Extra-dimensional models on the lattice and the Layer Phase

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OUTLINE

- Introductory hints on the extra dimensional theories
- Models with extra dimension on the lattice; inclusion of anisotropic couplings in the action
- Pure U(1) model in 5D and the Layer phase
- 5D anisotropic Abelian-Higgs model:
  The Higgs Layer phase
- 5D SU(2)-Higgs model in the adjoint representation
- Remarks & Conclusions

Extra dimensions

- As far as it is known, there is not any **fundamental principle** for which the space-time should be 3+1 dimensional… Hence, it might be “legitimate” to assume *extra* spatial dimensions.

- It can not be denied that, up to now, the theoretical exploration of extra dimensional models is rather *speculative* than based on well established facts.

- Nevertheless, the construction of the extra dimensional models is guided by the requirement of physical consistency in order to show possible ways on how the extra dimensions can be revealed (or why are *hidden*) and to connect the extra-dimensional models with a more **fundamental** theory in order to solve long-standing theoretical problems.

Kaluza-Klein proposal (1920, 1926):

- Consider a five-dimensional flat space with the fifth dimension compactified
- The photon resides in the extra dimension
- The unification of gravity with electromagnetism can be achieved

Extra dimensions *à la* Kaluza-Klein are hidden as a result of their size :

$l \sim 10^{-33}$ cm.

Need for energies as high as the Planck energy to make them detectable.

Large Extra dimensions

The idea of models with Large Extra dimensions is connected with the

Brane world picture

The ordinary matter (with exception of gravitons) is localised to a 3-D submanifold (**brane**) embedded in a fundamental multi-dimensional space (**bulk**)

It is assumed that the brane picture is an *effective* model representation. It comes from a high-energy theory which appears at a fundamental scale M_*

The large extra dimensions should be so large as to non contradict the observations

↓

For compactified extra dimensions their size can be as big as a fraction of millimeter

Large Extra dimensions (continue)

One of the main **motivations** for the Brane picture used in higher dimensional models is to give an answer to the :

Hierarchy Problem

***Why* … $M_{Pl} >> M_{EW}$?**

Possible answer: The fundamental scale is **$M_* \sim$ 1TeV** in a theory defined in a 4+n space

ADD model
(Arkani-Hamed, Dimopoulos & Dvali Phys.Lett.B 1998)

RS models
(Randall & Sundrum Phys.Rev.Lett. 1998)

ADD model

♦ Consider a ***flat*** brane (no brane tension) embedded in a 4+n space-time
- Standard matter resides on the brane
- Gravity spreads in the bulk

♦ The extra dimensions are compactified

♦ The metric does *not* depend on the extra coordinates

♦ Define a unique fundamental scale $\mathbf{M_*}$

⇩

Volumetric scaling: $$\mathbf{M_{Pl}^2 = M_*^{n+2}\, V_\delta}$$

… It is sufficient to assume $n \geq 2$ and R~0.1 mm for getting $M_* \geq 1$ TeV

RS model

- Consider a five-dimensional anti de Sitter space
  - the extra dimension obeys to
    $z = z + 2\pi$ & $z \rightarrow -z$ (S^1/Z_2 orbifolded)
  - negative cosmological constant in the bulk

- Assume the existence of two branes resided along the extra dimension and having opposite value of tension.
  - Matter is assumed on the branes while the gravity can be spread in the bulk

It is shown that the metric admitted for this set-up takes the form:

$$ds^2 = \omega^2(z)\,\eta_{\mu\nu}dx^\mu dx^\nu - dz^2 \qquad (\text{with } \omega^2(z) = e^{-kz})$$

The extra dimension becomes **warped** as a price of maintaining a 4D Lorentz space at every point of the extra dimension (non-factorizable geometry).

The model provides an ***exponential hierarchy*** expressed by the distance which separates the two branes along the extra coordinate:

$$\mathbf{M_{Pl} \sim e^{kz_c}\ M_*}$$

(Non) Renormalisability

The couplings defined in extra dimensional theories are of negative mass dimension …

↓

The ***current attitude*** is to make the …conjecture to

- treat these theories as effective theories for energy scales up to

$$E \sim 1/g^{1/n}$$

(n is the mass dimension of the coupling)

- For bigger energies the effective theory has to be replaced by a more fundamental theory.

(4+1)-dimensional pure U(1) gauge model

The gauge field in extra dimensional space

- What about a possible extension of the gauge field in the bulk?
- Is there any possibility of gauge localisation on the brane in the RS set-up?

The answer given in terms of analytical work is *negative* …

(**Pomarol, Phys.Lett.B2000**
Davoudias, Hewett & Rizzo, Phys.Lett.B2000)

But …

since the localisation of the gauge field on the brane may involve *strong coupling dynamics* along the extra dimensions, the non-perturbative methods are necessary for giving an answer.

Abelian Gauge field in the RS background

- Assume the RS metric : $ds^2 = e^{-kz}\left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2\right) - dz^2$

- Consider a 5-dim **abelian gauge model** in the background of the RS metric:

$$S_{gauge} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\, F_{MN}F_{KQ}\, g^{MK}g^{NQ}$$

$$= \int d^4x dz\left(-\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda}\eta^{\mu\kappa}\eta^{\nu\lambda} - \frac{1}{2g_5^2}e^{-kz}\, F_{\mu5}F_{\nu5}\eta^{\mu\nu}\right)$$

- In the Euclidean space the gauge action reads:

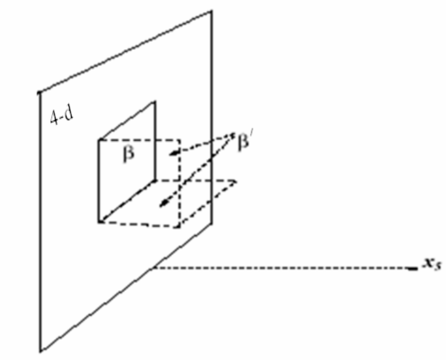
$$S^E_{gauge} = \int d^5x\left(\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda} + \frac{1}{2g_5^2}e^{-k|x_5|}\, F_{\mu5}F_{\mu5}\right) \quad \mu,\nu = 1,\ldots,4$$

⇨ We get a model with *two different couplings*, one defined on the 4D subspace and the second (***bigger***) along the extra dimension.

P.D., K. Farakos, A. Kehagias and G. Koutsoumbas
NPB 2001

On the lattice we define the pure U(1) Wilson gauge action with **anisotropic** couplings:

$$S_{gauge} = \beta \sum_{x,1\le\mu<\nu\le 4}\left(1 - Re\,U_{\mu\nu}(x)\right) + \beta' \sum_{x,1\le\mu\le 4}\left(1 - Re\,U_{\mu 5}(x)\right)$$



Link variables: $U_M = \left\{ U_\mu = e^{i a_s \bar{A}_\mu},\ U_5 = e^{i a_5 \bar{A}_5} \right\}$

Plaquettes:

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x + a_s\hat{\mu})U_\mu(x + a_s\hat{\nu})U_\nu(x)$$

$$U_{\mu 5}(x) = U_\mu(x)U_5(x + a_s\hat{\mu})U_\mu(x + a_5\hat{5})U_5(x)$$

The **Fu-Nielsen** proposal for ***dimensional reduction*** from lattice anisotropic models (couplings $\beta \neq \beta'$)

(**Fu and Nielsen NPB 1984, 1985**)

Phase diagram becomes richer: A new phase appears …

LAYER PHASE (I)

- For $\beta \neq 0$ and $\beta' = 0$ $\Longrightarrow$ phase diagram of the 4D isotropic model
  - Confining phase
  - 4D Coulomb phase

- For $\beta = \beta'$ $\Longrightarrow$ phase diagram of the 5D isotropic model
  - Confining phase
  - 5D Coulomb phase

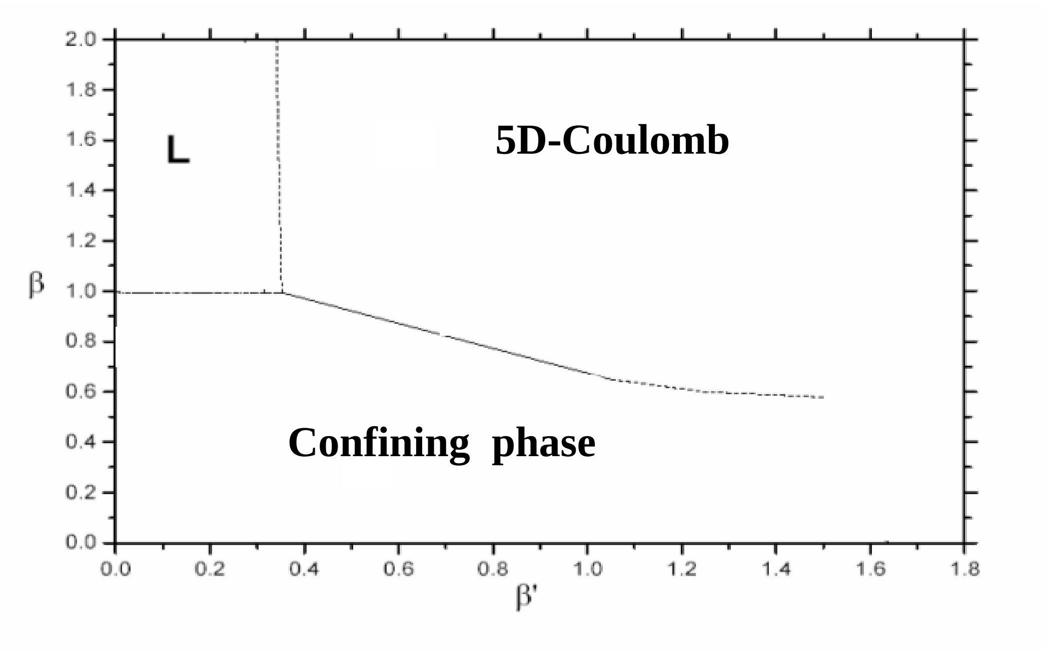
- For $\beta \neq \beta'$ $\Longrightarrow$ phase diagram of the 5D anisotropic model
  - Confining phase
  - **Layer phase**
    - 4D Coulomb phase
    - Confining extra dimension
  - 5D Coulomb phase

LAYER PHASE (II)

The potential between heavy test charges is closely connected with the **Wilson loops**:

1. $W_{\mu\nu}(L_1, L_2) \approx \exp[-\sigma L_1 L_2]$ (Confining phase, $1 \le \mu, \nu \le 5$)
2. $W_{\mu\nu}(L_1, L_2) \approx \exp[-\tau(L_1 + L_2)]$ (Coulomb phase, $1 \le \mu, \nu \le 5$)
3. $W_{\mu\nu}(L_1, L_2) \approx \exp[-\tau(L_1 + L_2)]$ } Layer phase
4. $W_{\mu 5}(L_1, L_2) \approx \exp[-\sigma'(L_1 + L_2)]$ } Layer phase

Phase Diagram from Mean Field analysis



Fu and Nielsen
NPB 1984, 1985

Phase Diagram — Monte Carlo Study

Basic order parameters

- 4D (or space) plaquette : $\hat{P}_S \equiv \frac{1}{6N^5} \sum_{x,1\le\mu<\nu\le4} \cos(F_{\mu\nu}(x))$ $\quad P_S = \langle \hat{P}_S \rangle$
- extra dimension (or transverse) plaquette : $\hat{P}_5 \equiv \frac{1}{4N^5} \sum_{x,1\le\mu\le4} \cos(F_{\mu5}(x))$ $\quad P_5 = \langle \hat{P}_5 \rangle$

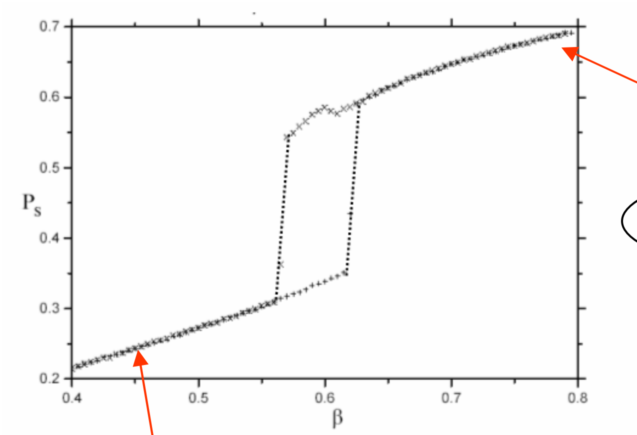
Behaviour of the Plaquette in the two limits of the coupling values

$$P = \begin{cases} \beta/2 + O(\beta^2) & [\,\beta<<1,\ \text{strong coupling}\,] \\ 1 - \frac{1}{D\beta} + O(\beta^{-2}) & [\,\beta>>1,\ \text{weak coupling}\,] \end{cases}$$

(D is the dimension of the space)

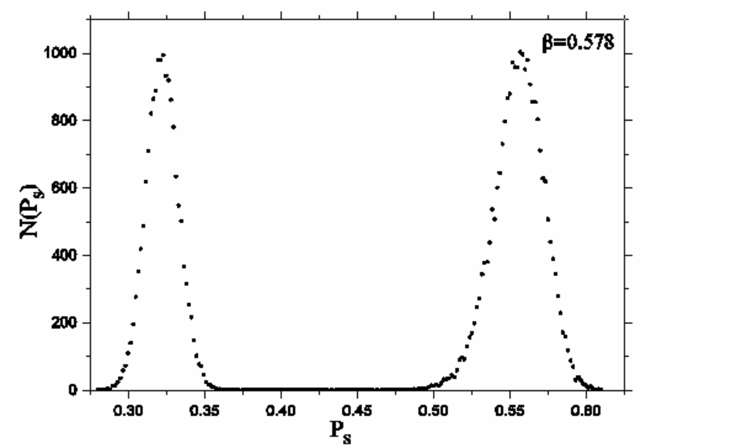
Confining – 5D Coulomb Phase Transition

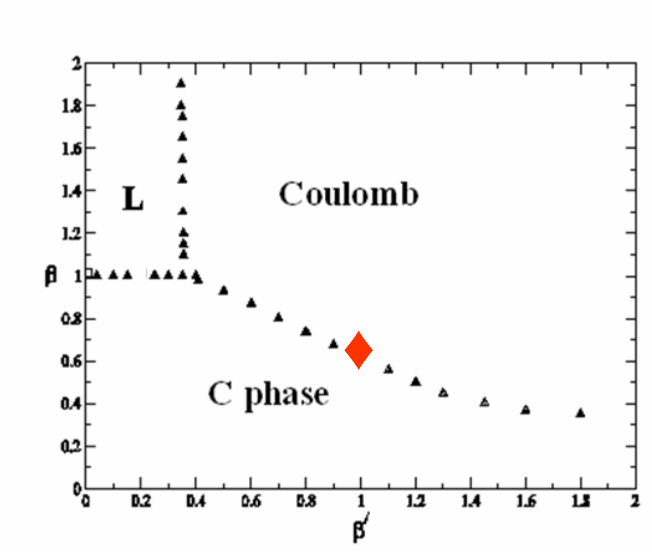
β′=1.0



~ [1-(1/5β)] : 5D-Coulomb phase

~ β /2: Confining phase





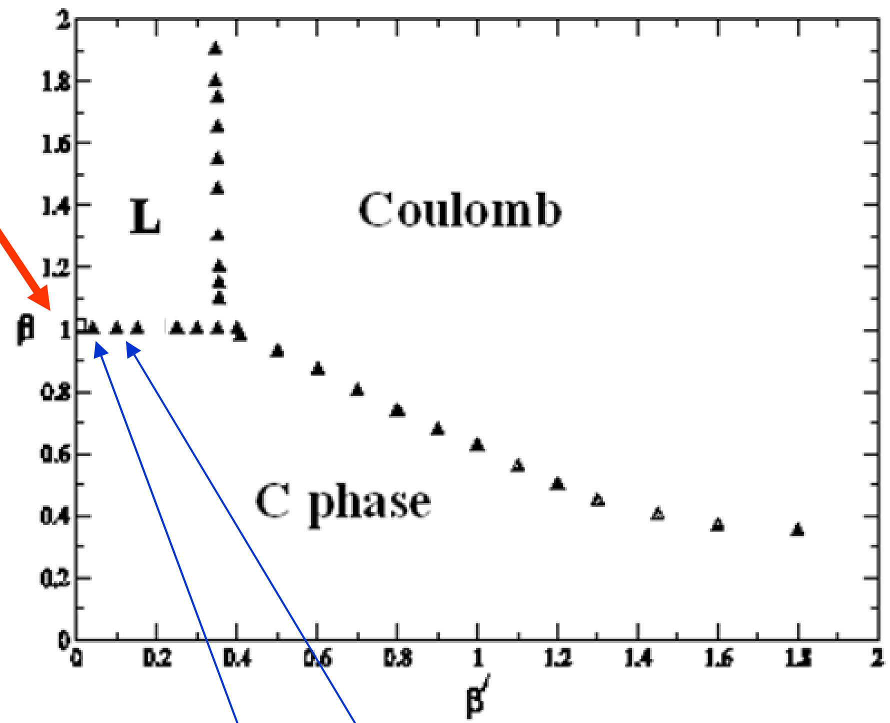
- **P.D.,Farakos, Kehagias and Koutsoumbas NPB 2001**
- **Hulsebos, Korthals-Altes and Nicolis NPB 1995**

Big hysteresis loop

Two-state signal → **1st order PT**

Confining – Layer Phase Transition

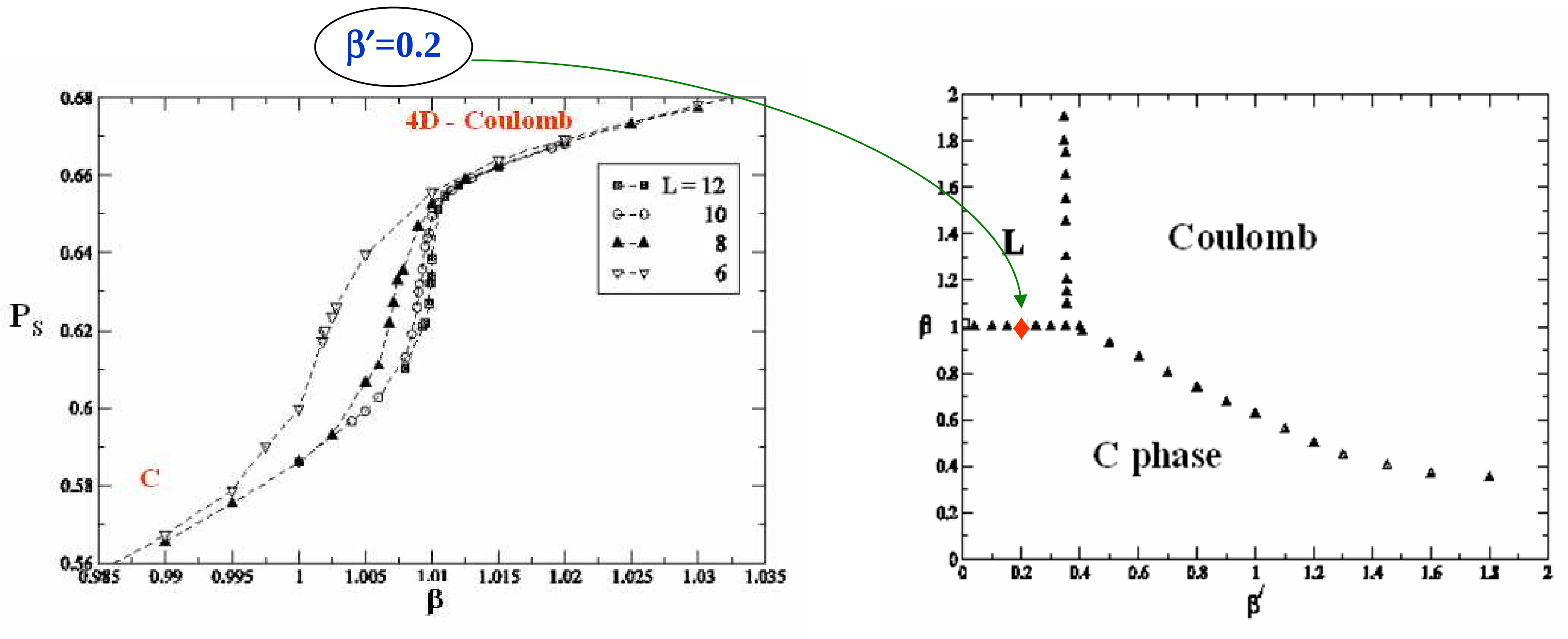
There is very strong evidence that the
4D Confining – Coulomb phase transition
is **first order** and takes place at
β=1.0111331(21) (β'=0).
[**Arnold, Bunk, Lippert and Schilling 2003**]



- **What does it happen as $\beta' > 0$** ?

We do an extensive study of the PT for two values of **β'=0.01 and 0.2**
while we let β variable **.**

Confining – Layer Phase Transition (continue)



- As the lattice volume becomes bigger the transition becomes steeper
- For **β<<1, P_S** $\rightarrow$ **$\beta/2$** : Confining phase
- For **β>>1, P_S** $\rightarrow$ **(1-1/4β)** : 4D-Coulomb phase
- For **all** β, **$P_5 \approx \beta'/2$=0.1 (constant)** $\rightarrow$ **extra dimension is confined !!**

Identification of the phases using the helicity modulus

The **helicity modulus** is an order parameter; it gives the response of the system to an external elecrtomagnetic flow. It is defined through the second derivative of the free energy: [**Vettorazzo and de Forcrand, NPB 2004**]

$$h(\beta) = \left.\frac{\partial^2 F(\Phi)}{\partial \Phi^2}\right|_{\Phi=0}$$

In the confinement phase the system does not feel any changes due to the external flux. On the contrary in the Coulomb phase the system reveals a response.

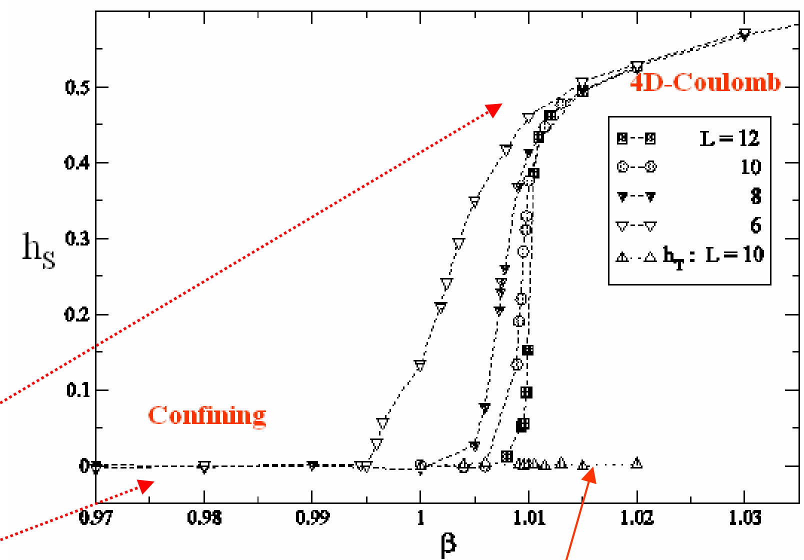
- $h(\beta) \neq 0$ in the Coulomb phase
- $h(\beta) = 0$ in the confining phase

The space-like and the transverse-like helicity modulus are:

$$h_s(\beta) = \frac{1}{(L_\mu L_\nu)^2}\left(\left\langle \sum_{P'} (\beta \cos(F_{\mu\nu}))\right\rangle - \left\langle \left(\left\langle \sum_{P'} (\beta \sin(F_{\mu\nu}))\right\rangle\right)^2 \right\rangle\right)$$

$$h_5(\beta') = \frac{1}{(L_\mu L_5)^2}\left(\left\langle \sum_{P'} (\beta' \cos(F_{\mu 5}))\right\rangle - \left\langle \left(\left\langle \sum_{P'} (\beta' \sin(F_{\mu 5}))\right\rangle\right)^2 \right\rangle\right)$$

β′=0.2

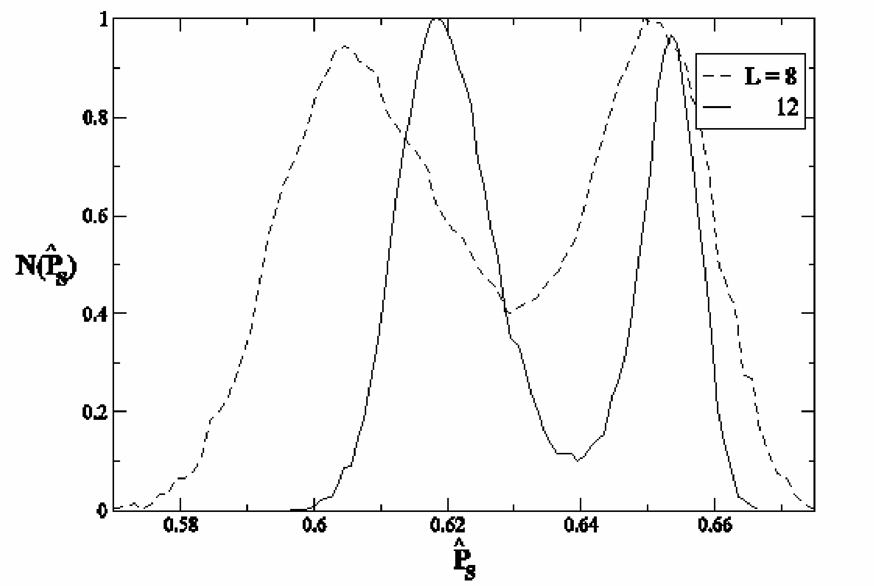


Helicity modulus in the transverse direction, **h_T** takes zero value for all values of β.

Confinement along the extra dimension

Confining – Layer Phase Transition (continue)

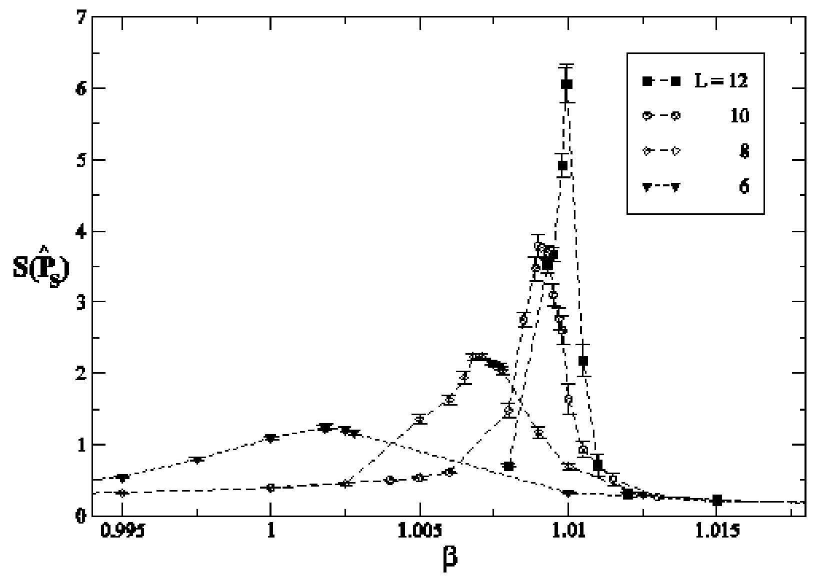
- Distribution of the space-like plaquette:
  Two-state signal



- **Susceptibility** of the space-like plaquette

$$S(\hat{P}_s) \equiv V\left(\left\langle \hat{P}_s^2 \right\rangle - \left\langle \hat{P}_s \right\rangle^2 \right)$$

A ***net increase with the volume*** appears

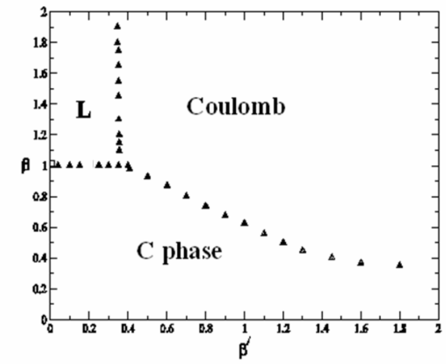


Confining – Layer Phase Transition (continue)

The relative analysis shows that:

- The transition between the Confining phase and the Layer phase is of **1st order** (though weak) as it happens to be the phase transition between the confining and the Coulomb phases for the 4D model.

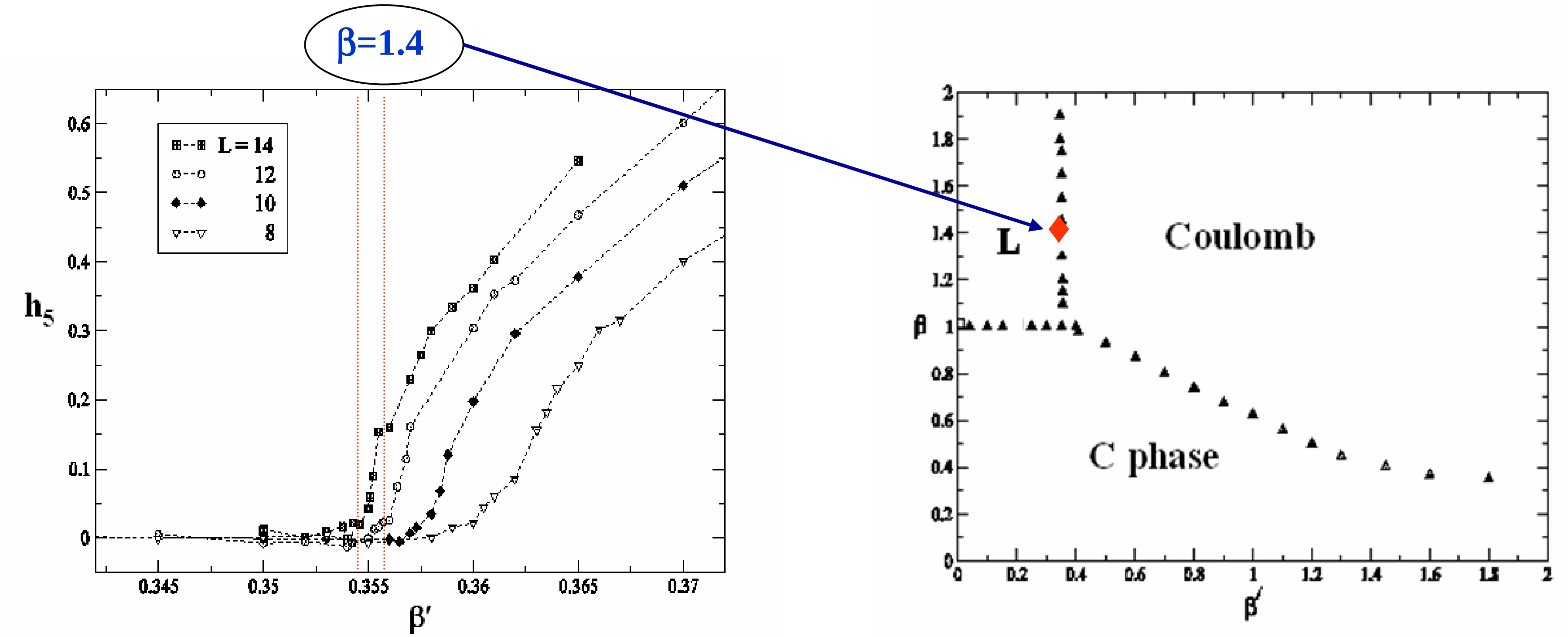
- The transition points of the 4D gauge coupling almost lie on the horizontal line the initial point of which is critical value of the pure 4D model.



- Hence the Layer phase is completely <u>distinguished</u> from the confining one.

P.D., Farakos, Vrentzos
Phys.Rev.D 2006

Layer – 5D Coulomb Phase Transition

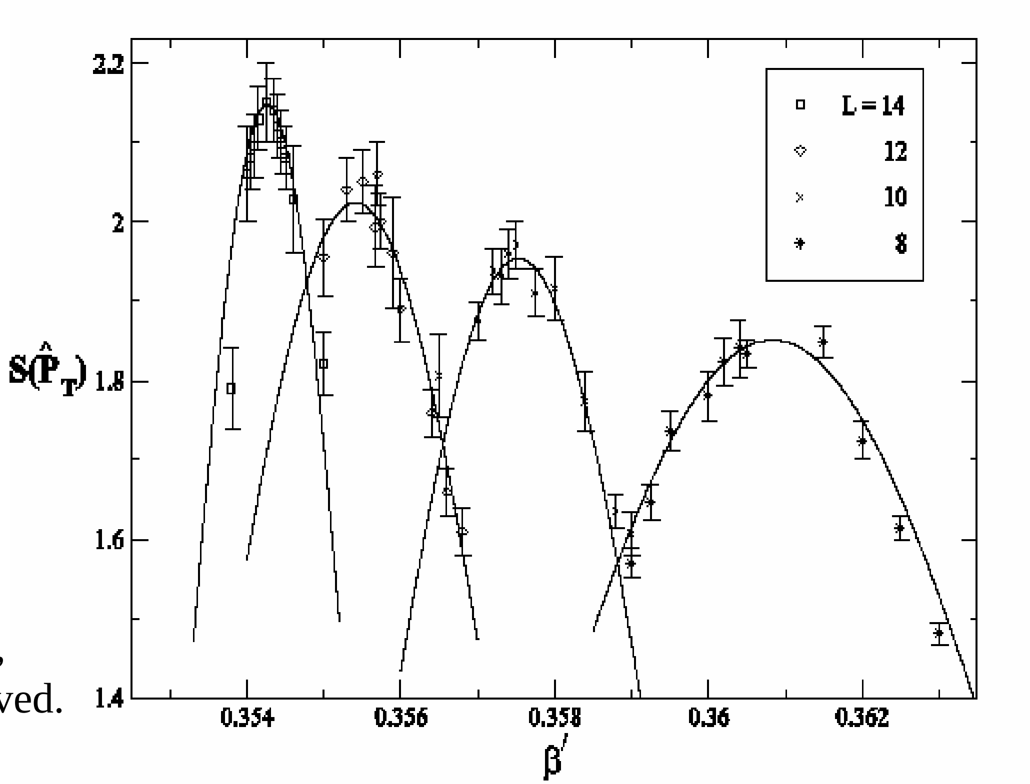


► **As the system passes to the 5D-Coulomb phase the extra fifth dimension ceases to be confined : h_5 passes from zero to a non-zero value**

Layer – 5D Coulomb Phase Transition (continue)

- The susceptibility of the tranverse-like plaquette is:

$$S(\hat{P}_5) \equiv V\left(\left\langle \hat{P}_5^2 \right\rangle - \left\langle \hat{P}_5 \right\rangle^2 \right)$$



- An increase of the susceptibility peaks, though weak, with the volume is observed.

$$S_{max} \sim V^{\delta}, \quad \delta < 1$$

Layer – 5D Coulomb Phase Transition (continue)

- A finite scaling analysis assumes:

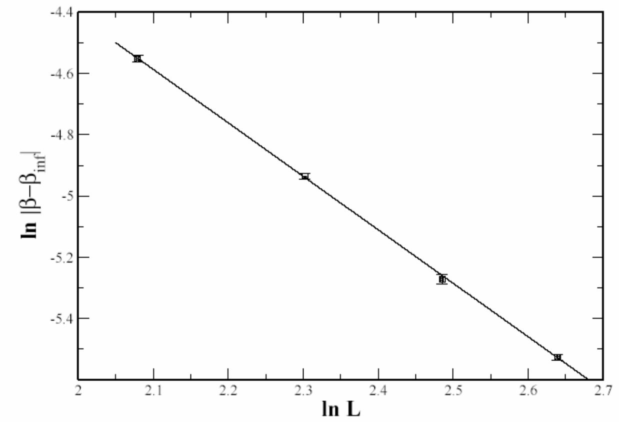
$$\beta'(L) = \beta'_{\infty}(1 + C_1 L^{-(1/\nu)})$$

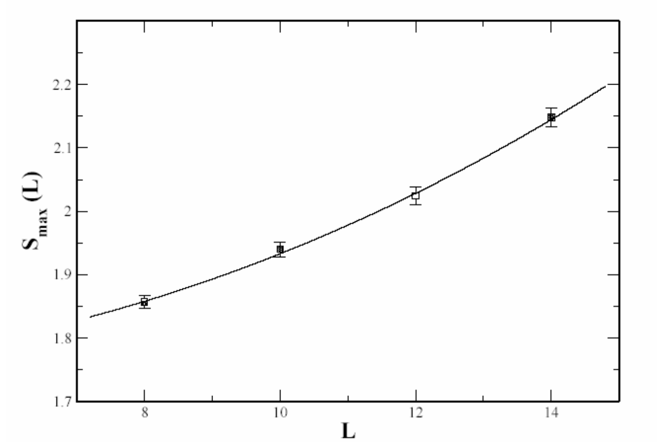
$$S_{max} = C_0 + C_1 L^{\gamma/\nu}$$

$\nu = 0.57(5)$ $\beta'_{\infty} = 0.35028(53)$ $\gamma = 1.24(44)$

$\delta = 0.44(15)$

Evidence for a 2nd PT





* Volumes bigger than V=14^5 have to be used in order to confirm this evidence

It can be shown numerically on the lattice:

- how it can be identified a phase (Layer) which is coulombic on the four-dimensional subspace while it exhibits confinement along the extra dimension

- The Layer phase is stable and it is well separated from the confinement phase and the five-dimensional Coulomb phase

- the potential between two test charges in the layer phase is that of a 4D Coulomb interaction (~1/r) and it is distinguishable from the potential of the 5D Coulomb phase (which goes as $\sim 1/r^2$)

(Farakos and Vrentzos, Phys.Rev.D 2008)

(4+1)-dimensional Abelian Higgs model

5D anisotropic U(1)-Higgs model

- Assume a U(1)-Higgs model in five dimensions in the RS background

- A general metric of the RS type (warped extra dimension) is written as:

$$ds^2 = \omega^2(z)\left[dx_0^2 - d\bar{x}^2\right] - dz^2 \qquad \left(\omega^2(z) \to 0 \quad \text{for} \quad z \to \infty\right)$$

- Write the action of a five-dimensional Abelian Higgs model (in this background):

$$\begin{aligned} S &= S_{gauge} + S_{scalar} \\ &= -\frac{1}{4g_5^2}\int d^5x\sqrt{g}\,F_{MN}F_{K\Lambda}\,g^{MK}\,g^{N\Lambda} + \int d^5x\sqrt{g}\left[D_M\Phi^* D_N\Phi\, g^{MN} - V(\Phi)\right] \\ &= \int d^4x dz\left[-\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda}\eta^{\mu\kappa}\eta^{\nu\lambda} - \frac{a^2(z)}{2g_5^2}F_{\mu 5}F_{\nu 5}\eta^{\mu\nu}\right] + \\ &\quad + \int d^4x dz\left[D_\mu\Phi^* D_\nu\Phi\,\eta^{\mu\nu} - a^4(z)D_z\Phi^* D_z\Phi - a^4(z)V(\Phi)\right] \end{aligned}$$

$(M, N = 0,\ldots,4 \text{ and } \mu,\nu = 0,\ldots,3)$

Make the rescaling $\alpha(z)\Phi = \varphi$ and consider a quartic scalar potential.
The scalar action reads:

$$S_{scalar} = \int d^4x dz \left[D_\mu \varphi^* D^\mu \varphi - a^2(z) D_z \varphi^* D_z \varphi - M(z)^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2 \right]$$

$$\left(\text{with} \quad M(z)^2 = a^2(z)\, m^2 + [a'(z)]^2 + \tfrac{1}{2} [a^2(z)]'' \right)$$

Translate the action on the lattice and we get:

$$\begin{aligned} S_{Lattice} &= S_{gauge} + S_{scalar} \\ &= \beta_g \sum_{x, 1\le\mu<\nu\le 4} \left(1 - \cos U_{\mu\nu}(x)\right) + \beta'_g \sum_{x,\mu} \left(1 - \cos U_{\mu 5}(x)\right) \\ &\quad + \beta_h \sum_{x, 1\le\mu<\nu\le 4} \left[\varphi_L(x) - U_{\hat\mu}(x)\varphi_L(x + a\hat\mu)\right]^* \left[\varphi_L(x) - U_{\hat\mu}(x)\varphi_L(x + a\hat\mu)\right] \\ &\quad + \beta'_h \sum_x \left[\varphi_L(x) - U_{\hat 5}(x)\varphi_L(x + a\hat 5)\right]^* \left[\varphi_L(x) - U_{\hat 5}(x)\varphi_L(x + a\hat 5)\right] \\ &\quad + \sum_x m_L^2 \varphi_L^*(x)\varphi_L(x) + \beta_R \left(\varphi_L^*(x)\varphi_L(x)\right)^2 \end{aligned}$$

$$(\text{we set}: \ 2^{1/2} a^{3/2} \varphi = \varphi_L)$$

The lattice couplings obey certain equations which depend on the warp factor:

$$\beta'_g = a^2(x_5)\beta_g \qquad \beta'_h = a^2(x_5)\beta_h \qquad \lambda = \frac{4\beta_R a}{\beta_h^2} \qquad a^2 M^2(x_5) = \frac{2}{\beta_h} m_L^2$$

Due to the assumed function of the warp factor, the couplings for both the gauge and the scalar fileds along the extra dimension are strongly coupled.

We adopt a ***simplified*** version for the lattice model under study: the couplings along the extra dimension do not depend on the extra coordinate but they are allowed to get different values from the couplings defined on the four dimensional subspace. The lattice action reads:

$$
\begin{aligned}
S_{\mathrm{Lattice}} &= S_{\mathrm{gauge}} + S_{\mathrm{scalar}} \\
&= \beta_g \sum_{x,1\le\mu<\nu\le 4} \left(1-\cos U_{\mu\nu}(x)\right) + \beta_g' \sum_{x,\mu} \left(1-\cos U_{\mu 5}(x)\right) \\
&\quad + \beta_h \sum_x \mathrm{Re}\left[4\varphi_L^*(x)\varphi_L(x) - \sum_{1\le\mu\le 4}\varphi_L^*(x)U_{\hat{\mu}}\varphi_L(x+a\hat{\mu})\right] \\
&\quad + \beta_h' \sum_x \mathrm{Re}\left[4\varphi_L^*(x)\varphi_L(x) - \varphi_L^*(x)U_{\hat{5}}\varphi_L(x+a\hat{5})\right] \\
&\quad + \sum_x \left[(1-2\beta_R - 4\beta_h - \beta_h')\varphi_L^*(x)\varphi_L(x) + \beta_R\left(\varphi_L^*(x)\varphi_L(x)\right)^2\right]
\end{aligned}
$$

an extension of the initial proposal of Fu and Nielsen to the Abelian Higgs model

- For the **full phase diagram** we need to study five couplings (!)

$$\beta_g,\ \beta'_g,\ \beta_h,\ \beta'_h \text{ and } \beta_R$$

- We can make a "reasonable" compromise :

  - keep small: $\beta'_h << 1$
  - take $\beta_g < 1$ in order to have the possibility to get a confinement phase in the four-dimensional subspace.
  - Choose different values for β_R. As β_R decreases the phase transitions become stronger.
  - Study the phase diagram in terms of β'_g and β_h

Order Parameters

- 4D (or space) plaquette) : $$P_S \equiv \left\langle \frac{1}{6N^5} \sum_{x,1\le\mu<\nu\le 4} \cos\left(F_{\mu\nu}(x)\right) \right\rangle$$
- extra dimension (or transverse) plaquette : $$P_5 \equiv \left\langle \frac{1}{4N^5} \sum_{x,1\le\mu\le 4} \cos\left(F_{\mu 5}(x)\right) \right\rangle$$
- 4D (or space) link: $$L_S \equiv \left\langle \frac{1}{4N^5} \sum_{x,1\le\mu\le 4} \cos\left(\chi(x+\hat{\mu}) + A_{\hat{\mu}} - \chi(x)\right) \right\rangle$$
- extra dimension (or transverse) link : $$L_5 \equiv \left\langle \frac{1}{N^5} \sum_{x,1\le\mu\le 4} \cos\left(\chi(x+\hat{5}) + A_{\hat{5}} - \chi(x)\right) \right\rangle$$
- Higgs field measure squared: $$R^2 = \frac{1}{N^5} \sum_x \rho^2(x)$$

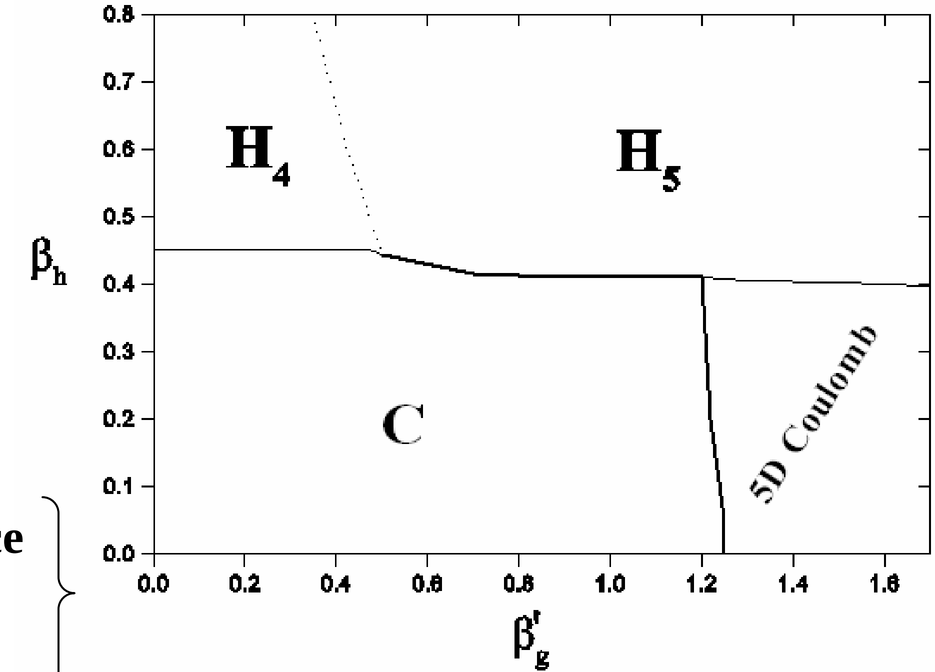
[define: $\varphi_L = \rho(x)\exp(i\chi(x))$]

The Phase Diagram

- C: Confining phase
- 5D Coulomb phase
- H_5: five dimensional Higgs phase
- H_4: four dimensional Higgs phase

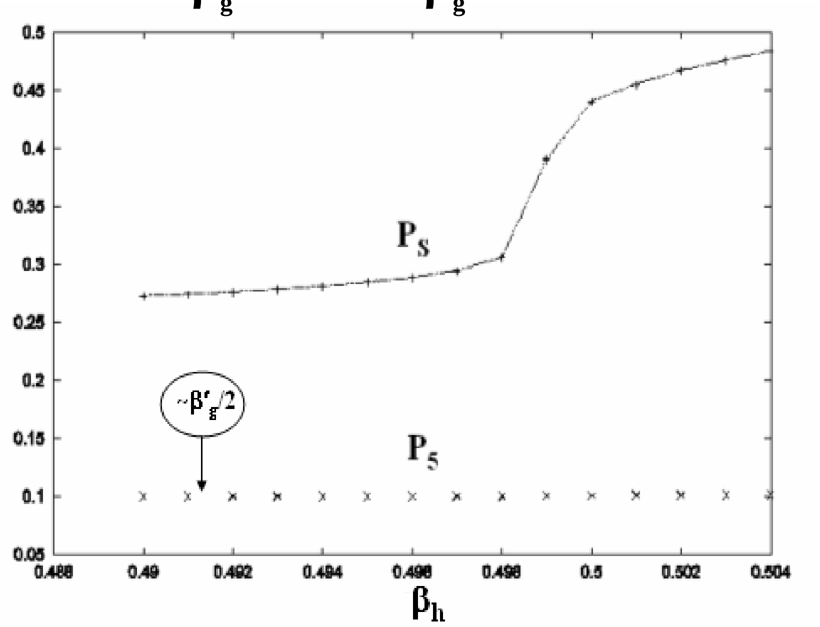
$$\left\{ \begin{array}{c} \text{broken symmetry on 4D subspace} \\ \oplus \\ \text{confinement along the extra dim} \end{array} \right\}$$

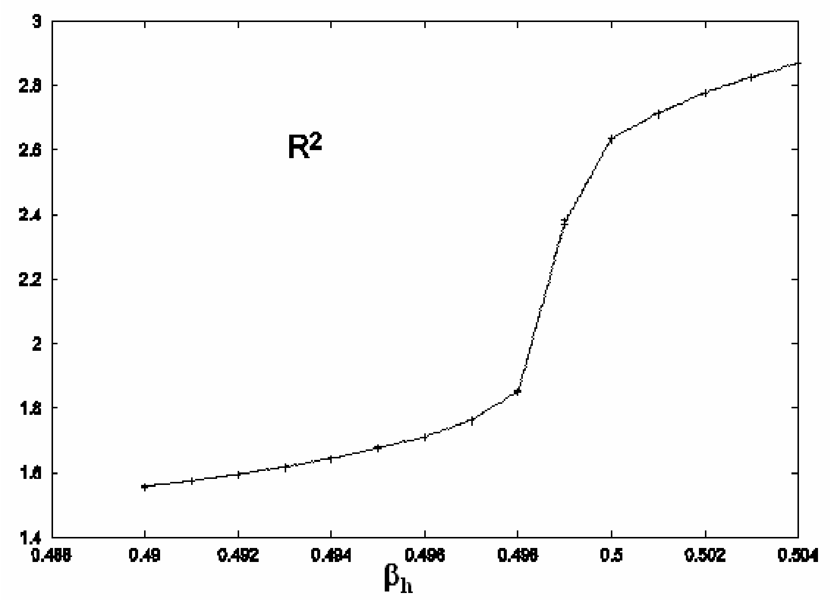
$\beta_R = 0.1 \quad \beta_g = 0.5 \quad \beta_h' = 0.001$

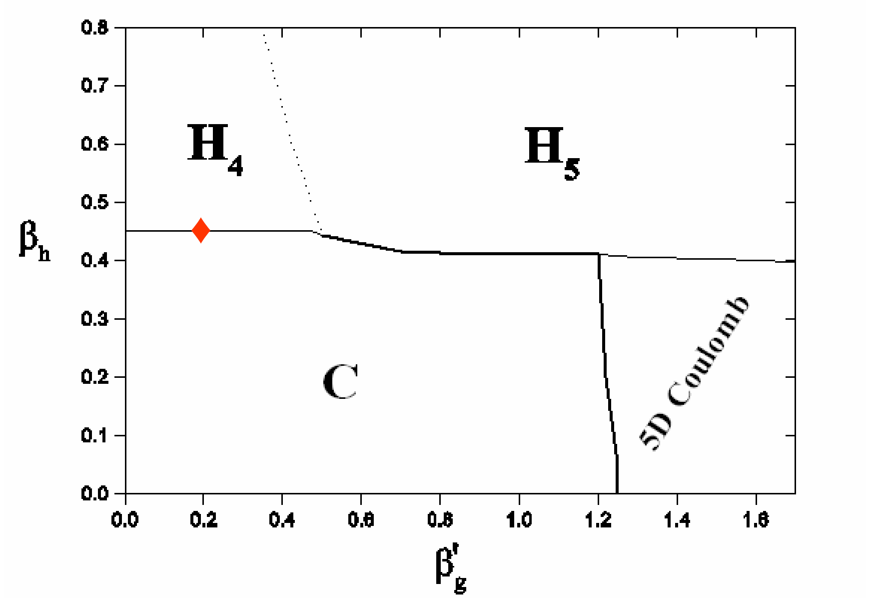


Confining - H_4 phase transition

$\beta_g' = 0.2 \quad \beta_g = 0.5$



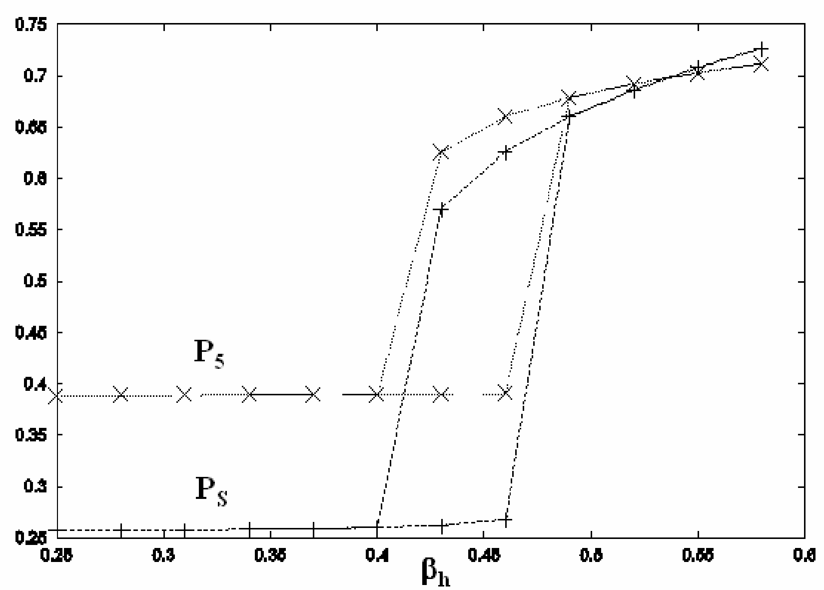


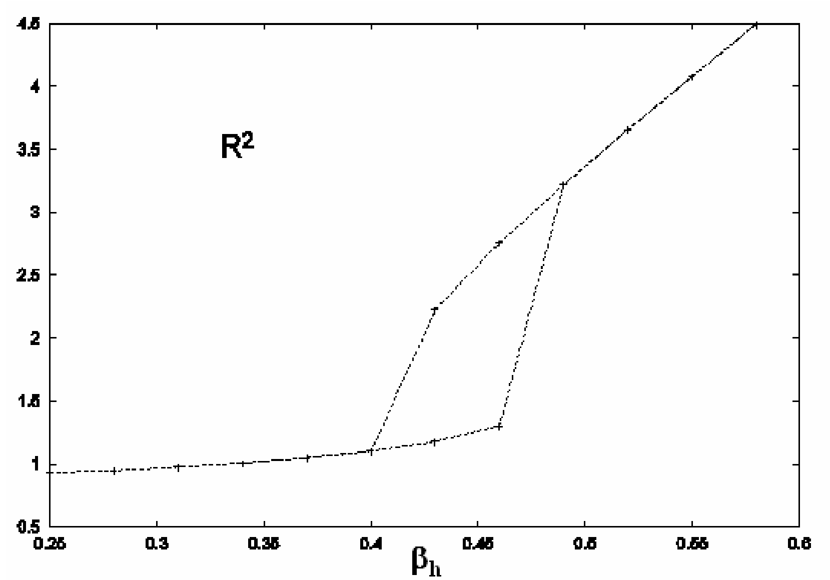


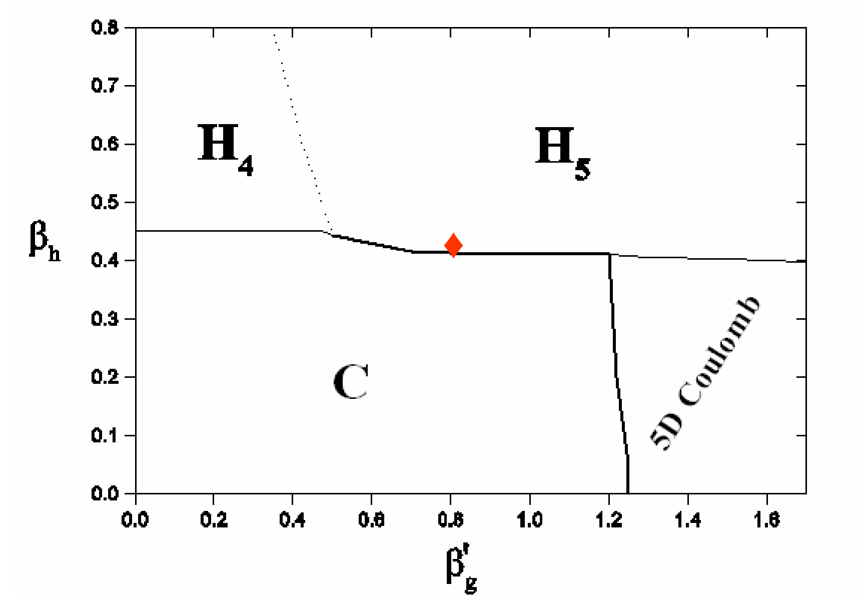
- P_5 does not "feel" the phase transition It remains in the confined regime
- The same happens to L_5
- R^2 passes from a value ~1 (unbroken) to values > 1 (broken phase)

Confining-H_5 phase transition

$\beta'_g = 0.8 \qquad \beta_g = 0.5$

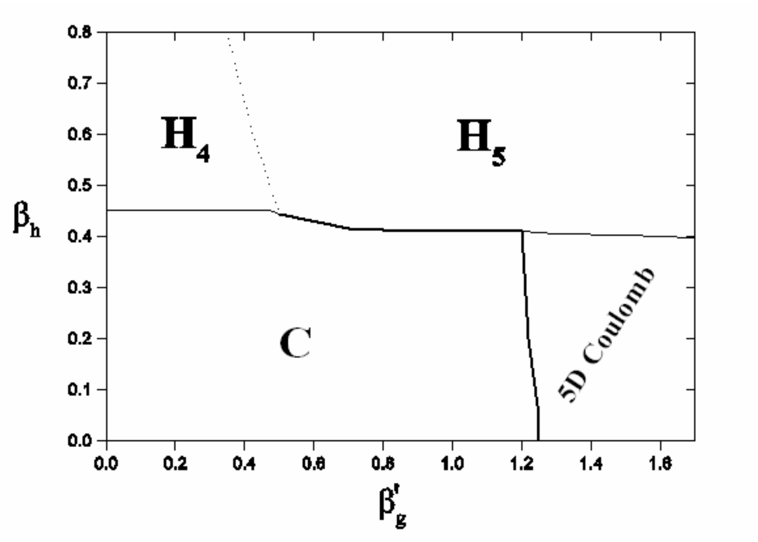






- Both P_5 and P_S do“feel” the phase transition
- R^2 passes from a value ~1 (unbroken) to values > 1 (broken phase)
- Big hysteresis loop: 1st order PT

Interesting features of the 5D Abelian-Higgs model with anisotropic couplings



- There is a ***stable*** phase with broken symmetry on the four-dimensional subspace and confinement along the extra-fifth dimension.

- Strong evidence can be provided that for a certain value of the scalar self-coupling (β_R=0.155 (2)) the transition from the strongly coupled phase to the layer Higgs phase becomes **2nd order**.

- The separation of H_4 from the H_5 phase seems to be a **crossover** (for a definite conclusion bigger volumes are needed)

(4+1)-dimensional SU(2)- Higgs model
in the adjoint representation

5D SU(2)-Higgs in the adjoint representation

- It is a “toy-model” based on the Georgi-Glashow model

- The SU(2) symmetry breaks to U(1)

  ▶ 1 massless photon
  ▶ 2 massive W-bosons
  ▶ 1 massive scalar neutral field

  } In the Higgs phase

- The 3D model is “equivalent” with the SU(2) pure model at finite temperature after dimensional reduction due to the integration of the heavy modes
  It presents two phases (confining + Higgs)
  [**Hart, Philipsen, Stack and Teper Phys.Lett.B 1997**]

- A first numerical study of the 4D model can be found in
  [**Mitrjushkin and Zadorozhny, Phys.Lett.B 1986**]

The lattice model

The action is :

$$
\begin{aligned}
S^{5D}_{Lattice} = {} & \beta_g \sum_{x,1\le\mu<\nu\le 4} \left(1-\frac{1}{2}\mathrm{Tr}U_{\mu\nu}(x)\right) + \beta'_g \sum_{x,\mu}\left(1-\frac{1}{2}\mathrm{Tr}U_{\mu 5}(x)\right) \\
& + \beta_h \sum_{x,\mu}\left(\frac{1}{2}\mathrm{Tr}\left[\Phi^2(x)\right] - \frac{1}{2}\mathrm{Tr}\left[\Phi(x)U_\mu(x)\,\Phi(x+\mu)U^+_\mu(x)\right]\right) \\
& + \beta'_h \sum_{x}\left(\frac{1}{2}\mathrm{Tr}\left[\Phi^2(x)\right] - \frac{1}{2}\mathrm{Tr}\left[\Phi(x)U_5(x)\Phi\ (x+\hat{5})U^+_5(x)\right]\right) \\
& + (1-2\beta_R-4\beta_h-\beta'_h)\sum_x \frac{1}{2}\mathrm{Tr}\left[\Phi^2(x)\right] + \beta_R \sum_x \left(\frac{1}{2}\mathrm{Tr}\left[\Phi^2(x)\right]\right)^2
\end{aligned}
$$

The Links are defined by: $U_\mu = e^{igA_\mu}$ $\quad U_5 = e^{igA_5}$

The gauge potential and the matter fields are represented by 2x2 Hermitian matrices:

$$A_\mu = A^a_\mu \sigma_a \quad \text{and} \quad \Phi = \Phi^a \sigma_a \quad (\sigma_a: \text{ Pauli matrices})$$

Five couplings: $\beta_g,\ \beta'_g,\ \beta_h,\ \beta'_h$ and β_R

Order Parameters

- 4D (or space) plaquette) :

$$P_S \equiv \left\langle \frac{1}{6N^5} \sum_{x,1\le\mu<\nu\le 4} \mathrm{Tr} U_{\mu\nu}(x) \right\rangle$$

- extra dimension (or transverse) plaquette :

$$P_5 \equiv \left\langle \frac{1}{4N^5} \sum_{x,1\le\mu\le 4} \mathrm{Tr} U_{\mu 5}(x) \right\rangle$$

- 4D (or space) link:

$$L_S = \left\langle \frac{1}{4N^5} \sum_{x,1\le\mu\le 4} \Bigg(\frac{1}{2} \mathrm{Tr}\Big[\Phi(x) U_\mu(x) \Phi x) + \mu) U_\mu^+(x)\Big] / \frac{1}{2} \mathrm{Tr}\Big[\Phi^2(x)\Big] \Bigg) \right\rangle$$

- extra dimension (or transverse) link :

$$L_5 = \left\langle \frac{1}{N^5} \sum_{x} \Bigg(\frac{1}{2} \mathrm{Tr}\Big[\Phi(x) U_5(x) \Phi x) + \hat{5}) U_5^+(x)\Big] / \frac{1}{2} \mathrm{Tr}\Big[\Phi^2(x)\Big] \Bigg) \right\rangle$$

- Higgs field measure squared:

$$R^2 = \left\langle \frac{1}{N^5} \sum_{x} \Bigg(\frac{1}{2} \mathrm{Tr}\Big[\Phi^2(x)\Big] \Bigg) \right\rangle$$

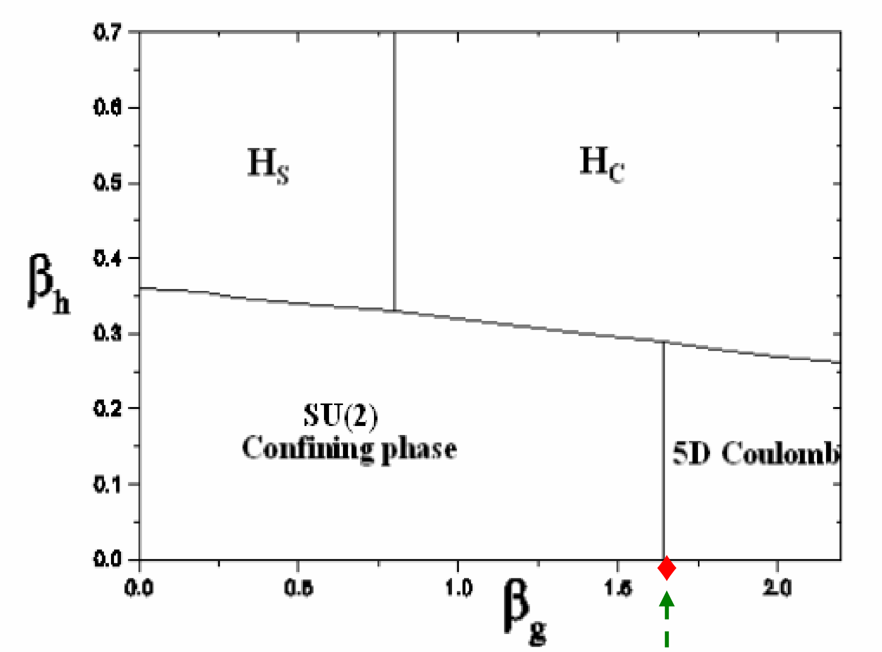
The phase diagram of the model with isotropic couplings

Take: $\beta_g = \beta'_g, \quad \beta_h = \beta'_h$

The phase diagram in the parametric-space of β_g and β_h for $\beta_R=0.01$ is:

New Higgs phases of the 5D model

- **H_S:** five dimensional Higgs phase with the U(1) symmetry in the confining regime
- **H_C:** five dimensional Higgs phase with the U(1) symmetry in the Coulomb regime



The pure SU(2) transition point, β_g**~1.63**
[**Creutz, Phys.Rev.Lett. 1979**]

Relationship of the Layer phase and the dimensionality

A lattice model defined in D=d+n dimensions can provide a Layer phase as long as the d-dimensional model can be found in at least two phases one of which is the Coulomb phase.

Fu and Nielsen NPB 1984, 1985

★ Therefore it is reasonable that a 4D layer phase can exist in the 5D anisotropic U(1) gauge model since the phase diagram of the four-dimensional model consists of two phases : confining and Coulomb.

★ On the contrary, the 5D anisotropic SU(2) gauge model can not furnish a layer phase since it is known that the 4D SU(2) gauge model does **not** have a well defined Coulomb phase. However it can exist a 5D layer phase of SU(2) in a 6D space.

➡ But, what does it happen when the scalar field is added?

The phase diagram of the model with anisotropic couplings

- For $\beta'_g = 0$ and $\beta'_h = 0$ the model becomes four-dimensional. It has a confining phase (for β_h small) and a four-dimensional Higgs phase (for β_h big). The U(1) symmetry that survives in the Higgs phase shows a phase transition between the strong coupled and the weak phase (following the β_g value).

- The parameter β_R "controls" the strength of the phase transitions

- • Switch on β'_g and β'_h :

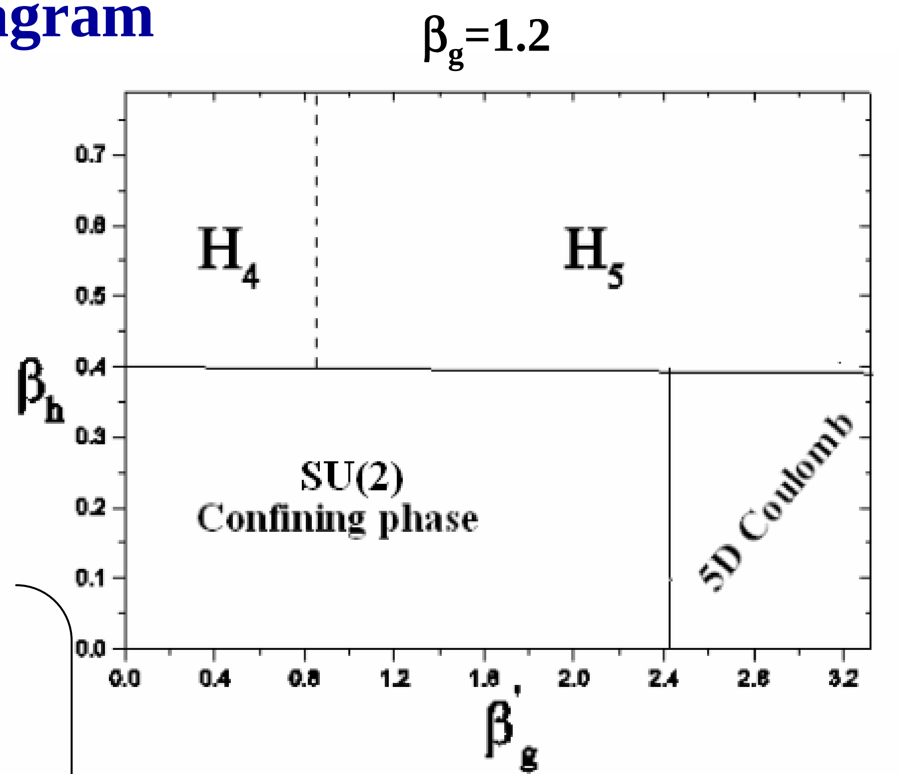
    ► Keep β'_h small (=0.01).

    ► Vary β_g in the interval for which the confined phase can be present ($\beta_g < 1.63$).

• Give the Phase Diagram in terms of β_h and β'_g

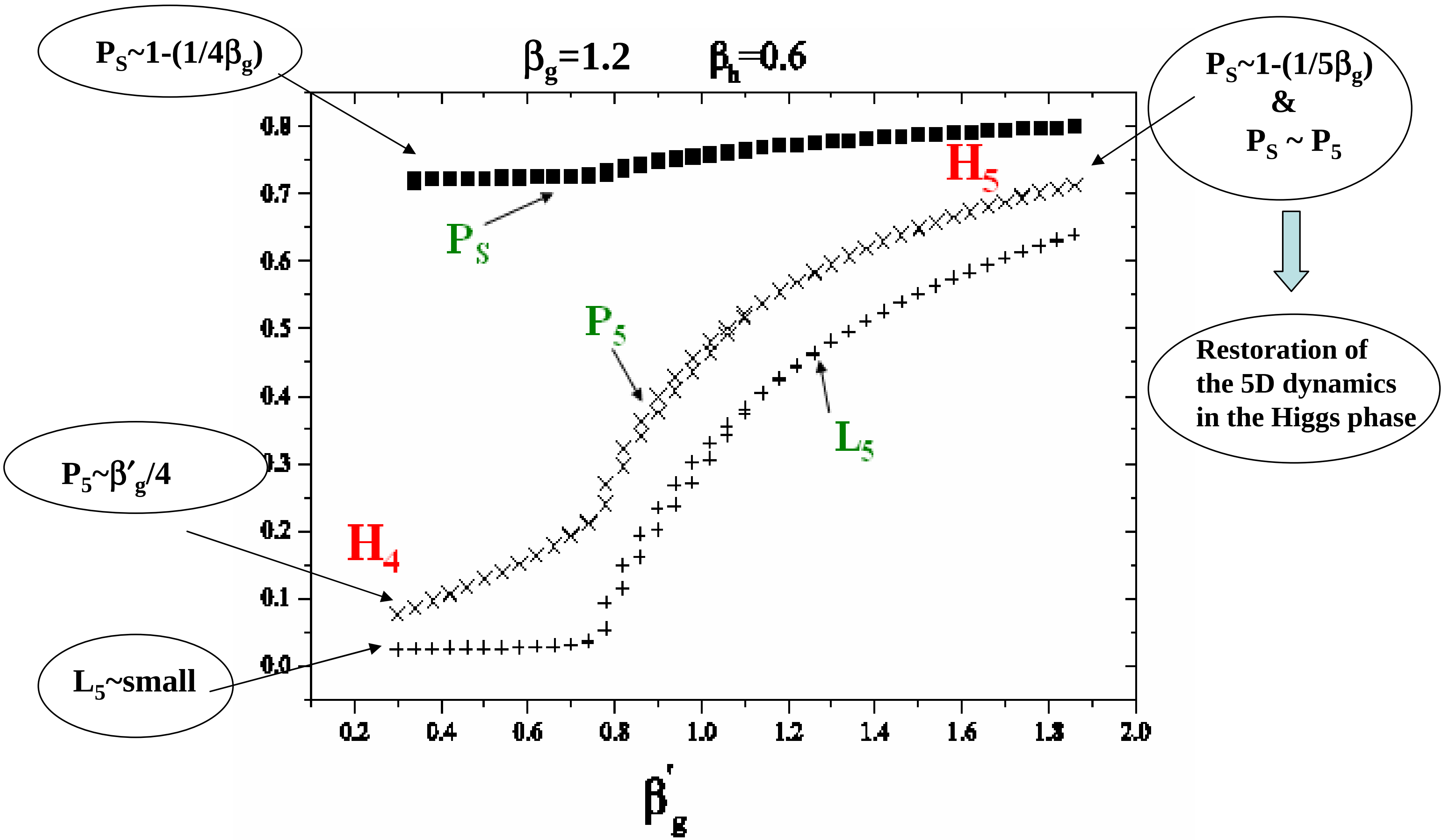
The Phase Diagram

- SU(2) Confining phase
- 5D Coulomb phase (for SU(2))
- H_5: (five dimensional Higgs phase $\oplus$ U(1) unbroken in 5D)
- H_4: (four dimensional Higgs phase $\oplus$ U(1) unbroken in 4D space $\oplus$ SU(2) confinement along the extra dim)



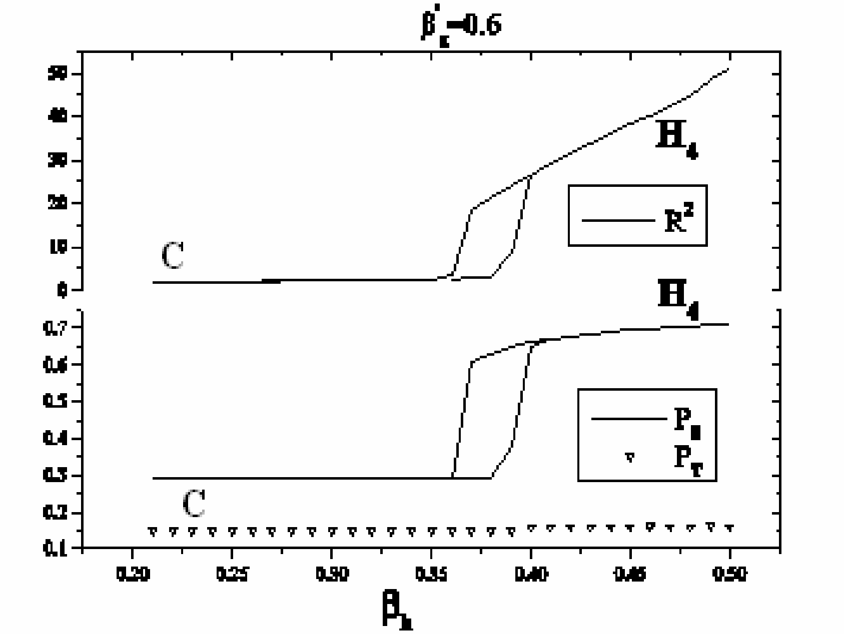
	4D space	5^{th} dimension
Conf	SU(2)–strong: $P_S \sim \frac{\beta_g}{4}$	SU(2)–strong: $P_5 \sim \frac{\beta'_g}{4}$
5D Ch	SU(2)–Coulomb: $P_S \sim 1 - \frac{3}{5\beta_g}$	SU(2)–Coulomb: $P_5 \sim 1 - \frac{3}{5\beta'_g}$
H_4	U(1)–4D Coulomb: $P_S \sim 1 - \frac{1}{4\beta_g}$	SU(2)–strong: $P_5 \sim \frac{\beta'_g}{4}$
H_5	U(1)–5D Coulomb: $P_S \sim 1 - \frac{1}{5\beta_g}$	U(1)–5D Coulomb: $P_5 \sim 1 - \frac{1}{5\beta'_g}$

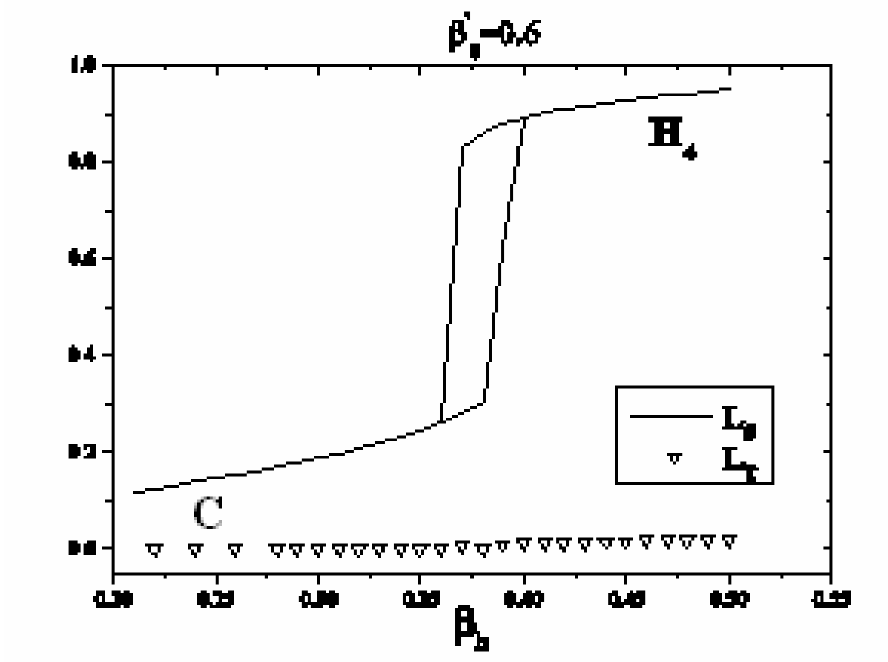
H_4 – H_5 phase transition

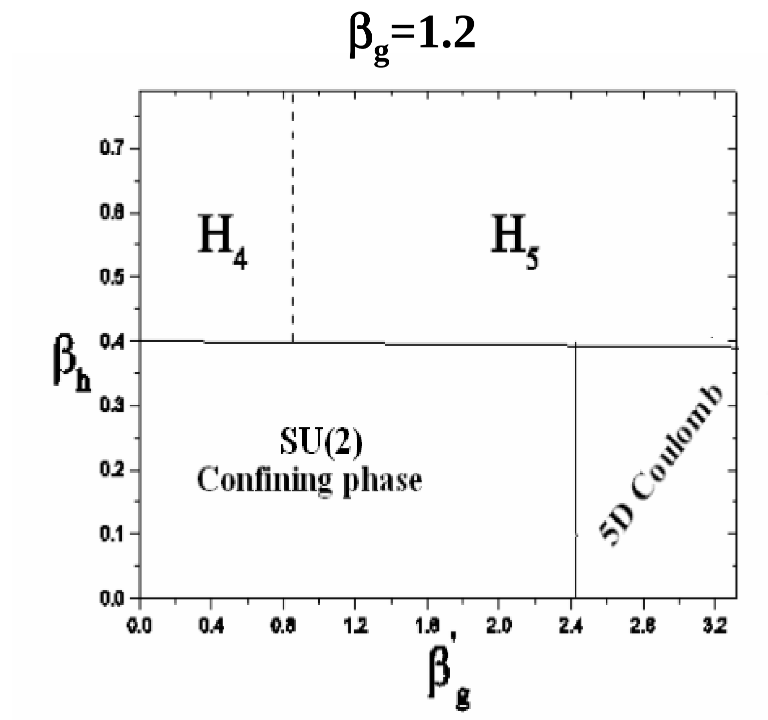


► For both **H_4** and **H_5** the order parameter **R^2 >> O(1)**

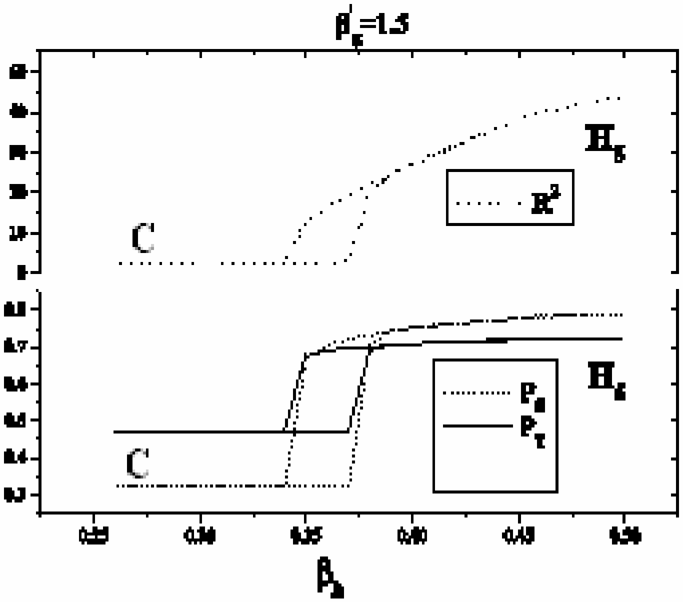
Confining - H_4 phase transition

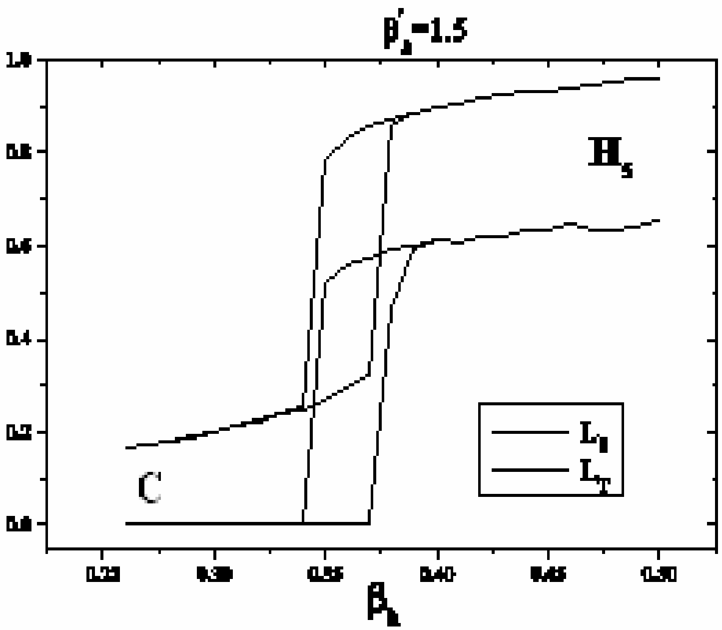


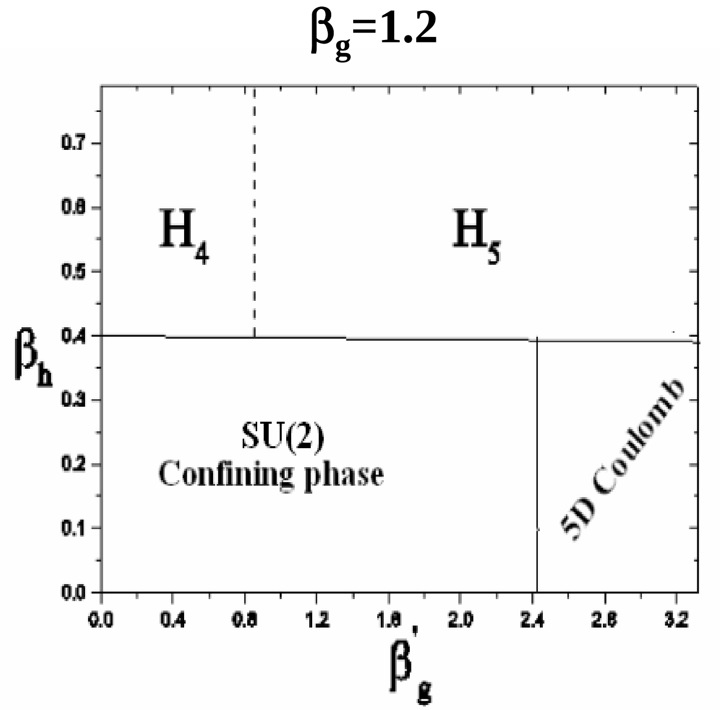




Confining - H_5 phase transition







- The Higgs phase $\mathbf{H_4}$ is located in a four-dimensional subspace embedded in a five-dimensional space.

- A ***massless*** photon is found to be “localized” on the four-dimensional subspace

- The ***confinement*** along the extra dimension due to the SU(2) interaction serves for the “screening” of the extra dimension.

$\mathbf{H_4}$ is a four-dimensional Layer phase formed in a five-dimensional “bulk”

It shares the characteristics of the analytical solution of the ***Dvali-Shifman model*** (Phys.Lett.B 1997) for the localisation of a massless photon on a two-dimensional wall out of which the interaciton is confined by the SU(2) interaction.

Remarks & Conclusions

- **(D+1)-dimensional gauge models with anisotropic couplings can reveal a new kind of D-dimensional phase which we call Layer. It is a D-dimensional Coulomb phase accompanied by confinement along the extra dimension.**

- **The necessary condition for the layer phase formation is that the D-dimensional gauge model must already have two distinct phases. Hence the minimum dimensionality is D=4 for the pure U(1) model and D=5 for the pure SU(2) model.**

- **Extra dimensional lattice gauge models with anisotropic couplings can be "inspired" in an extra dimensional space described by the RS metric.**

- **The study of the 5D-Abelian Higgs model with anisotropic couplings shows that a Layer phase exists in the broken phase: we get a set of 4-dimensional subspaces in the Higgs phase which do not comunicate due to confinement along the extra direction.**

- **The 5D anisotropic SU(2)-Higgs model in the adjoint representation shows two main features:**
  - ► **The inclusion of the scalar field is responsible for the formation of a 4–dimensional layer (higgs) phase in a model with non-abelian dynamics**
  - ► **The confinement along the extra dimension is of the non-abelian type**

- **The existence of the layer phase in the phase diagram of (4+1)D lattice gauge models with anisotropic couplings supports the conjecture of an effectively four-dimensional world embedded in a bulk of extra dimensions.**