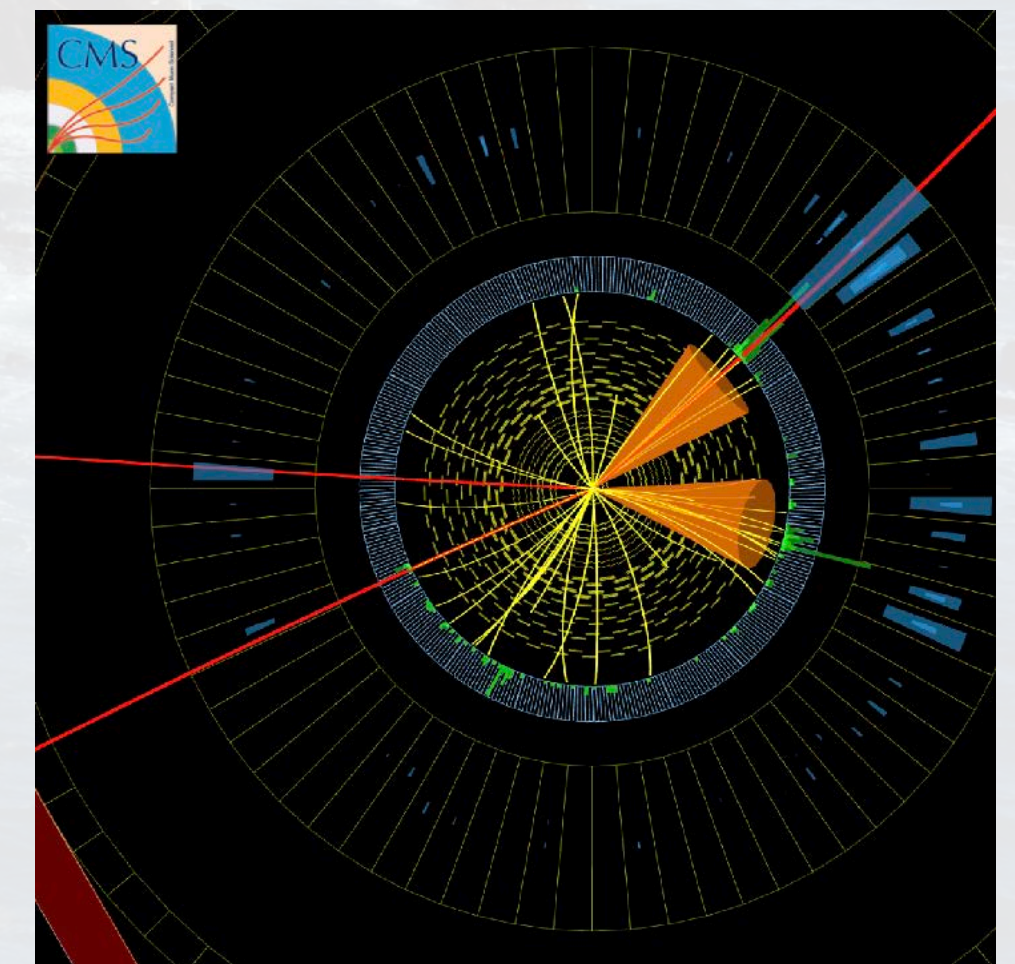
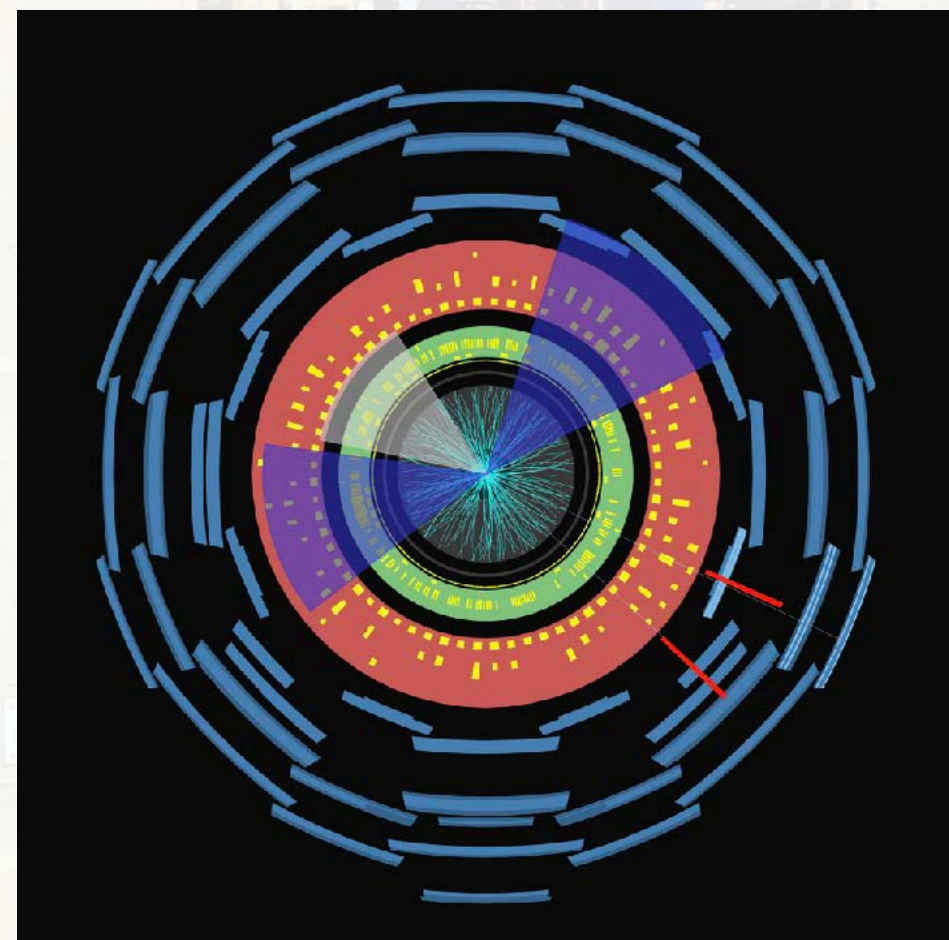
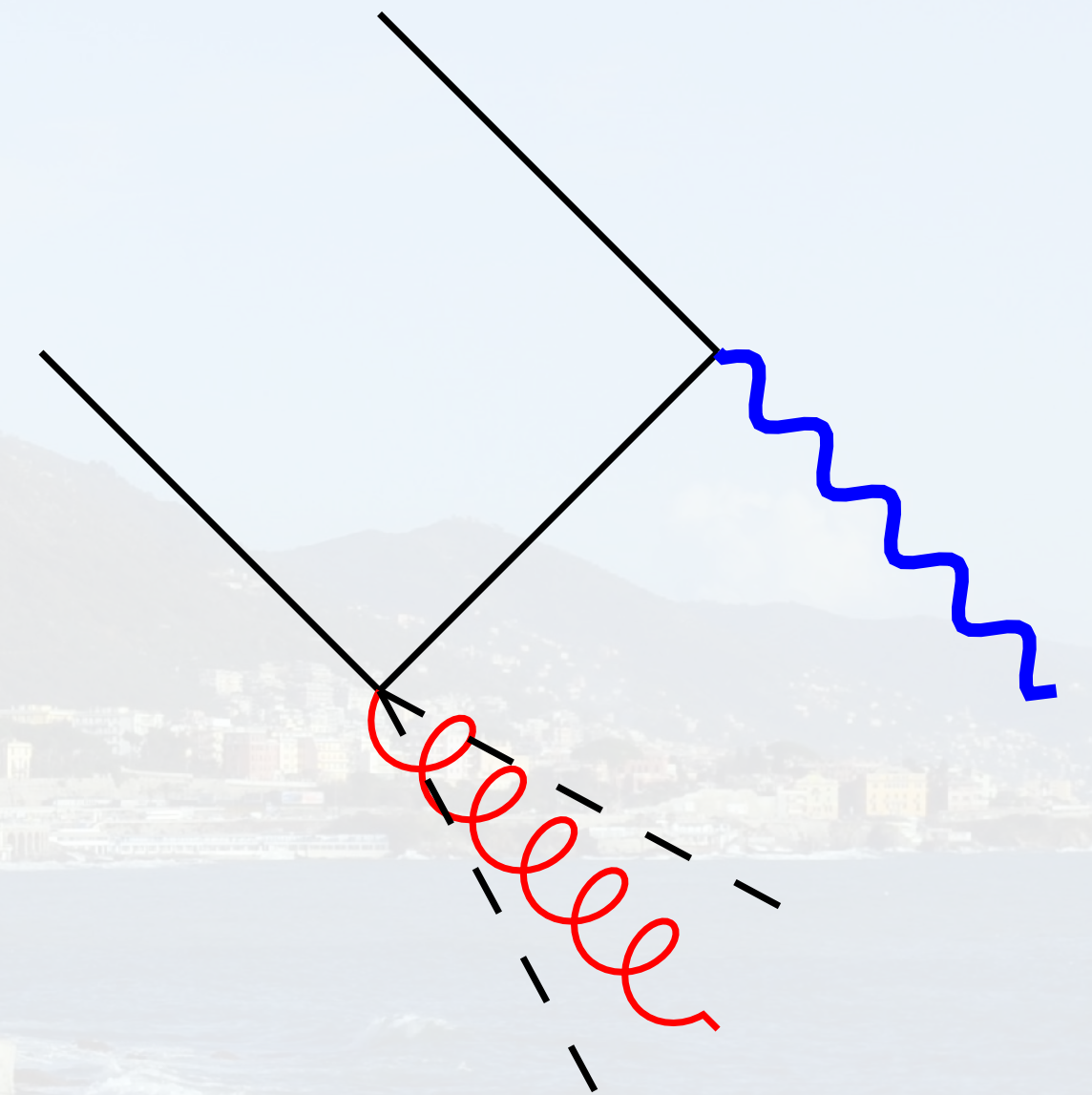
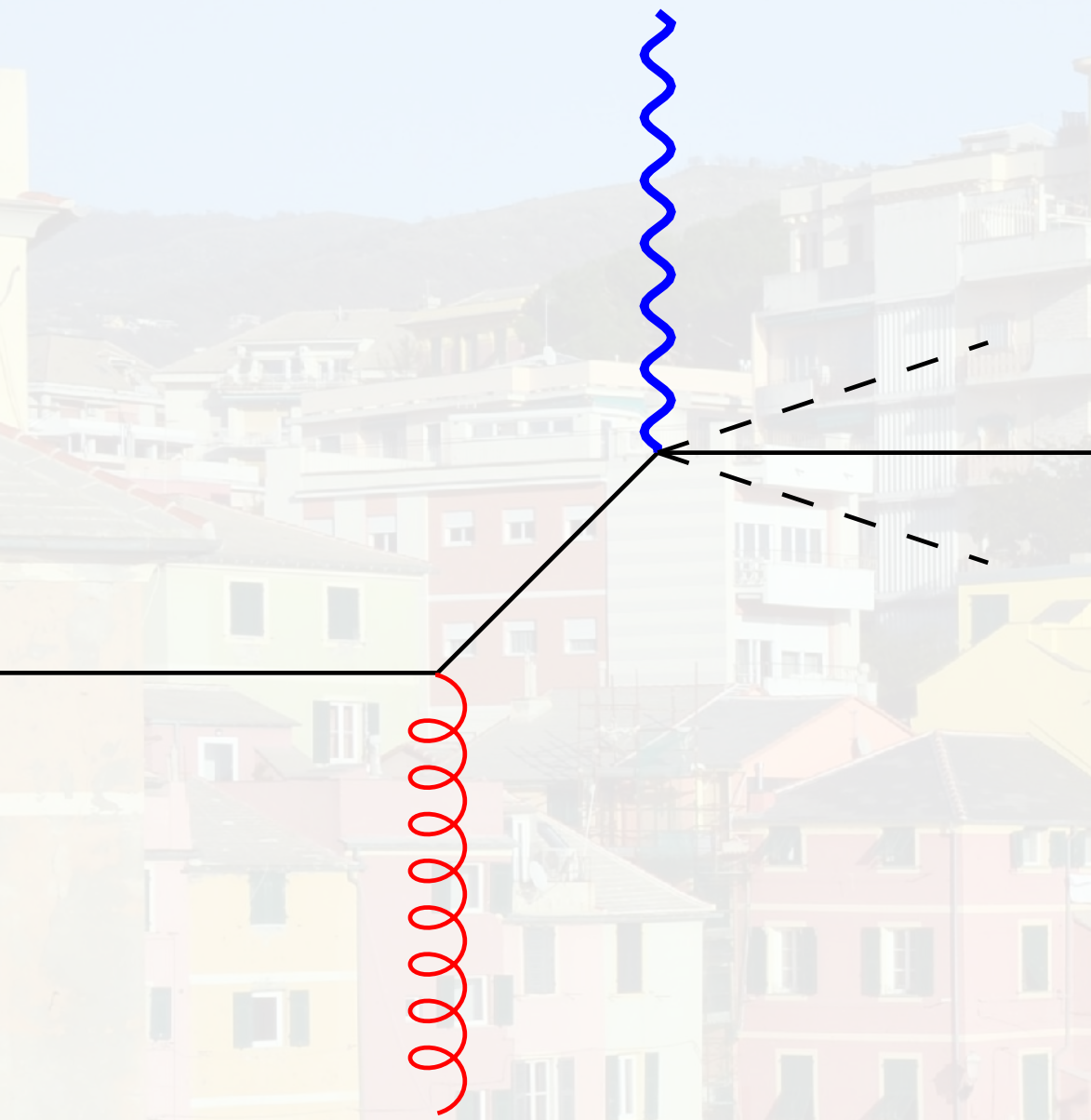


# Precise predictions for $V$ +jet production

Giovanni Stagnitto  
(Milano Bicocca University & INFN)



Seminari di Fisica Teorica, Genova, 26/02/2025





# Outlook of this talk

1. (Rather long and pedagogical) introduction
2. Flavoured jets at the LHC:  
 $Z+c$ -jet and  $W+c$ -jet production
3. Towards NNLO+PS for  $V$ +jet:  
improving slicing methods for  $V$ +jet production

*Biased selection of recent results where I personally contributed.  
Minimal inclusion of references, apologies for any relevant omission.*

# Outlook of this talk

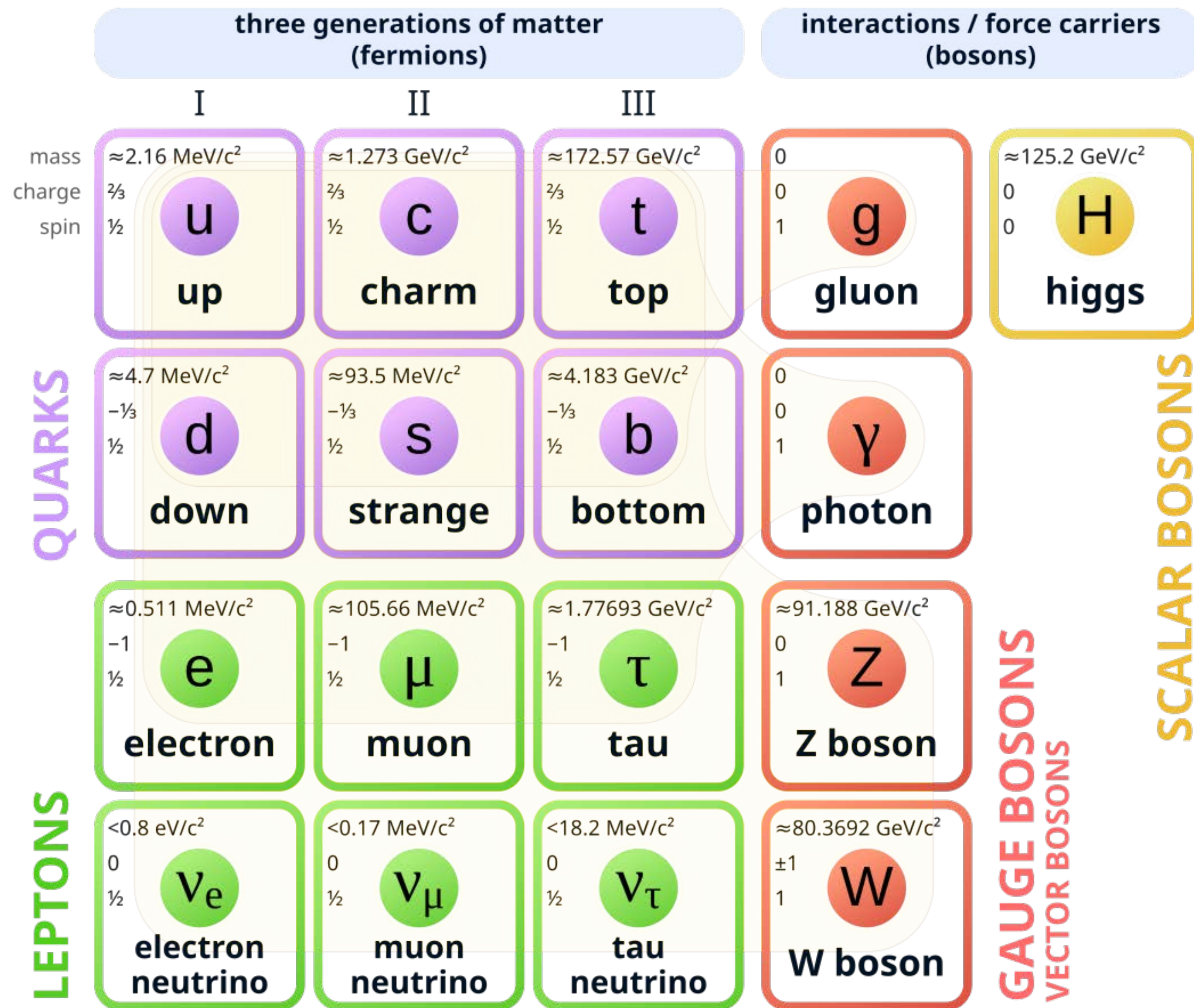
1. (Rather long and pedagogical) introduction

2. Flavoured jets at the LHC:  
 $Z+c$ -jet and  $W+c$ -jet production

3. Towards NNLO+PS for  $V$ +jet:  
improving slicing methods for  $V$ +jet production

*Biased selection of recent results where I personally contributed.  
Minimal inclusion of references, apologies for any relevant omission.*

# Standard Model of Elementary Particles



**Very successful theory, BUT:**

Open *experimental* puzzles:

- what is dark matter?
- origin of neutrino masses?
- what is dark energy?
- baryogenesis?
- ...

Open *theory* puzzles:

- origin of EWSB?
- hierarchy problem?
- origin of flavour?
- ...

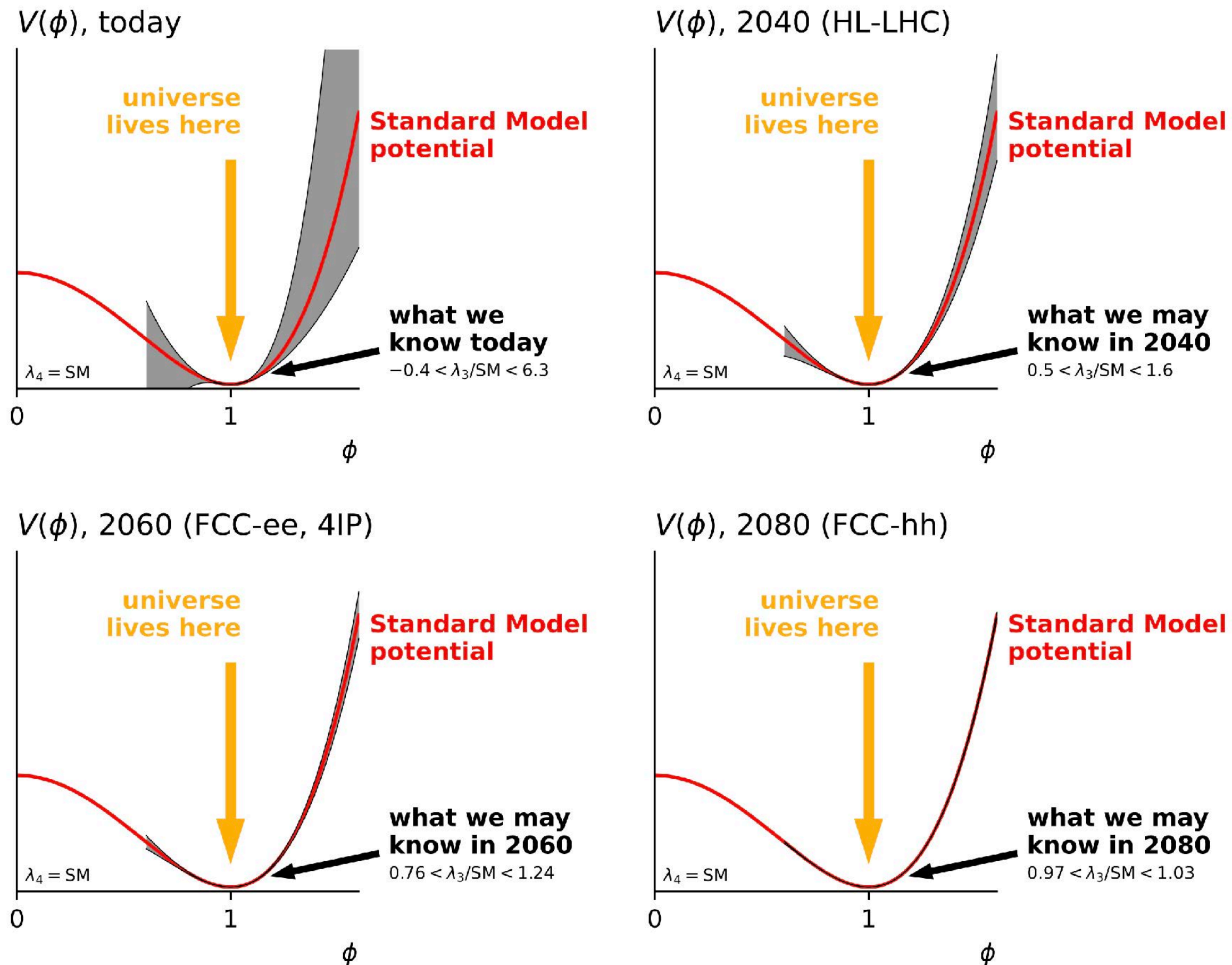
Plethora of Beyond Standard Model (BSM) scenarios to offer solutions



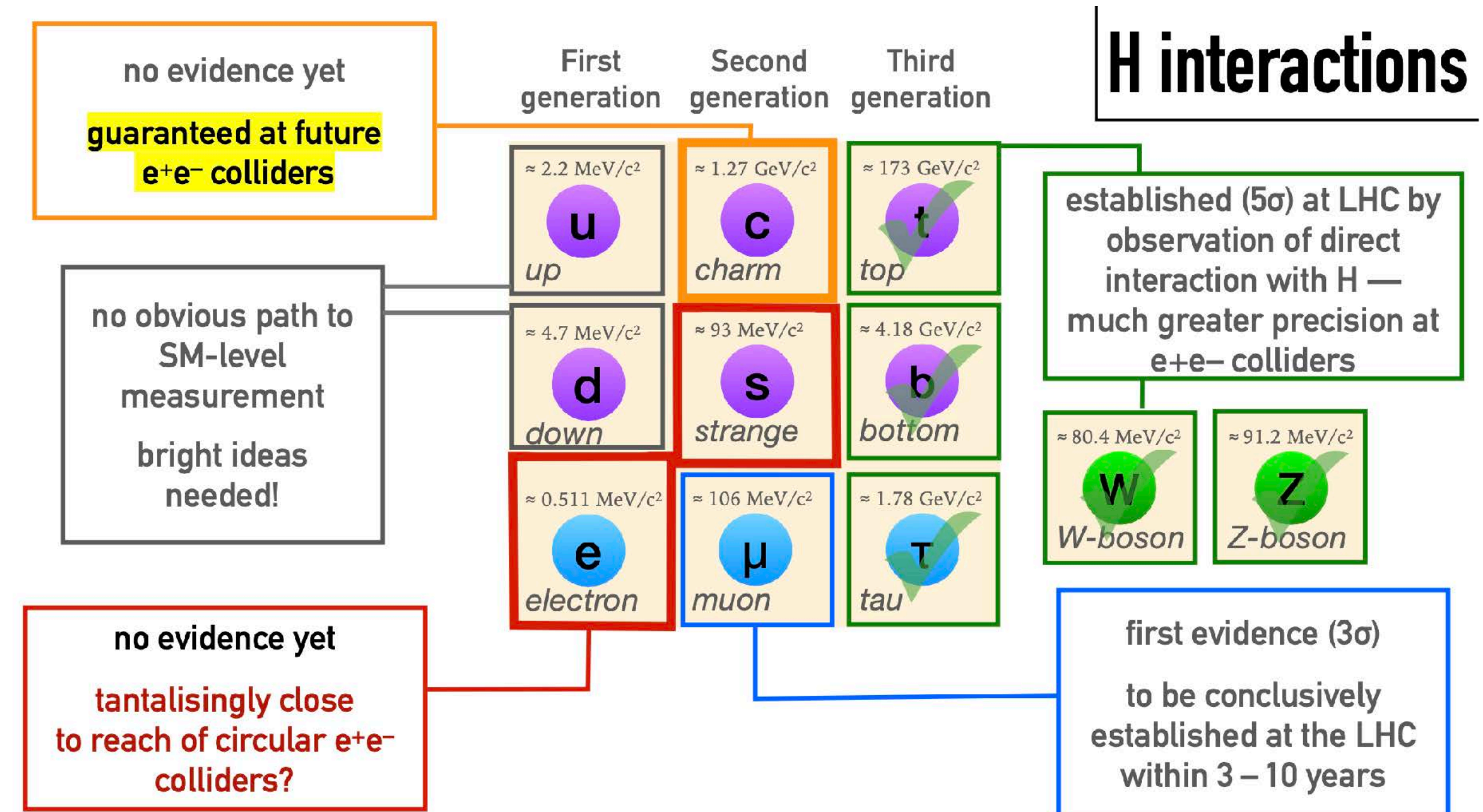
# Higgs physics: a “guaranteed discovery”

We have just started the exploration of the Higgs sector of the SM

## Higgs potential



## Yukawa interactions

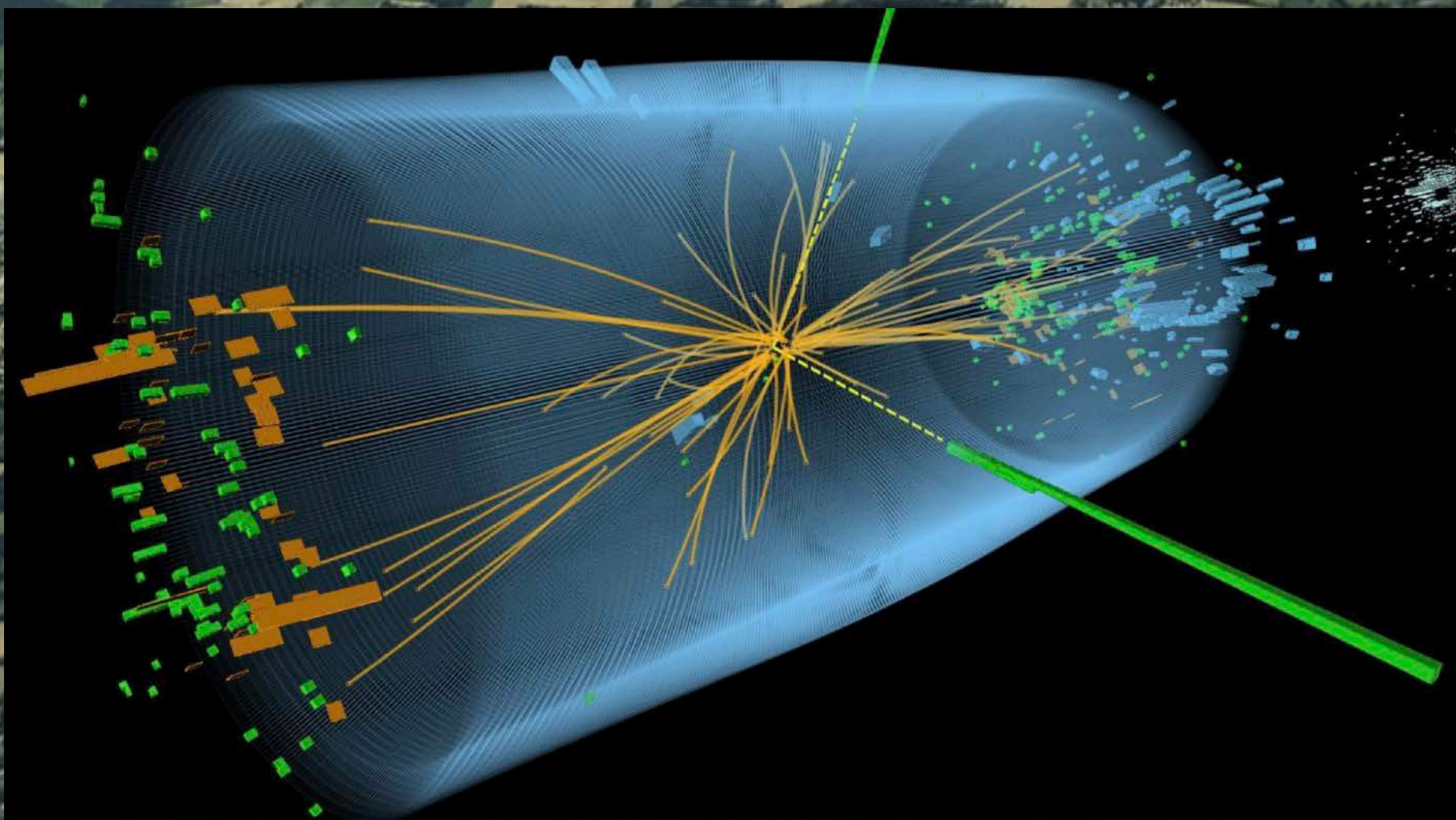
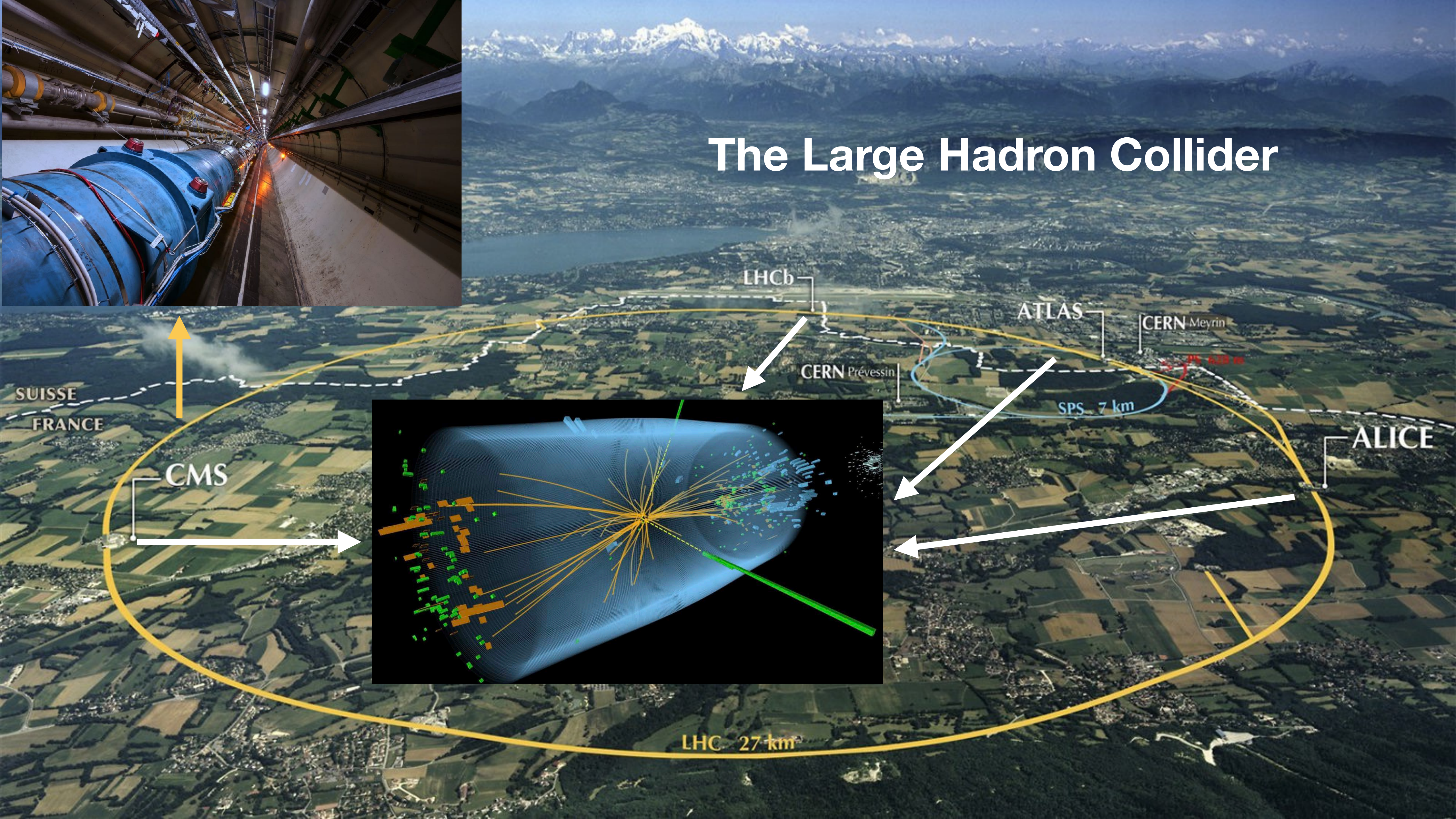


G. Salam





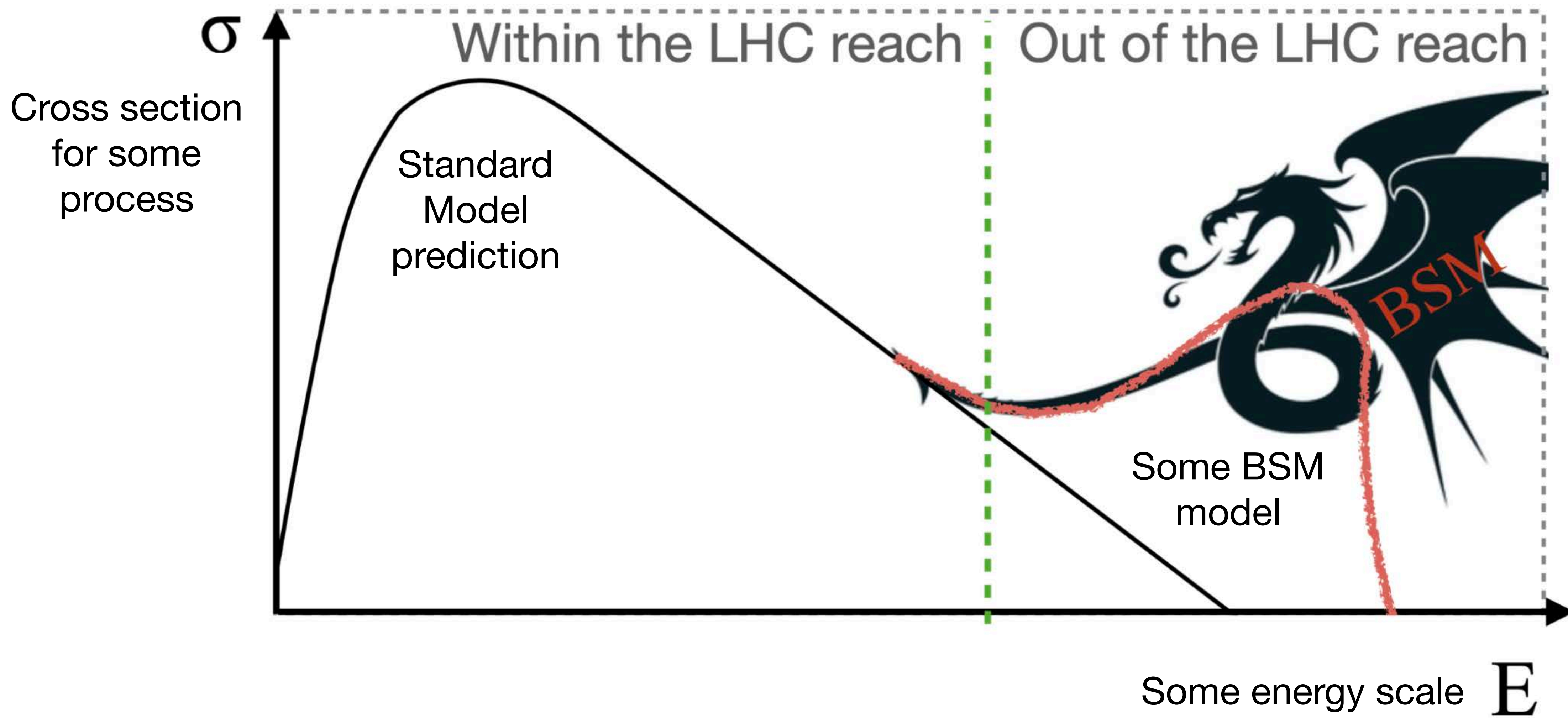
# The Large Hadron Collider





# Why precise Standard Model Phenomenology is important?

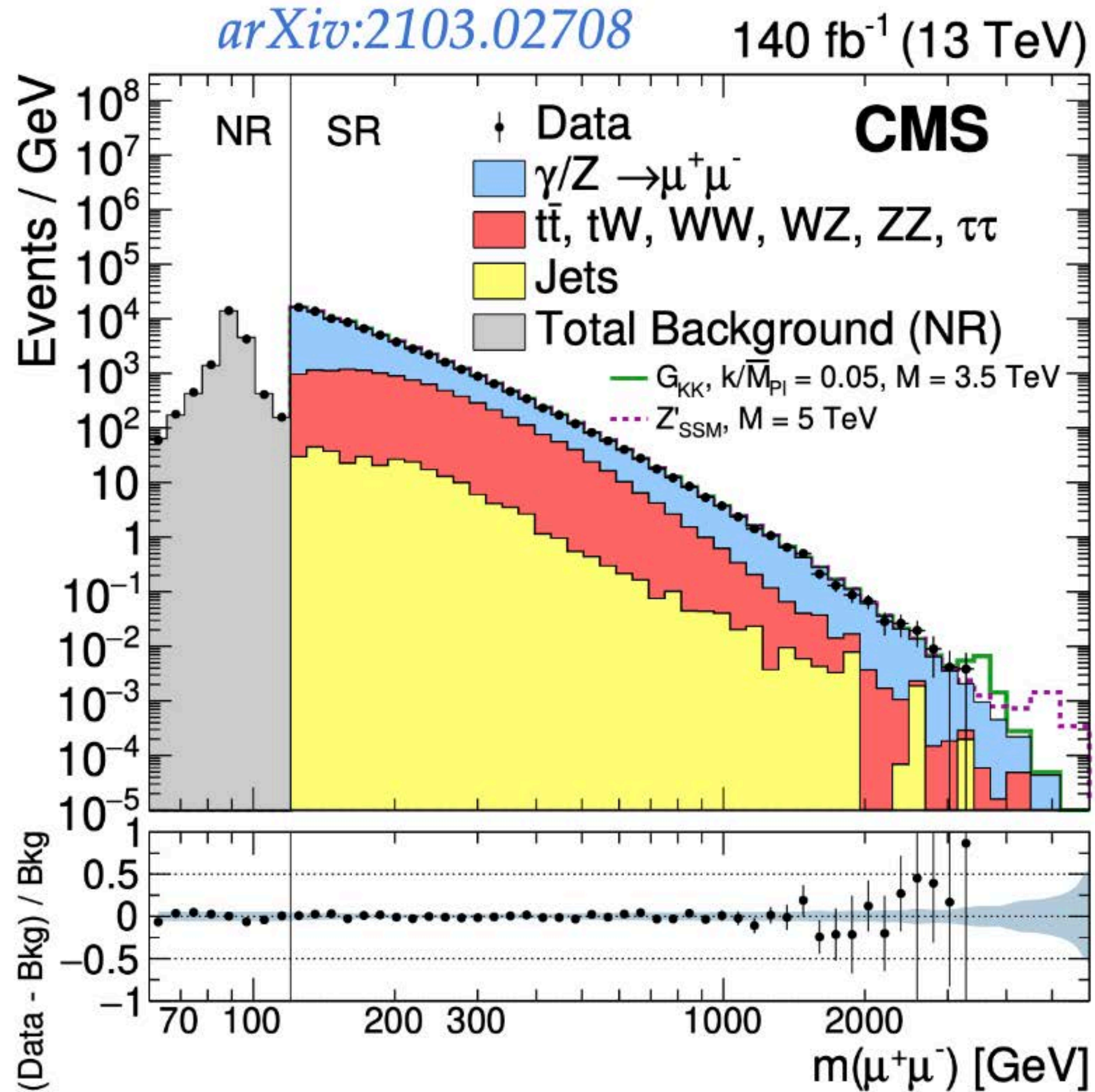
*Direct searches*





# Why precise Standard Model Phenomenology is important?

## Direct searches



mass window [GeV]	stat. unc. 140fb <sup>-1</sup>	stat. unc. 3ab <sup>-1</sup>
600 < m <sub>μμ</sub> < 900	1.4%	0.2%
900 < m <sub>μμ</sub> < 1300	3.2%	0.6%



# Why precise Standard Model Phenomenology is important?

Indirect searches e.g. value of mass of  $W$ -boson

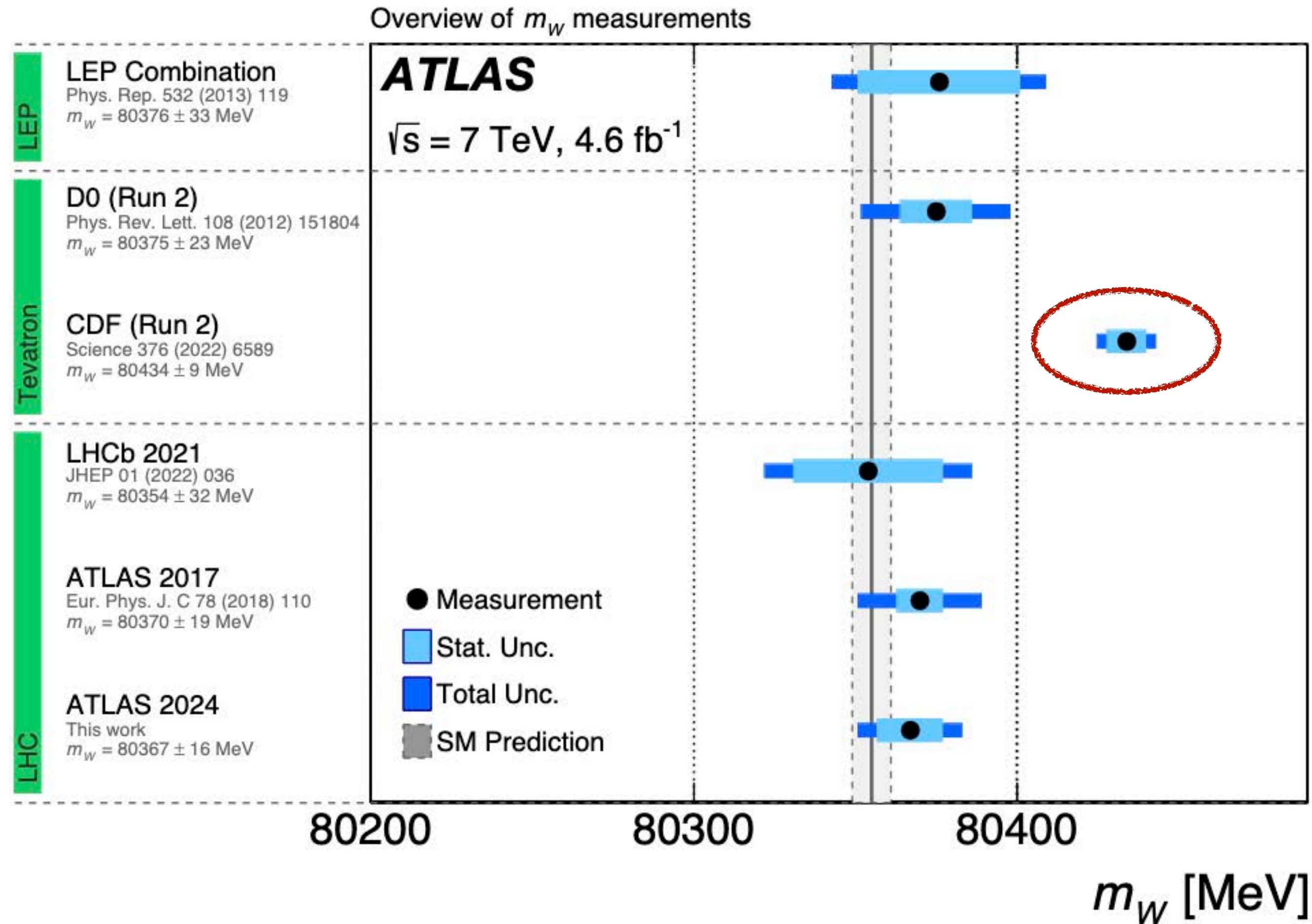


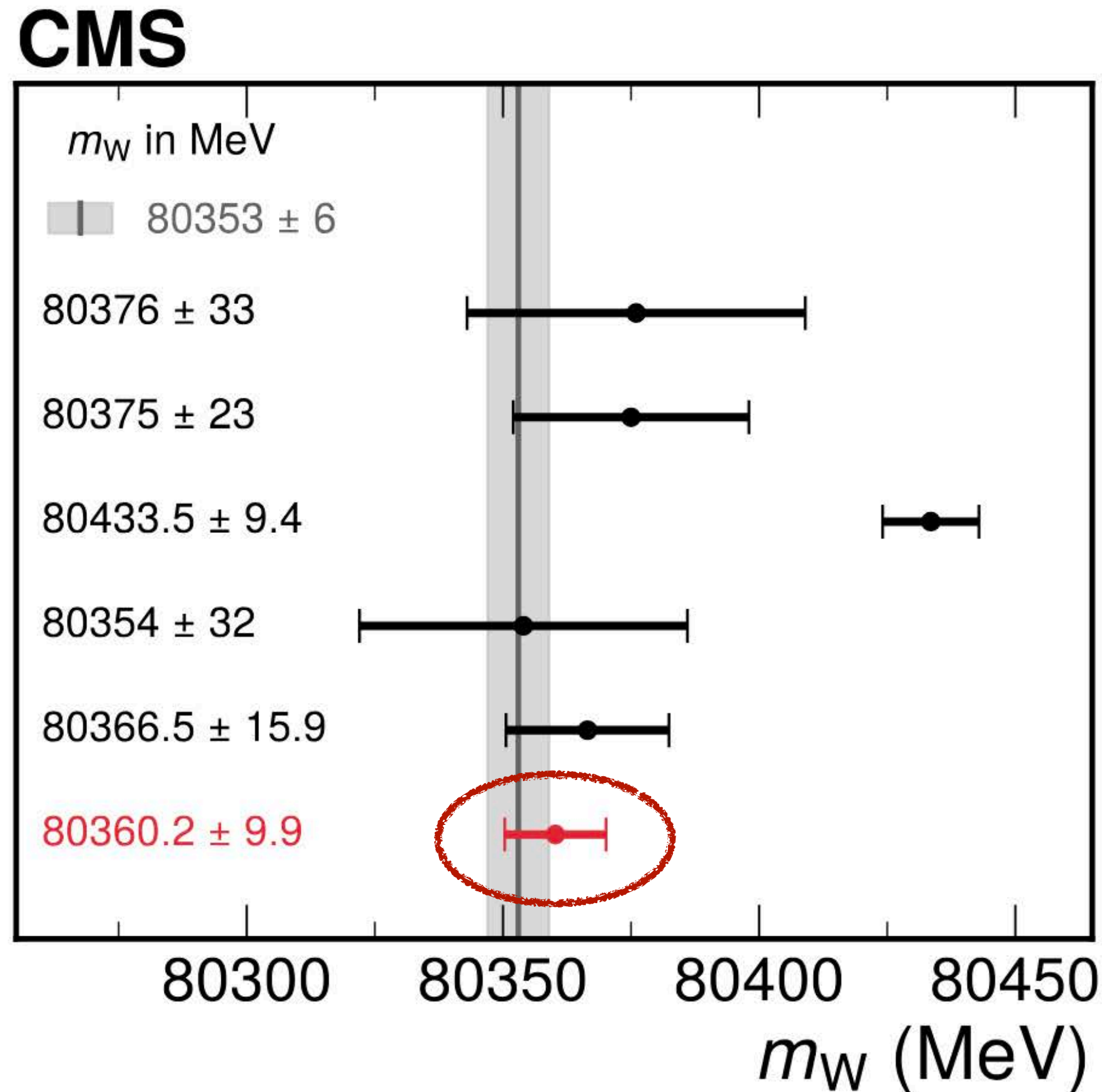
Table 2. Uncertainties on the combined  $M_W$  result. **CDFII uncertainties**

Source	Uncertainty (MeV)	
Lepton energy scale	3.0	Exp. systematics
Lepton energy resolution	1.2	
Recoil energy scale	1.2	
Recoil energy resolution	1.8	
Lepton efficiency	0.4	
Lepton removal	1.2	
Backgrounds	3.3	Th. systematics
$p_T^Z$ model	1.8	
$p_T^W/p_T^Z$ model	1.3	
Parton distributions	3.9	
QED radiation	2.7	
$W$ boson statistics	6.4	Statistics
Total	9.4	



# Why precise Standard Model Phenomenology is important?

*Indirect searches e.g. value of mass of  $W$ -boson*



Electroweak fit  
PRD 110 (2024) 030001

LEP combination  
Phys. Rep. 532 (2013) 119

D0  
PRL 108 (2012) 151804

CDF  
Science 376 (2022) 6589

LHCb  
JHEP 01 (2022) 036

ATLAS  
arXiv:2403.15085

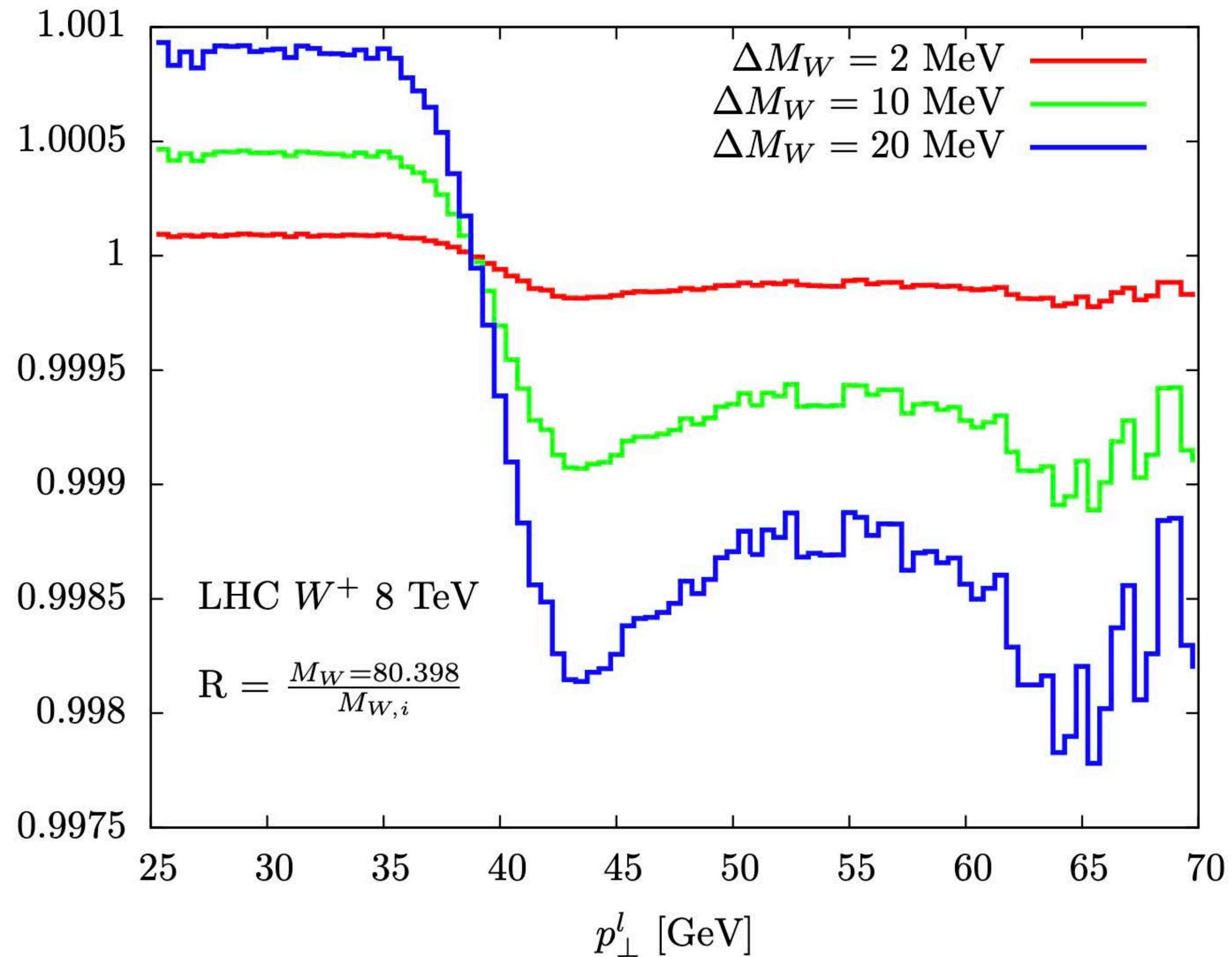
**CMS**  
*This work*

Source of uncertainty	Impact (MeV)			
	Nominal		Global	
	in $m_Z$	in $m_W$	in $m_Z$	in $m_W$
Muon momentum scale	5.6	4.8	5.3	4.4
Muon reco. efficiency	3.8	3.0	3.0	2.3
W and Z angular coeffs.	4.9	3.3	4.5	3.0
Higher-order EW	2.2	2.0	2.2	1.9
$p_T^V$ modeling	1.7	2.0	1.0	0.8
PDF	2.4	4.4	1.9	2.8
Nonprompt-muon background	—	3.2	—	1.7
Integrated luminosity	0.3	0.1	0.2	0.1
MC sample size	2.5	1.5	3.6	3.8
Data sample size	6.9	2.4	10.1	6.0
Total uncertainty	13.5	9.9	13.5	9.9



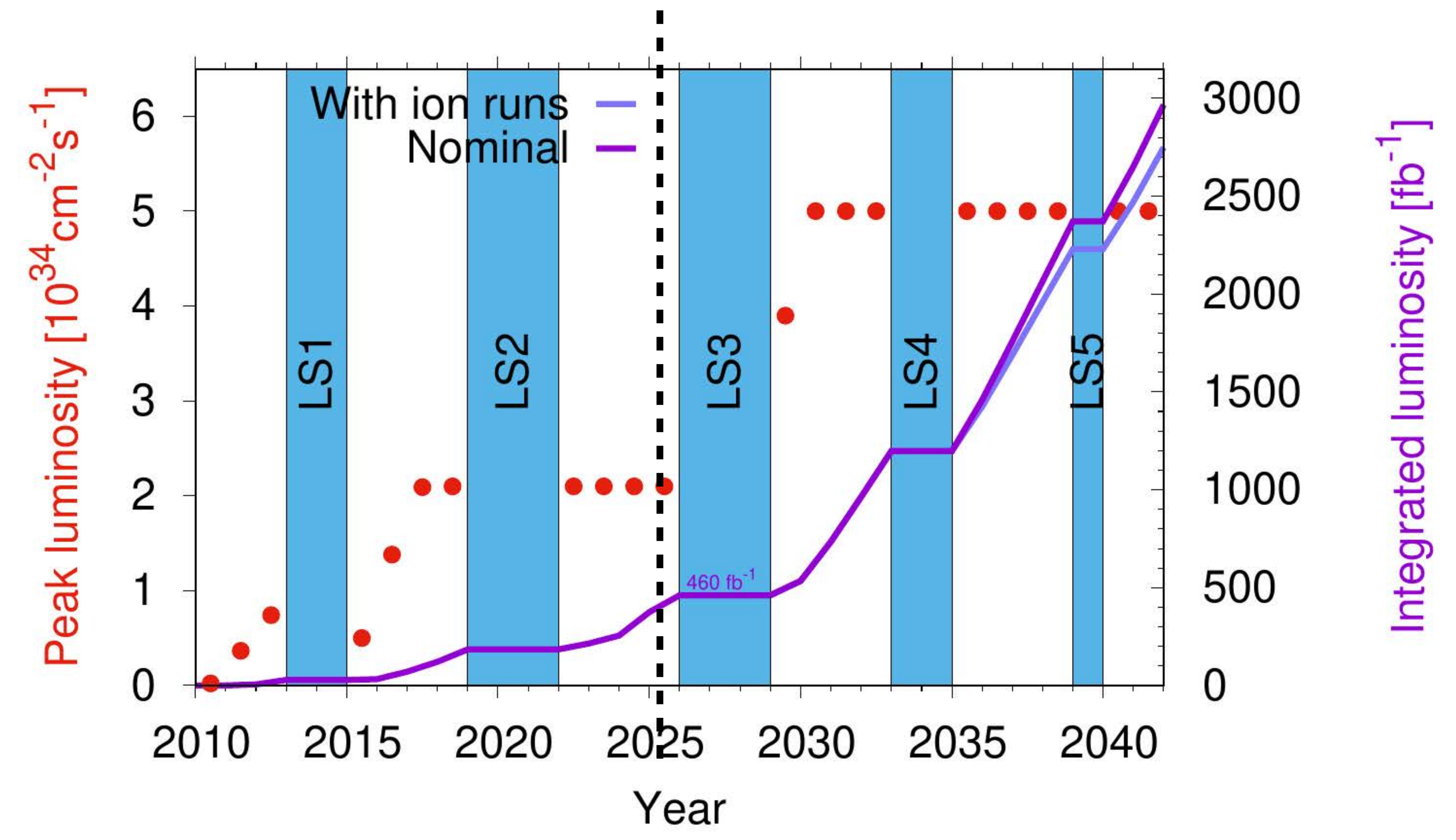
# Why precise Standard Model Phenomenology is important?

*Indirect searches e.g. value of mass of  $W$ -boson*



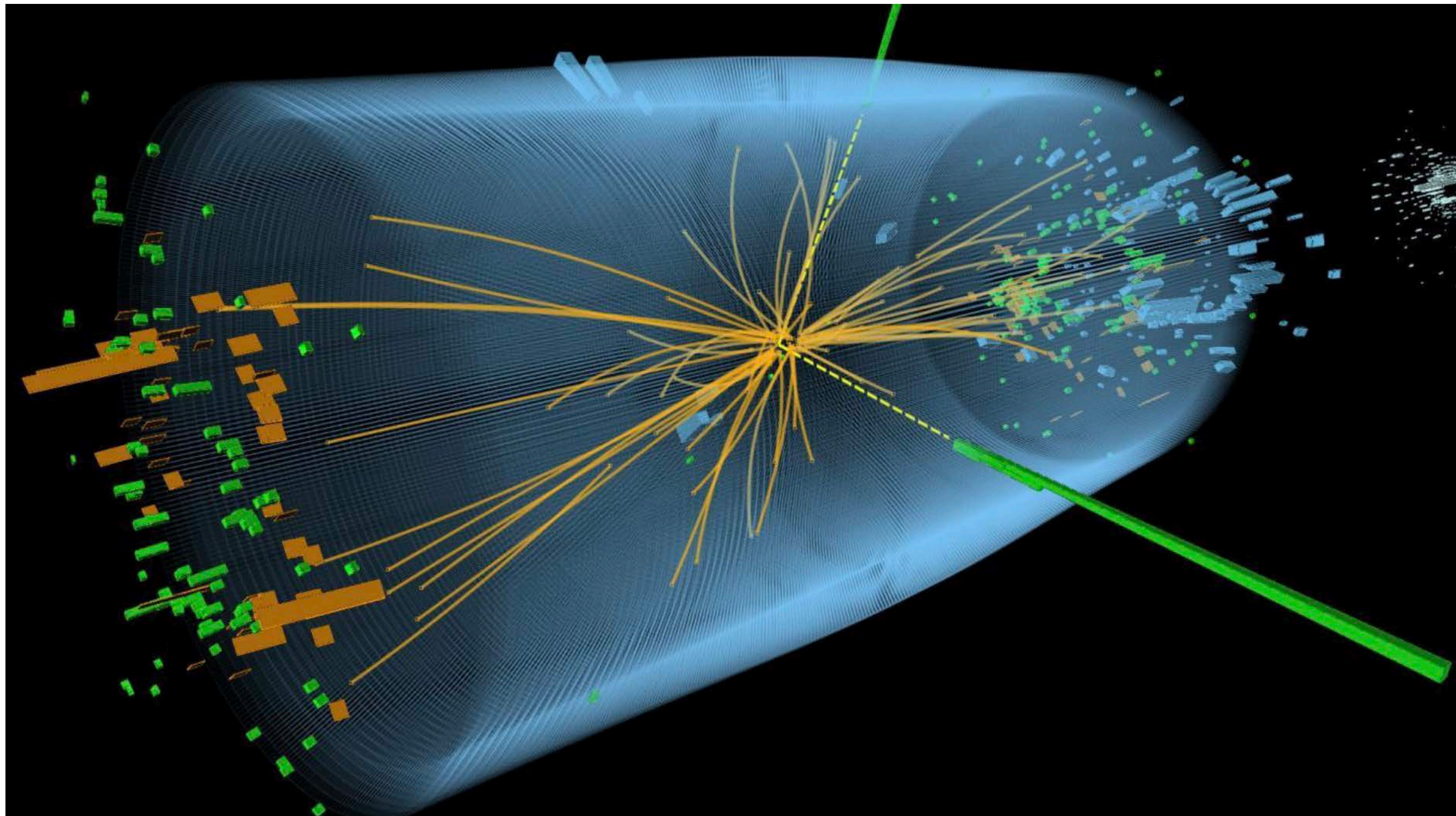


**~90% of LHC collisions yet to be delivered!**



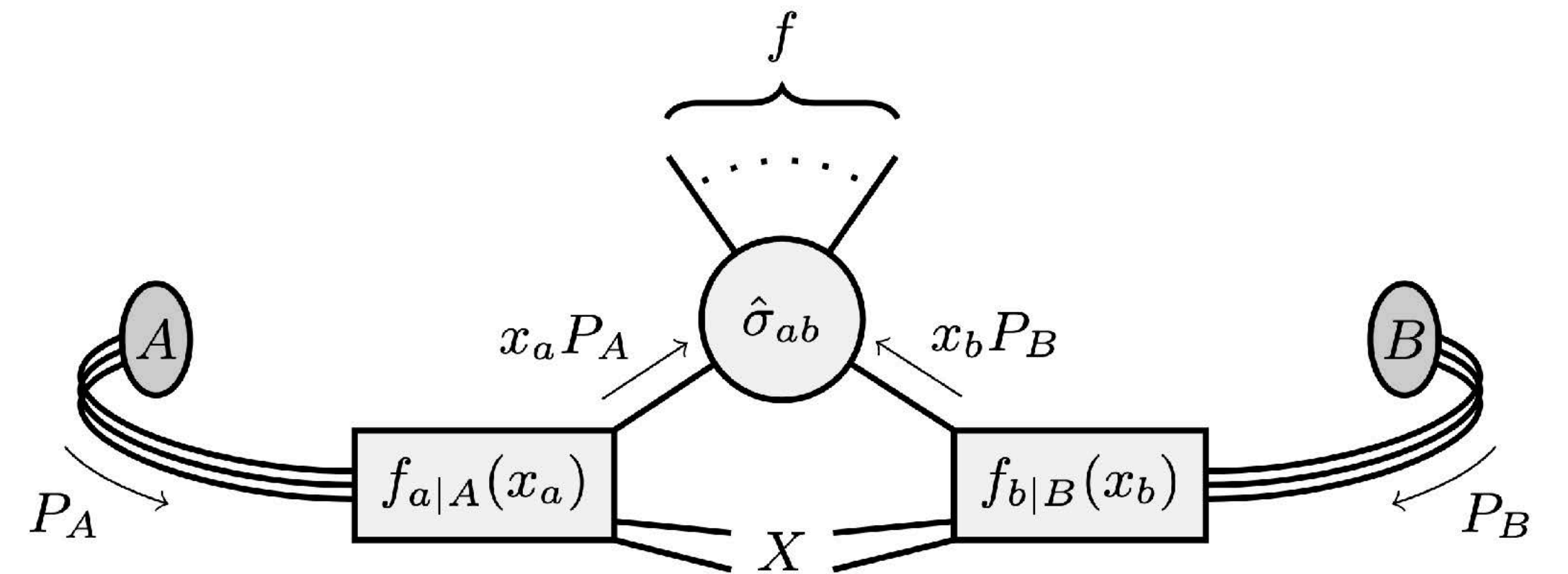
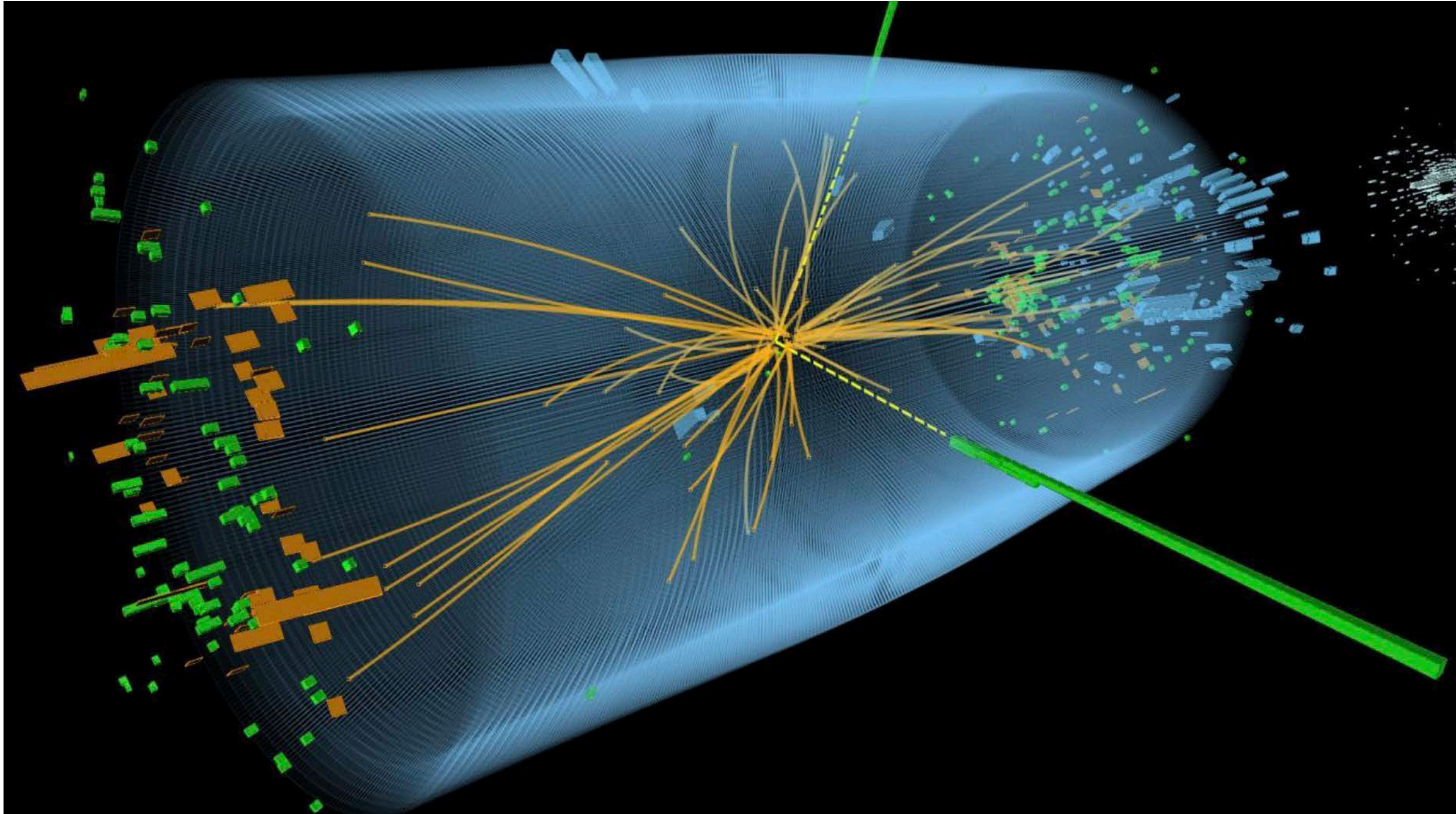


# How can we describe collider events?





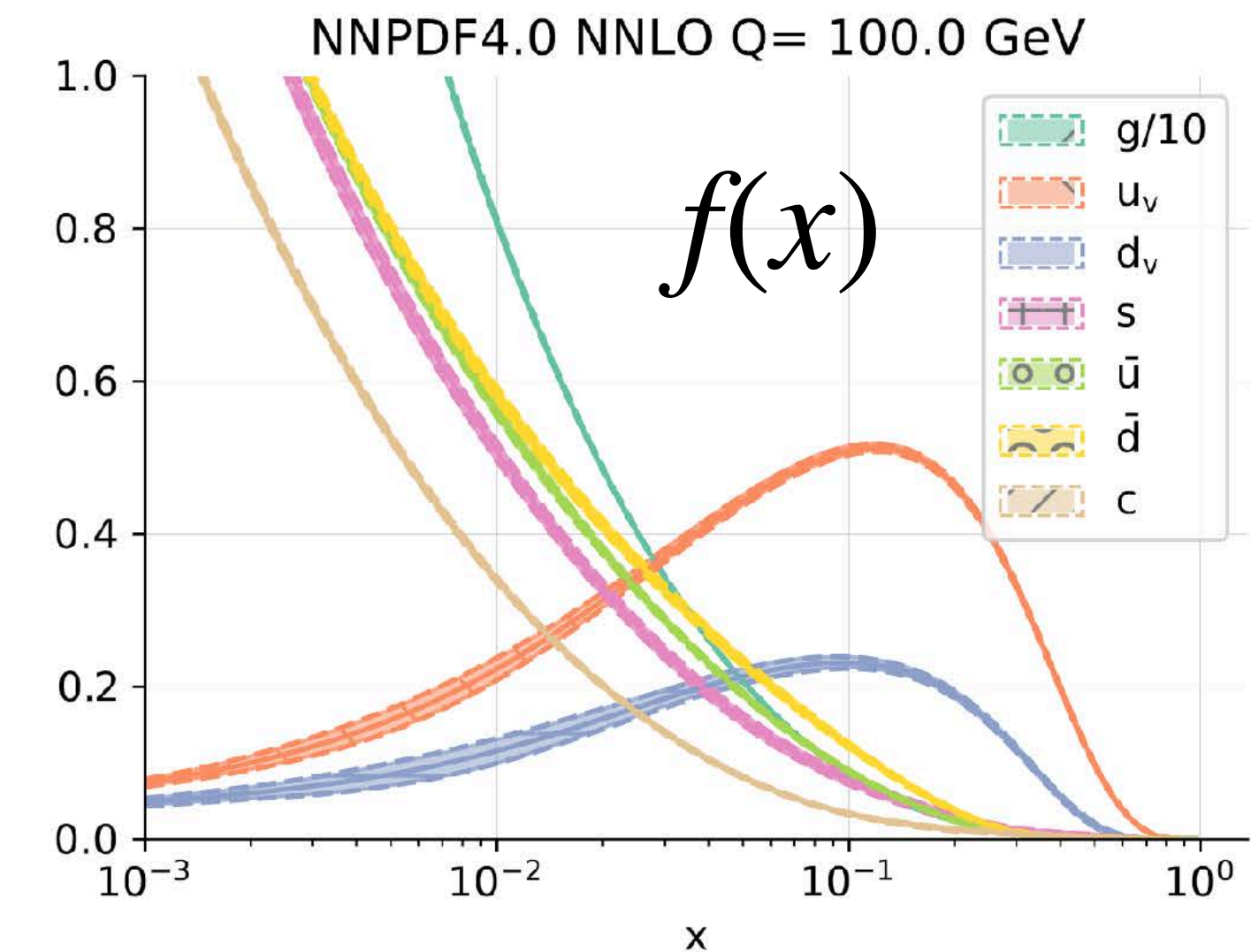
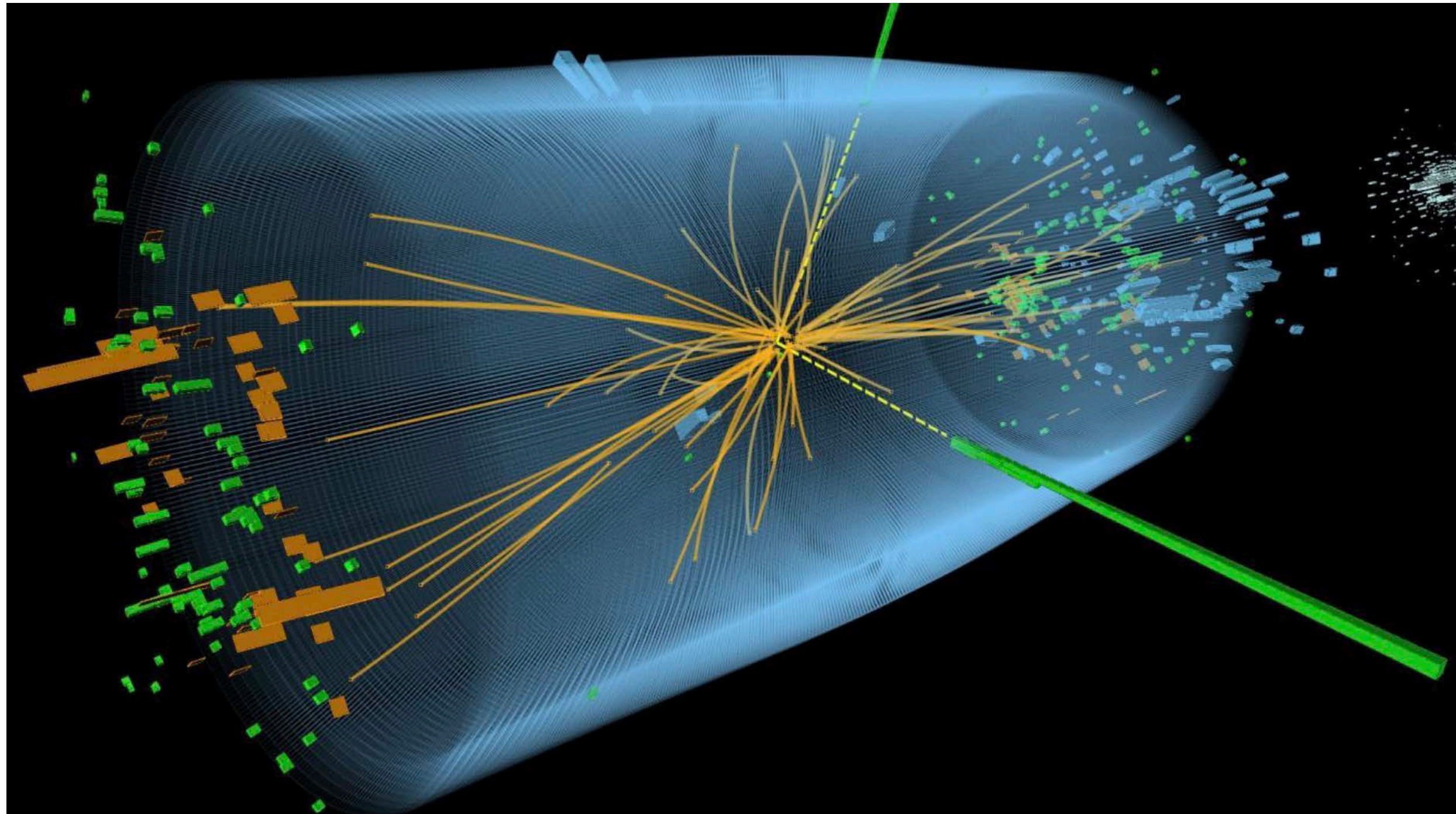
# Collider events and their theoretical description



$$d\sigma_{AB} = \sum_{ab} f_{a|A} \otimes f_{b|B} \otimes d\hat{\sigma}_{ab}$$

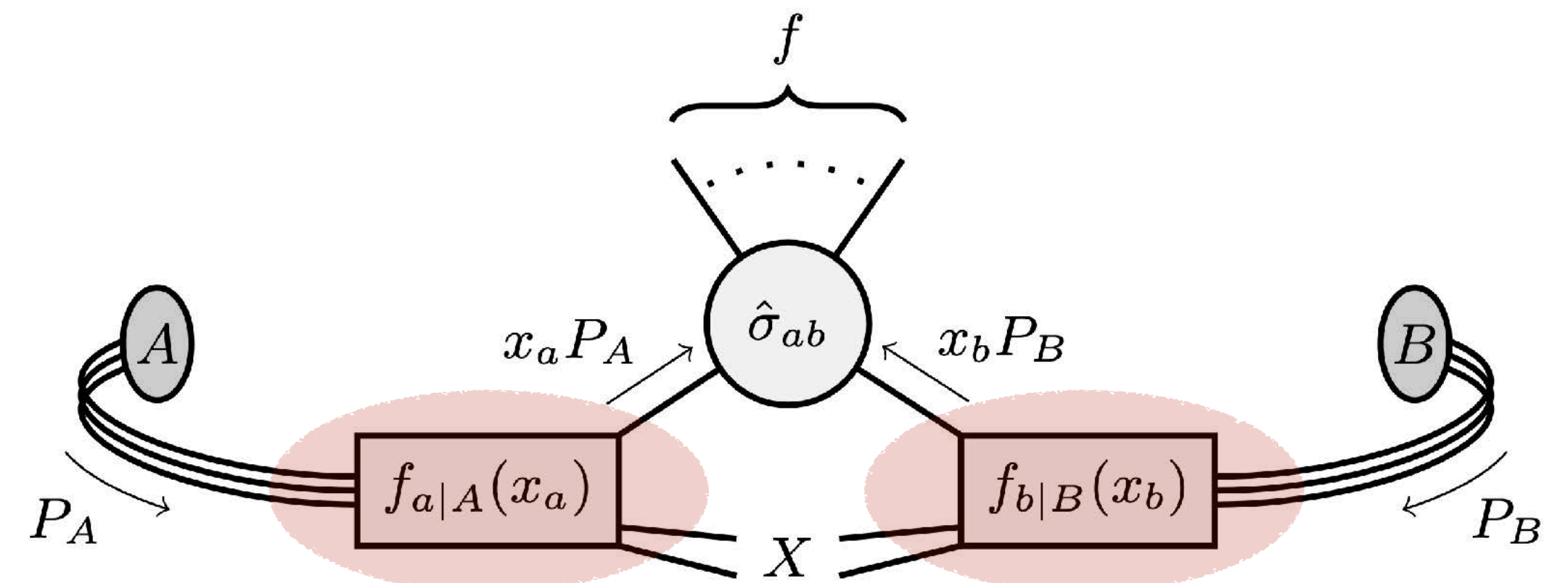


# Collider events and their theoretical description (1/3)



## Parton Distributions Functions (PDFs)

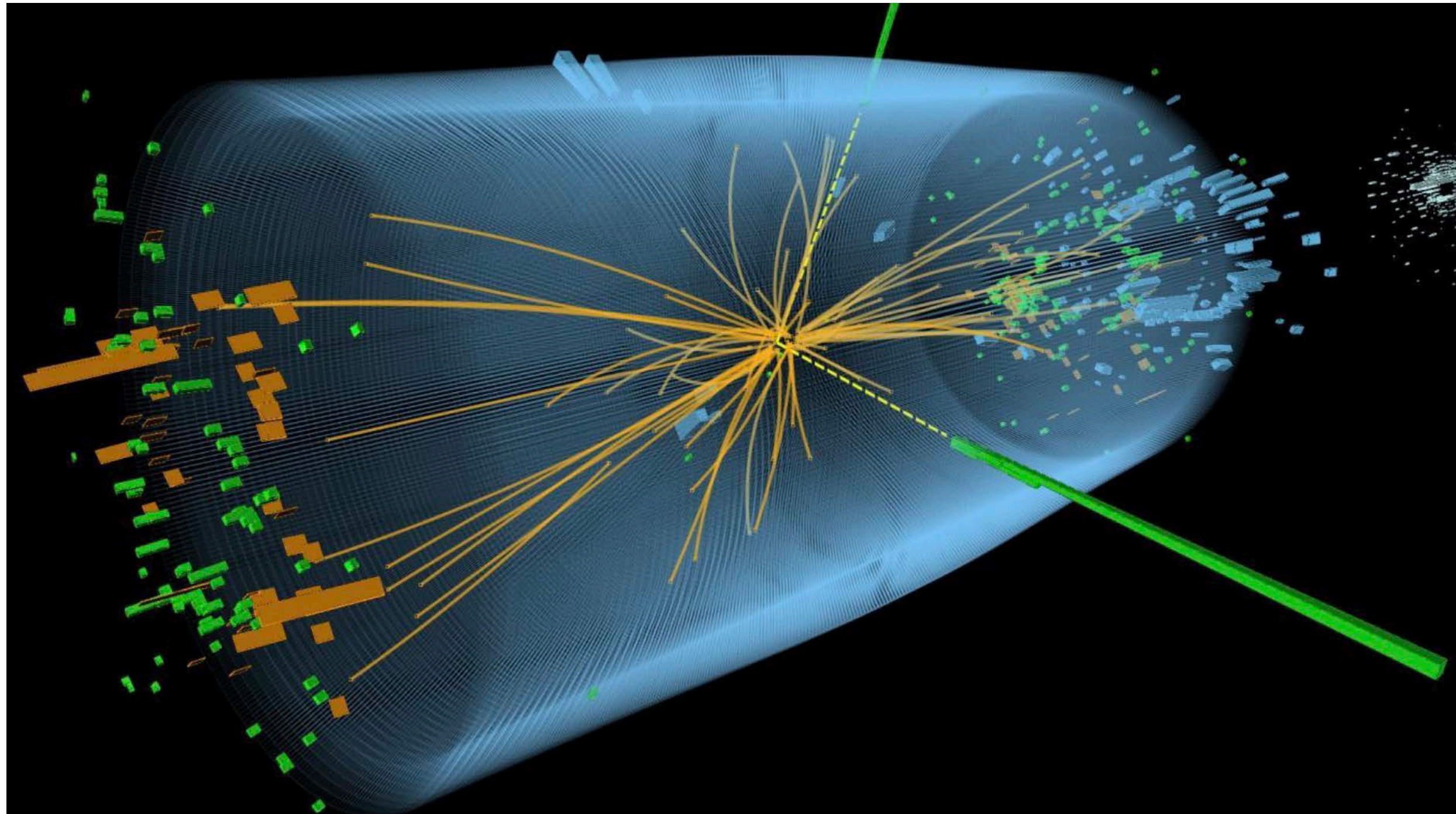
Non-perturbative functions describing momentum distribution of quark and gluons inside the proton



$$d\sigma_{AB} = \sum_{ab} \underline{f_{a|A}} \otimes \underline{f_{b|B}} \otimes d\hat{\sigma}_{ab}$$



# Collider events and their theoretical description (2/3)

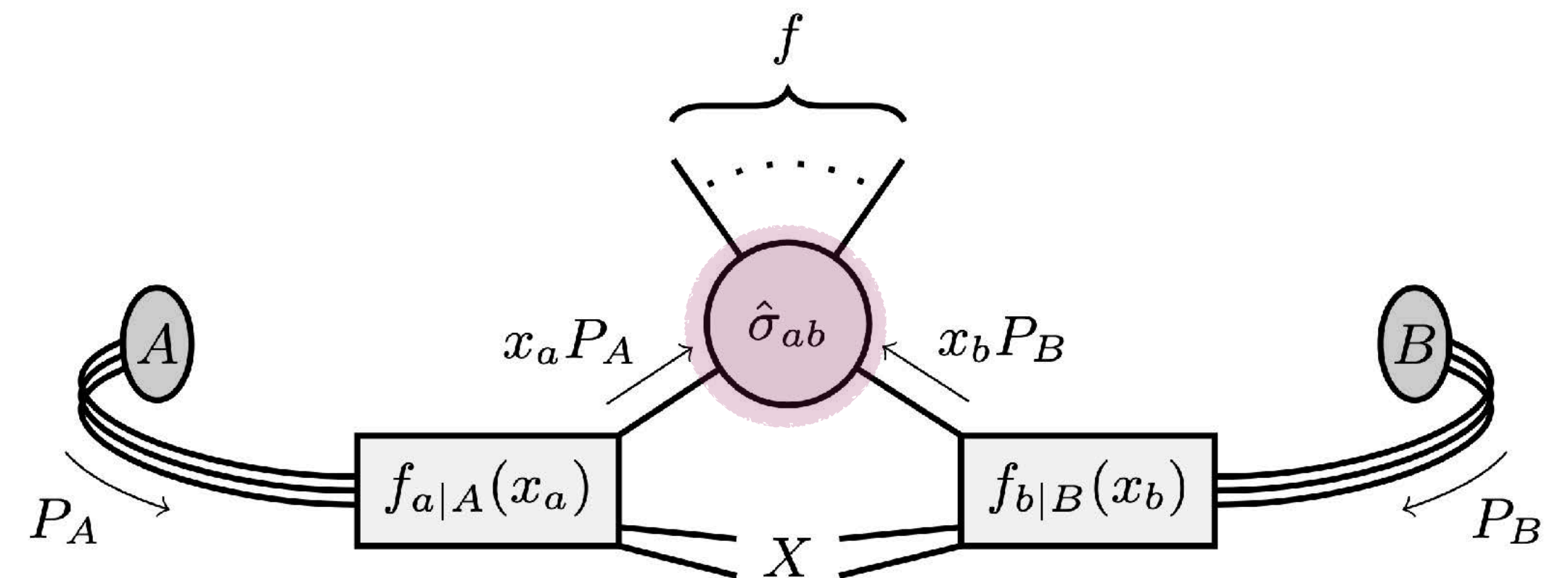


## Short-distance cross section

The strong coupling constant  $\alpha_s$  is small at high energies,  $\alpha_s(Q \sim 100 \text{ GeV}) \sim 0.1$ , so we can work in perturbation theory

$$\begin{array}{cccc}
 \text{LO} & \text{NLO} & \text{NNLO} & \text{N}^3\text{LO} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \hat{\sigma} = \hat{\sigma}_0 + \alpha_s \hat{\sigma}_1 + \alpha_s^2 \hat{\sigma}_2 + \alpha_s^3 \hat{\sigma}_3 + \mathcal{O}(\alpha_s^4)
 \end{array}$$

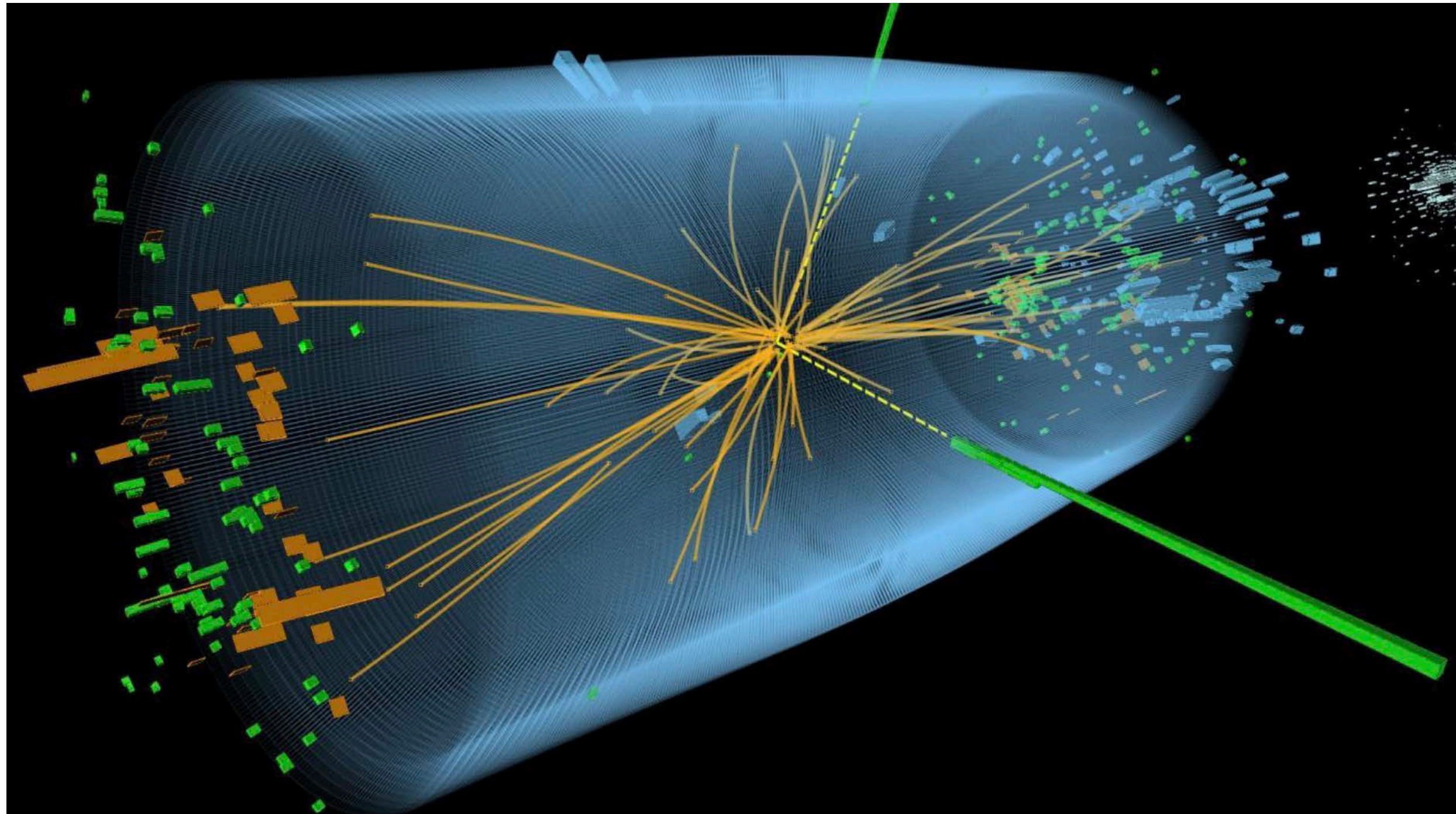
$\hat{\sigma}_k$  is the  $\text{N}^k\text{LO}$  cross section



$$d\sigma_{AB} = \sum_{ab} f_{a|A} \otimes f_{b|B} \otimes d\hat{\sigma}_{ab}$$

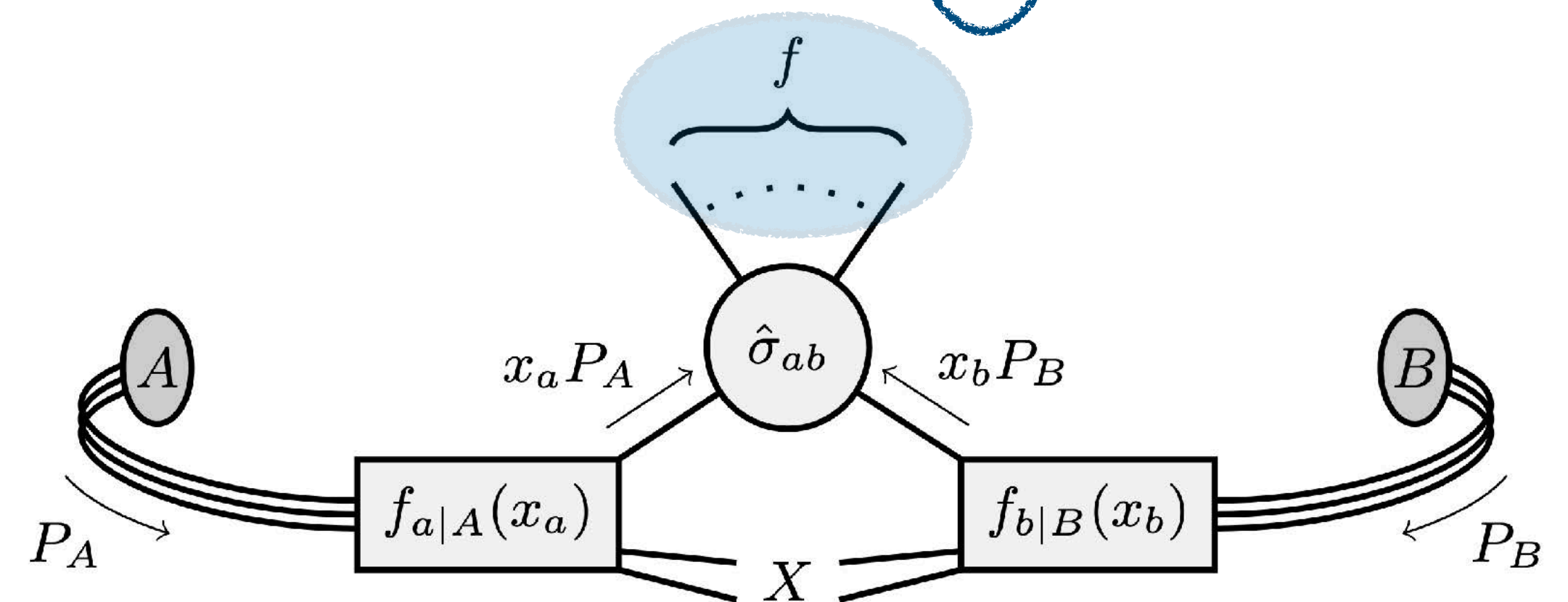
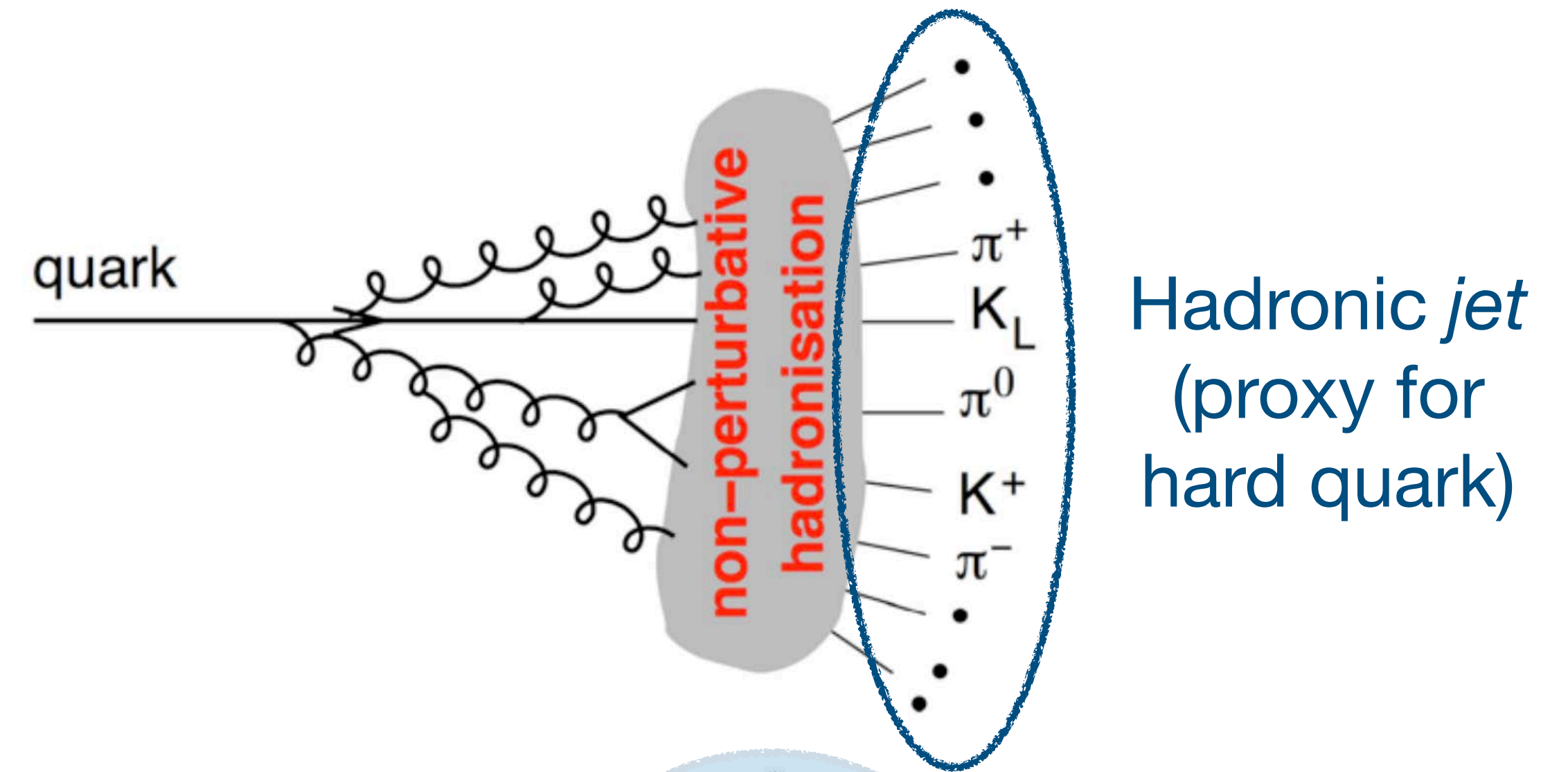


# Collider events and their theoretical description (3/3)



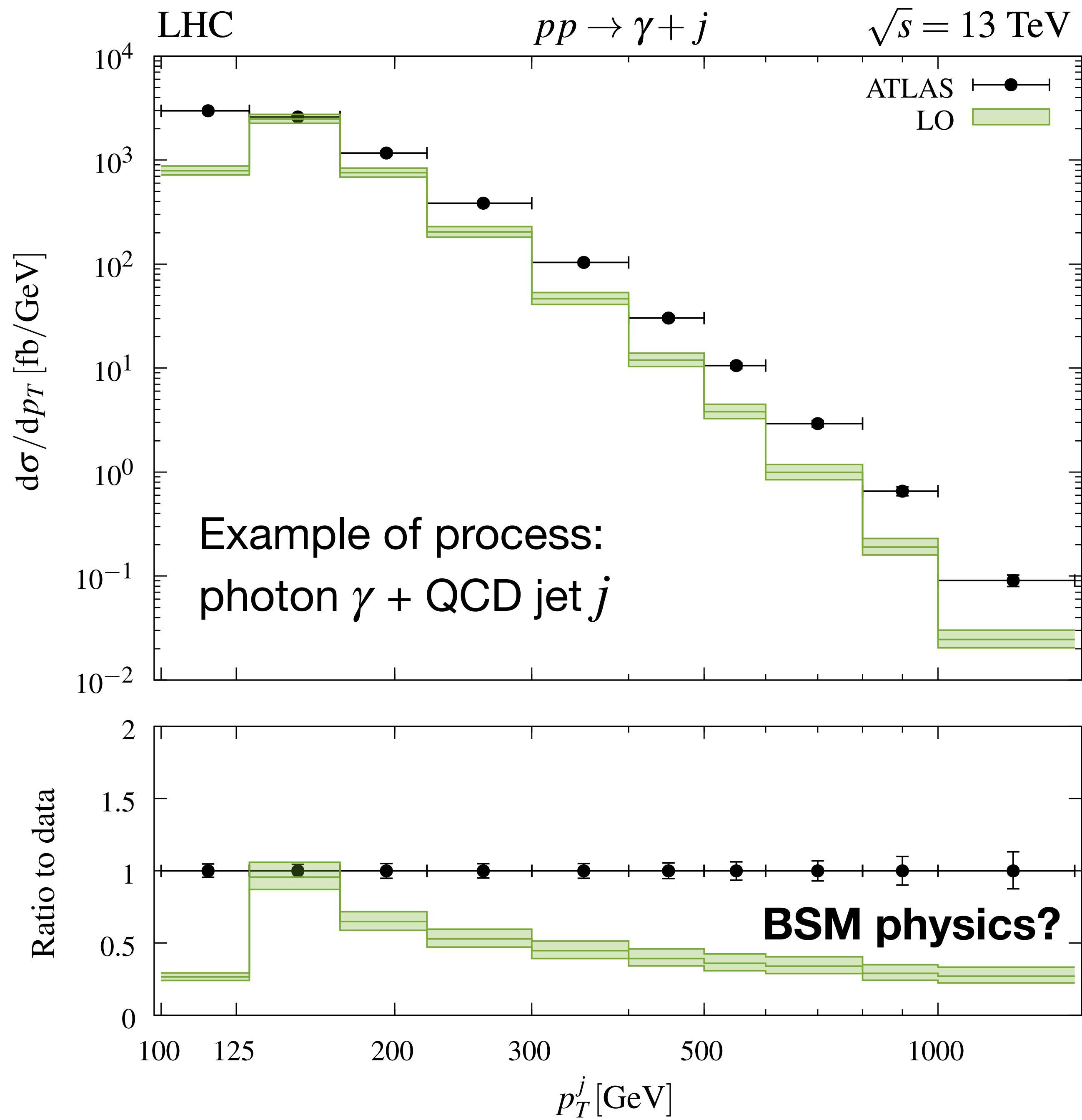
Experiments do not see quarks and gluons,  
but composite QCD particles (*hadrons*)

Measurement and predictions in terms of *jets*  
(~ clusters of particles close in angle)



$$\underline{d\sigma}_{AB} = \sum_{ab} f_{a|A} \otimes f_{b|B} \otimes \underline{d\hat{\sigma}}_{ab}$$

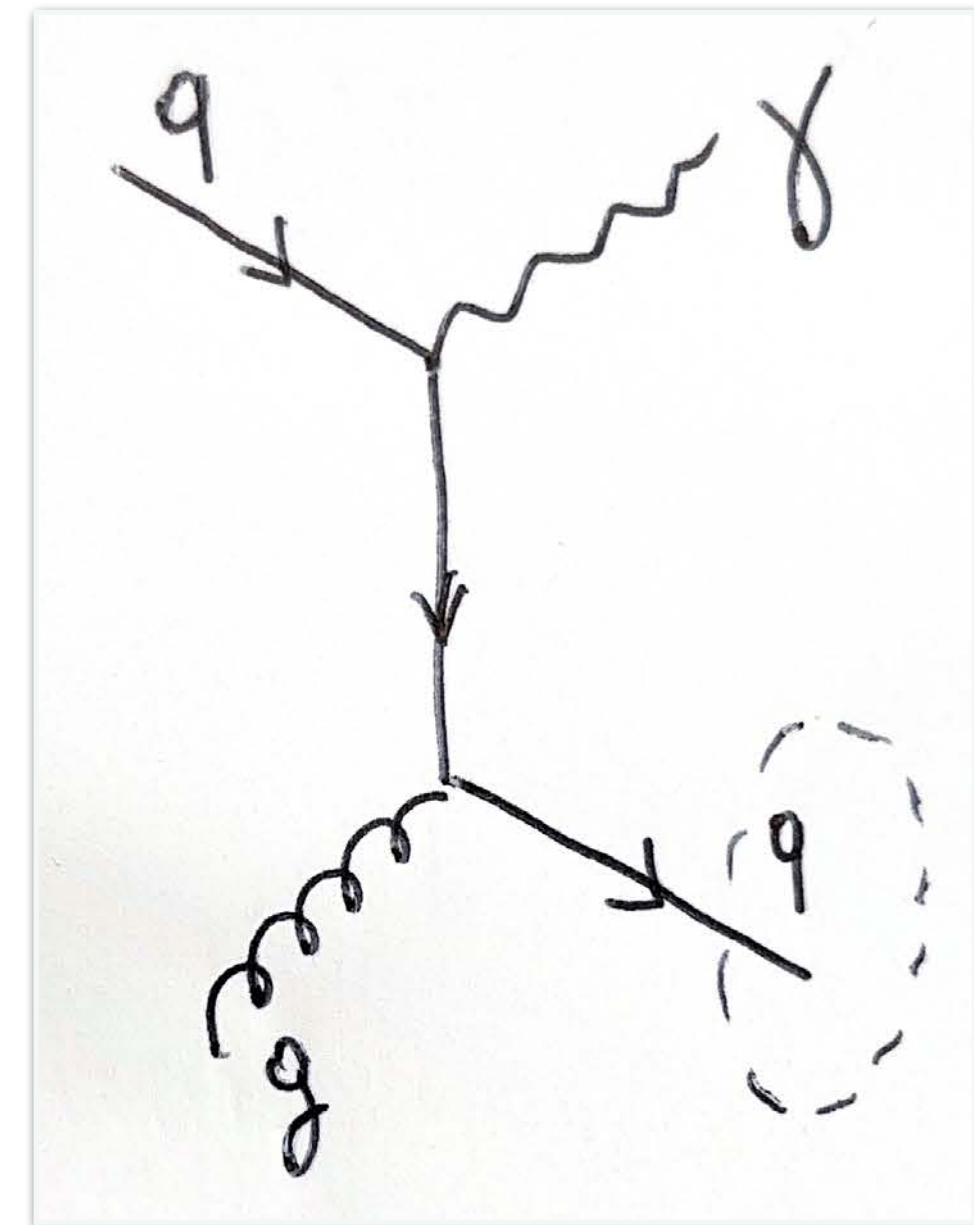




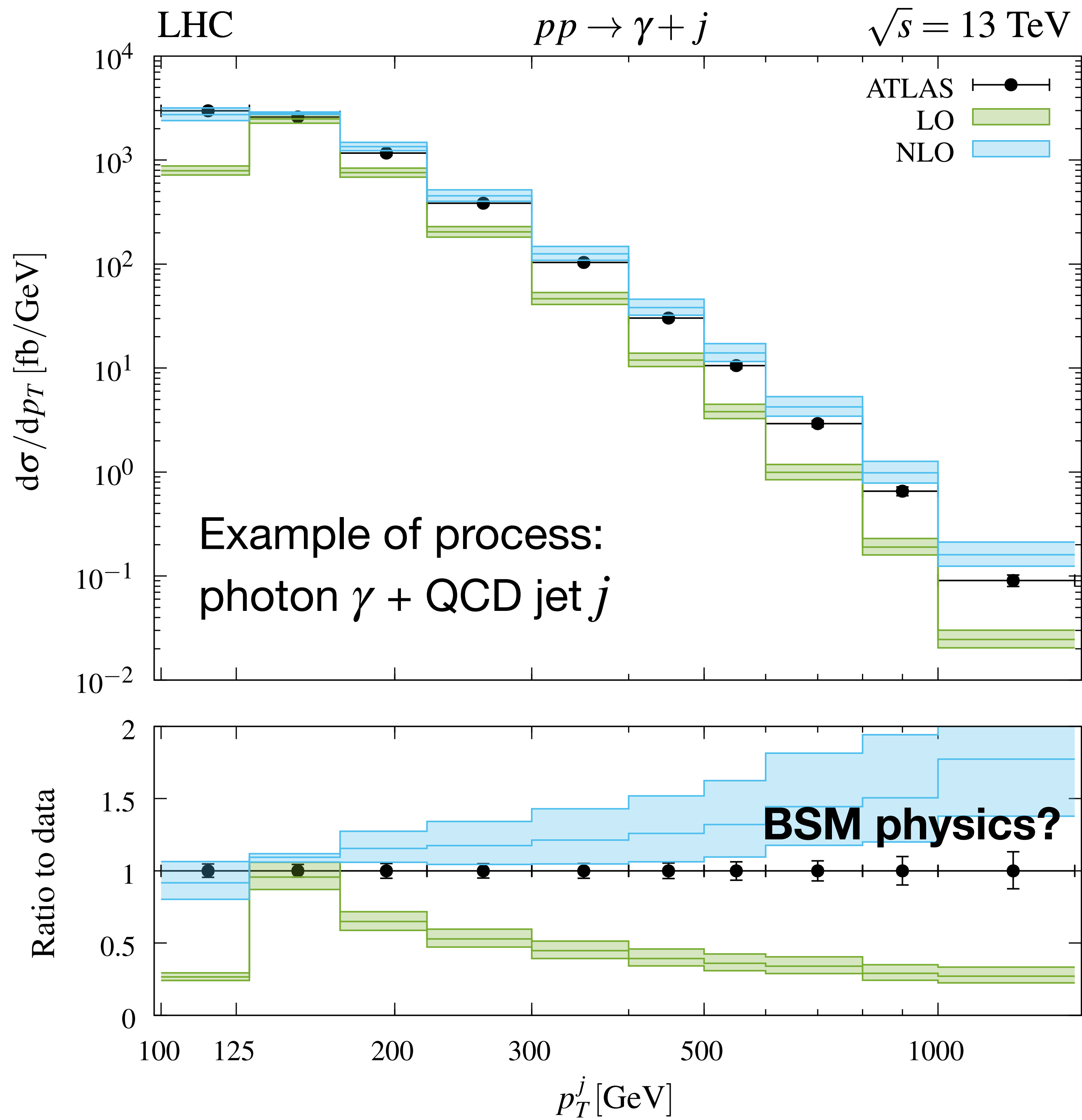
## Precise description of hard scattering

Leading Order (LO)

$$\sigma = \underline{\sigma_0} + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \mathcal{O}(\alpha_s^3)$$



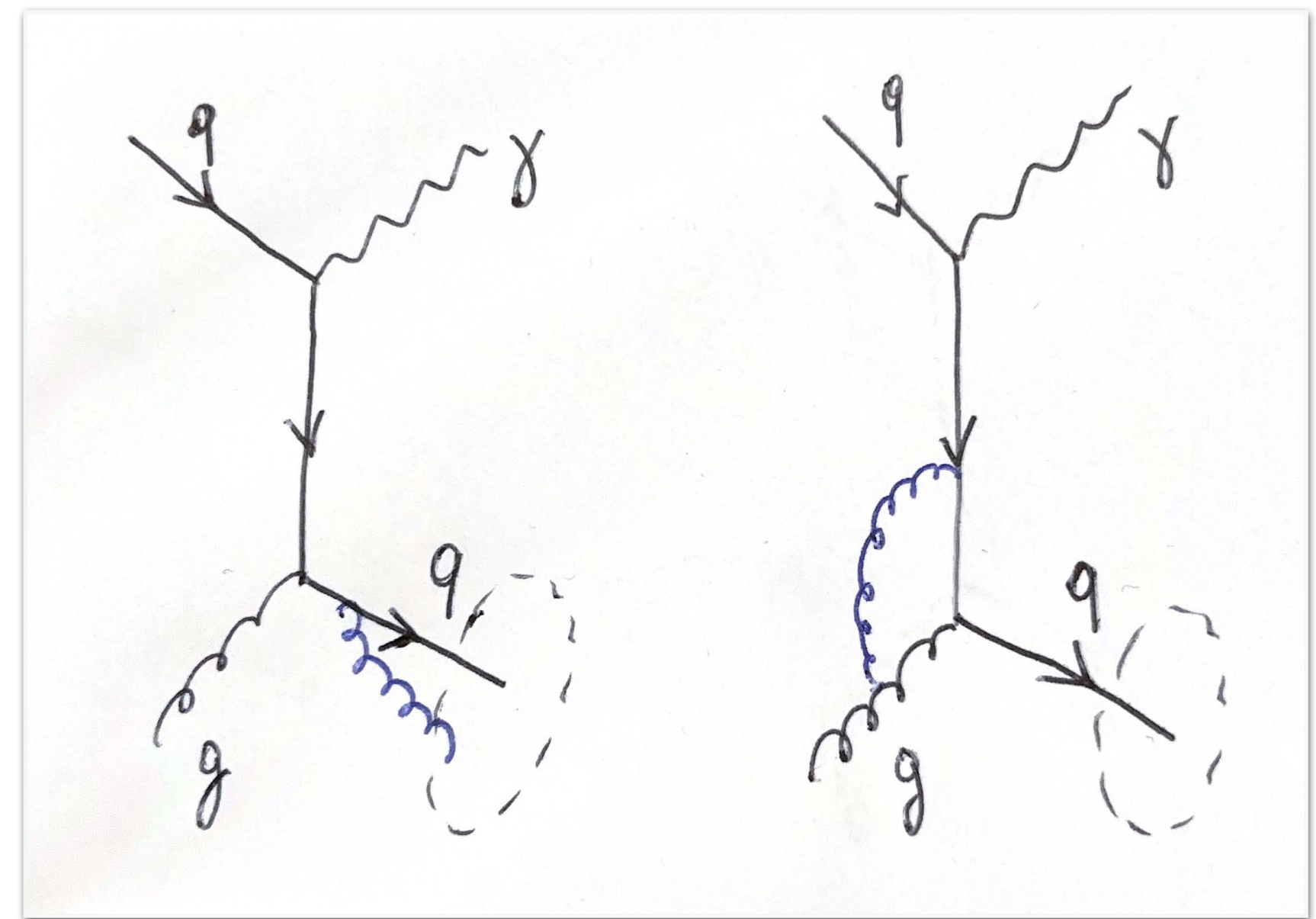




## Precise description of hard scattering

Next-to-Leading Order (NLO)

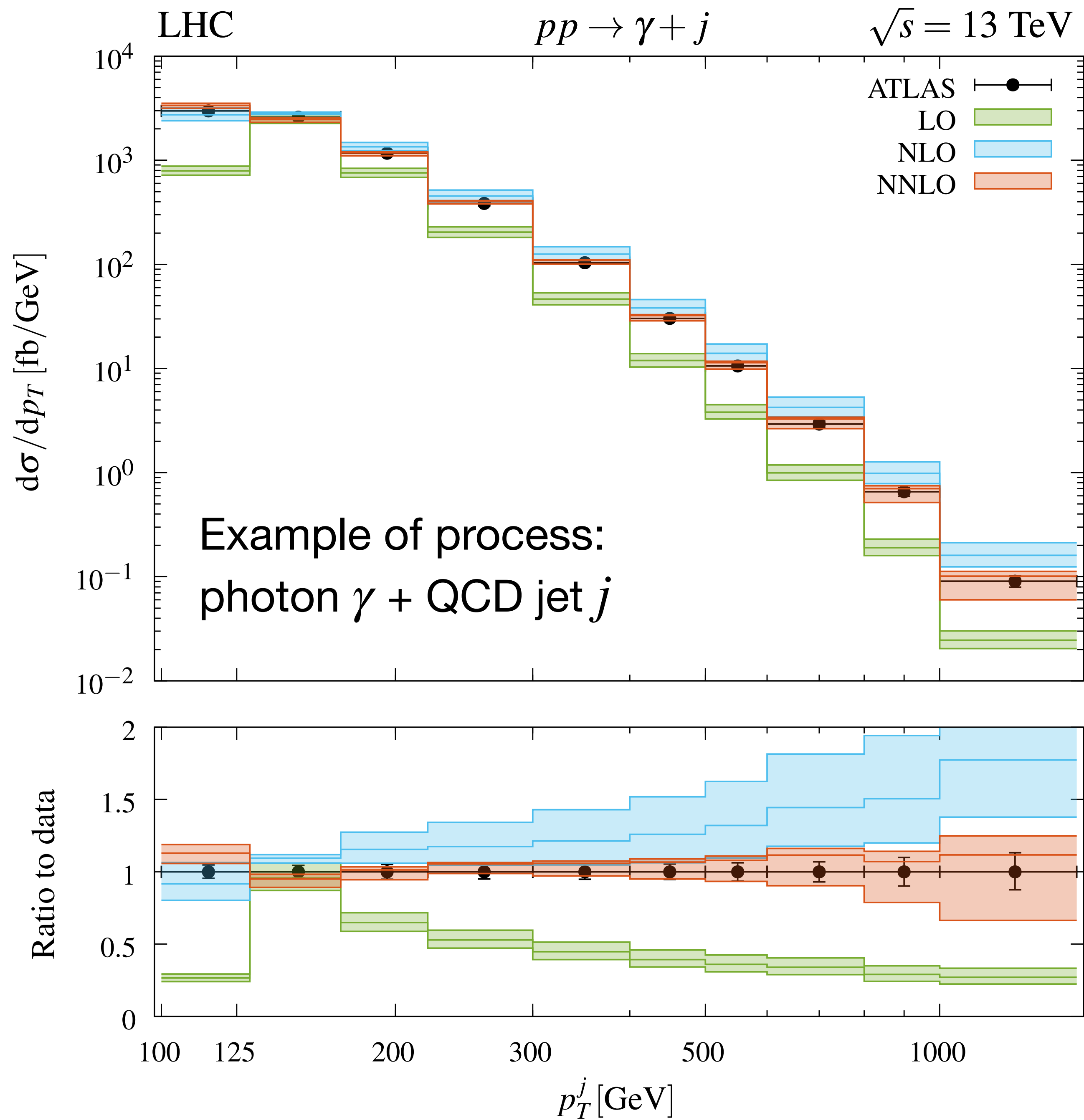
$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \mathcal{O}(\alpha_s^3)$$



Real gluon

Virtual gluon

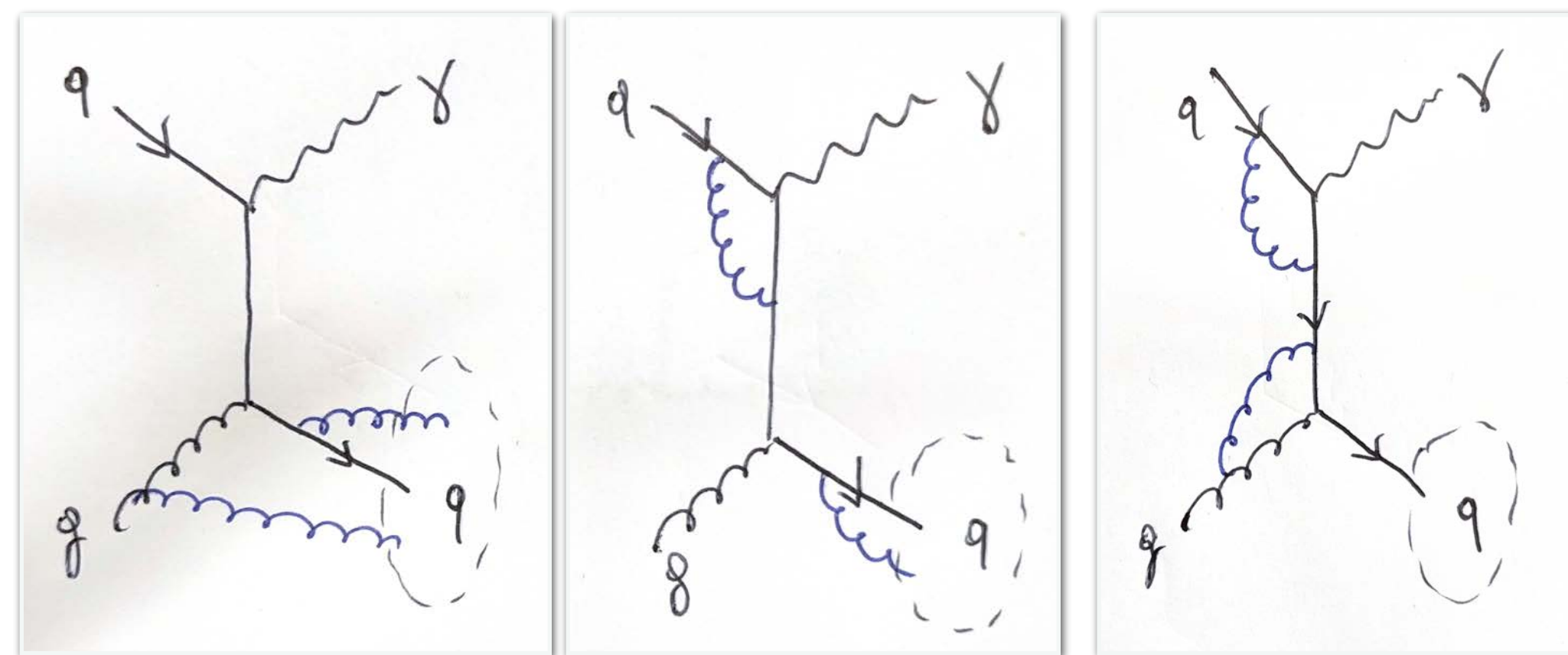




## Precise description of hard scattering

Next-to-Next-to-Leading Order (NNLO)

$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \mathcal{O}(\alpha_s^3)$$



*Real-real*

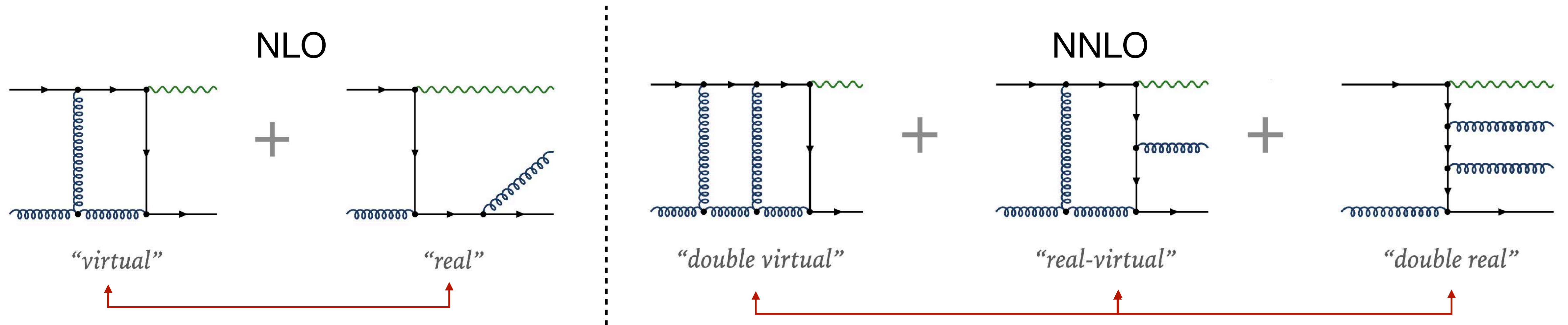
*Real-virtual*

*Virtual-virtual*



# Fixed-order calculations: dealing with infrared singularities in QCD

Infrared singularities appearing when particles are *soft* (low energy) and/or *collinear* (close in angle)



Toy model: parameter  $\epsilon$  acting as regulator;  $F$  complicated "measurement" function over the "emission phase space"  $x$  (e.g. energy of emitted particle or angle between particles)

$$\text{Real} \quad \text{Implicit divergence as } x \rightarrow 0 \quad (\text{regulated by } x^\epsilon) \quad I = \lim_{\epsilon \rightarrow 0} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\} \quad \text{Virtual} \quad \text{Explicit divergence} \quad (\text{pole in } 1/\epsilon)$$

Two class of methods to tackle this issue: *subtraction* or *slicing*



# Fixed-order calculations: dealing with infrared singularities in QCD

Two class of methods to tackle this issue: *subtraction* or *slicing*

**Subtraction:** add and subtract a term

mimicking the divergent limit under integration AND simple to integrate.  
Analytical cancellation of poles and leftover integral can be integrated numerically

$$I = \lim_{\epsilon \rightarrow 0} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon (F(x) - F(0)) + \int_0^1 \frac{dx}{x} x^\epsilon F(0) - \frac{1}{\epsilon} F(0) \right\}$$
$$= \int_0^1 \frac{dx}{x} (F(x) - F(0)),$$

Established NLO methods: Catani-Seymour, FKS, ...

NNLO methods: antenna subtraction, sector-improved residue subtraction, nested soft-collinear subtraction, local analytic sector subtraction, ...





## Antenna subtraction implemented in the NNLOJET code



Stay tuned for the first public release: <https://nnlojet.hepforge.org/>

**NNLOJET: a parton-level event generator for  
jet cross sections at NNLO QCD accuracy**



NNLOJET Collaboration

A. Huss<sup>1,\*</sup>, L. Bonino<sup>2</sup>, O. Braun-White<sup>3</sup>, S. Caletti<sup>4</sup>, X. Chen<sup>5</sup>, J. Cruz-Martinez<sup>1</sup>,  
J. Currie<sup>3</sup>, R. Gauld<sup>6</sup>, W. Feng<sup>2</sup>, E. Fox<sup>3</sup>, G. Fontana<sup>2</sup>, A. Gehrmann-De Ridder<sup>2,4</sup>,  
T. Gehrmann<sup>2</sup>, E.W.N. Glover<sup>3</sup>, M. Höfer<sup>7</sup>, P. Jakubčik<sup>2</sup>, M. Jaquier<sup>8</sup>, M. Löchner<sup>2</sup>,  
F. Lorkowski<sup>2</sup>, I. Majer<sup>4</sup>, M. Marcoli<sup>3</sup>, P. Meinzinger<sup>2</sup>, J. Mo<sup>2</sup>, T. Morgan<sup>3</sup>, J. Niehues<sup>3,9</sup>,  
J. Pires<sup>10</sup>, C. Preuss<sup>11</sup>, A. Rodriguez Garcia<sup>4</sup>, K. Schönwald<sup>2</sup>, V. Sotnikov<sup>2</sup>,  
R. Schürmann<sup>2</sup>, G. Stagnitto<sup>12</sup>, D. Walker<sup>3</sup>, S. Wells<sup>3</sup>, J. Whitehead<sup>13</sup>, T.Z. Yang<sup>2</sup> and  
H. Zhang<sup>8</sup>

Processes available in version 1.0,  
all at NNLO accuracy:

- $pp \rightarrow Z+\text{jet}$
- $pp \rightarrow W^\pm+\text{jet}$
- $pp \rightarrow H+\text{jet}$
- $pp \rightarrow \gamma+\text{jet}$
- $pp \rightarrow 1/2 \text{ jet}$
- $e^\pm p \rightarrow 2/3 \text{ jets}$
- $e^+e^- \rightarrow 2/3 \text{ jets}$



# Fixed-order calculations: dealing with infrared singularities in QCD

Two class of methods to tackle this issue: *subtraction* or *slicing*

**Slicing:** introduce a small parameter  $\delta$ , such that:

- for  $x > \delta$ , we avoid all divergences (and we can compute it numerically)
- for  $x < \delta$ , we can approximate it analytically (usually neglecting powers  $\delta^{n>0}$ )

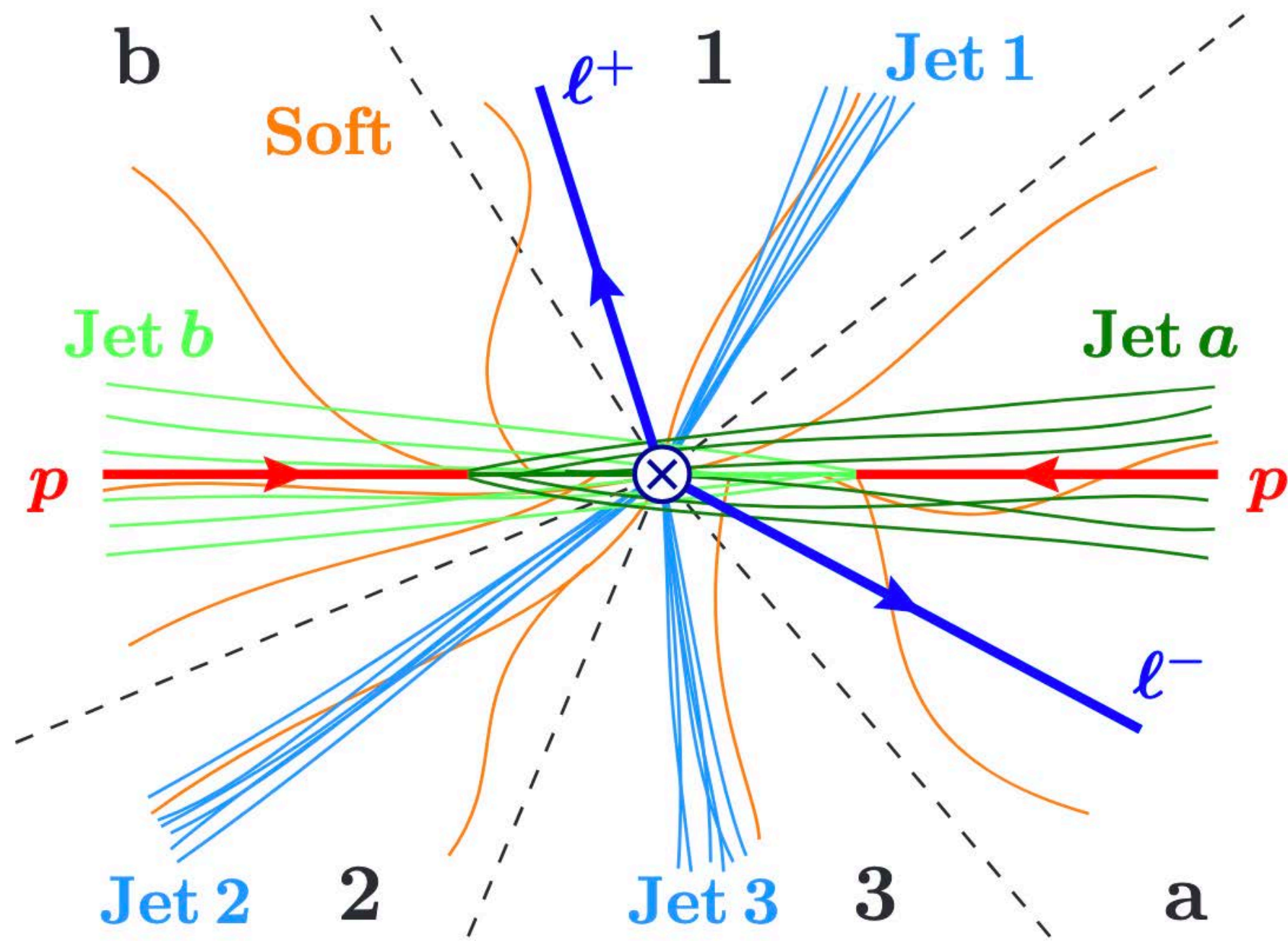
$$\begin{aligned} I &= \lim_{\epsilon \rightarrow 0} \left\{ \int_0^{\delta} \frac{dx}{x} x^\epsilon F(0) + \int_{\delta}^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\} \\ &= \lim_{\epsilon \rightarrow 0} \left\{ \cancel{\frac{1}{\epsilon} F(0)} + \log(\delta) F(0) + \int_{\delta}^1 \frac{dx}{x} x^\epsilon F(x) - \cancel{\frac{1}{\epsilon} F(0)} \right\} \\ &= \log(\delta) F(0) + \int_{\delta}^1 \frac{dx}{x} F(x). \end{aligned}$$

Variants at NNLO:  $q_T$ -subtraction,  $N$ -jettiness subtraction, ...



# $N$ -jettiness: a global event shapes at hadron colliders

[Stewart, Tackmann, Waalewijn '10]



$$\tau_N = \frac{2}{Q^2} \sum_k \min \{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \dots, q_N \cdot p_k \}$$

$q_a, q_b$  and  $q_1, \dots, q_N$  are massless reference momenta

It vanishes for exactly  $N$  infinitely narrow jets

The limit  $\tau_N \rightarrow 0$  encapsulates all the singularities of the  $V + N$ -jet process

-> it can be used as slicing variable for fixed-order calculations



# Fixed-order calculations: dealing with infrared singularities in QCD

$$\int_0^1 \frac{dx}{x} (F(x) - F(0))$$

## SUBTRACTION

### PROS:

- *local* subtraction (more stable numerically)
- general and/or automated formulations

### CONS:

- numerical implementation challenging
- integration of counter-terms may be tough (either analytically or numerically)

$$\log(\delta)F(0) + \int_\delta^1 \frac{dx}{x} F(x)$$

## SLICING

### PROS:

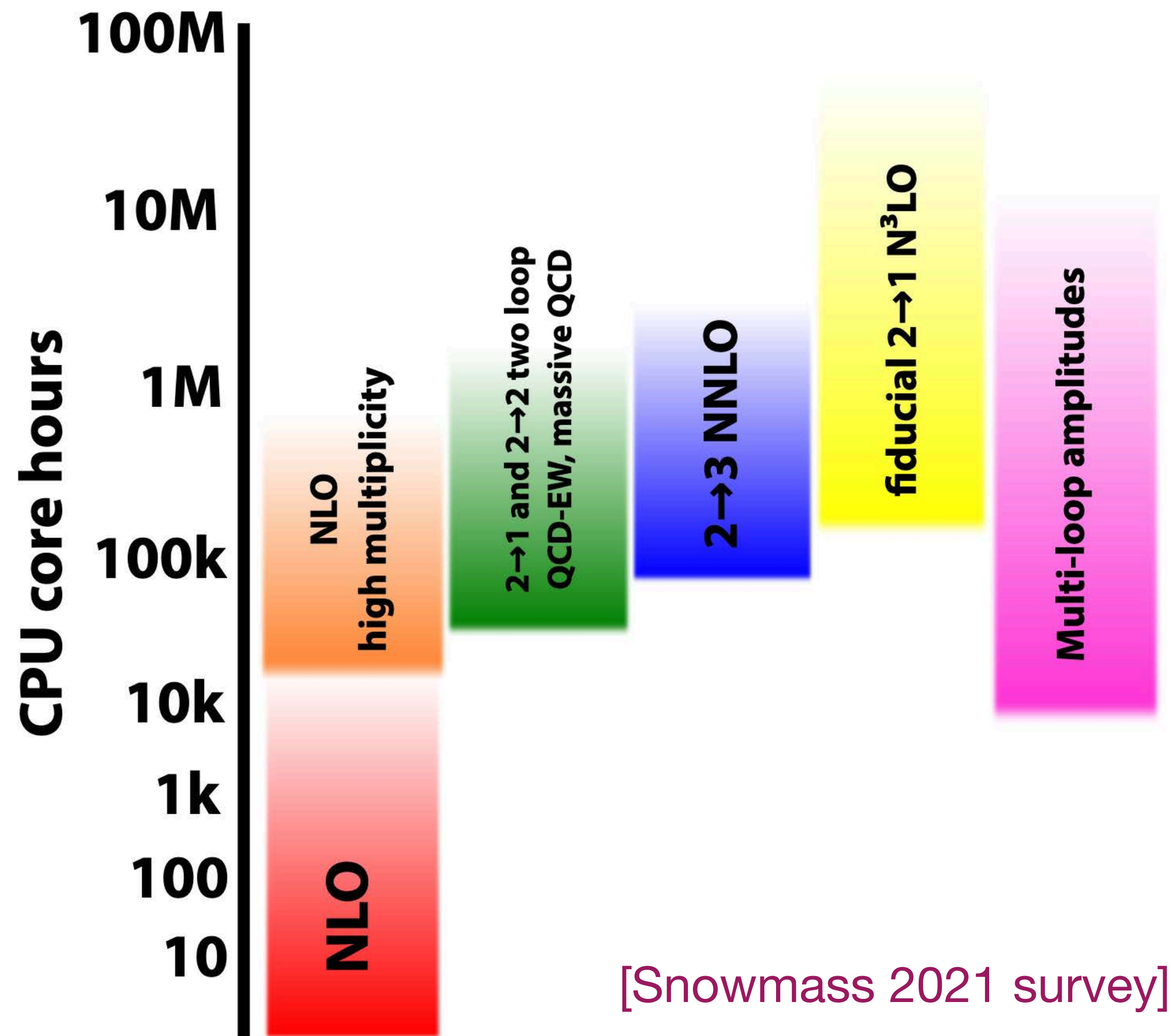
- simpler to implement numerically
- matching to parton showers easier

### CONS:

- *global* subtraction (large numerical cancellations)
- need analytical knowledge of  $\delta \rightarrow 0$
- extrapolation for obtain the  $\delta \rightarrow 0$  limit

Both techniques require significant computing resources at NNLO or at NLO with many particles





## Open questions

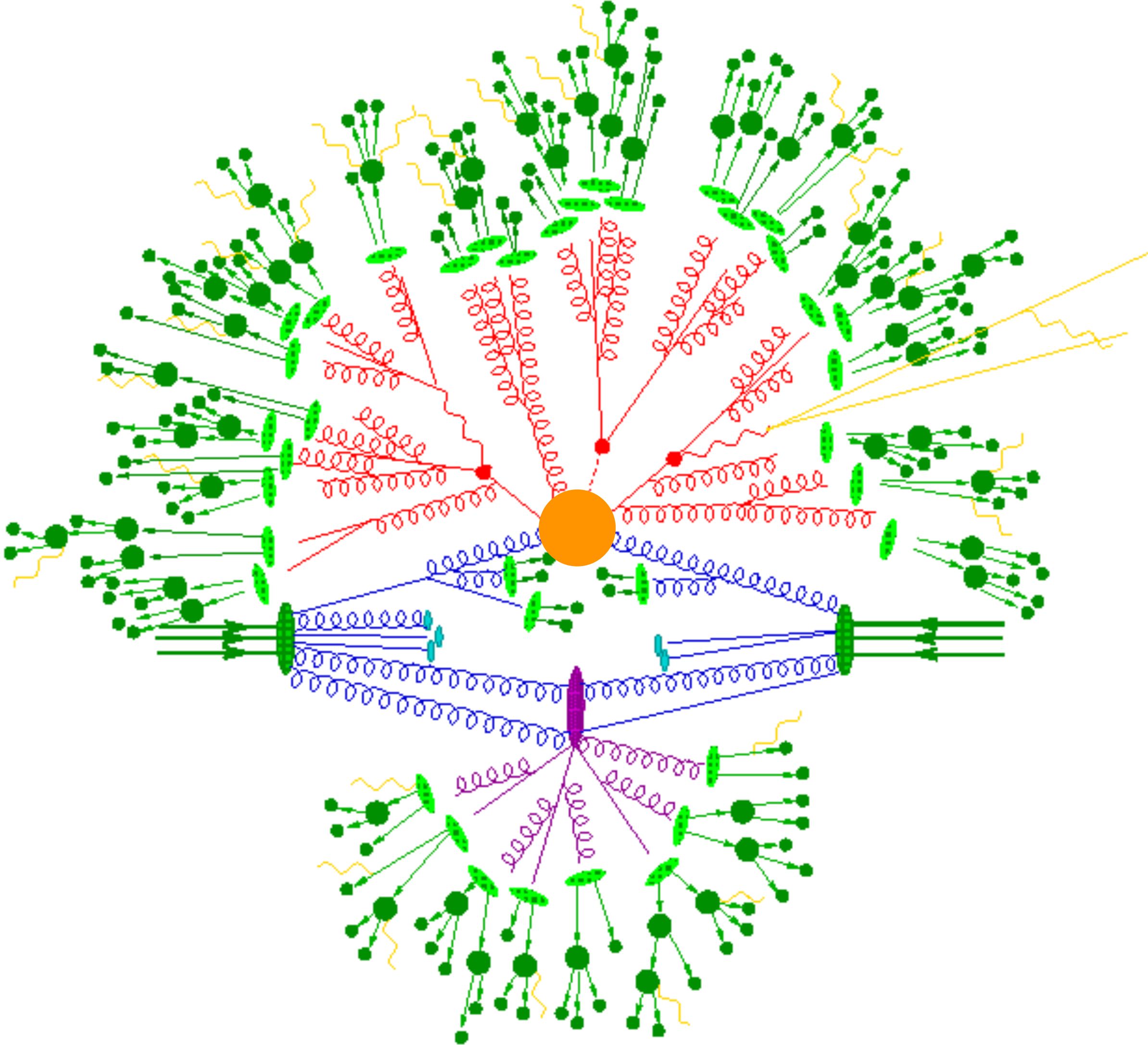
How to efficiently store/deliver predictions?  
E.g. grids or (un)weighted events?

How to optimise Monte Carlo integration?  
E.g. machine learning?

How to exploit advantages of GPUs?  
E.g. Redesign our (Fortran) codes?

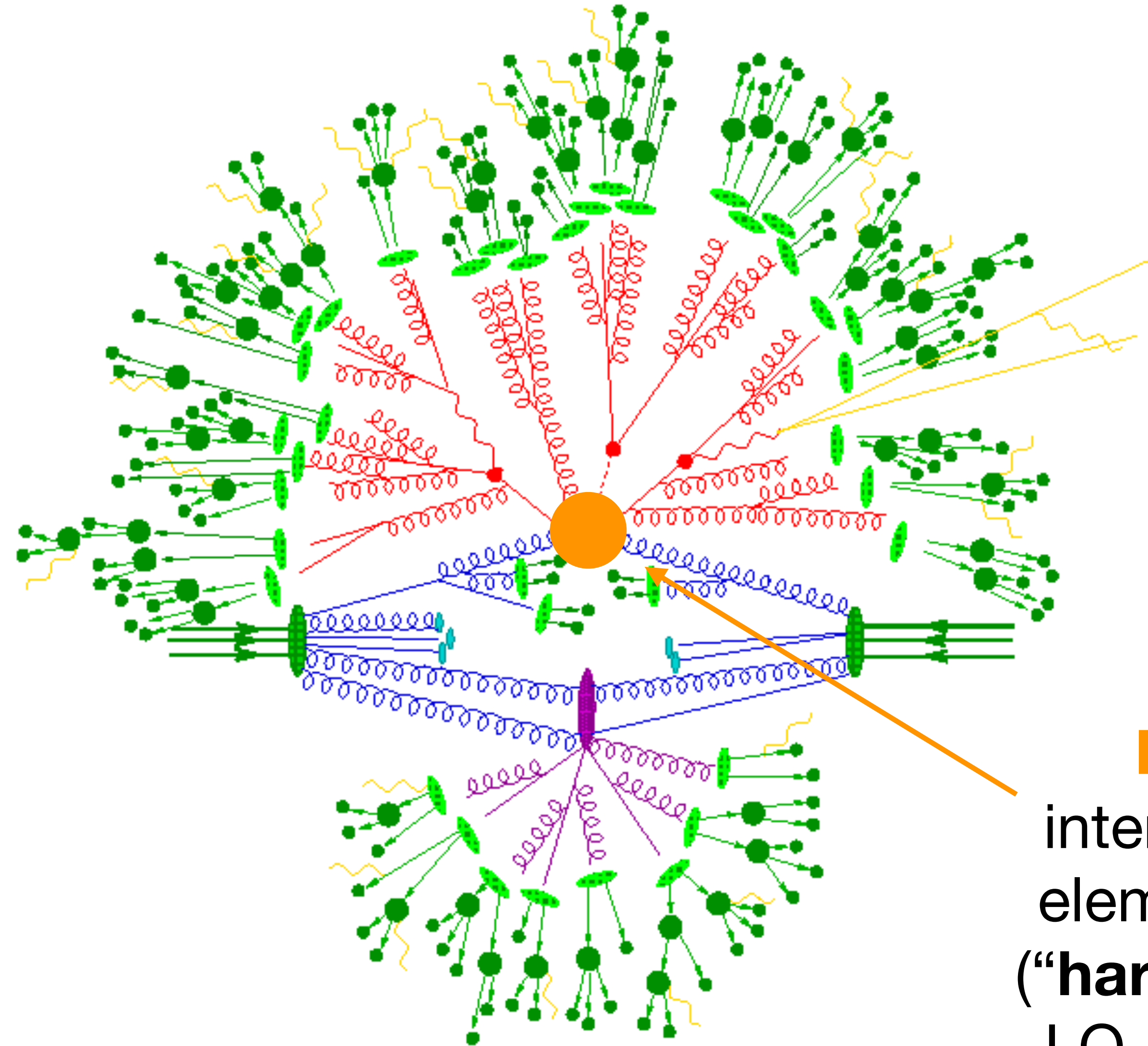


# From hard scattering to realistic events





# From hard scattering to realistic events

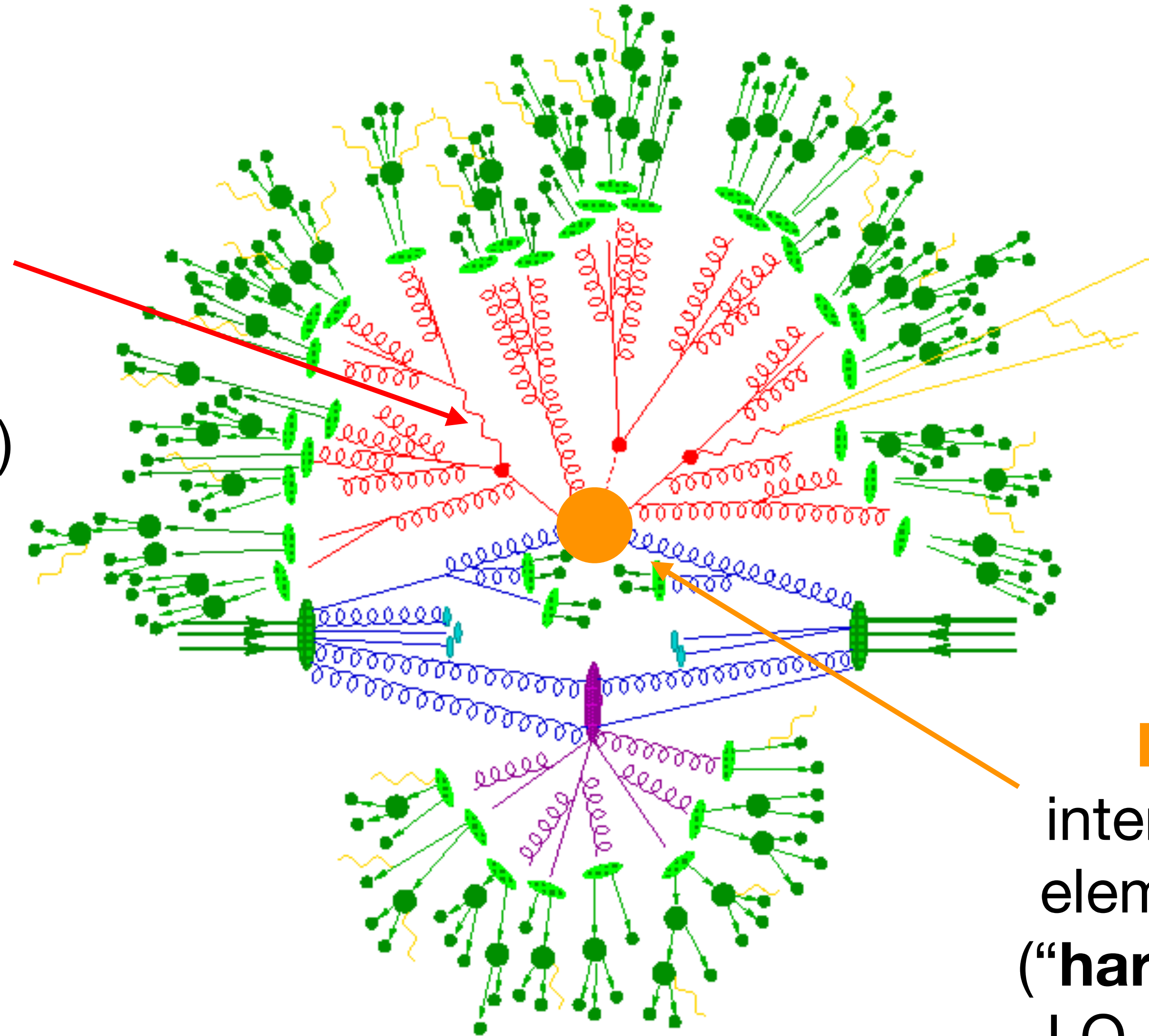


**High-energy**  
interaction between  
elementary particles  
("hard scattering") at  
LO, NLO, NNLO, ...



# From hard scattering to realistic events

**From high-energy to low-energy** with emission of (mostly) collinear and/or soft quark and gluons (“**parton shower**”, PS)

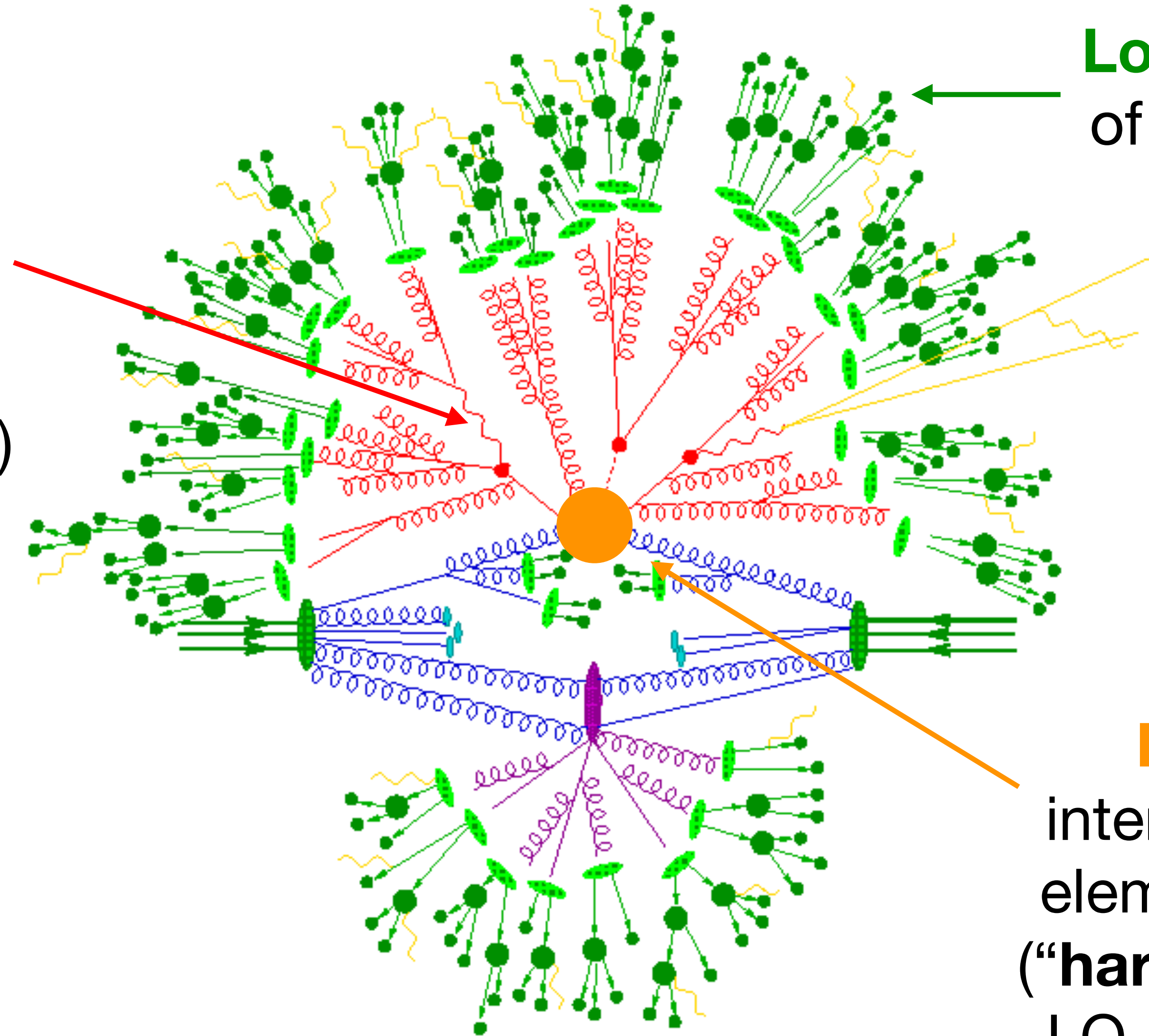


**High-energy** interaction between elementary particles (“**hard scattering**”) at LO, NLO, NNLO, ...



# From hard scattering to realistic events

**From high-energy to low-energy** with emission of (mostly) collinear and/or soft quark and gluons (“**parton shower**”, PS)



**Low-energy** formation of composite particles (“**hadronization**”)

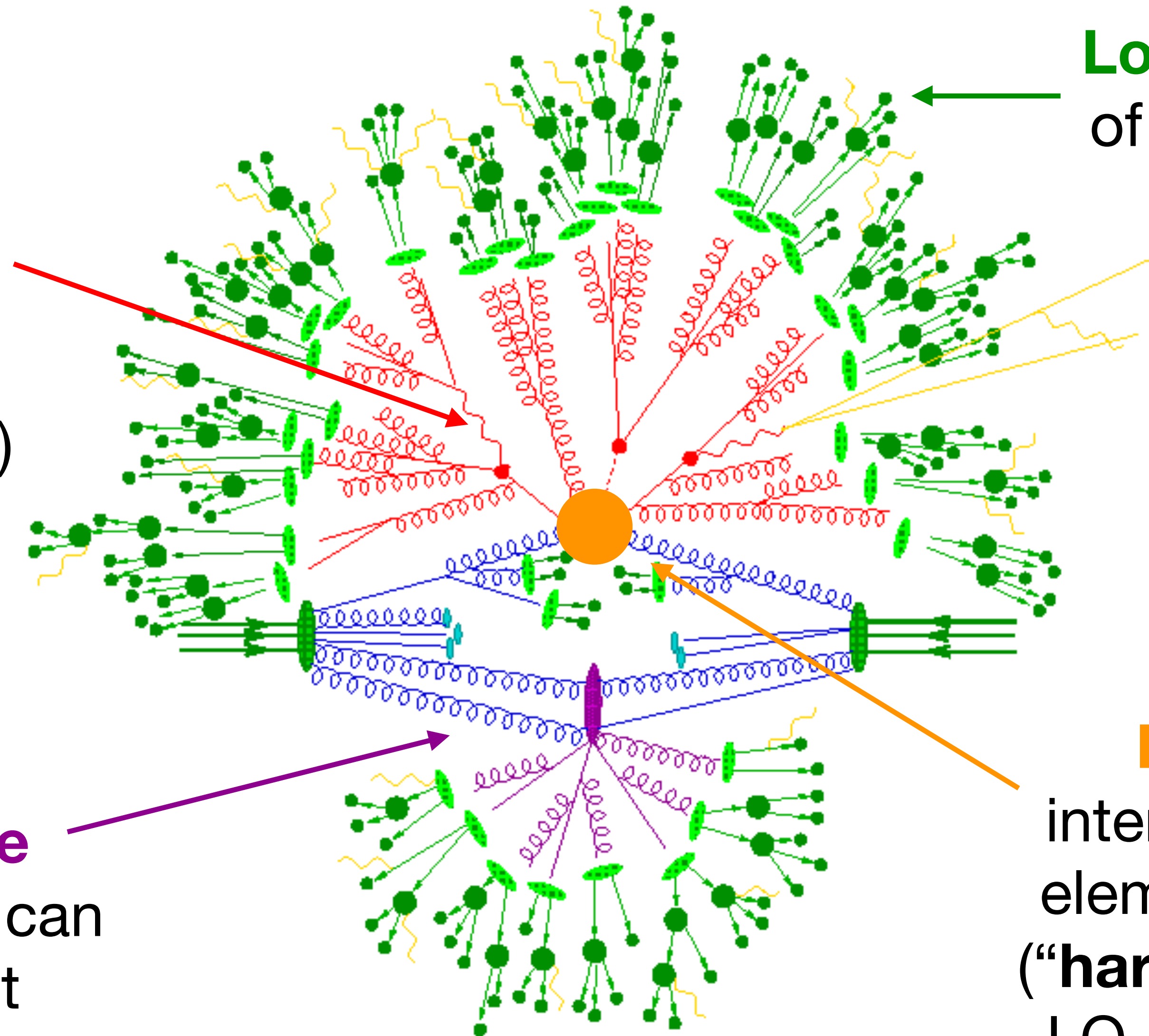
**High-energy** interaction between elementary particles (“**hard scattering**”) at LO, NLO, NNLO, ...



# From hard scattering to realistic events

**From high-energy to low-energy** with emission of (mostly) collinear and/or soft quark and gluons (“**parton shower**”, PS)

**Low-energy** formation of composite particles (“**hadronization**”)



**Multiple Particle Interactions** (MPI) can pollute the event

**High-energy** interaction between elementary particles (“**hard scattering**”) at LO, NLO, NNLO, ...

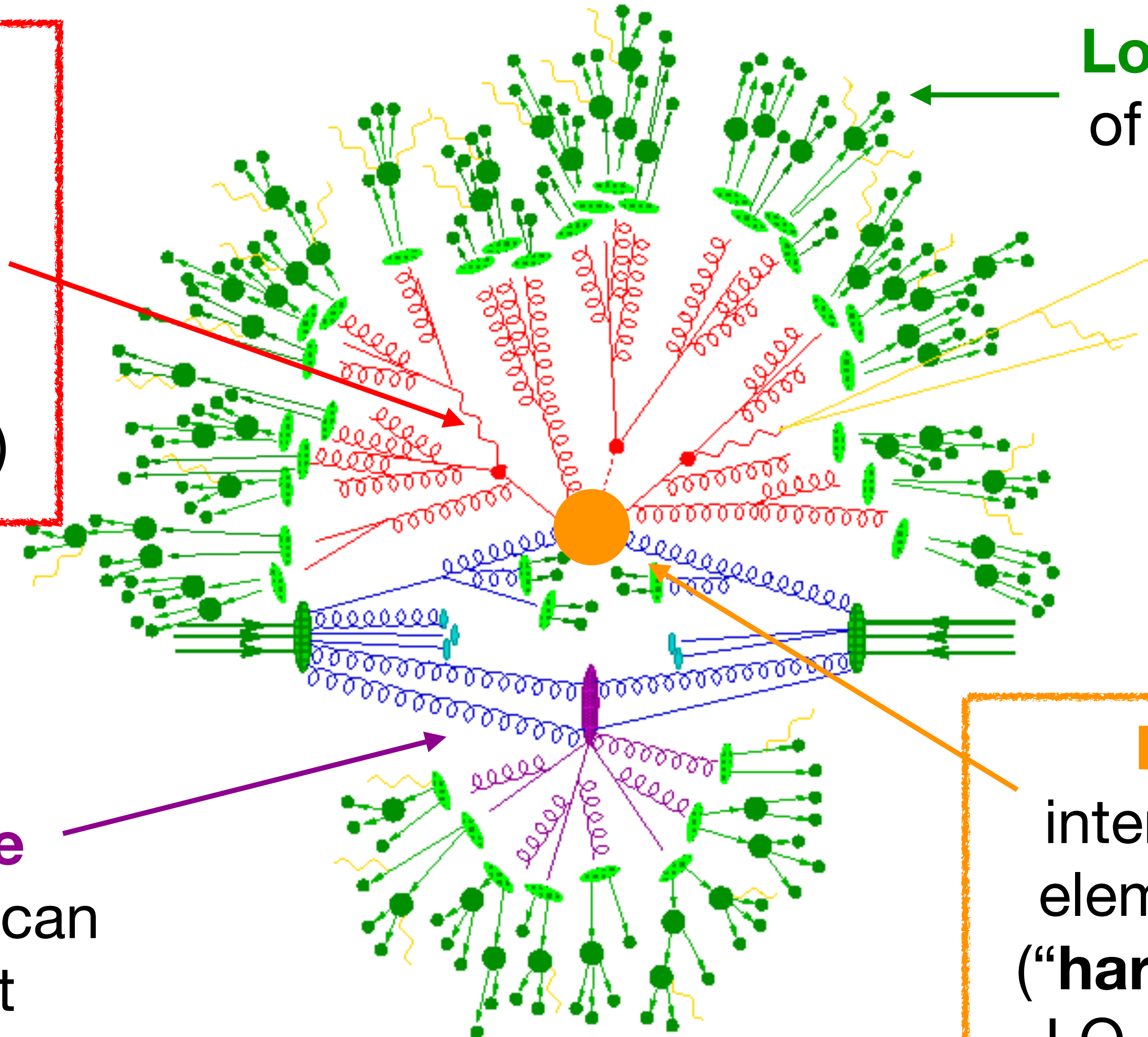


# From hard scattering to realistic events

**From high-energy to low-energy** with emission of (mostly) collinear and/or soft quark and gluons (“**parton shower**”, PS)

Approximate multiple emissions

**Multiple Particle Interactions** (MPI) can pollute the event

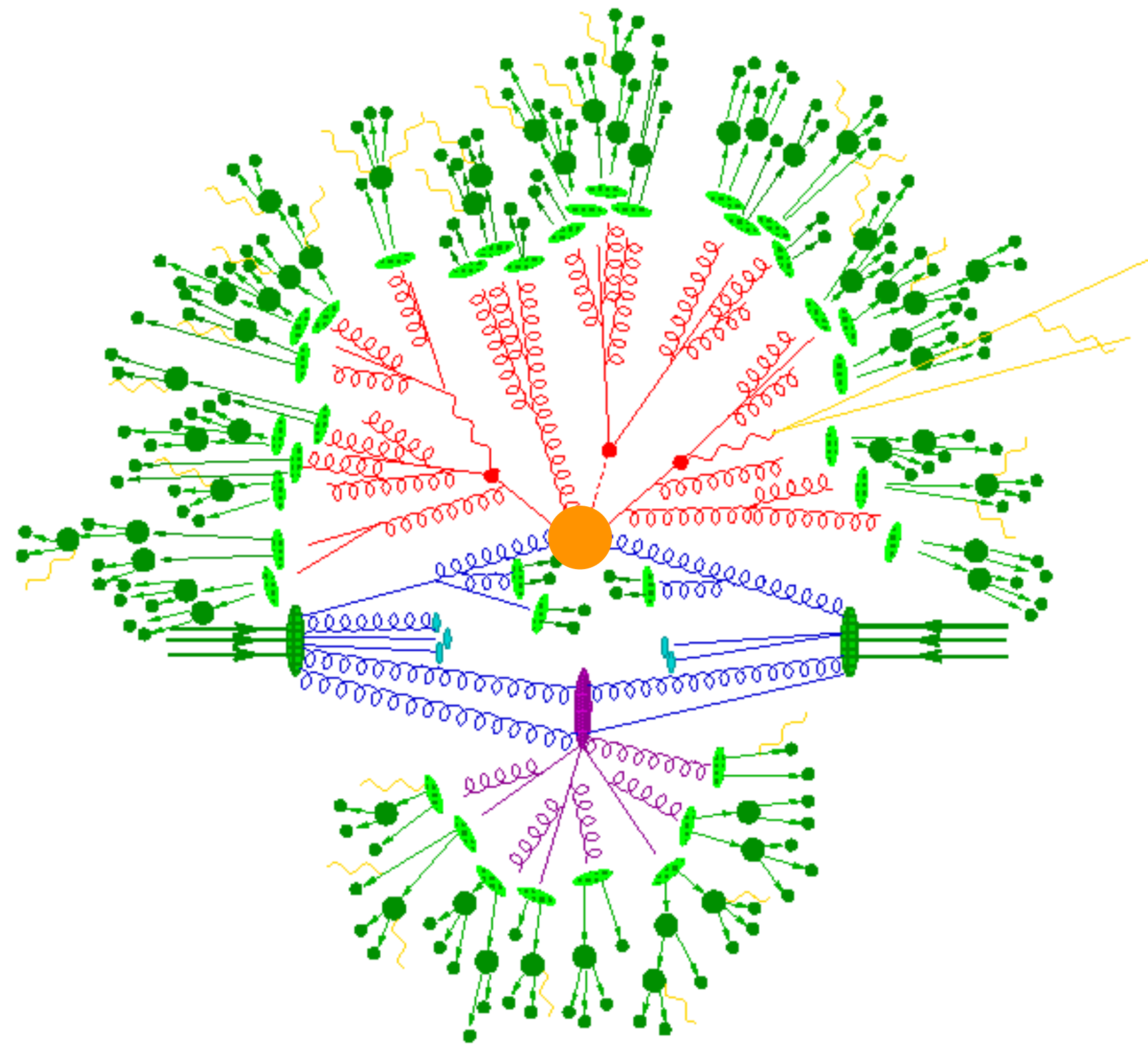


**Low-energy** formation of composite particles (“**hadronization**”)

Exact 1/2/3/... emissions

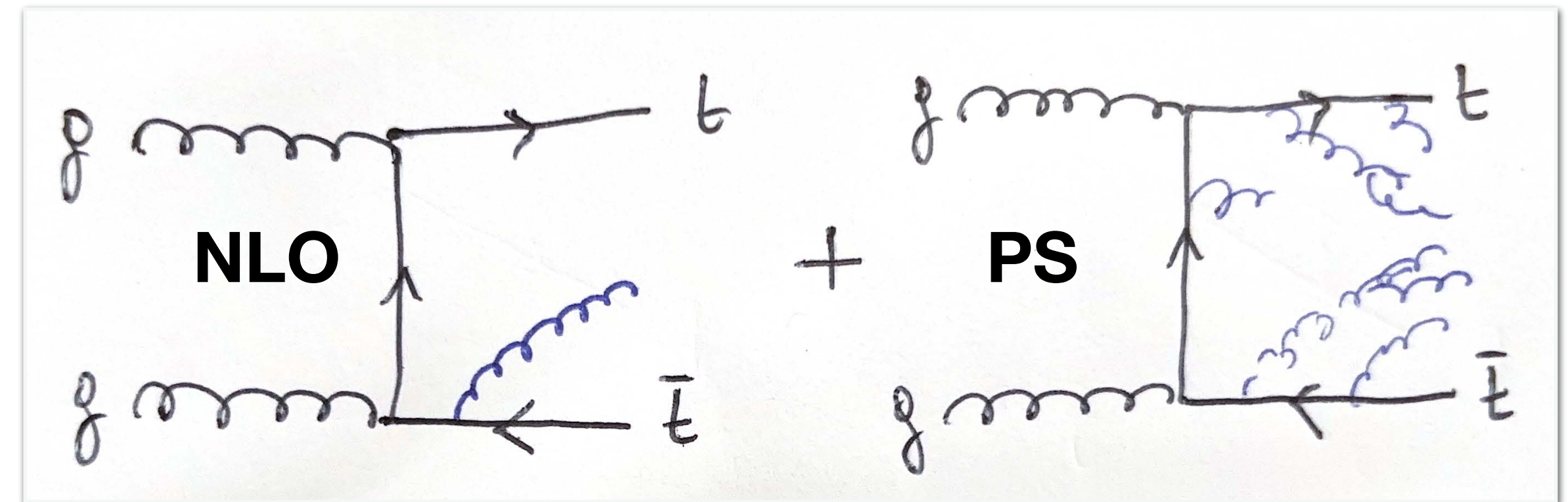
**High-energy** interaction between elementary particles (“**hard scattering**”) at LO, NLO, NNLO, ...





**How to match**  
**fixed-order** (LO, NLO, NNLO)  
 to **parton shower** (PS)  
 without losing accuracy  
 and without “double counting”?

Example: top + anti-top production



*Exact one*  
 gluon emission

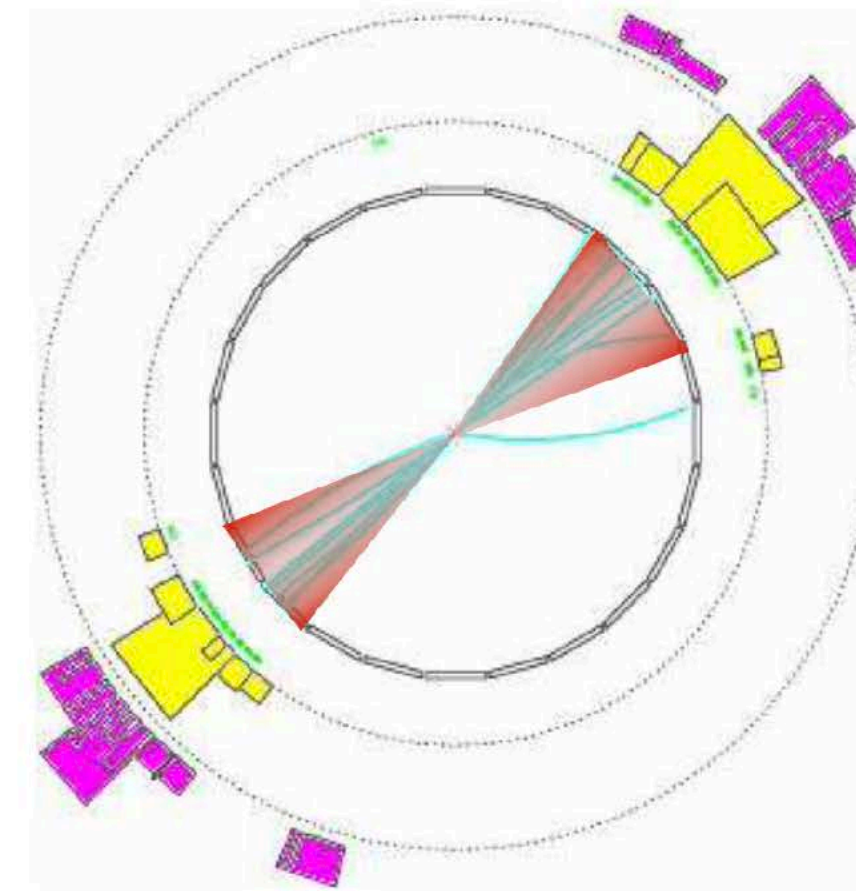
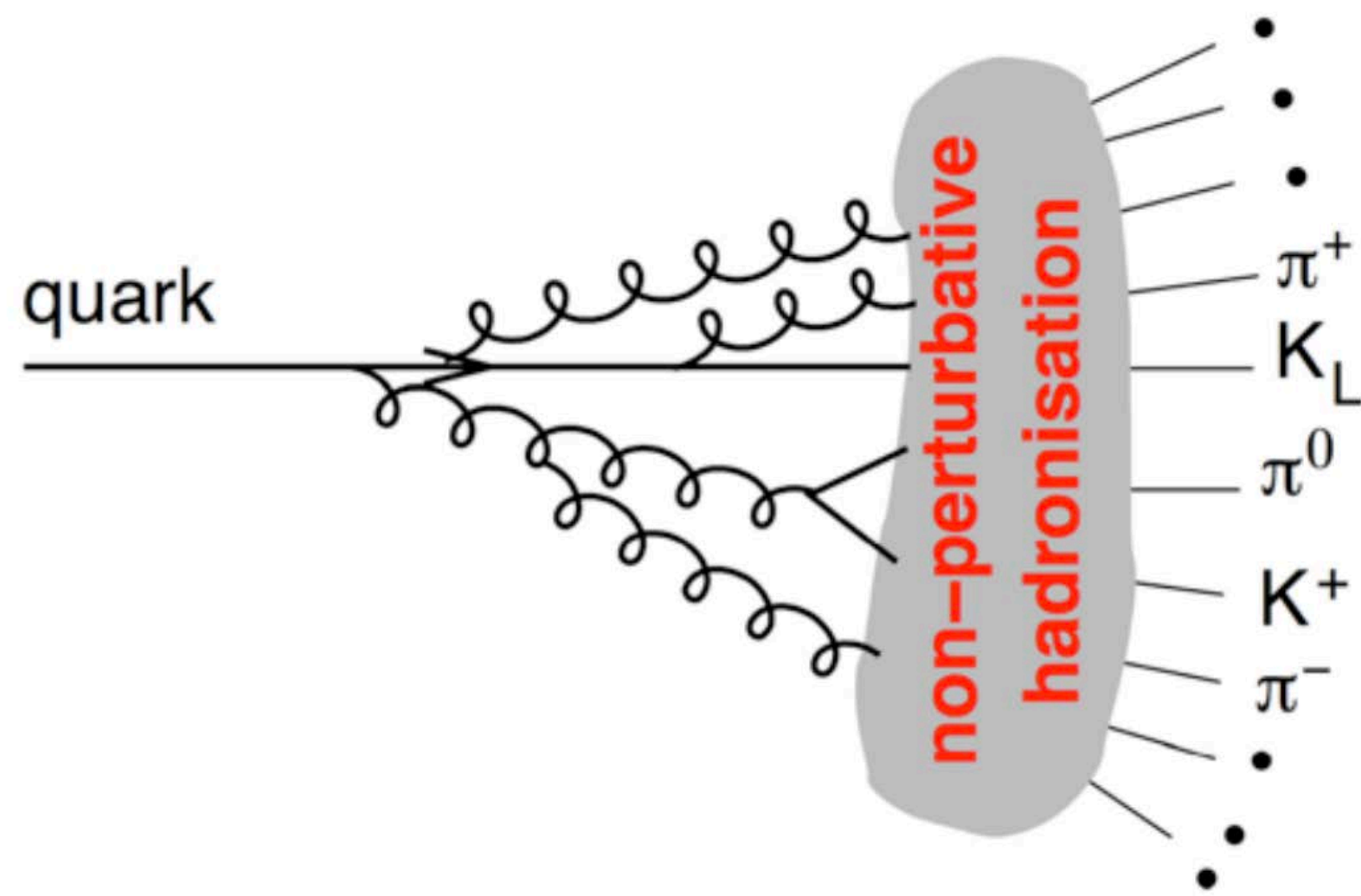
*Approximate multiple*  
 gluon emissions

Well established methods for  
**NLO+PS** predictions:  
 MC@NLO, POWHEG, ...

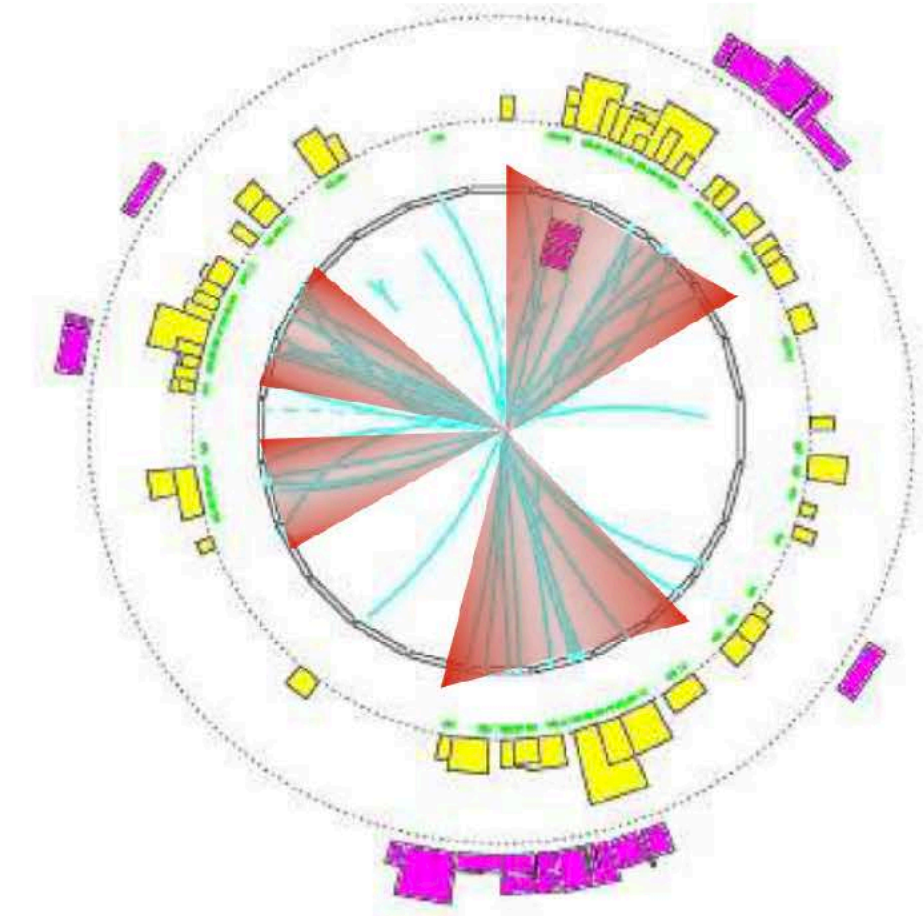
Methods for **NNLO+PS** predictions  
 still in development:  
 GENEVA, MiNNLOPS, ...



# On the definition of jets



2 clear jets



3 jets?  
or 4 jets?

*Naive definition:*  
**collimated bunch of hadrons**  
flying roughly in the same direction

*Proper definition:*  
a collection of hadrons  
defined by means of a **jet algorithm**

*“Jet [definitions] are legal contracts between theorists and experimentalists”*

MJ Tannenbaum



# Jet definition must satisfy **IRC safety**

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged**:

$$O(X; p_1, \dots, p_n, p_{n+1} \rightarrow 0) \rightarrow O(X; p_1, \dots, p_n)$$

$$O(X; p_1, \dots, p_n \parallel p_{n+1}) \rightarrow O(X; p_1, \dots, p_n + p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:  
**soft emissions and collinear splittings must not change the hard jets**



# Jets at the LHC usually defined by means of a sequential clustering algorithm

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2} \quad d_{iB} = p_{ti}^{2p}$$

Distance between particles  $i$  and  $j$

Distance between particle  $i$  and the beam

**p = 1**  $k_t$  algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187  
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

**p = 0** Cambridge/Aachen algorithm

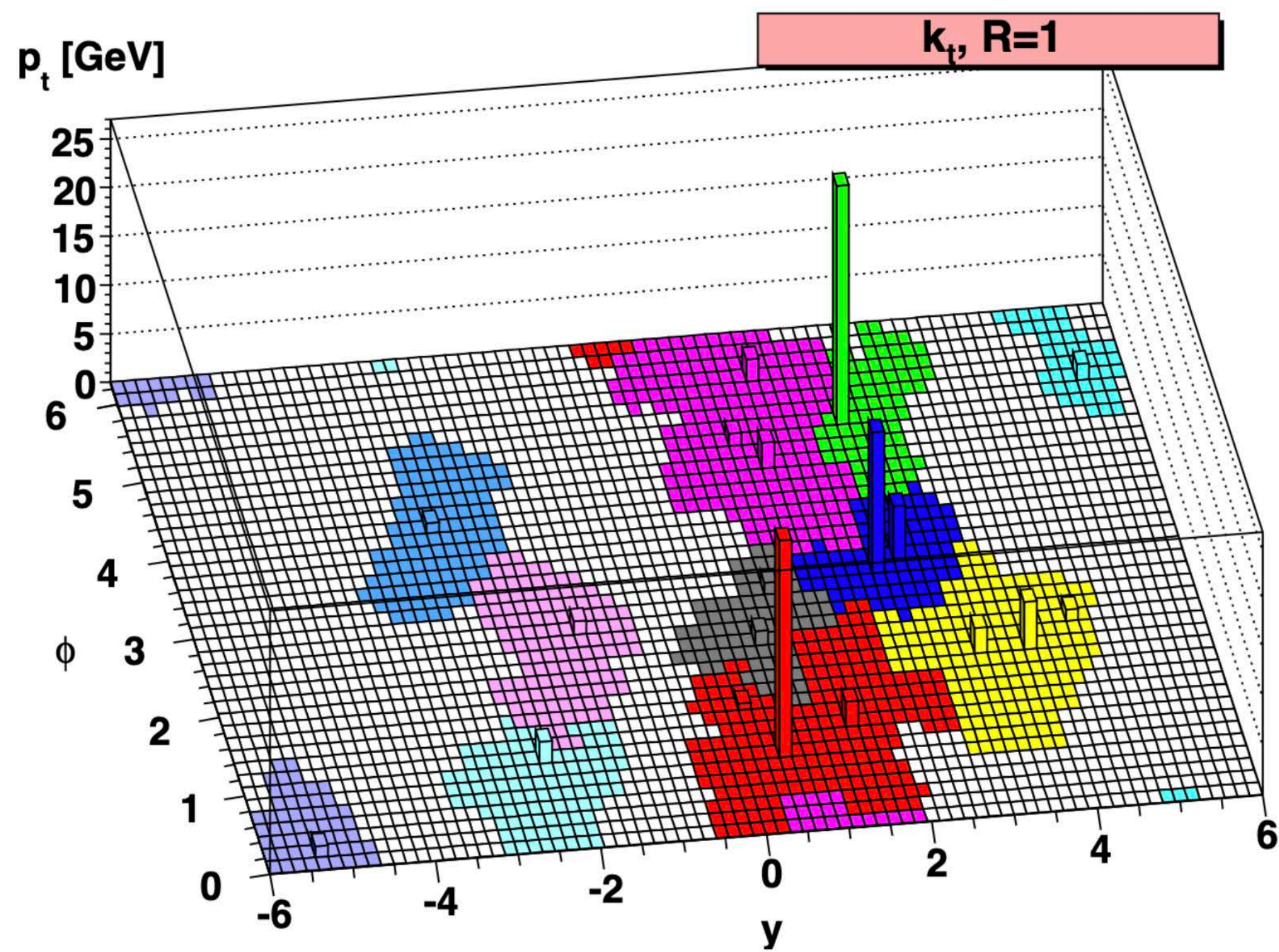
Y. Dokshitzer, G. Leder, S. Moretti and B. Webber, JHEP 08 (1997) 001  
M. Wobisch and T. Wengler, hep-ph/9907280

**p = -1** anti- $k_t$  algorithm

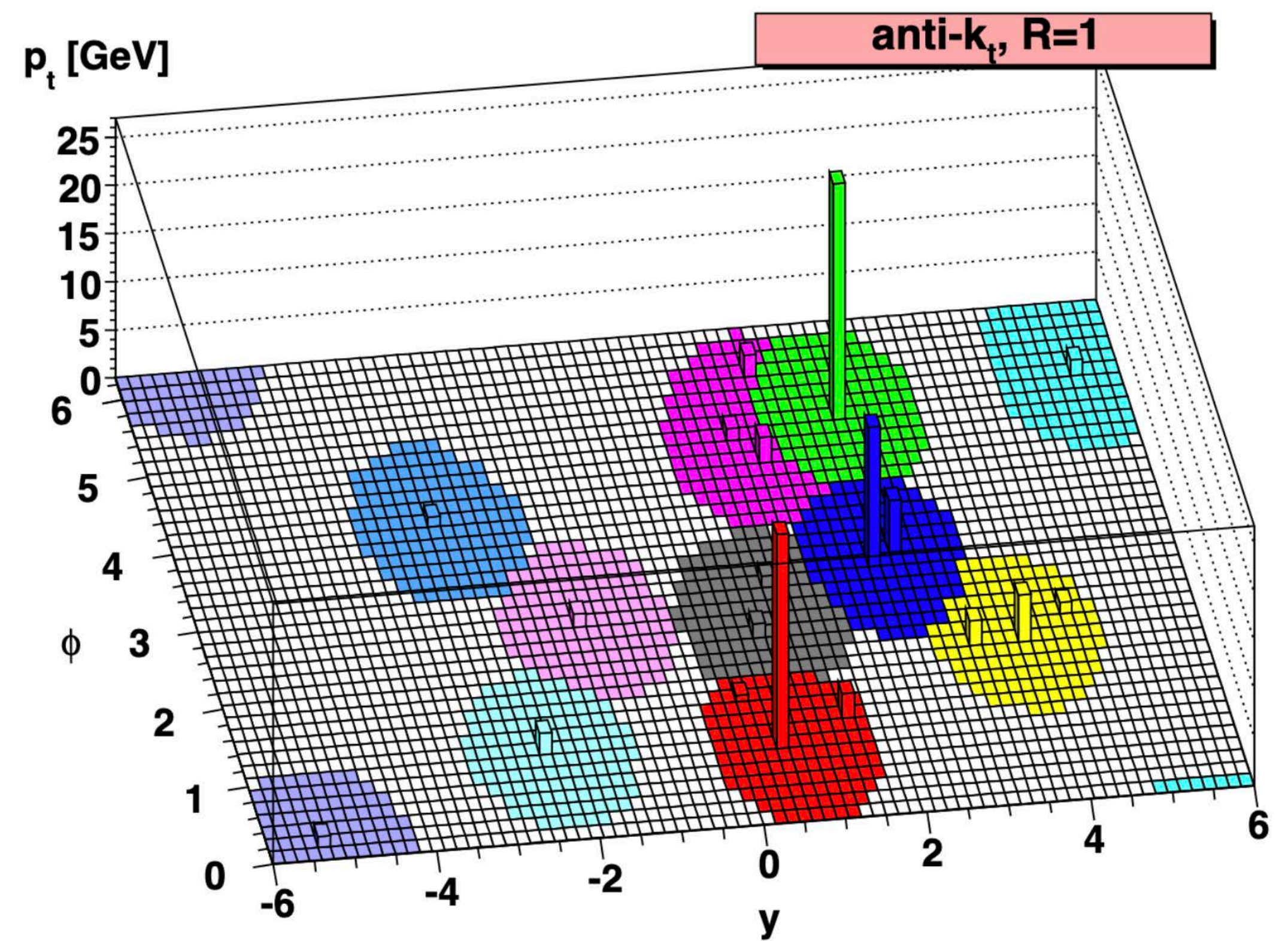
MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti- $k_t$  pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**





**$p = |$**   $k_t$  algorithm



**$p = -|$**  anti- $k_t$  algorithm

The anti- $k_t$  acts as IRC safe “cone” algorithm



# Outlook of this talk

1. (Rather long and pedagogical) introduction

2. Flavoured jets at the LHC:  
 $Z+c$ -jet and  $W+c$ -jet production

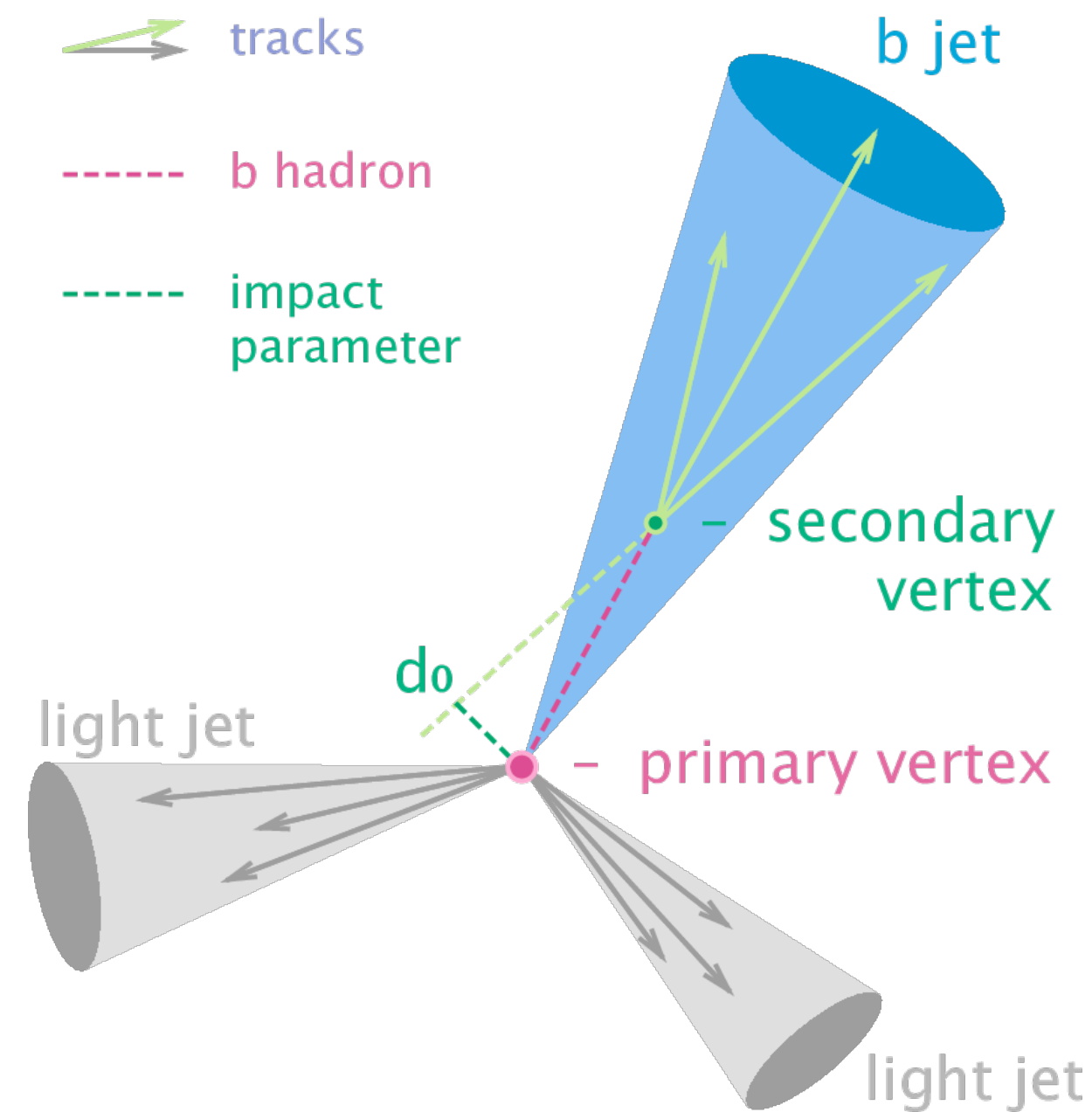
3. Towards NNLO+PS for  $V$ +jet:  
improving slicing methods for  $V$ +jet production

*Biased selection of recent results where I personally contributed.  
Minimal inclusion of references, apologies for any relevant omission.*

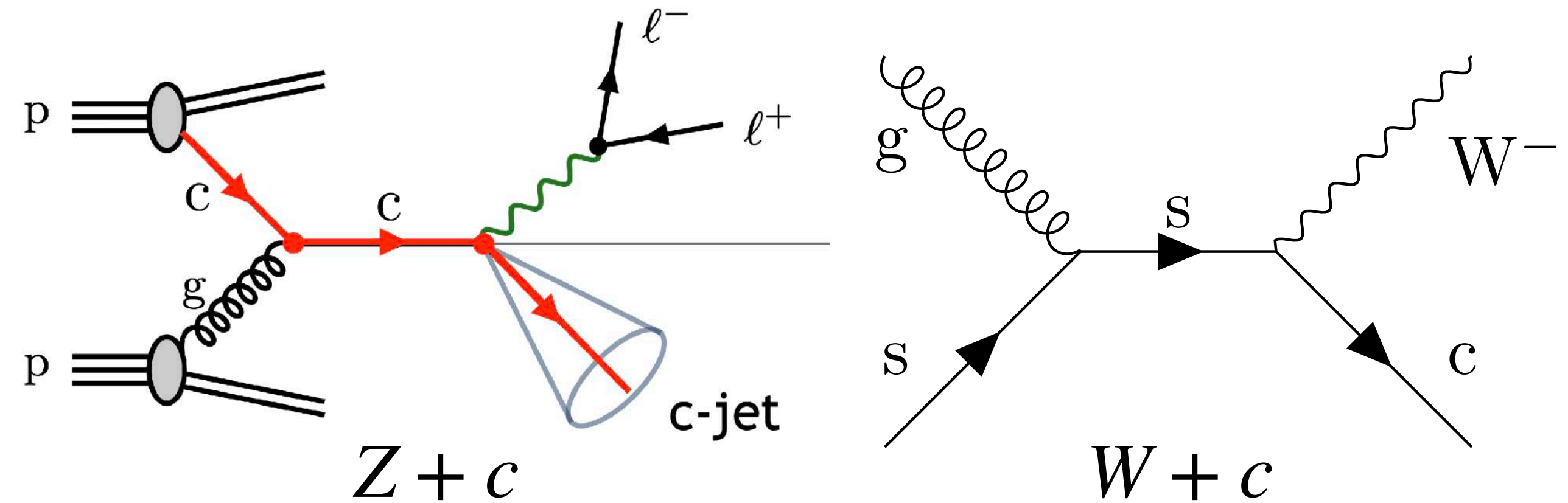


# What do we mean by *flavoured jets*?

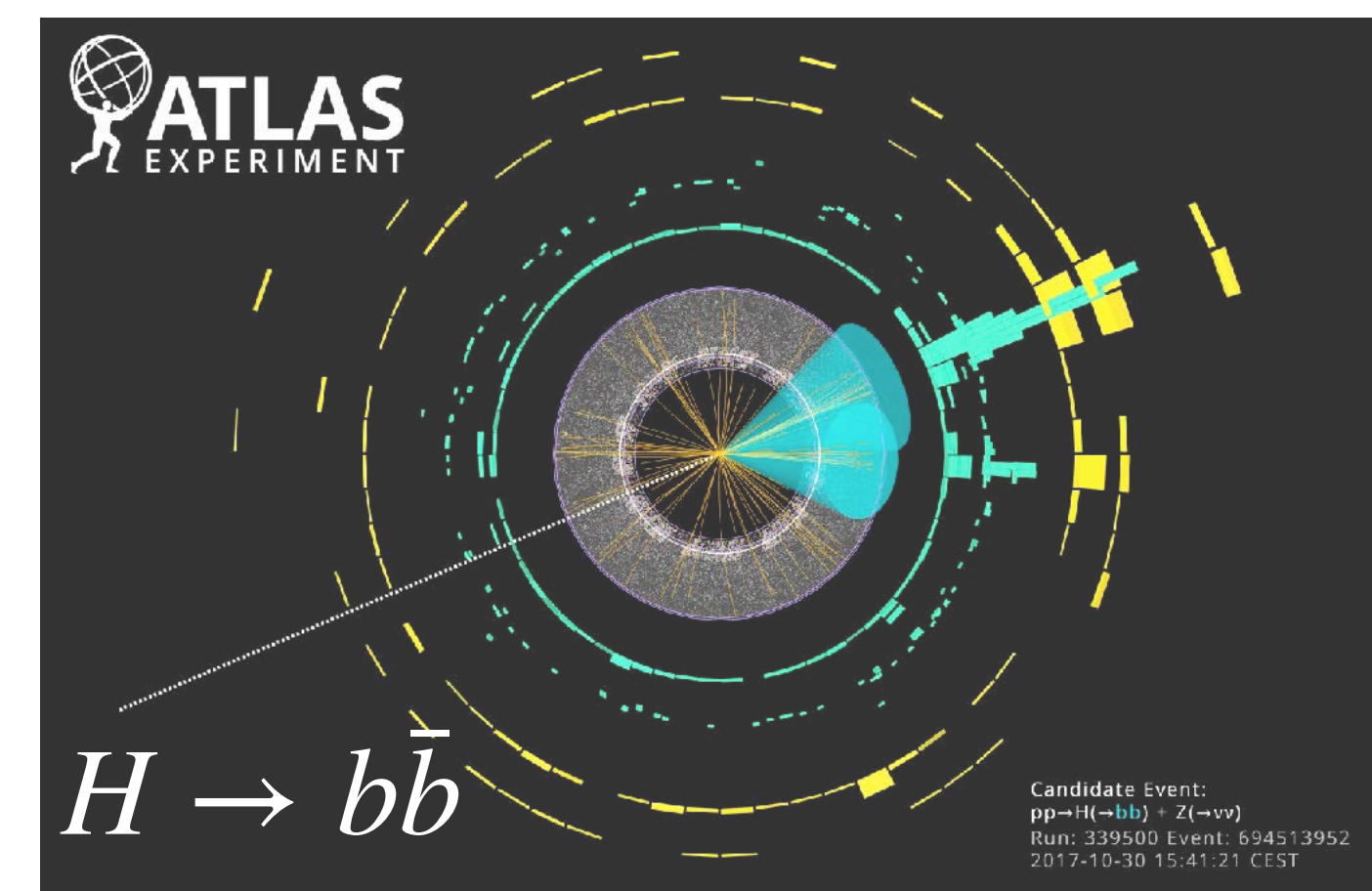
Jets initiated by charm ( $m \sim 1.5$  GeV) or bottom ( $m \sim 4.2$  GeV) quarks that leaves specific signatures in the detector



e.g. lifetime of B-hadron long enough to travel a macroscopic distance before decaying



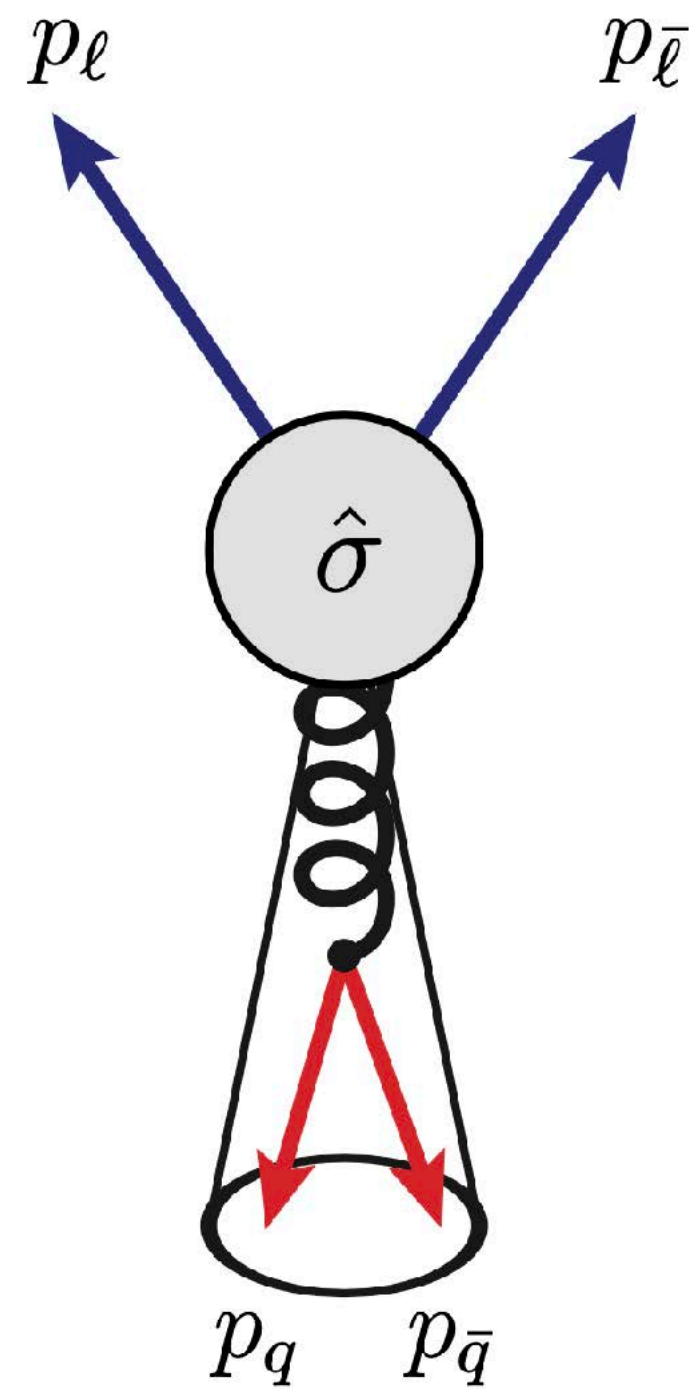
Flavoured jets useful to disentangle quark flavours inside the proton and special role in Higgs physics



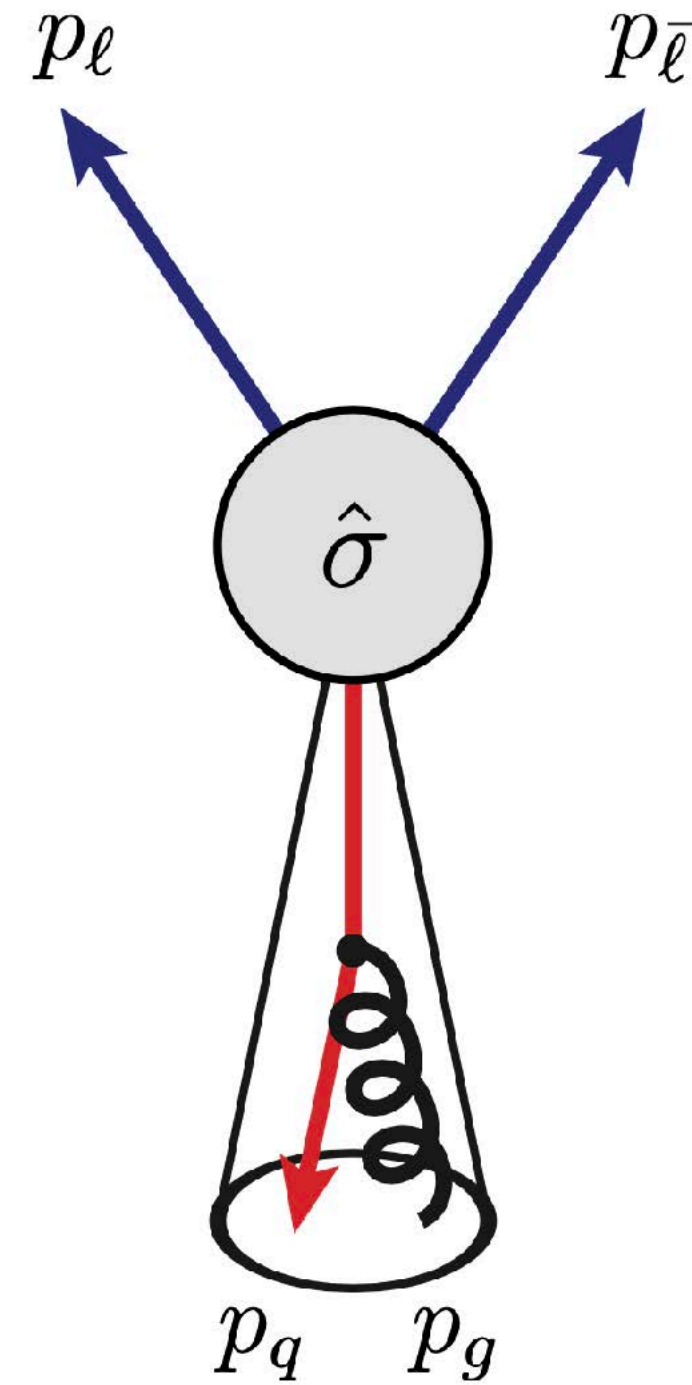


# Problem: definition of flavoured jet in perturbative QCD calculations

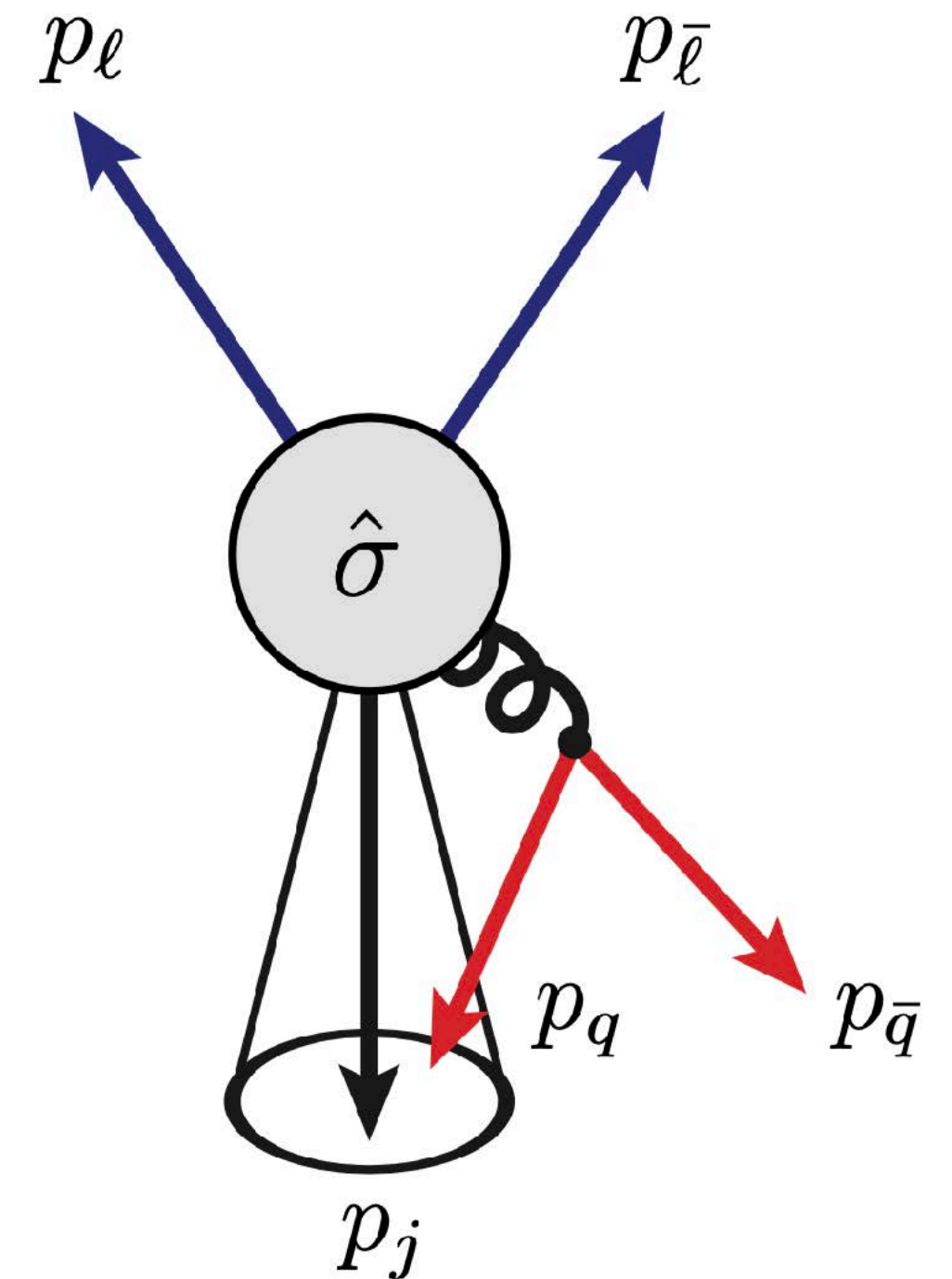
“A ( $anti-k_T$ ) jet is flavoured if it contains a flavoured quark inside it with some minimal energy”



$g \rightarrow q\bar{q}$  is always flavoured  
even in the collinear limit  
-> **collinear unsafe**



$q \rightarrow qg$  collinear with a hard gluon  
leads to a flavourless jet  
-> **collinear unsafe**



Soft large-angle  $g \rightarrow q\bar{q}$   
polluting the flavour of other jets  
-> **soft unsafe**



## Solution: new generation of infrared safe flavoured jet algorithms (2022 - ...)

4 new proposals, IRC safe to all orders (or up to high order) with exact (or close to exact) anti- $k_t$  kinematics

[Caletti, Larkoski, Marzani, Reichelt (2205.01109)] **SDF**

[Czakon, Mitov, Poncelet (2205.11879)] **CMP**

[Gauld, Huss, GS (2208.11138)] **GHS**

[Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2306.07314)] **IFN**

**A FastJet implementation of all algorithms available  
in fjcontrib from version 1.101**

<https://fastjet.hepforge.org/contrib/>



# Solution: new generation of infrared safe flavoured jet algorithms (2022 - ...)

4 new proposals, IRC safe to all orders (or up to high order) with exact (or close to exact) anti- $k_t$  kinematics

[Caletti, Larkoski, Marzani, Reichelt (2205.01109)] **SDF**

[Czakon, Mitov, Poncelet (2205.11879)] **CMP**

[Gauld, Huss, GS (2208.11138)] **GHS**

[Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (2306.07314)] **IFN**

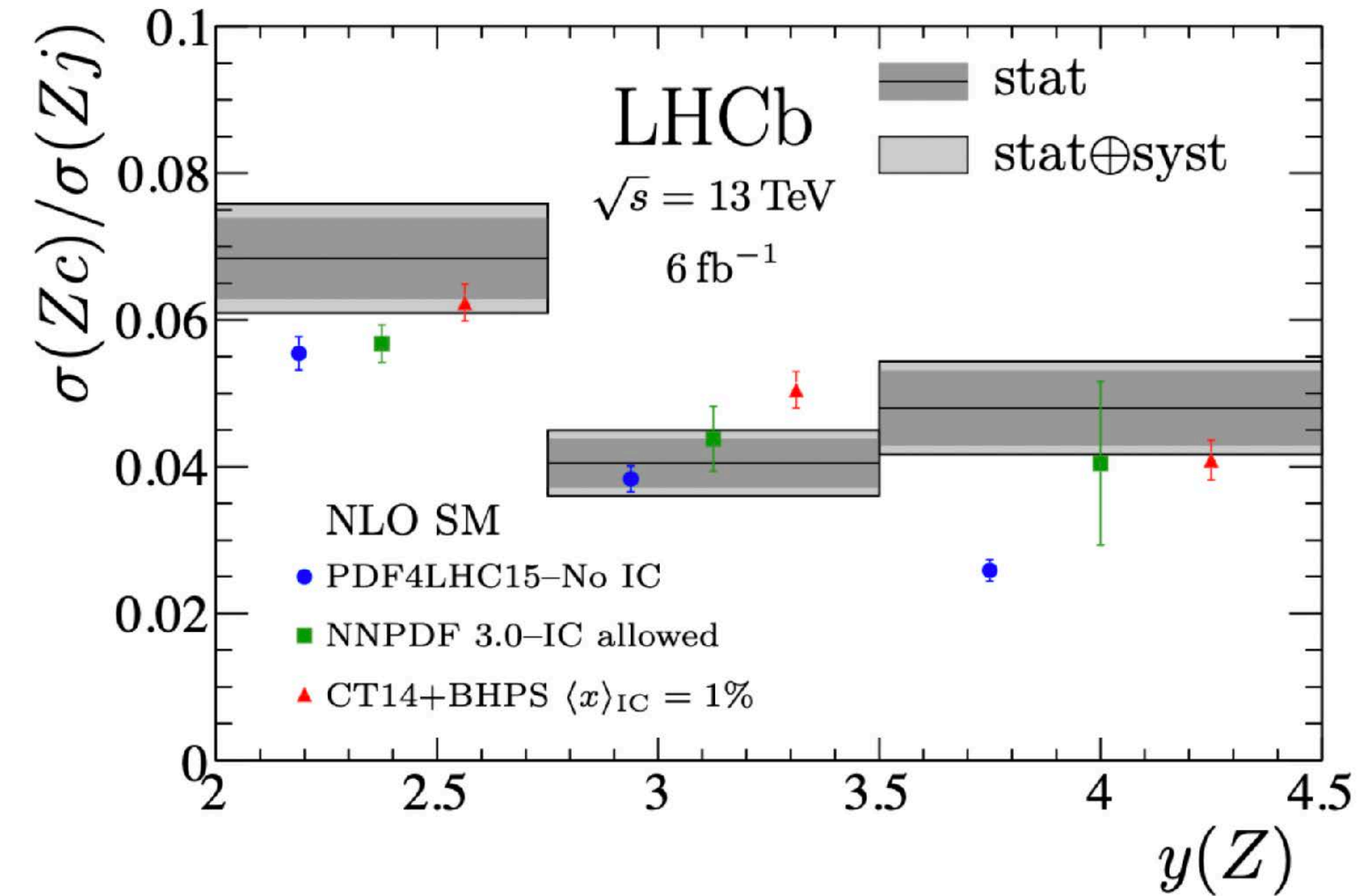
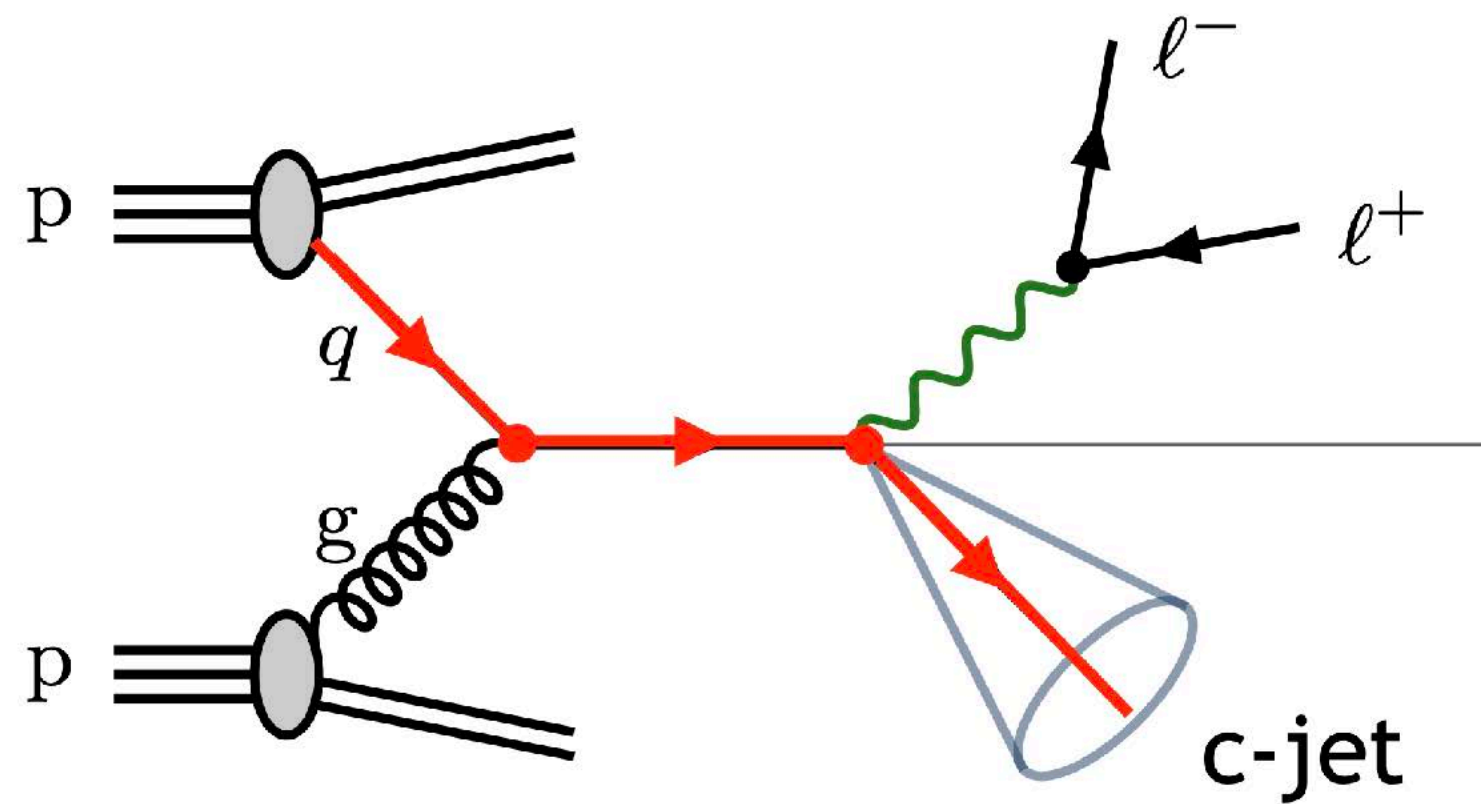
**Here I will focus on some QCD NNLO results  
about  $Z+c$ -jet [2302.12844] and  $W+c$ -jet [2311.14991]  
with GHS algorithm**

These NNLO predictions were obtained with NNLOJET  
by tracking flavour of final-state particles in all layers of the calculation



# $Z + c$ -jet in the forward region (LHCb)

Measurement sensitive to **intrinsic charm in the proton**



$Z$ bosons	$p_T(\mu) > 20 \text{ GeV}, 2.0 < \eta(\mu) < 4.5, 60 < m(\mu^+ \mu^-) < 120 \text{ GeV}$
Jets	$20 < p_T(j) < 100 \text{ GeV}, 2.2 < \eta(j) < 4.2$
Charm jets	$p_T(c \text{ hadron}) > 5 \text{ GeV}, \Delta R(j, c \text{ hadron}) < 0.5$
Events	$\Delta R(\mu, j) > 0.5$



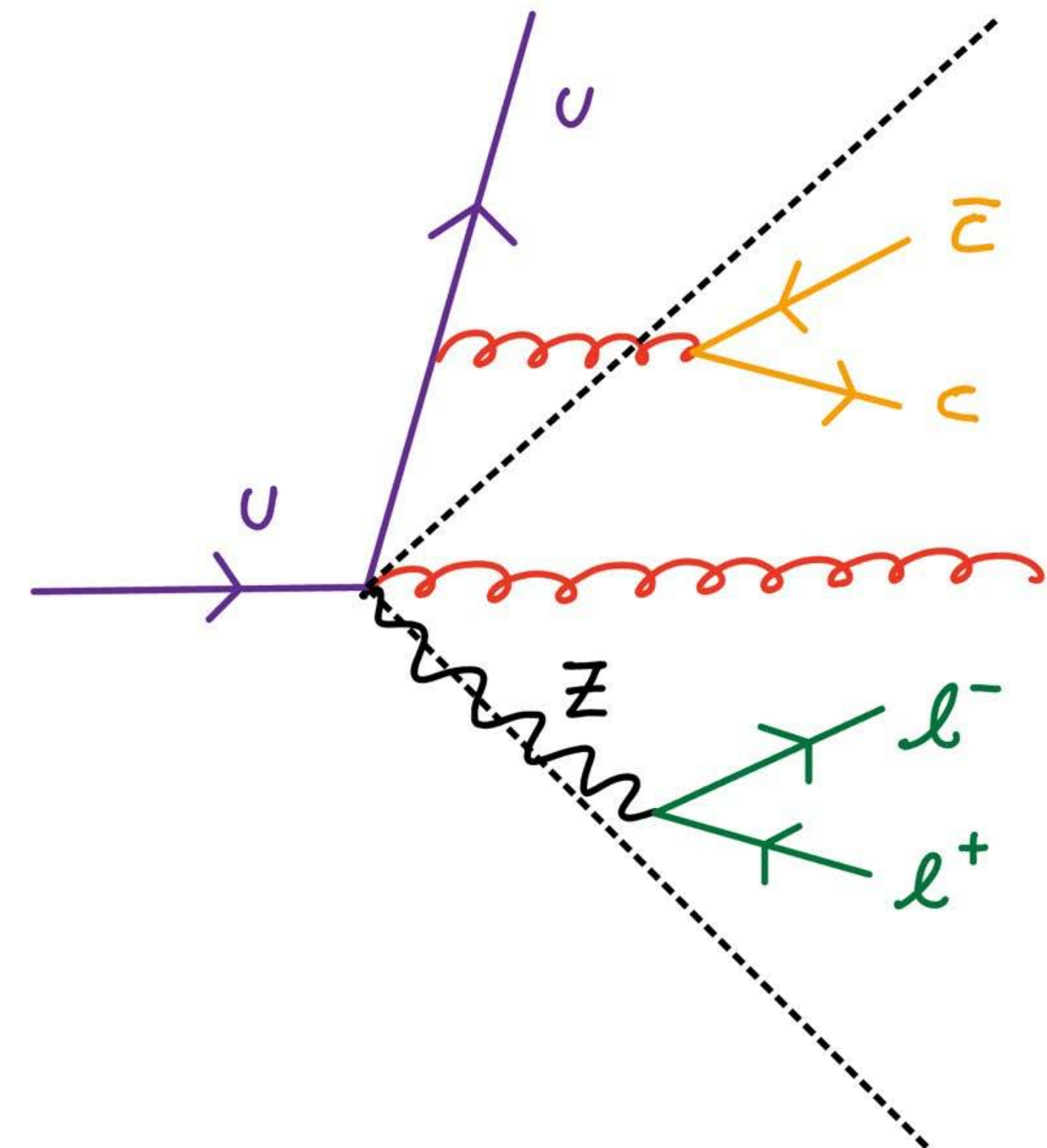
Very unique fiducial region of the measurement:

$Z$ bosons	$p_T(\mu) > 20 \text{ GeV}, 2.0 < \eta(\mu) < 4.5, 60 < m(\mu^+ \mu^-) < 120 \text{ GeV}$
Jets	$20 < p_T(j) < 100 \text{ GeV}, 2.2 < \eta(j) < 4.2$
Charm jets	$p_T(c \text{ hadron}) > 5 \text{ GeV}, \Delta R(j, c \text{ hadron}) < 0.5$
Events	$\Delta R(\mu, j) > 0.5$

We explore a theory-driven cut:

$$p_T(Z + \text{jet}) < p_{T,\text{jet}}$$

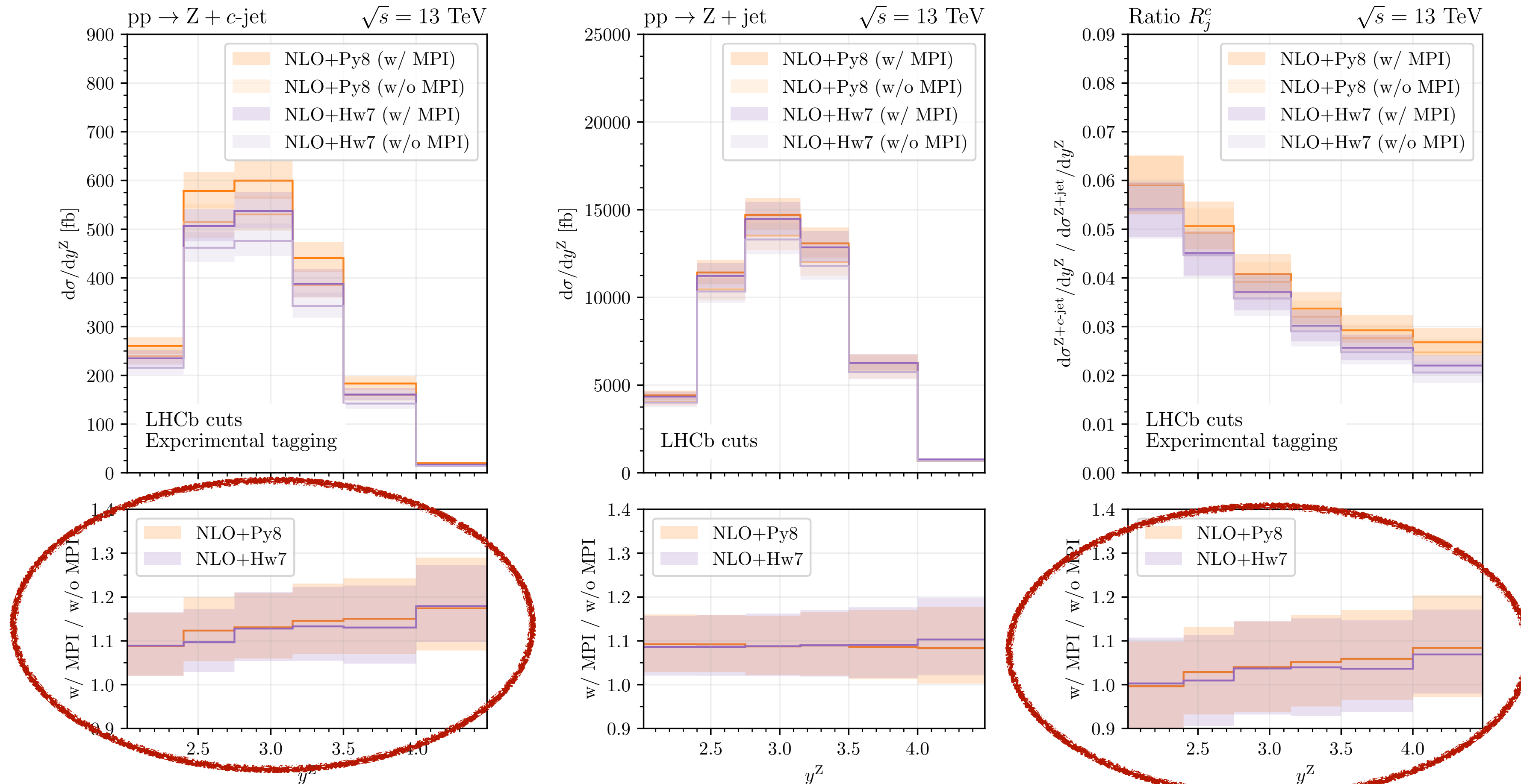
At Born level, the  $p_T$  of the  $Z$ +jet system vanishes, hence the cut limits the hard QCD radiation outside the LHCb acceptance in a dynamical way.



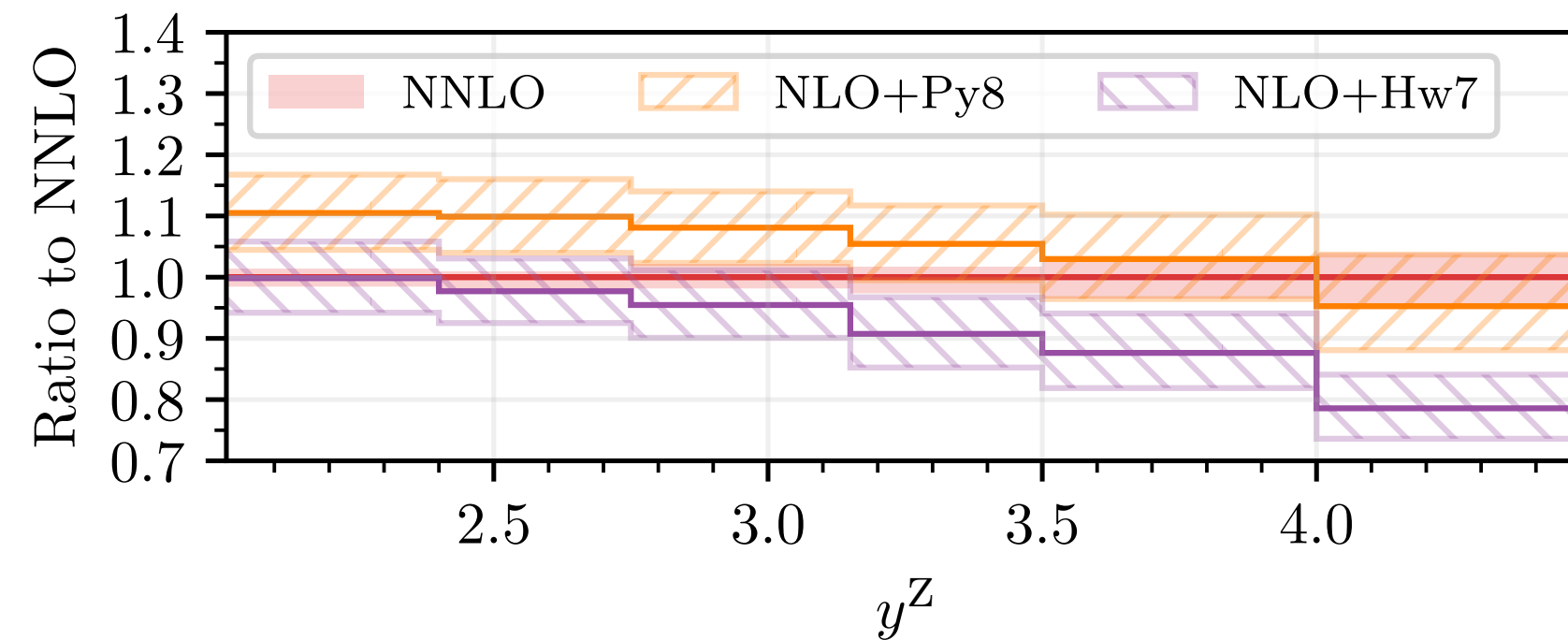
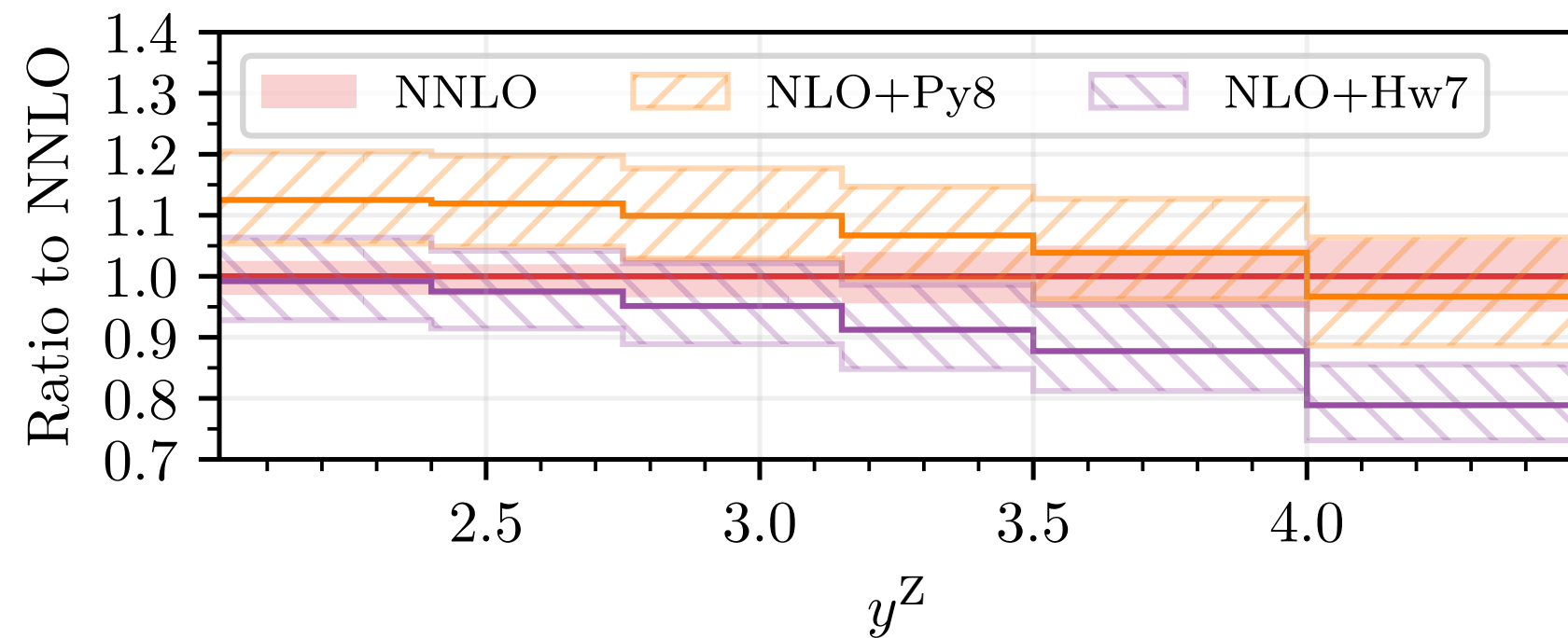
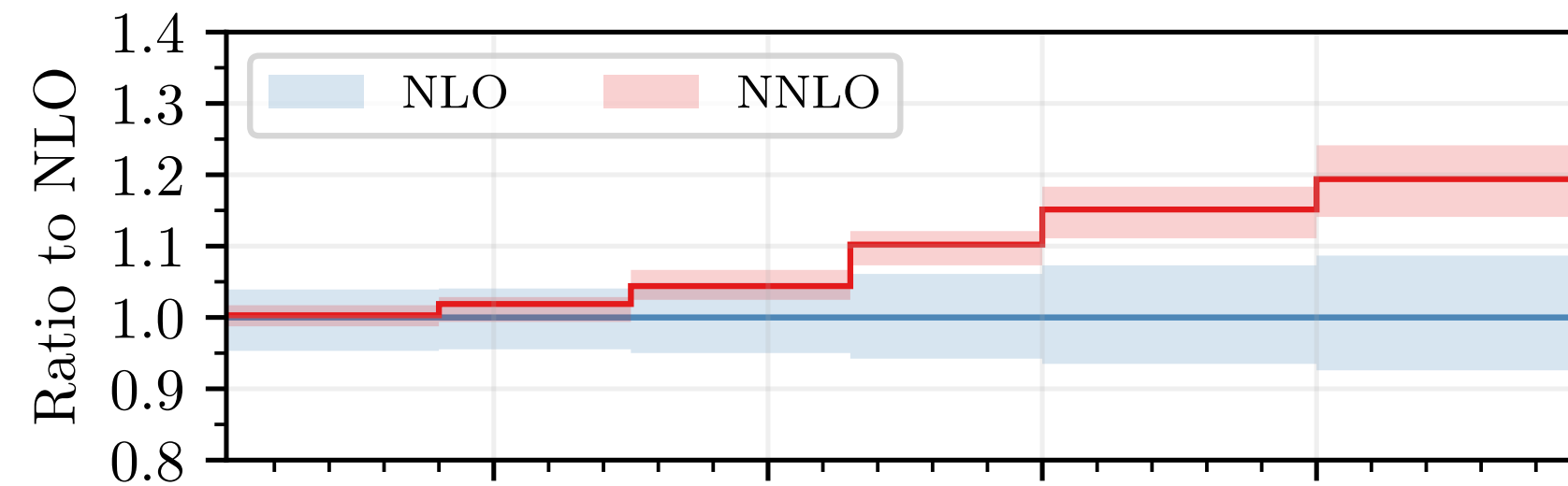
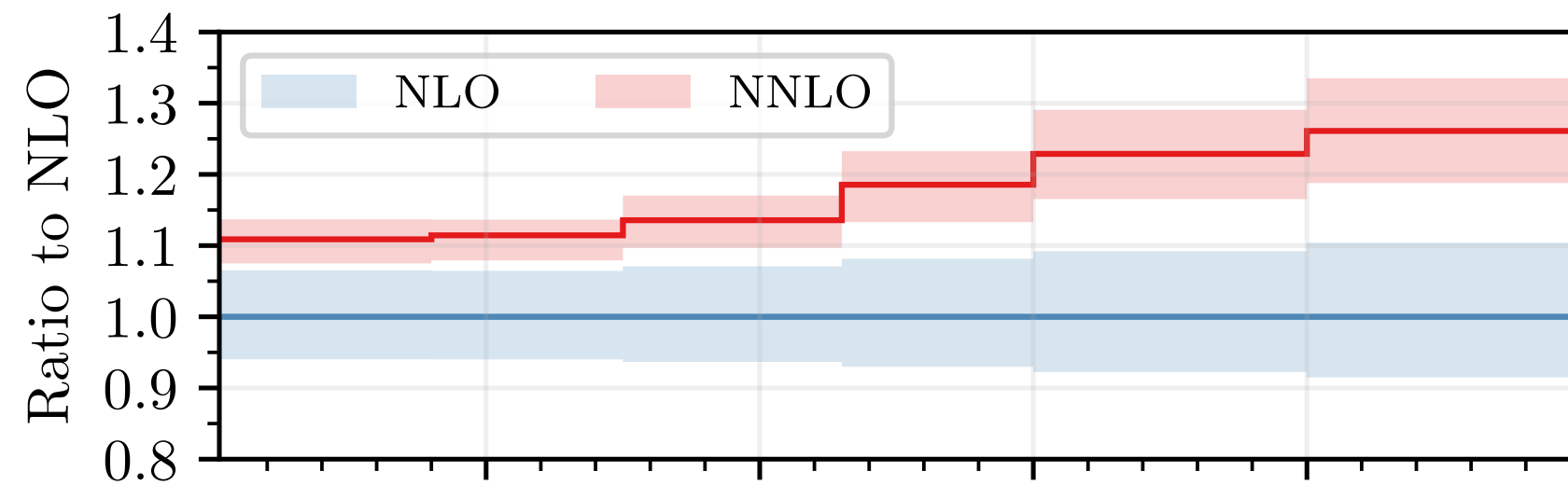
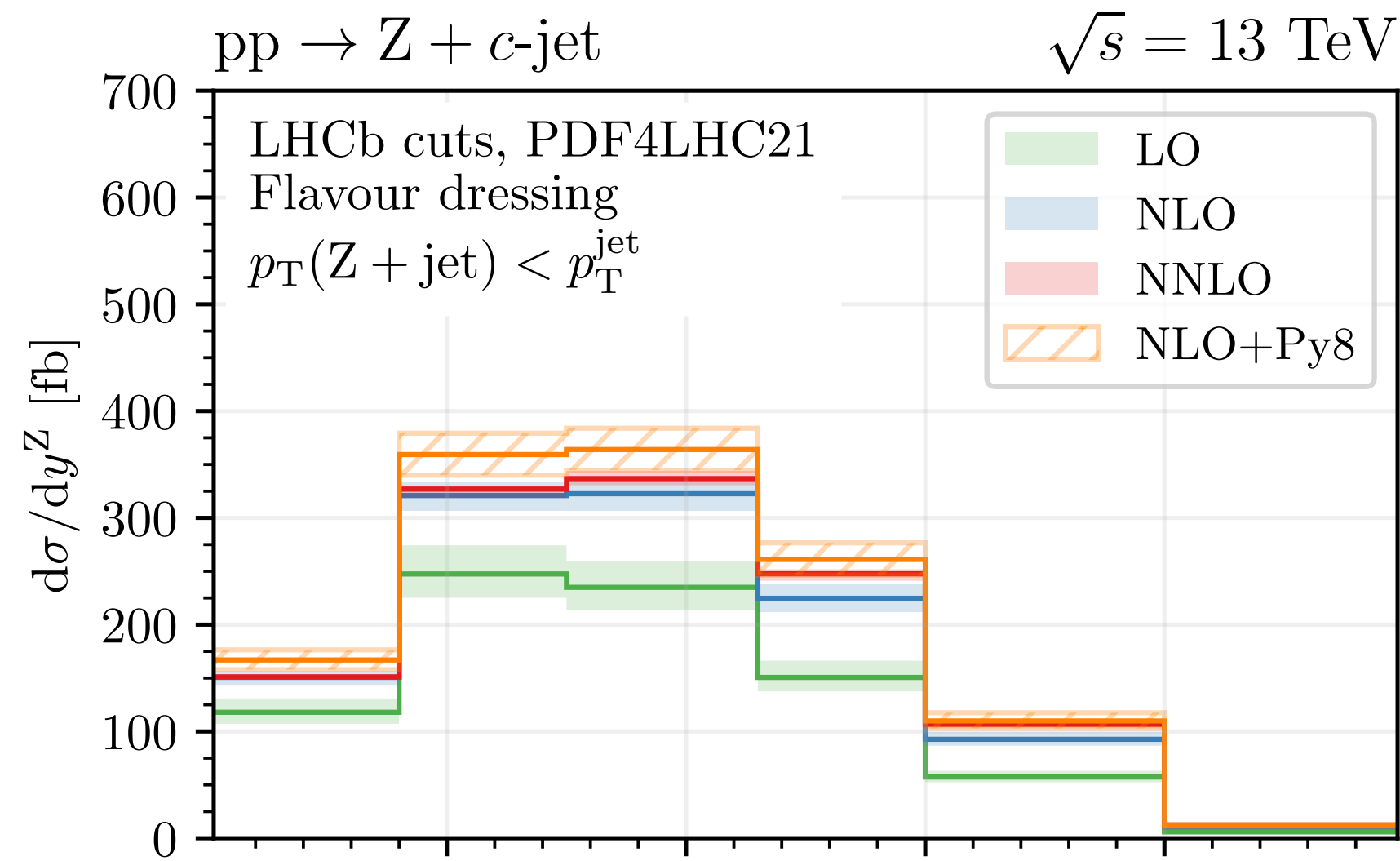
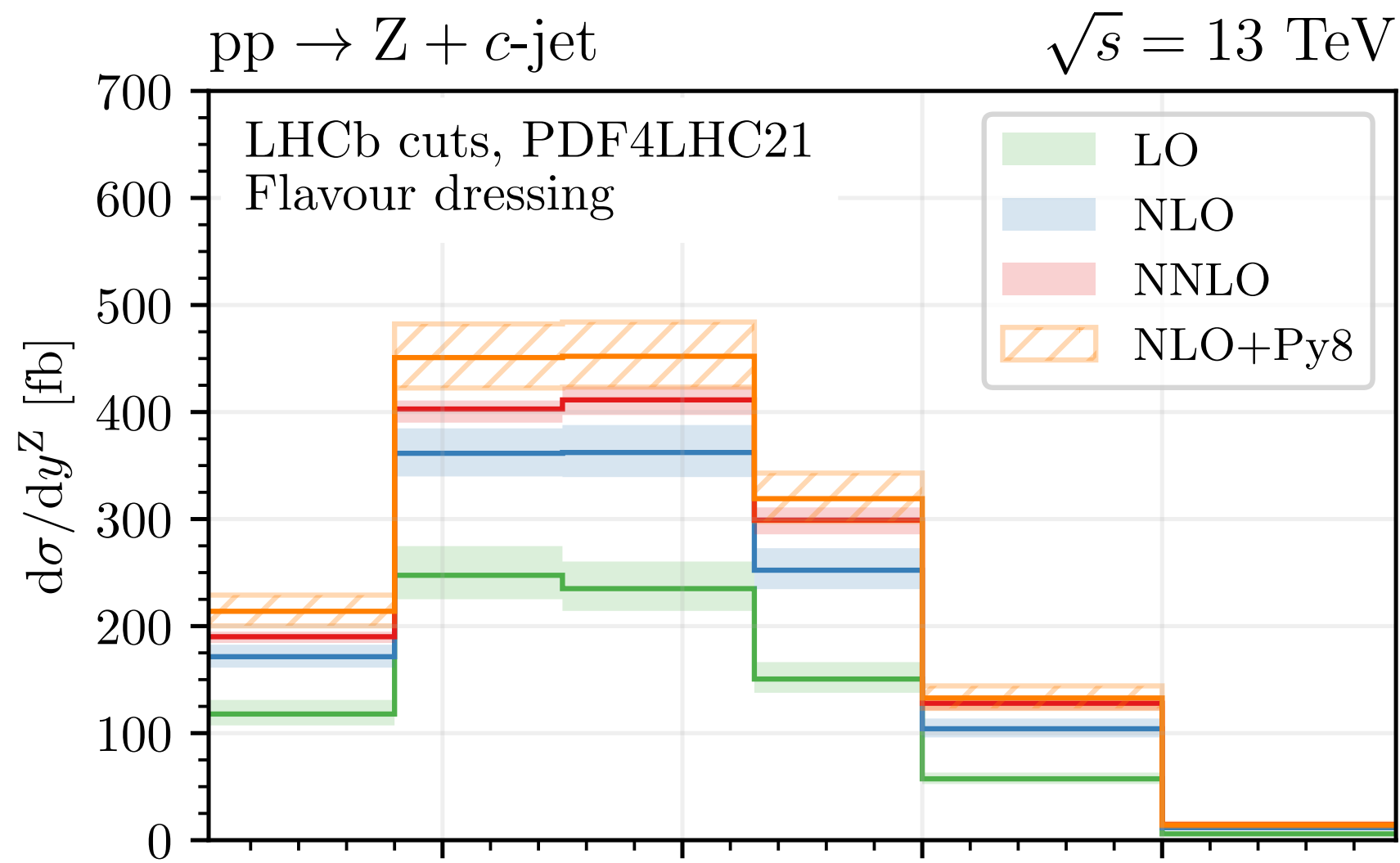


# We refrain from making a comparison to the LHCb data

- 1) definition of flavoured jet adopted by LHCb not IRC safe
- 2) significant contamination from MPI





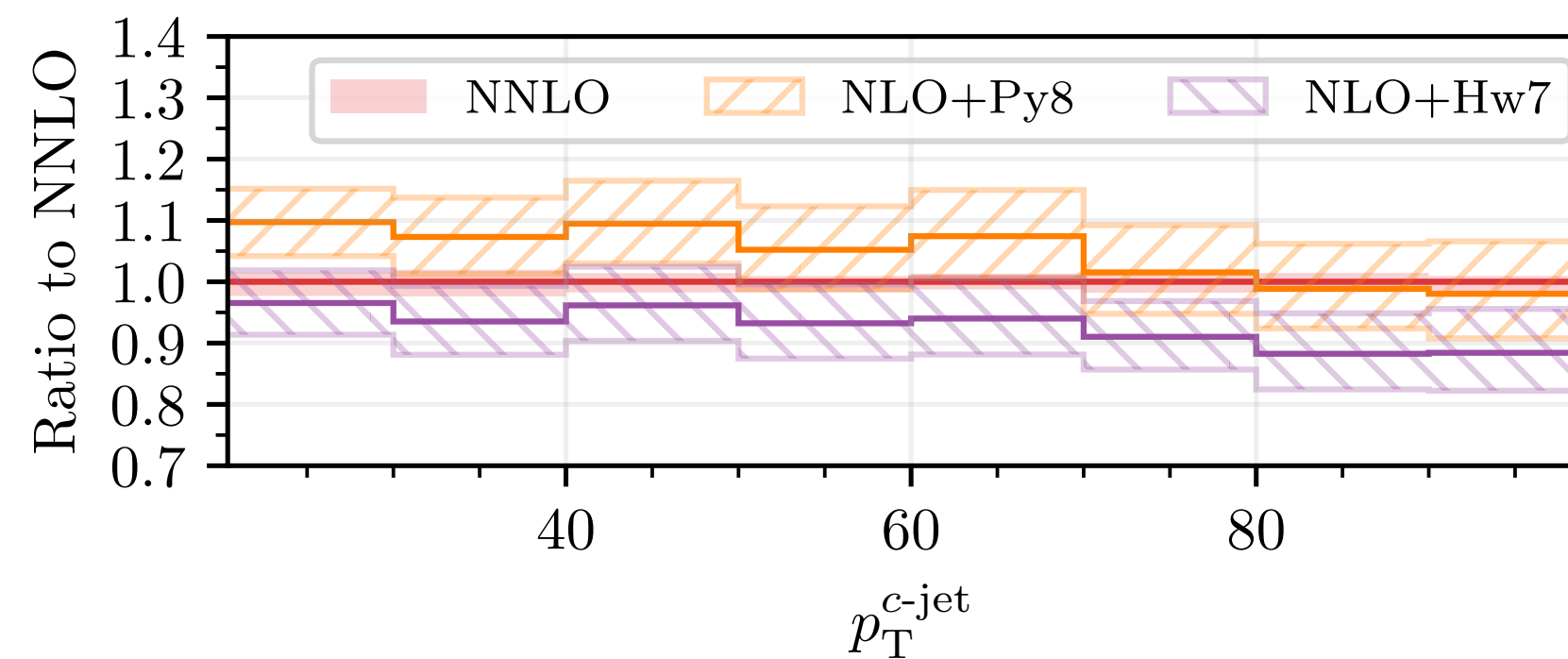
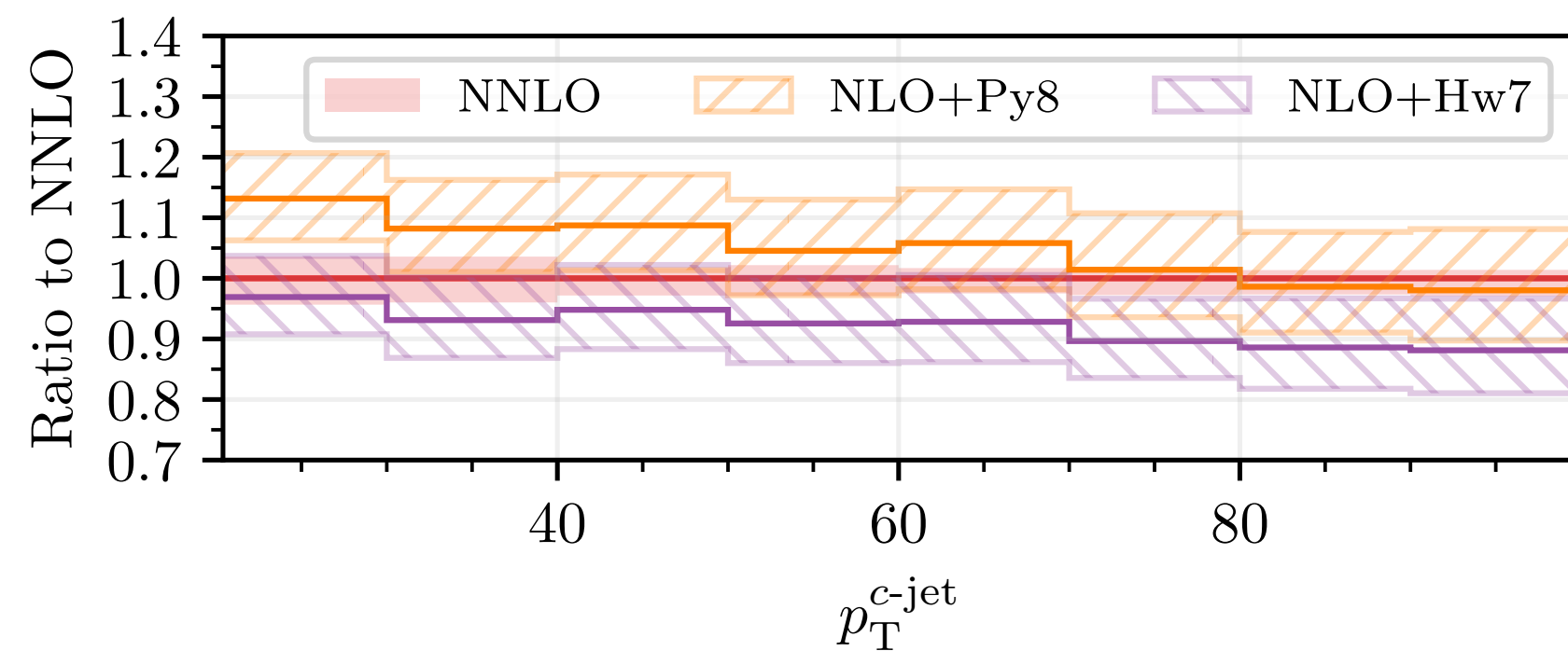
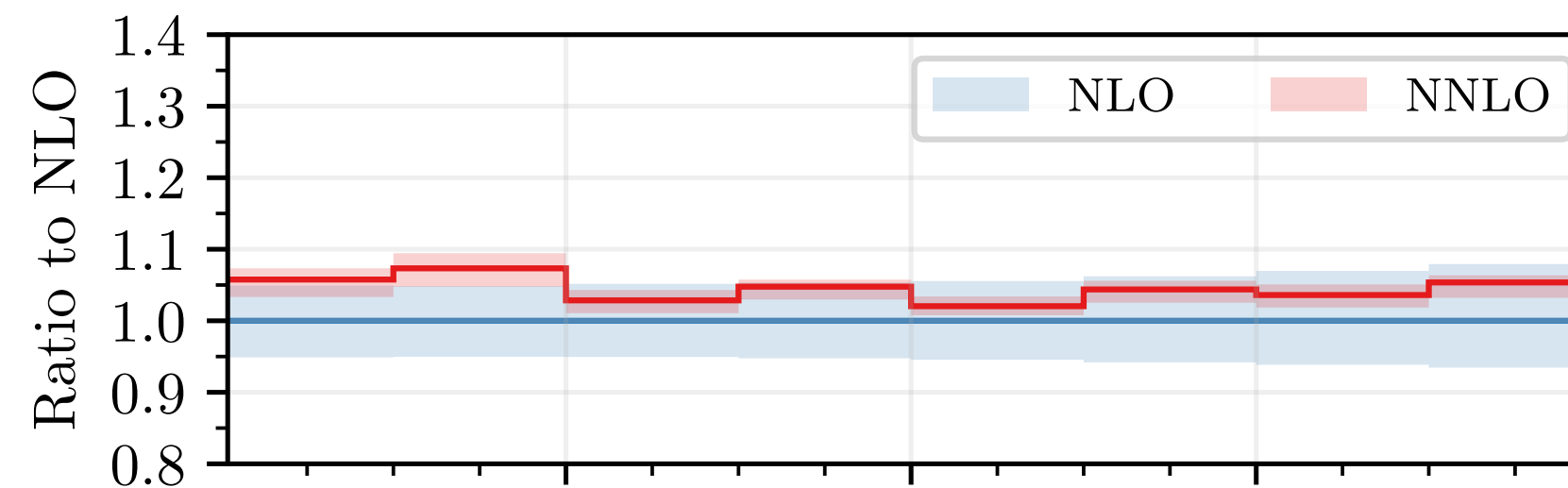
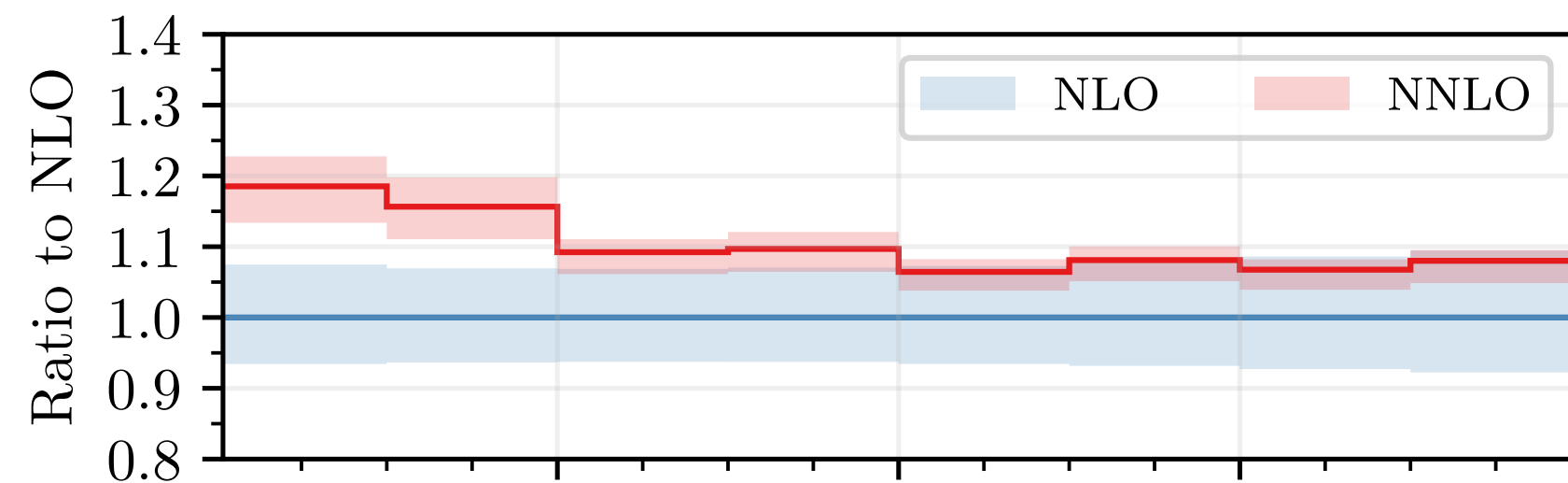
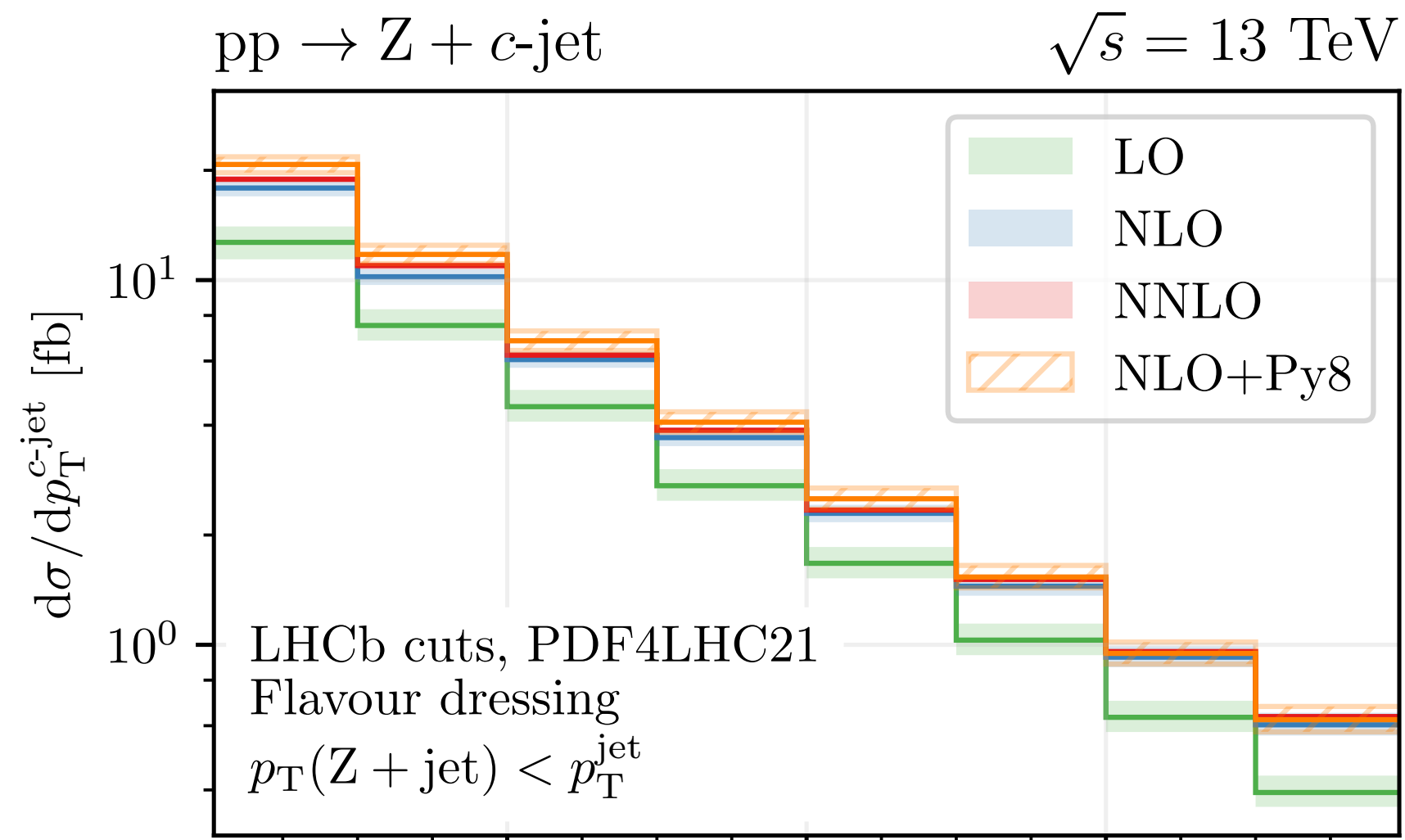
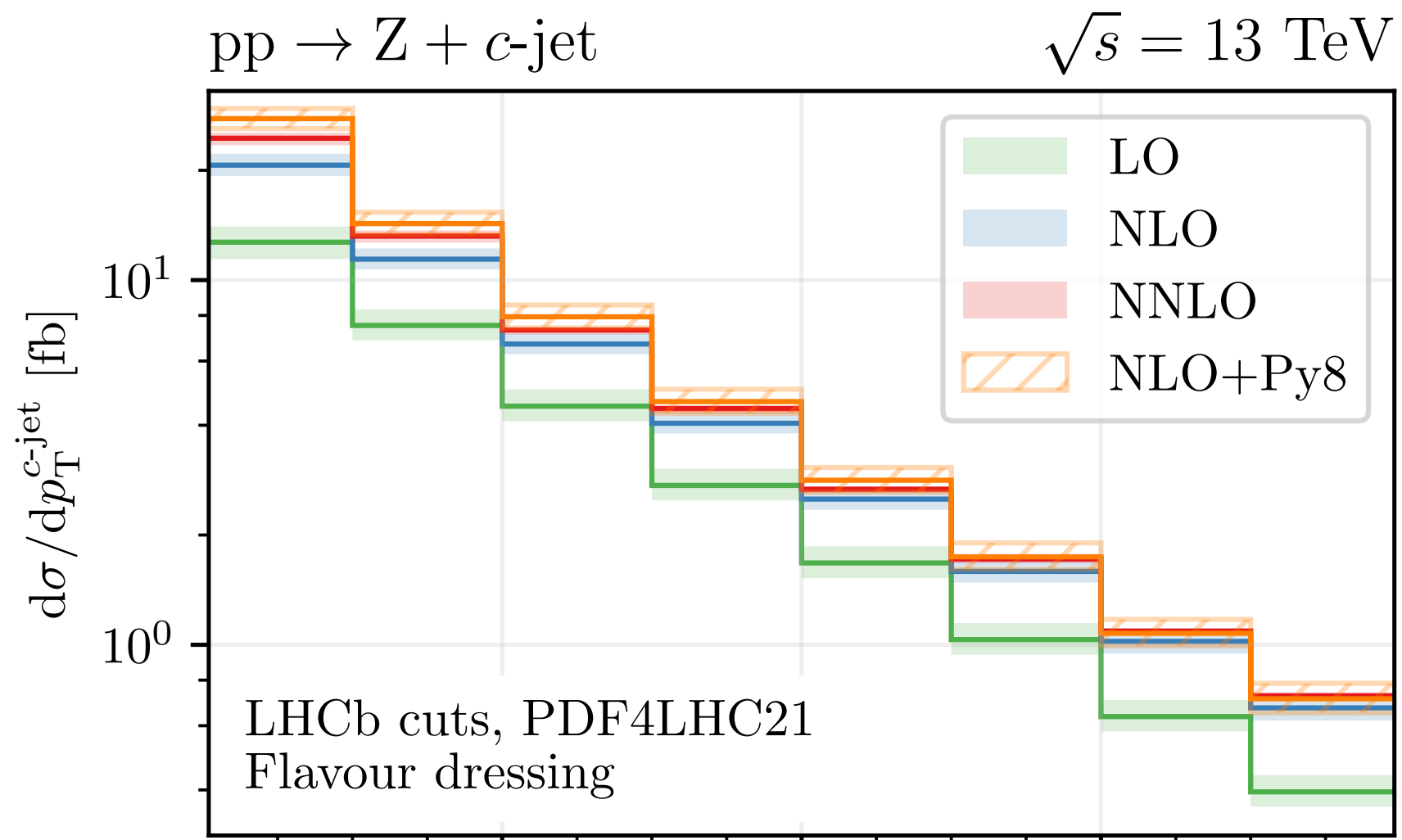


**NNLO lies between  
NLO+PS predictions  
with different PS.**

**Reduction of theory  
uncertainties by a  
factor of 2.**

Theory-driven cut  
improves perturbative  
convergence.





**NNLO lies between  
NLO+PS predictions  
with different PS.**

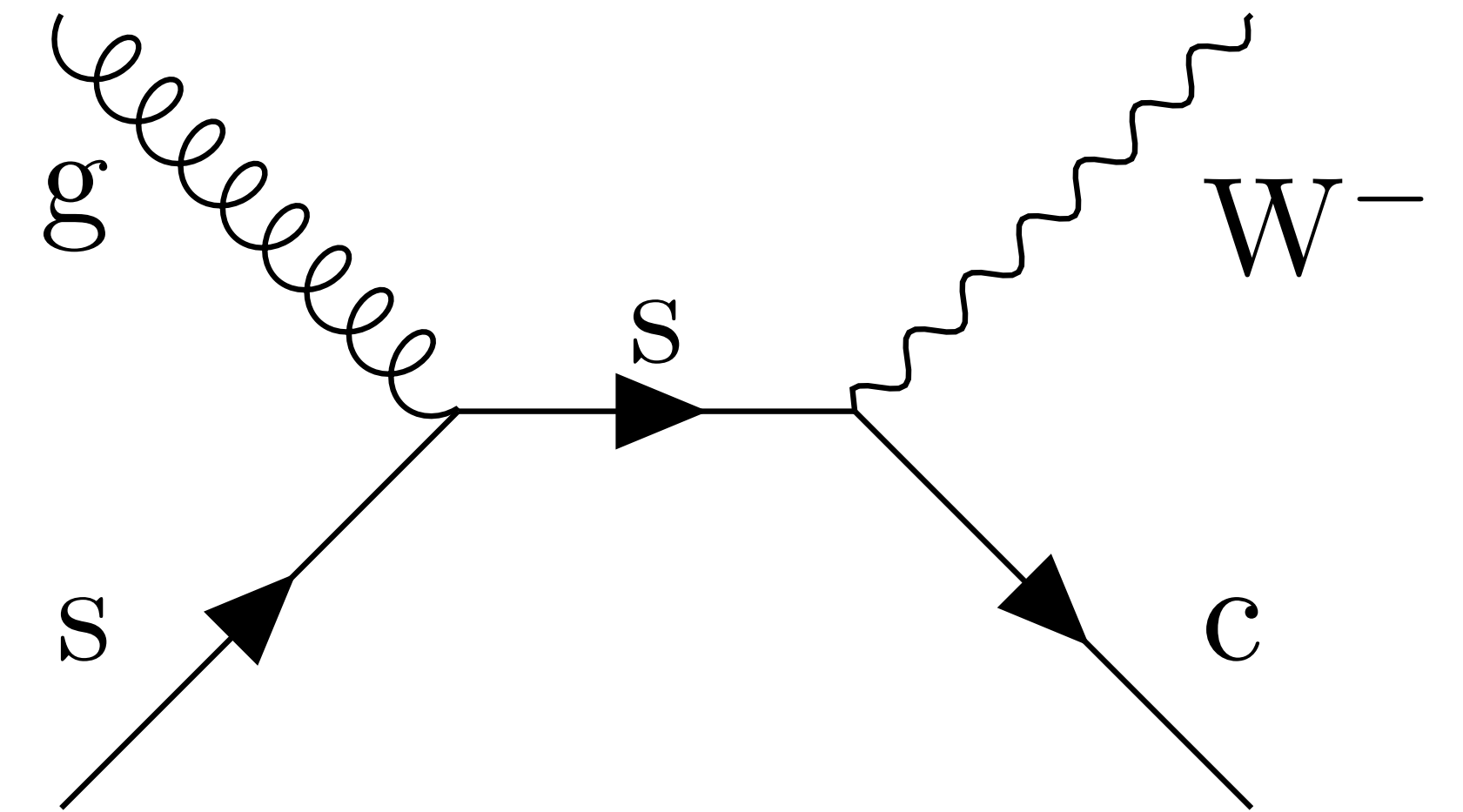
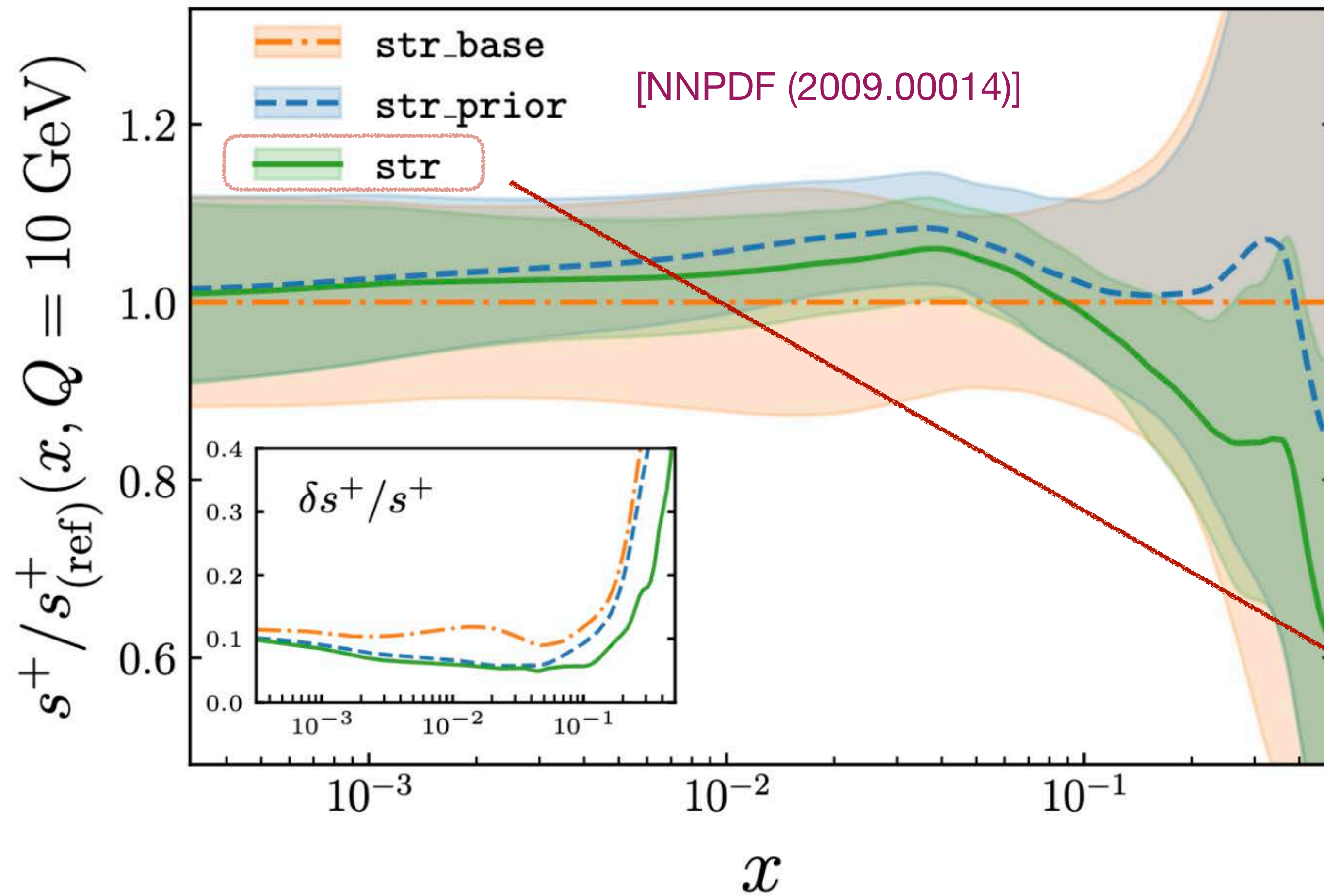
**Reduction of theory  
uncertainties by a  
factor of 2.**

Theory-driven cut  
improves perturbative  
convergence.



# $W + c$ -jet in the central region (ATLAS/CMS)

Unique probe into the **strange PDF**



contain [ATLAS (1402.6263)] and [CMS (1310.1138)] 7 TeV data



## Fiducial region, LHC 13 TeV

Jet defined with anti- $k_t$ ,  $R = 0.4$ , flavour assignment with GHS algorithm

$$p_{T,\ell} > 27 \text{ GeV}, \quad |y_\ell| < 2.5, \quad p_{T,j} > 20 \text{ GeV}, \quad |\eta_j| < 2.5,$$

$$E_{T,\text{miss}} > 20 \text{ GeV}, \quad M_{T,W} > 45 \text{ GeV}, \quad \Delta R(j, \ell) > 0.4.$$

We keep the full CKM matrix in our results!

### Definitions:

*Inclusive*: at least one c-jet

*Exclusive*: exactly one c-jet

OS: events with lepton from W-decay with opposite charge of that the c-jet

SS: events with lepton from W-decay with same charge of that the c-jet

OS-SS subtraction should remove events where the charm is radiatively generated



## Results for fiducial cross sections

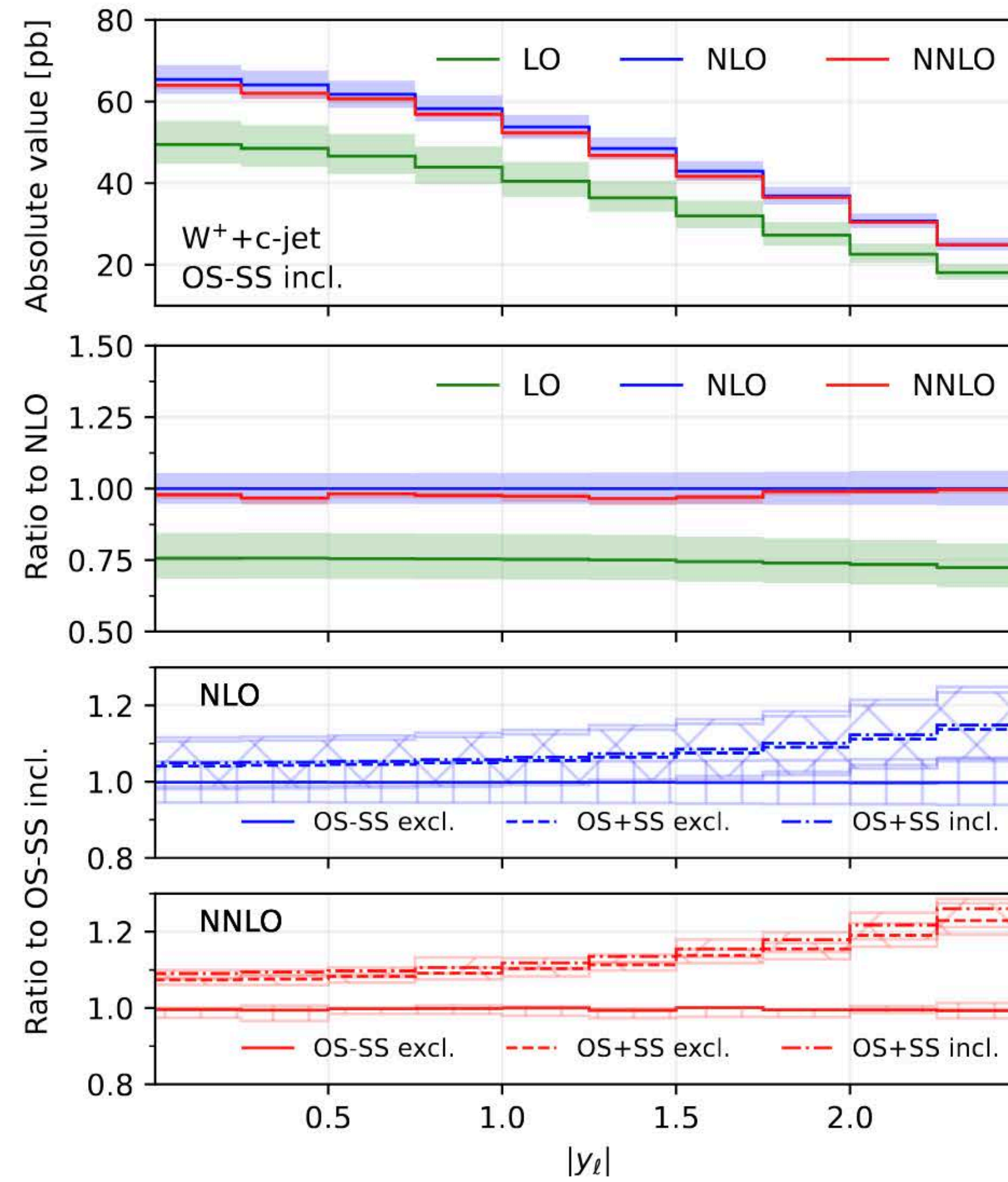
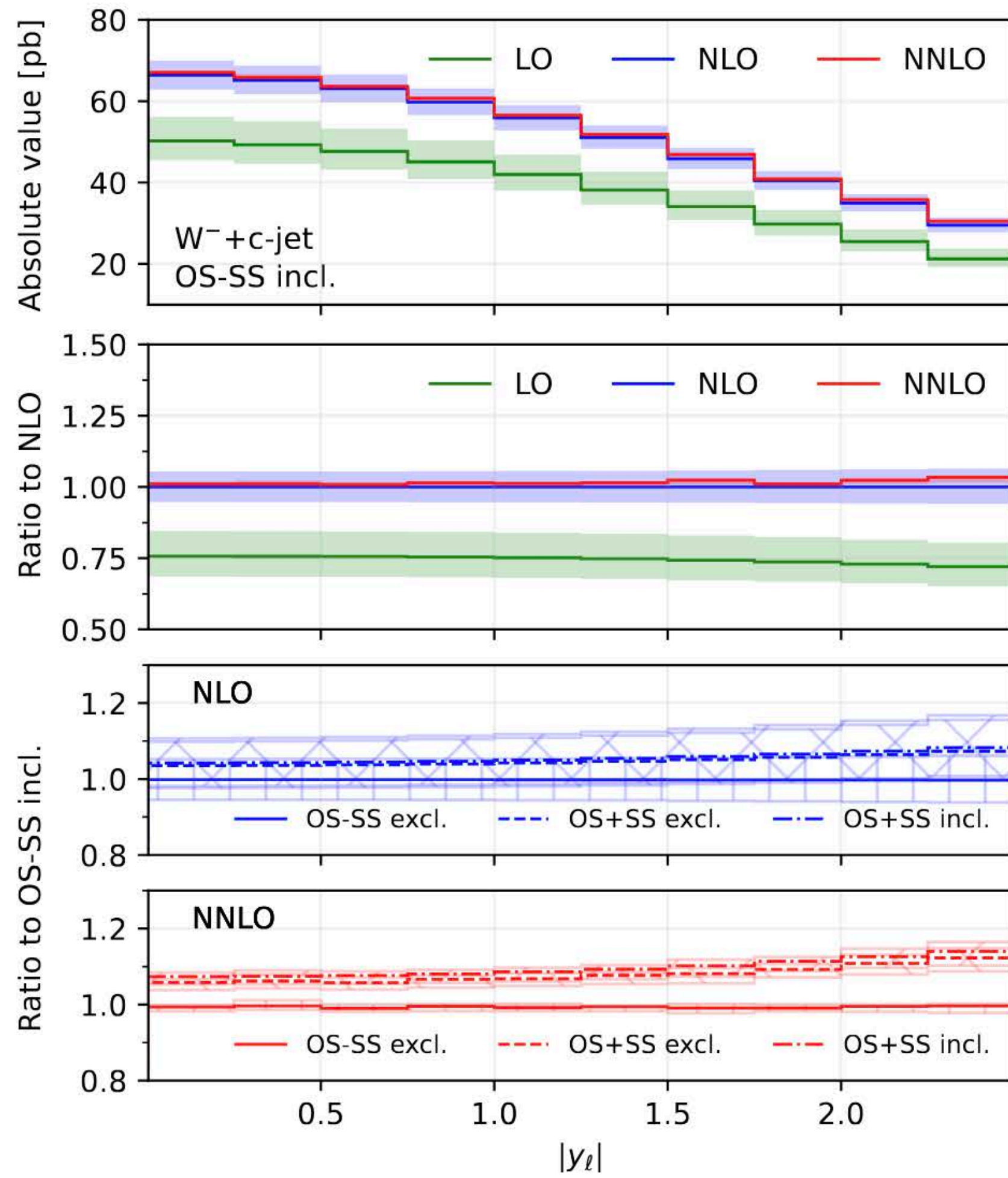
$W^+ + c\text{-jet}$	OS–SS incl.	OS–SS excl.	OS+SS incl.	OS+SS excl.
$\sigma^{\text{LO}}$	$91.34(1)^{+11.7\%}_{-9.5\%}$	$91.34(1)^{+11.7\%}_{-9.5\%}$	$91.34(1)^{+11.7\%}_{-9.5\%}$	$91.34(1)^{+11.7\%}_{-9.5\%}$
$\Delta\sigma^{\text{NLO}}$	30.45(4)	30.24(4)	39.23(4)	38.12(4)
$\sigma^{\text{NLO}}$	$121.79(4)^{+5.6\%}_{-5.4\%}$	$121.58(4)^{+5.6\%}_{-5.4\%}$	$130.56(4)^{+6.9\%}_{-6.3\%}$	$129.46(4)^{+6.8\%}_{-6.2\%}$
$\Delta\sigma^{\text{NNLO}}$	−2.3(8)	−2.7(7)	4.5(7)	3.2(7)
$\sigma^{\text{NNLO}}$	$119.5(8)^{+0.4\%}_{-1.8\%}$	$119.0(7)^{+0.1\%}_{-1.6\%}$	$135.1(8)^{+1.2\%}_{-1.9\%}$	$132.7(7)^{+0.6\%}_{-1.5\%}$
$W^- + c\text{-jet}$	OS–SS incl.	OS–SS excl.	OS+SS incl.	OS+SS excl.
$\sigma^{\text{LO}}$	$95.782(4)^{+11.7\%}_{-9.5\%}$	$95.782(4)^{+11.7\%}_{-9.5\%}$	$95.782(4)^{+11.7\%}_{-9.5\%}$	$95.782(4)^{+11.7\%}_{-9.5\%}$
$\Delta\sigma^{\text{NLO}}$	32.244(8)	32.004(8)	39.011(8)	38.043(8)
$\sigma^{\text{NLO}}$	$128.026(9)^{+5.7\%}_{-5.5\%}$	$127.786(9)^{+5.7\%}_{-5.5\%}$	$134.794(9)^{+6.6\%}_{-6.1\%}$	$133.826(9)^{+6.5\%}_{-6.0\%}$
$\Delta\sigma^{\text{NNLO}}$	2.9(5)	2.5(5)	8.2(5)	7.1(5)
$\sigma^{\text{NNLO}}$	$130.9(5)^{+0.7\%}_{-1.5\%}$	$130.3(5)^{+0.9\%}_{-1.5\%}$	$143.0(5)^{+1.5\%}_{-2.5\%}$	$141.0(5)^{+1.1\%}_{-2.4\%}$

**Reduction of theory  
uncertainties at  
increasing orders**

**Smaller NNLO  
corrections for  
OS-SS subtraction**



# Results for differential distributions



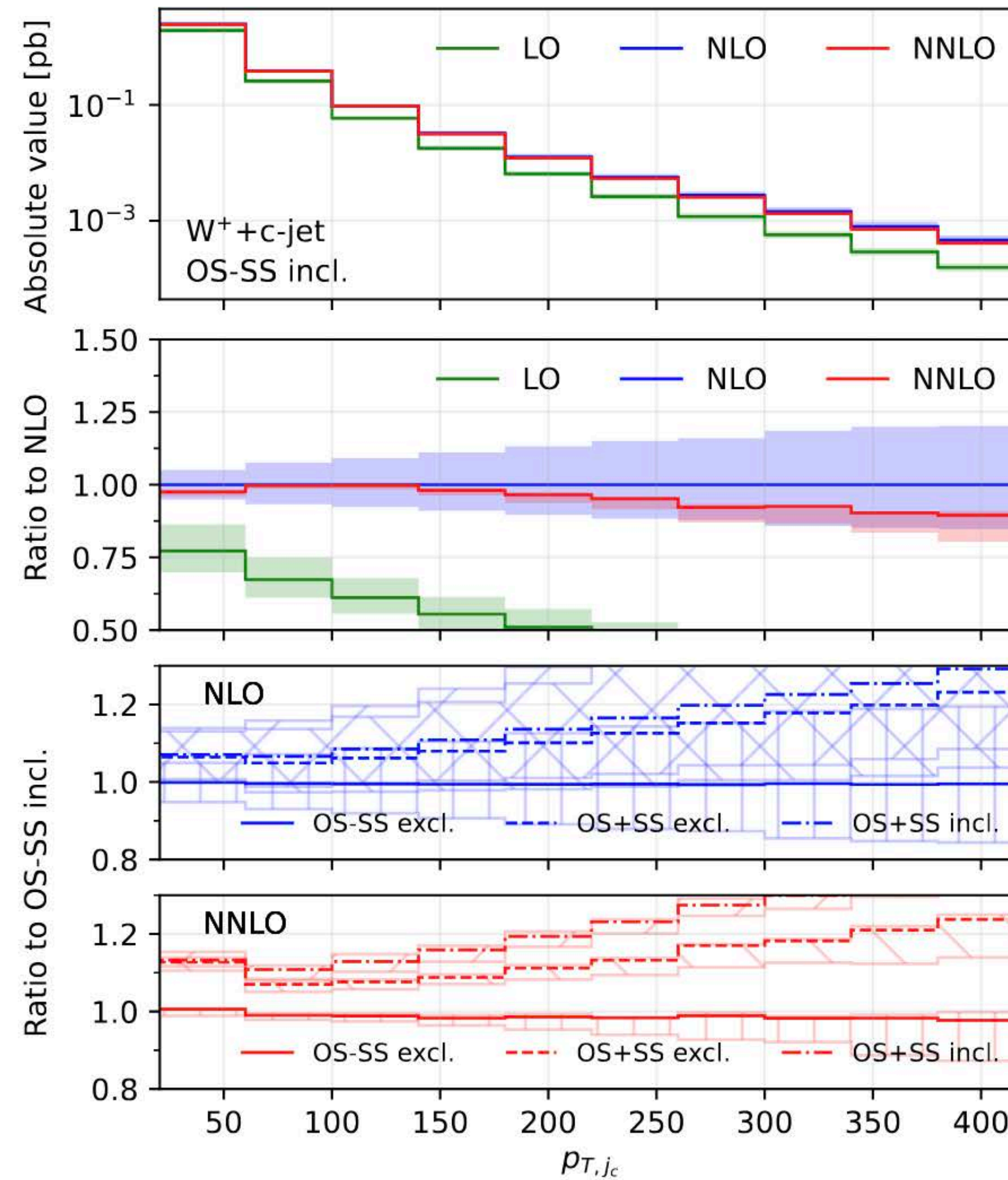
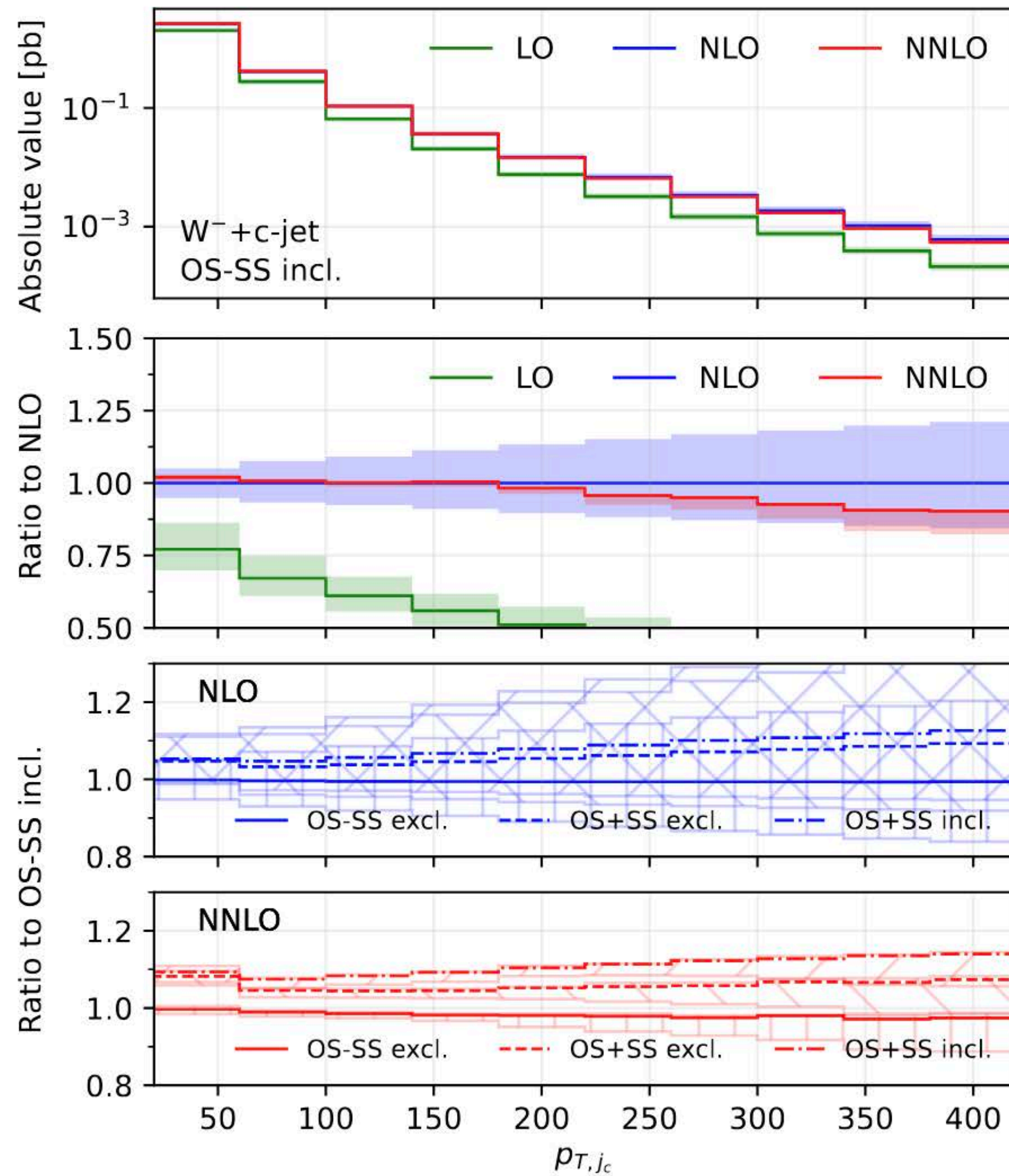
Good perturbative  
convergence

OS-SS incl. very similar  
to OS-SS excl.  
(OS-SS very efficient in  
discarding events with  
more than one c-jet)

OS-SS vs. OS+SS  
increasing at large values  
of  $|y_\ell|$  and  $p_{T,j_c}$ ,  
difference more  
pronounced at NNLO



# Results for differential distributions



Good perturbative  
convergence

OS-SS incl. very similar  
to OS-SS excl.  
(OS-SS very efficient in  
discarding events with  
more than one c-jet)

OS-SS vs. OS+SS  
increasing at large values  
of  $|y_\ell|$  and  $p_{T,jc}$ ,  
difference more  
pronounced at NNLO



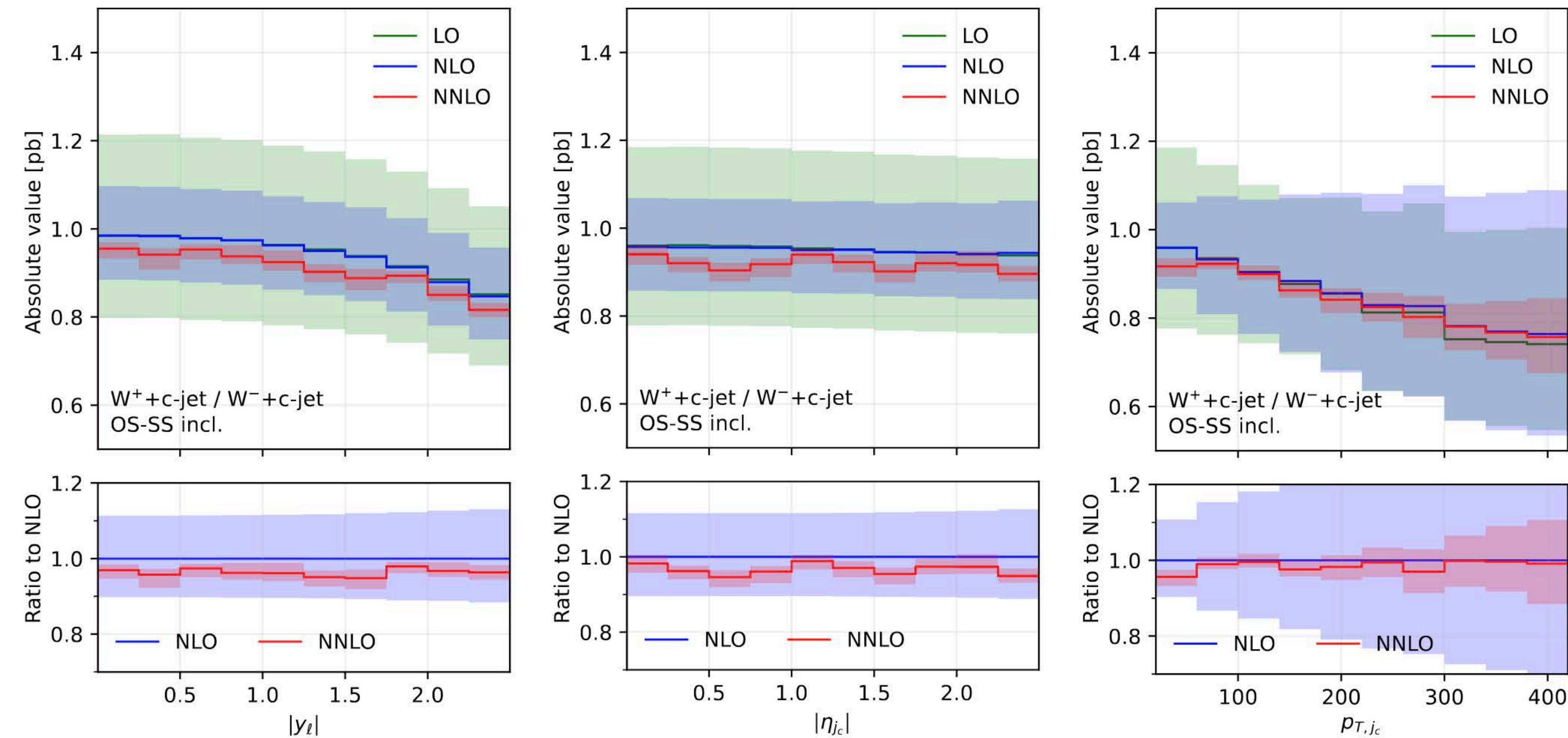
# Results for ratio $W^+ + \text{c-jet} / W^- + \text{c-jet}$

$\mathcal{R}_c^\pm$	OS-SS incl.	OS-SS excl.	OS+SS incl.	OS+SS excl.
LO	$0.9536(3)^{+23.4\%}_{-19.0\%}$	$0.9536(3)^{+23.4\%}_{-19.0\%}$	$0.9536(3)^{+23.4\%}_{-19.0\%}$	$0.9536(3)^{+23.4\%}_{-19.0\%}$
NLO	$0.951(1)^{+13.9\%}_{-12.1\%}$	$0.952(1)^{+13.6\%}_{-11.9\%}$	$0.968(1)^{+13.9\%}_{-12.1\%}$	$0.967(1)^{+13.6\%}_{-11.9\%}$
NNLO	$0.91(1)^{+1.9\%}_{-2.7\%}$	$0.91(1)^{+1.6\%}_{-2.5\%}$	$0.94(1)^{+3.7\%}_{-3.3\%}$	$0.94(1)^{+3.0\%}_{-2.6\%}$

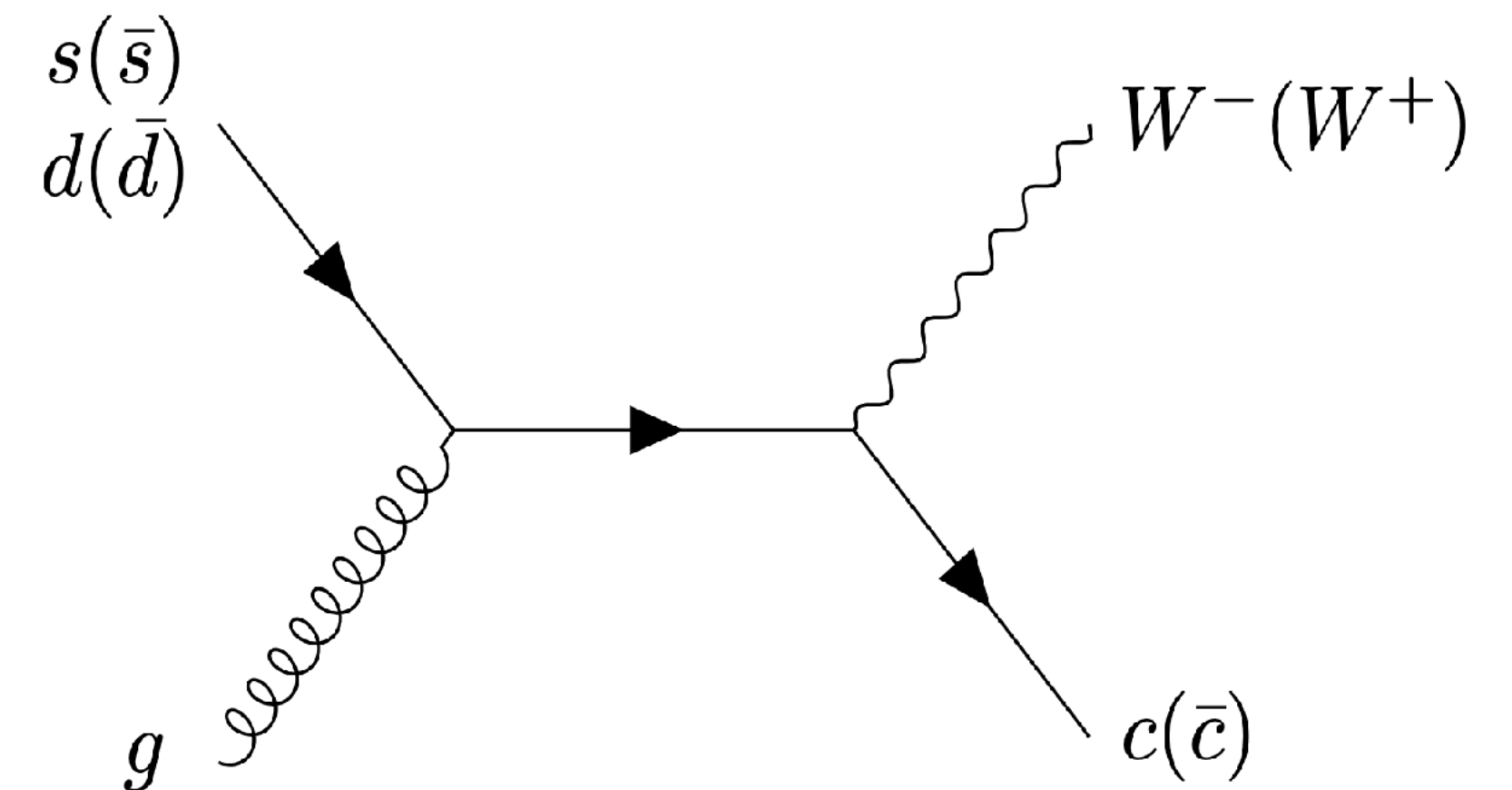
**Ratio always smaller than 1**

**i.e.  $W^-$  always larger than  $W^+$**

Physical interpretation at LO: contribution from the  $gs(\bar{s})$  channel similar, because  $s$  and  $\bar{s}$  PDF are similar. But subdominant  $gd$  channel is numerically different from  $g\bar{d}$  channel, because  $d$  PDF feature a valence component!



Very nice convergence





## Channel breakdown for fiducial cross sections

$W^- + c\text{-jet}$	OS LO	OS NLO	SS NLO	OS NNLO	SS NNLO
$c(\bar{c})s(\bar{s})$	0.0	-0.1225(3)	0.4852(2)	-0.05(2)	0.842(3)
$c(\bar{c})c(\bar{c})$	0.0	0.2158(1)	0.2062(2)	0.360(2)	0.351(1)
$c(\bar{c})q(\bar{q})$	0.0	1.2392(3)	1.3132(4)	1.958(4)	2.088(4)
$s(\bar{s})q(\bar{q})$	0.0	-0.651(3)	0.03134(1)	-1.1(2)	0.0537(2)
$s(\bar{s})s(\bar{s})$	0.0	-0.2549(3)	0.0	-0.42(3)	0.0
$q(\bar{q})q(\bar{q})$	0.0	1.0314(7)	0.9838(4)	1.73(2)	1.676(6)
$gq(\bar{q})$	8.9255(6)	12.700(1)	0.0	12.7(2)	0.405(3)
$gs(\bar{s})$	86.857(4)	123.002(8)	0.0	128.9(3)	-0.0353(6)
$gc(\bar{c})$	0.0	0.0	0.0	-0.14(2)	-0.057(2)
$gg$	0.0	-6.355(3)	0.0	-8.31(1)	0.0
total	95.782(5)	130.806(1)	3.020(1)	135.6(5)	5.324(9)

Dominant channel is  $gs(\bar{s})$ , followed by  $gq(\bar{q})$

The  $c(\bar{c})c(\bar{c})$ ,  $c(\bar{c})q(\bar{q})$  and  $q(\bar{q})q(\bar{q})$  channels are very similar between SS and OS: then OS-SS basically removes them



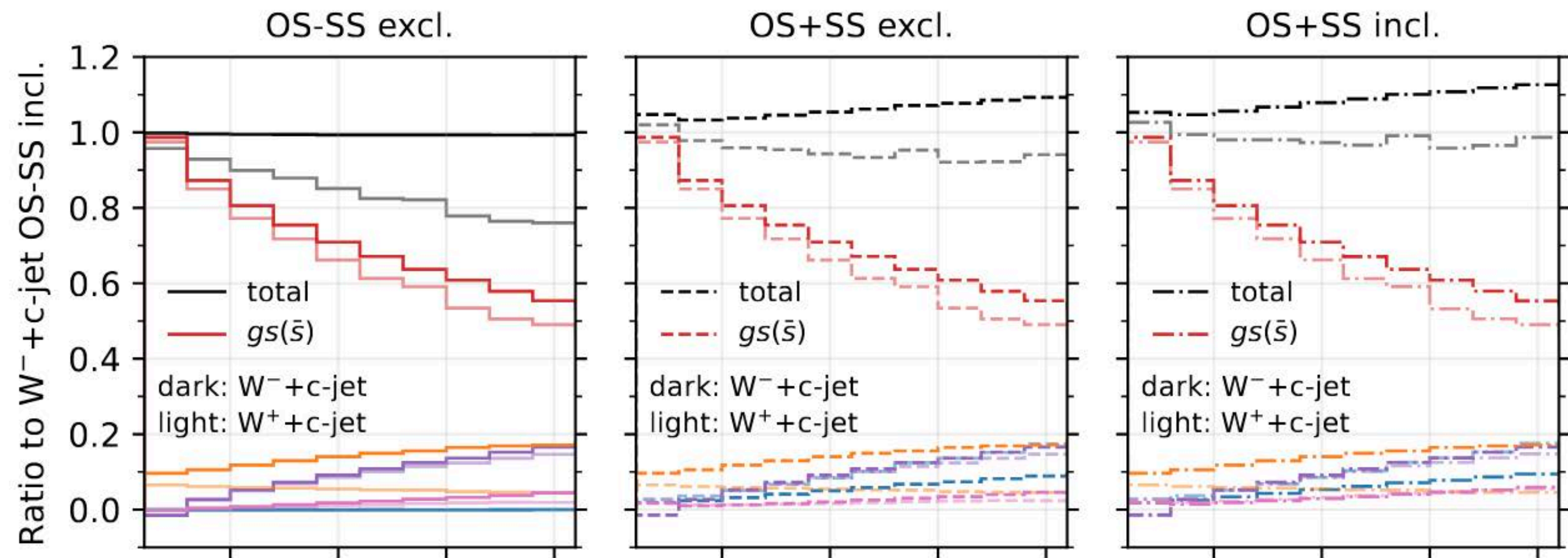
## Channel breakdown for fiducial cross sections

$W^+ + c\text{-jet}$	OS LO	OS NLO	SS NLO	OS NNLO	SS NNLO
$c(\bar{c})s(\bar{s})$	0.0	-0.1191(9)	0.4752(4)	-0.13(2)	0.838(1)
$c(\bar{c})c(\bar{c})$	0.0	0.2151(3)	0.2047(3)	0.3316(5)	0.3246(6)
$c(\bar{c})q(\bar{q})$	0.0	1.948(3)	1.988(4)	2.945(6)	3.038(6)
$s(\bar{s})q(\bar{q})$	0.0	-0.649(9)	0.0673(1)	-1.9(3)	0.1157(3)
$s(\bar{s})s(\bar{s})$	0.0	-0.258(1)	0.0	-0.55(5)	0.0
$q(\bar{q})q(\bar{q})$	0.0	1.431(2)	1.409(2)	2.35(2)	2.423(6)
$gq(\bar{q})$	5.8299(7)	8.257(2)	0.0	10.1(4)	0.508(4)
$gs(\bar{s})$	85.51(1)	121.04(3)	0.0	126.3(6)	-0.0430(4)
$gc(\bar{c})$	0.0	0.0	0.0	0.02(2)	-0.0293(7)
$gg$	0.0	-6.34(1)	0.0	-13.62(6)	0.0
total	91.34(1)	125.51(4)	4.146(4)	125.9(7)	7.17(1)

Dominant channel is  $gs(\bar{s})$ , followed by  $gq(\bar{q})$

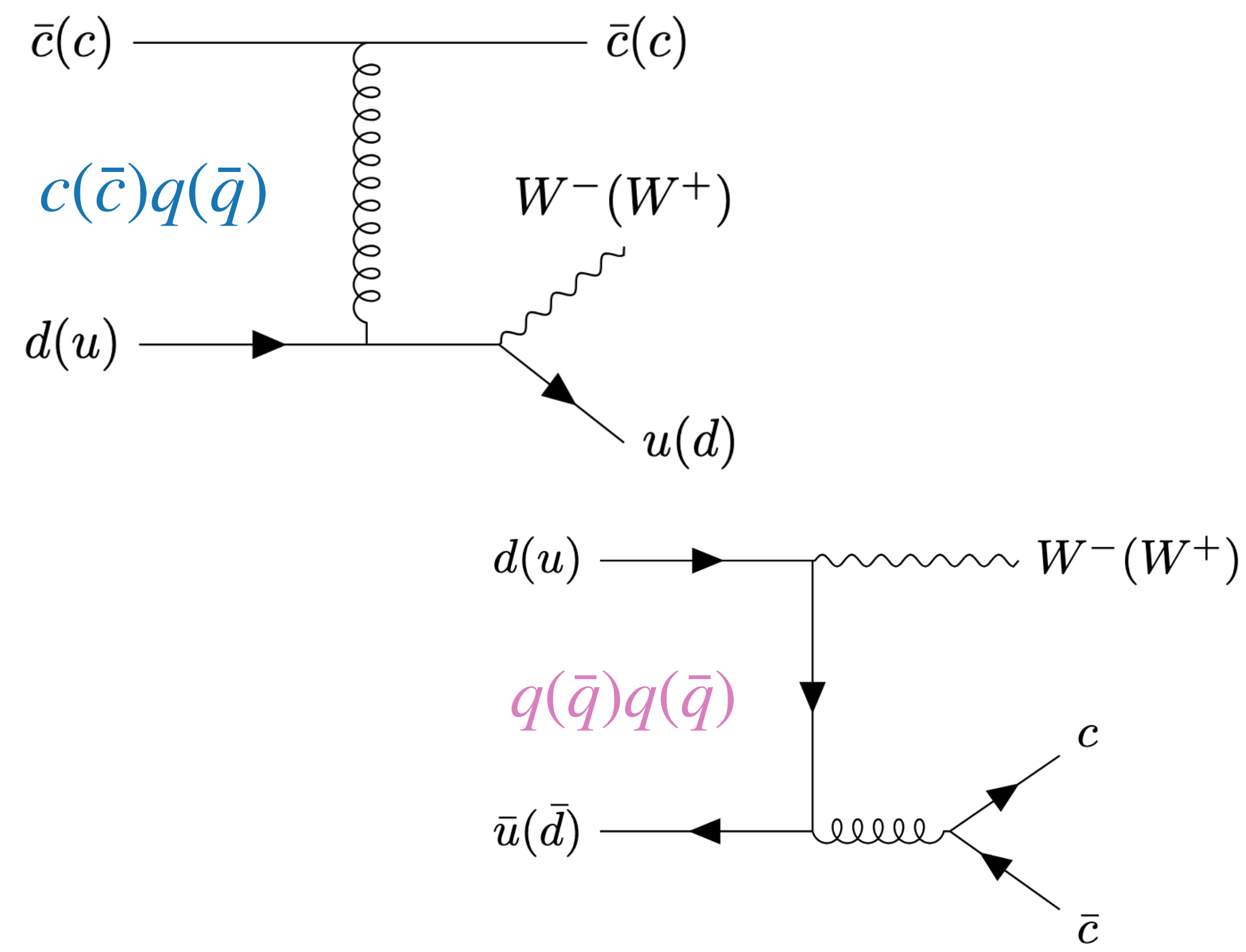
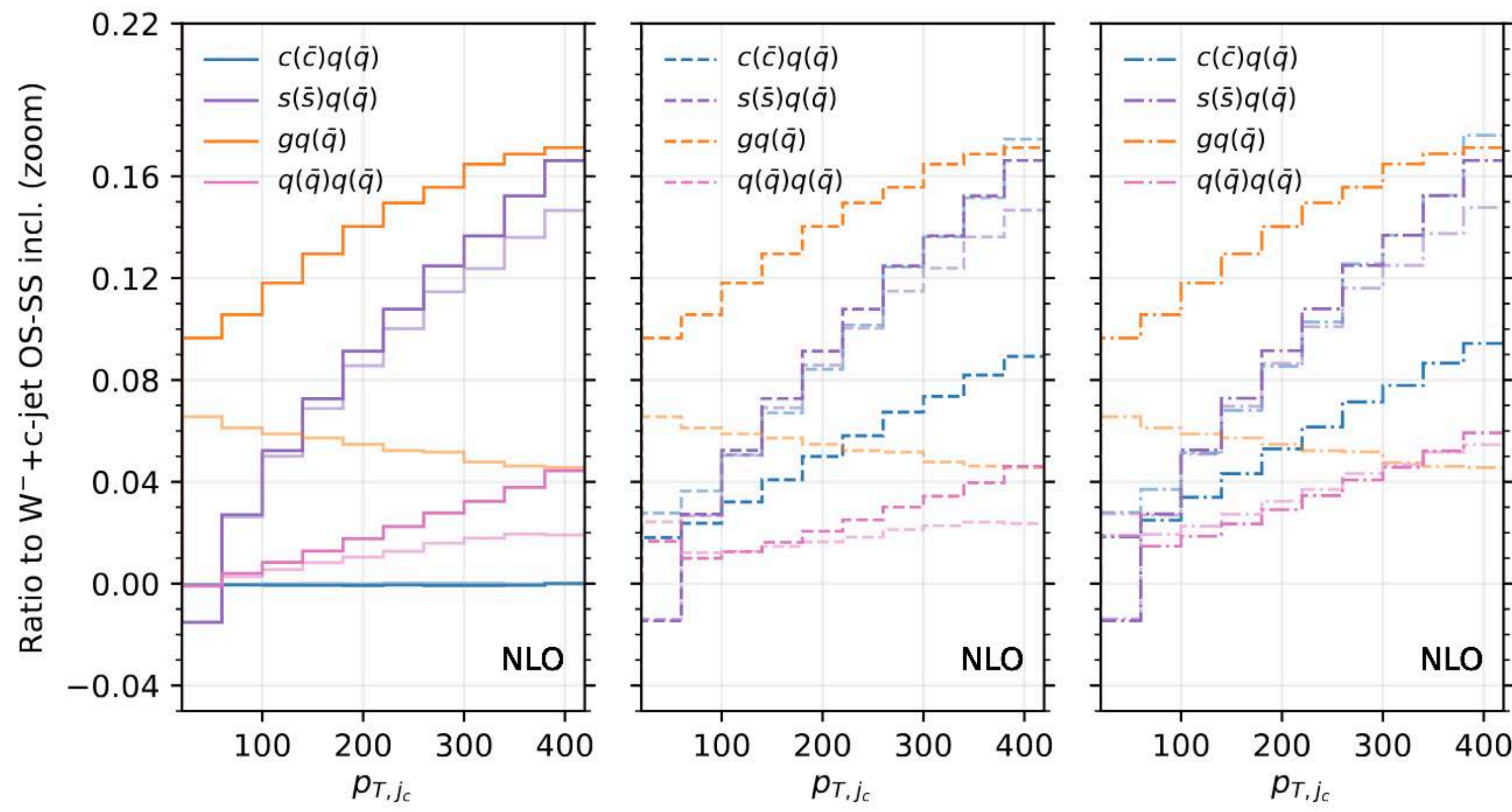
The  $c(\bar{c})c(\bar{c})$ ,  $c(\bar{c})q(\bar{q})$  and  $q(\bar{q})q(\bar{q})$  channels are very similar between SS and OS: then OS-SS basically removes them



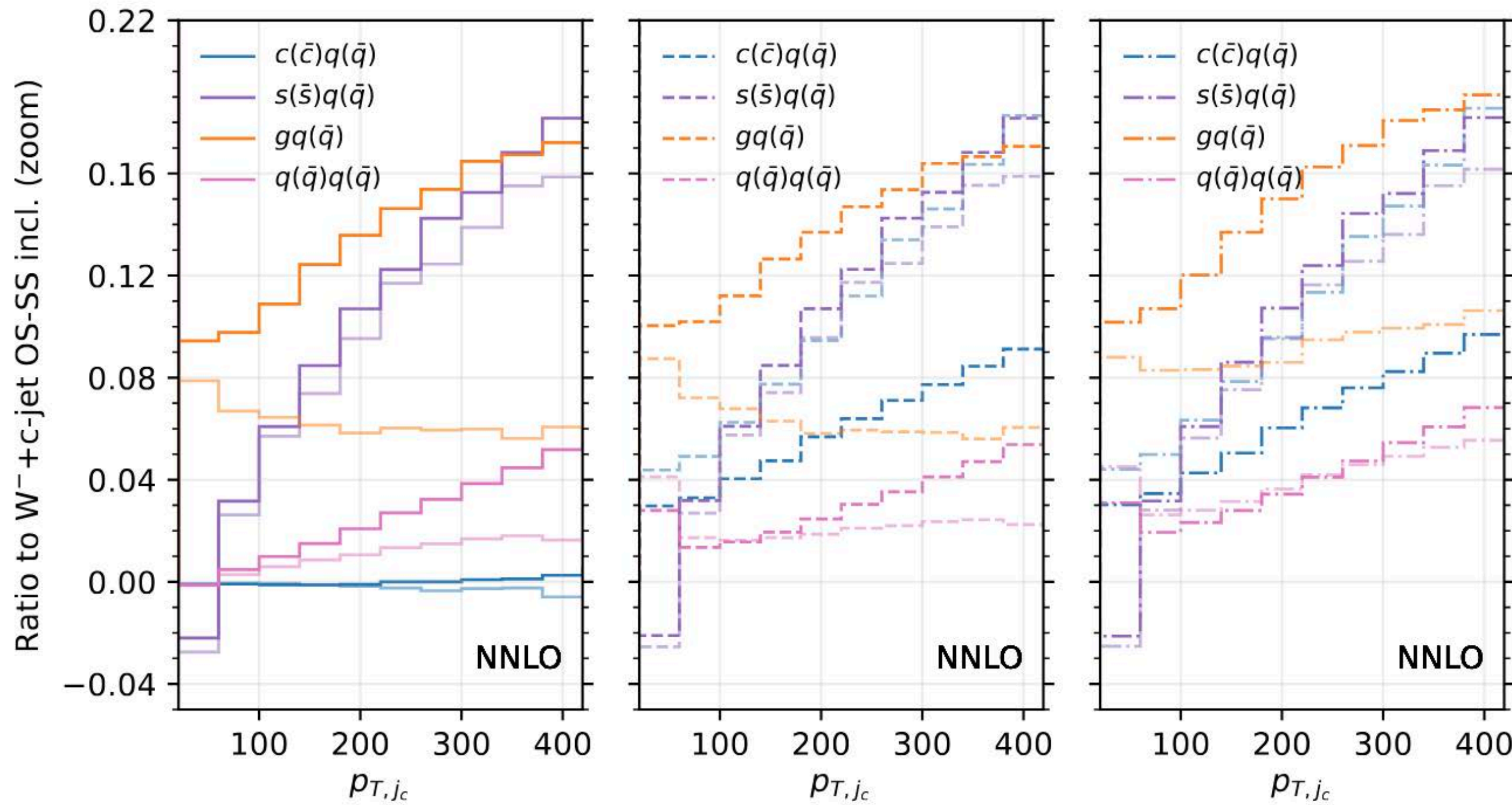
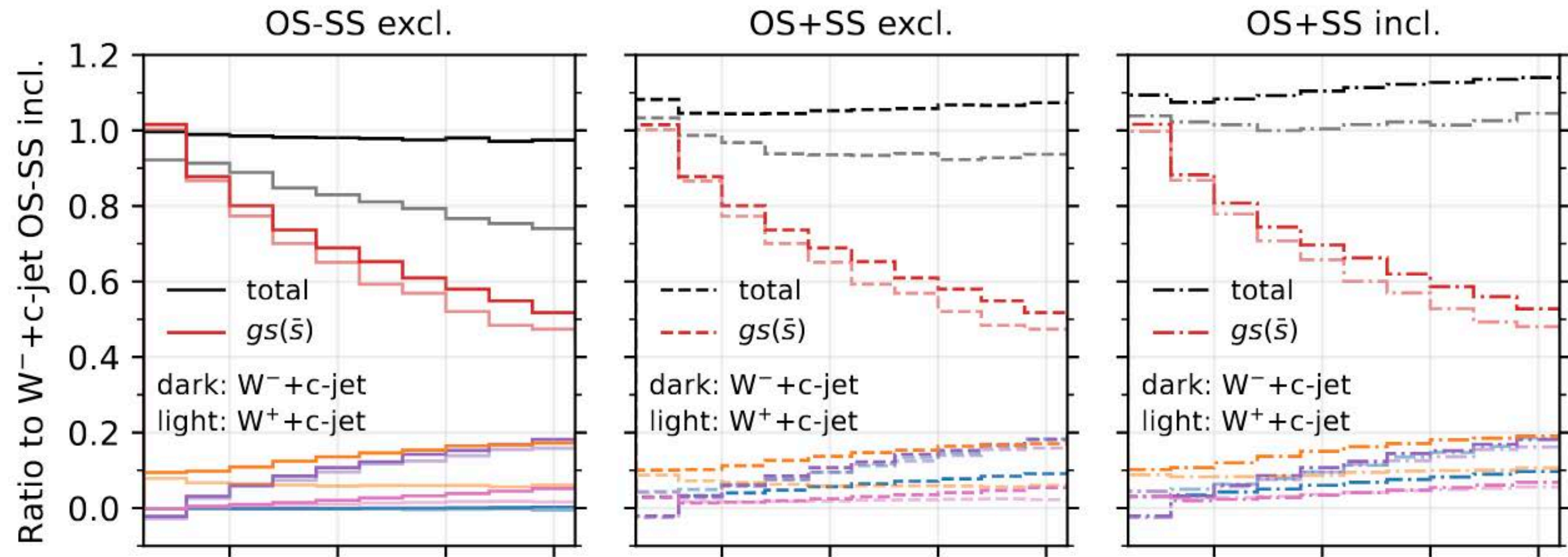


**Difference OS-SS vs. OS+SS**  
**driven by  $c(\bar{c})q(\bar{q})$  channel**  
**Difference OS+SS excl. vs. OS+SS incl.**  
**driven by  $q(\bar{q})q(\bar{q})$  channel**

Due to the difference between the size of  
 $u$  valence PDF and  $d$  valence PDF  
( $u$  valence is twice the  $d$  valence)

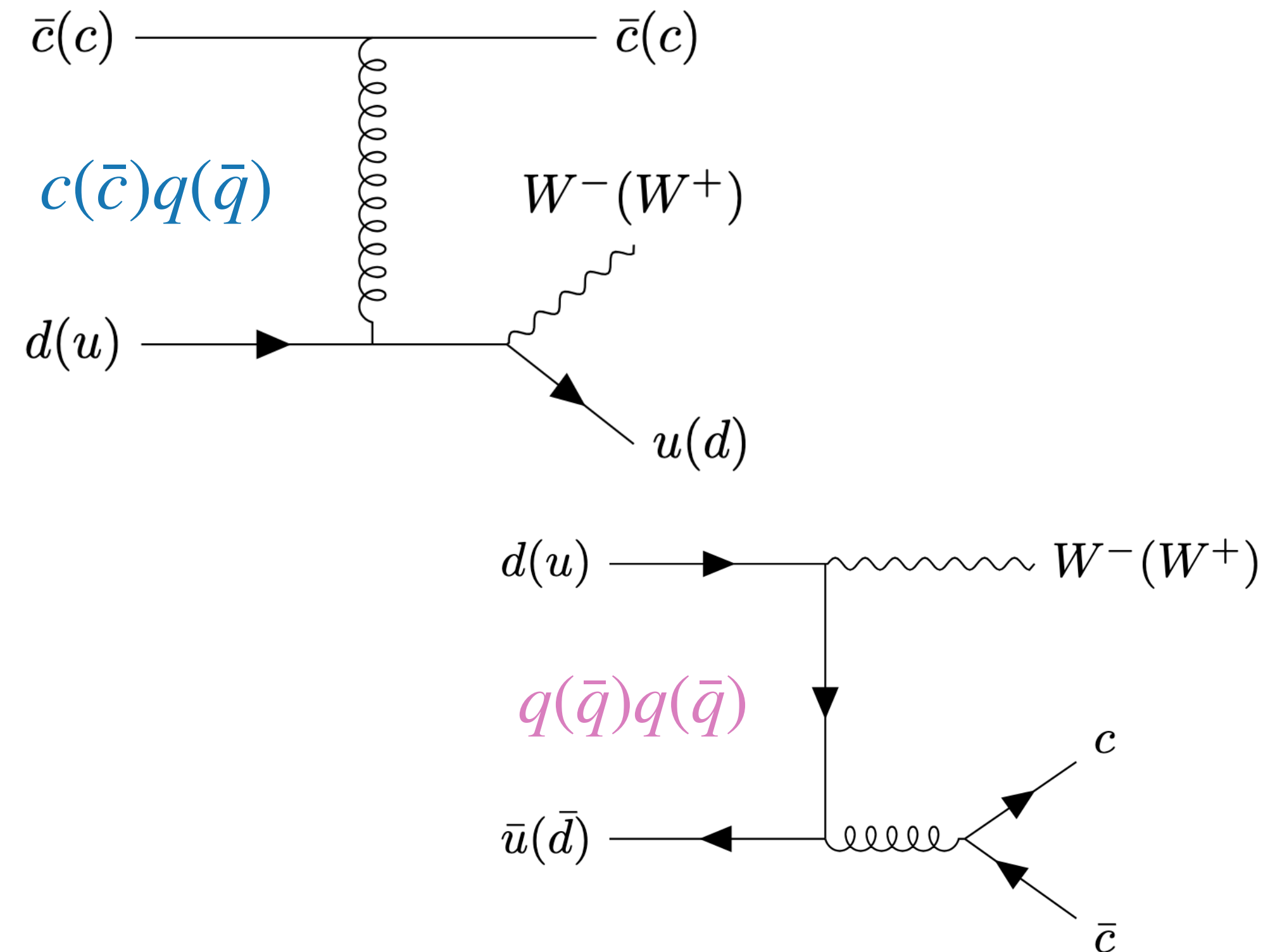






**Difference OS-SS vs. OS+SS**  
**driven by  $c(\bar{c})q(\bar{q})$  channel**  
**Difference OS+SS excl. vs. OS+SS incl.**  
**driven by  $q(\bar{q})q(\bar{q})$  channel**

Due to the difference between the size of  
 $u$  valence PDF and  $d$  valence PDF  
( $u$  valence is twice the  $d$  valence)





# Outlook of this talk

1. (Rather long and pedagogical) introduction

2. Flavoured jets at the LHC:  
 $Z+c$ -jet and  $W+c$ -jet production

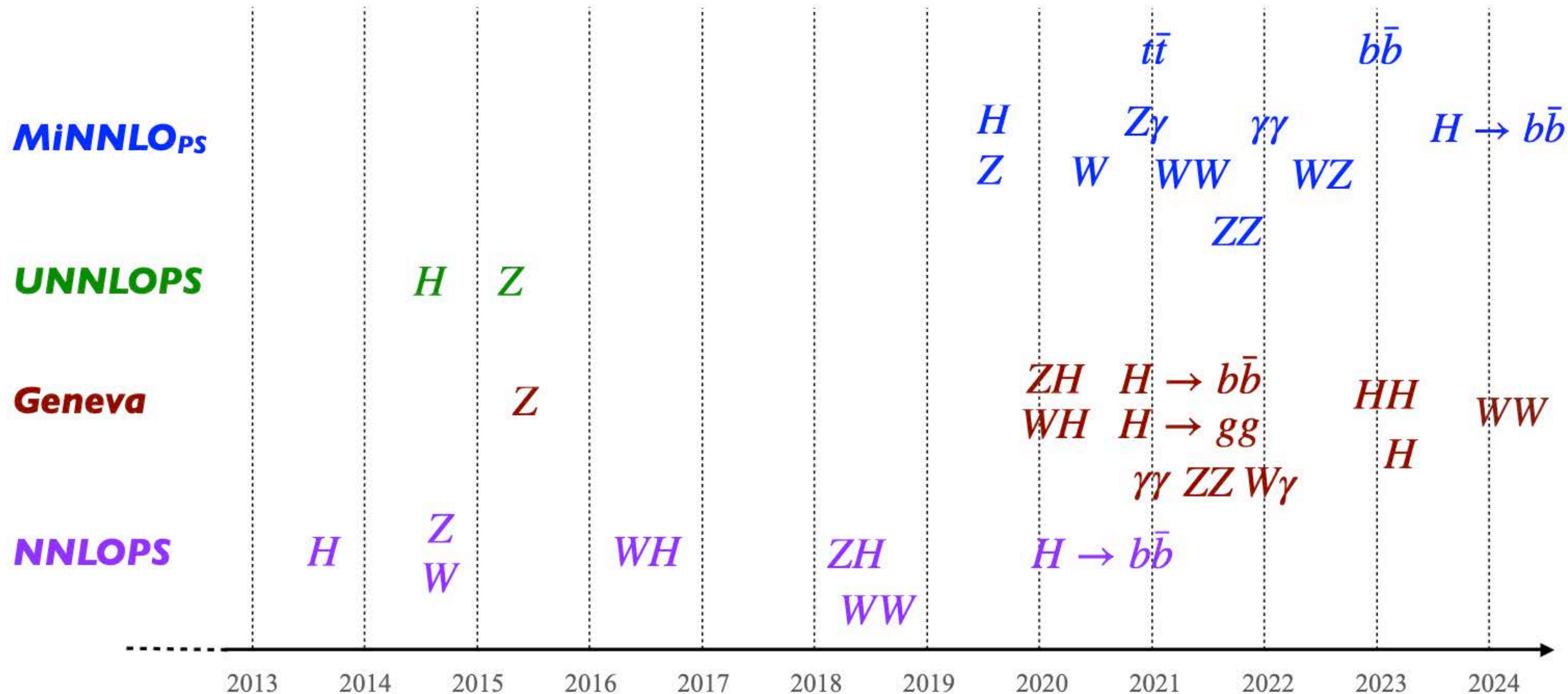
3. Towards NNLO+PS for  $V$ +jet:  
improving slicing methods for  $V$ +jet production

*Biased selection of recent results where I personally contributed.  
Minimal inclusion of references, apologies for any relevant omission.*



# Status of NNLO+PS

M. Wiesemann

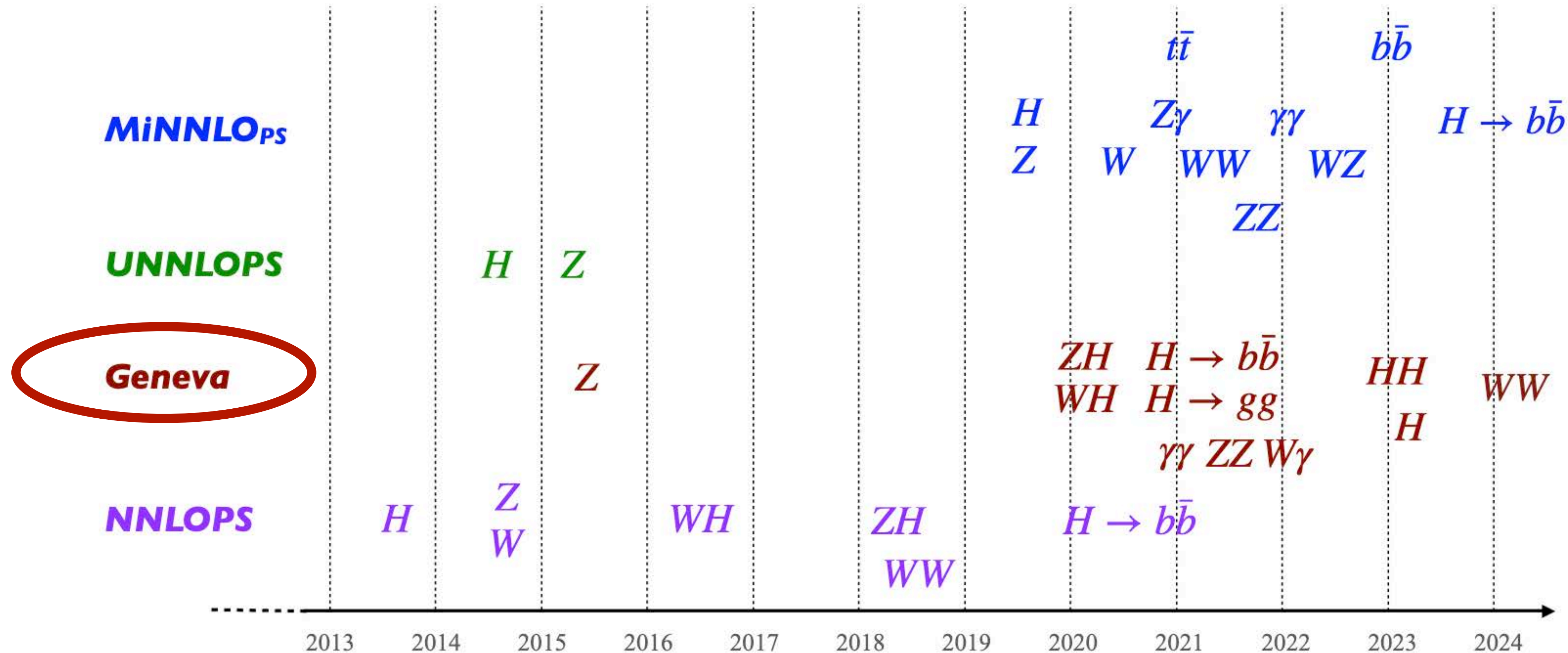


Impressive results in the recent years, but  
limited to processes with colour-singlets or heavy quarks in the final state



# Status of NNLO+PS

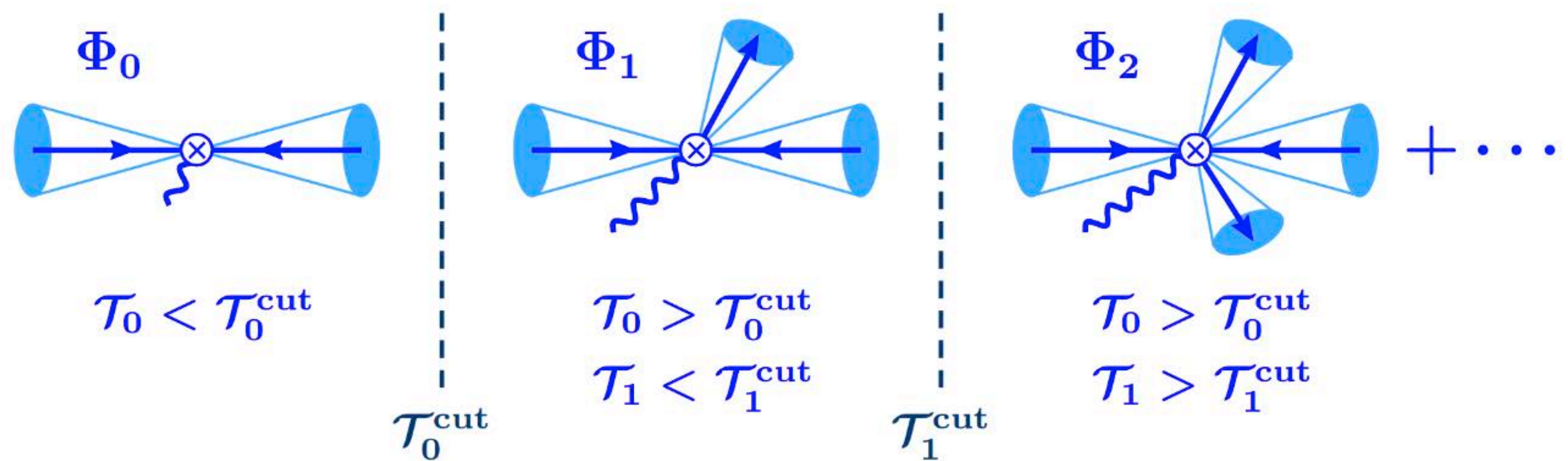
M. Wiesemann



Impressive results in the recent years, but  
limited to processes with colour-singlets or heavy quarks in the final state



## GENEVA in a nutshell (for colour-singlet production)



Division into 0/1/2-jet events dictated by **resolution variable(s)**  $\mathcal{T}_N$

Originally developed for  $N$ -jettiness  $\mathcal{T}_N$ , but later extended to colour-singlet  $q_T$  [Alioli, Bauer et al. '21] and leading-jet  $p_T$  [Gavardi, Lim et al. '23]

As  $\mathcal{T}_N$ s regulate IR divergences, large logarithms appear: resummation is required!

$\mathcal{T}_0$  resummed up to NNLL',  $\mathcal{T}_1$  up to NLL



# How to extend GENEVA to vector boson plus jet production?

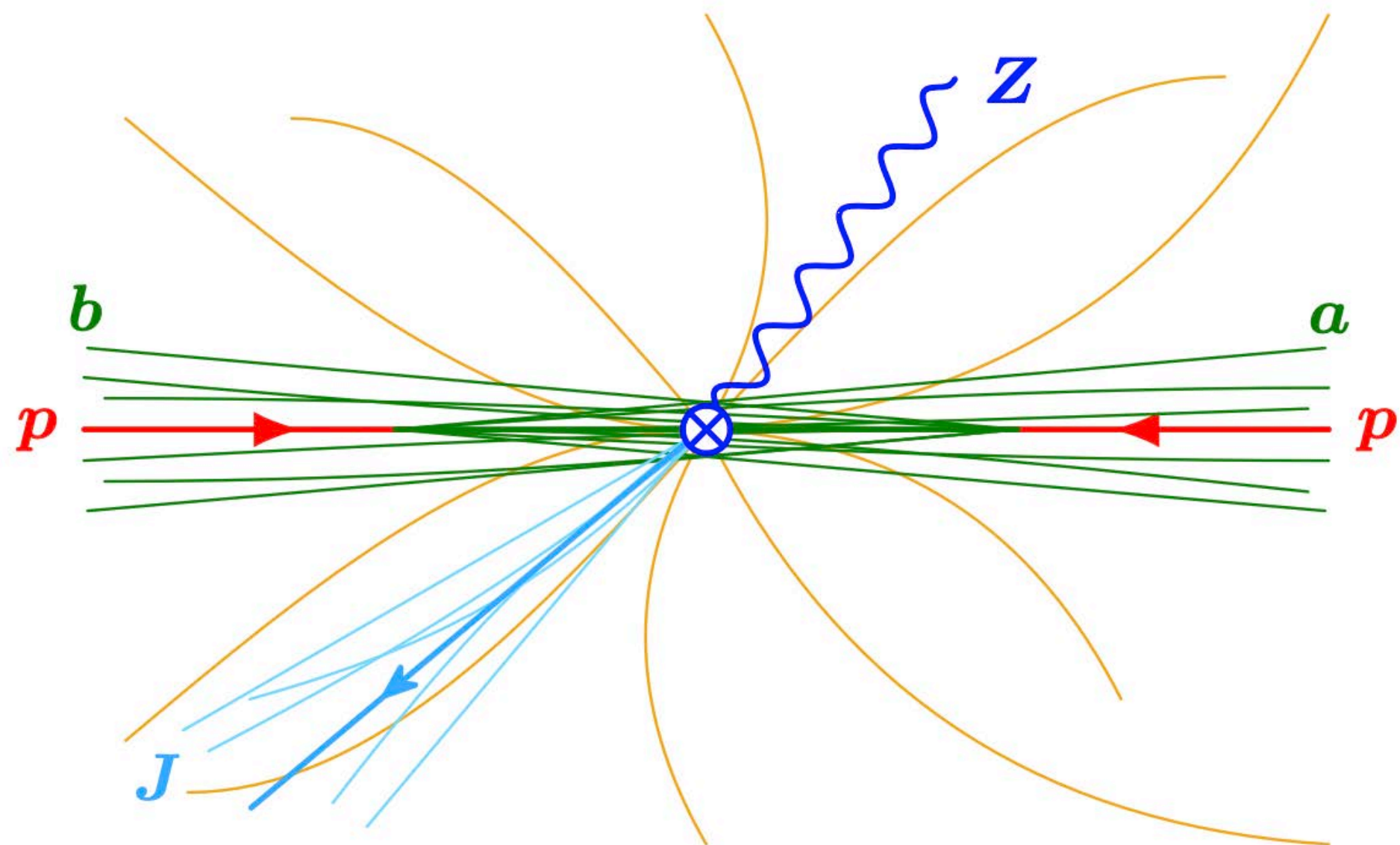
First step: resummation of one-jettiness  $\mathcal{T}_1$ , performed up to N<sup>3</sup>LL

[Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, Rahn '23]

$$\mathcal{T}_1 = \sum_k \min \left\{ \frac{2q_a \cdot k}{Q_a}, \frac{2q_b \cdot k}{Q_b}, \frac{2q_J \cdot k}{Q_J} \right\}$$

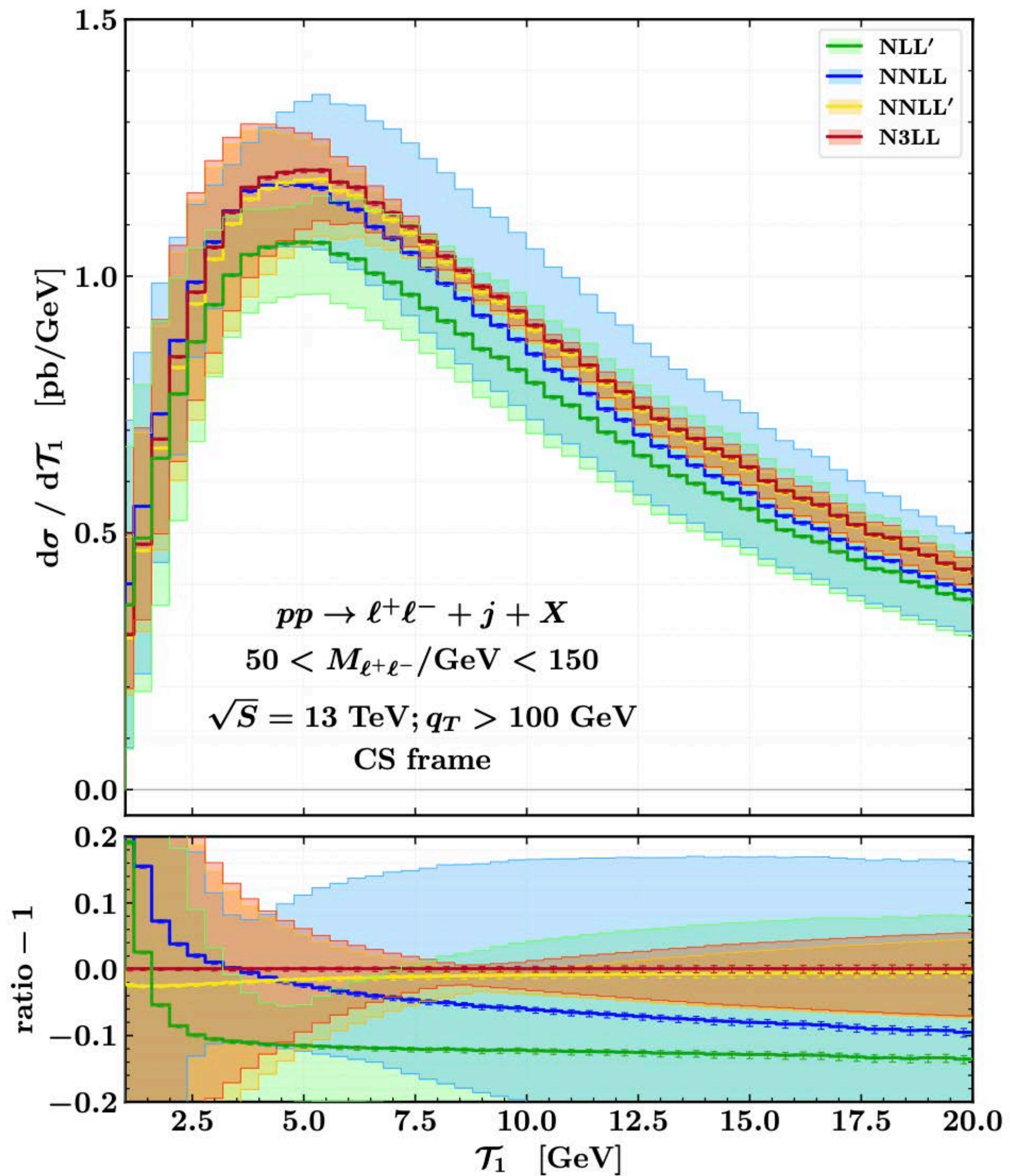
$$Q_i = 2\rho_i E_i$$

Freedom in precise definition of  $\mathcal{T}_1$ :  
 dependence on reference frame;  
 dependence on definition of jet axis  
 (e.g. obtained recursively with exclusive clustering or a priori with inclusive clustering)

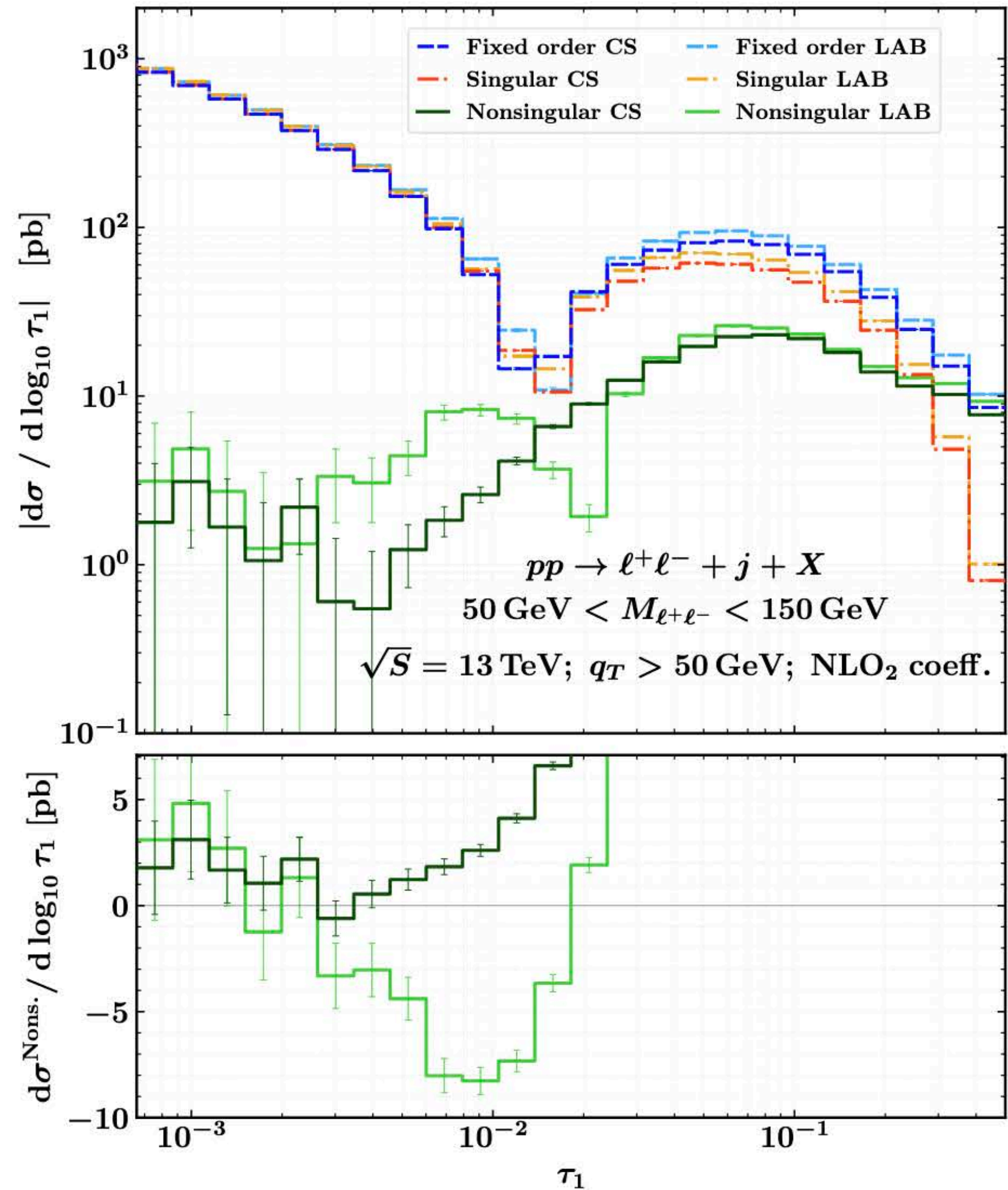


frames	$\rho_{a,b}$	$\rho_J$
Lab	1	1
Color Singlet (CS)	$e^{\pm Y_V}$	$(e^{Y_V} p_J^- + e^{-Y_V} p_J^+) / E_J$
Underlying Born (UB)	$e^{\pm Y_{VJ}}$	$(e^{Y_{VJ}} p_J^- + e^{-Y_{VJ}} p_J^+) / E_J$





NLL'  $\rightarrow$  NNLL  $\rightarrow$  NNLL' sizeable  
 NNLL'  $\rightarrow$  N<sup>3</sup>LL minor effect



Fixed-order approaches singular as  $\tau_1 \rightarrow 0$   
 Power corrections behave better in the CS frame

In order to have a finite  
 Born for  $Z+\text{jet}$ ,  
 one adopts a cut  
 on  $q_T$  or on  $\mathcal{T}_0$

Nonsingular =  
 Fixed order - Singular

$$\tau_1 = \mathcal{T}_1 / m_T$$

$$m_T \equiv \sqrt{M_{\ell^+\ell^-}^2 + q_T^2}$$



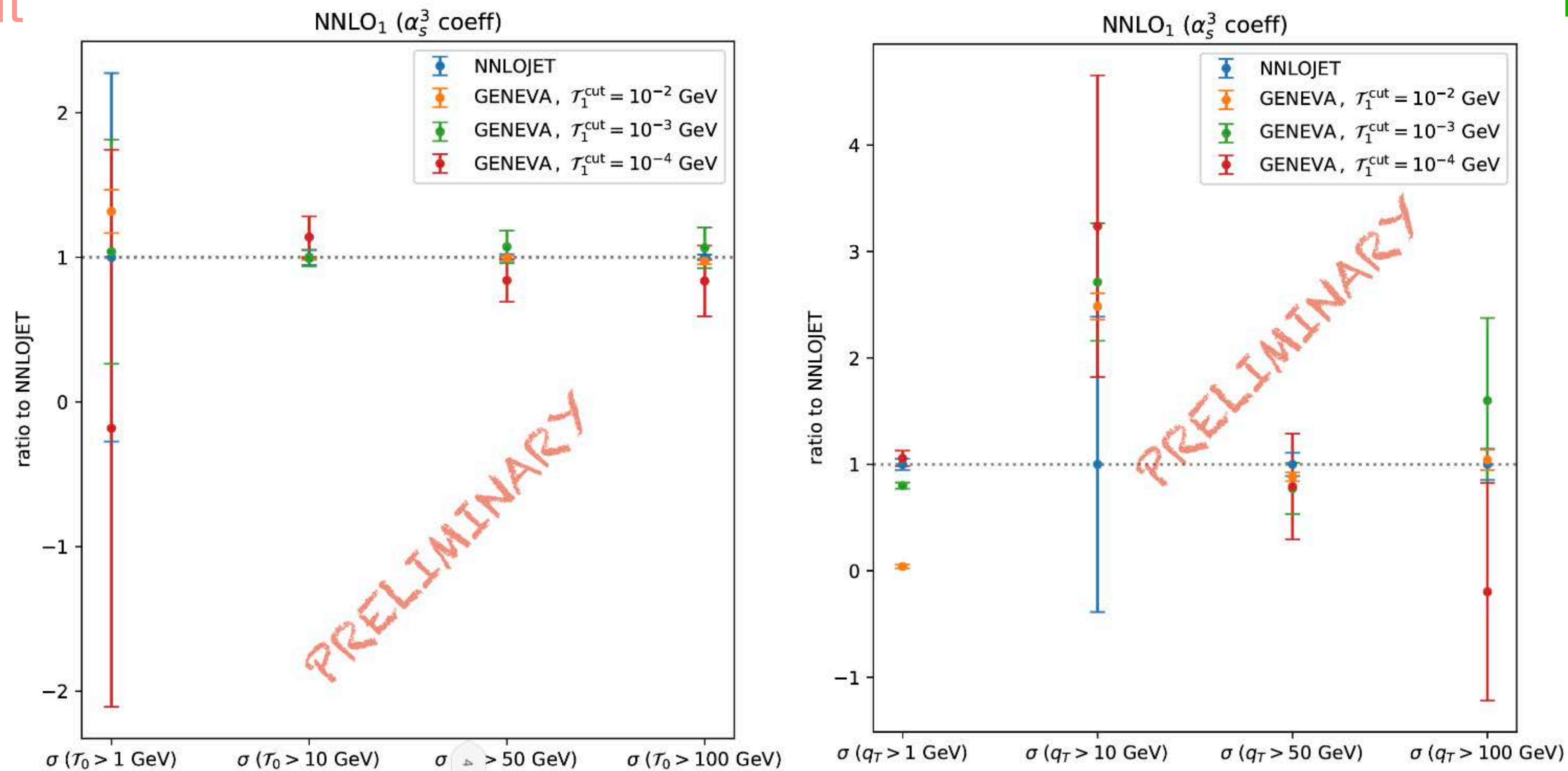
## Second step: recover the NNLO fixed-order result with slicing [Alioli, Billis, Broggio, GS, in preparation]

$$O^{\delta\text{NNLO}_1}(\Phi_1) = \underbrace{\frac{d\sigma^{\text{N3LL}}}{d\Phi_1}(\mathcal{T}_1^{\text{cut}})}_{\mathcal{O}(\alpha_s^3)} O(\Phi_1) + \int_{\mathcal{T}_1^{\text{cut}}}^{\mathcal{T}_1^{\text{max}}} \frac{d\Phi_2}{d\Phi_1} \frac{d\sigma^{\delta\text{NLO}_2}}{d\Phi_2} O(\Phi_{\{2,3\}})$$

Analytic cumulant expanded NLO with local FKS subtraction

Below the cut,  
resummed result  
integrated and  
expanded

Above the cut,  
fixed-order result  
for  $Z+2$ -jets

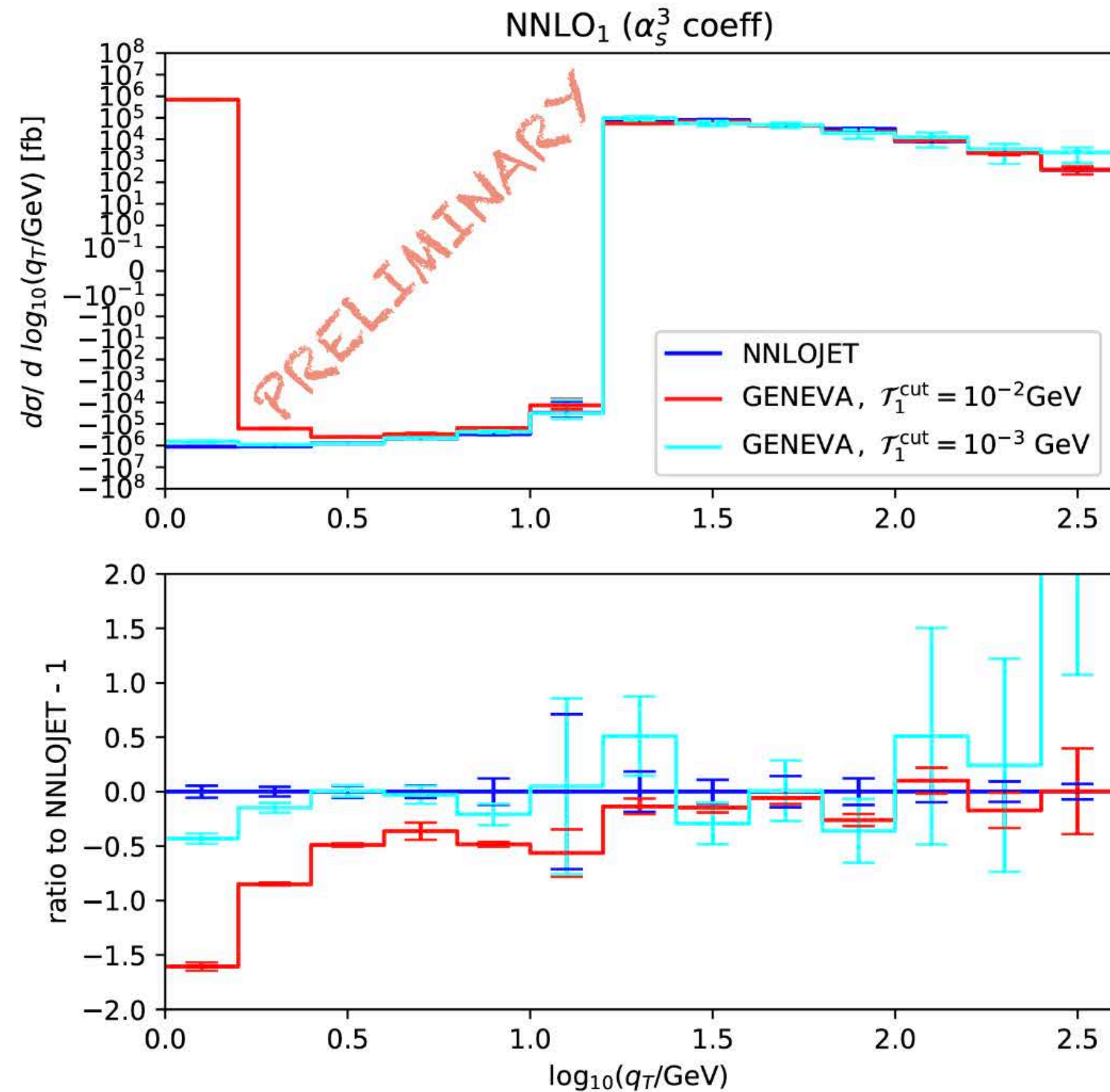




Second step: recover the NNLO fixed-order result with slicing  
 [Alioli, Billis, Broggio, GS, in preparation]

$$O^{\delta\text{NNLO}}_1(\Phi_1) = \underbrace{\frac{d\sigma^{\text{N3LL}}}{d\Phi_1}(\mathcal{T}_1^{\text{cut}})}_{\mathcal{O}(\alpha_s^3)} O(\Phi_1) + \int_{\mathcal{T}_1^{\text{cut}}}^{\mathcal{T}_1^{\text{max}}} \frac{d\Phi_2}{d\Phi_1} \frac{d\sigma^{\delta\text{NLO}_2}}{d\Phi_2} O(\Phi_{\{2,3\}})$$

Analytic cumulant expanded                      NLO with local FKS subtraction



Smaller  $\tau_1^{\text{cut}}$  closer to NNLOJET (=local subtraction),  
 but with increased numerical errors

Larger  $\tau_1^{\text{cut}}$  more stable,  
 but not reproducing NNLOJET result at small  $q_T$

How can we improve the slicing?

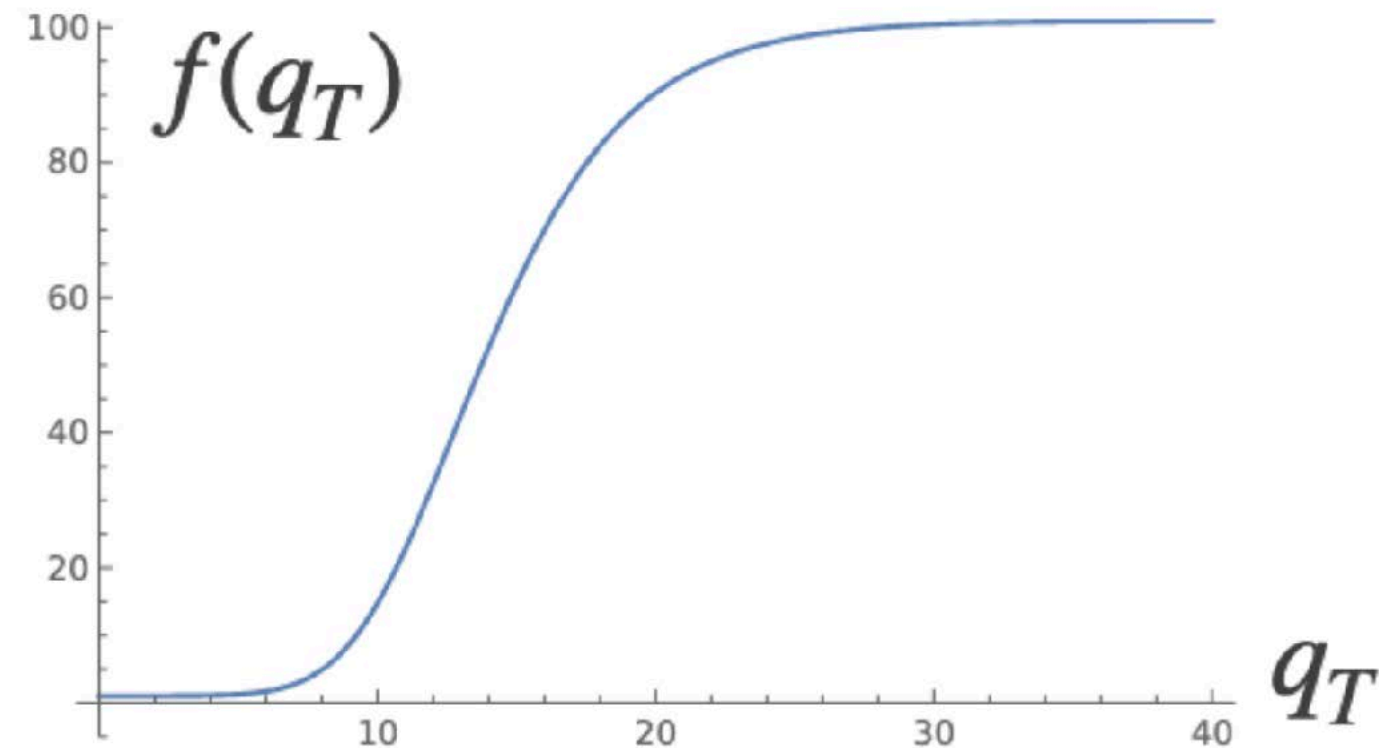


## How can we improve the slicing?

### Dynamical cuts:

It is a multiscale problem!  
 We would like to avoid large logs  
 between  $\tau_1^{\text{cut}}$  and other scales e.g.  $q_T$

$$\mathcal{T}_1^{\text{cut}} = \min\{10^{-4} f(q_T), \mathcal{T}_0/2\}$$



$\tau_1^{\text{cut}}$  smoothly interpolating  
 between  $10^{-2}$  and  $10^{-4}$

### Local subtraction in $\tau_1$ :

we can subtract the singular spectrum  
 locally in  $\tau_1$  between  $\tau_1^{\text{cut}}$  and  $\tau_1^{\text{IR}} \ll \tau_1^{\text{cut}}$

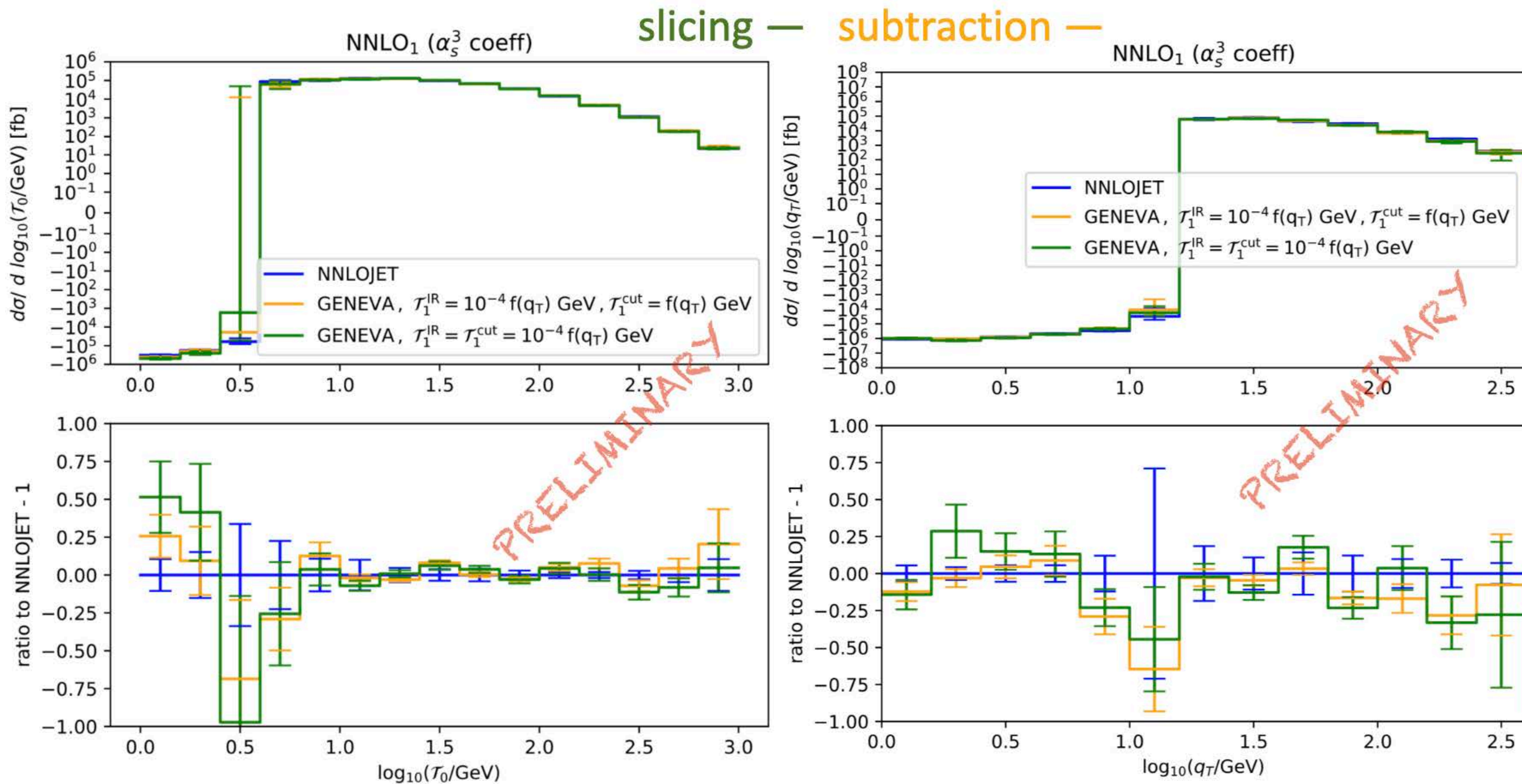
$$O^{\delta\text{NNLO}_1}(\Phi_1) = \frac{d\sigma^{\text{N3LL}}}{d\Phi_1}(\mathcal{T}_1^{\text{cut}}) \Big|_{\mathcal{O}(\alpha_s^3)} O(\Phi_1) + \int_{\mathcal{T}_1^{\text{cut}}}^{\mathcal{T}_1^{\text{max}}} \frac{d\Phi_2}{d\Phi_1} \frac{d\sigma^{\delta\text{NLO}_2}}{d\Phi_2} O(\Phi_{\{2,3\}})$$

$$+ \int_{\mathcal{T}_1^{\text{IR}}}^{\mathcal{T}_1^{\text{cut}}} \frac{d\Phi_2}{d\Phi_1} \left[ \frac{d\sigma^{\delta\text{NLO}_2}}{d\Phi_2} O(\Phi_{\{2,3\}}) - \frac{d\sigma^{\text{N3LL}}}{d\Phi_1 d\mathcal{T}_1} \Big|_{\mathcal{O}(\alpha_s^3)} \mathcal{P}(z, \varphi) O(\Phi_1) \right]$$

The subtraction term can be any approximation of the exact NNLO result with the same singular behaviour

(here, we adopt the singular spectrum times a normalised splitting function  $\mathcal{P}$  to make it differential in the higher-multiplicity phase-space)





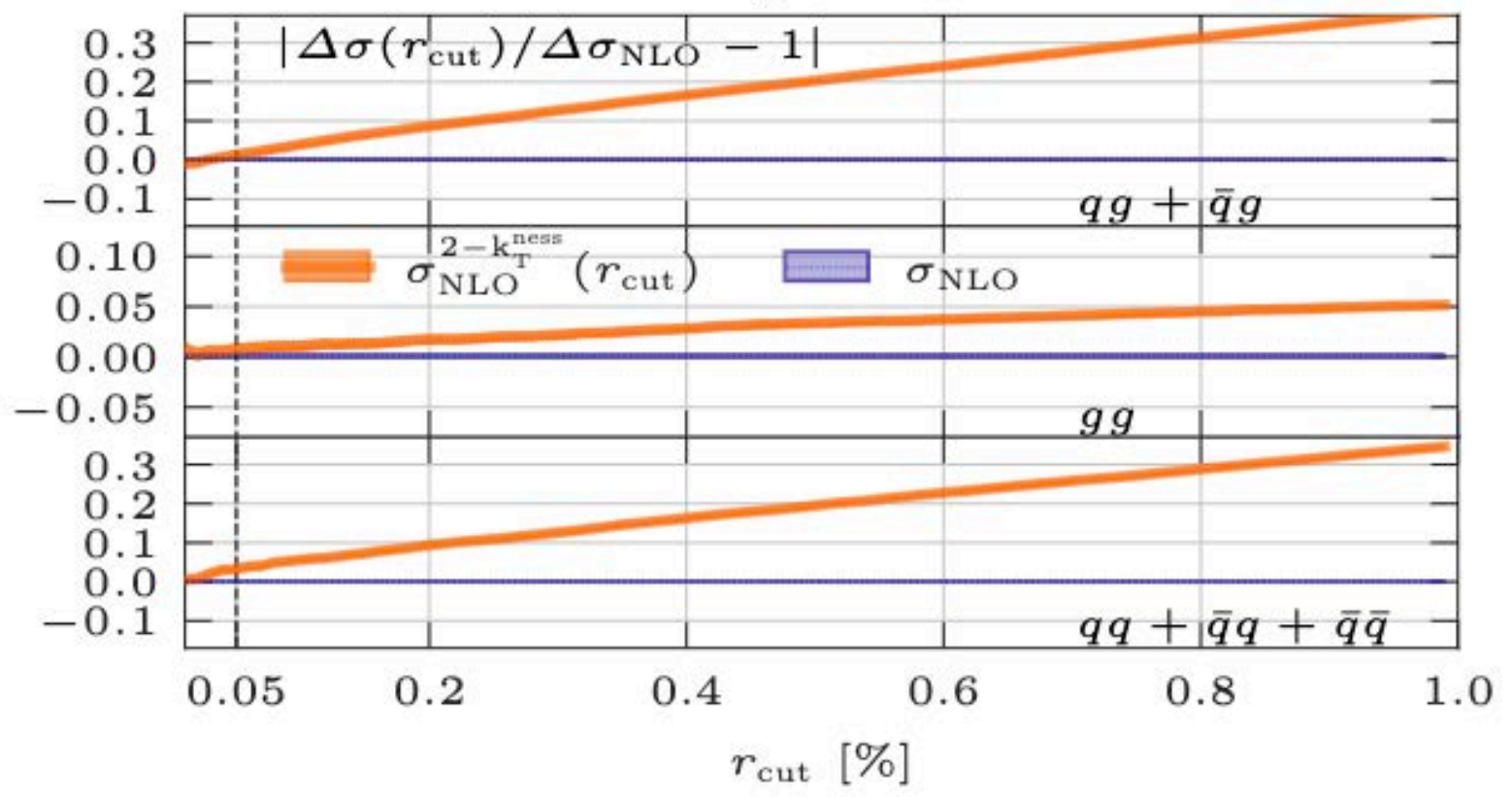
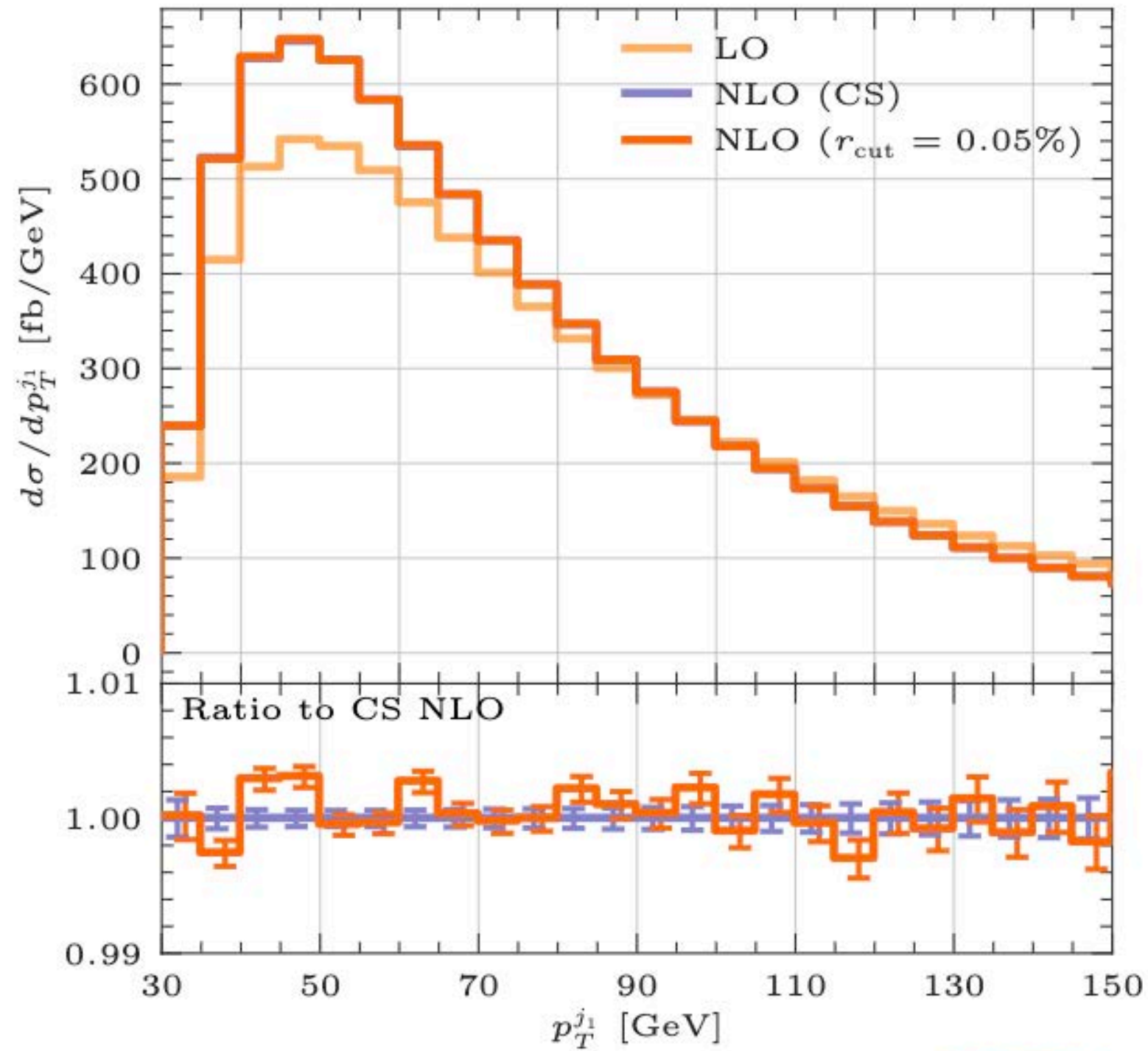


## Next steps towards NNLO+PS for $Z$ +jet:

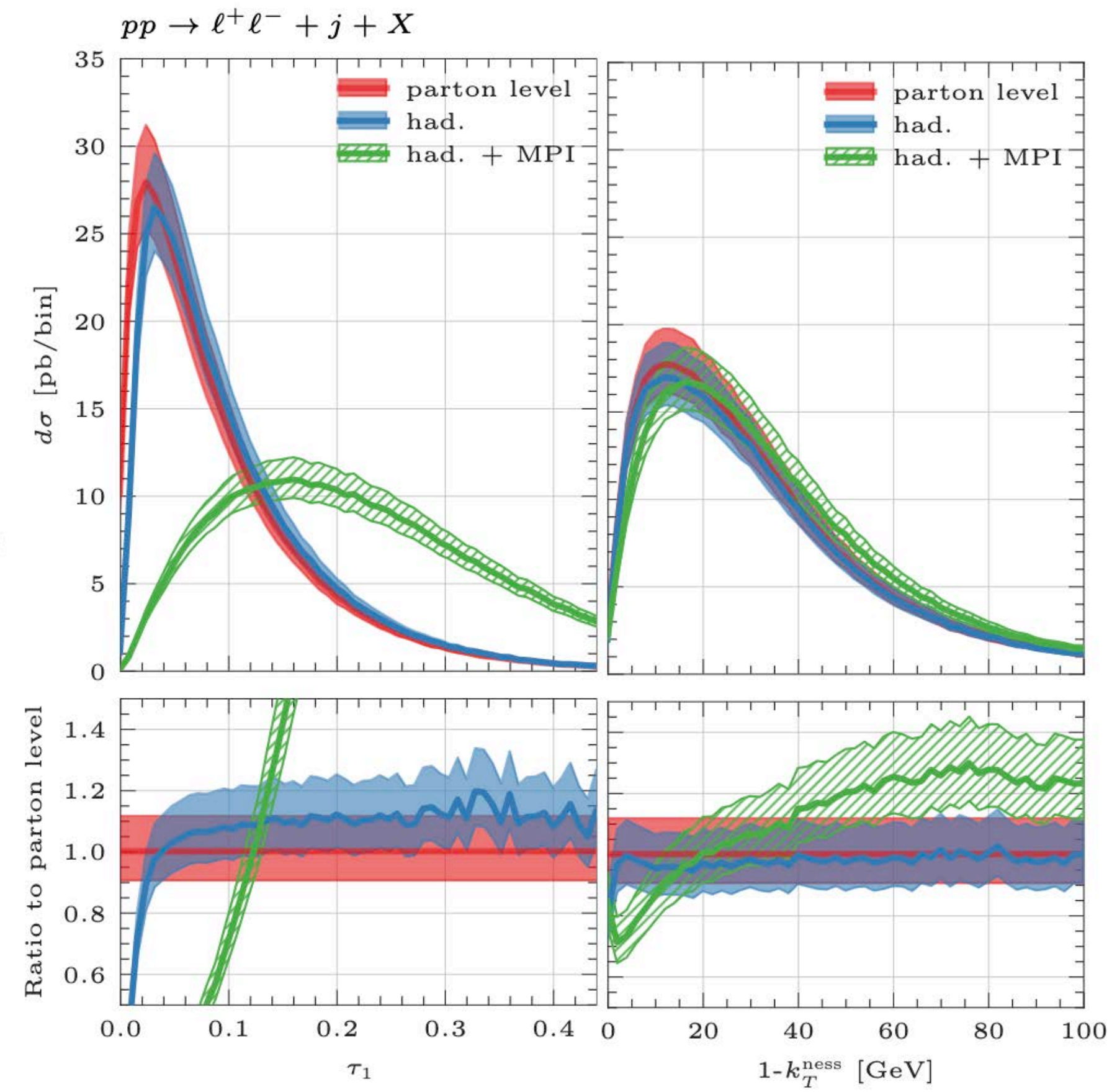
- $\mathcal{T}_1$ -preserving mapping
- splitting functions  $\mathcal{P}_{2\rightarrow 3}(\Phi_2)$ 
  - resummation of  $\mathcal{T}_2$
  - interface to parton shower
- **alternative resolution variables?**



$$pp \rightarrow \ell^+ \ell^- + 2j + X$$



e.g.  $k_T^{\text{ness}}$ , based on exclusive  $k_T$ -clustering algorithm  
 [Buonocore, Grazzini, Haag, Rottoli, C. Savoini '22,'23]



More stable than  $\mathcal{T}_1$   
 under non-pert. effects  
 (had. and MPI)

All ingredients at NLO,  
 extension to NNLO WIP

If available, resummation  
 up to NNLL' would allow  
 for usage in NNLO+PS  
 frameworks



# Conclusions

*“At the LHC, with enough luminosity, any measured observable will show a deviation from theory predictions”*

A wise man

We have just entered the precision era of colliders.

**We know very well the Standard Model, but not enough.**

Precision implies not only to push accuracy of predictions, but also to revisit basic assumptions and develop new strategies.

To claim percent-level accuracy on SM predictions, there are a lot of things to improve:

- accuracy of hard-scattering (first N3LO results appearing)
- matching to parton shower (progresses towards NNLO+PS for processes with jets)
- accuracy of parton showers (new generation of PS with higher logarithmic accuracy)
- understanding of non-perturbative effects (?)