Distinguishing models with $\mu \to e$ observables

Based on MA, S. Davidson, S. Lavignac 2308.16897, 2401.06214

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CLFV Searches with the Mu2e experiment: LNF workshop

12/06/25

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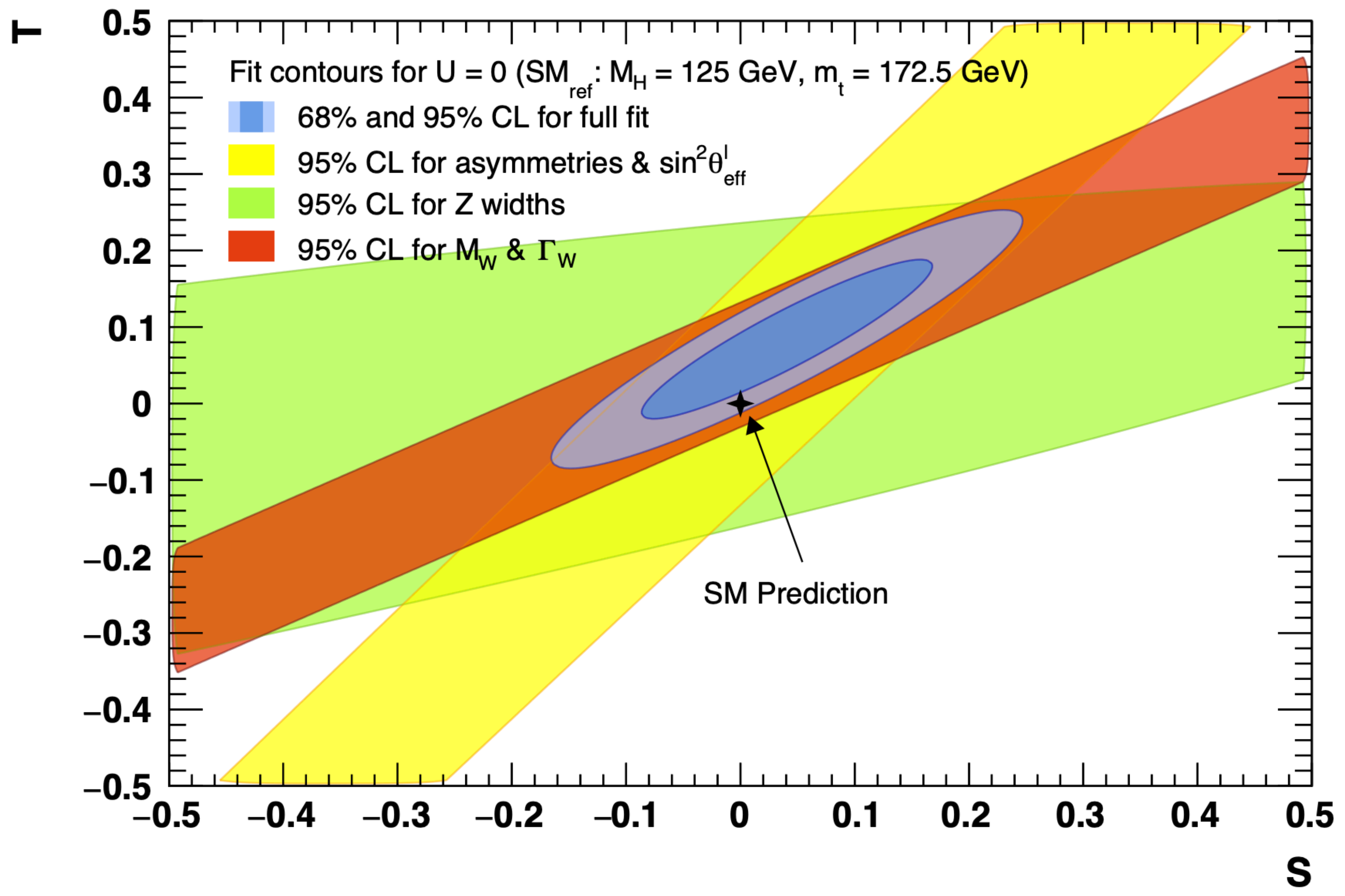

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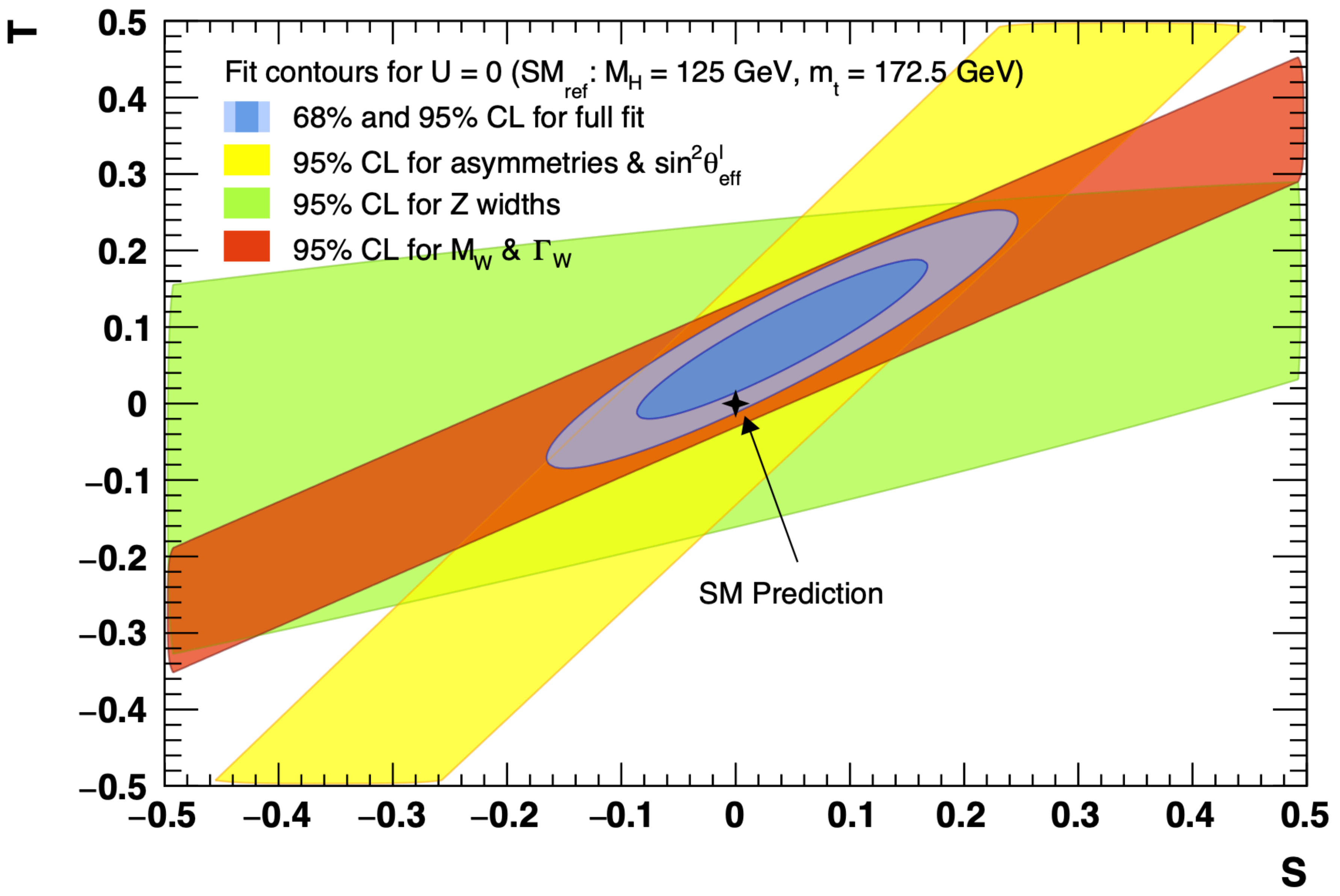

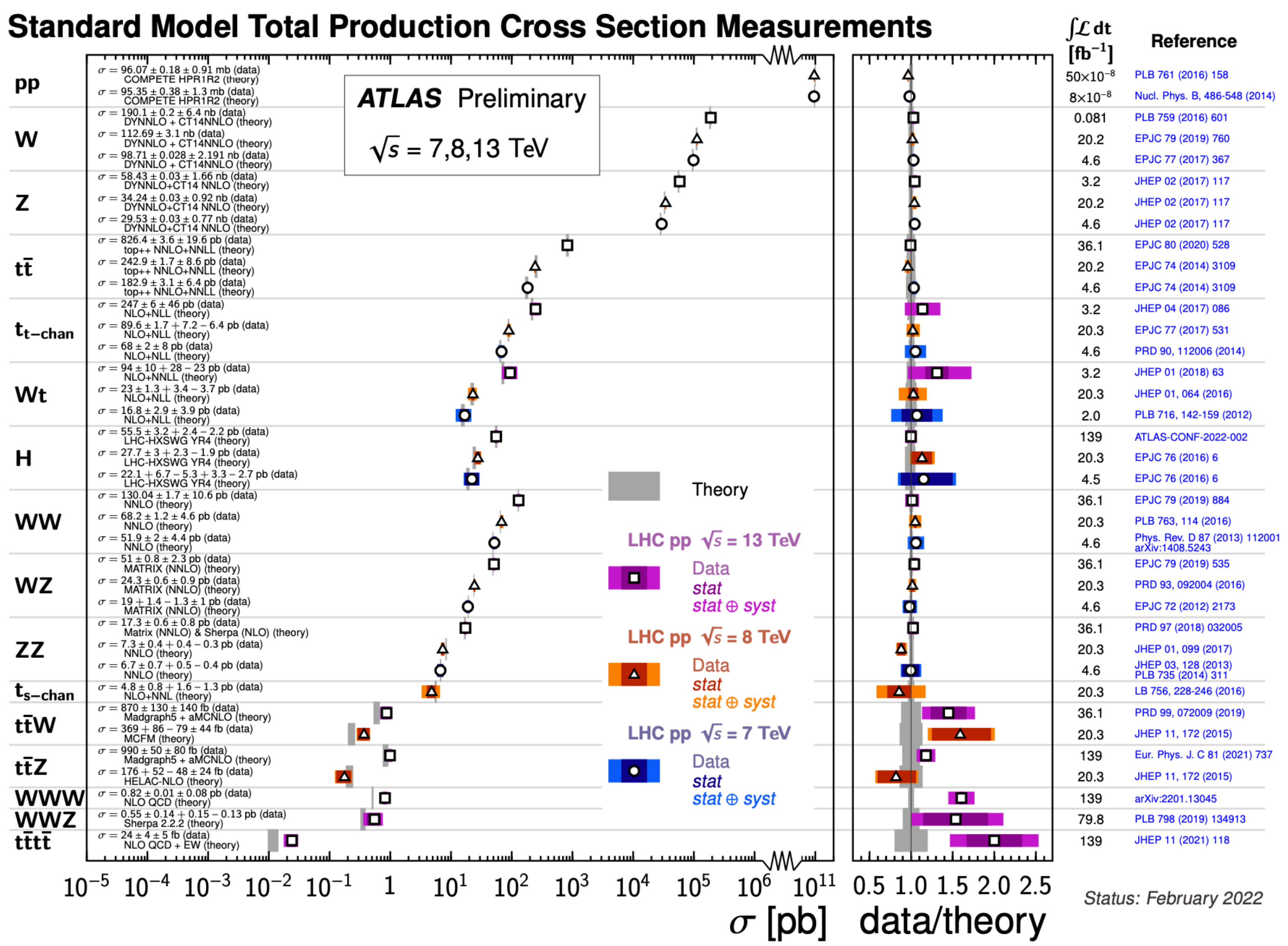

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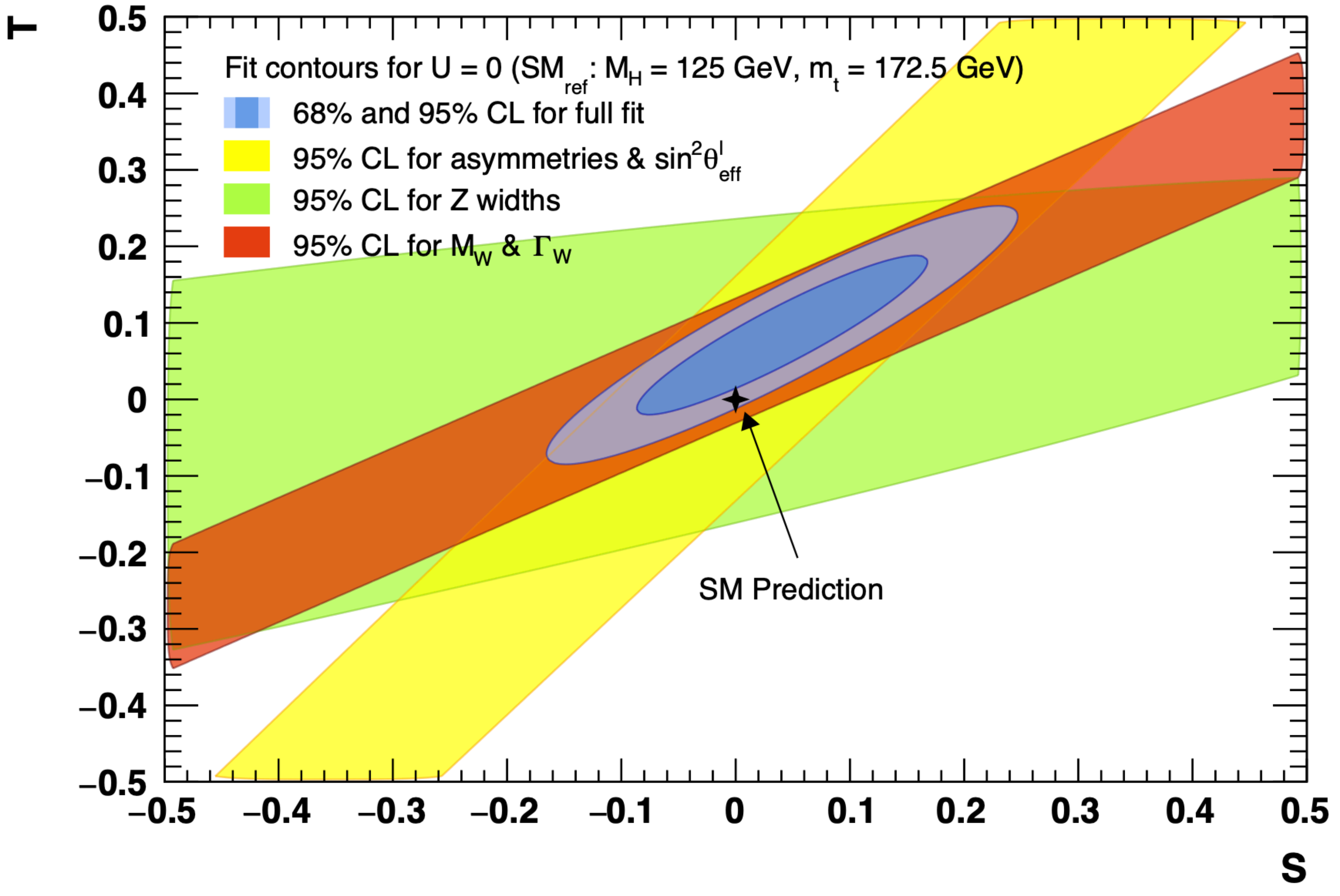

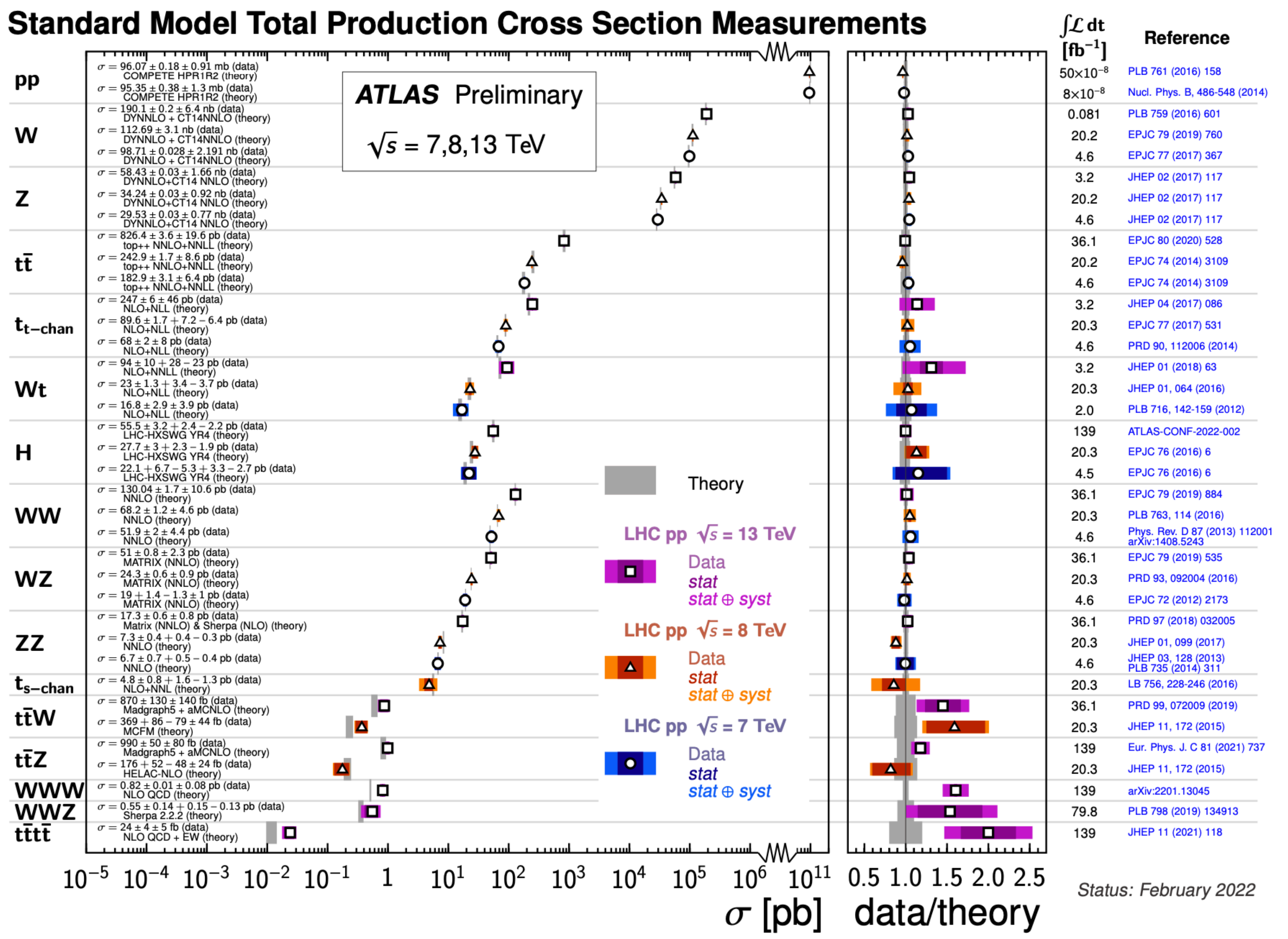

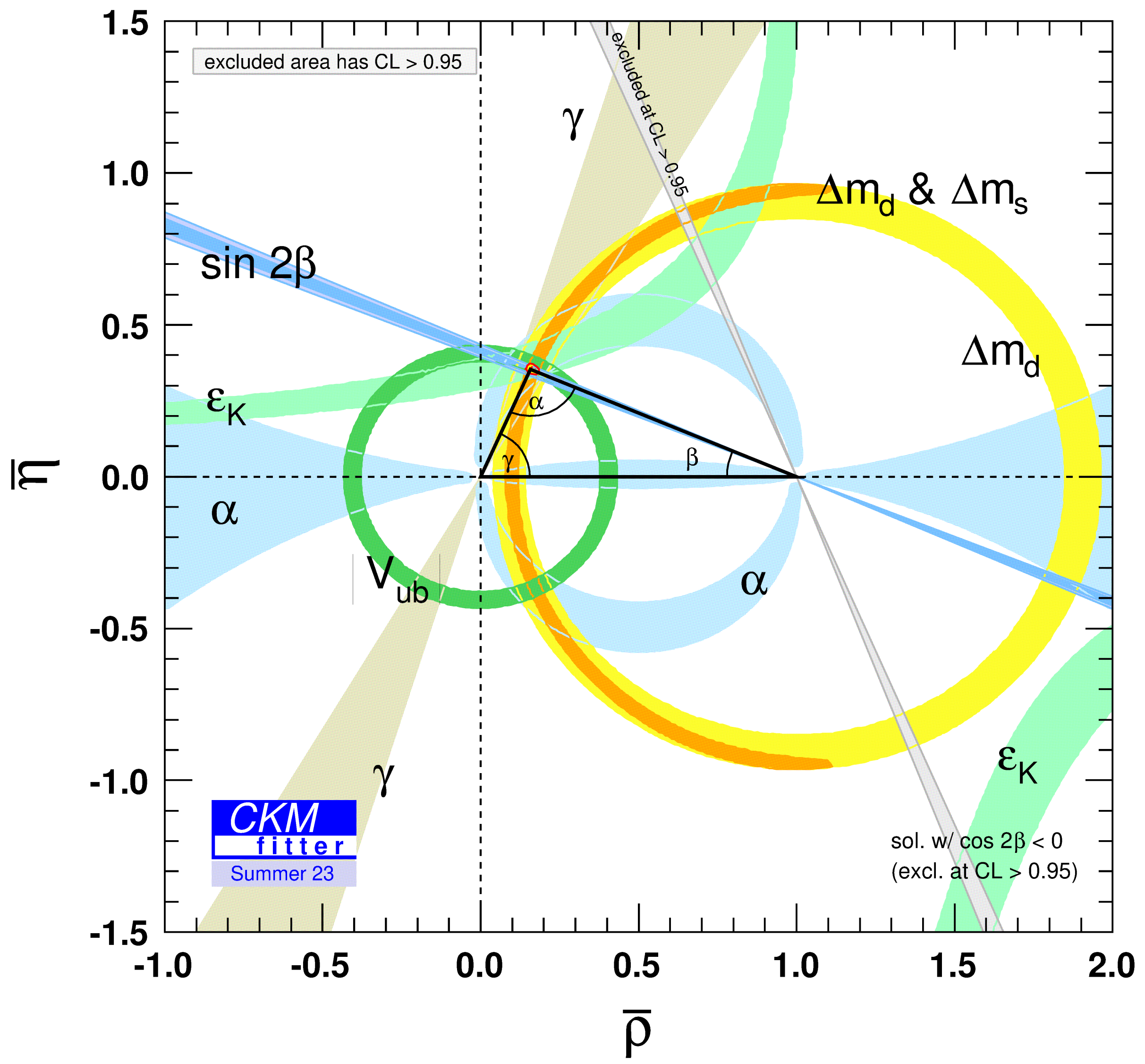

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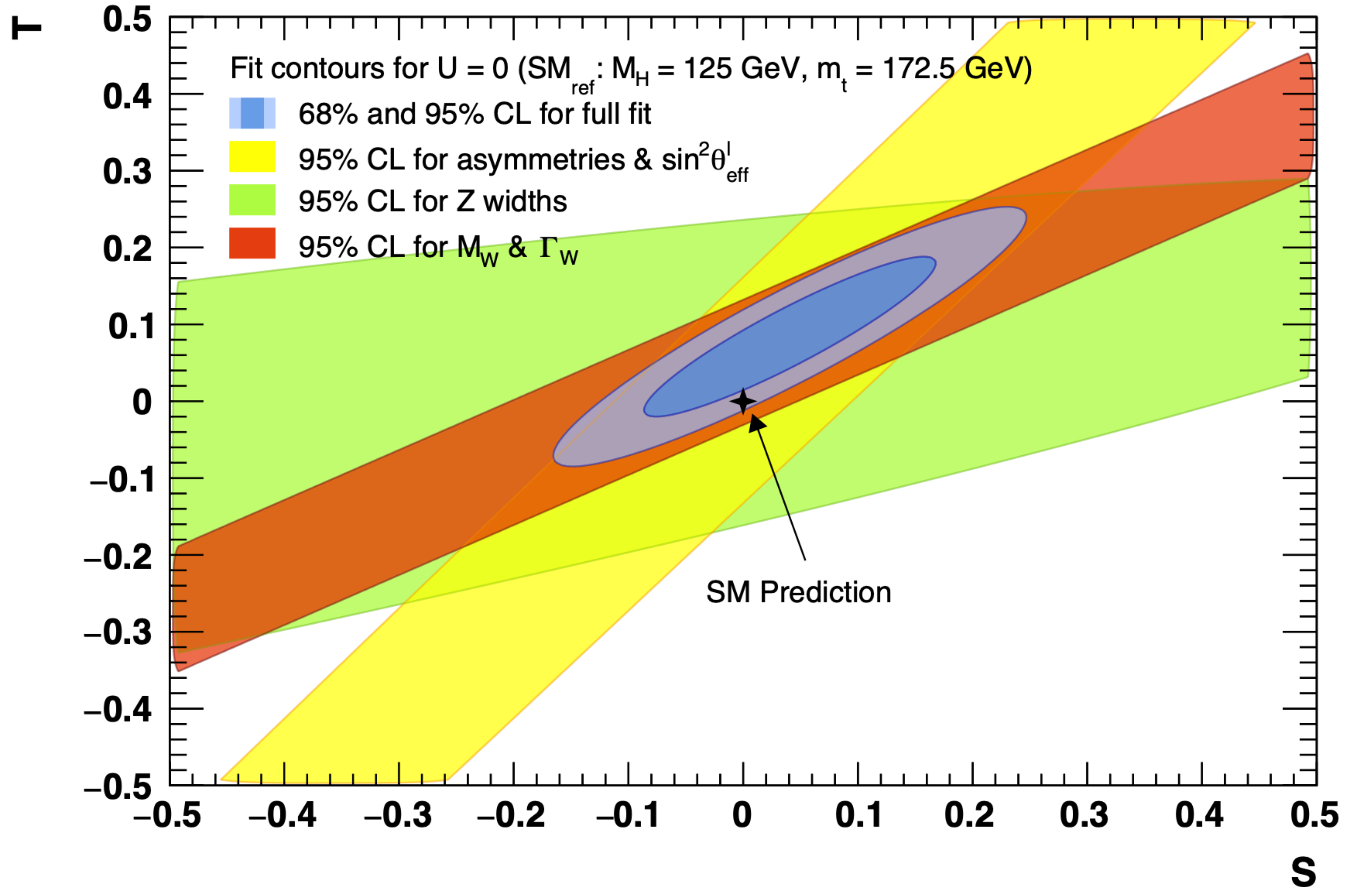
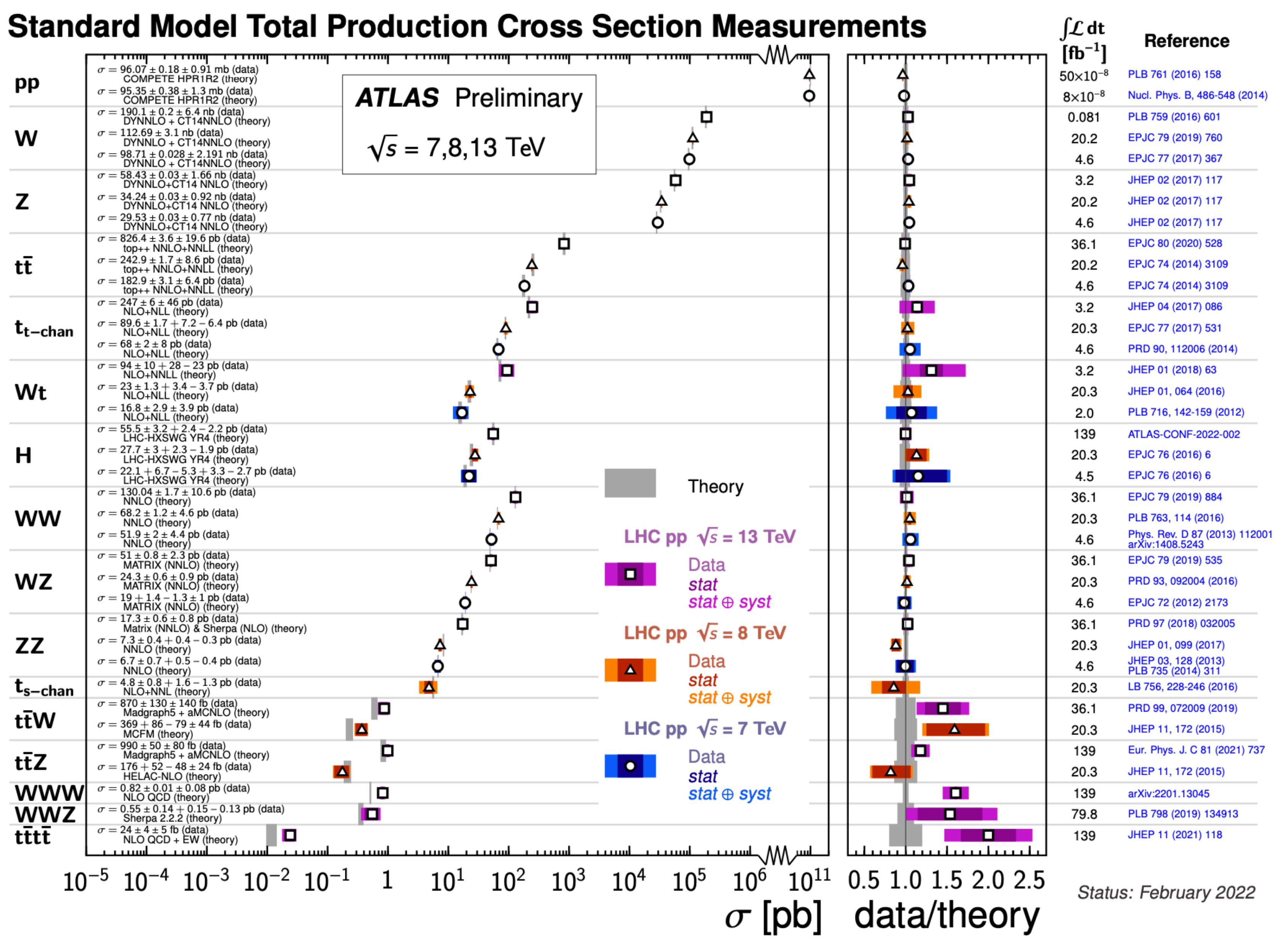
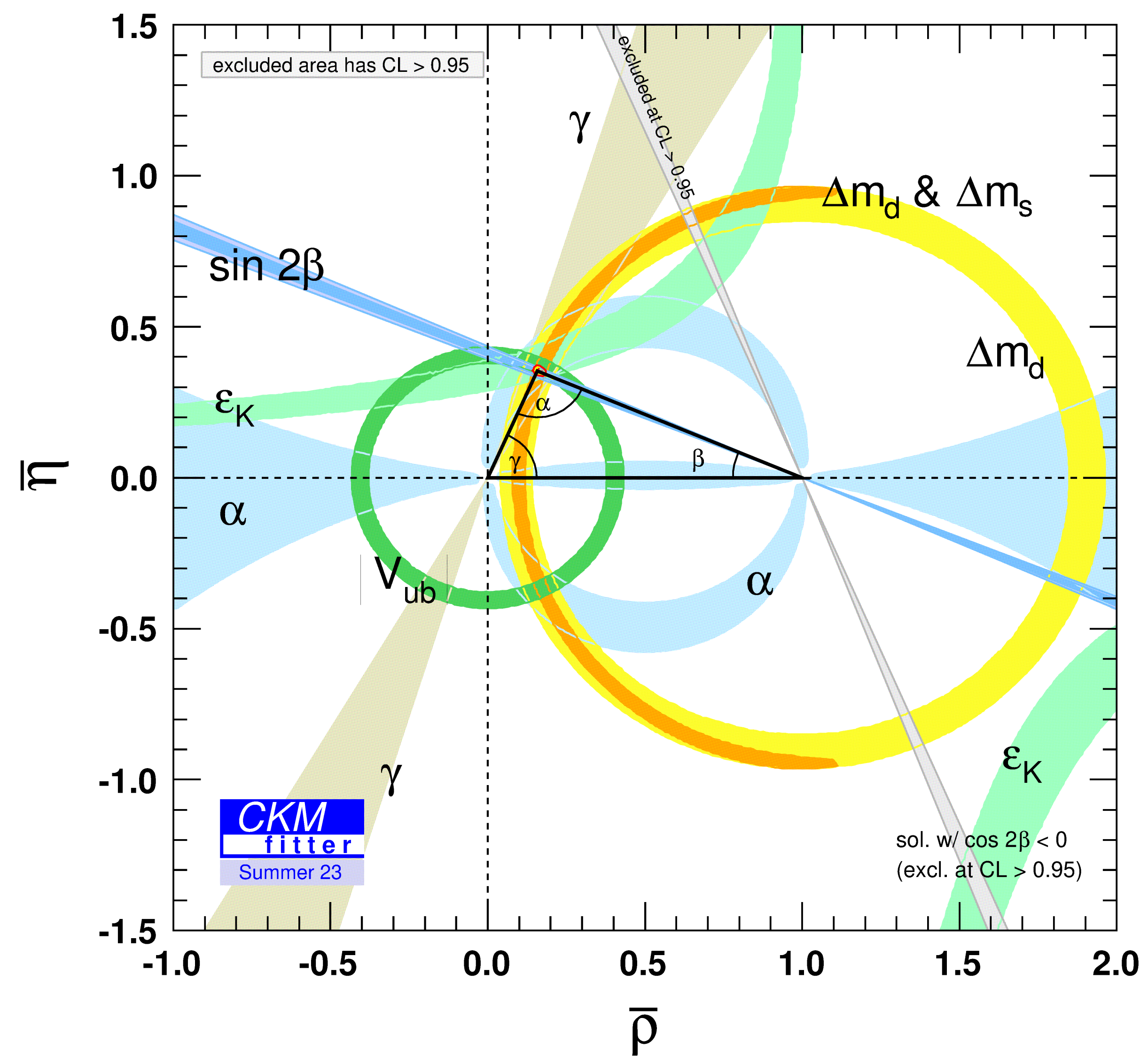


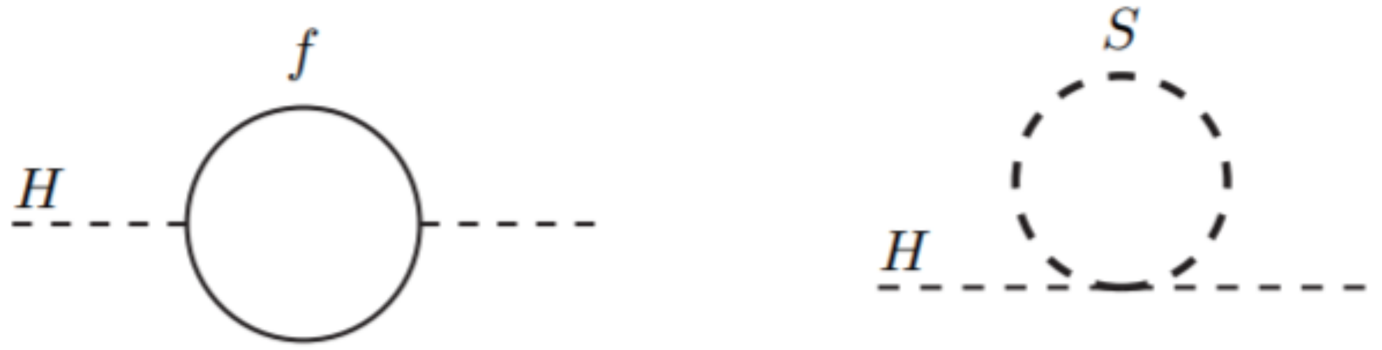


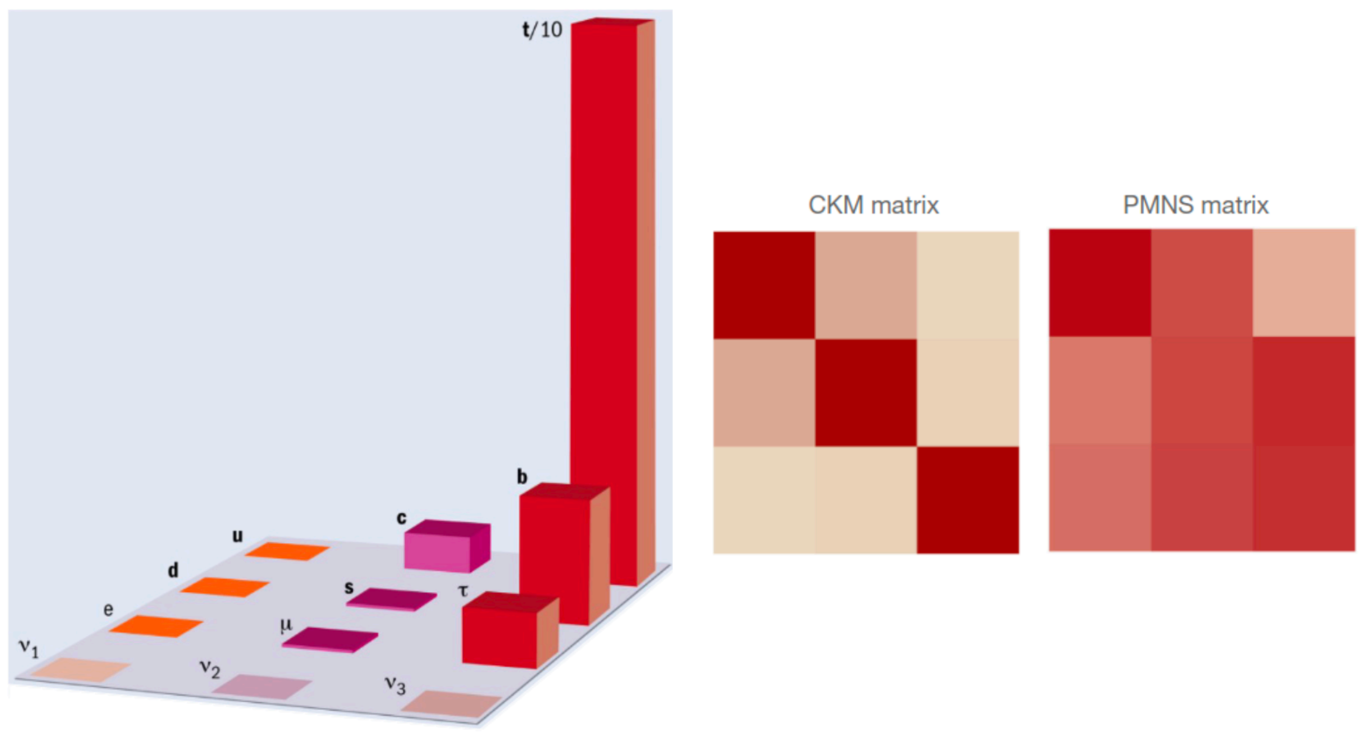


- The Standard Model of Particle Physics is extremely successful in explaining a wide variety of phenomena
- No evidence of new states
- **Yet we know that it cannot be the full story…**

The need for Beyond SM physics

- Strong CP Problem
- Hierarchy Problem
- Flavour puzzle
- …





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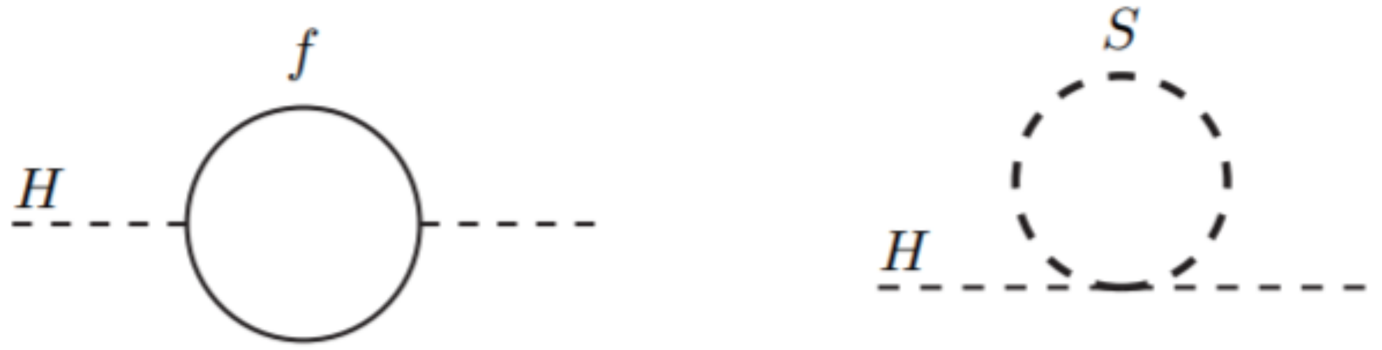
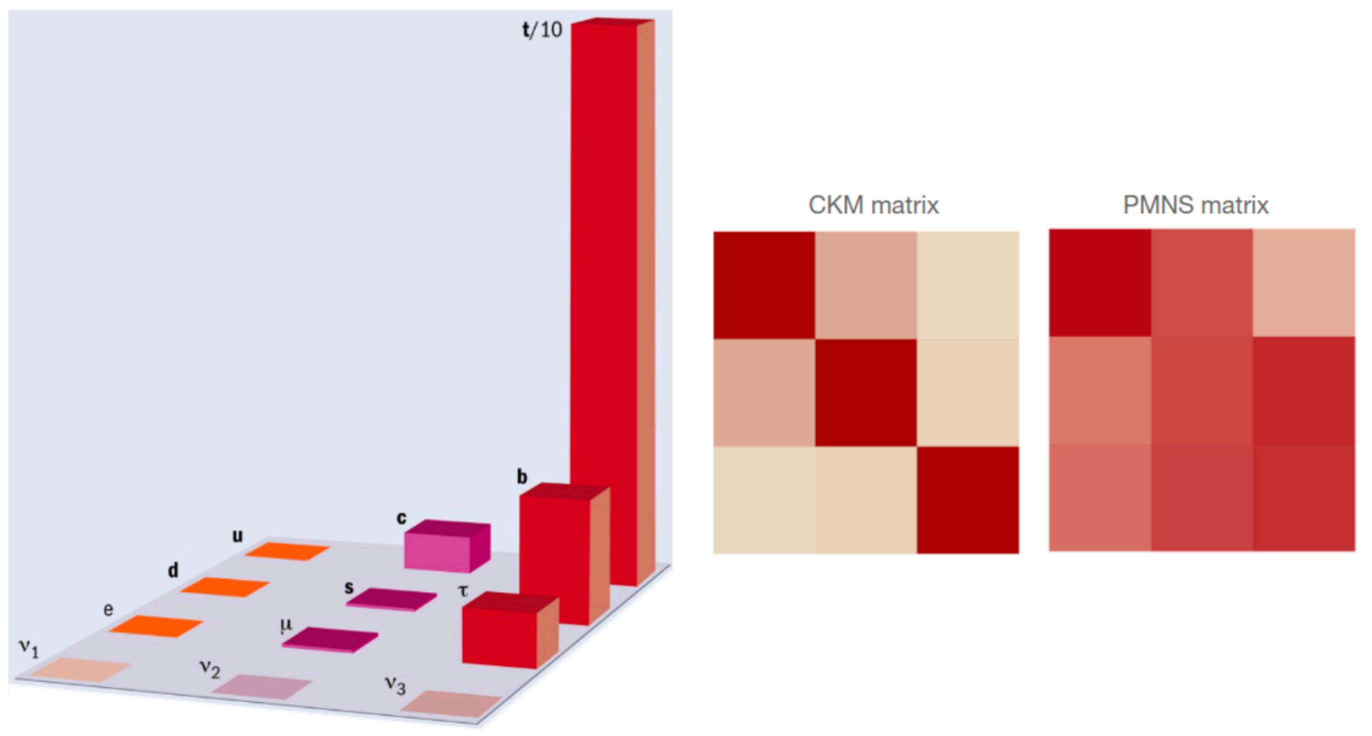
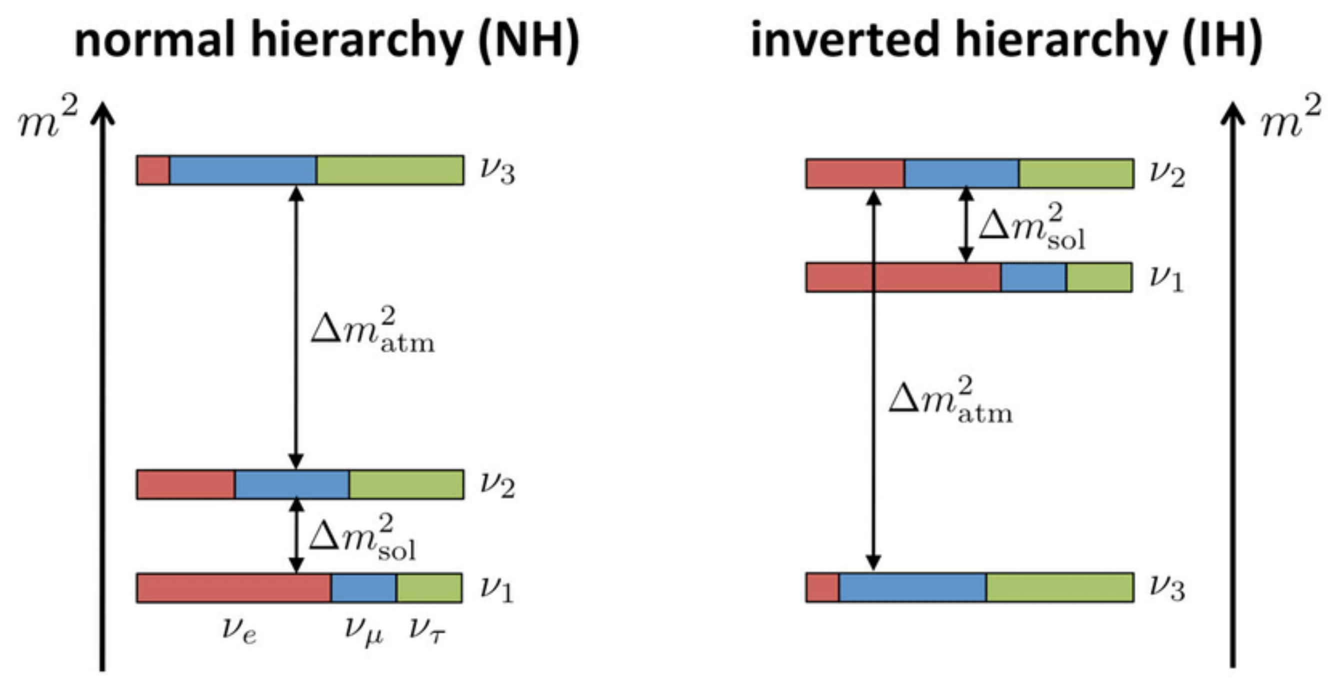
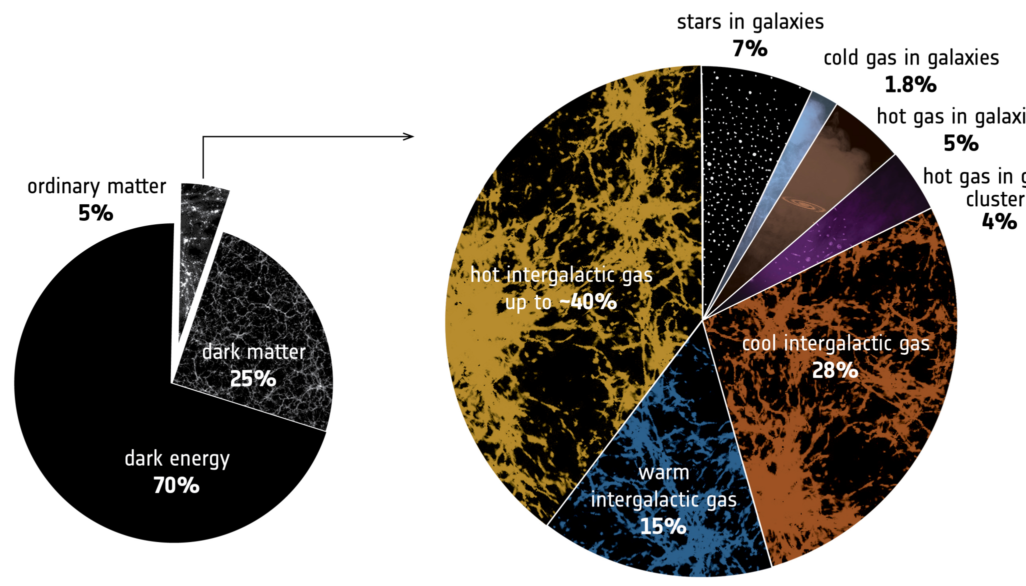




- Neutrino masses
- Dark matter
- Baryon asymmetry of the Universe





$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$$

Neutrino masses imply Lepton Flavour Violation

The Standard Model Lagrangian (without right-handed neutrinos) is accidentally invariant under a phase rotation of each **lepton flavor** $U(1)_{L_\alpha}$

$$\ell_\alpha = \begin{pmatrix} \nu_\alpha \\ \alpha_L \end{pmatrix}, e_\alpha = \alpha_R \quad \text{with} \quad \alpha = e, \mu, \tau$$

$$U(1)_{L_\alpha} : \begin{cases} \ell_\alpha \to e^{i\chi_\alpha} \ell_\alpha \\ e_\alpha \to e^{i\chi_\alpha} e_\alpha \end{cases}$$

Neutrino masses break all three symmetries

$$\begin{tikzcd} & \nu_\tau \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\ \nu_\mu \arrow[rr, leftrightarrow] & & \nu_e \end{tikzcd}$$

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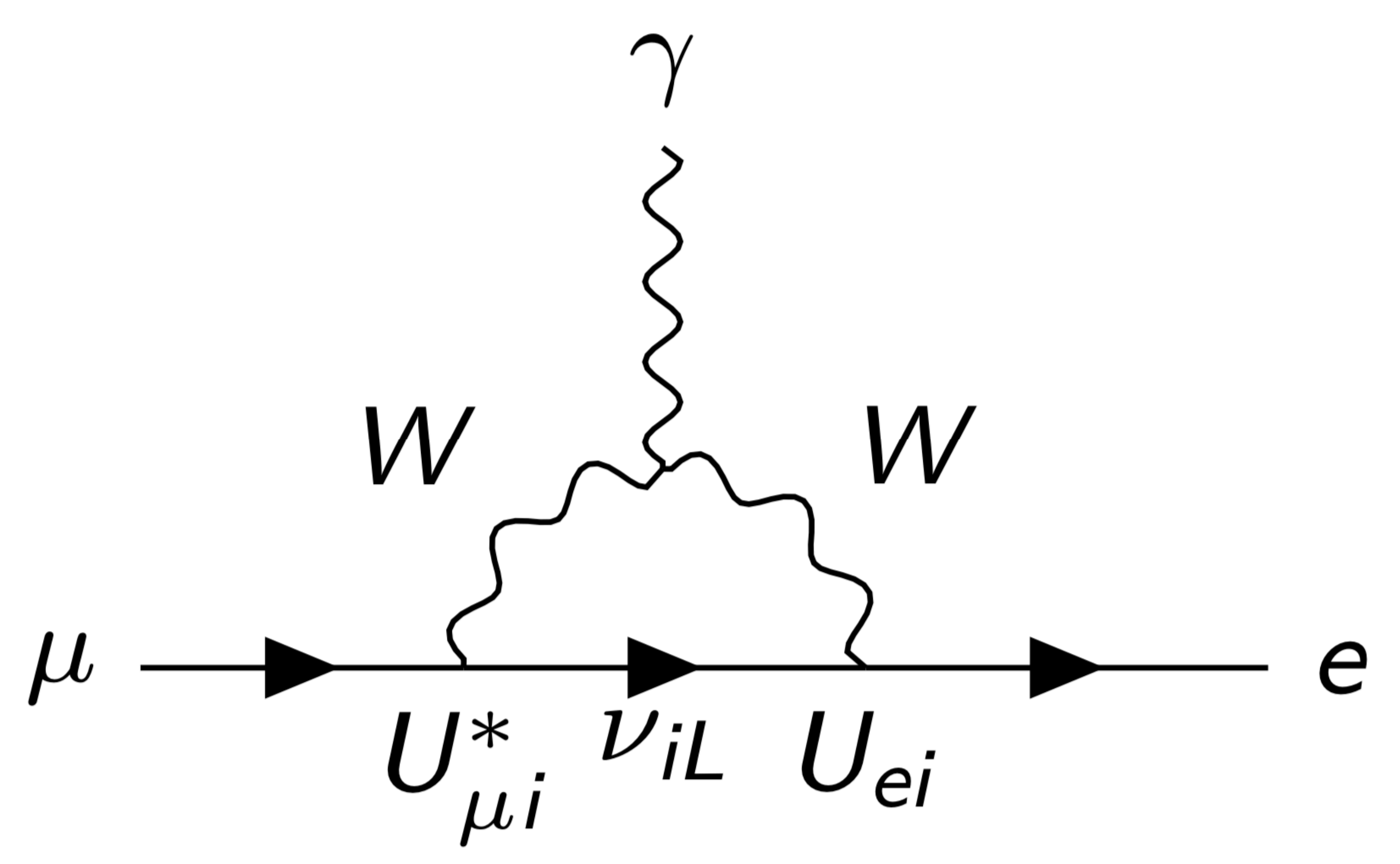
Since there is no symmetry that forbids it, **lepton flavour violation in the charged sector is inevitable**:

$$\mu^\pm \to e^\pm \gamma \qquad \tau^\pm \to e^\pm e^+ e^- \qquad h \to \tau^\pm \mu^\mp \ldots$$

must happen, but at what rates?

Charged Lepton Flavour Violation (cLFV)



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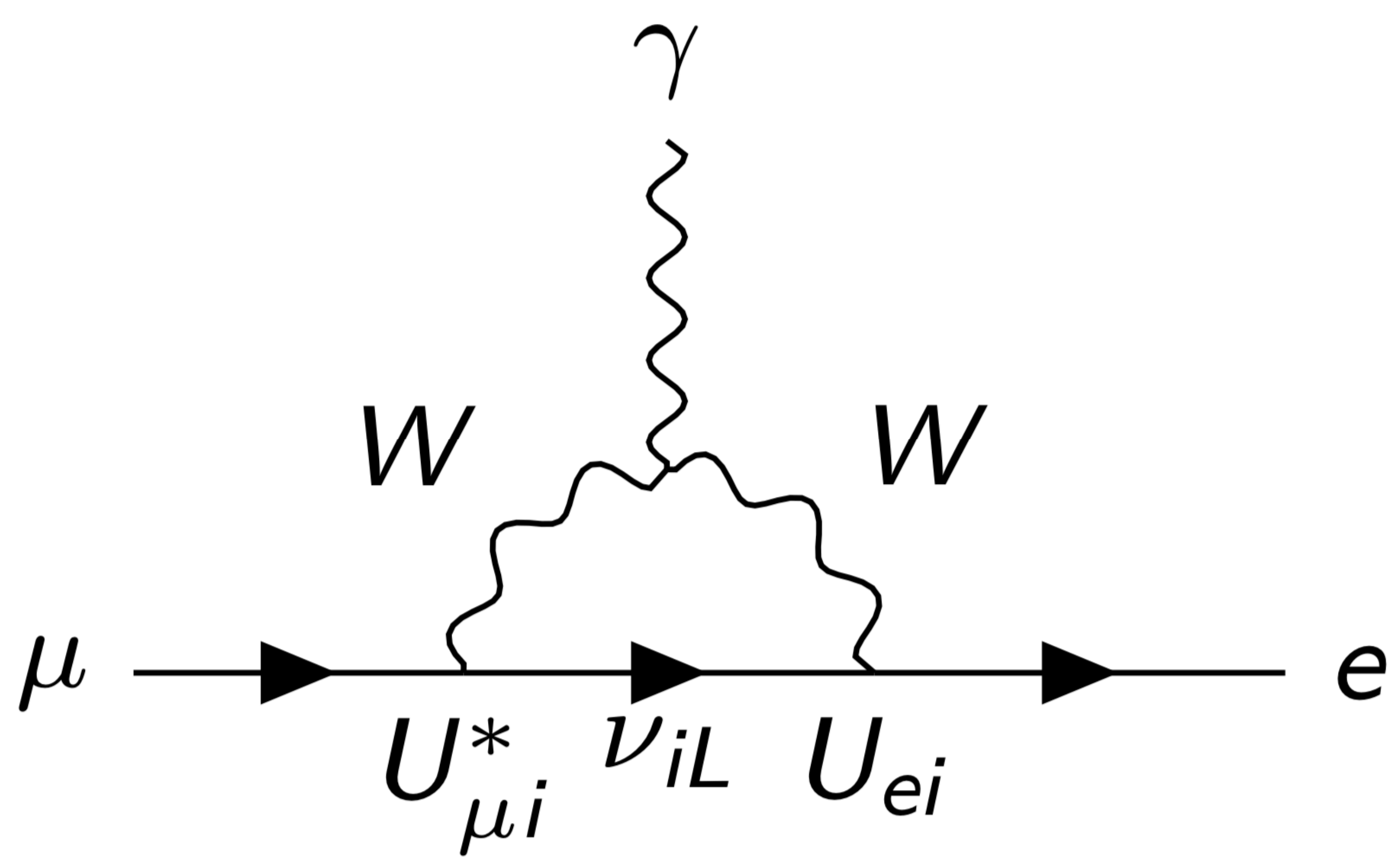

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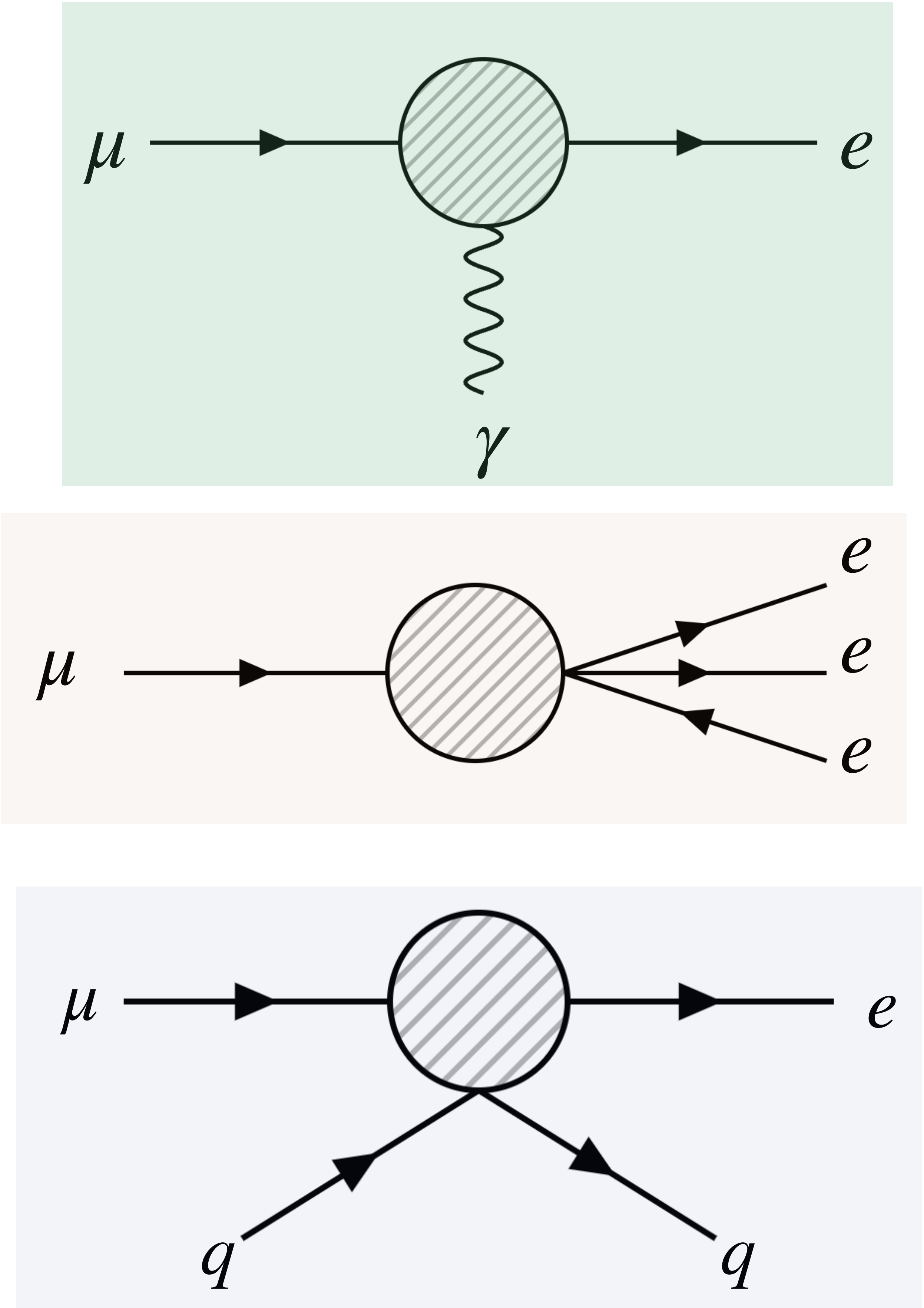
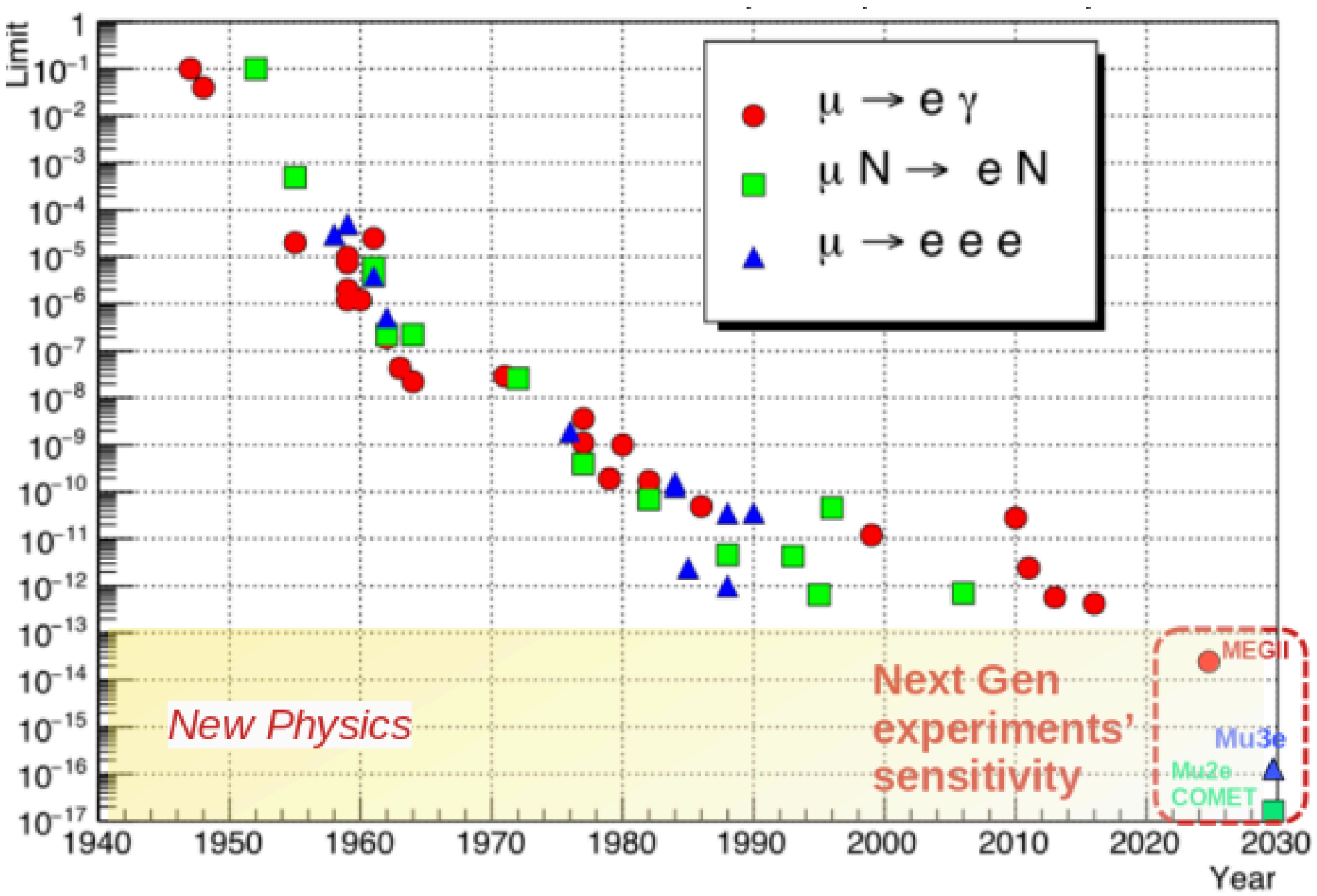
- **An observation of LFV would be a clear signature of new physics**
- **It could shed light on the mechanism behind neutrino masses**
- **Many models** that address unresolved puzzles (independently from neutrino masses) **predict potentially observable LFV signals**

$\mu \to e\gamma,\ \mu \to 3e,\ \mu A \to eA$

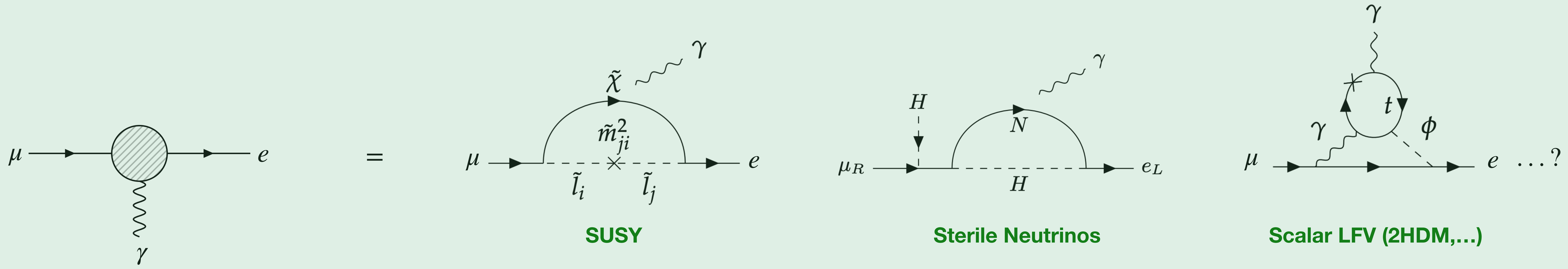
- The possibility of extremely intense muon beams make these the **golden channels for LFV**
- Experimental sensitivities are expected to be **improved by up to five orders of magnitude**





$\mu \to e\gamma,\ \mu \to 3e$

- $\mu \to e + \gamma$ $Br(\mu \to e\gamma) < 3 \times 10^{-13}$ (MEG+MEGII) $\to$ $Br(\mu \to e\gamma) \sim 6 \times 10^{-14}$ (MEGII)



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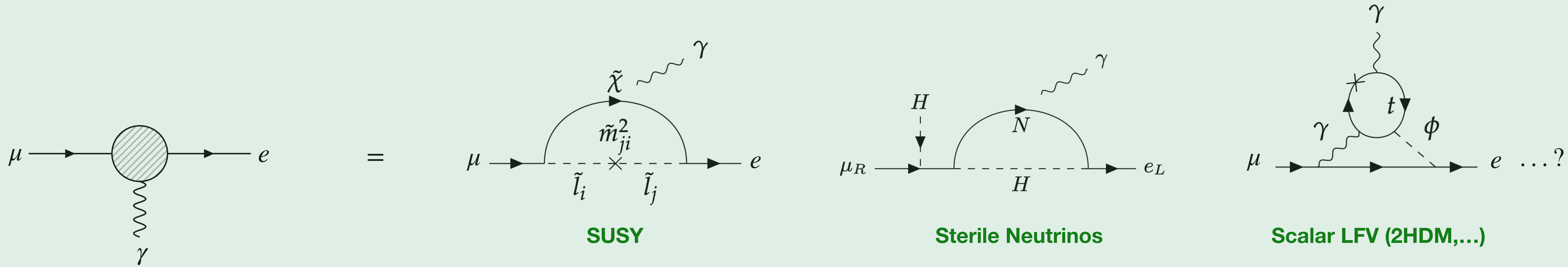
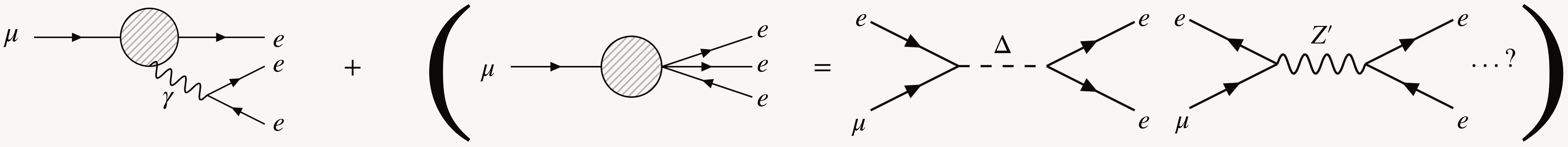


SUSY — Sterile Neutrinos — Scalar LFV (2HDM,...)

- $\mu \to e + \bar{e} + e$ $\quad Br(\mu \to e\bar{e}e) < 10^{-12}$ (SINDRUM) $\to$ $Br(\mu \to e\bar{e}e) \sim 10^{-16}$ (Mu3e)



Type-II seesaw — Extra gauge bosons

$\mu \to e$ conversion in nuclei

- The muon gets captured by the (Z, A) nucleus and tumbles down to the 1s state

- If there are LFV interactions with nucleons, an electron can be emitted without a neutrino (conversion)

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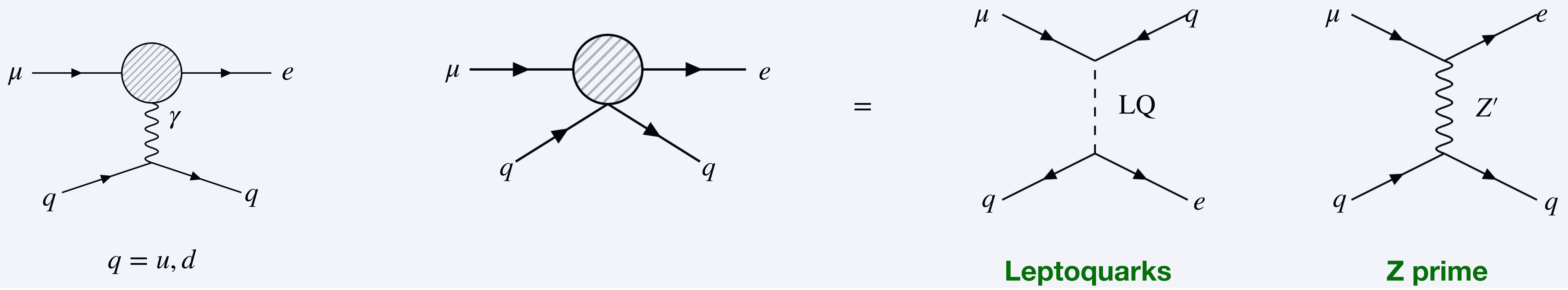

A jungle of possible models that give LFV…

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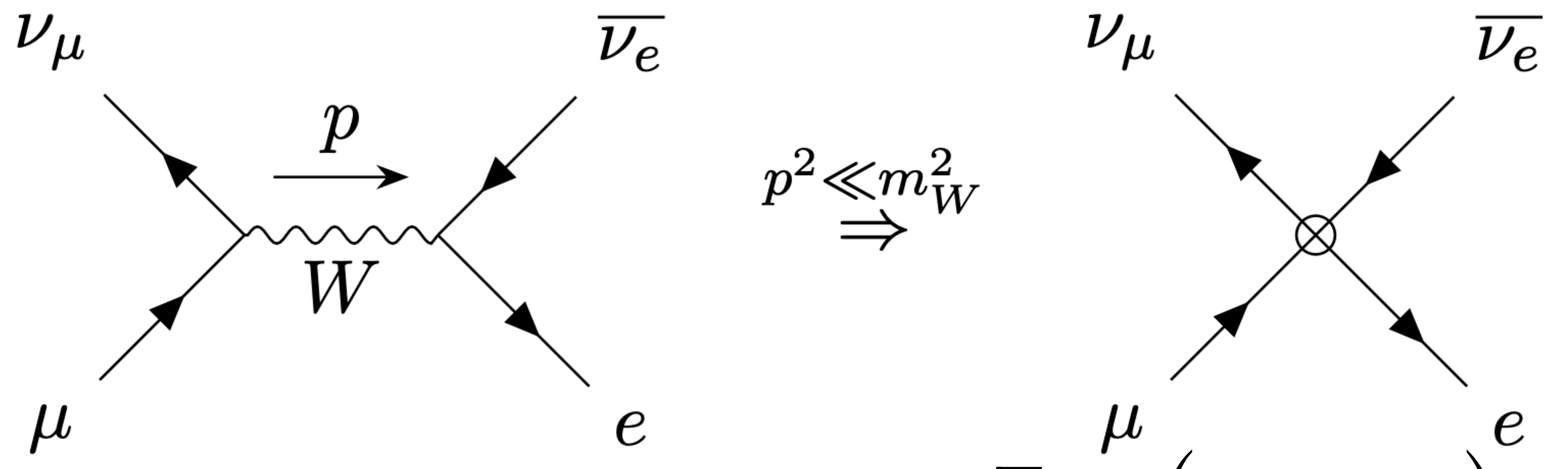
Fermi theory is the low energy limit of the SM:



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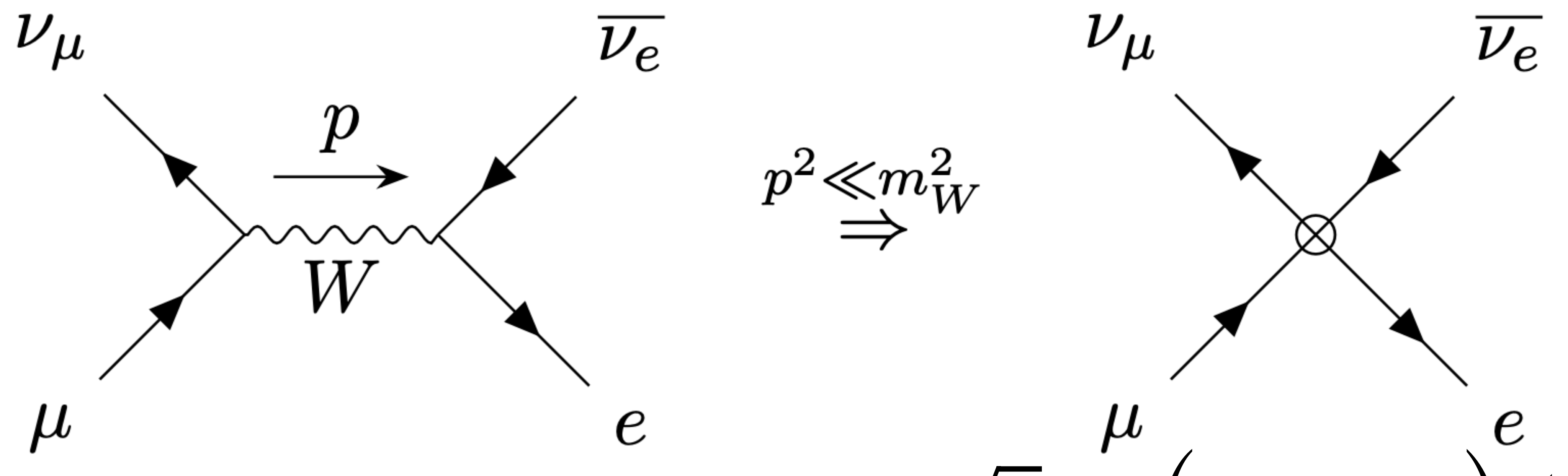

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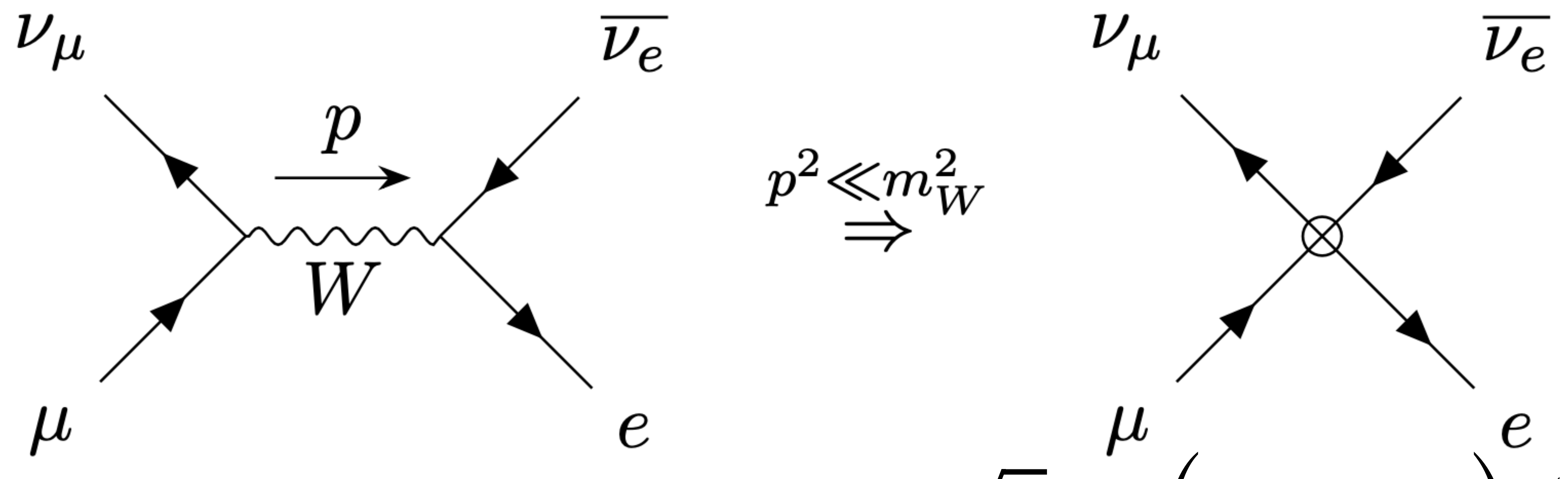

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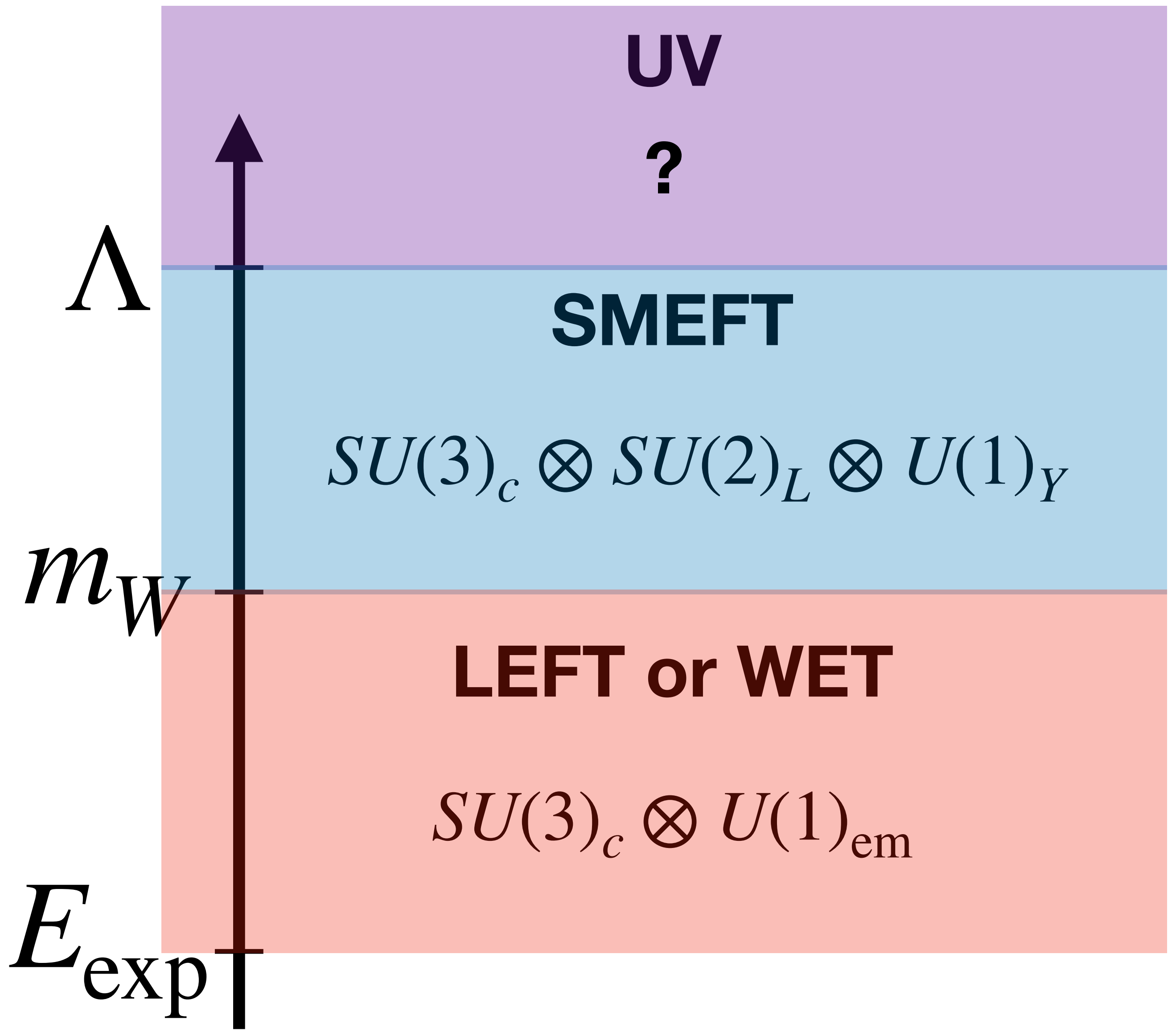

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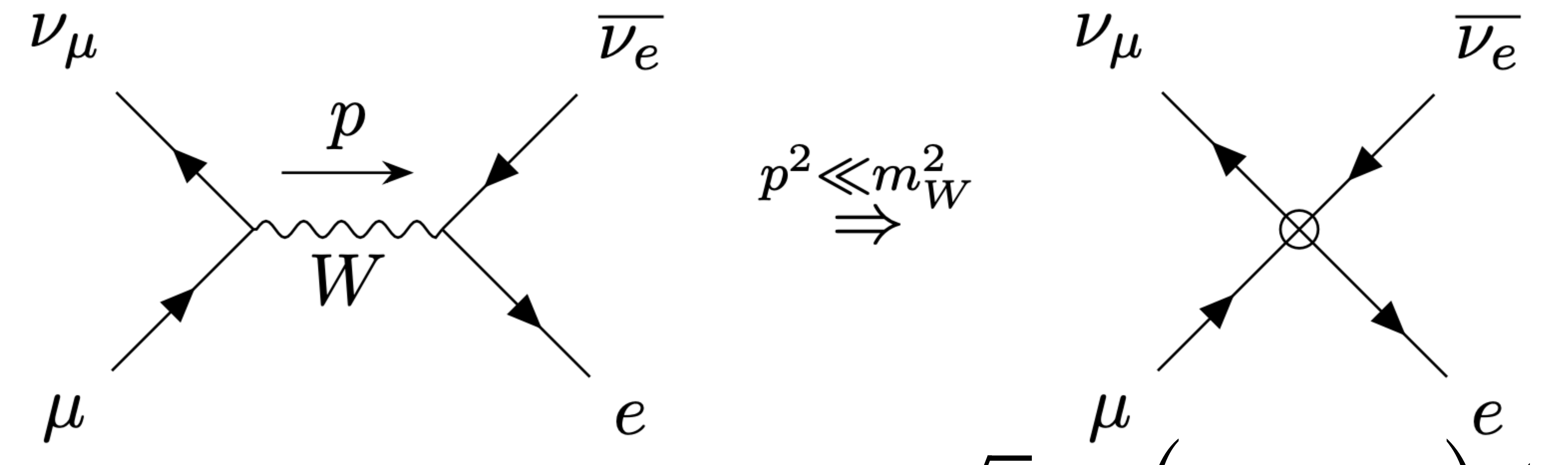

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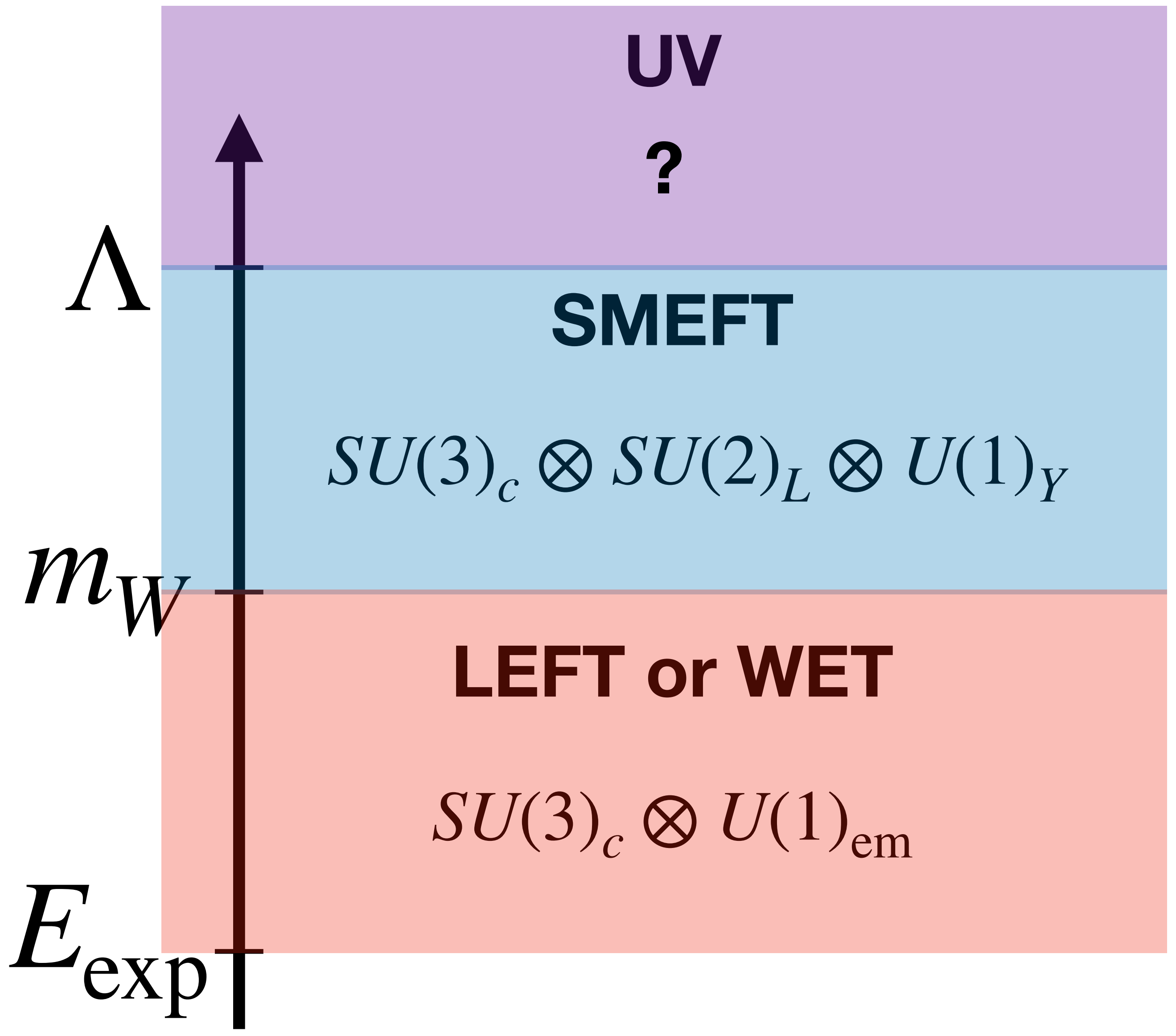
C_n **: Wilson Coefficients = short-distance heavy physics**

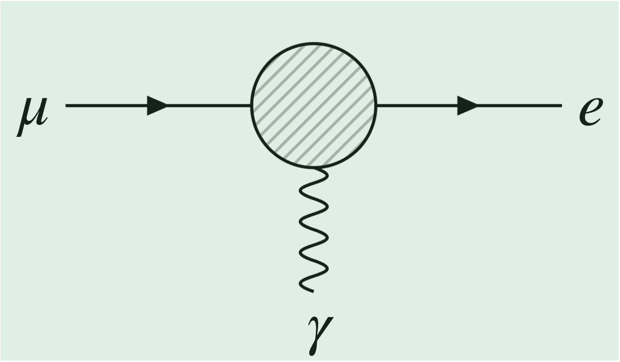
$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_{n>4} \frac{C_n \mathcal{O}_n}{\Lambda^{n-4}}$$

$\mathcal{O}_n$ **: Contact interactions among light fields**



EFT and LFV observables

- **Observables calculated in terms of the operator coefficients**



$$\delta\mathscr{L}_{\mu\to e\gamma} = \frac{m_\mu}{\Lambda^2}(C^{e\mu}_{D,R}\overline{e}\sigma_{\alpha\beta}P_R\mu + C^{e\mu}_{D,L}\overline{e}\sigma_{\alpha\beta}P_L\mu)F^{\alpha\beta}$$

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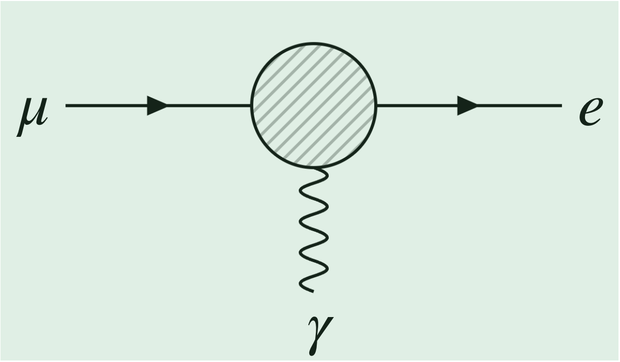

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$$Br(\mu\to e\gamma) = 384\pi^2\left(\frac{v}{\Lambda}\right)^4(|C^{e\mu}_{D,R}|^2 + |C^{e\mu}_{D,L}|^2) < 4.2\times 10^{-13} \longrightarrow \left(\frac{v}{\Lambda}\right)^2 |C^{e\mu}_{D,X}| < 10^{-8}$$

$v^2 = (2\sqrt{2}G_F)^{-1} \sim (174\ \text{GeV})^2$

$\Lambda \gtrsim 10^4 v\ (\text{if } C_D \sim 1)$

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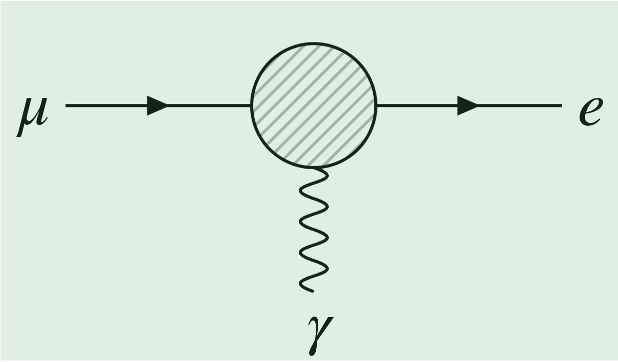

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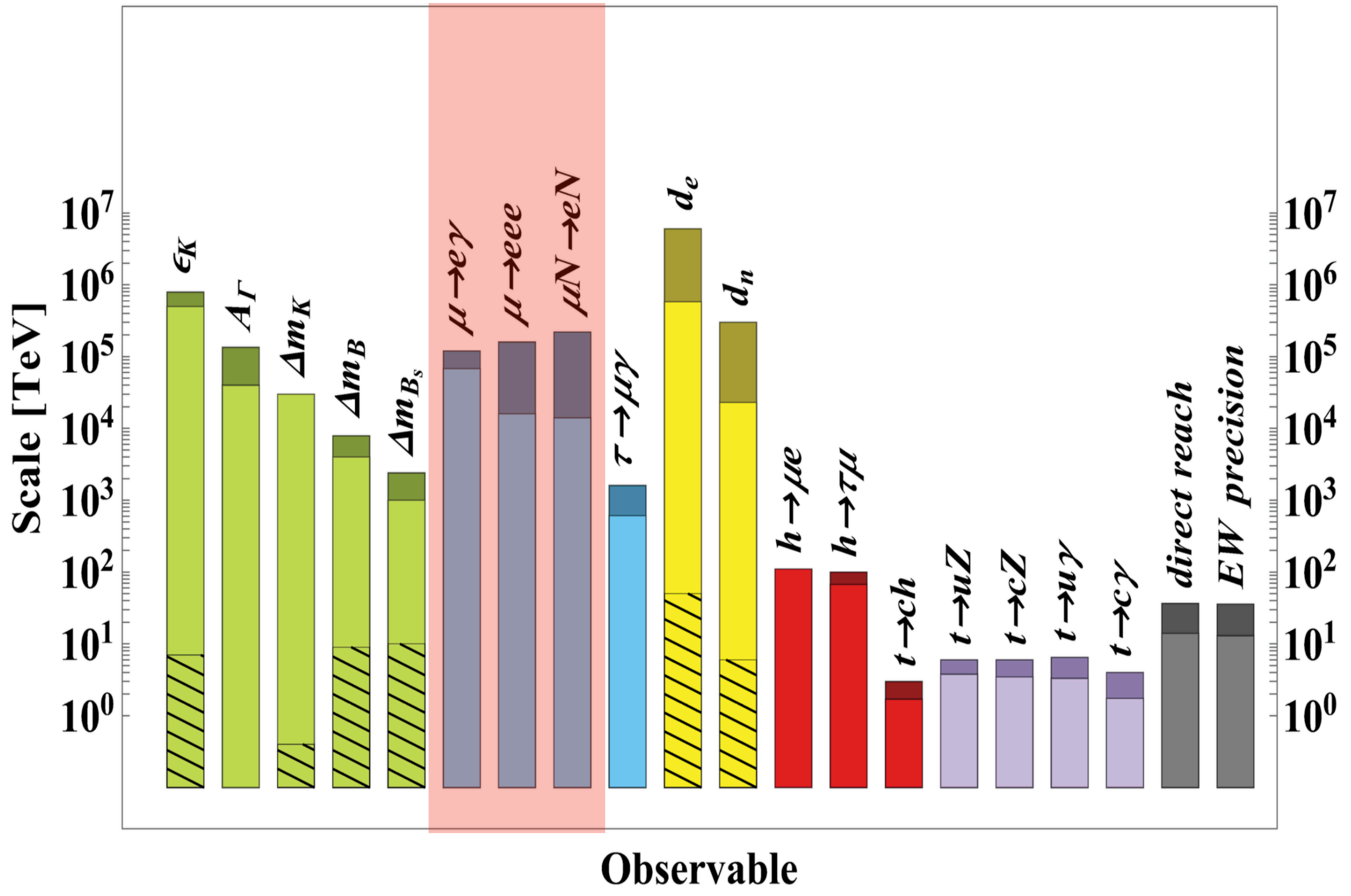
LFV observables sensitive to large New Physics scale



More than Branching Ratios

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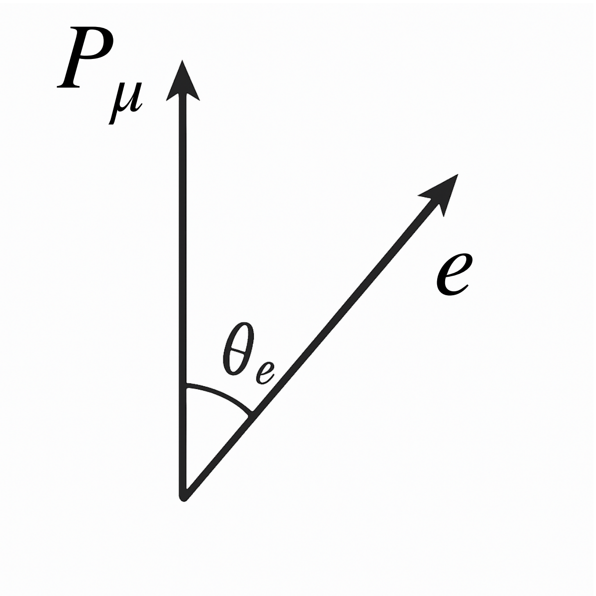

Muon polarization vector

Angle between e momentum and $\vec{P}_\mu$

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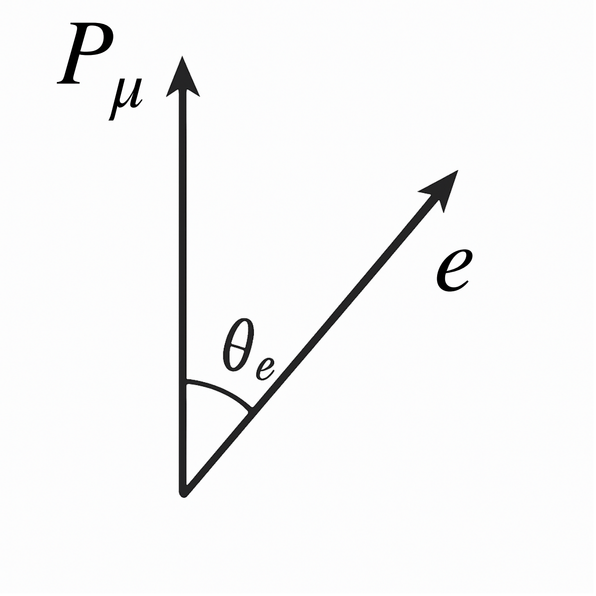

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Angle between e momentum and $\vec{P}_\mu$

- Possible to distinguish $\mu \to e_{L,R}$ conversion with polarised muons

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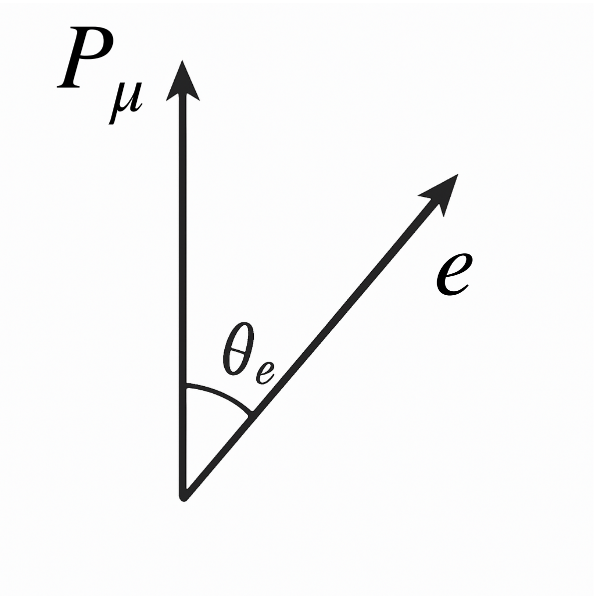

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- Angular observables and Dalitz plots can assist in distinguishing operators in $\mu \to 3e$

Low-energy EFT for $\mu \to e$

$$\mathscr{L}_{|m_\mu} = \frac{1}{v^2} \sum_{X\in\{L,R\}} \left[C^{e\mu}_{D,X}(m_\mu \bar{e}\sigma^{\alpha\beta}P_X\mu)F_{\alpha\beta} + C^{e\mu ee}_{S,XX}(\bar{e}P_X\mu)(\bar{e}P_Xe) + C^{e\mu ee}_{V,LX}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_Xe) + C^{e\mu ee}_{V,RX}(\bar{e}\gamma^\alpha P_R\mu)(\bar{e}\gamma_\alpha P_Xe) + C_{Alight,X}\mathcal{O}_{Alight,X} + C_{Aheavy\perp,X}\mathcal{O}_{Aheavy\perp,X} \right] + h.c.$$

Low-energy EFT for $\mu \to e$

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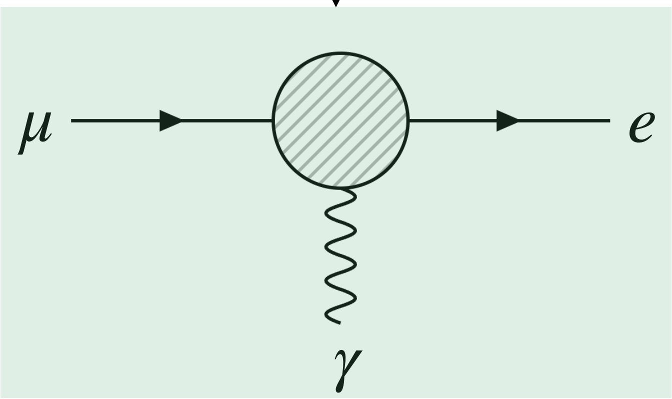
Sum on Chiralities



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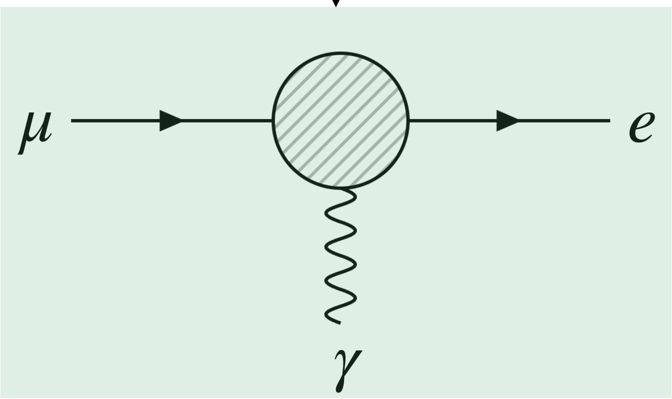

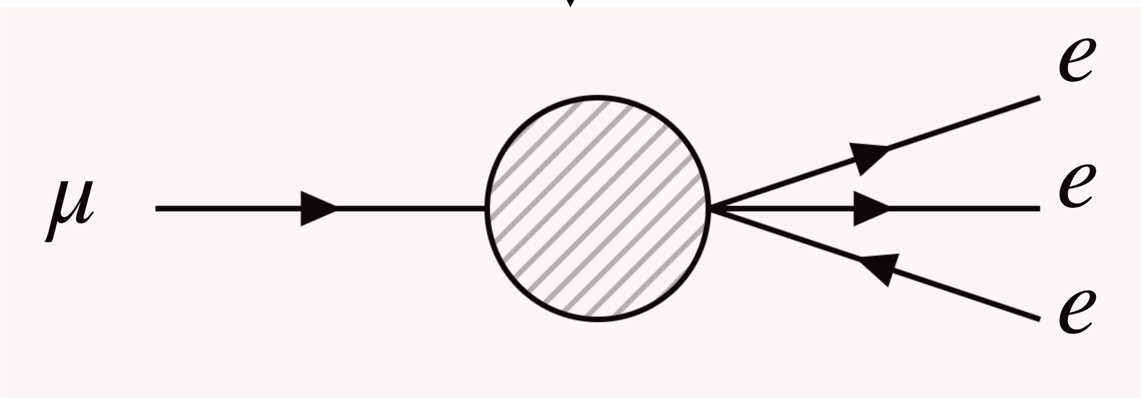

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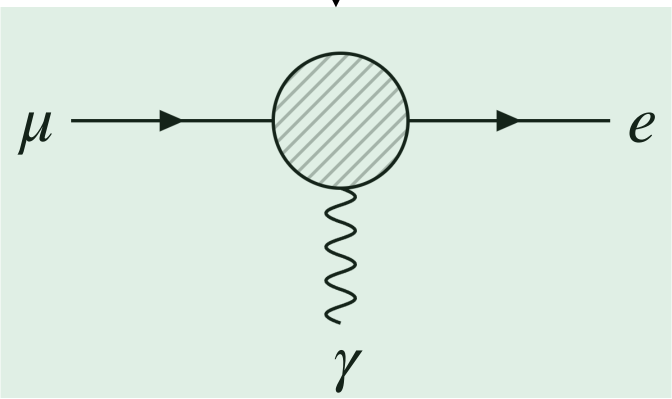

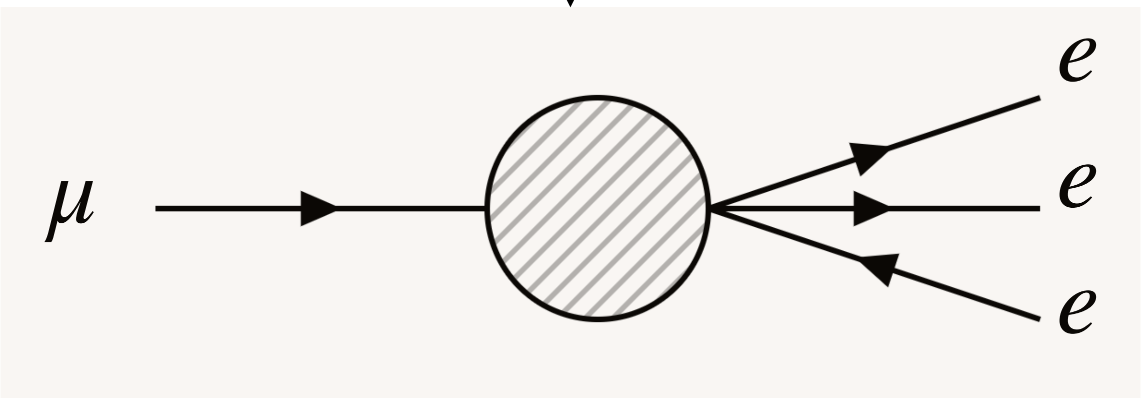

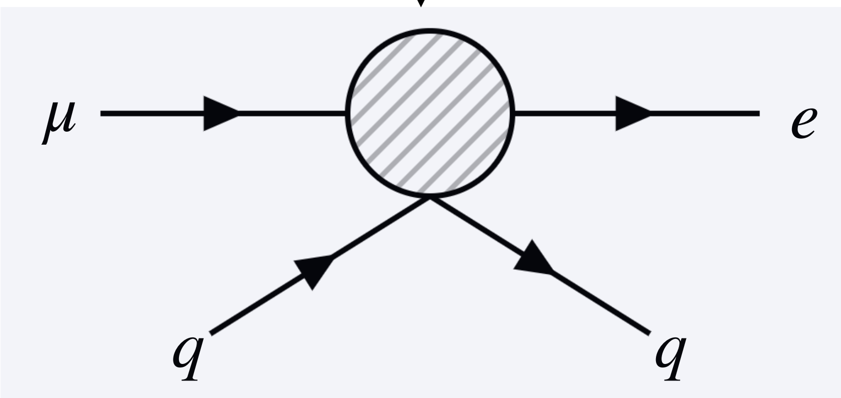

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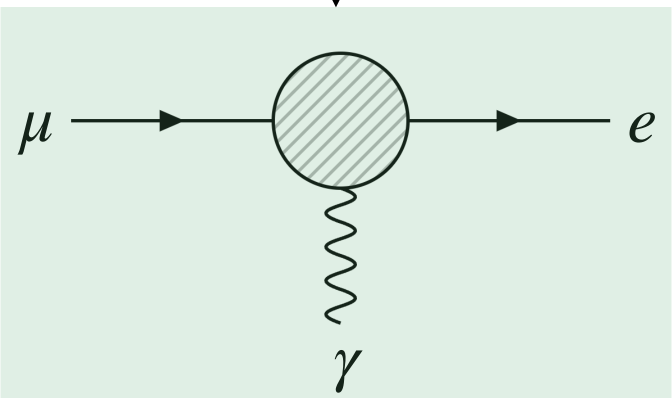
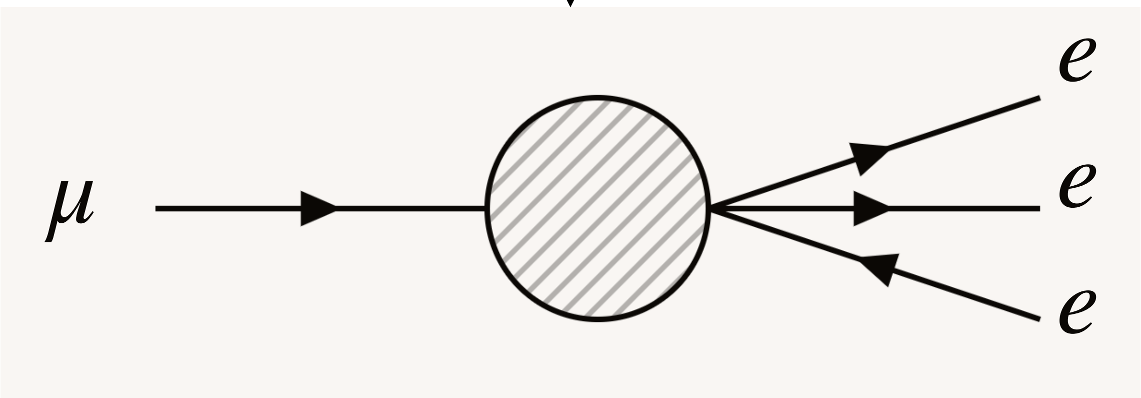
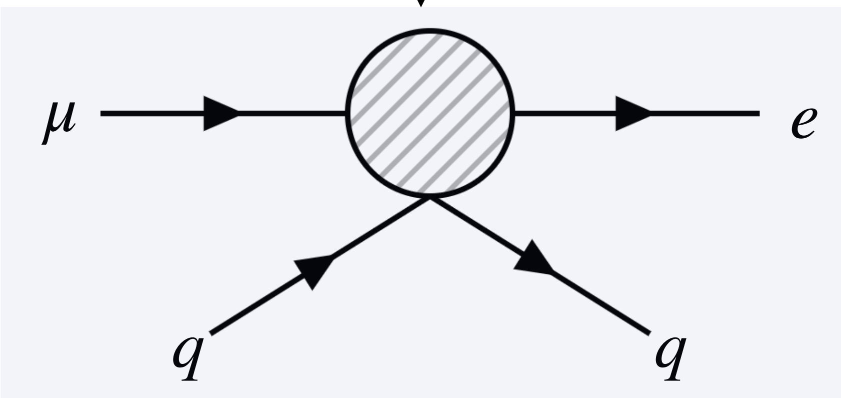
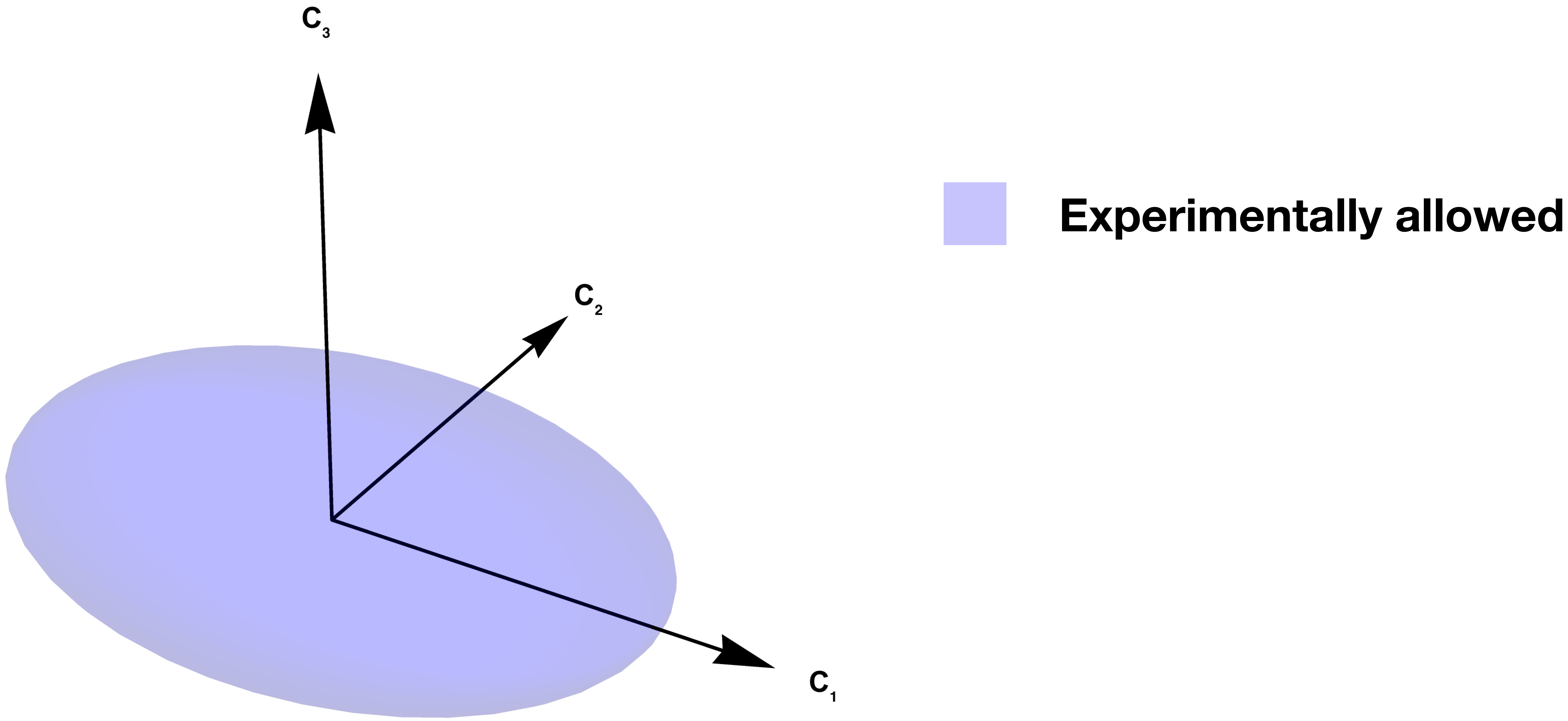






- There are 12 operators coefficients that are constrained by data to lie in the interior of a 12-d ellipse in coefficient space



Intermezzo: Including loops

- Observables are sensitive to low-energy ($E \sim m_\mu$) operators that contribute at tree-level

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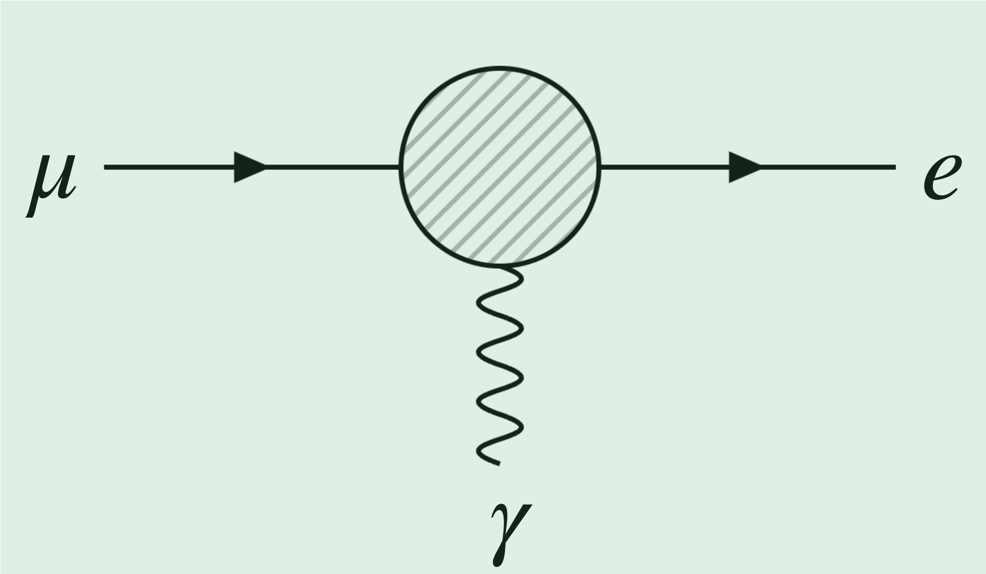

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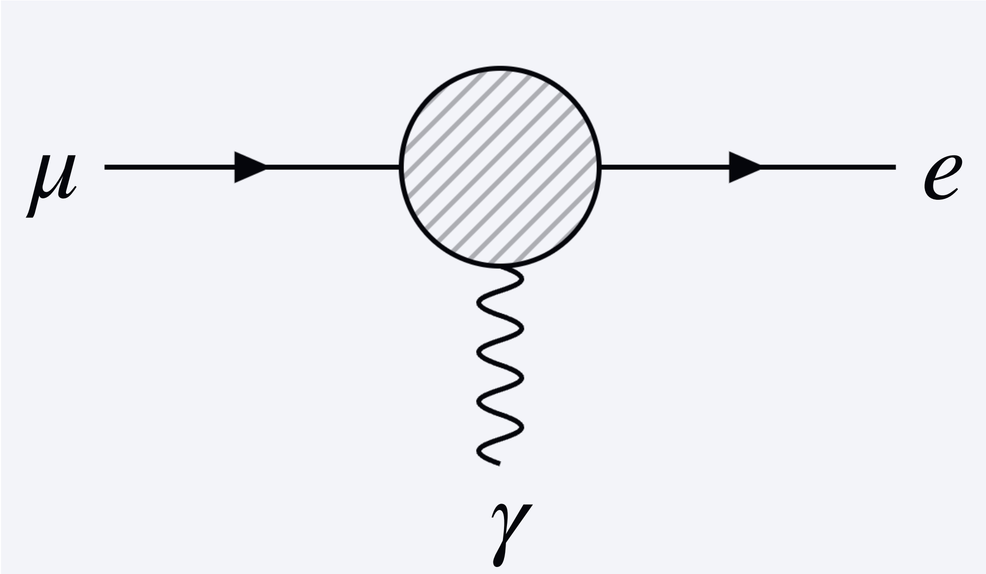

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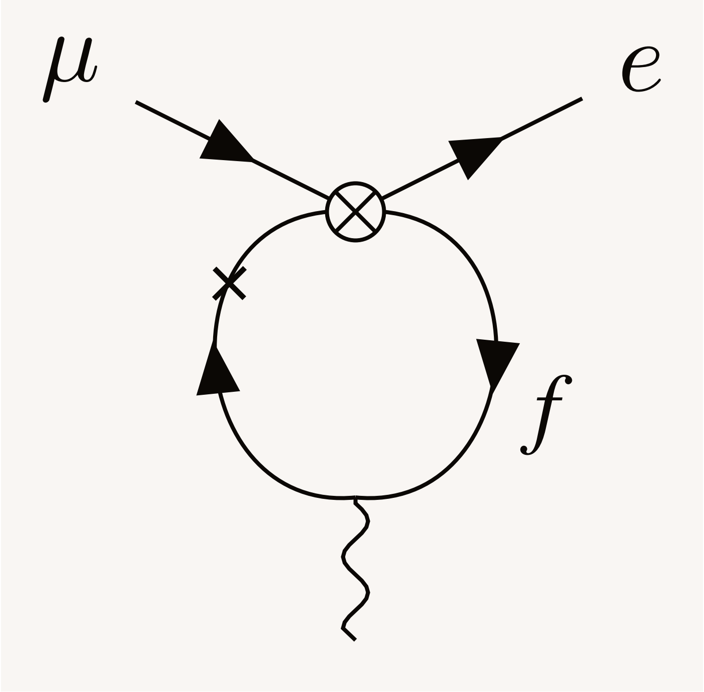

$$\times \log\left(\frac{\Lambda}{m_\mu}\right) + \cdots$$

$E \sim m_\mu$ $\qquad$ $E \sim \Lambda$ $\qquad$ $E \sim \Lambda$

- This is described by the Renormalization Group Equations (RGEs)

$$\boxed{\vec{C}(m_\mu) = \vec{C}(\Lambda) \cdot U(m_\mu, \Lambda)}$$

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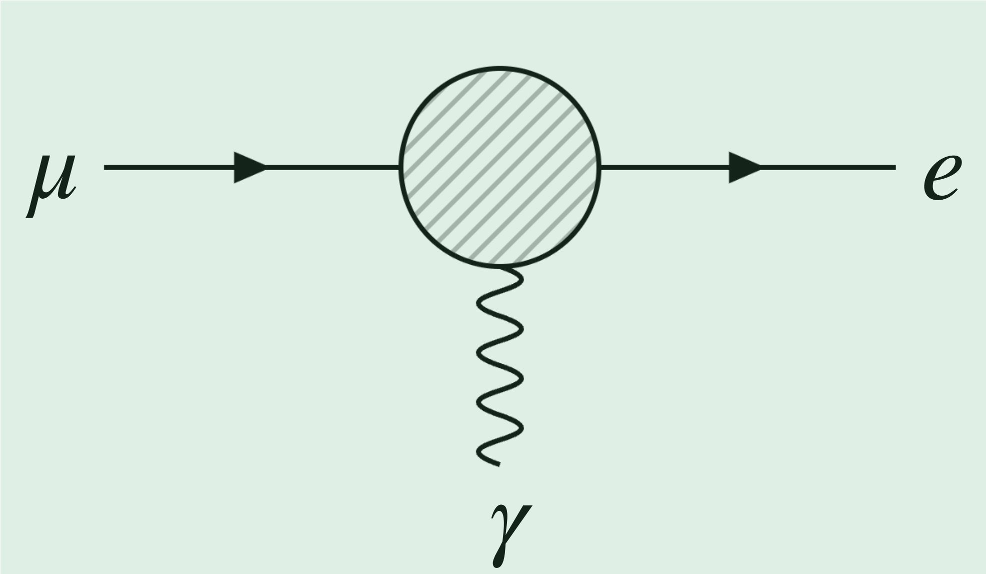

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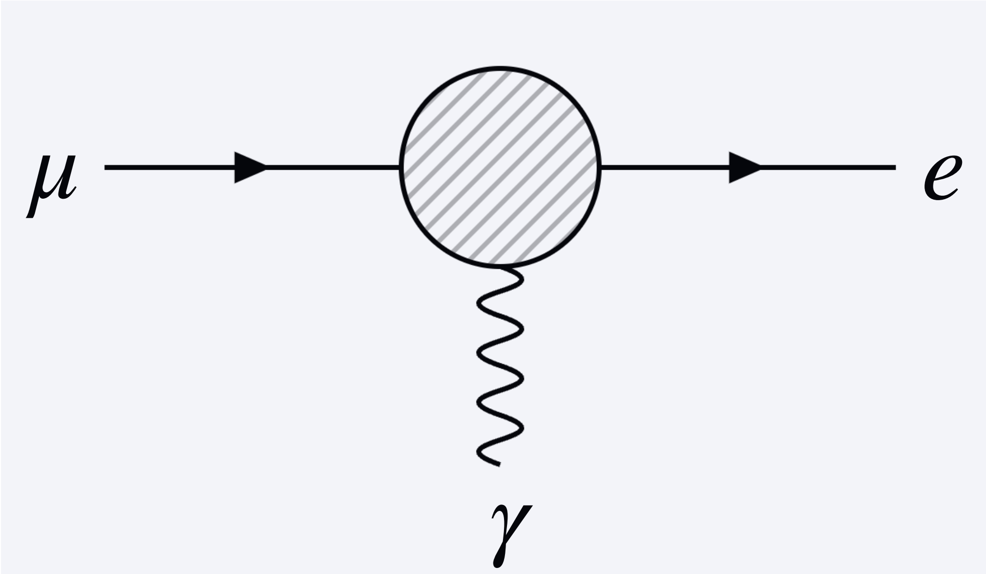

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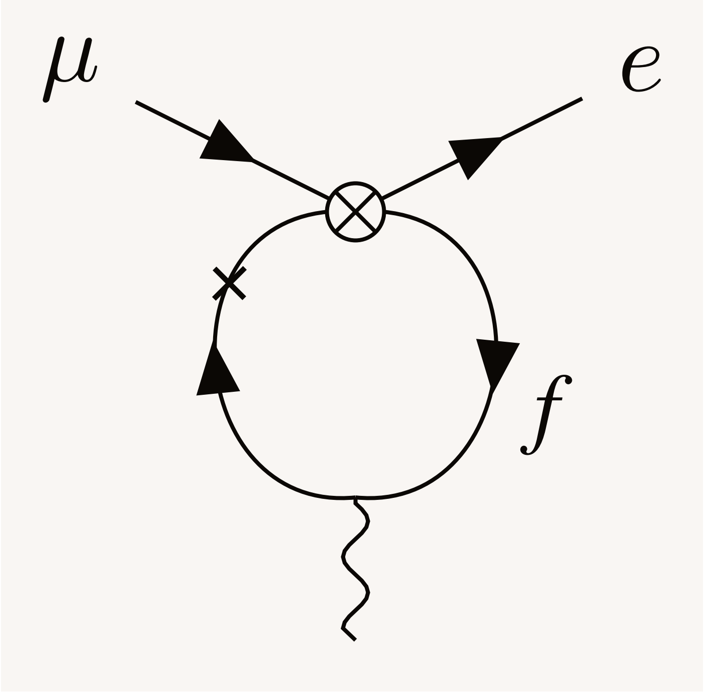

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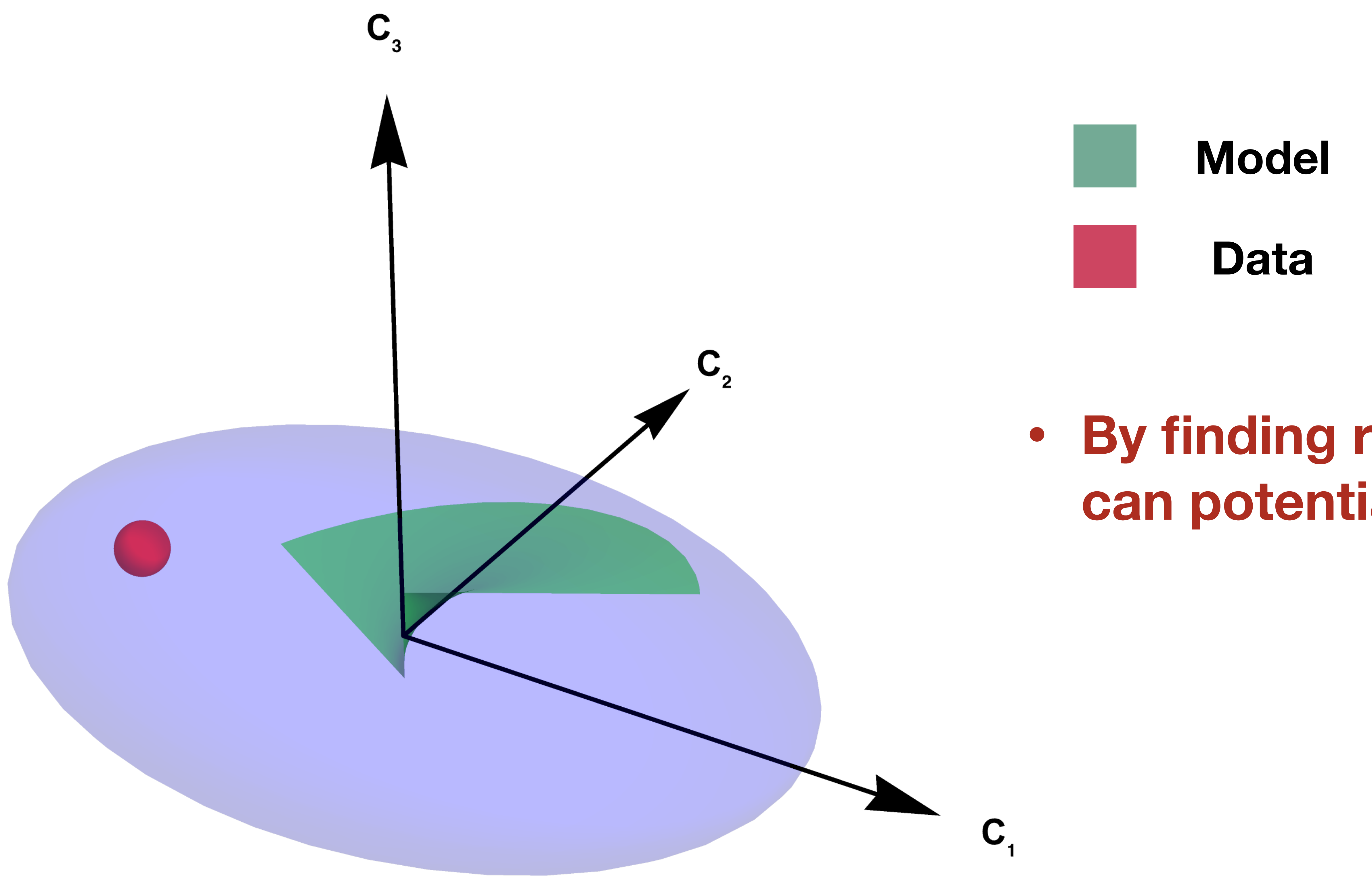
- **We always look at the low-energy coefficients because are directly related to data**

Excluding models?

- Combine everything data ($\mu \to e_X\gamma$, $\mu \to e_X\bar{e}_Y e_Z$, $\mu A \to e_X A \times 2$) can tell us = in principle determine the 12 low-energy coefficients

- Suppose we observe a combination of $\mu \to e$ processes in upcoming experiments

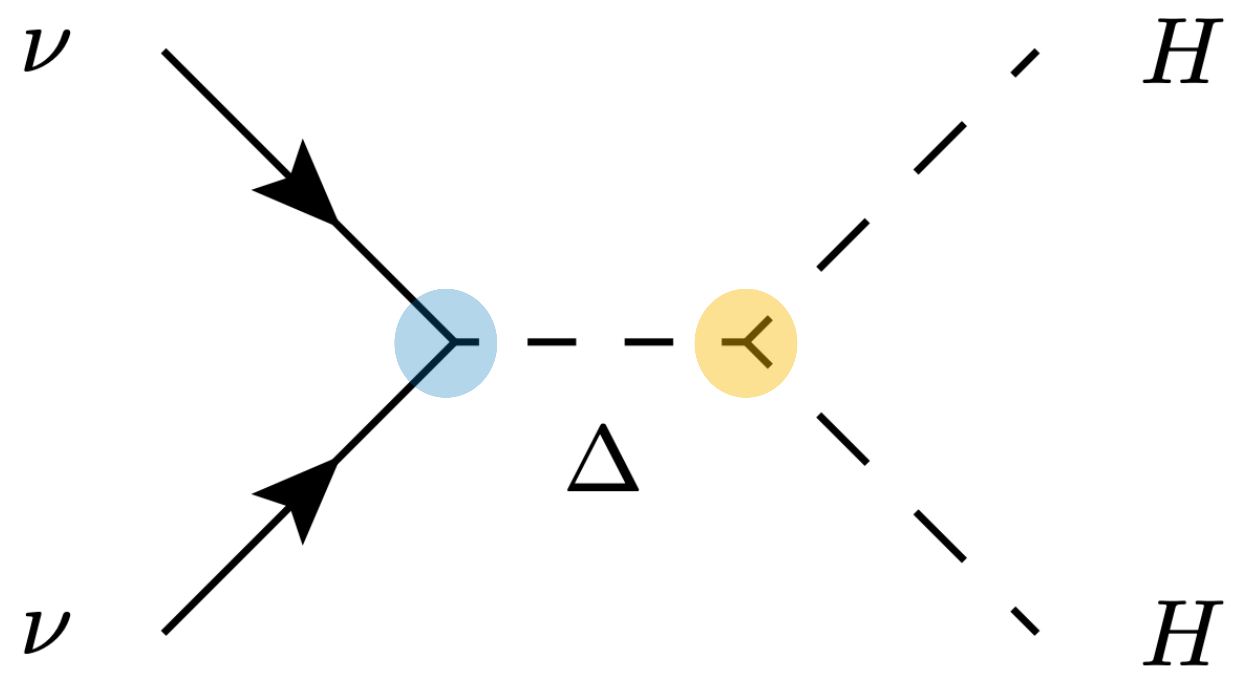


- **By finding regions where models CANNOT be, we can potentially exclude them**

Apply this recipe to popular New Physics models

Type-II seesaw (SM+Triplet Δ)

$$\mathscr{L} \supset F_{\alpha\beta}\overline{\ell^c_\alpha}\epsilon\Delta\cdot\tau\ell_\beta + M_\Delta\lambda_H H^T\epsilon\Delta\cdot\tau H + \ldots$$

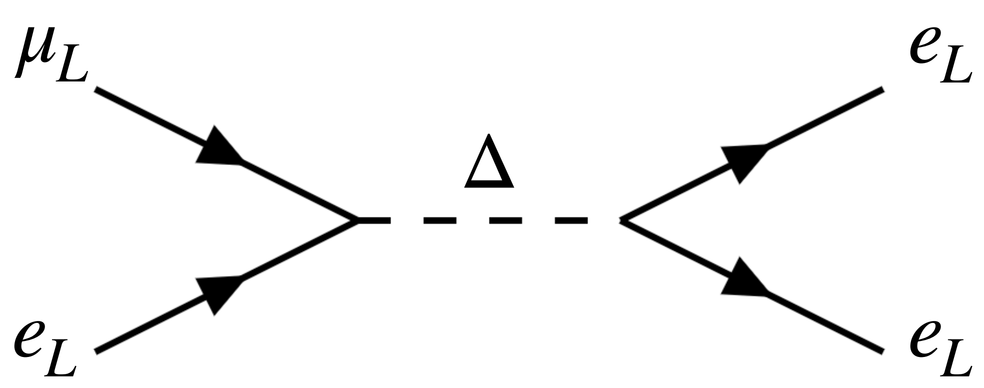


$$[m_\nu]_{\alpha\beta} \sim 0.03\text{ eV } F_{\alpha\beta}\frac{\lambda_H}{10^{-12}}\frac{\text{TeV}}{M_\Delta}$$

- Simple neutrino mass model
- Neutrino masses directly related to the triplet Yukawas
- But the overall scale, the ordering/lightest mass and phases are unknown

Type-II seesaw: LFV coefficients

- Tree-level coupling with left-handed leptons



$$C_{V,LL}^{e\mu ee} \qquad (\mu \to e_L \bar{e}_L e_L)$$

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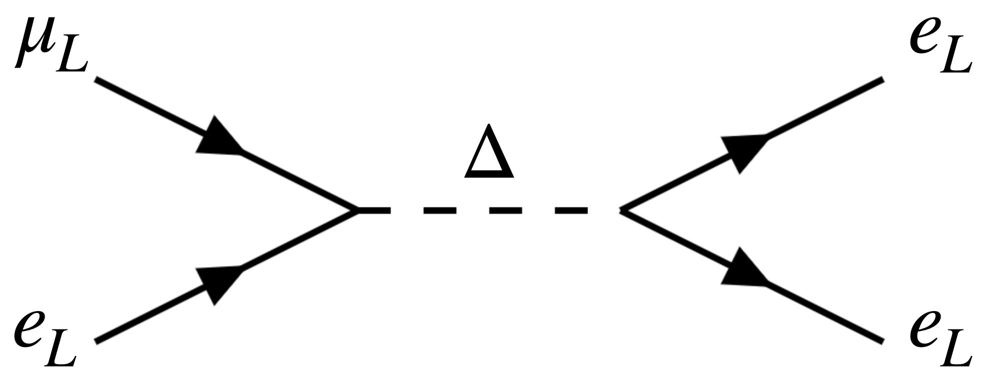
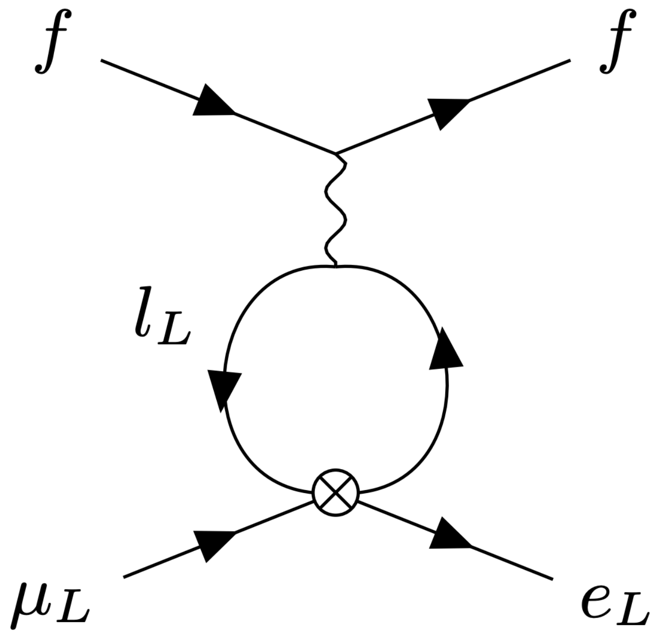


$C^{e\mu ee}_{V,LL}$ $(\mu \to e_L \bar{e}_L e_L)$

- Loop induced coefficients with quarks and right-handed leptons



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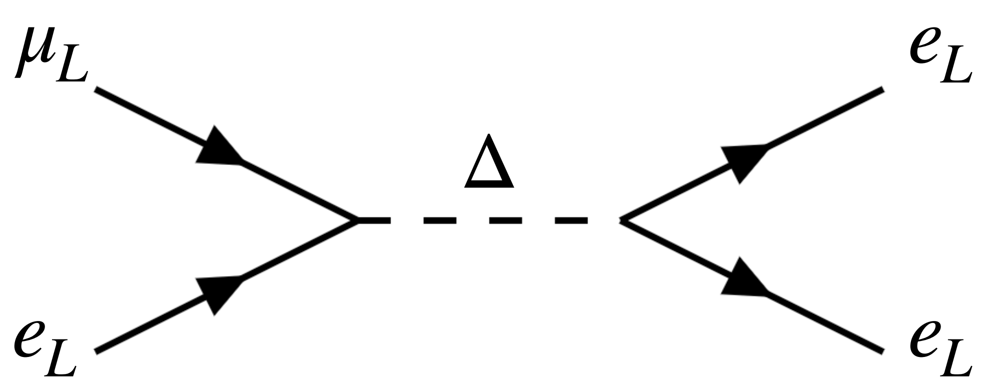

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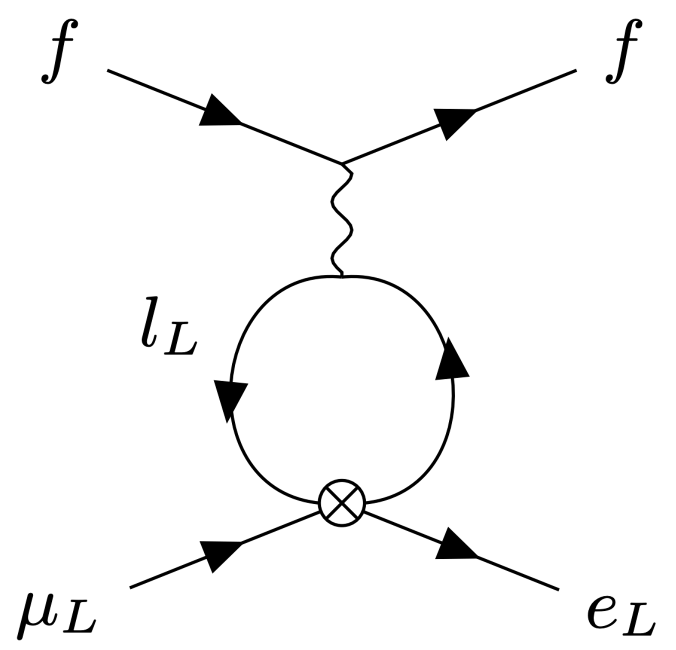
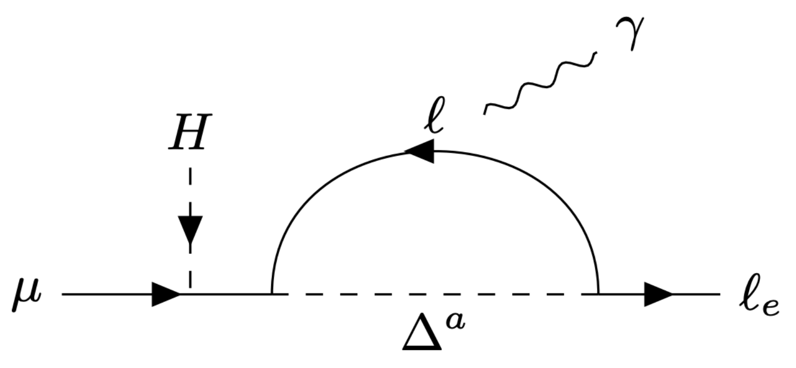
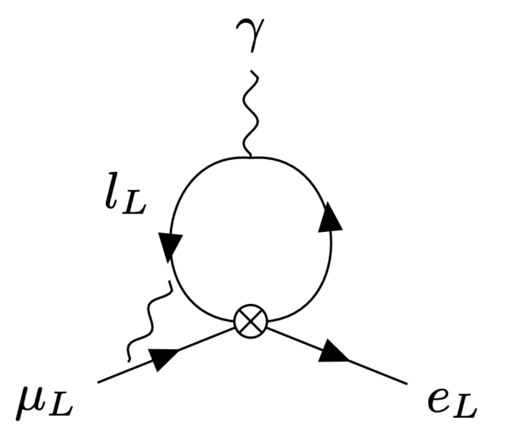


$C^{e\mu ee}_{V,LX}$ $(\mu \to e_L \bar{e}_X e_X)$

$C^{e\mu}_{A,L}$ $(\mu A \to e_L A)$

- Contribution to the dipole with left-handed out-going electrons





$C^{e\mu}_{D,R}$ $(\mu \to e_L \gamma)$

Type-II: where does it live in the observable ellipse?

- **Cannot predict sizable $\mu \to e_R$** (because triplet couples to left-handed leptons and these are suppressed by m_e)

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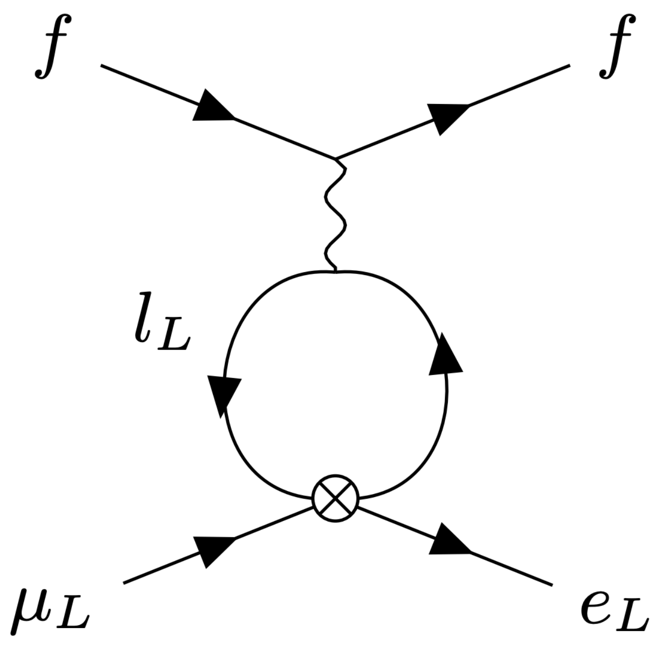

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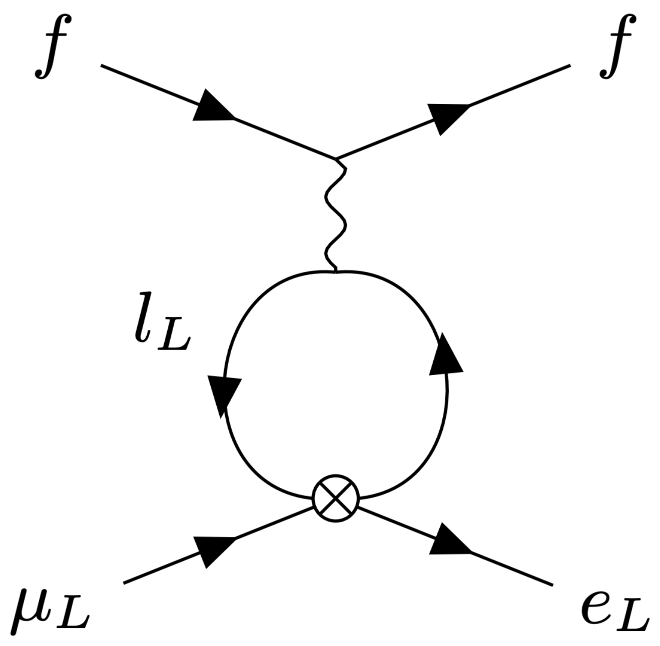

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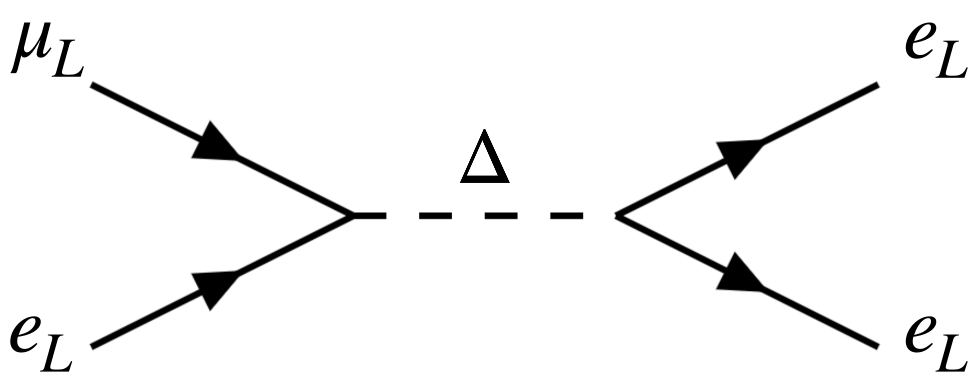

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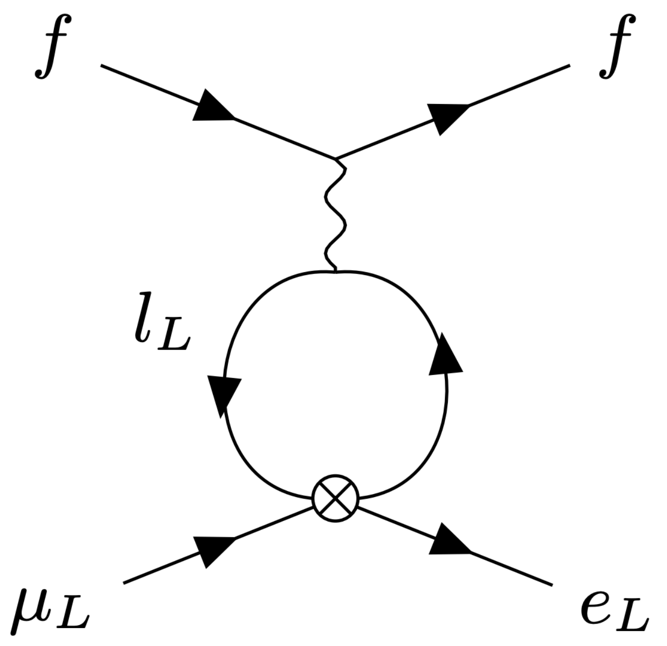

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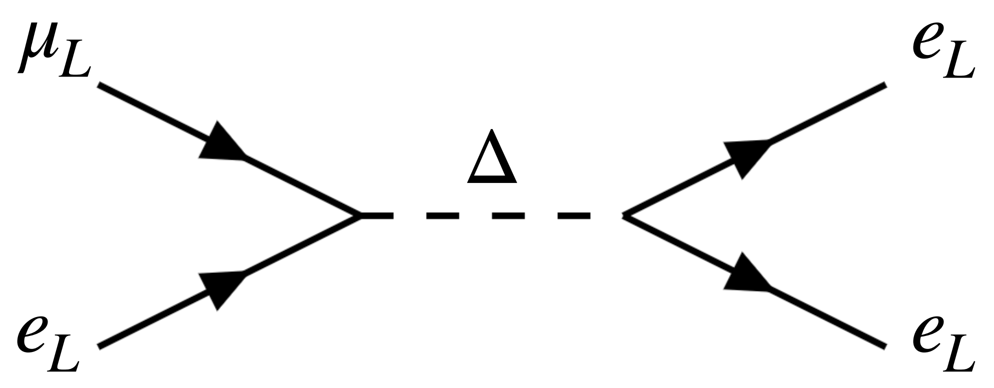

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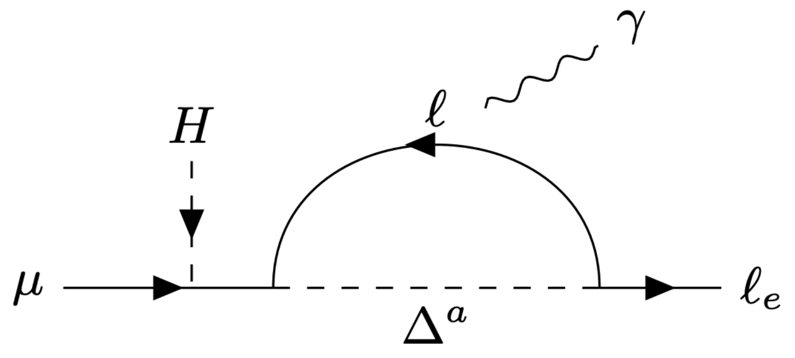
- Surprisingly also **the dipole can be suppressed** (although it requires some tuned cancellations and high m_ν — but we want to know what the model cannot do!)



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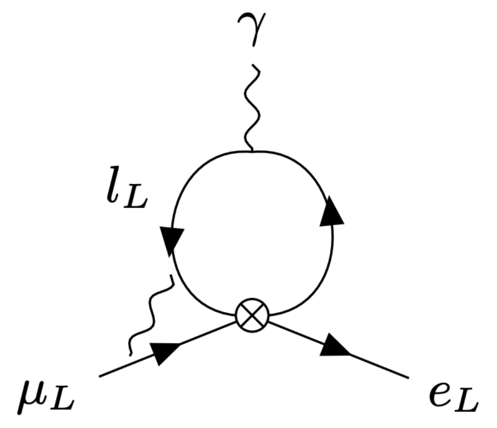

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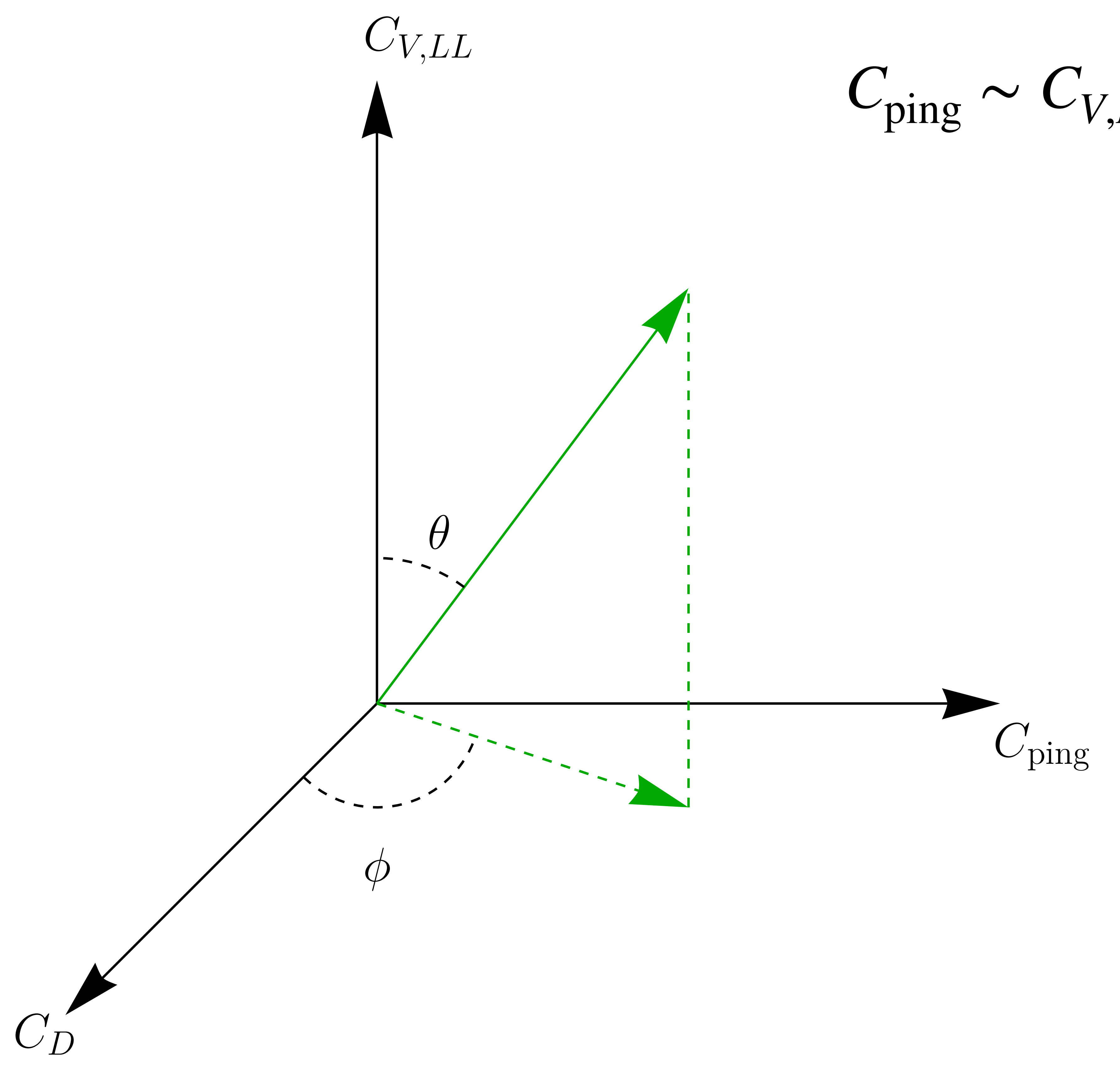

$$C_{\text{ping}} \sim C_{V,LR}, C_{A,L}$$

- Explore the space for the non-trivial coefficients (absolute values)

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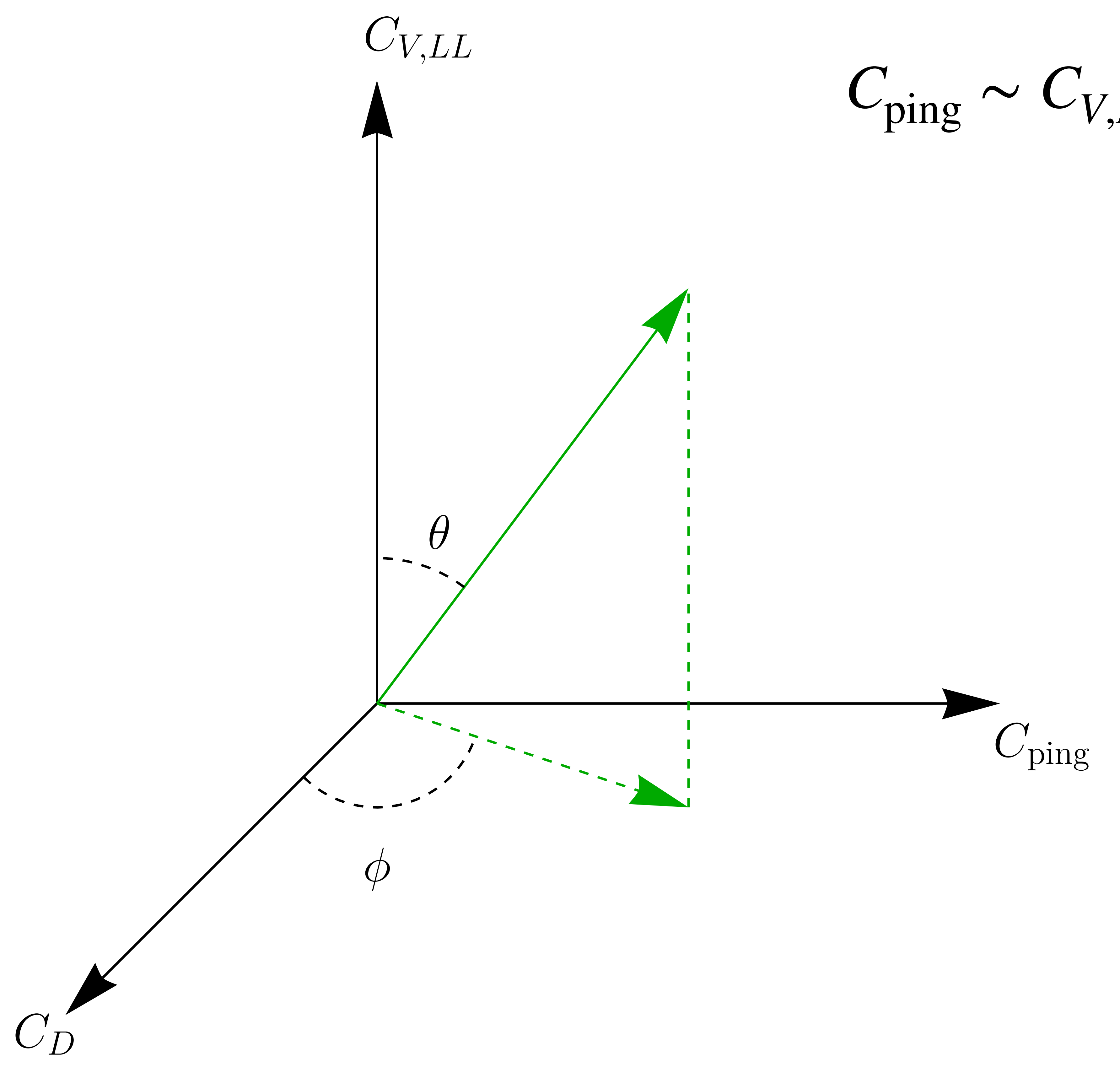

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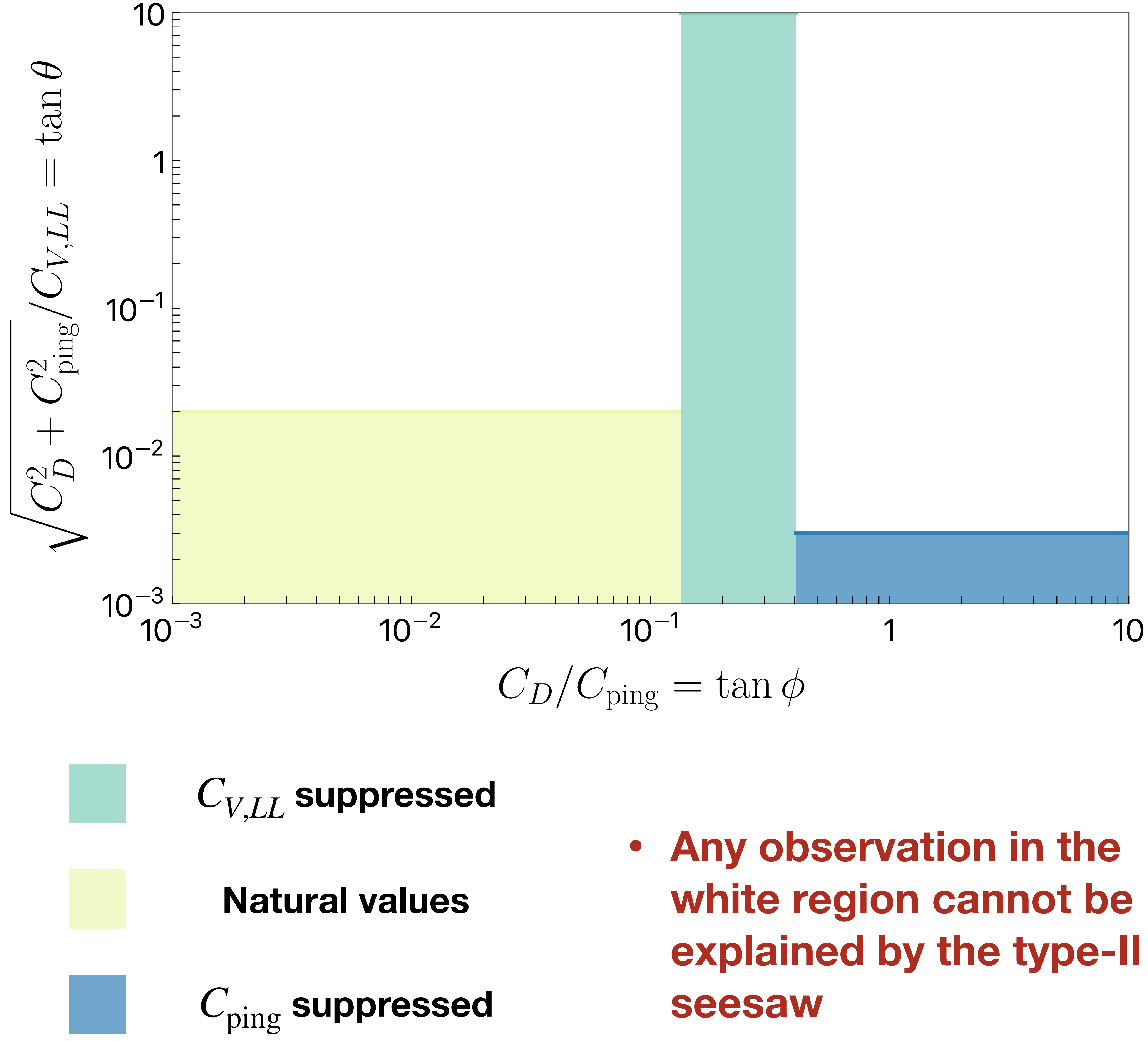

- Explore the space for the non-trivial coefficients (absolute values)

- **Any observation in the white region cannot be explained by the type-II seesaw**

Inverse seesaw

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- Add to the Lagrangian pairs of gauge singlet fermions N, S with opposite chirality

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$$\delta\mathscr{L}_{NS} = i\bar{N}\partial N + i\bar{S}\partial S - \left(Y_\nu^{\alpha a} \left(\bar{\ell}_\alpha \tilde{H} N_a\right) + M_{ab}\bar{S}_a N_b + \frac{1}{2}\mu_{ab}\overline{S_a}S_b^c + \mathrm{h.c}\right)$$

Lepton Number Violating

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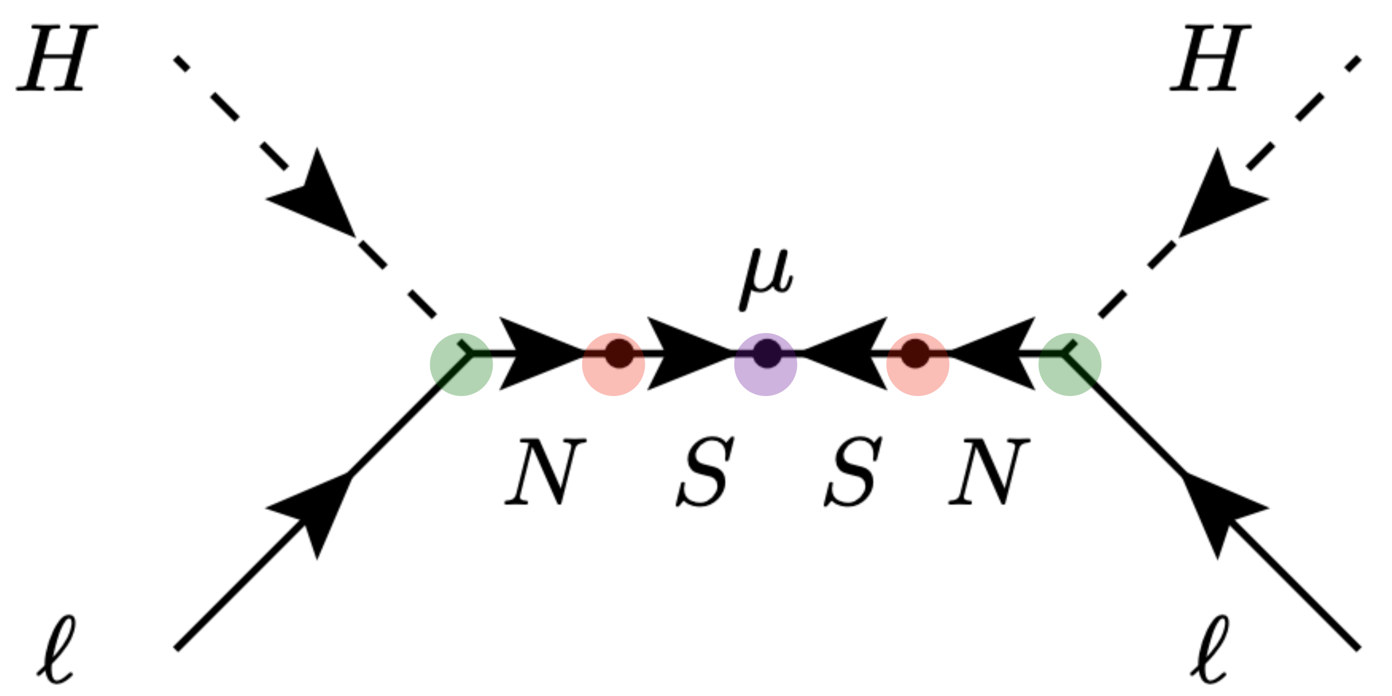

$Y_\nu v,\ \mu \ll M$

$$m_\nu = v^2 \left(Y_\nu \left(M^{-1}\right) \mu \left(M^T\right)^{-1} Y_\nu^T \right)$$

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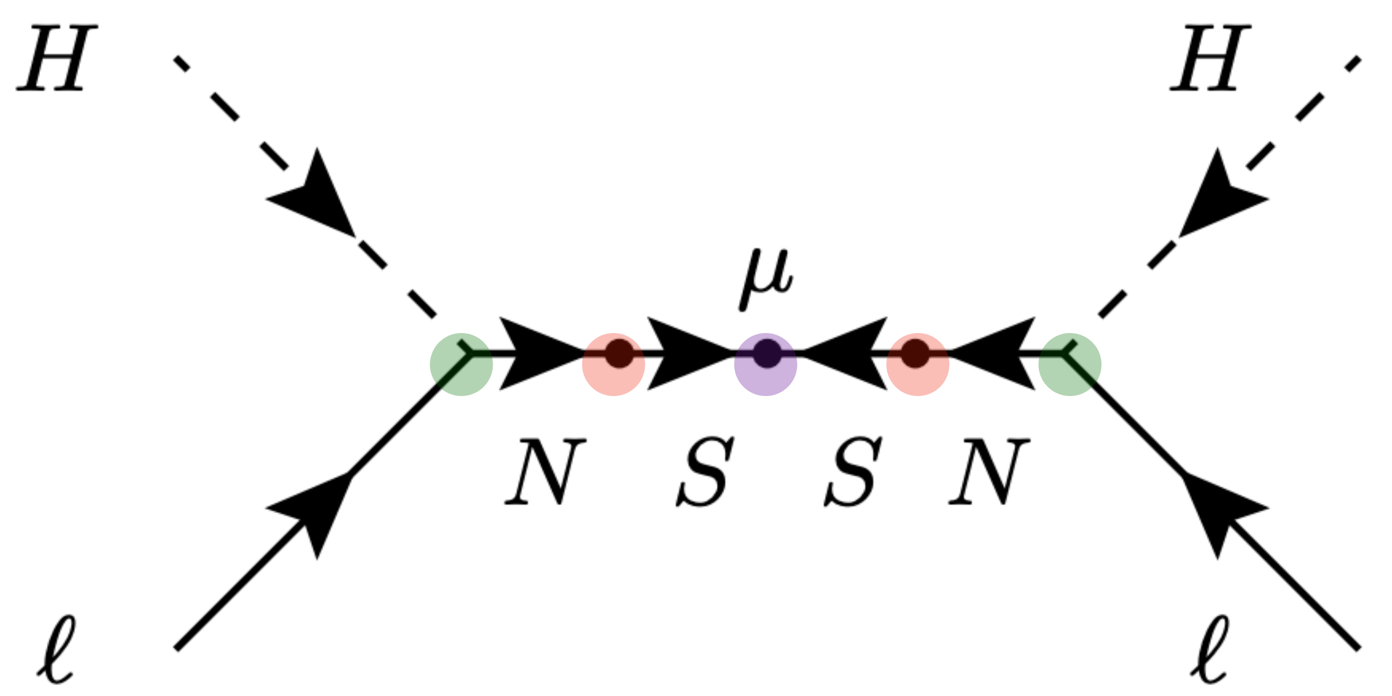
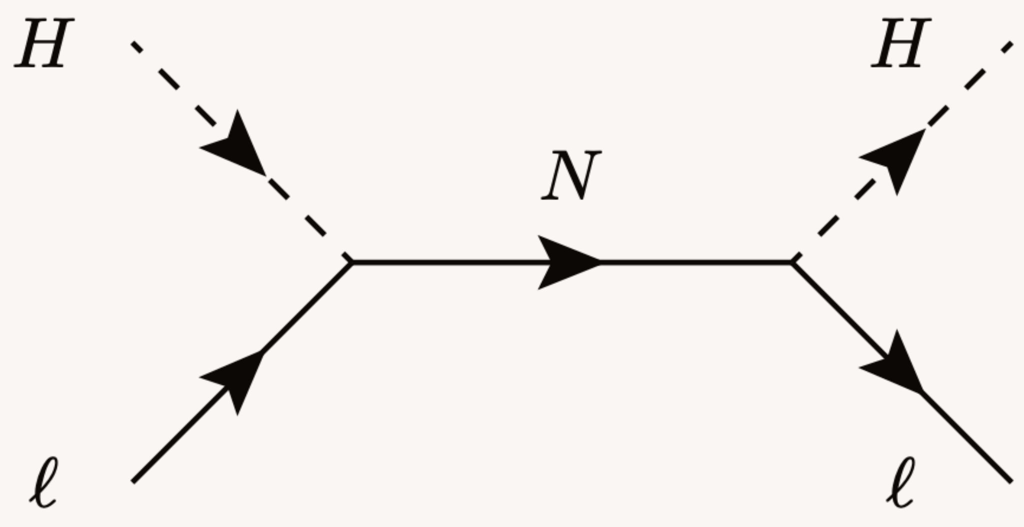


$$Y_\nu v,\ \mu \ll M \quad \Longrightarrow \quad m_\nu = v^2 \left(Y_\nu \left(M^{-1}\right) \mu \left(M^T\right)^{-1} Y_\nu^T \right)$$

- Can **suppress m_ν with μ while keeping Y_ν large**. With enough pairs of sterile, **Y_ν is independent from neutrino masses**

Inverse seesaw LFV



- PMNS non-unitary contribution, leads to modified $W - l - \nu$ couplings

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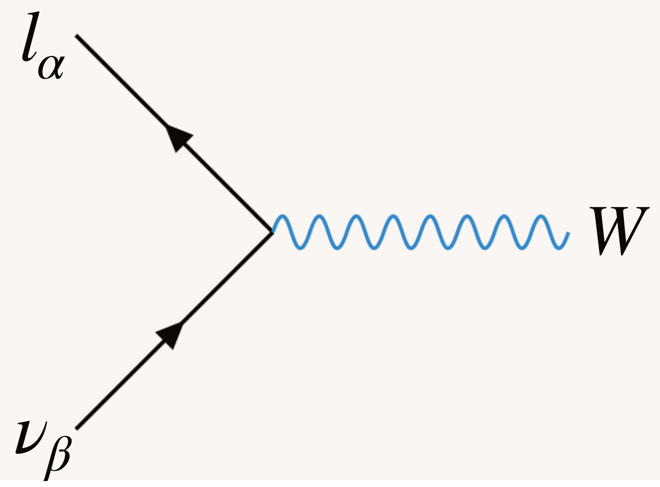
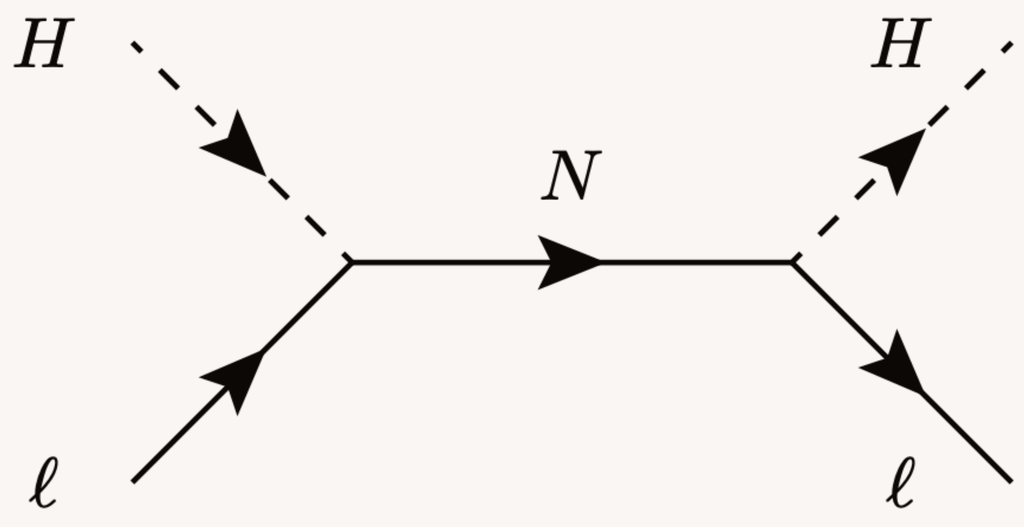


Inverse seesaw LFV



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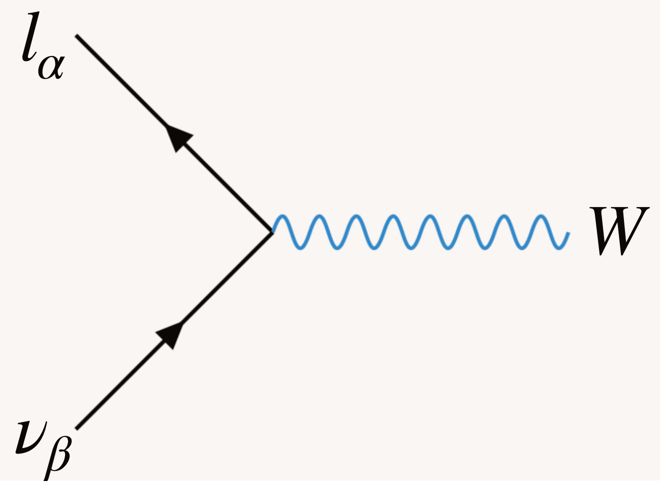
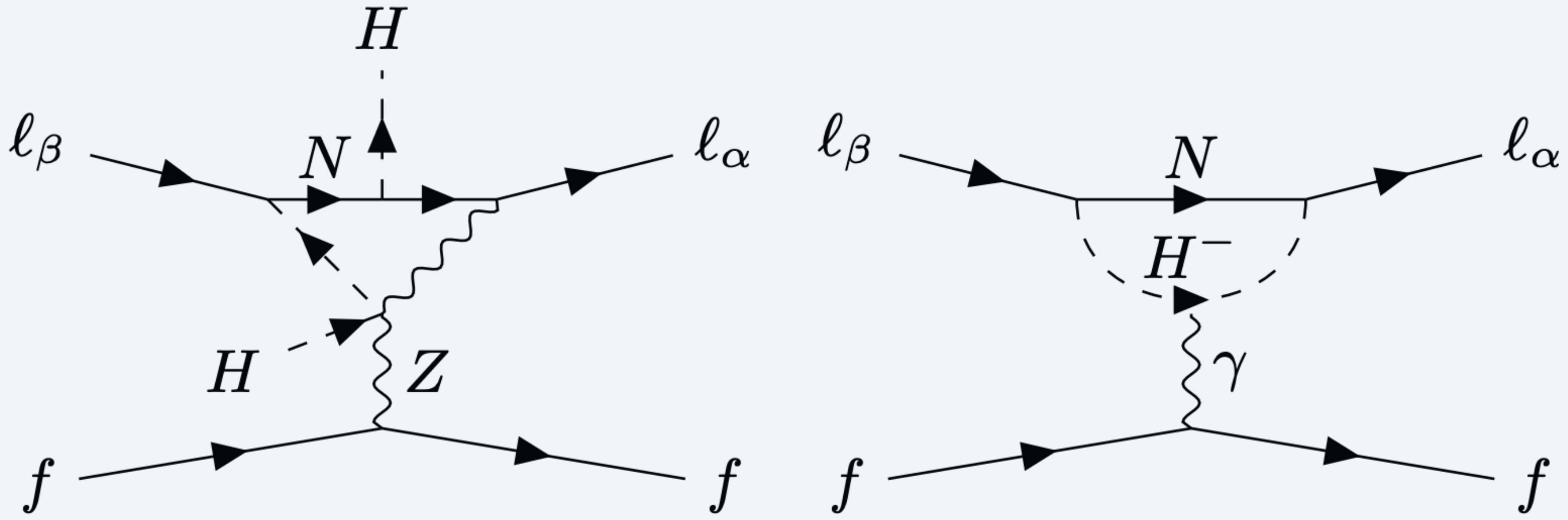
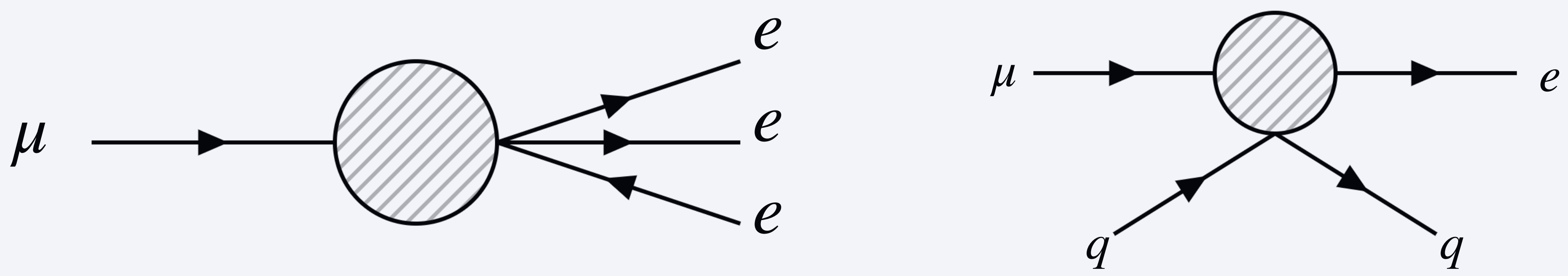


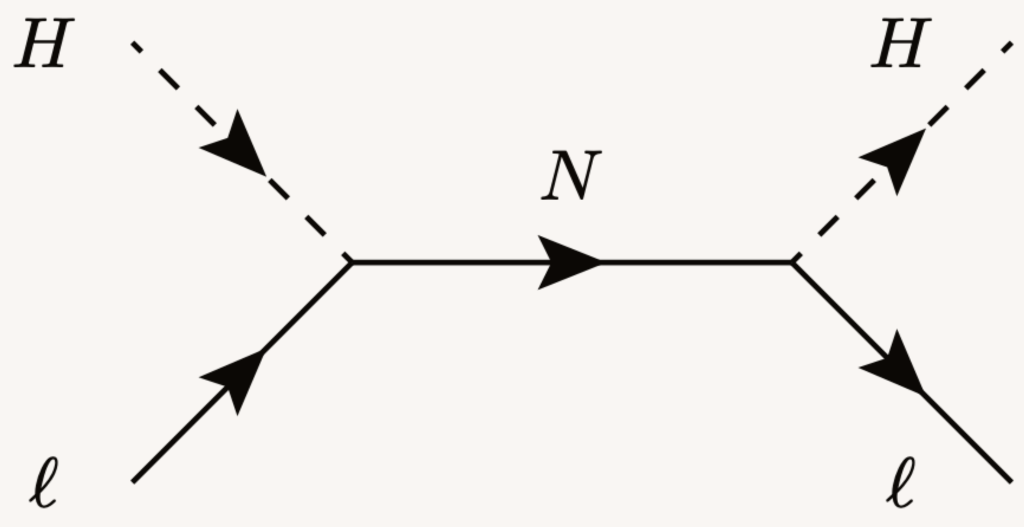


- Match onto four-vector operators

$$C^{\alpha\beta ff}_{V,LX} \propto \frac{\alpha_e}{4\pi} v^2 (Y_\nu M_a^{-2} Y_\nu^\dagger)_{\alpha\beta}(\log(m_W^2/M_a^2) + \text{const.})$$

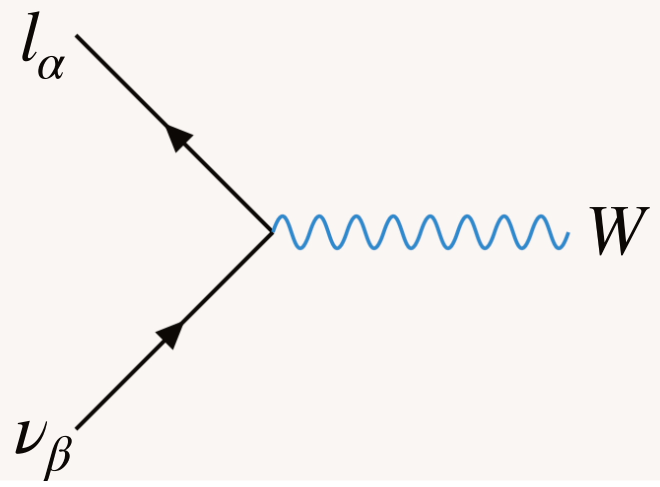


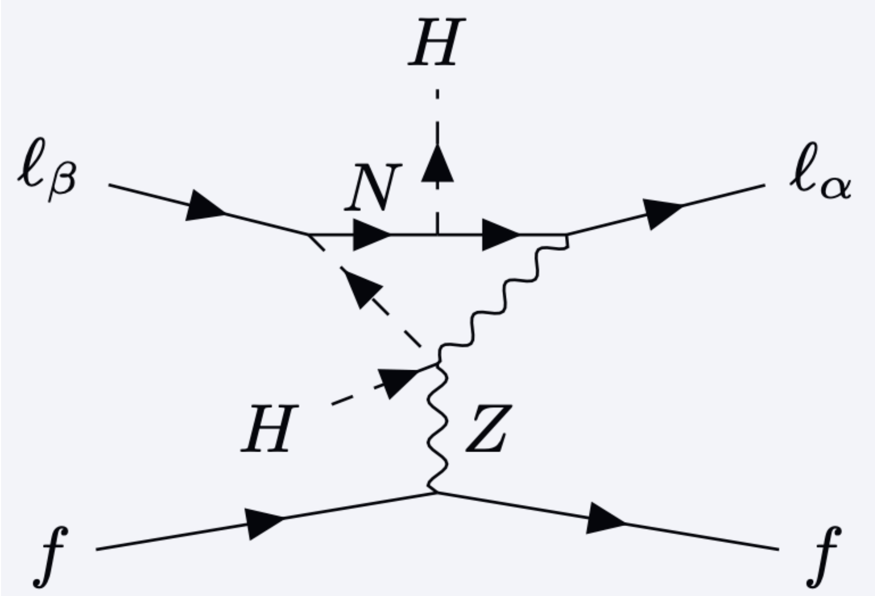
Inverse seesaw LFV

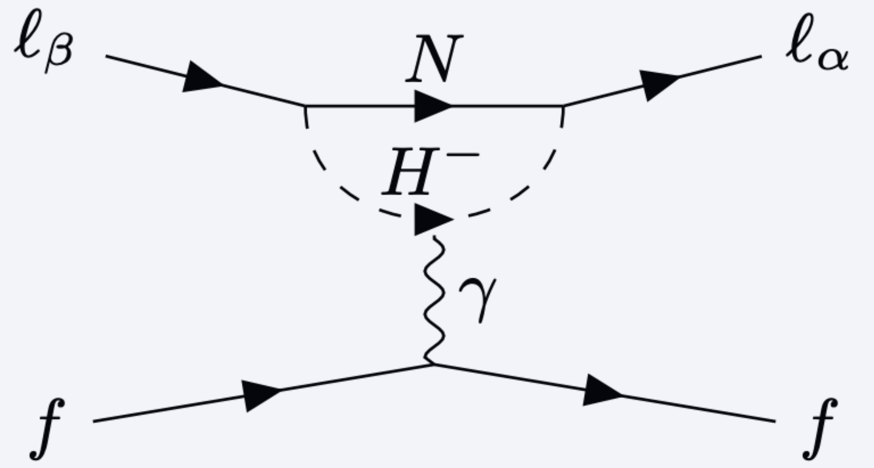


- PMNS non-unitary contribution, leads to modified $W - l - \nu$ couplings

$$\propto v^2 (Y_\nu M^{-2} Y_\nu^\dagger)_{\alpha\beta}$$







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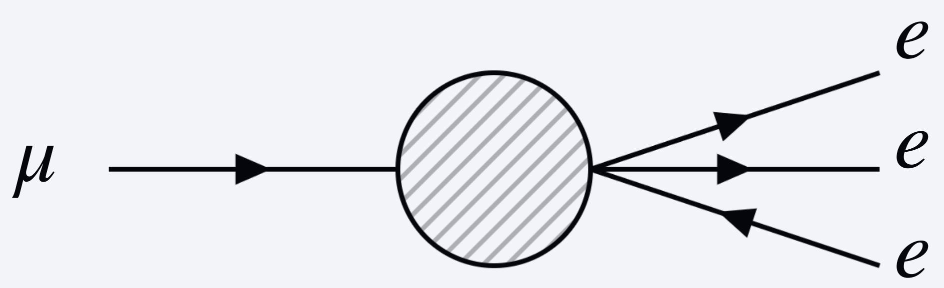
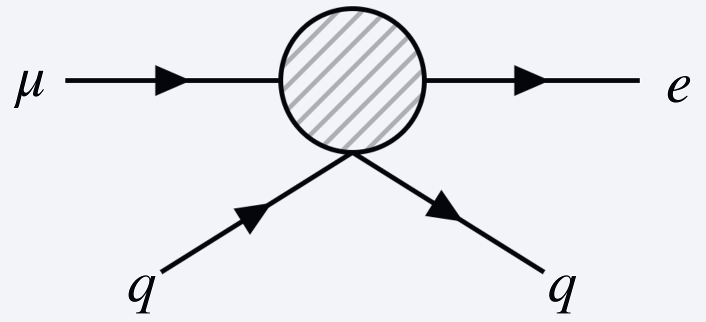
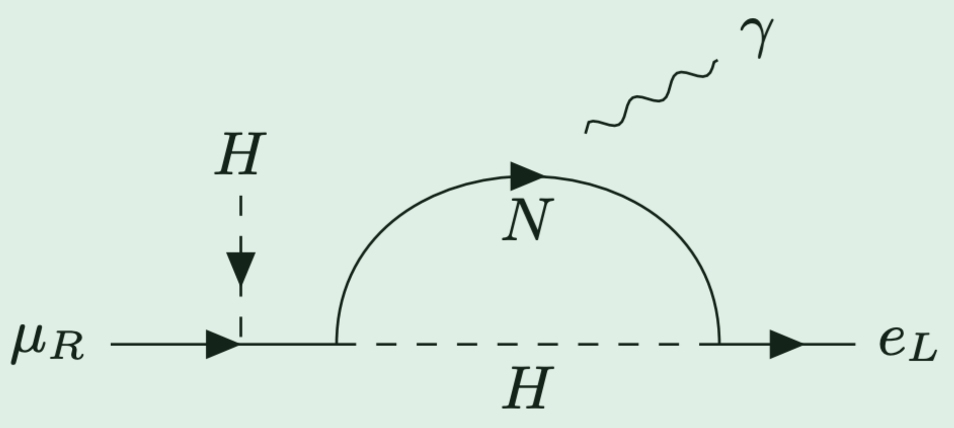
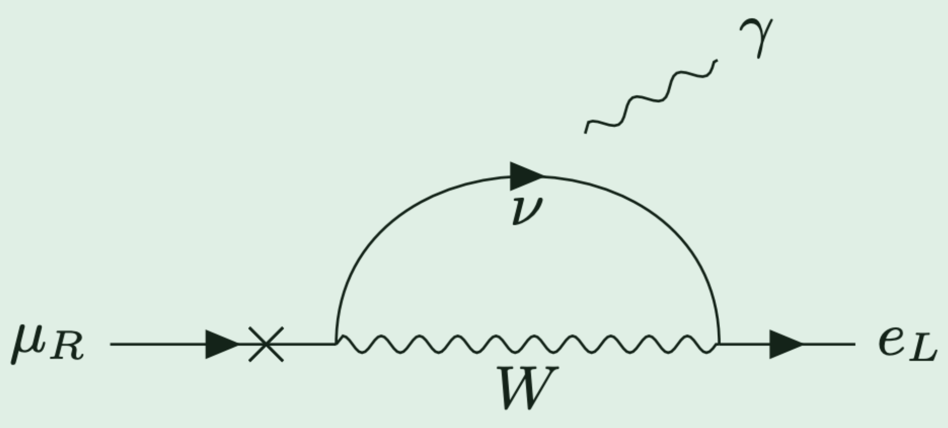


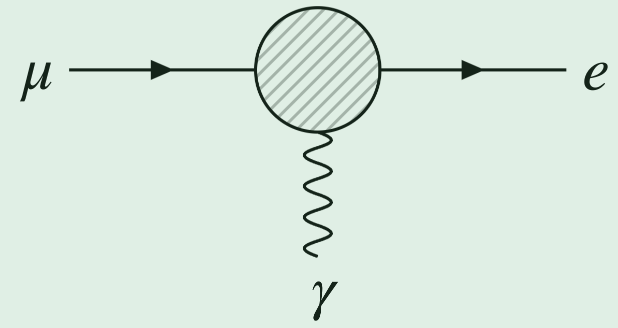






- Match onto the dipole

$$C^{\alpha\beta}_{D,R} \propto \frac{\alpha_e}{4\pi e} v^2 (Y_\nu M^{-2} Y_\nu^\dagger)_{\alpha\beta}$$



Inverse seesaw LFV

- So far, EFT coefficients proportional to two matrix element: $v^2(Y_\nu M^{-2} Y_\nu^\dagger)_{\alpha\beta}$, $v^2(Y_\nu M_a^{-2} Y_\nu^\dagger)_{\alpha\beta}(\log(m_W^2/M_a^2) + \text{const.})$

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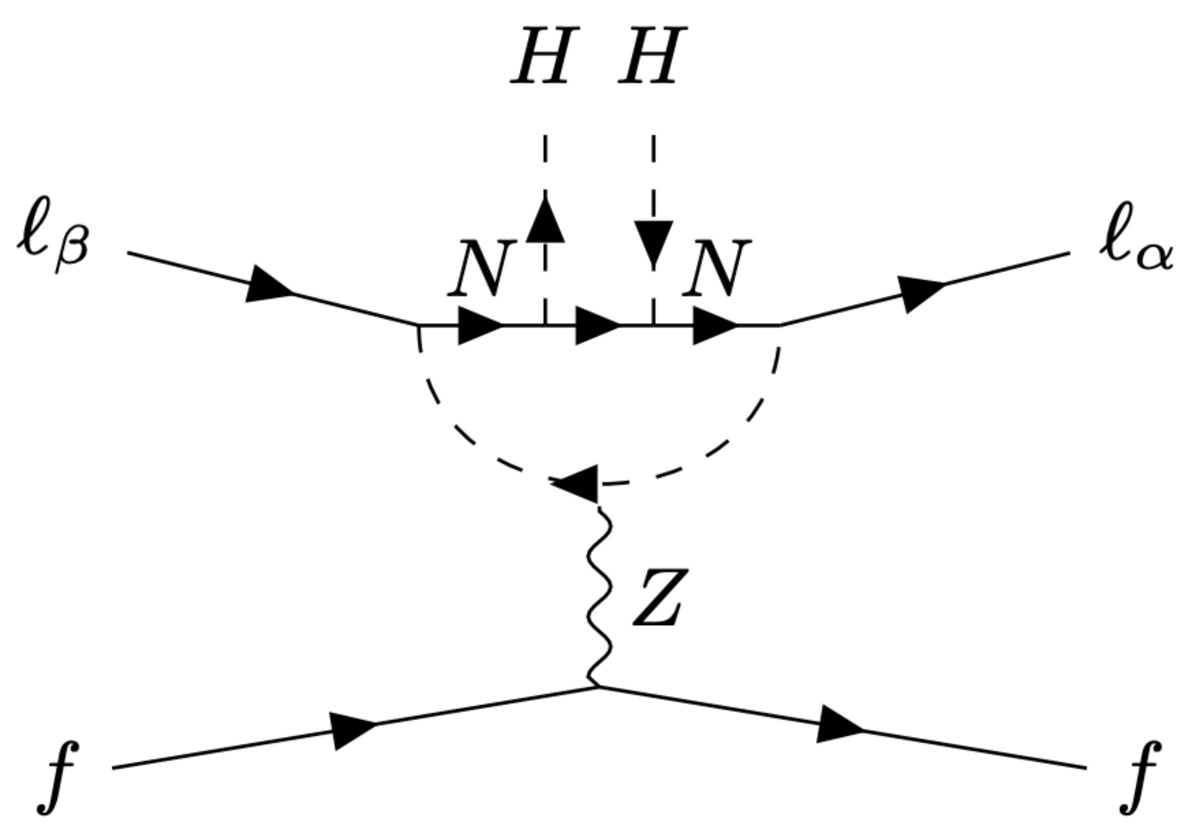
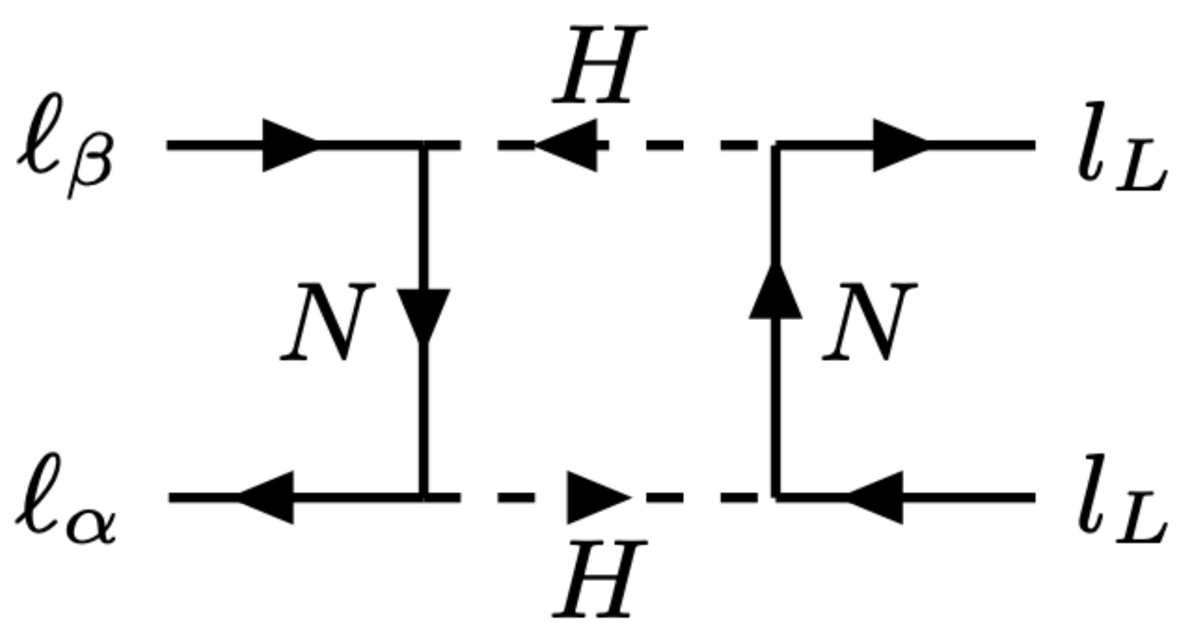
- But Yukawas can be large, and there are diagrams $\mathcal{O}(Y_\nu^4)$



$$\propto \left[Y_\nu \left(Y_\nu^\dagger Y_\nu \right)_{ab} \frac{1}{M_a^2 - M_b^2} \ln\left(\frac{M_a^2}{M_b^2}\right) Y_\nu^\dagger \right]_{\alpha\beta}$$



$$\propto Y_\nu^{\alpha a} Y_\nu^{*\beta a} Y_\nu^{lb} Y_\nu^{*lb} \frac{1}{M_a^2 - M_b^2} \ln\left(\frac{M_a^2}{M_b^2}\right)$$

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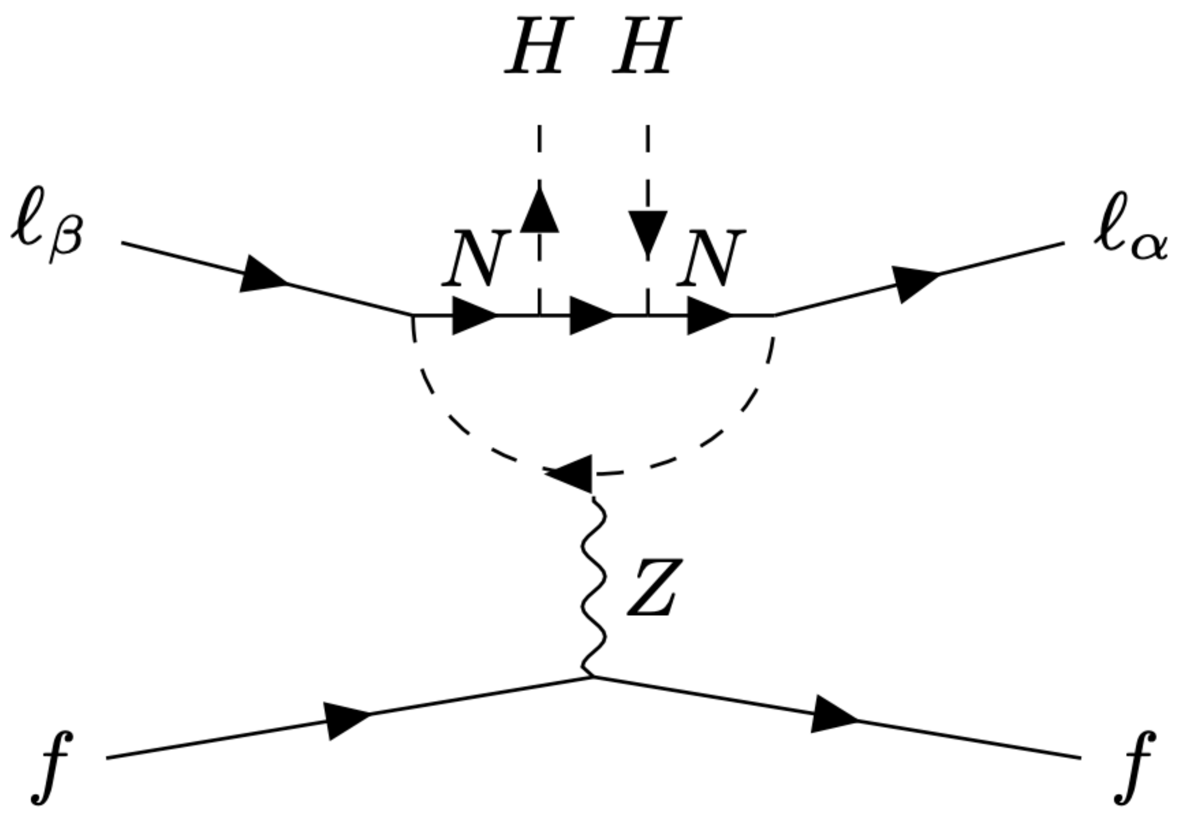

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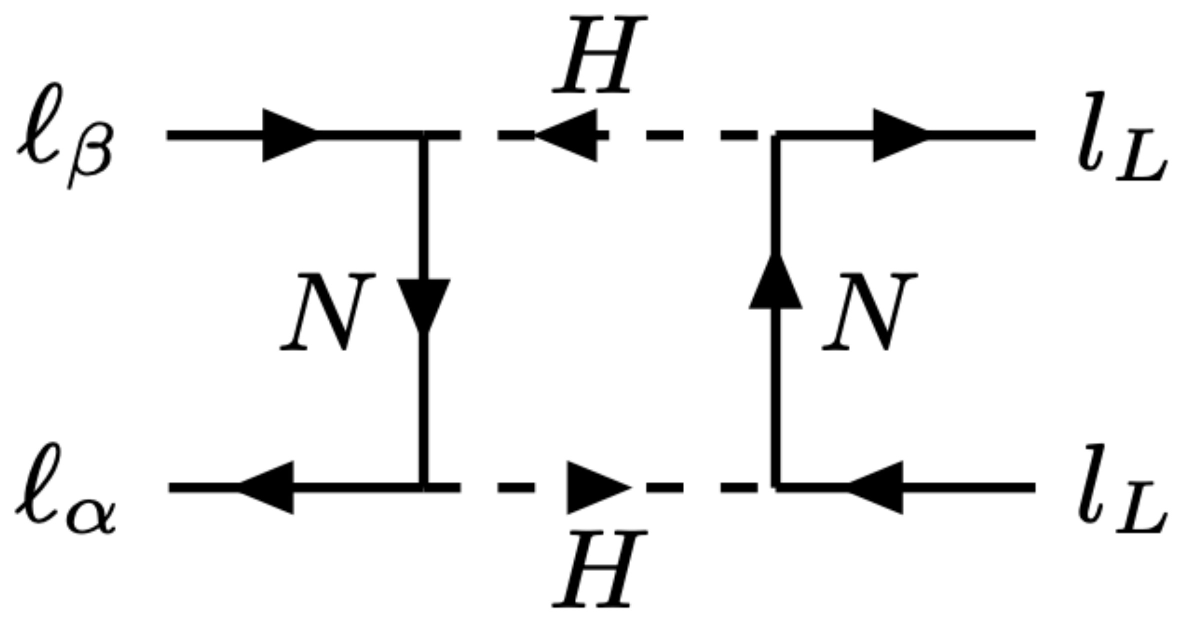

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- In general, **four "invariants" parametrize the size of the $\mu \to e$ coefficients**

Inverse seesaw: $\mu \to e$ ellipse

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$$C^{e\mu}_{D,R},\ C^{e\mu ee}_{V,LL},\ C^{e\mu ee}_{V,LR},\ C^{e\mu}_{Alight,L}$$

Parametrized in terms of

$$v^2(Y_\nu M^{-2} Y_\nu^\dagger)_{e\mu}$$

$$v^2(Y_\nu M_a^{-2} Y_\nu^\dagger)_{e\mu}(\ln(m_W^2/M_a^2) + \text{const.})$$

$$[Y_\nu \left(Y_\nu^\dagger Y_\nu\right)_{ab} \frac{1}{M_a^2 - M_b^2} \ln\left(\frac{M_a^2}{M_b^2}\right) Y_\nu^\dagger]_{e\mu}$$

$$Y_\nu^{ea} Y_\nu^{*\mu a} Y_\nu^{eb} Y_\nu^{*eb} \frac{1}{M_a^2 - M_b^2} \ln\left(\frac{M_a^2}{M_b^2}\right)$$

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$C_{D,R}^{e\mu}$, $C_{V,LL}^{e\mu ee}$, $C_{V,LR}^{e\mu ee}$, $C_{Alight,L}^{e\mu}$ **Parametrized in terms of**

$$[Y_\nu \left(Y_\nu^\dagger Y_\nu\right)_{ab} \frac{1}{M_a^2 - M_b^2} \ln\left(\frac{M_a^2}{M_b^2}\right) Y_\nu^\dagger]_{e\mu}$$

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- For generic sterile masses **the model can completely fill the experimentally allowed 4-D ellipse for the four coefficients**

- Possibly, **the four fermion combination contributing to $\mu \to e$ conversion on heavy target is predicted once the four are measured**

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- If sterile neutrinos have quasi-degenerate masses $M_a^2 - M_b^2 \sim \mathcal{O}(v^2)$, $\mu \to e$ **observables depend on only two/three coupling combinations**

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$(v^2/M^2)(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)_{e\mu}$

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$v^2(Y_\nu M^{-2} Y_\nu^\dagger)_{e\mu}(Y_\nu Y_\nu^\dagger)_{ee}$

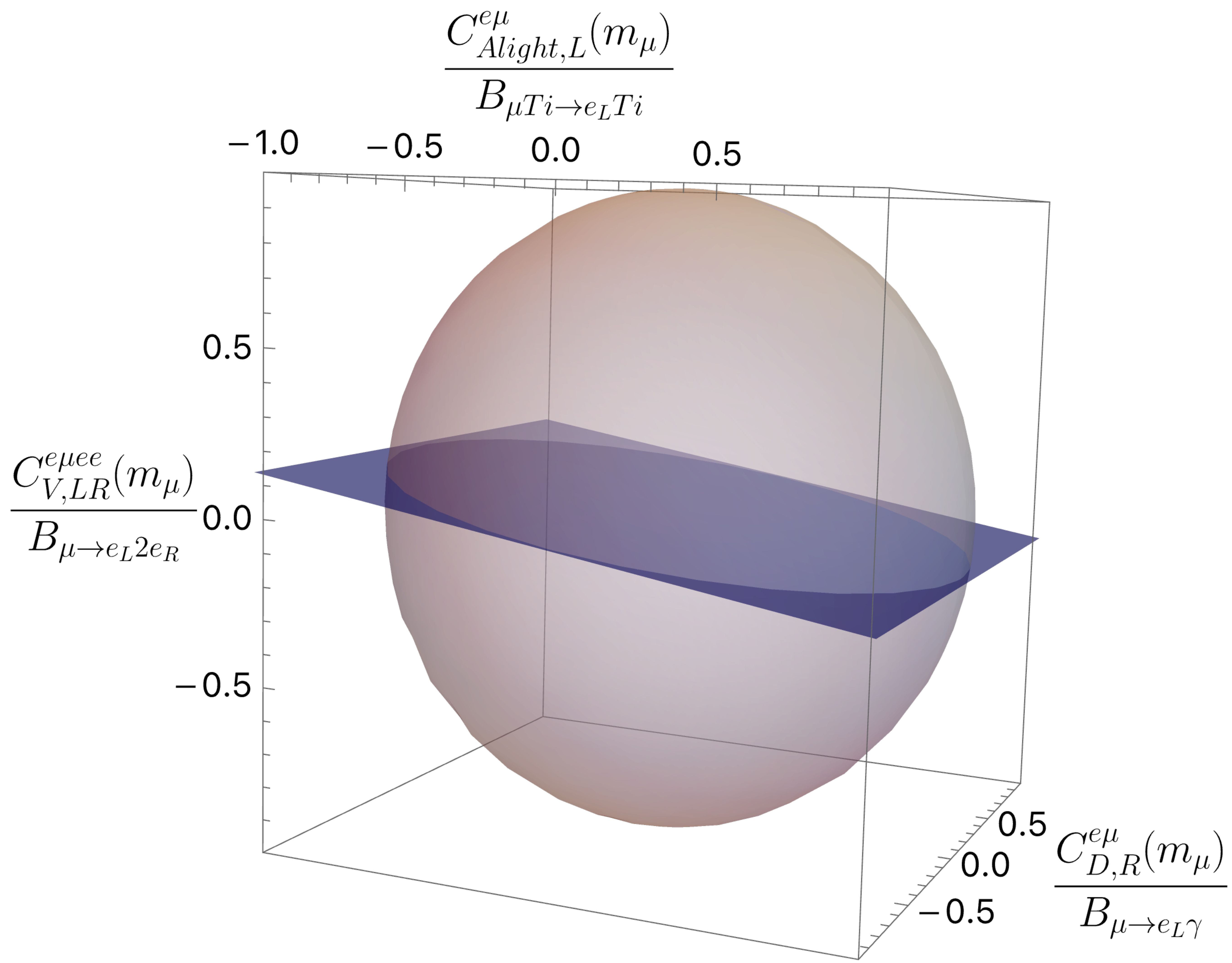
- One coefficient is known once two/three of them are measured

$$C_{V,LR}^{e\mu ee} = -2.4 C_{Alight,L}^{e\mu} + 0.02 C_{D,R}^{e\mu}$$

$$C_{V,LL}^{e\mu ee} = 2.4 C_{Alight,L}^{e\mu} + c_d C_{D,R}^{e\mu} \qquad -1.99 \lesssim c_d \lesssim -0.57$$

Inverse seesaw: $\mu \to e$ with quasi-degenerate sterile neutrinos

Any point outside the plane is not compatible with the inversee seesaw



Conclusions

- Lepton Flavour Violation is new physics that must exist because we see it in neutrino oscillations
- $\mu \to e$ observables are the most promising channels for a discovery thanks to the impressive experimental sensitivities of the upcoming searches
- By parametrising data in a bottom-up EFT, we have (in principle) more observables than just branching ratios, which correspond to 12 directions in the Wilson coefficient space
- Analysing what regions of this 12-d coefficient space models can reach we could have a way to distinguish/exclude models by combination of $\mu \to e$ observations/non-observations
- Is it possible for upcoming $\mu \to e$ to exclude popular TeV-scale models: type-II seesaw, inverse seesaw

Back-up Slides

Low-energy basis

$$\mathcal{O}^{ll}_{V,YY} = (\bar{e}\gamma^\alpha P_Y\mu)(\bar{l}\gamma_\alpha P_Y l), \quad \mathcal{O}^{ll}_{V,YX} = (\bar{e}\gamma^\alpha P_Y\mu)(\bar{l}\gamma_\alpha P_X l)$$

$$\mathcal{O}^{ll}_{S,YY} = (\bar{e}P_Y\mu)(\bar{l}P_Y l) \qquad \mathcal{O}^{\tau\tau}_{S,YX} = (\bar{e}P_Y\mu)(\bar{\tau}P_X\tau)$$

$$\mathcal{O}^{\tau\tau}_{T,YY} = (\bar{e}\sigma^{\alpha\beta}P_Y\mu)(\bar{\tau}\sigma_{\alpha\beta}P_Y\tau)$$

$$\mathcal{O}^{qq}_{V,YY} = (\bar{e}\gamma^\alpha P_Y\mu)(\bar{q}\gamma_\alpha P_Y q) \quad , \quad \mathcal{O}^{qq}_{V,YX} = (\bar{e}\gamma^\alpha P_Y\mu)(\bar{q}\gamma_\alpha P_X q)$$

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$$\mathcal{O}^{qq}_{T,YY} = (\bar{e}\sigma^{\alpha\beta}P_Y\mu)(\bar{q}\sigma_{\alpha\beta}P_Y q)$$

$$\mathcal{O}_{D,L} = m_\mu \overline{e_R}\sigma^{\alpha\beta}\mu_L F_{\alpha\beta} \qquad m_\mu \overline{e_L}\sigma^{\alpha\beta}\mu_R F_{\alpha\beta}$$

$$\mathcal{O}_{GG,Y} = \frac{1}{v}(\bar{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \quad , \quad \mathcal{O}_{G\tilde{G},Y} = \frac{1}{v}(\bar{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$$

$$\mathcal{O}_{GGV,Y} = \frac{1}{v^2}(\bar{e}\gamma_\sigma P_Y\mu)G_{\alpha\beta}\partial_\beta G^{\alpha\sigma} \quad , \quad \mathcal{O}_{G\tilde{G}V,Y} = \frac{1}{v^2}(\bar{e}\gamma_\sigma P_Y\mu)G_{\alpha\beta}\partial_\beta \widetilde{G}^{\alpha\sigma}$$

$$\mathcal{O}_{FF,Y} = \frac{1}{v}(\bar{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \quad , \quad \mathcal{O}_{F\tilde{F},Y} = \frac{1}{v}(\bar{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

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where $l \in \{e, \mu\}, q \in \{u, d, s, c, b\}$

SMEFT basis dimension six

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D^\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu}$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G^A_{\mu\nu}$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W^I_{\mu\nu}$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

$8: (\not{B}) + \text{h.c.}$	
Q_{duql}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(d_p^{\alpha} C u_r^\beta)(q_s^{j\gamma} C l_t^k)$
Q_{qque}	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(q_p^{j\alpha} C q_r^{k\beta})(u_s^\gamma C e_t)$
Q_{qqql}	$\epsilon_{\alpha\beta\gamma}\epsilon_{mn}\epsilon_{jk}(q_p^{m\alpha} C q_r^{j\beta})(q_s^{k\gamma} C l_t^n)$
Q_{duue}	$\epsilon_{\alpha\beta\gamma}(d_p^{\alpha} C u_r^\beta)(u_s^\gamma C e_t)$

$\mu \to e$ Rates

$$BR(\mu \to e\gamma) = 384\pi^2(|C^{e\mu}_{DL}|^2 + |C^{e\mu}_{DR}|^2)$$

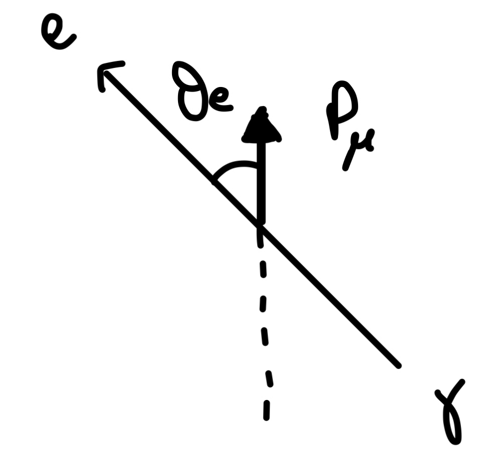
$$BR(\mu \to e\bar{e}e) = \frac{|C^{e\mu ee}_{S,LL}|^2 + |C^{e\mu ee}_{S,RR}|^2}{8} + 2|C^{e\mu ee}_{V,RR} + 4eC^{e\mu}_{D,L}|^2 + 2|C^{e\mu ee}_{V,LL} + 4eC^{e\mu}_{D,R}|^2$$
$$+(64\ln\frac{m_\mu}{m_e} - 136)(|eC^{e\mu}_{D,R}|^2 + |eC^{e\mu}_{D,L}|^2) + |C^{e\mu ee}_{V,RL} + 4eC^{e\mu}_{D,L}|^2 + |C^{e\mu ee}_{V,LR} + 4eC^{e\mu}_{D,R}|^2$$

$$BR_{SI}(\mu A \to eA) = B_A(|d_A C^{e\mu}_{DR} + C_{A,L}|^2 + |d_A C^{e\mu}_{DL} + C_{A,R}|^2)$$

Polarising the muon to distinguish operators

Kuno, Okada hep-ph/9909265

$$\frac{dB\left(\mu \to e\gamma\right)}{d\left(\cos\theta_e\right)} = 192\pi^2 \left(\frac{v}{\Lambda}\right)^4 \left[\left|C_{D,R}\right|^2 \left(1 - P_\mu \cos\theta_e\right) + \left|C_{D,L}\right|^2 \left(1 + P_\mu \cos\theta_e\right)\right]$$



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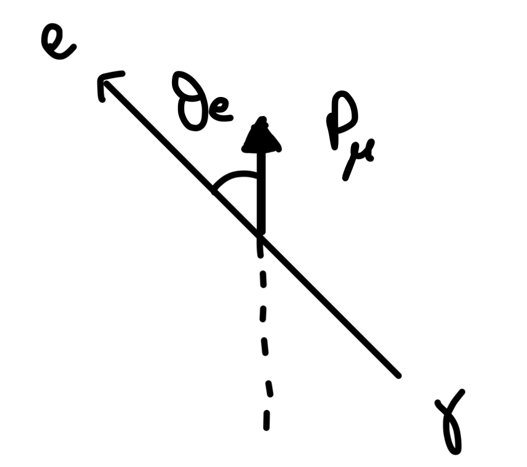
Muon polarization vector

Angle between e momentum and $\overrightarrow{P}_\mu$



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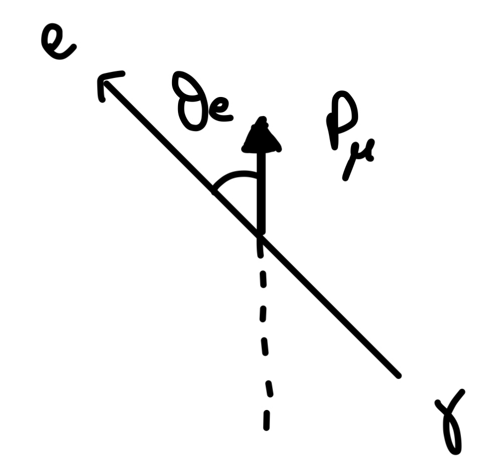
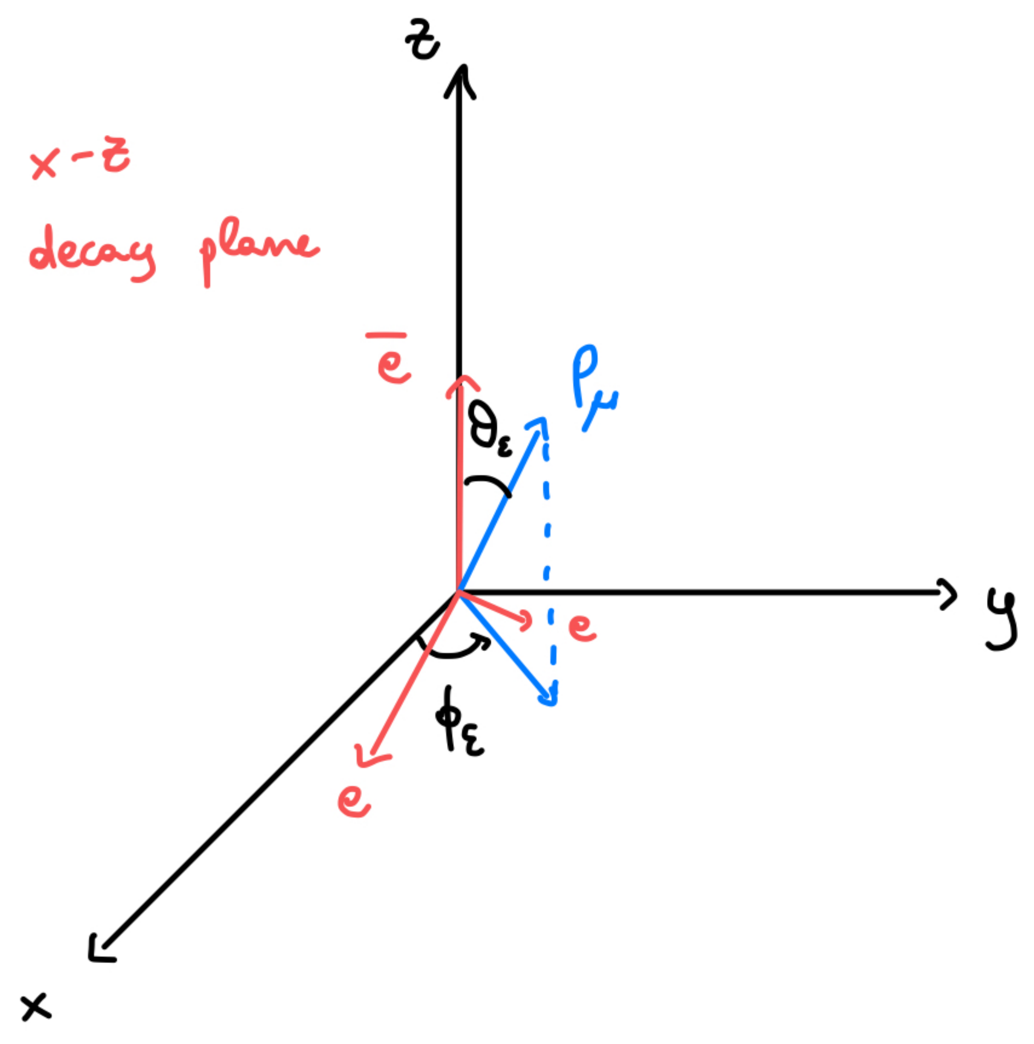


Petcov, Bolton 2204.03468

$$\frac{dB_{\mu\to 3e}}{dx_1 dx_2 d\Omega_\varepsilon} = \frac{3}{2\pi}\left[C_1\left(x_1, x_2\right) + C_2\left(x_1, x_2\right) P_\mu \cos\theta_\varepsilon + C_3\left(x_1, x_2\right) P_\mu \sin\theta_\varepsilon \cos\phi_\varepsilon + C_4\left(x_1, x_2\right) P_\mu \sin\theta_\varepsilon \sin\phi_\varepsilon\right]$$



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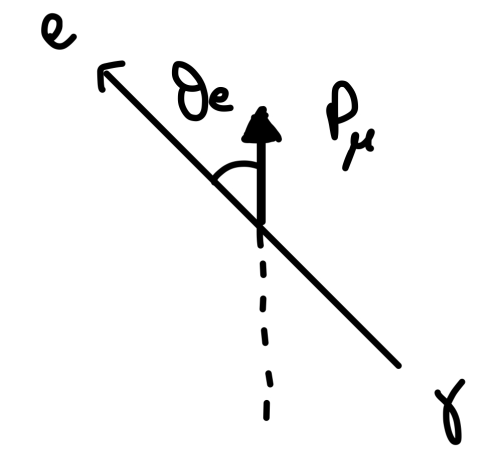

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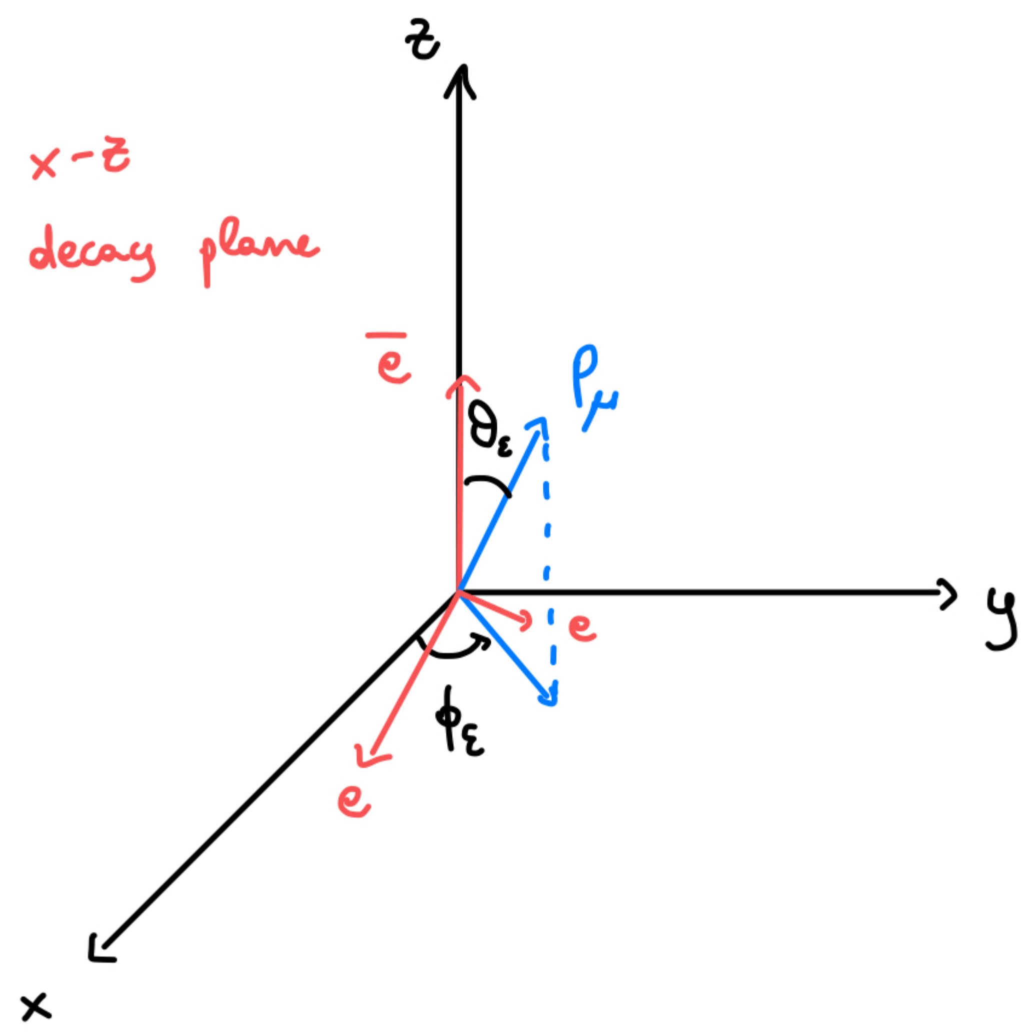
Can distinguish $C_{V,LX}, C_{V,LX}, C_{S,R}$ from $C_{V,RX}, C_{V,RX}, C_{S,L}$ but not scalars from vectors

CP asymmetries are also measureable (phase between dipoles and vectors)



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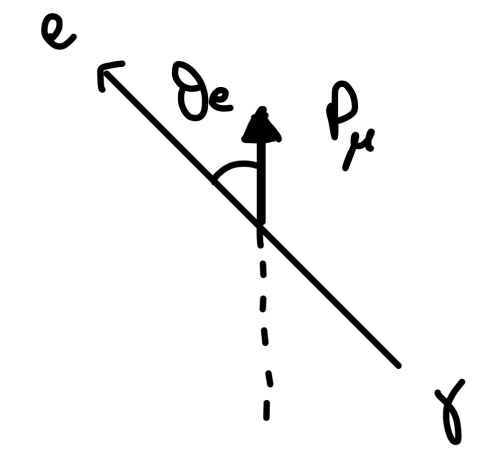

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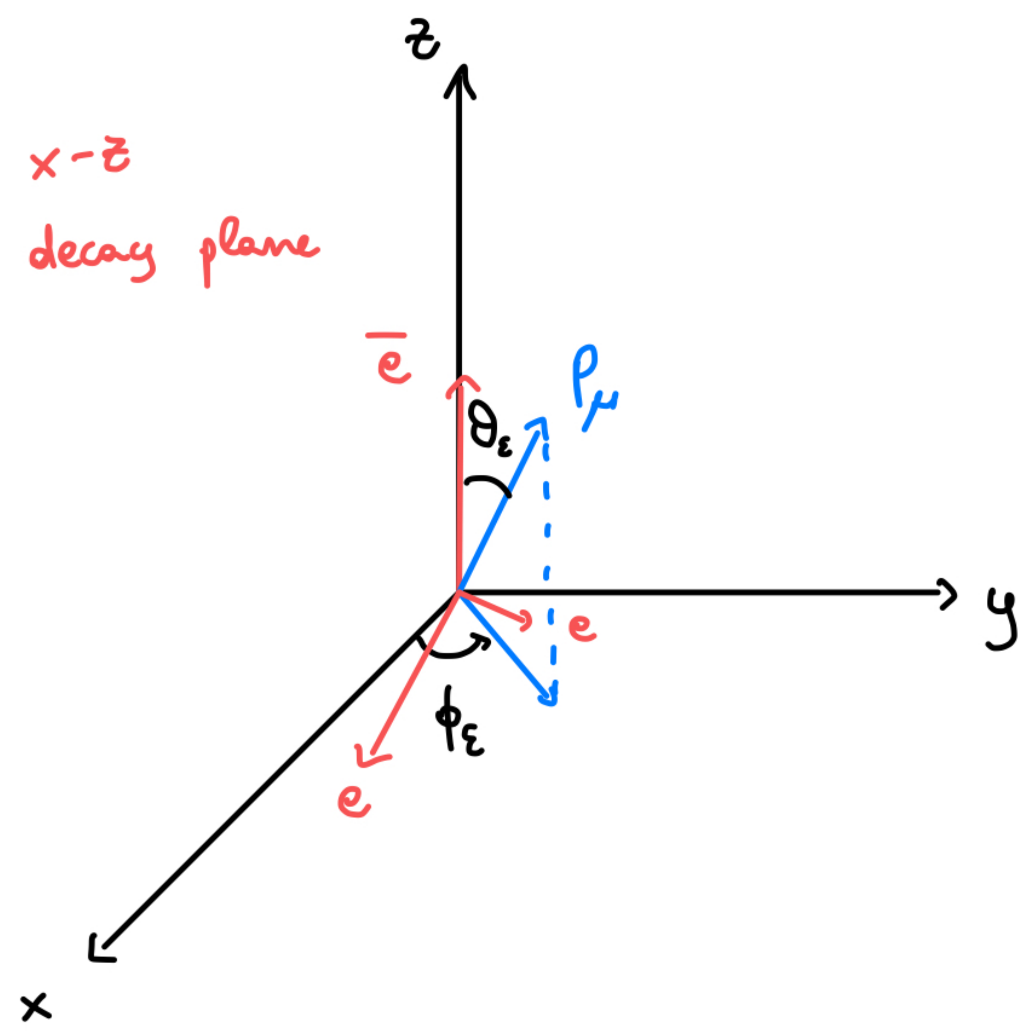
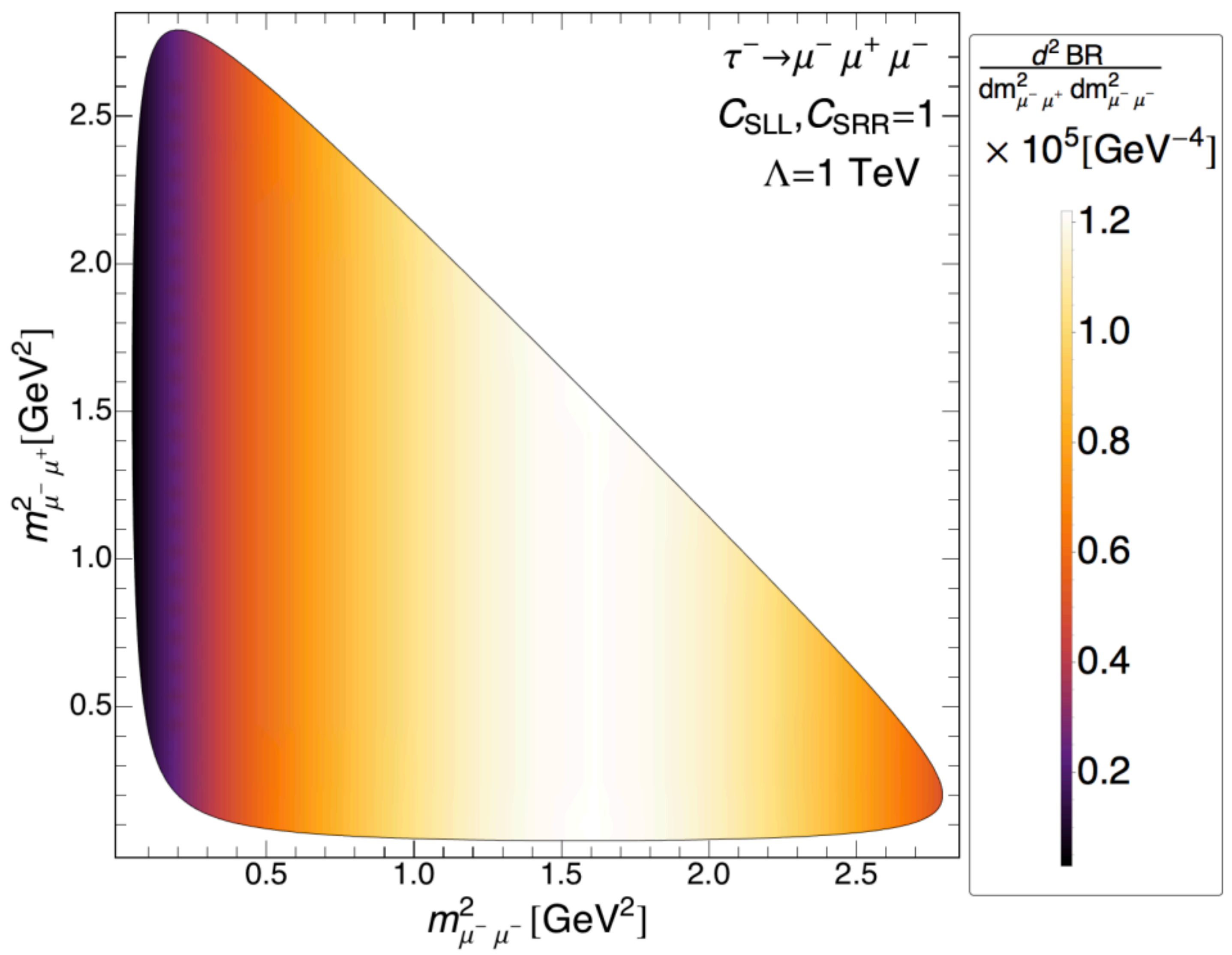


- Dalitz plots for the three body also possible to distinguish operators (vector vs scalar)

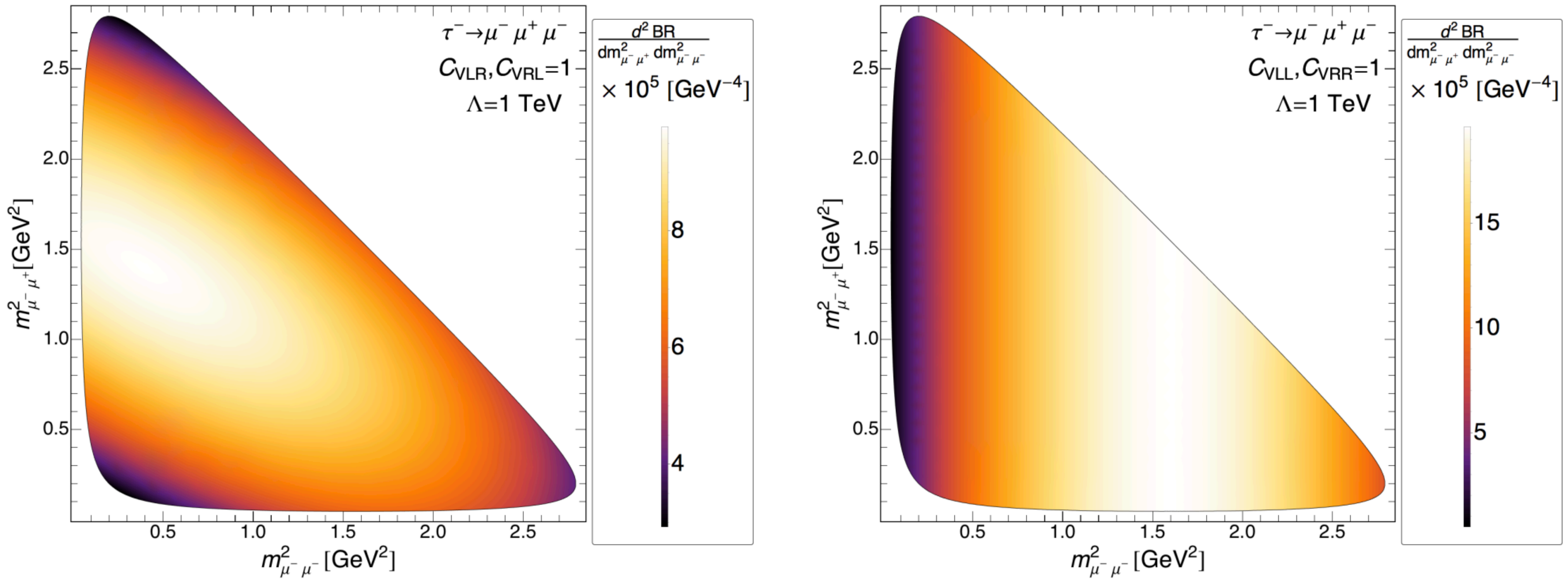
Three body decay: Dalitz plots

Celis, Passemar, Cirigliano 1403.5781

- Dalitz plots could also assist in distinguishing operators



Scalars



Vectors

Type-II coefficients

- We list here the EFT coefficients in the type-II seesaw

$$C^{e\mu}_{DR} = \frac{3e}{128\pi^2}\Big[\frac{[m_\nu m_\nu^\dagger]_{e\mu}}{\lambda_H^2 v^2}\Big(1+\frac{32}{27}\frac{\alpha_e}{4\pi}\ln\frac{M_\Delta}{m_\tau}\Big)+\frac{116\alpha_e}{27\pi}\ln\frac{m_\tau}{m_\mu}\sum_{\alpha\in e\mu}\frac{[m_\nu]_{\mu\alpha}[m_\nu^*]_{e\alpha}}{\lambda_H^2 v^2}\Big] \qquad v = 174\ \mathrm{GeV}$$

$$C^{e\mu ee}_{V,LL} = \frac{[m_\nu^*]_{\mu e}[m_\nu]_{ee}}{2\lambda_H^2 v^2} + \frac{\alpha_e}{3\pi\lambda_H^2 v^2}\left[m_\nu^\dagger \ln\left(\frac{M_\Delta}{m_\alpha}\right) m_\nu\right]_{\mu e}$$

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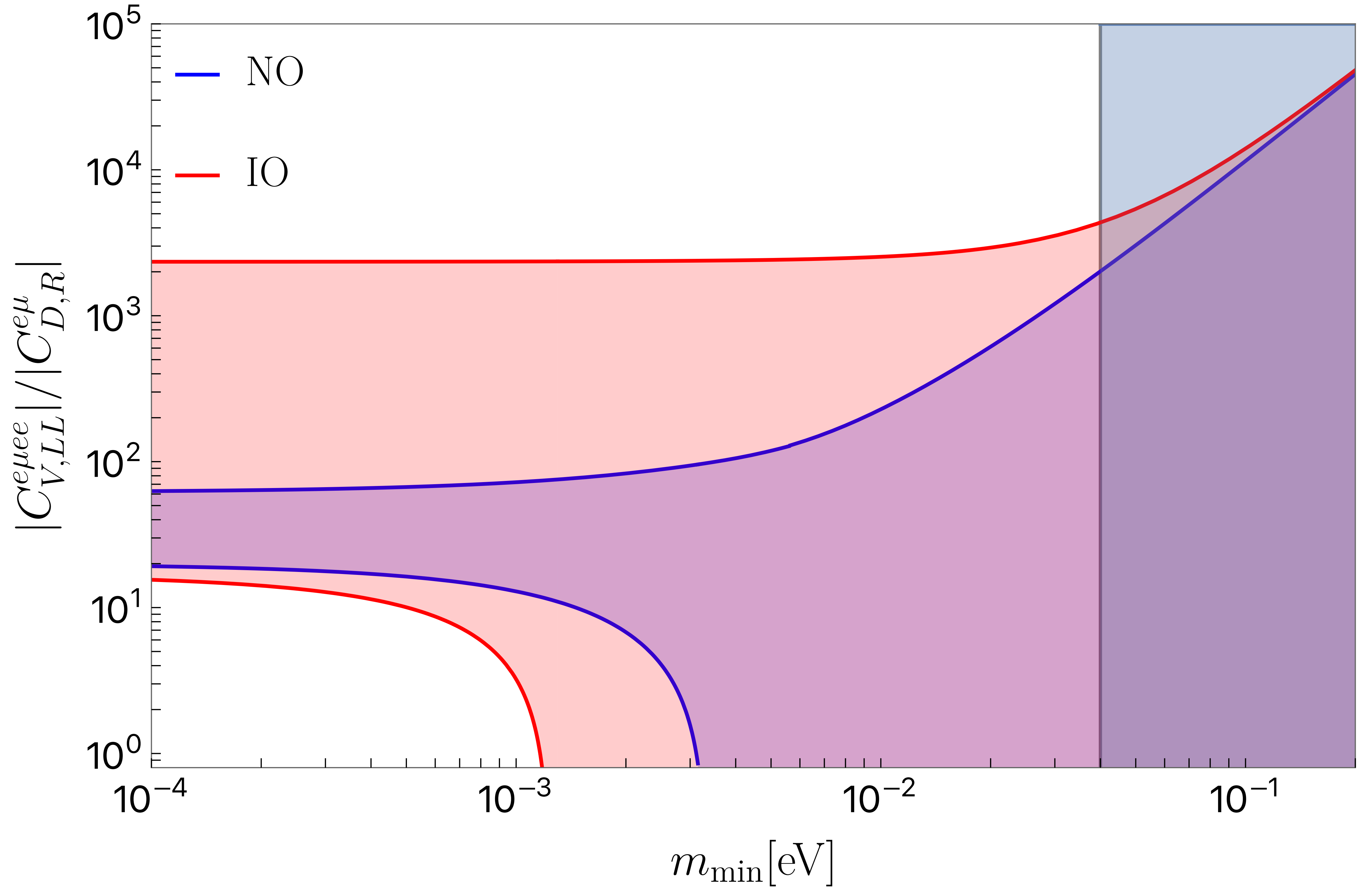
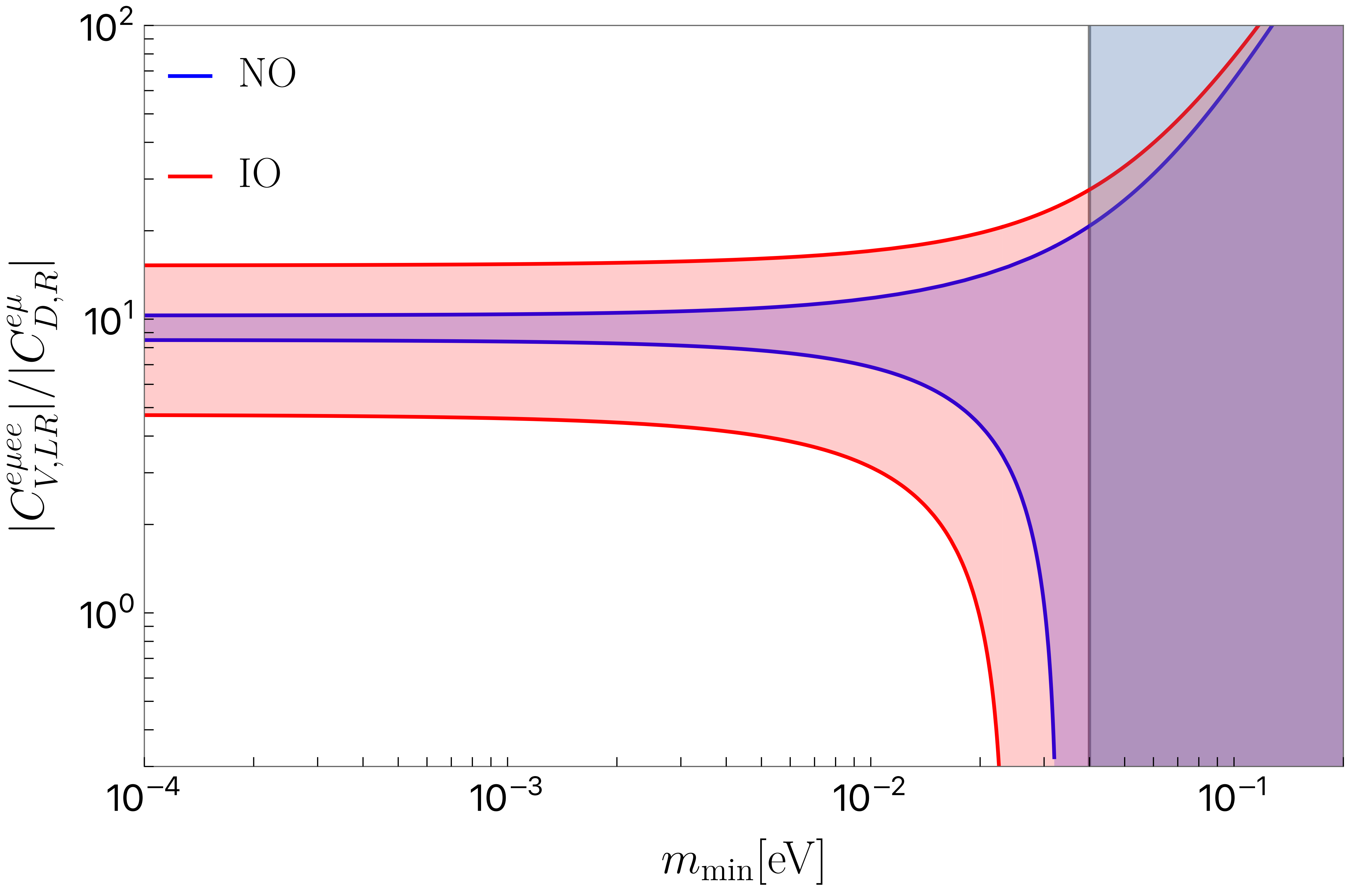
Type-II seesaw and neutrino mass scale

- Wilson coefficients are function of the neutrino mass scale, hence one can constrain their size knowing $m_{\min}$





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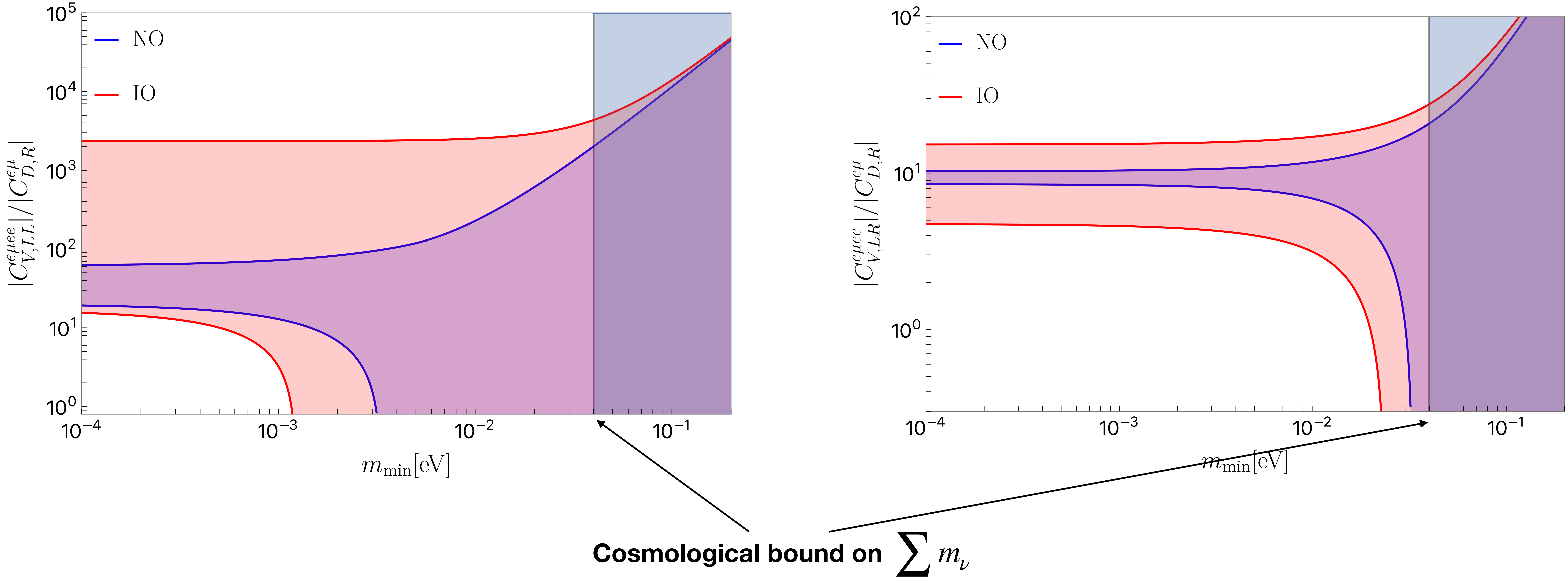

Inverse seesaw

- We list here the EFT coefficients in the type-I seesaw

$$C^{e\mu ee}_{V,LR} \simeq v^2 \frac{\alpha_e}{4\pi}\left(1.5[Y_\nu M_a^{-2}\left(\frac{11}{6}+\ln\left(\frac{m_W^2}{M_a^2}\right)\right)Y_\nu^\dagger]_{e\mu} - 2.7[Y_\nu (Y_\nu^\dagger Y_\nu)_{ab}\frac{1}{M_a^2-M_b^2}\ln\left(\frac{M_a^2}{M_b^2}\right)Y_\nu^\dagger]_{e\mu} + \mathcal{O}\left(\frac{\alpha_e}{4\pi}\right)\right)$$

$$C^{e\mu}_{Alight,L} \simeq v^2 \frac{\alpha_e}{4\pi}\left(-0.6[Y_\nu M_a^{-2}\left(\frac{11}{6}+\ln\left(\frac{m_W^2}{M_a^2}\right)\right)Y_\nu^\dagger]_{e\mu} + 1.1[Y_\nu (Y_\nu^\dagger Y_\nu)_{ab}\frac{1}{M_a^2-M_b^2}\ln\left(\frac{M_a^2}{M_b^2}\right)Y_\nu^\dagger]_{e\mu} + \mathcal{O}\left(\frac{\alpha_e}{4\pi}\right)\right)$$

$$C^{e\mu ee}_{V,LL} \simeq v^2 \frac{\alpha_e}{4\pi}\left(-1.8[Y_\nu M_a^{-2}\left(\frac{11}{6}+\ln\left(\frac{m_W^2}{M_a^2}\right)\right)Y_\nu^\dagger]_{e\mu} + 2.7[Y_\nu (Y_\nu^\dagger Y_\nu)_{ab}\frac{1}{M_a^2-M_b^2}\ln\left(\frac{M_a^2}{M_b^2}\right)Y_\nu^\dagger]_{e\mu}\right.$$

$$\left. +2.5 Y_\nu^{ea} Y_\nu^{*\mu a} Y_\nu^{eb} Y_\nu^{*eb}\frac{1}{M_a^2-M_b^2}\ln\left(\frac{M_a^2}{M_b^2}\right) + \mathcal{O}\left(\frac{\alpha_e}{4\pi}\right)\right)$$

$$C^{e\mu}_{D,R} \simeq -\frac{v^2}{2}\left(\frac{\alpha_e}{4\pi e}\right)[Y_\nu M^{-2} Y_\nu^\dagger]_{e\mu}$$

Scalar Singlet Leptoquark

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- Add to the SM a scalar SU(2) singlet leptoquark S with a mass $m_{LQ} \sim$ TeV (can fit the $R_{D^{(*)}}$ anomaly)

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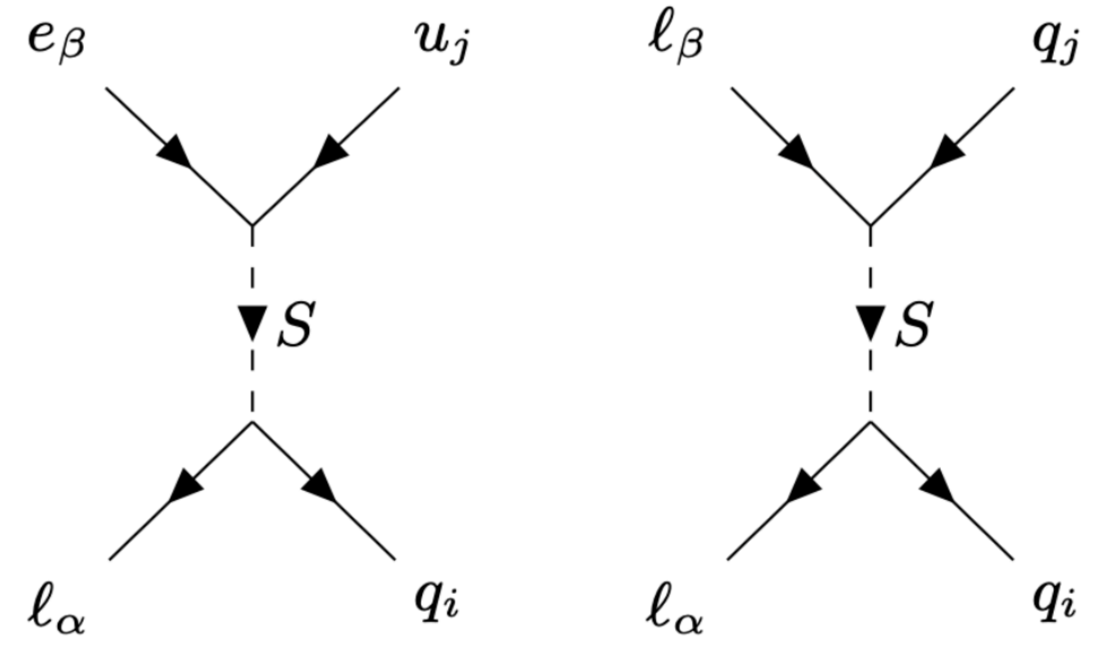

Scalar, tensors and vectors + RGEs mixing

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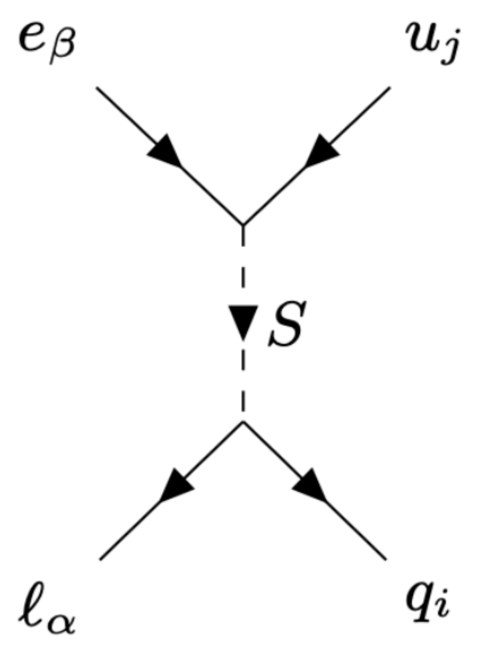

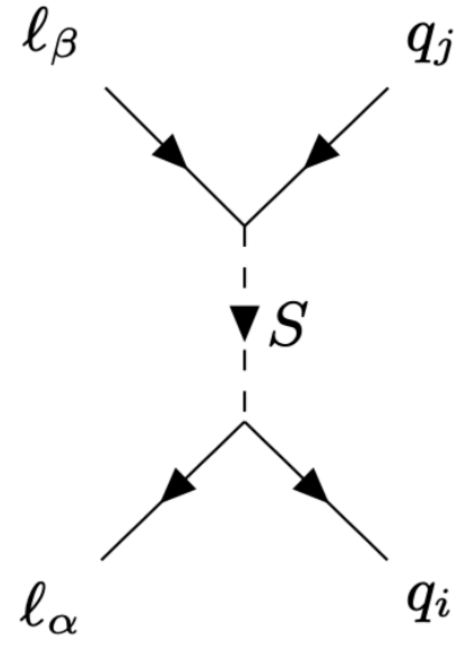

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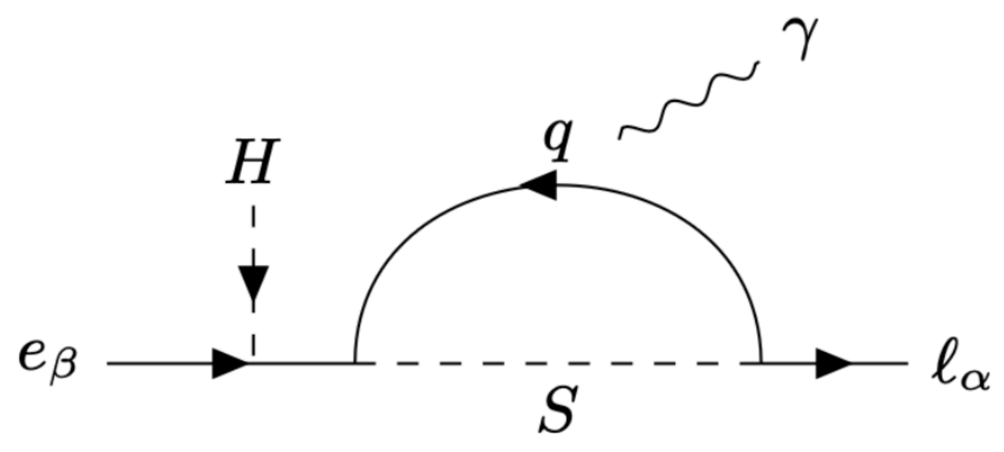

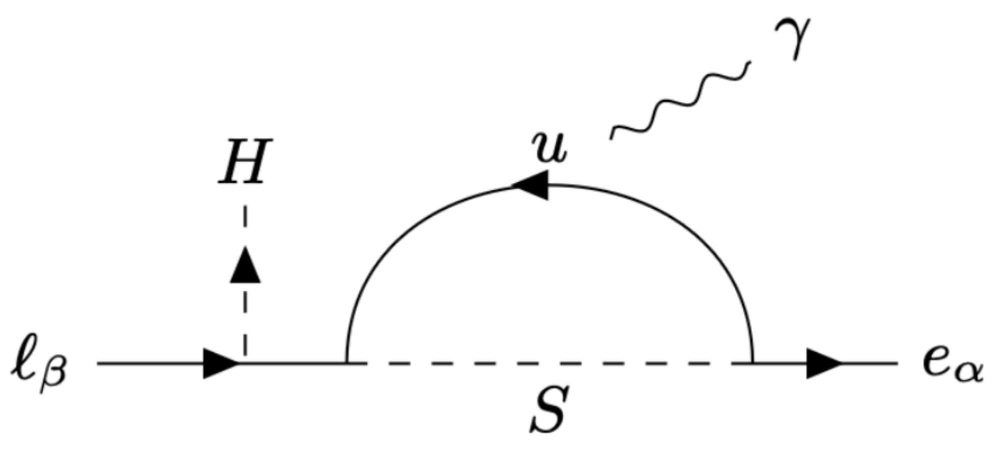

Dipole with left-handed and right-handed outgoing leptons

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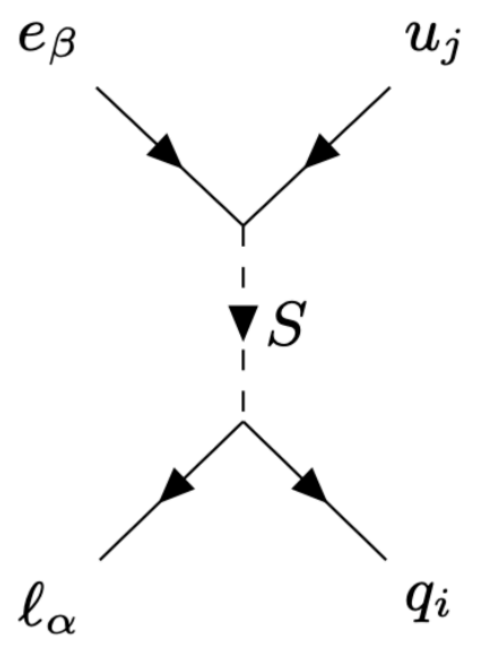

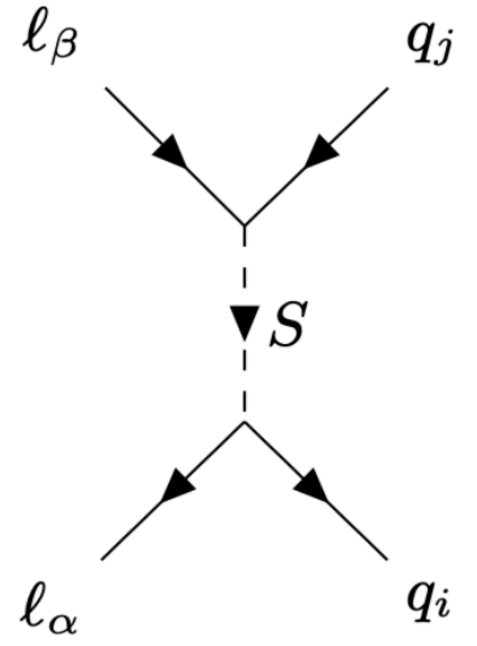

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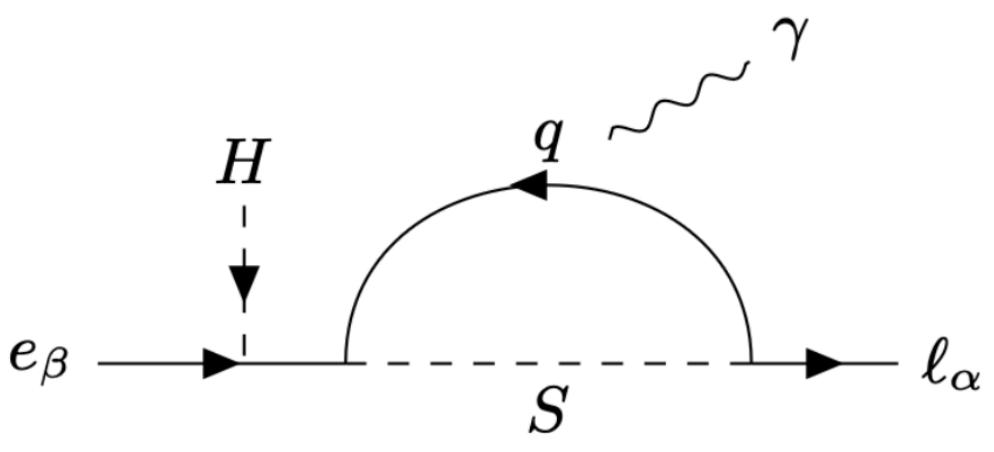

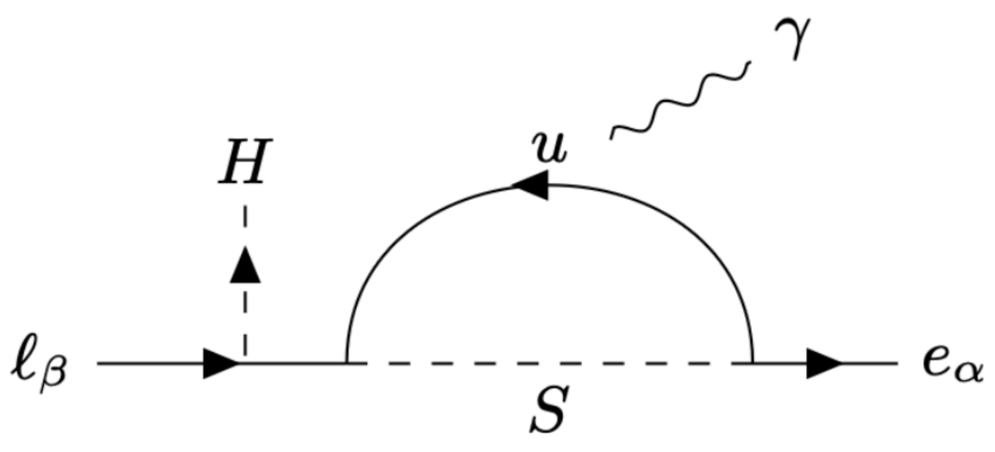

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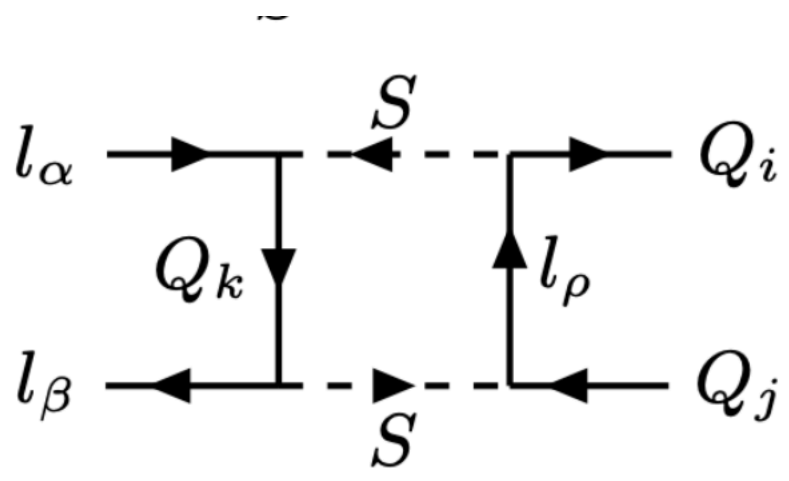

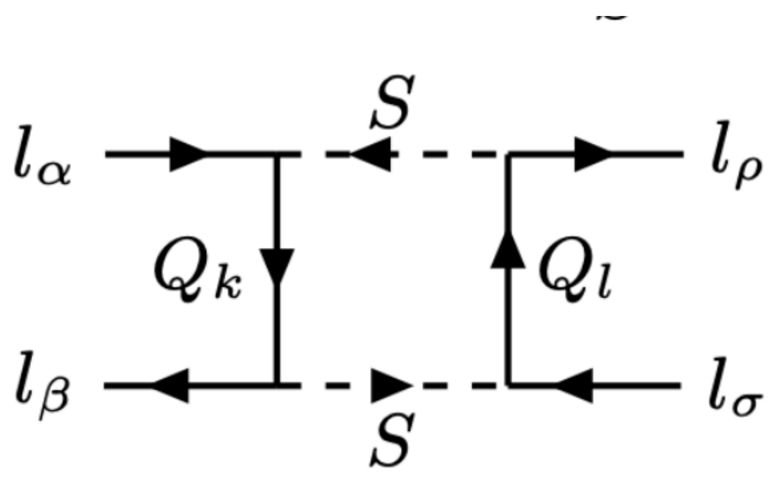

$\mathcal{O}(\lambda^4)$ matching contribution to scalars and vectors

Scalar Singlet Leptoquark: $\mu \to e$

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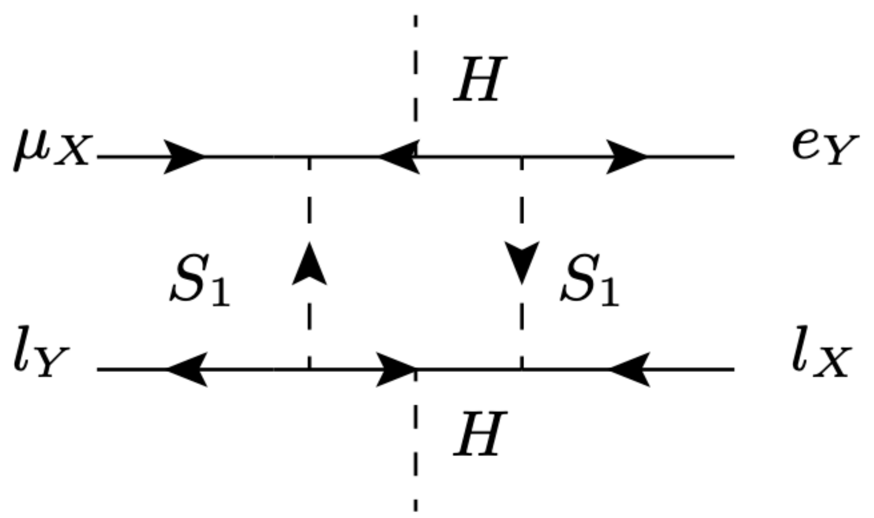

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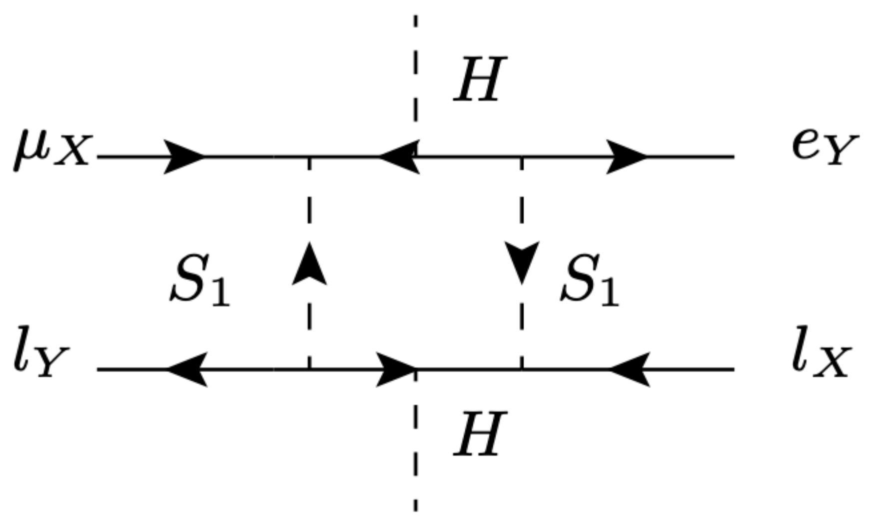

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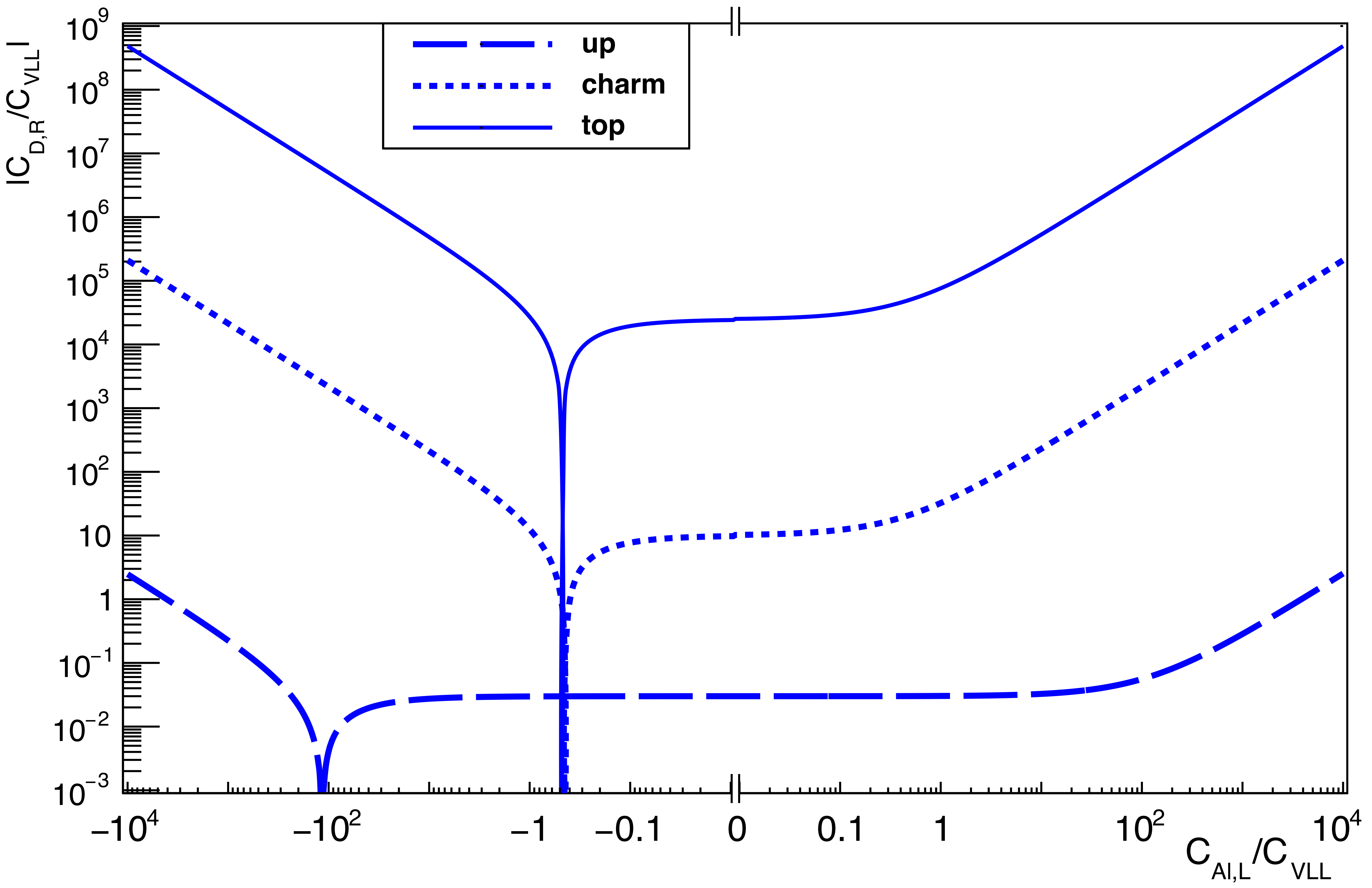
- The leptoquark could in principle fill completely the remaining 10-dimension of the $\mu \to e$ observables (too many parameters…)

Leptoquark: $\mu \to e$ with one-generation-at-a-time



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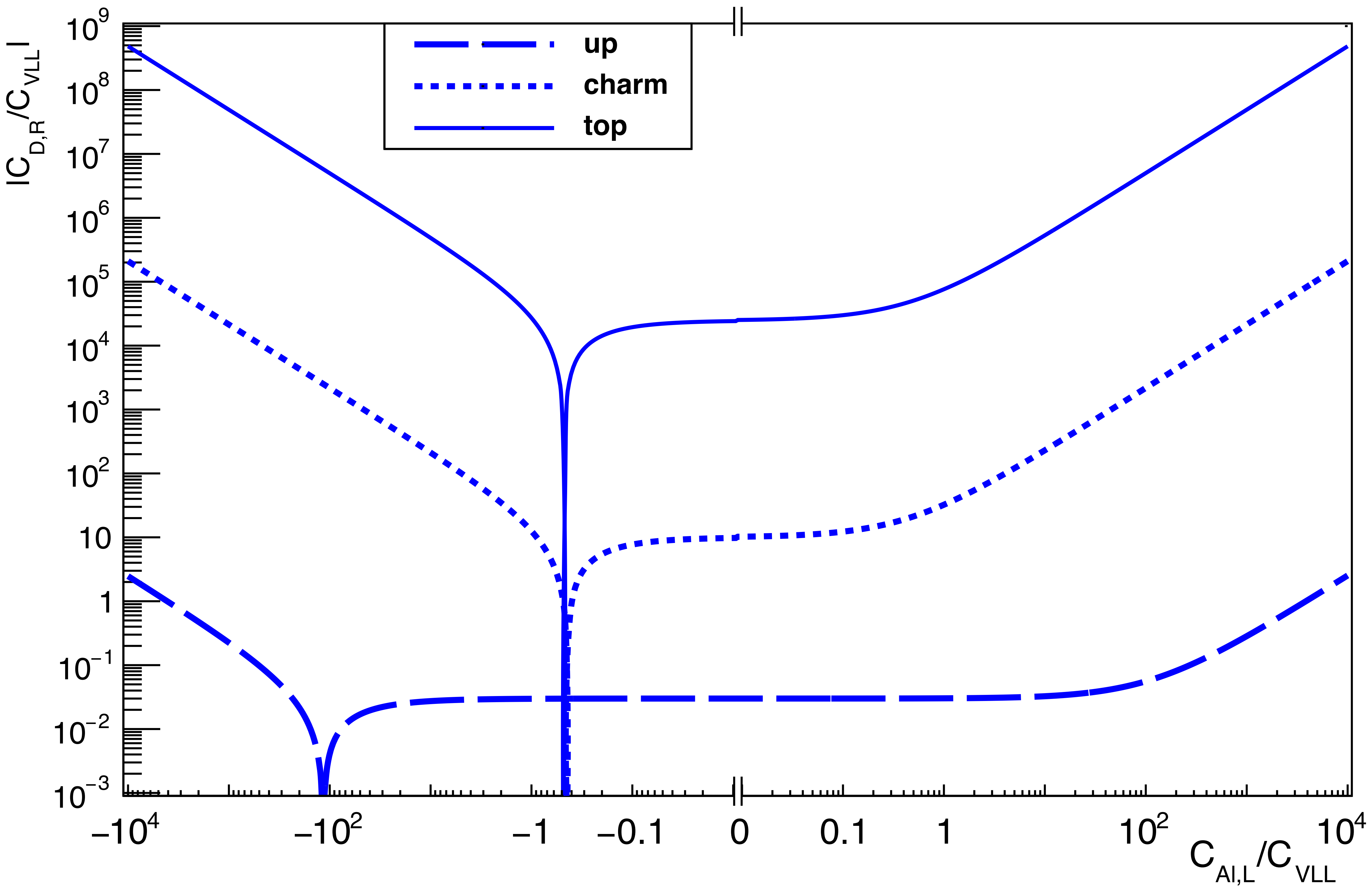

- Assuming the leptoquark to couple only with one generation at a time reduces the number of “invariants”
- But cancellations between coefficients are in principle possible, so combination of observations could exclude “generic” parameter points, but not the model