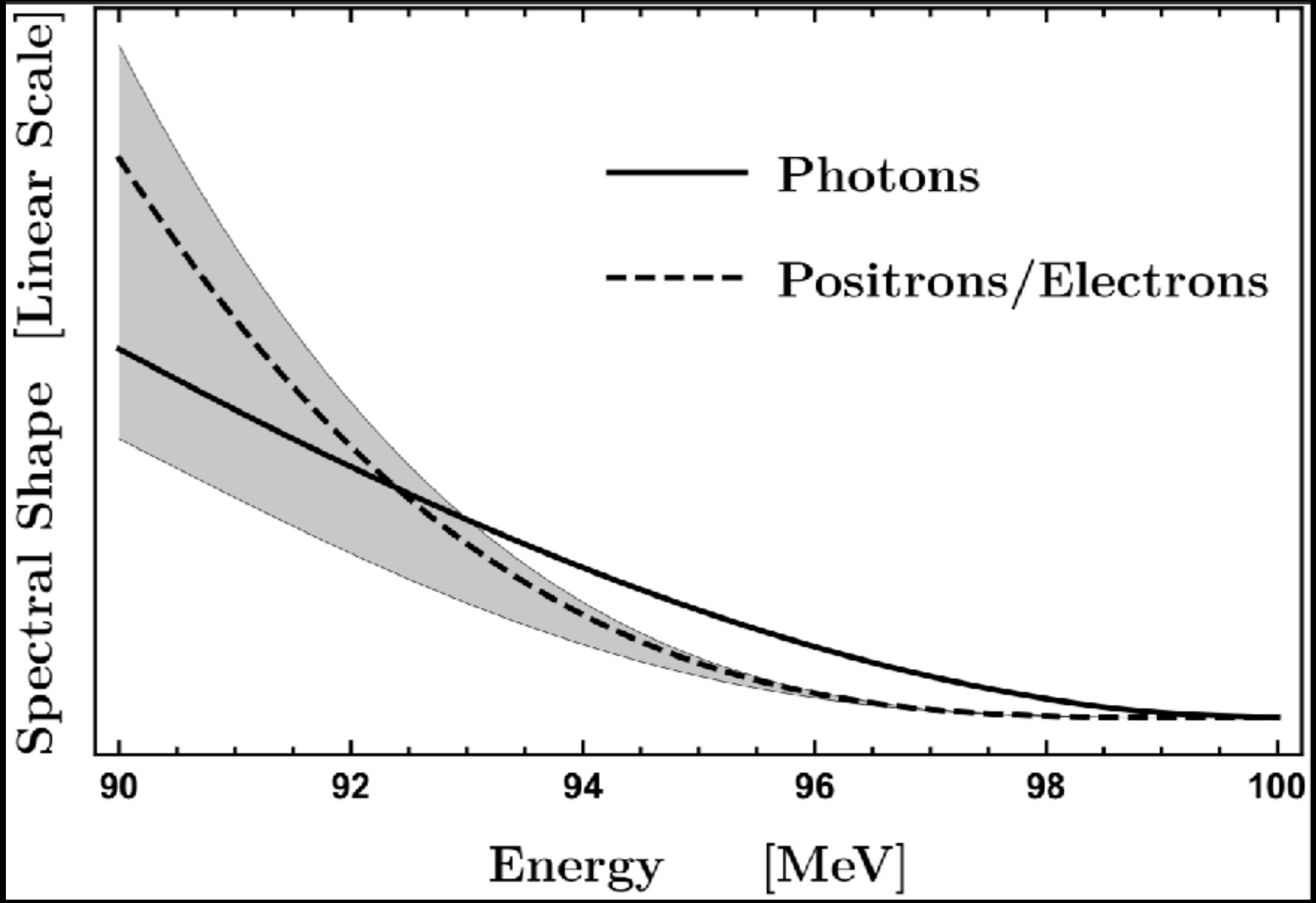




- This function + photon spectrum predicts the positron spectrum



- We estimate errors by treating unknown matrix elements as Gaussian distributed with 50% uncertainty.

Main conclusions:

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- Reliable approximation as $E_+ \to E_\gamma - m_e$ or equivalently as $T_- \to 0$. The small parameters we use are T_-/E_γ and m_e/E_γ .

- Near the end point there is a calculable function for internal conversion.

The real photon spectrum

Summary:

The RMC spectrum must be a linear superposition of different photon spectra each with their own endpoints. Simple phase space arguments yield insight into the spectral shape of RMC at high energies.

What does ``Closure Approximation” mean in the context of the published TRIUMF dataset?

Overly constrained spectral *ansatz*.

Only free parameter (besides normalization) is $k_{\max}$.

Naturally one fits $k_{\max}$ to data but then....

Fitted spectrum predicts ZERO events in perfectly physical regions of phase space

Primakoff & the "Closure Approximation"

REVIEWS OF MODERN PHYSICS VOLUME 31, NUMBER 3

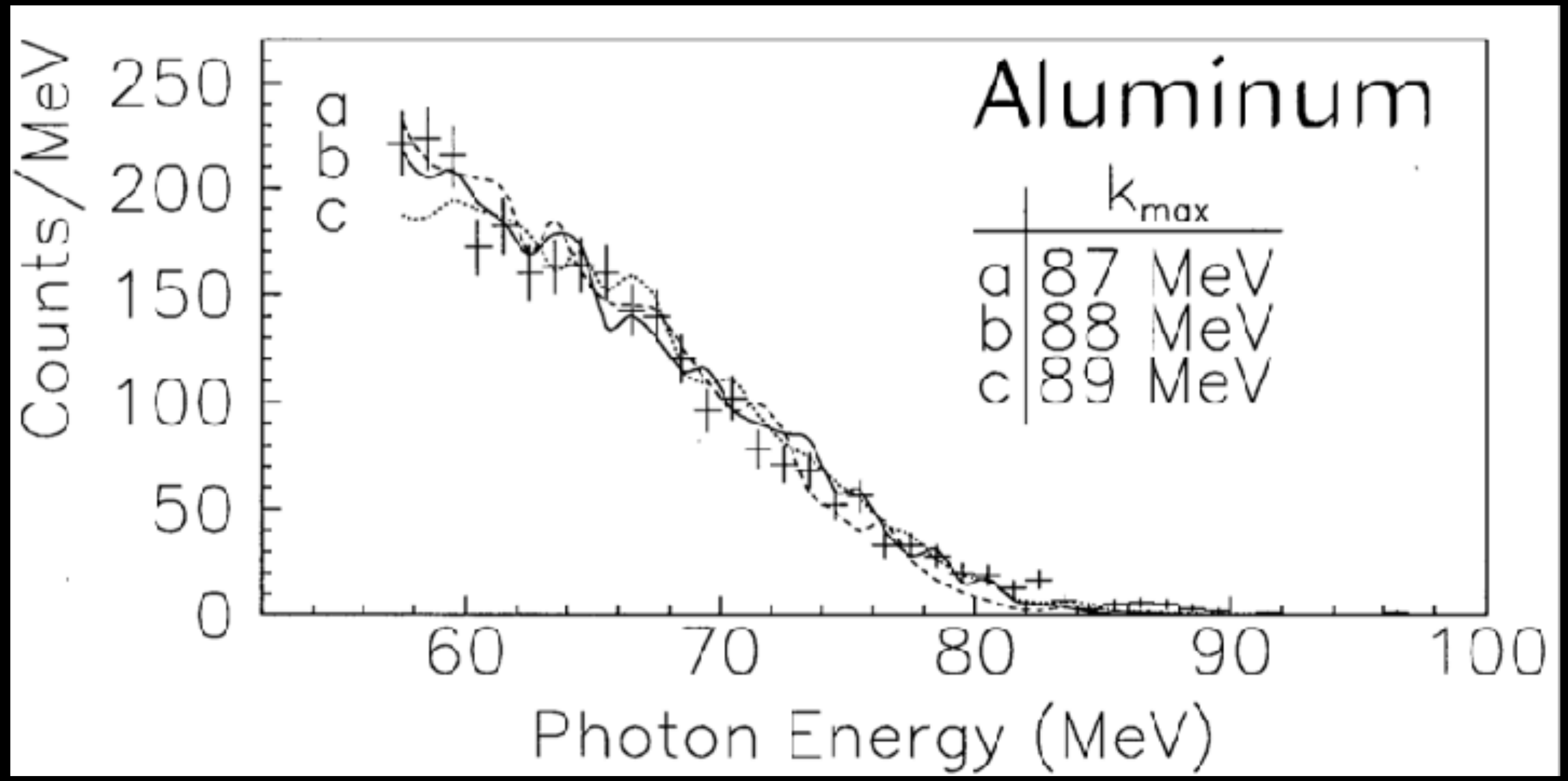
Theory of Muon Capture*

H. PRIMAKOFF

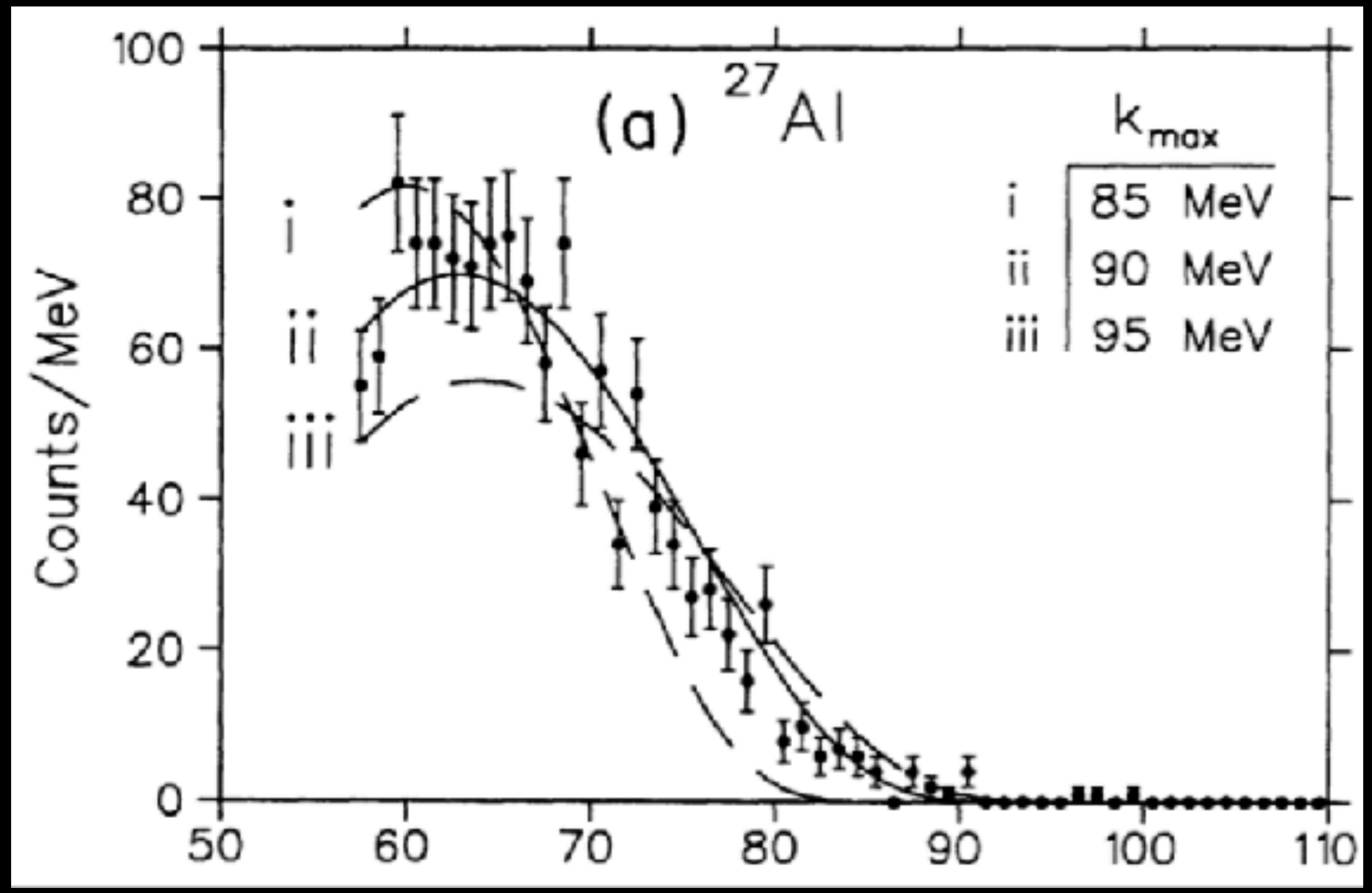
Department of Physics, Washington University, St. Louis 5, Missouri

$$x = k/k_{\max}$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}k} \propto x(1-x)^2 \times \left[1 - 2x(1-x)\right]$$



Bergbusch M.Sc. Thesis (1994)



Armstrong *et. al.* PRC **46:**3 1094-1106 (1993)

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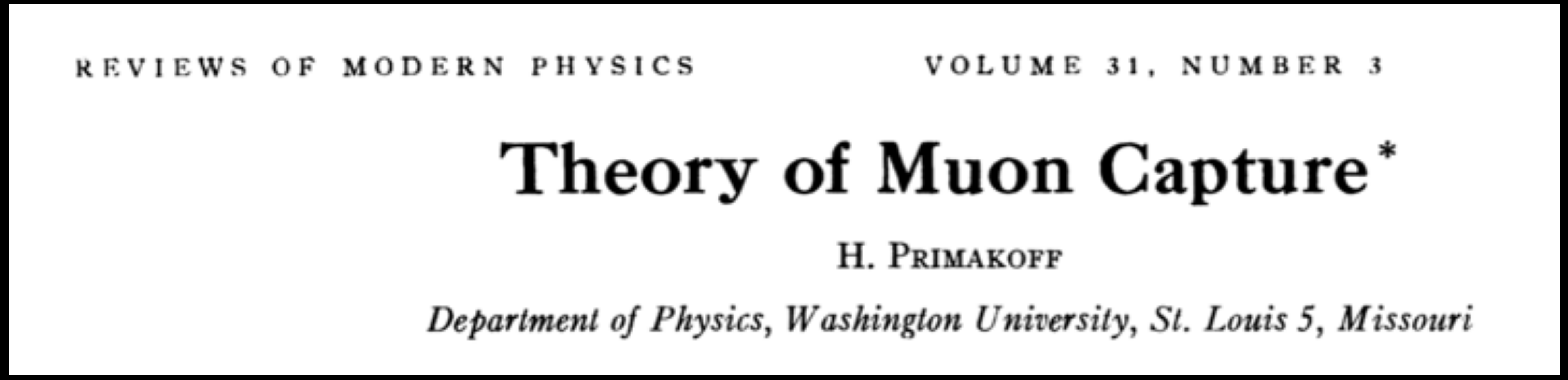
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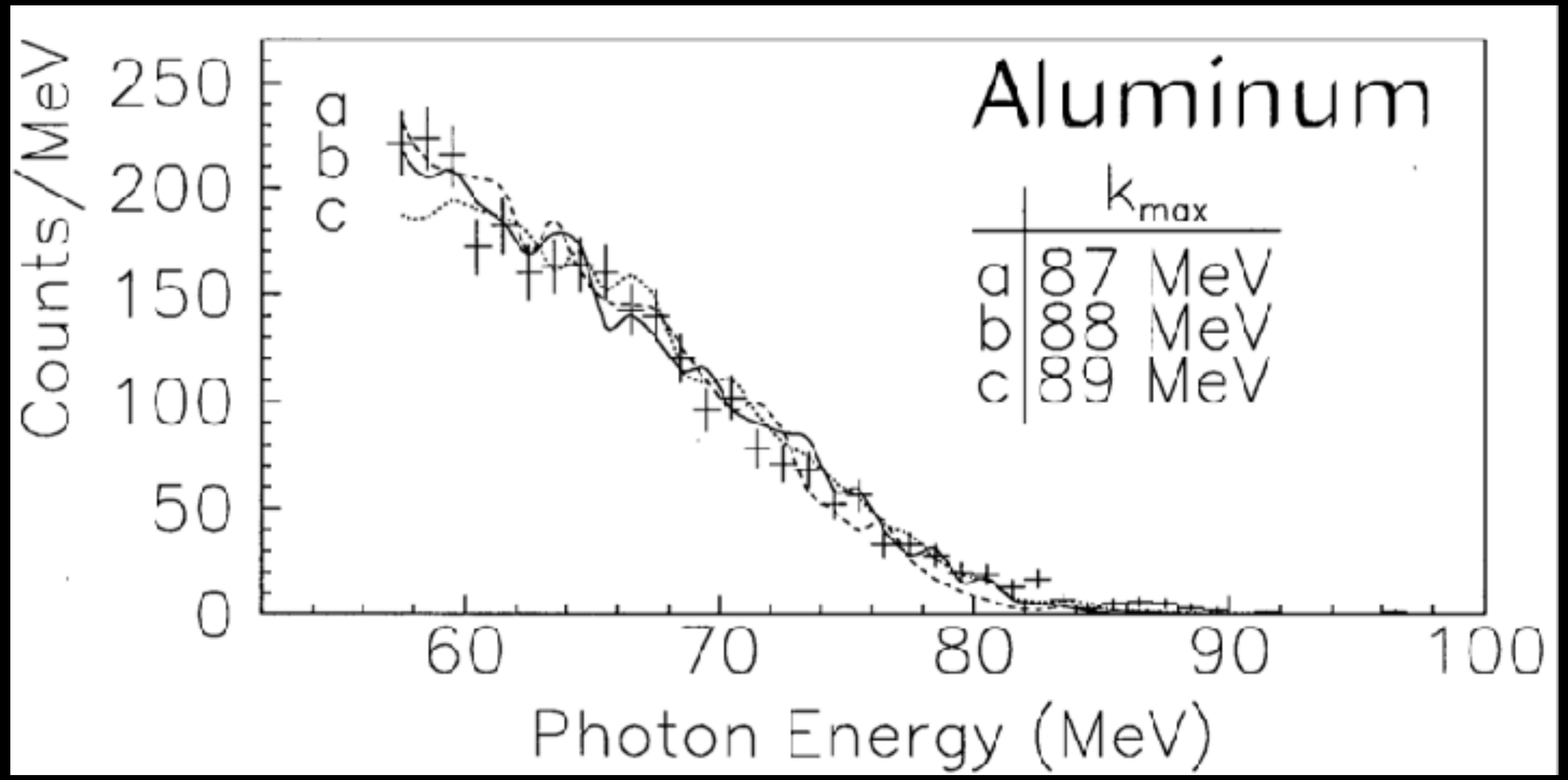
$\nu\gamma$ Phase Space



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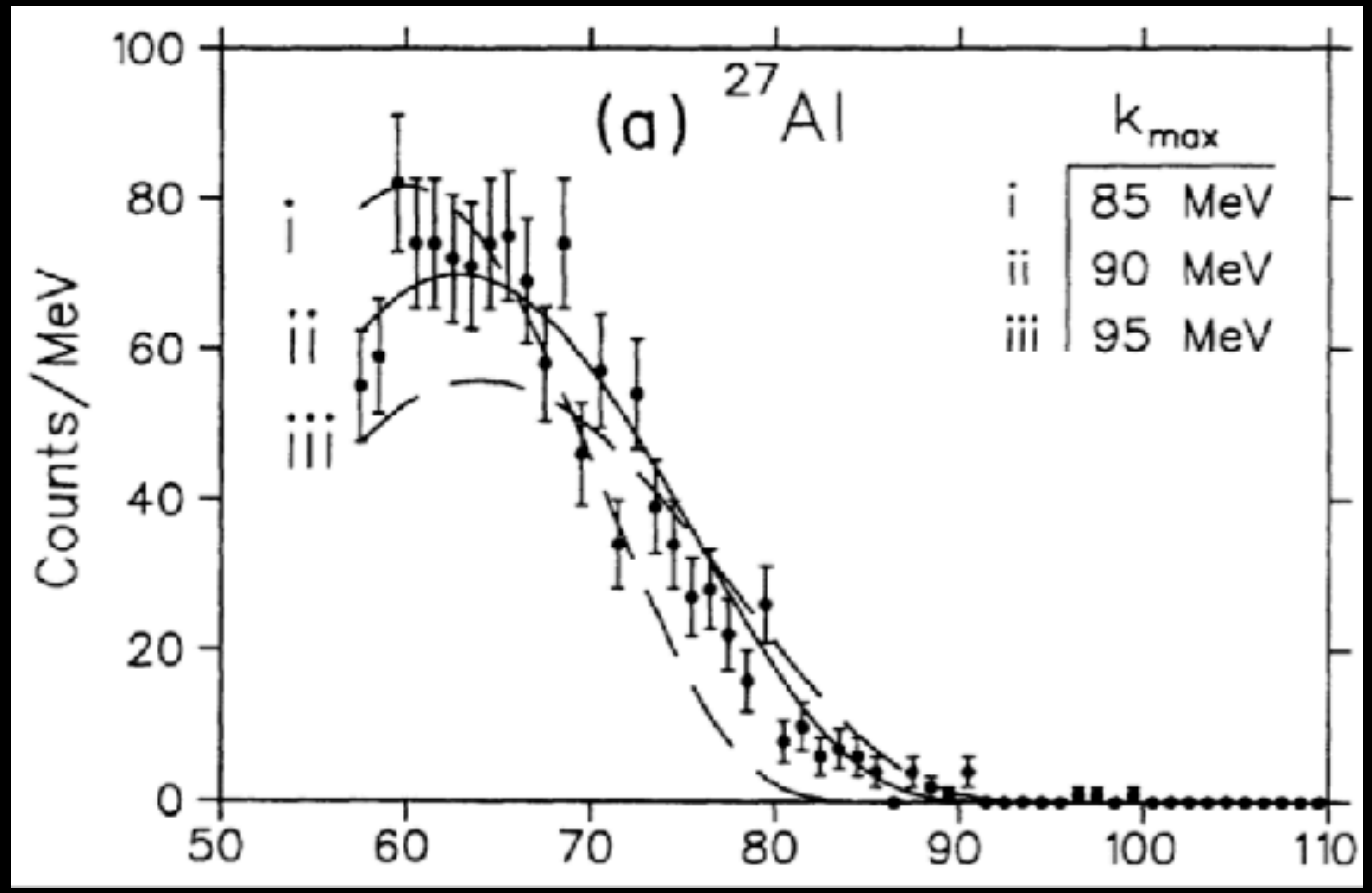

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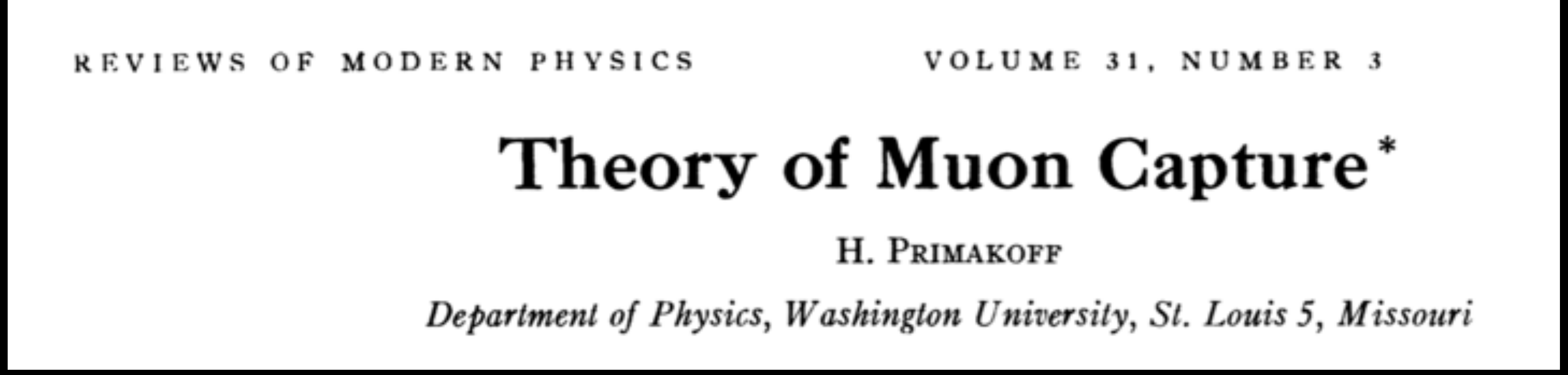
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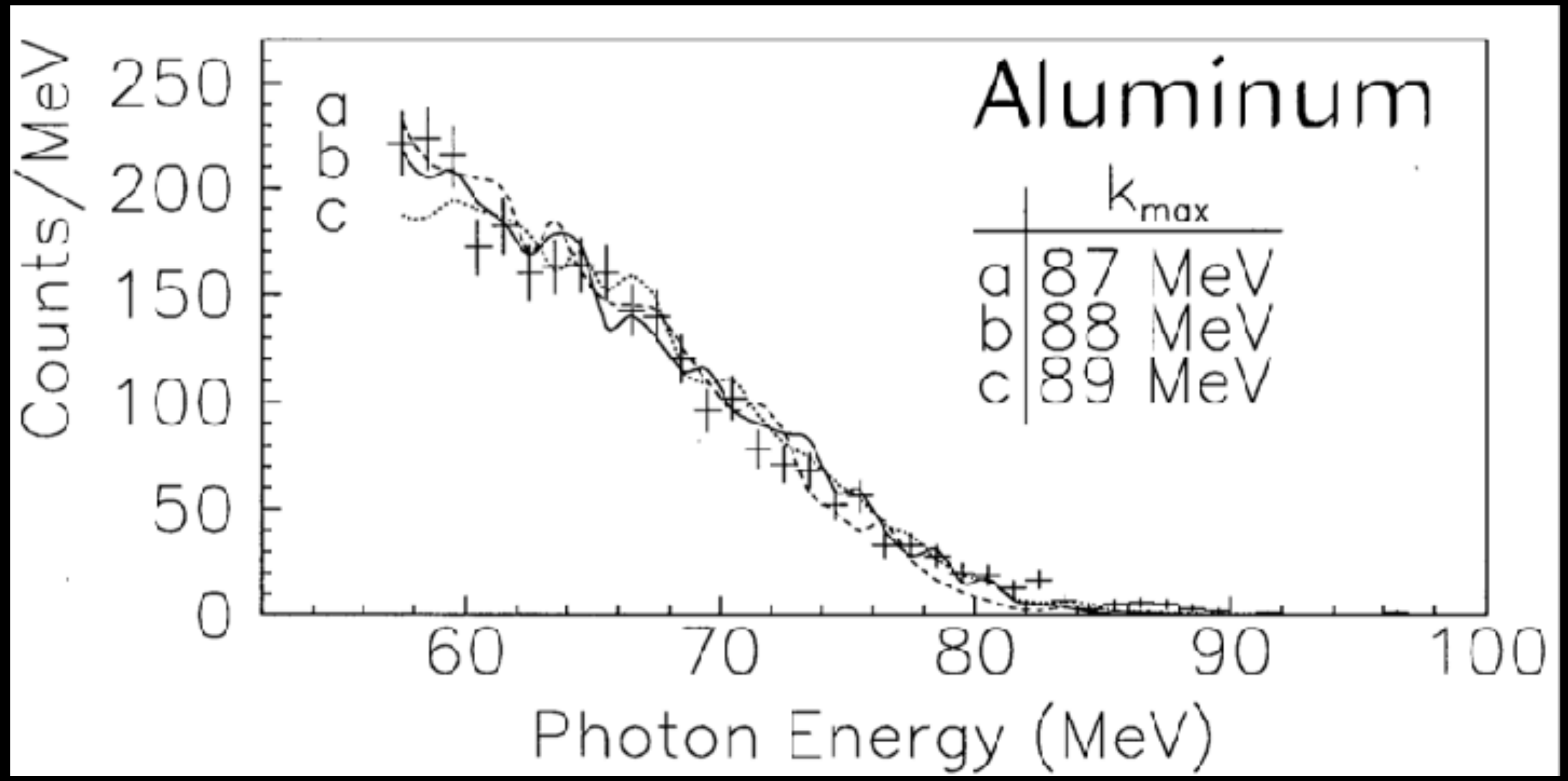
$\nu\gamma$ Phase Space

Estimates vary



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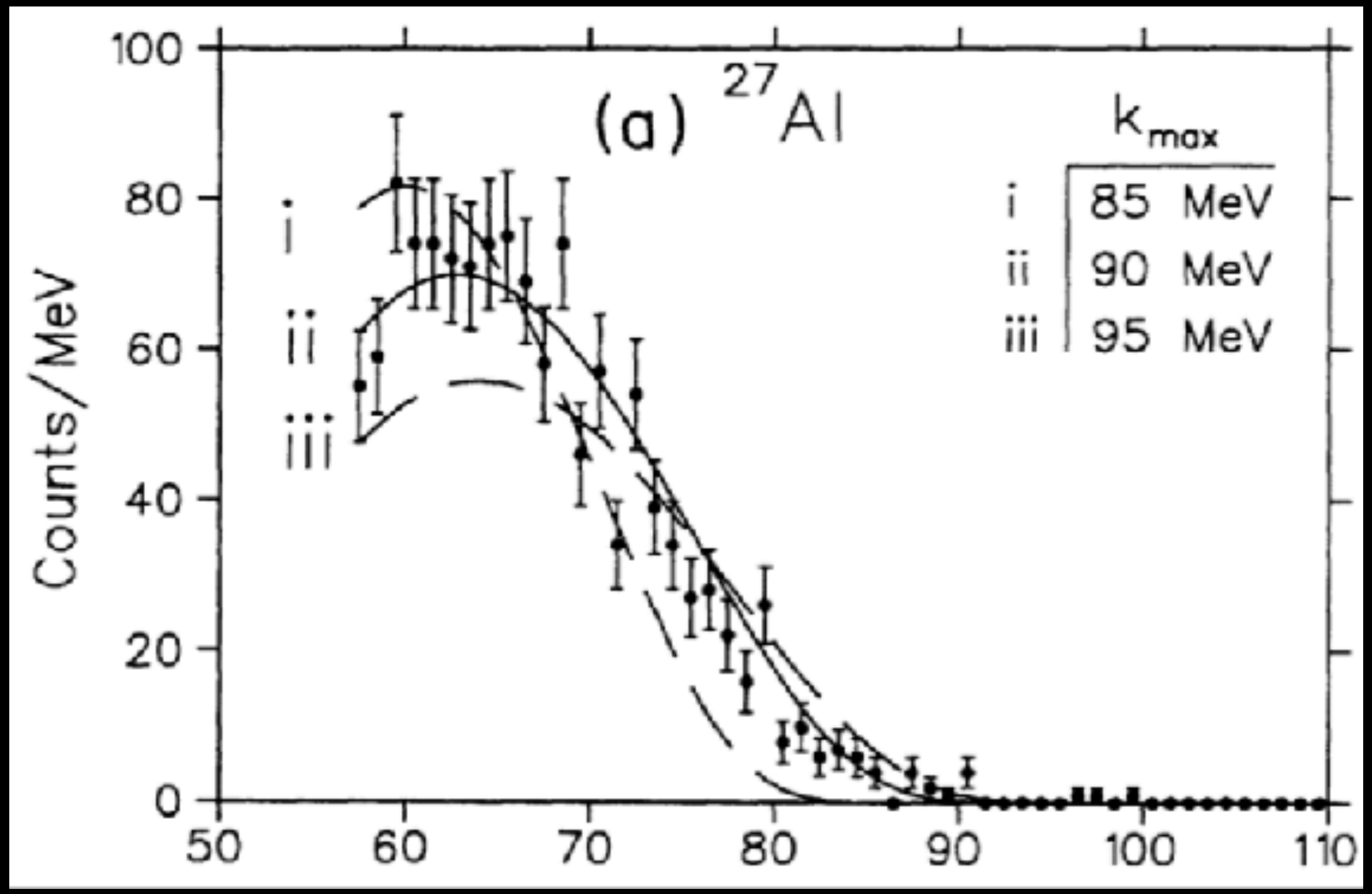

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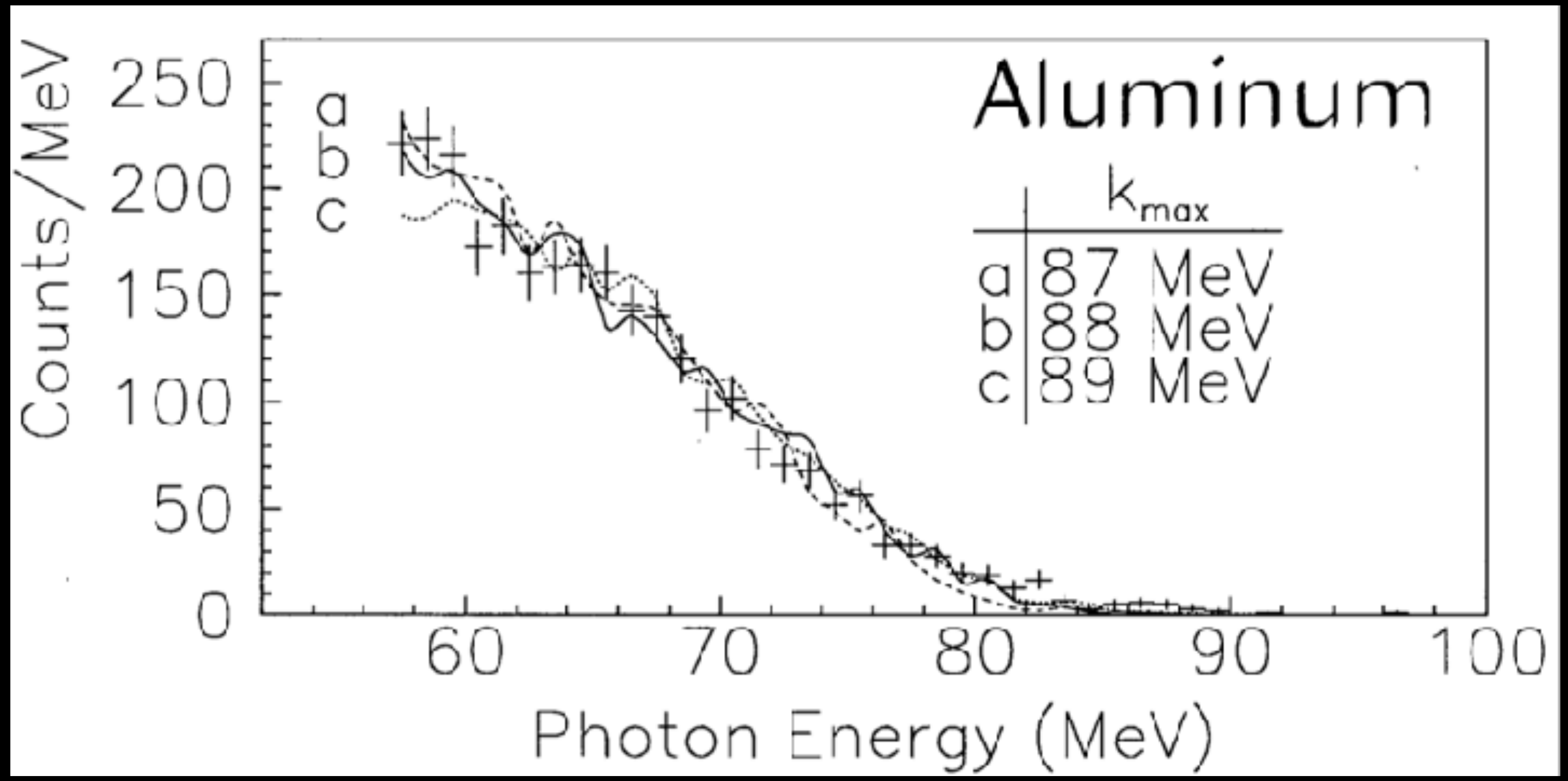
$\nu\gamma$ Phase Space   Estimates vary

TRIUMF experiments fit their spectra by varying k_{max} .



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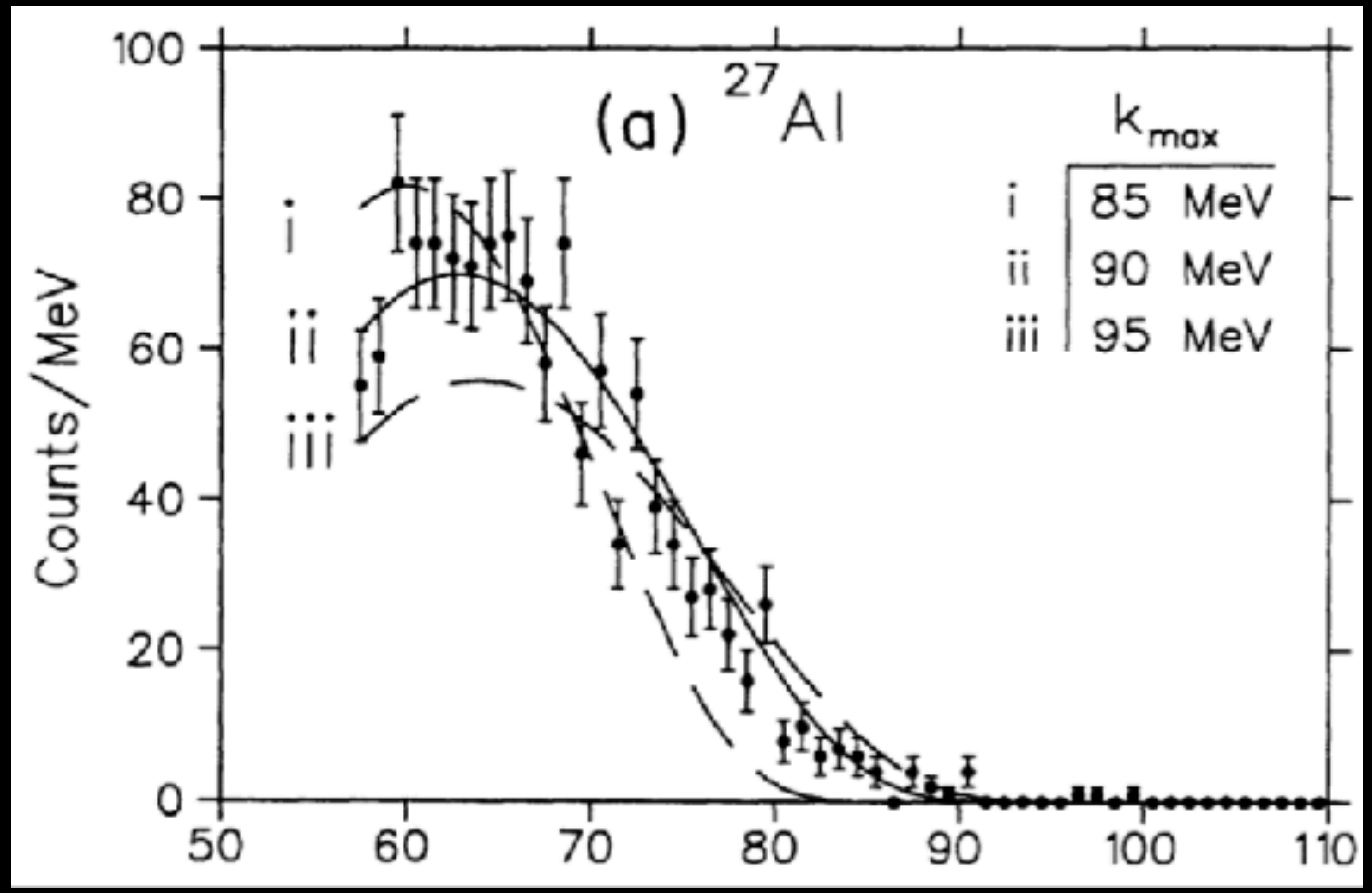

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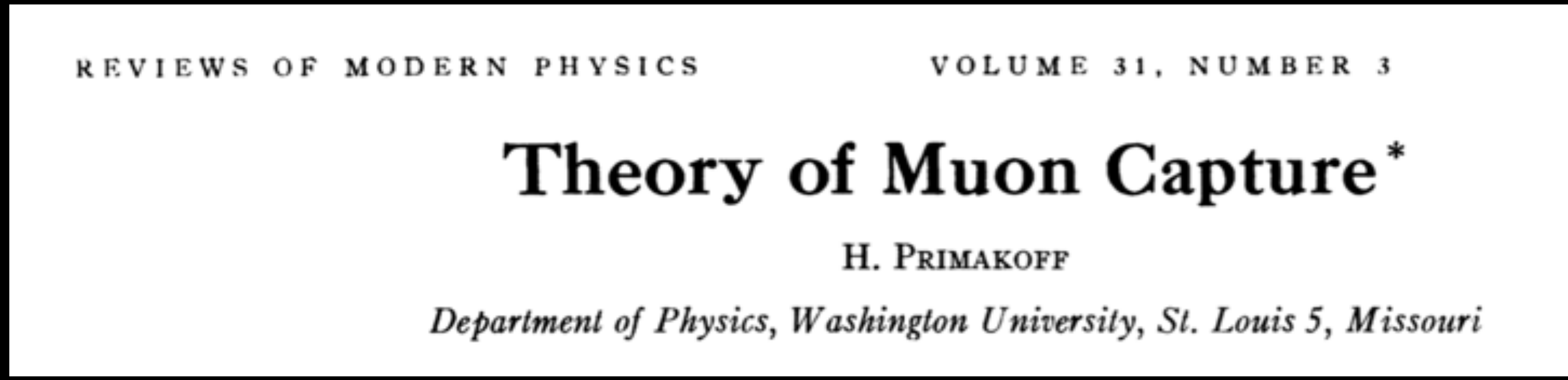
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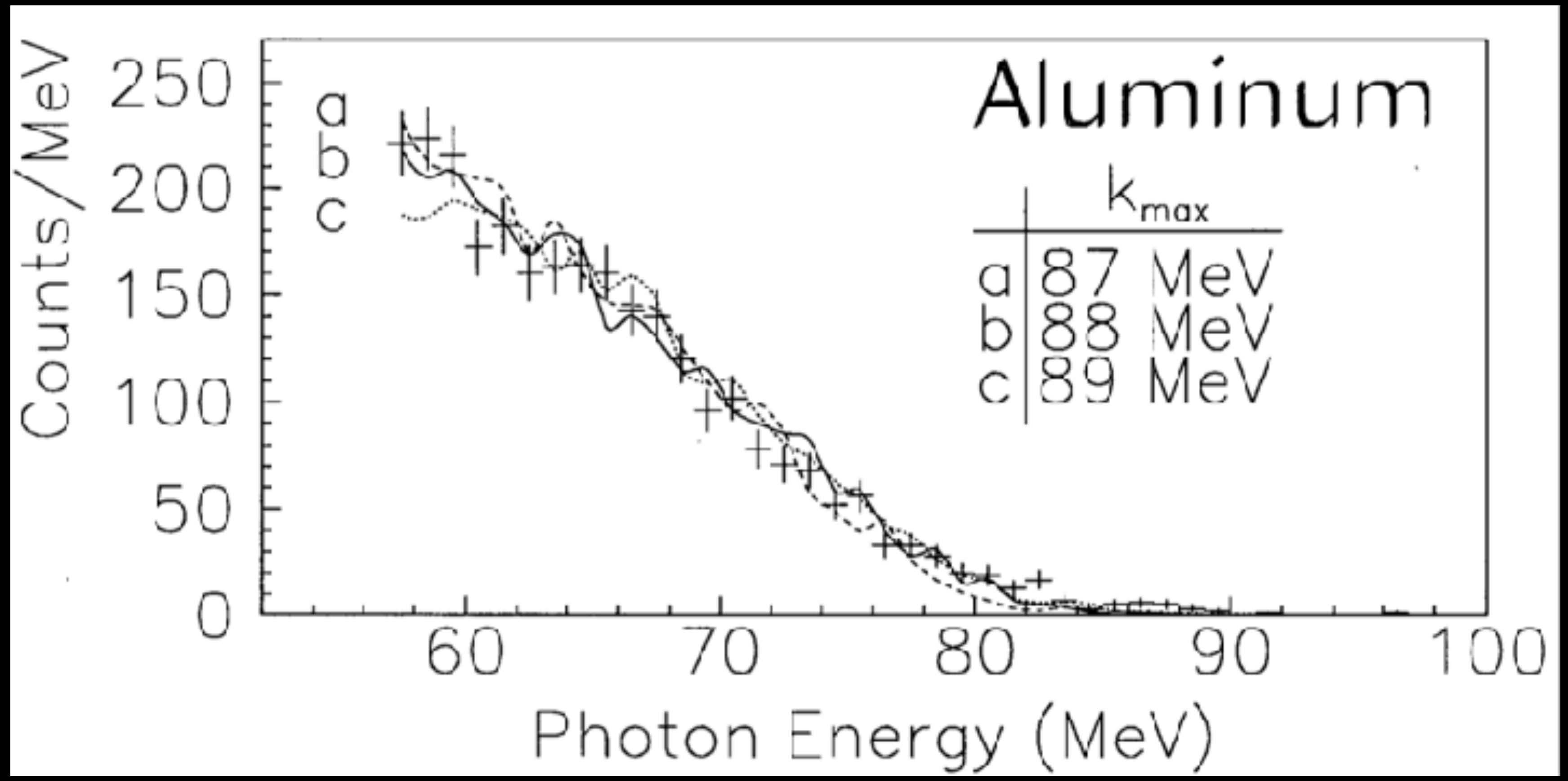
$\nu\gamma$ Phase Space — Estimates vary

TRIUMF experiments fit their spectra by varying k_{max} .

This does not provide a reliable extrapolation to the endpoint.



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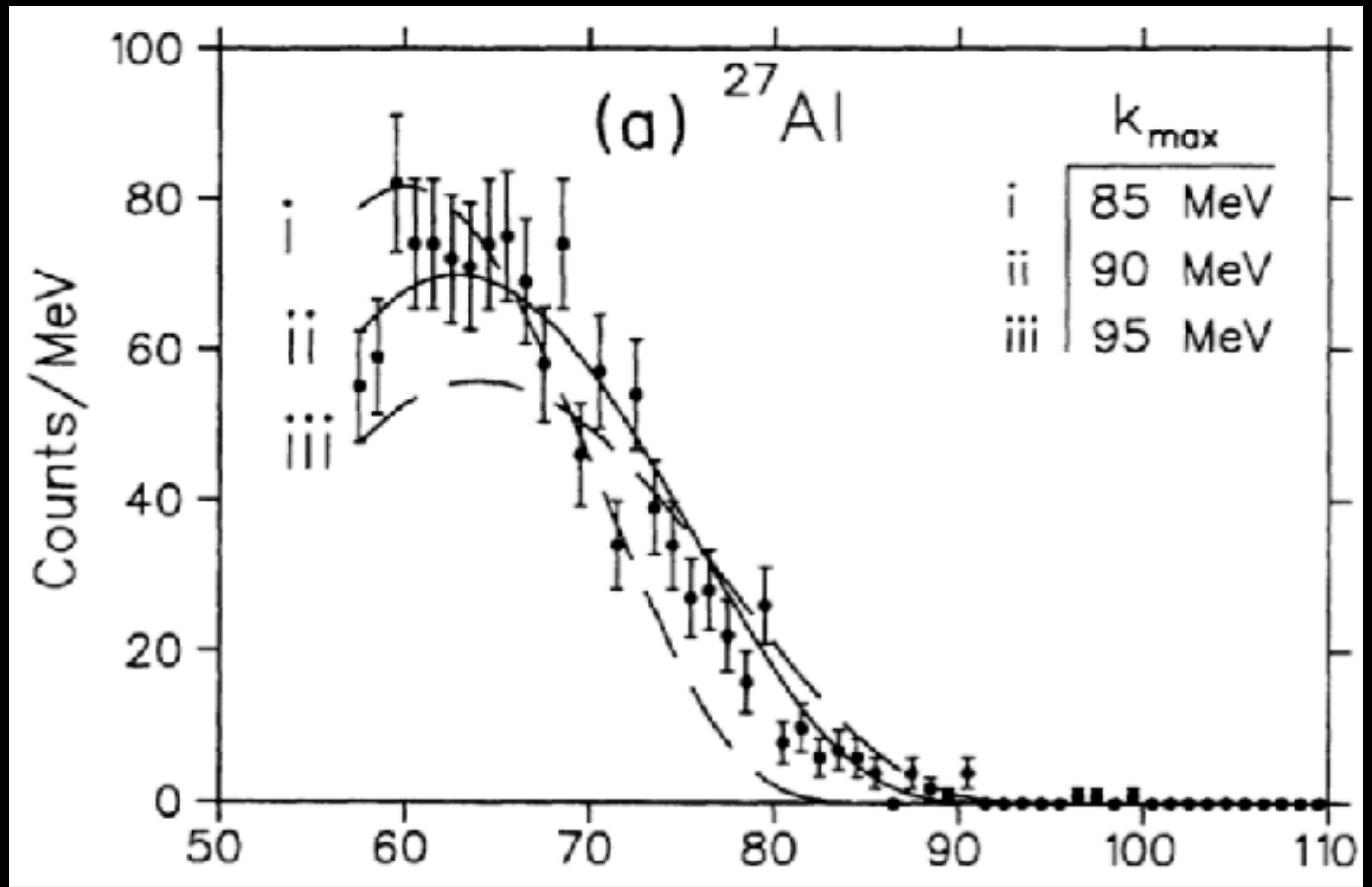

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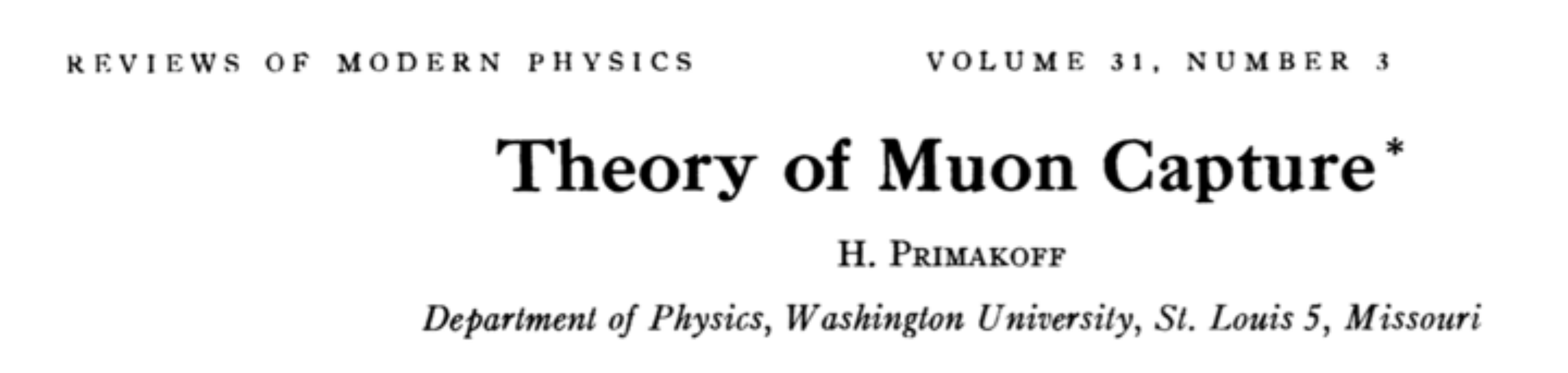
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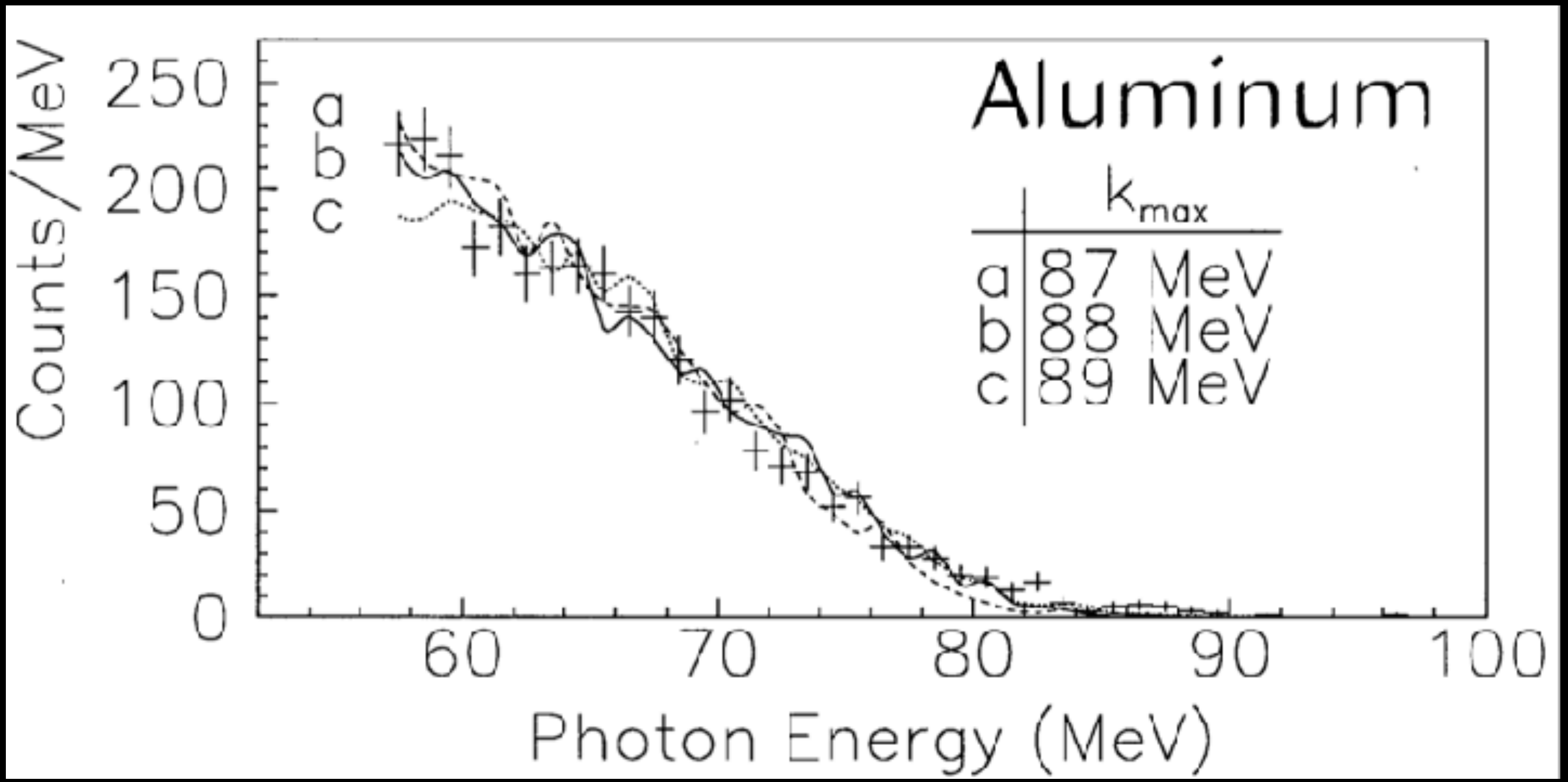

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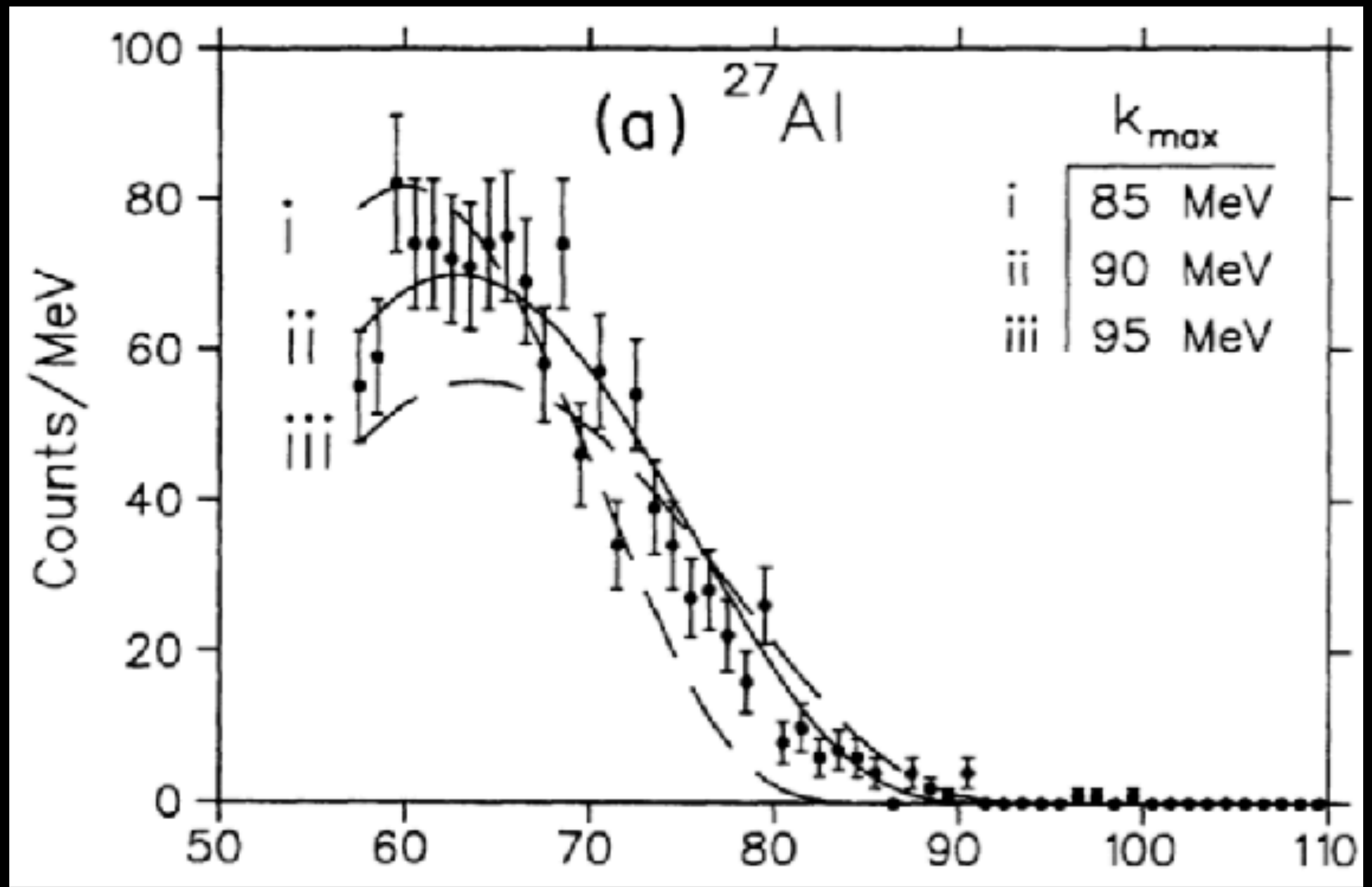
Best fit k_{max} is less than the kinematic end point.



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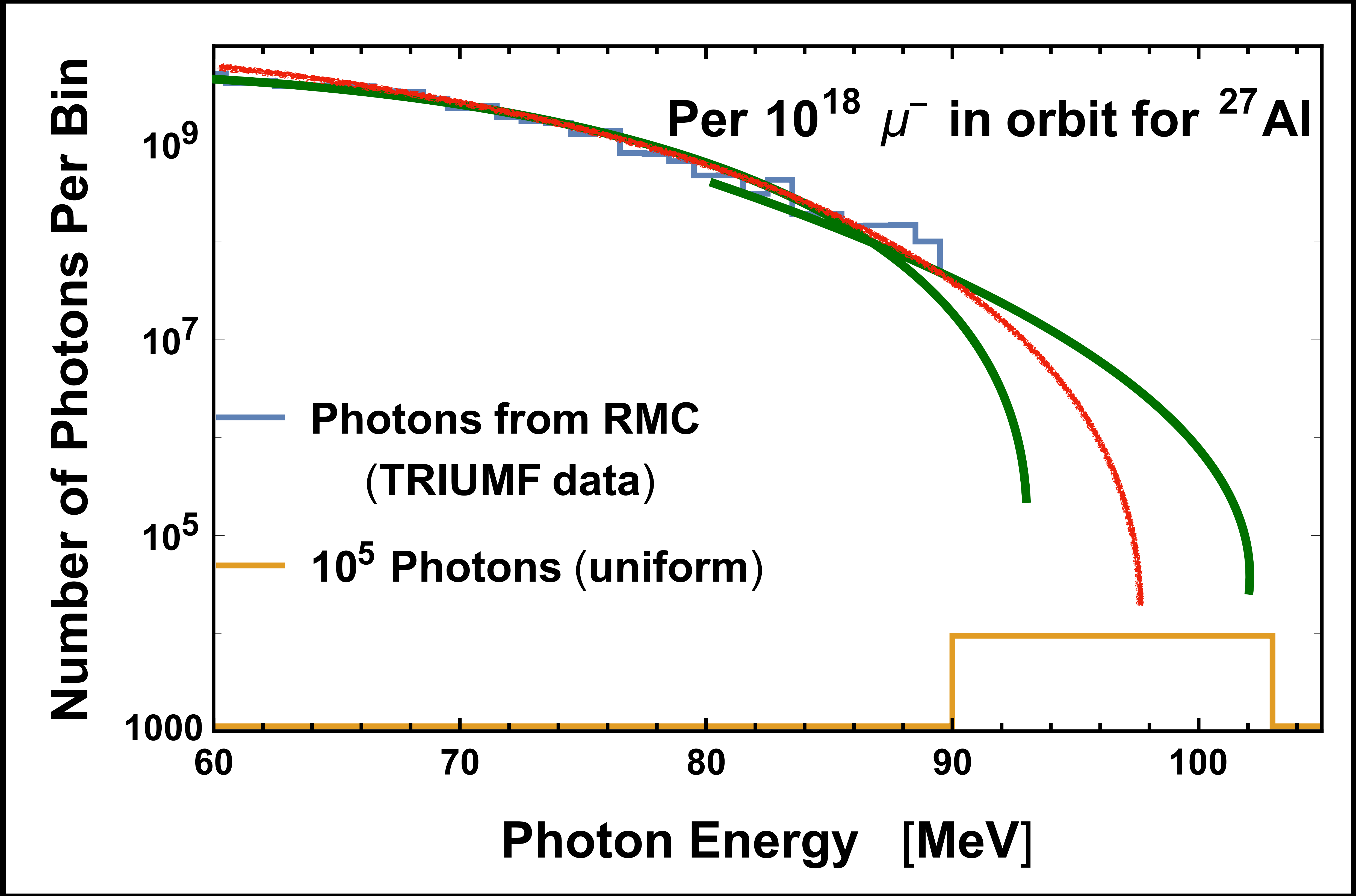
This is a problem for Mu2e!

Green and red are hard to distinguish in TRIUMF DATA

Hugely different in Mu2e energy range.

Spectral features can distort e.g., DIO fit.

Very important for LNV search.



Some data that can help inform our thinking

TABLE XI. Summary of the yield (in %) of all possible muon capture reactions in ^{27}Al.

Reaction	Observed γ-ray yield	Estimated ground-state transition	Missing yields	Total yield
$^{27}\mathrm{Al}(\mu^-, \nu)^{27}\mathrm{Mg}$	10(1)	0	3	13
$^{27}\mathrm{Al}(\mu^-, \nu n)^{26}\mathrm{Mg}$	53(5)	4	4	61
$^{27}\mathrm{Al}(\mu^-, \nu 2n)^{25}\mathrm{Mg}$	7(1)	3	2	12
$^{27}\mathrm{Al}(\mu^-, \nu 3n)^{24}\mathrm{Mg}$	2	3	1	6
$^{27}\mathrm{Al}(\mu^-, \nu pxn)^{26\text{-}23}\mathrm{Na}$	2	2	1	5
$^{27}\mathrm{Al}(\mu^-, \nu \alpha xn)^{23\text{-}21}\mathrm{Ne}$	1	2	0	3
Total	75(5)	14	11	100

Table XI are rough estimates, but the exercise is useful because the sum must be 100%.

Measday *et. al.* Phys. Rev. C **76** 035504 (2007)

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- Suggests that TRIUMF data is dominantly teaching you about nucleon knockout.

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Some data that can help inform our thinking

- Ordinary muon capture data suggests 1-nucleon knockout is dominant.
- Suggests that TRIUMF data is dominantly teaching you about nucleon knockout.
- The end-point for the 1-n knockout spectrum is 6.44 MeV lower than for 0-n knockout!

TABLE XI. Summary of the yield (in %) of all possible muon capture reactions in ^{27}Al.

Reaction	Observed γ-ray yield	Estimated ground-state transition	Missing yields	Total yield
$^{27}\mathrm{Al}(\mu^-, \nu)^{27}\mathrm{Mg}$	10(1)	0	3	13
$^{27}\mathrm{Al}(\mu^-, \nu n)^{26}\mathrm{Mg}$	53(5)	4	4	61
$^{27}\mathrm{Al}(\mu^-, \nu 2n)^{25}\mathrm{Mg}$	7(1)	3	2	12
$^{27}\mathrm{Al}(\mu^-, \nu 3n)^{24}\mathrm{Mg}$	2	3	1	6
$^{27}\mathrm{Al}(\mu^-, \nu pxn)^{26\text{-}23}\mathrm{Na}$	2	2	1	5
$^{27}\mathrm{Al}(\mu^-, \nu \alpha xn)^{23\text{-}21}\mathrm{Ne}$	1	2	0	3
Total	75(5)	14	11	100

Table XI are rough estimates, but the exercise is useful because the sum must be 100%.

Measday *et. al.* Phys. Rev. C **76** 035504 (2007)

Endpoint spectrum from kinematics

Inclusive spectrum is sum of exclusive spectra

$$a\ \mu \to X\ \nu\gamma \qquad \mathrm{d}\Gamma = \sum_b \mathrm{d}\Gamma_b$$

Qualitativley different final states

No nucleon (0-n) knockout

$$^{27}\mathrm{Al}\ \mu \to {}^{27}\mathrm{Mg}\ \nu\gamma$$
$$^{27}\mathrm{Al}\ \mu \to {}^{27}\mathrm{Mg}^*\ \nu\gamma$$

One nucleon (1-n) knockout

$$^{27}\mathrm{Al}\ \mu \to {}^{26}\mathrm{Mg} + 1n +\ \nu\gamma$$
$$^{27}\mathrm{Al}\ \mu \to {}^{26}\mathrm{Mg}^* + 1n +\ \nu\gamma$$

Endpoint spectrum from first principles

No nucleon knockout

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

Ignore nuclear recoil

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathcal{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

Ignore nuclear recoil

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\frac{\mathrm{d}\Phi_2}{2M_B} \left\langle |\mathcal{M}_{ba}|^2 \right\rangle (2\pi)\delta(\Sigma E)$$

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

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$$E_\nu = k_{\max}(b) - k$$

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

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$$E_\nu = k_{\max}(b) - k$$

$$\mathrm{d}\Gamma_b \propto x_b(1-x_b)^2 \times \left\langle |\mathscr{M}_{ba}|^2 \right\rangle$$

Endpoint spectrum from first principles

No nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_3 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

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$$E_\nu = k_{\max}(b) - k$$

$$\mathrm{d}\Gamma_b \propto x_b(1 - x_b)^2 \times \left\langle |\mathscr{M}_{ba}|^2 \right\rangle$$

$$x_b = \frac{k}{k_{\max}(b)}$$

Endpoint spectrum from first principles

One nucleon knockout

Endpoint spectrum from first principles

One nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\mathrm{d}\Phi_4 \left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)^4 \delta^{(4)}(\Sigma P)$$

Endpoint spectrum from first principles

One nucleon knockout

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Endpoint spectrum from first principles

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$$E_\nu = k_{\max}(b) - k - T_n$$

Endpoint spectrum from first principles

One nucleon knockout

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$$E_\nu = k_{\max}(b) - k - T_n$$

Now we must integrate over neutron's phase space

Endpoint spectrum from first principles

One nucleon knockout

Endpoint spectrum from first principles

One nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\frac{\mathrm{d}\Phi_3}{2M_B}\left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)\delta(\Sigma E)$$

Endpoint spectrum from first principles

One nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\frac{\mathrm{d}\Phi_3}{2M_B}\left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)\delta(\Sigma E)$$

Take matrix element as constant

Endpoint spectrum from first principles

One nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\frac{\mathrm{d}\Phi_3}{2M_B}\left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)\delta(\Sigma E)$$

Take matrix element as constant

$$k\int \mathrm{d}p_n \frac{p_n^2}{2m_n}\left[k_{\max}(b) - \frac{p_n^2}{2m_n} - q\right]^2 \sim k\left(k_{\max}(b) - k\right)^{7/2} \sim x_b(1-x_b)^{7/2}$$

Endpoint spectrum from first principles

One nucleon knockout

$$\mathrm{d}\Gamma_b = \frac{1}{2M_A}\frac{\mathrm{d}\Phi_3}{2M_B}\left\langle |\mathscr{M}_{ba}|^2 \right\rangle (2\pi)\delta(\Sigma E)$$

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Restricted phase space changes functional dependence near the endpoint

Endpoint spectrum from first principles

No nucleon knockout

One nucleon knockout

Two nucleon knockout

Endpoint spectrum from first principles

No nucleon knockout

One nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1 - x_b)^{7/2} \quad \text{as} \quad k \to k_{\max}(b)$$

Two nucleon knockout

Endpoint spectrum from first principles

No nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1 - x_b)^2 \quad \text{as} \quad k \to k_{\text{max}}(b)$$

One nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1 - x_b)^{7/2} \quad \text{as} \quad k \to k_{\text{max}}(b)$$

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Endpoint spectrum from first principles

No nucleon knockout

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One nucleon knockout

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Two nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1-x_b)^5 \quad \text{as} \quad k \to k_{\mathrm{max}}(b)$$

Endpoint spectrum from first principles

No nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1-x_b)^2 \quad \text{as} \quad k \to k_{\mathrm{max}}(b)$$

One nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1-x_b)^{7/2} \quad \text{as} \quad k \to k_{\mathrm{max}}(b)$$

Two nucleon knockout

$$\frac{\mathrm{d}\Gamma_b}{\mathrm{d}k} \propto x_b(1-x_b)^5 \quad \text{as} \quad k \to k_{\mathrm{max}}(b)$$

} Generalization of Seargent's rule for weak interaction rates

Implications for extrapolation of TRIUMF data

- Simple exercise: Fit TRIUMF data but with update spectral shape that allows for endpoint photons

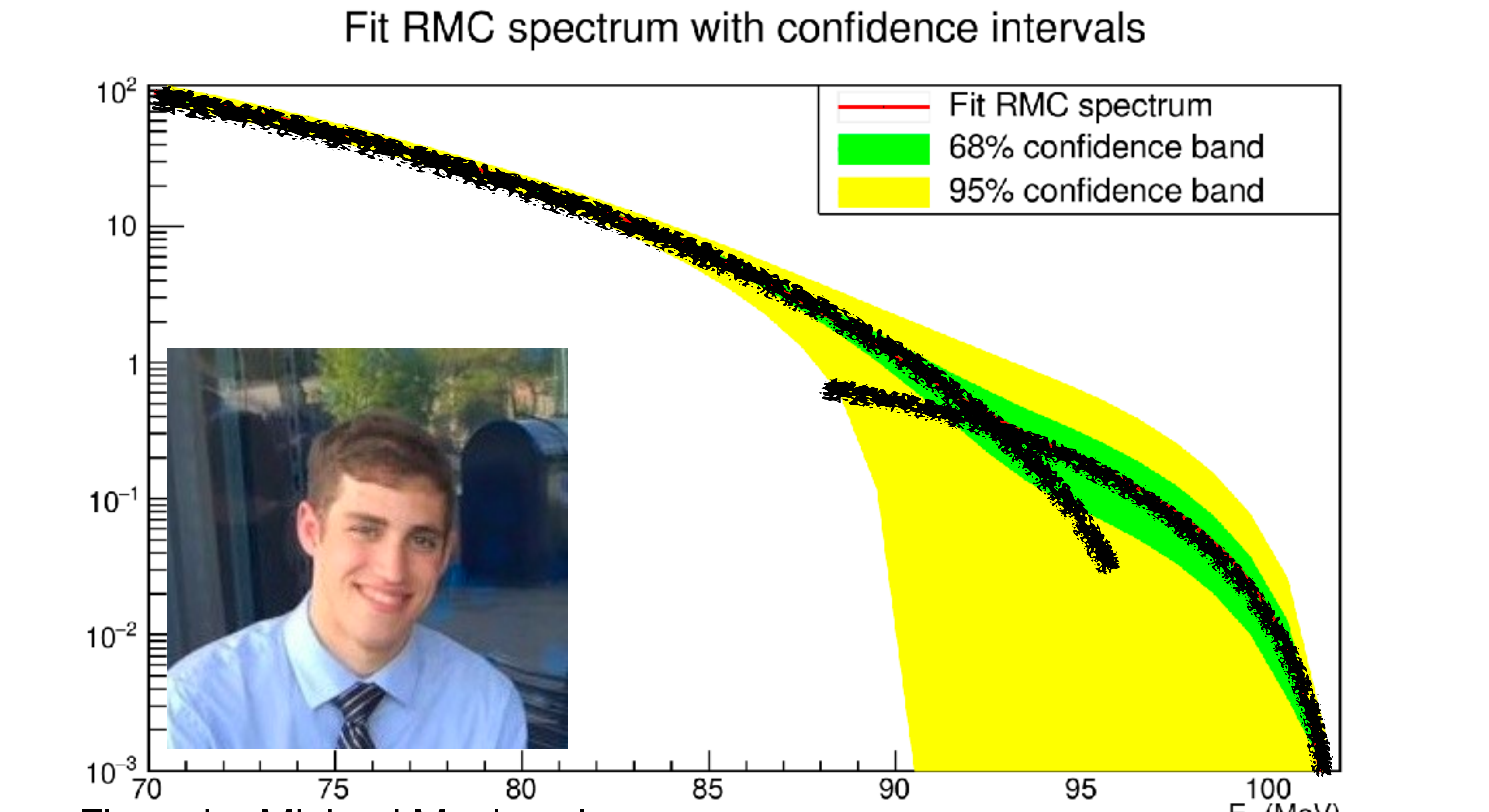

Figure by Michael Mackenzie

$$\frac{d\Gamma_b}{dk} \propto A_1 x_1 (1 - x_1)^{7/2} + A_0 x_0 (1 - x_0)^2$$

$$x_0 = \frac{k}{102\ \text{MeV}}$$

$$x_1 = \frac{k}{95.5\ \text{MeV}}$$

Implications for extrapolation of TRIUMF data

- Simple exercise: Fit TRIUMF data but with update spectral shape that allows for endpoint photons

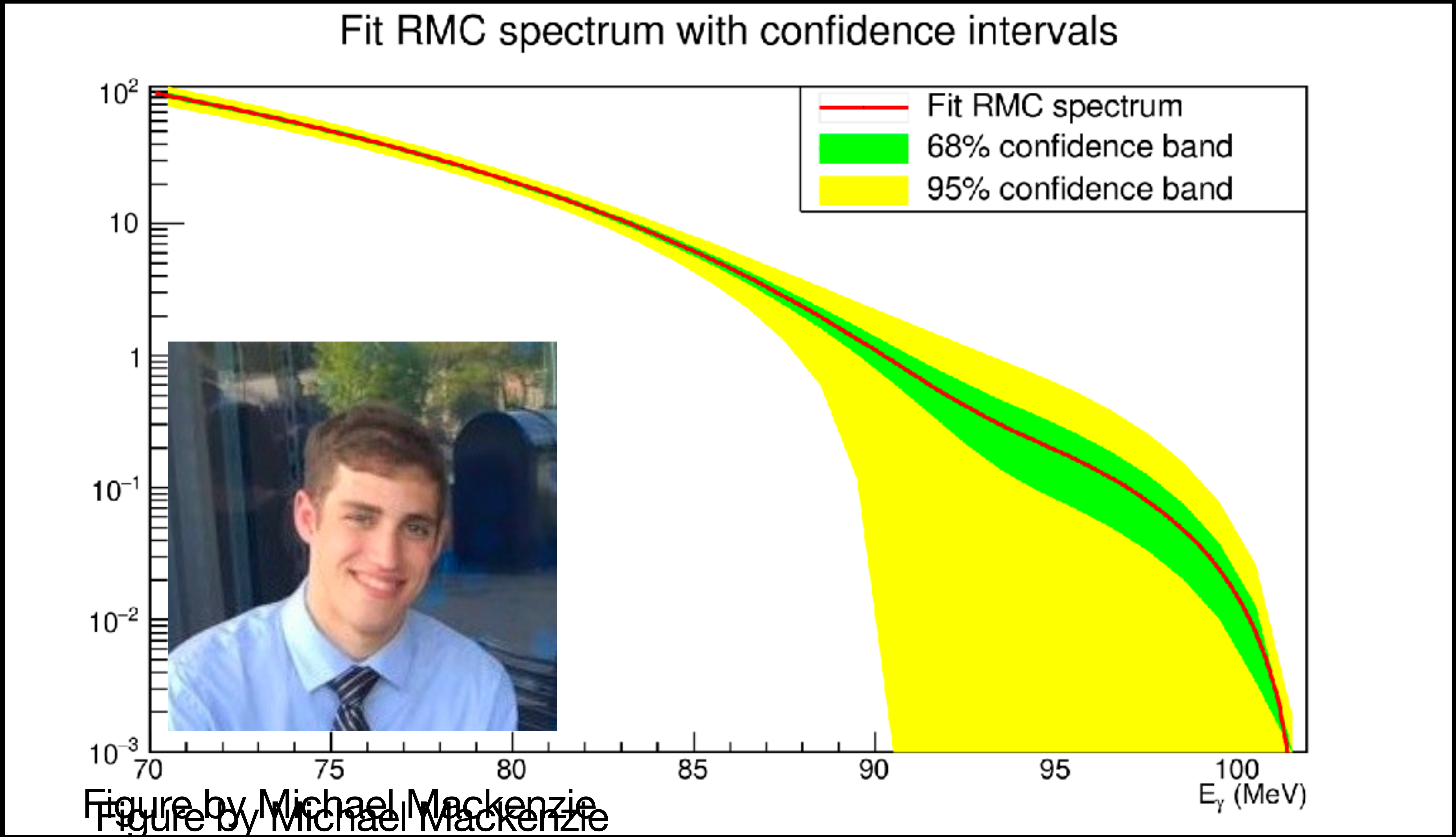

Figure by Michael Mackenzie
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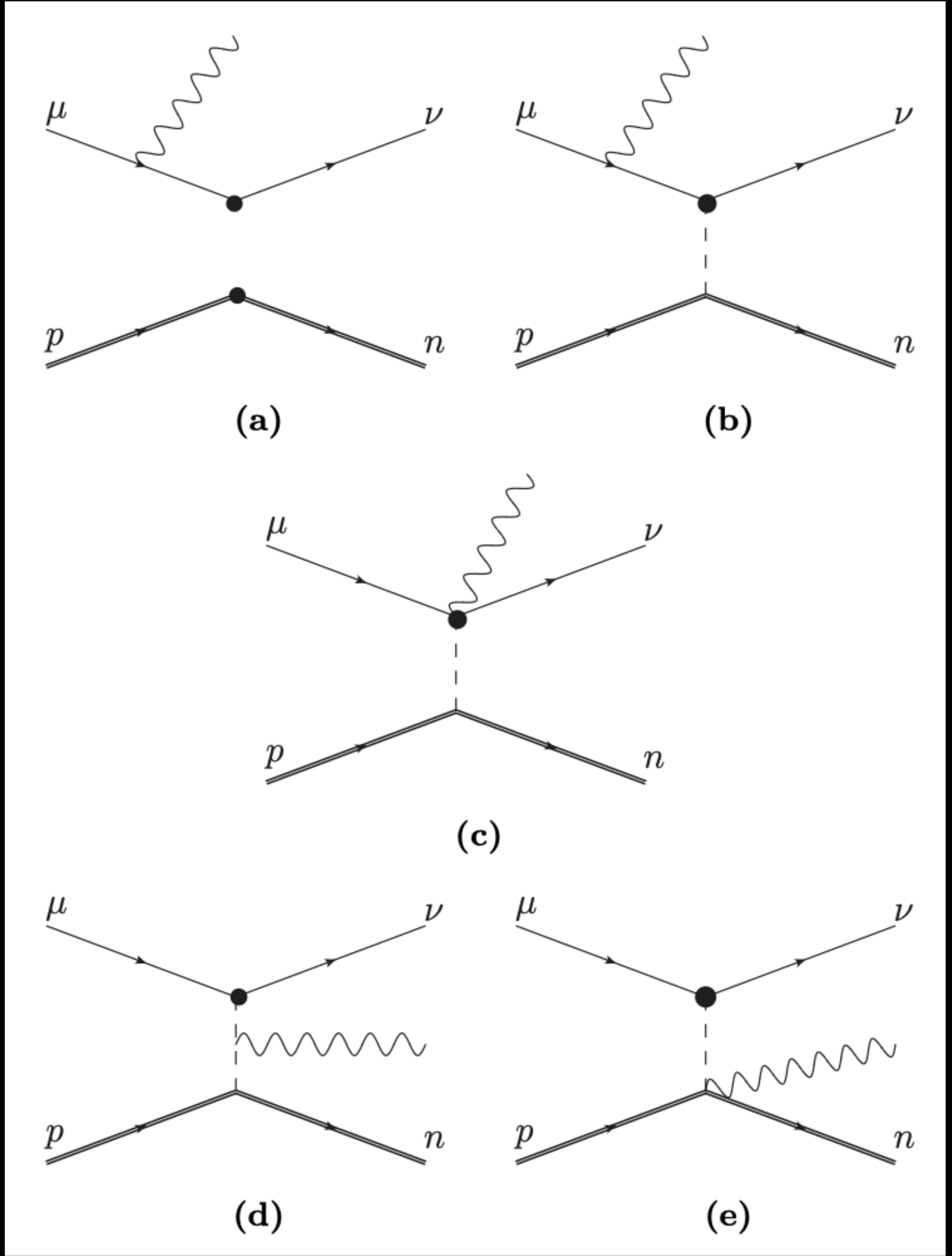
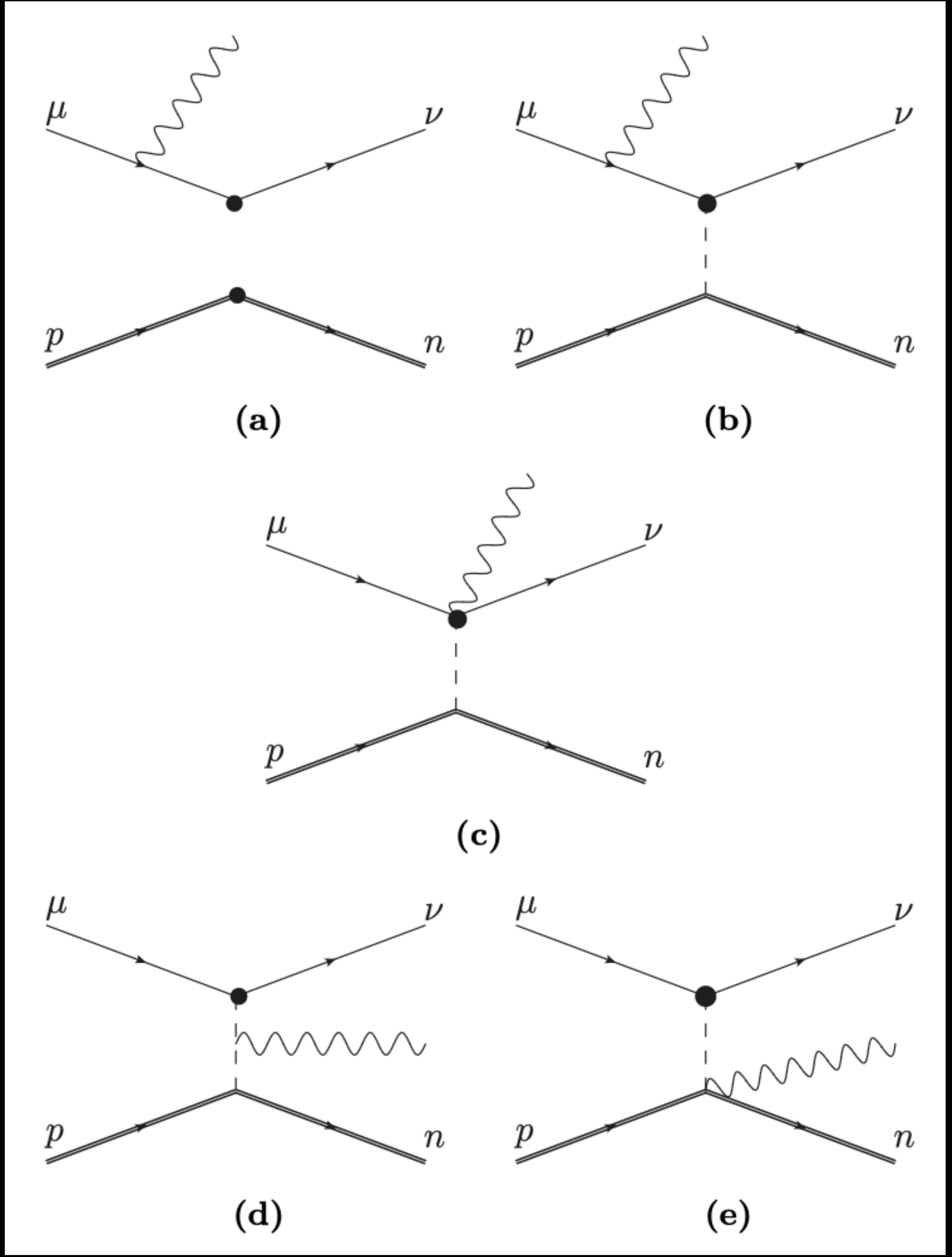
Chiral Perturbation Theory



Chiral Perturbation Theory

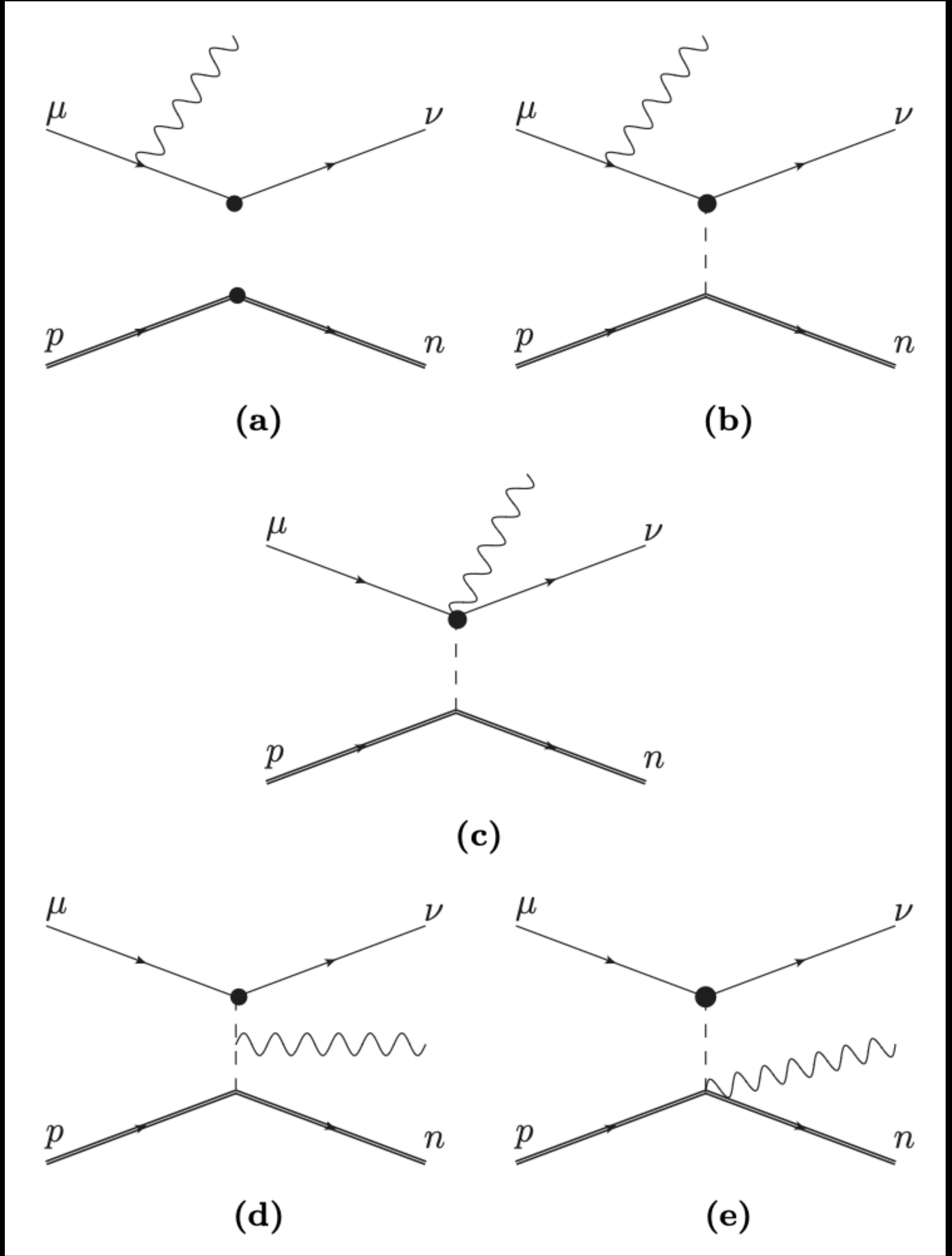
- Five graphs contribute at leading order on a nucleon (T. Meissner, F. Myhrer, K . Kubodera 1997)



Chiral Perturbation Theory

- Five graphs contribute at leading order on a nucleon (T. Meissner, F. Myhrer, K . Kubodera 1997)

- Embedding in a nucleus has not been done.

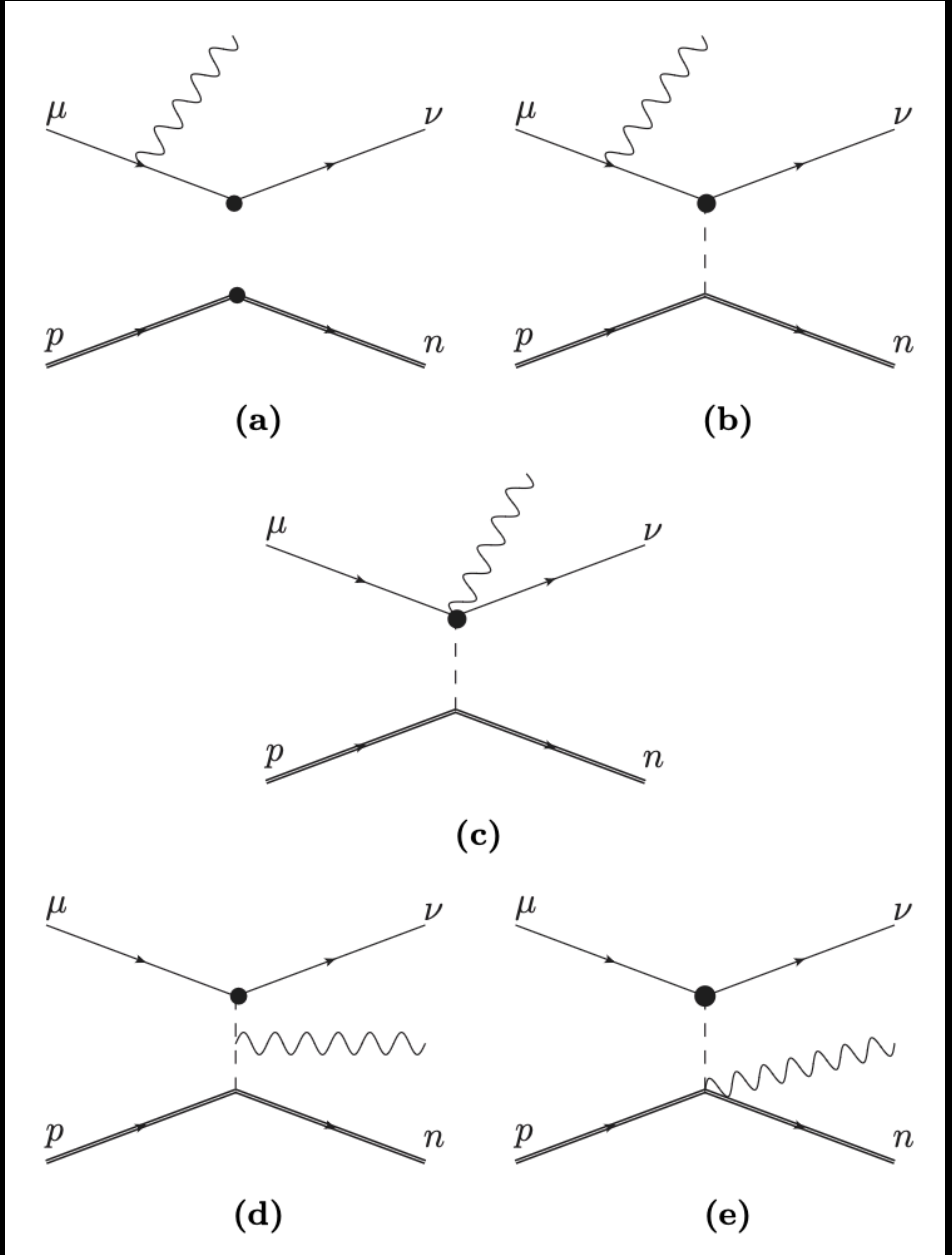


Chiral Perturbation Theory

- Five graphs contribute at leading order on a nucleon (T. Meissner, F. Myhrer, K . Kubodera 1997)

- Embedding in a nucleus has not been done.

- Only two leading order nuclear matrix elements needed.



New work: Nuclear Matrix Element input

Lotta Jokiniemi | TU Darmstadt

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- Lotta is an expert in ordinary muon capture.

New work: Nuclear Matrix Element input

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- Lotta is an expert in ordinary muon capture.
- Has computed all necessary matrix elements as a function of momentum transfer.

New work: Nuclear Matrix Element input

Lotta Jokiniemi | TU Darmstadt

- Lotta is an expert in ordinary muon capture.
- Has computed all necessary matrix elements as a function of momentum transfer.
- I need to finish the calculation with these as input.

Summary & Conclusions

Summary & Conclusions

- RMC: an important background at Mu2e.

- High energy positrons (both internal and external) can be predicted using the real photon spectrum.

- Real photon spectrum's shape is sculpted dominantly by phase space.

- Kinematic thresholds must be properly included.

- Hope to have new calculations by September.