Dimensional regularization and spurious gauge-invariance

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Outline

- Introduction and motivation
- Spurious axial invariance
- Applications at 1-loop
- Conclusions

Introduction

’t Hooft-Veltman (1972)

QFTs require regularization and renormalization.
Dimensional regularization is the most popular scheme because:

- It is efficiently applicable to high order calculations
- It regulates both UV and IR divergences
- It is a mass-independent scheme
- It is compatible with gauge invariance

Definition

1) Extend to d-dimensions (formally, d is complex!)

4-dimensions

(d-4)-dimensions

$$S \supset \int d^4x \ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \rightarrow \int d^dx \left[\frac{1}{2}\partial_{\bar{\mu}}\phi\partial^{\bar{\mu}}\phi + \frac{1}{2}\partial_{\hat{\mu}}\phi\partial^{\hat{\mu}}\phi\right]$$

2) Compute amplitudes in d-dimensions: they are meromorphic functions of d=4-ϵ.

Removes all 1/ε poles

$$e^{i\Gamma_{\text{reg}}[\xi_c]} = \int_{\text{1PI}} \mathcal{D}\xi \ e^{iS[\xi+\xi_c]+iS_{\text{ct}}[\xi+\xi_c]}$$

3) Take the 4-dimensional limit where $d \rightarrow 4$ **and all evanescent terms** disappear

$$\Gamma[\xi_c] \equiv \text{LIM}_{d\rightarrow 4}\,\Gamma_{\text{reg}}[\xi_c]$$

Systematically applicable at all loops!

Why does it work?

$$\mathcal{L} = (\partial\phi)^2 + \overline{\psi} i\gamma^\mu \partial_\mu \psi - y\overline{\psi}\psi\phi - \lambda\phi^4$$

$$[\phi] = \frac{d-2}{2}$$

$$[\psi] = \frac{d-1}{2}$$

$$\left.\begin{array}{l} [y] = 2 - \frac{d}{2} \\ [\lambda] = 4 - d \end{array}\right\} \Rightarrow$$

For d<4 all couplings are relevant:
control on UV divergences

Analogously, d>4 all couplings are irrelevant:
control on IR divergences

Complex d regulates
both UV and IR divergences!

It is a mass-independent scheme:
Great for Effective Field Theories!!!

No power-law divergences $\displaystyle \int \frac{d^d k}{(2\pi)^d} (k^2)^\alpha = 0$

→ no contamination from higher-dimensional operators $\displaystyle \frac{c_6}{\Lambda^2}\phi^6 \not\to \delta\lambda\phi^4$

→ RG evolution constrained by dimensional analysis

Respects QCD and QED

$$\mathcal{L} = \overline{\psi} i \gamma^\mu (\partial_\mu - i T^A A_\mu^A) \psi - \frac{1}{4} F_{\mu\nu}^A F^{A\,\mu\nu}$$

By extending this theory to d dimensions we have
d-dimensional gauge-invariance and
d-dimensional Lorentz invariance

Chirality?!

The notion of chirality does not exist at arbitrary d.

The γ_5 problem:

$$\left.\begin{aligned} &\{\gamma_\mu, \gamma_5\} = 0 \\ &\mathrm{Tr}\left(\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta \gamma_5\right) = -4i\epsilon_{\mu\nu\alpha\beta} \\ &\mathrm{Tr}\left(\Gamma_1 \Gamma_2\right) = \mathrm{Tr}\left(\Gamma_2 \Gamma_1\right) \end{aligned}\right\} \Rightarrow \quad (d-4)\mathrm{Tr}\left(\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta \gamma_5\right) = 0$$

d=4 is “singular”: we cannot
analytically continue these properties!

't Hooft-Veltman (1972)

Solution!

Levi-Civita and γ_5 are 4-dimensional objects

$$\gamma_5 \equiv \frac{i}{4!}\epsilon_{\bar{\mu}\bar{\nu}\bar{\alpha}\bar{\beta}}\gamma^{\bar{\mu}}\gamma^{\bar{\nu}}\gamma^{\bar{\alpha}}\gamma^{\bar{\beta}}$$

$$\left.\begin{array}{l} \gamma_5 \equiv \frac{i}{4!}\epsilon_{\bar{\mu}\bar{\nu}\bar{\alpha}\bar{\beta}}\gamma^{\bar{\mu}}\gamma^{\bar{\nu}}\gamma^{\bar{\alpha}}\gamma^{\bar{\beta}} \\ \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \end{array}\right\} \Rightarrow \begin{array}{l} \{\gamma_{\bar{\mu}}, \gamma_5\} = 0 \\ [\gamma_{\hat{\mu}}, \gamma_5] = 0 \end{array}$$

γ_5 is <u>not</u> anti-commuting

Breitenlohner and Maison showed that the above definition implies a consistent regularization at all orders in perturbation

Breitenlohner-Maison (1977)

A nuisance !

Consider a chiral transformation

$$\psi \to e^{i\alpha\gamma_5}\psi$$
$$\overline{\psi} \to \overline{\psi}e^{i\alpha\gamma_5}$$

$$S = \int d^dx\,\overline{\psi}i\gamma^\mu\partial_\mu\psi = \int d^dx\,\left[\overline{\psi}i\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\psi + \overline{\psi}i\gamma^{\hat{\mu}}\partial_{\hat{\mu}}\psi\right]$$

$$\int d^dx\,\left[\overline{\psi}i\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\psi + \overline{\psi}e^{2i\alpha\gamma_5}i\gamma^{\hat{\mu}}\partial_{\hat{\mu}}\psi\right]$$

Chiral symmetries are explicitly broken, even if gauged!!!

A classical anomaly!

$$\delta S = -\int d^d x \, \alpha \boxed{\left[2\overline{\psi}\gamma_5\gamma^{\hat{\mu}}\partial_{\hat{\mu}}\psi\right]}$$

Unavoidable: d-dim kinetic term mixes L with R → explicit breaking of chiral symmetry.
Evanescent: the anomaly must vanish as d → 4.

Is it strange that chiral symmetries are broken by the classical action?
No, it had to be this way!

$$e^{i\Gamma[\xi_c]} = \int_{1\mathrm{PI}} \mathcal{D}\xi \; e^{iS[\xi+\xi_c]}$$

Let us prove Ward identities: $\xi' = e^{iT\alpha}\xi$

$$e^{i\Gamma[\xi'_c]} = \int_{1\mathrm{PI}} \mathcal{D}\xi \; e^{iS[\xi+\xi'_c]} = \int_{1\mathrm{PI}} \mathcal{D}\xi' \; e^{iS[\xi'+\xi'_c]}$$

Crucially, in Dimensional Regularization the measure is always invariant:

$$\mathcal{D}\xi' = e^{i\int d^dx\,\alpha J}\mathcal{D}\xi$$

$$J = \mathrm{Tr}[T]\delta^{(d)}(0)$$

$$\delta^{(d)}(0) = \int \frac{d^dk}{(2\pi)^d} \equiv 0 \Rightarrow \begin{cases} \mathcal{D}\xi' = \mathcal{D}\xi \\ e^{i\Gamma[\xi'_c]} = \int_{\mathrm{1PI}} \mathcal{D}\xi\ e^{iS[\xi+\xi_c]+i\delta S[\xi+\xi_c]} \end{cases}$$

1) Anomalies (ex: chiral anomaly in QCD) ⇔ non-invariance of the regularised action

$$\delta\Gamma[\xi_c] = \frac{\int_{1\mathrm{PI}} \mathcal{D}\xi \; e^{iS[\xi+\xi_c]} \delta S[\xi + \xi_c]}{\int_{1\mathrm{PI}} \mathcal{D}\xi \; e^{iS[\xi+\xi_c]}}$$

At infinitesimal level the variation of the 1PI effective action is given by the matrix elements of the classical anomaly (Quantum Action Principle)

— Symmetries of the classical action hold at all orders (4-dim Lorentz, vector-like, CP, P).
— What happens to anomalous symmetries?
Spurious (gauge, non-abelian axial) or Physical (abelian axial, scale invariance)

2) For axial symmetries δS is evanescent (ϵ), the anomaly must be multiplied by $1/\epsilon$ div:
$\rightarrow$ it is finite and local $\rightarrow$ may be removed by a counterterm.

When a consistent regularization breaks a symmetry, we have a spurious anomaly

Spurious anomaly

$$\delta\Gamma_{\rm reg}|_{(n)} = \mathcal{A}|_{(n)}$$

We can define a symmetric 1PI effective action as

$$\Gamma_{\rm inv}[\xi_c]|_{(n)} = \Gamma_{\rm reg}[\xi_c]|_{(n)} + \Delta S_{\rm ct}^{\rm Fin}[\xi_c]|_{(n)}$$

$$\delta(\Delta S_{\rm ct}^{\rm Fin}|_{(n)}) = -\mathcal{A}|_{(n)}$$

An appropriate counterterm exists as long as

$$D^{abc} = \mathrm{tr}(T_L^a\{T_L^b, T_L^c\}) - \mathrm{tr}(T_R^a\{T_R^b, T_R^c\}) = 0\,.$$

Georgi-Glashow (1972)

No new anomalies emerge in perturbation theory (even beyond renormalizable). See, e.g., Gomis-Weinberg (1995)

Luscher (1999)

Breaking due to Dim-Reg is artificial ⇛ the anomaly can be removed via counterterms.

Tonin et al. (1977)

Another “nuisance”!

Consider a chiral gauge theory

Different by def.

$$A_\mu^A \overline{\psi} \gamma^\mu (T_L^A P_L + T_R^A P_R)\psi$$

$$P_L = \frac{1}{2}(1-\gamma_5)$$
$$P_R = \frac{1}{2}(1+\gamma_5)$$

These are still projectors,
But they do not commute with the Lorentz generators.

d-dimensional Lorentz is broken
Only 4-dimensional Lorentz is preserved (real world)

Implications

The BMHV (Breitenlohner, Maison, ’t Hooft, Veltman) prescription is perfectly consistent but

1) γ_5 is not always anti-commuting → More care in loop computations
2) We have to add appropriate counterterms order by order → Rather annoying procedure (Only chiral symmetry, Lorentz is fine)
3) The constraining power of symmetries is lost in intermediate steps → A nuisance: Symmetry is no more of any guidance?!

Many alternatives have been proposed to avoid these implications:

None of them has a definition of γ_5

None of them has been shown to be consistent at all orders

Most popular:

Naive Dimensional Regularization: γ_5 is anti-commuting, but diagrams are treated differently.

Kreimer's scheme (KKS): γ_5 is anti-commuting, but the trace is not cyclic.

$$\{\gamma_\mu, \gamma_5\} = \begin{cases} 0 & \text{Naive} \\ 2\gamma_{\hat{\mu}}\gamma_5 & \text{BMHV} \end{cases}$$

They differ by
an evanescent term
(Compensated by loops)

<u>They are supposed to be a "trick" to avoid the introducing the counterterms.</u>
You can "use them if you know what you are doing" (Altarelli?)

Do we?!

Alternatives imply ambiguities:

1) At 4-loops (!!!) the QCD beta function in the SM can acquire different values… Zoller (2015) Bednyakov, Pikelner (2016)

2) In QCD, the Chern-Simons current mixes with the axial current at 2-loops (must add a new counterterm anyway!) Chen (2023)

3) Disagreement already at 1-loop in the evaluation of g_1 (deep inelastic scattering) Manohar (Private communication)

We cannot implement these alternative prescriptions on a code and be done with it!

Implications

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Spurious axial-invariance

Explicitly broken symmetries are still useful, if we know how they are broken.

In massive QCD, the chiral $SU(3)_L \times SU(3)_R$ symmetry can be restored treating the quark mass as a field (spurion)

$$M \to LMR^\dagger$$

We can do the same in Dimensional Regularization

Under a (global) chiral symmetry

$$\left.\begin{array}{l} P_L\psi \to LP_L\psi \\ P_R\psi \to RP_R\psi \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \overline{\psi}\gamma^{\bar{\mu}}\psi \to \overline{\psi}\gamma^{\bar{\mu}}\psi \\ \overline{\psi}\gamma^{\hat{\mu}}\psi \to \overline{\psi}\gamma^{\hat{\mu}}(R^\dagger LP_L + L^\dagger RP_R)\psi \end{array}\right.$$

We therefore introduce a new field (spurion) Ω that transforms as

$$\Omega \to L\Omega R^\dagger$$

We have thus formally recovered the axial symmetries:

$$\boxed{\overline{\psi}i\gamma^{\bar{\mu}}\partial_{\bar{\mu}}\psi + \overline{\psi}i\gamma^{\hat{\mu}}(\Omega^\dagger P_L + \Omega P_R)\partial_{\hat{\mu}}\psi}$$

In the end what matters is the 4-dimensional limit

Under a local chiral symmetry

$$\left.\begin{array}{l} P_L\psi \to L(\bar{x})P_L\psi \\ P_R\psi \to R(\bar{x})P_R\psi \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \overline{\psi}\gamma^{\bar{\mu}}\psi \to \overline{\psi}\gamma^{\bar{\mu}}\psi \\ \overline{\psi}\gamma^{\hat{\mu}}\psi \to \overline{\psi}\gamma^{\hat{\mu}}(R^\dagger L P_L + L^\dagger R P_R)\psi \end{array}\right.$$

We therefore introduce a new field (spurion) Ω that transforms as

$$\Omega \to L(\bar{x})\Omega R^\dagger(\bar{x})$$

We have thus formally recovered the axial symmetries:

$$\boxed{\overline{\psi}i\gamma^{\bar{\mu}}D_{\bar{\mu}}\psi + \overline{\psi}i\gamma^{\hat{\mu}}(\Omega^\dagger P_L + \Omega P_R)\partial_{\hat{\mu}}\psi}$$

Finally, the regularised action is written as

Dynamical fields

$$S^{\text{Reg}}[\xi,\Omega] = S_{\text{Bos}} + S_{\text{Yuk}} + \frac{1}{2}\int d^dx\,\{\overline{\psi} i\gamma^{\bar{\mu}} D_{\bar{\mu}}\psi + \overline{\psi} i\gamma^{\hat{\mu}}(\Omega^\dagger P_L + \Omega P_R)\partial_{\hat{\mu}}\psi + \text{hc}\}$$

Pure boson part
(No issues)

Yukawa sector
(No issues)

The standard BMHV action is recovered setting $\Omega = 1$.

Conserved global symmetries:

— **SO(1,3)xSO(d-4)**. There is no need of Lorentz-restoring counterterms.
— **Spurious CP and P** (under which generators transform)
— **Spurious chiral rotations**

What do we gain?!

See, e.g. Abbott

Adopting the background gauge (with gauge-covariant gauge-fixing),
all symmetries are linearly realized by construction:

$$\int_{1\mathrm{PI}} \mathcal{D}\xi \; e^{iS^{\mathrm{Reg}}[\xi+\xi_c,\Omega]}$$

— the divergences are symmetric
— the associated counterterms are symmetric
— the symmetry-restoring counterterms are symmetric

Alternatively:
Gauge-fixing leaves BRST ➔ (non-linear) Slavnov-Taylor Identities.

Martin-SanchezRuiz (2000)
SanchezRuiz (2003)
BeluscaMaito et al. (2020-2021)

$$e^{i\Gamma_{\rm inv}[\xi_c,\Omega]} \equiv \int_{\rm 1PI} \mathcal{D}\xi \; e^{iS^{\rm Reg}[\xi+\xi_c,\Omega]+S^{\rm Div}_{\rm ct}[\xi+\xi_c,\Omega]+S^{\rm Fin}_{\rm ct}[\xi+\xi_c,\Omega]}$$

The “Divergent counterterms” are derived as usual:
Non-symmetric divergent 1PI diagrams have external Ω’s

The “Finite counterterms” are defined so that the result is symmetric even if $\Omega = 1$. How is it done concretely?

Iteratively:

Assume we have found the symmetry-restoring counterterm at order $\hbar^{n-1}$:

$$e^{i\Gamma[\xi_c,\Omega]|_{(n)}} = \int_{1\text{PI}} \mathcal{D}\xi \; e^{iS^{\text{Reg}}[\xi+\xi_c,\Omega]+S_{\text{ct}}^{\text{Div}}[\xi+\xi_c,\Omega]|_{(n)}+S_{\text{ct}}^{\text{Fin}}[\xi+\xi_c,\Omega]|_{(n-1)}}$$

$$\Gamma[\xi_c,1]|_{(n)}$$

This is the object that standard BMHV gives.
It has a spurious anomaly

$$\delta(\Gamma[\xi_c,1]|_{(n)}) = \mathcal{A}|_{(n)}$$

$$\Gamma_{\text{inv}}[\xi_c,1]|_{(n)} \equiv \Gamma[\xi_c,1]|_{(n)} + \Delta S_{\text{ct}}^{\text{Fin}}[\xi,1]|_{(n)}$$

We want to find this

On the other hand, with Ω turned on, all chiral symmetries are preserved:

$$\Gamma[\xi_c, \Omega]|_{(n)} = \Gamma_{\not{\Omega}}[\xi_c]|_{(n)} + \Gamma_{\Omega}[\xi_c, \Omega]|_{(n)}$$

No dependence on Ω

This is invariant ONLY because of Ω

$$\delta(\Gamma_{\not{\Omega}}[\xi_c]|_{(n)}) = 0$$

$$\delta(\Gamma_{\Omega}[\xi_c, \Omega]|_{(n)}) = 0$$

The ξ variation is compensated by that of Ω

They are separately invariant under the symmetries of the theory

What does this mean for the $\Omega = 1$ theory?!

$$\delta(\Gamma[\xi_c, 1]|_{(n)}) = \delta(\Gamma_\Omega[\xi_c, 1]|_{(n)}) = \mathcal{A}|_{(n)} \equiv -\delta(\Delta S^{\text{Fin}}_{\text{ct}}[\xi_c, 1]|_{(n)})$$

The “symmetry-restoring counterterm” is just the opposite of the Ω-dependent part of the 1PI action

$$\Delta S^{\text{Fin}}_{\text{ct}}[\xi_c, 1]|_{(n)} = -\Gamma_\Omega[\xi_c, 1]|_{(n)}$$

$$\Gamma_{\text{inv}}[\xi_c, 1]|_{(n)} = \Gamma_{\not\Omega}[\xi_c]|_{(n)}$$

Symmetry-restoring counterterms at 1-loop

Consider a general renormalizable gauge theory with scalars and fermions:

$$\mathcal{L} = \mathcal{L}_{\text{Bos}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{Kin}}$$

$$\mathcal{L}_{\text{Bos}} = -\frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu} + (D_\mu \phi)^\dagger_a (D^\mu \phi)_a - V(\phi)$$

$$\mathcal{L}_{\text{Yuk}} = -Y^a_{ij} \overline{\Psi}_i P_R \Psi_j \phi_a + \text{hc}$$

$$\mathcal{L}_{\text{Kin}} = \frac{1}{2} \left\{ \overline{\Psi} i \gamma^{\bar{\mu}} D_{\bar{\mu}} \Psi + \overline{\Psi} i \gamma^{\hat{\mu}} (\Omega^\dagger P_L + \Omega P_R) \partial_{\hat{\mu}} \Psi + \text{hc} \right\}$$

The non-symmetric counterterms (Finite and Divergent) are found among the operators involving Ω (analogy with pions in QCD!)

Operators with vectors and no Levi-Civita

$$\times \frac{1}{16\pi^2}$$

$\boldsymbol{D^4}$	
$\langle L_{\bar{\mu}\bar{\nu}} \Omega R^{\bar{\mu}\bar{\nu}} \Omega^\dagger \rangle$	0
$i\langle L_{\bar{\mu}\bar{\nu}} D^{\bar{\mu}}\Omega D^{\bar{\nu}}\Omega^\dagger + R_{\bar{\mu}\bar{\nu}} D^{\bar{\mu}}\Omega^\dagger D^{\bar{\nu}}\Omega \rangle$	$-\frac{1}{2}$
$\langle D_{\bar{\mu}}\Omega D^{\bar{\mu}}\Omega^\dagger D_{\bar{\nu}}\Omega D^{\bar{\nu}}\Omega^\dagger + D_{\bar{\mu}}\Omega^\dagger D^{\bar{\mu}}\Omega D_{\bar{\nu}}\Omega^\dagger D^{\bar{\nu}}\Omega \rangle$	$-\frac{1}{6}$
$\langle D_{\bar{\mu}}\Omega D_{\bar{\nu}}\Omega^\dagger D^{\bar{\mu}}\Omega D^{\bar{\nu}}\Omega^\dagger \rangle$	$+\frac{1}{12}$
$\langle D_{\bar{\mu}} D^{\bar{\mu}}\Omega D_{\bar{\nu}} D^{\bar{\nu}}\Omega^\dagger \rangle$	0
$\langle D_{\bar{\mu}} D_{\bar{\nu}}\Omega D^{\bar{\mu}} D^{\bar{\nu}}\Omega^\dagger \rangle$	$+\frac{1}{6}$

$\boldsymbol{\psi^2 D}$		$\boldsymbol{\psi^2 \phi}$	
$\overline{\Psi}\gamma^{\bar{\mu}} T_L^A \Omega i D_{\bar{\mu}}\Omega^\dagger T_L^A P_L \Psi + \text{P.c.}$	1	$[\overline{\Psi} T_R^A \Omega^\dagger \Phi \Omega^\dagger T_L^A P_L \Psi + \text{P.c.}] + \text{h.c.}$	-2
$\overline{\Psi}\gamma^{\bar{\mu}} Y^a \Omega^\dagger i D_{\bar{\mu}}\Omega [Y^a]^\dagger P_L \Psi + \text{P.c.}$	$\frac{1}{2}$		

Operators with fermions

$\boldsymbol{\phi^4}$		$\boldsymbol{\phi^2 D}$	
$\langle (\Phi\Omega^\dagger)^4 \rangle + \text{h.c.}$	$-\frac{1}{12}$	$\langle \Phi D_{\bar{\mu}}\Omega^\dagger \Phi D^{\bar{\mu}}\Omega^\dagger \rangle + \text{h.c.}$	$+\frac{1}{3}$
$\langle (\Phi\Omega^\dagger)^2 \Phi\Phi^\dagger \rangle + \text{h.c.}$	$-\frac{2}{3}$	$\langle (\Phi\Omega^\dagger)^2 D_{\bar{\mu}}\Omega D^{\bar{\mu}}\Omega^\dagger \rangle + \text{h.c.}$	$-\frac{1}{3}$
		$\langle \Phi\Phi^\dagger D_{\bar{\mu}}\Omega D^{\bar{\mu}}\Omega^\dagger + \Phi^\dagger\Phi D_{\bar{\mu}}\Omega^\dagger D^{\bar{\mu}}\Omega \rangle$	$-\frac{1}{3}$
		$\langle \Phi\Omega^\dagger D_{\bar{\mu}}\Omega \Phi^\dagger \Omega D^{\bar{\mu}}\Omega^\dagger \rangle$	$+\frac{1}{3}$
		$\langle D_{\bar{\mu}}\Phi D^{\bar{\mu}}\Omega^\dagger \Phi\Omega^\dagger + D_{\bar{\mu}}\Phi\Omega^\dagger \Phi D^{\bar{\mu}}\Omega^\dagger \rangle + \text{h.c.}$	$+\frac{2}{3}$
		$\langle D_{\bar{\mu}}\Phi\Omega^\dagger D^{\bar{\mu}}\Phi\Omega^\dagger \rangle + \text{h.c.}$	$+\frac{1}{6}$
		$\langle \Phi \overleftrightarrow{D}_{\bar{\mu}} \Phi^\dagger \Omega D^{\bar{\mu}}\Omega^\dagger + \Phi^\dagger \overleftrightarrow{D}_{\bar{\mu}} \Phi\Omega^\dagger D^{\bar{\mu}}\Omega \rangle$	$+\frac{1}{3}$

Operators with scalars and no fermions

$$\Phi_{ij} \equiv Y^a_{ij}\phi_a$$

Annoying
But systematic

Operators with Levi-Civita

As in the chiral Lagrangian, the only term involving Levi-Civita is the Wess-Zumino-Witten term:

$$S_{\rm ct}^{\rm Fin}[\xi,\Omega]\Big|_{\rm WZW} = \frac{n}{48\pi^2}\left\{\int {\rm d}^4x\ \epsilon^{\bar\mu\bar\nu\bar\alpha\bar\beta}\, Z_{\bar\mu\bar\nu\bar\alpha\bar\beta} + \dots\right\}$$

$$\begin{aligned} Z_{\bar\mu\bar\nu\bar\alpha\bar\beta} = \langle &-\Omega^\dagger\partial_{\bar\mu}L_{\bar\nu}L_{\bar\alpha}\Omega R_{\bar\beta} + \Omega\partial_{\bar\mu}R_{\bar\nu}R_{\bar\alpha}\Omega^\dagger L_{\bar\beta} \\ &- \partial_{\bar\mu}R_{\bar\nu}\Omega^\dagger L_{\bar\alpha}\Omega R_{\bar\beta} + \partial_{\bar\mu}L_{\bar\nu}\Omega R_{\bar\alpha}\Omega^\dagger L_{\bar\beta} \\ &+ i\Omega^\dagger L_{\bar\mu}L_{\bar\nu}L_{\bar\alpha}\Omega R_{\bar\beta} - i\Omega R_{\bar\mu}R_{\bar\nu}R_{\bar\alpha}\Omega^\dagger L_{\bar\beta} \\ &+ \frac{i}{2}\Omega^\dagger L_{\bar\mu}\Omega R_{\bar\nu}\Omega^\dagger L_{\bar\alpha}\Omega R_{\bar\beta} + \mathcal{O}(\partial\Omega)\rangle, \end{aligned}$$

Here n=1 cannot be affected by radiative corrections: exact at all orders

In the Standard Model (excluding H, for simplicity):

— QCD & QED are vector-like and manifest

— no terms with Levi-Civita, peculiarity of SU(2)xU(1)

— Contains all interactions that respect QCD & QED but violate SU(2)xU(1)

VVDD: $D_\mu W^-_\nu D^\mu W^{+\nu}$ $\quad \partial_\mu Z_\nu \partial^\mu Z^\nu$

VVVD: $iF^{\mu\nu} W^+_\mu W^-_\nu$ $\quad iD^\mu W^-_\mu W^+_\nu Z^\nu$ $\quad iD^\nu W^-_\mu W^+_\nu Z^\mu$ $\quad iD_\nu W^-_\mu W^{+\mu} Z^\nu$ $\quad +\mathrm{hc}$

VVVV: $(W^-_\mu W^{+\mu})^2$ $\quad (W^-_\mu W^{-\mu})(W^+_\nu W^{+\nu})$ $\quad (Z_\mu Z^\mu)^2$ $\quad (W^+_\mu Z^\mu)(W^-_\nu Z^\nu)$ $\quad (W^+_\mu W^{-\mu})(Z_\nu Z^\nu)$

ffW: $W^+_\mu \overline{f_u} \gamma^\mu P_L f_d$ $\quad W^+_\mu \overline{f_u} \gamma^\mu P_R f_d$ $\quad +\mathrm{hc}$

ffZ: $Z_\mu \overline{f} \gamma^\mu P_L f$ $\quad Z_\mu \overline{f} \gamma^\mu P_R f$ $\quad +\mathrm{hc}$

Example, fermion-gauge counterterms in the Standard Model:

$$
\begin{aligned}
- \quad \frac{g^3}{16\pi^2} \Bigg\{ & \frac{9-t_w^2}{36\sqrt{2}} \left[\bar{u}_L \gamma^\mu W_\mu^+ d_L + \bar{d}_L \gamma^\mu W_\mu^- u_L\right] \\
& + \frac{9-t_w^2}{72 c_w} \left[\bar{u}_L \gamma^\mu Z_\mu u_L - \bar{d}_L \gamma^\mu Z_\mu d_L\right] \\
& + \frac{1-t_w^2}{4\sqrt{2}} \left[\bar{\nu}_L \gamma^\mu W_\mu^+ e_L + \bar{e}_L \gamma^\mu W_\mu^- \nu_L\right] \\
& + \frac{1-t_w^2}{8 c_w} \left[\bar{\nu}_L \gamma^\mu Z_\mu \nu_L - \bar{e}_L \gamma^\mu Z_\mu e_L\right] \\
& + \frac{2 t_w^2}{9\sqrt{2}} \left[\bar{u}_R \gamma^\mu W_\mu^+ d_R + \bar{d}_R \gamma^\mu W_\mu^- u_R\right] \\
& - \frac{t_w^2}{18 c_w} \left[4 \bar{u}_R \gamma^\mu Z_\mu u_R - \bar{d}_R \gamma^\mu Z_\mu d_R\right] \\
& + \frac{t_w^2}{2 c_w} \bar{e}_R \gamma^\mu Z_\mu e_R \Bigg\} .
\end{aligned}
$$

Cornella-Feruglio-LV (2022)

Spurious d-dimensional Lorentz

Technically, the $SU(N)_L \times SU(N)_R \times U(1)_A$ symmetry is broken by $\gamma^{\hat{\mu}}$, an invariant tensor of $H = SU(3)_{L+R}$: $\gamma^{\hat{\mu}} = h\gamma^{\hat{\mu}}h^\dagger$.

The axial part is recovered introducing a coset representative

$$\sqrt{\Omega} \to L\sqrt{\Omega}h^\dagger = h\sqrt{\Omega}R^\dagger$$

and defining $\sigma = \sqrt{\Omega^\dagger}P_L + \sqrt{\Omega}P_R$, so that

$$\sigma\gamma^{\hat{\mu}}\sigma = \Omega^\dagger P_L + \Omega P_R$$

is covariant.

Technically, the $SO(1,d-1) \to SO(1,3) \times SO(d-4)$ symmetry is broken by γ_5, an invariant tensor of $H = SO(1,3)$: $\gamma_5 = S(\Lambda_4)\gamma_5 S^{-1}(\Lambda_4)$.

Lorentz is recovered introducing a coset representative

$$\Omega' \to \Lambda\Omega'\Lambda_4^{-1}(\Pi')$$

and defining $\Gamma_5 = S(\Omega')\gamma_5 S^{-1}(\Omega') \to S(\Lambda)\Gamma_5 S^{-1}(\Lambda)$, so that

$$\mathscr{P}_L = \frac{1}{2}(1-\Gamma_5),\ \mathscr{P}_R = \frac{1}{2}(1+\Gamma_5) \text{ and } \Sigma = \sqrt{\Omega^\dagger}\mathscr{P}_L + \sqrt{\Omega}\mathscr{P}_R$$

are covariant.

The following fermionic action is fully symmetric:

$$\int d^d x \left[\frac{1}{2}\overline{\Psi}_j \Sigma\gamma^\mu \Sigma i\mathcal{D}_\mu \Psi_j + Y_{ij}^a \overline{\Psi}_i \mathcal{P}_R \Psi_j \phi_a + \mathrm{hc}\right]$$

Conclusions

- ☑ BMHV is the only rigorous approach: safely automatized
- ☑ A spurious symmetry can be restored → <u>some order!</u>
- ☑ Outcome:
  - * Very efficient way of determining the symmetry-restoring counterterms
  - * Some of these counterterms are 1-loop exact
  - * …
- ☑ Much still to be done…