Positivity Constraints and Their Applications in SMEFT

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19/03/2025

Outline

Review on the SMEFT

- Introduction
- Global fit and inferring UV
- RG running and mixing

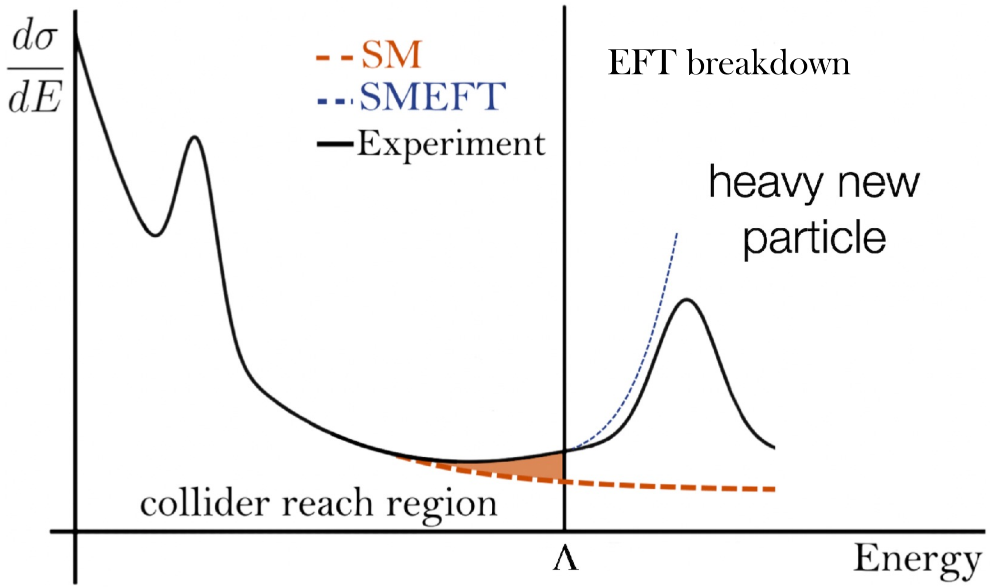
Positivity bounds

- Convex cone approach
- Pheno. of the positivity and inverse problem
- Constraints on anomalous dimension matrix

Summary

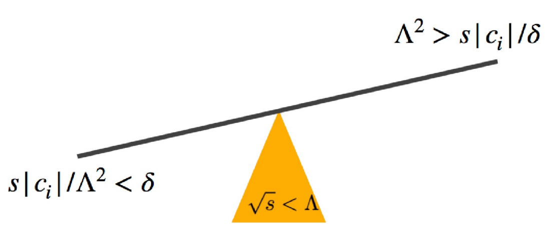
Why EFT ?

- BSM is low scale: direct searches for peaks, probe on-shell NP
- BSM outside the collider reach region: indirect search, precision physics, EFT



[M. Farina et al., 1609.08157]

- LEP experiment is an example of the precision physics: tests the SM electroweak sector near Z-pole, EWPOs
- EFT approach – sensitivity to NP is increased by the energy

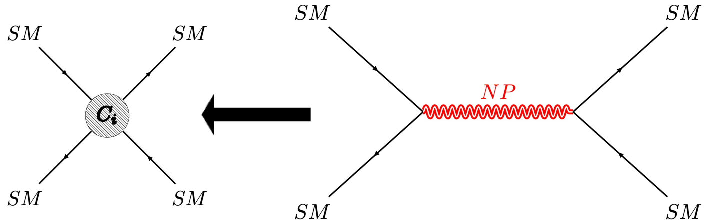


Energy helps precision

Introduction to SMEFT

- SMEFT calculates physical processes by adding high-dimensional operators

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \cdots$$



- Construction of the SMEFT basis
  - The Field content and Symmetries of SM
  - Remove redundance: integrate by part & Fierz identities
  - Equations of motion

- Bases have been constructed:
  - **Dim-6:**
    [B. Grzadkowski et al., 1008.4884]（Warsaw）
  - **Dim-7:**
    [L. Lehman, 1410.4193] [Liao & Ma, 1612.04527]
  - **Dim-8:**
    [H.-L. Li et al., 2005.00008] [C. Murphy, 2005.00059]
  - **Dim-9:**
    [H.-L. Li et al., 2007.07899] [Liao & Ma, 2007.08125]
  - **Hilbert Series**
    [Henning et al, 1512.03433] [Marinissen et al, 1512.03433]
  - **Automation Tools:**
    Sym2Int [Fonseca, 1703.05221]
    ABC4EFT [H.-L. Li et al, 2201.04639]
    AutoEFT [Harlander&Schaaf, 2309.15783]

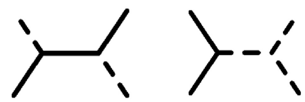
What is their relation to NP?

Tree-level UV dictionary of SMEFT

- Dim-5: origin of majorana-type neutrino masses:

$$\mathcal{O}_5 = \epsilon^{ik}\epsilon^{jl}(\ell_i^T C \ell_j) H_k H_l$$

Weinberg operator



Topology

Field	(J, C, W, Y)
S_6	(0, 1, 3, 1)
F_1	(1/2, 1, 1, 0)
F_5	(1/2, 1, 3, 0)

Three type of Seesaw models

- Dim-6: **[C. Arzt et al., 9405214] [J. de Blas et al., 1711.10391][N. Craiget al., 2001.00017]**

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
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$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
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		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
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Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
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- Dim-7 and Dim-8 EFT-UV dictionaries have also been obtained recently. **[X-X. Li et al., 2307.10380] [H-L. Li al., 2309.15933]**

- Recent development: one-loop dictionary of dim-6 operators **[G. Guedes&P. Olgoso, 2412.14253]**

Tree-level UV completion of SMEFT

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Fermion extension | Scalar extension | Vertor extension

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Can not be UV-completed at tree-level

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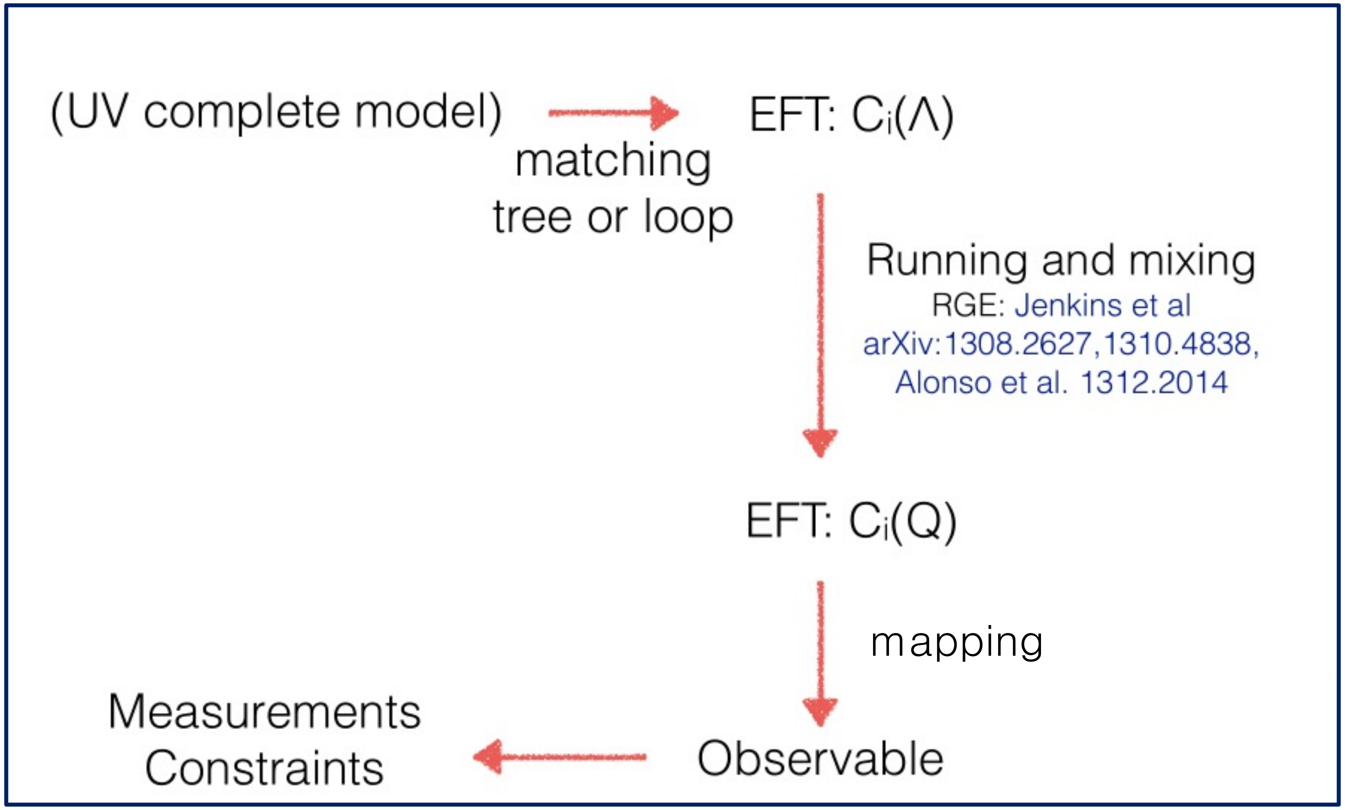
The UV perspective



- SMEFT pheon. in 3 steps

1. Matching
2. Running   Wilson [J. Aebischer et al, 1804.05033] DsixTools [J. F.-Martin et al, 2010.16341]
3. Mapping   Tools: FeynRules, MG5_aMC, Pythia etc

Matching UV model to SMEFT

➢ **Tree-level**:

- Full tree-level matchings up to dim-7 are known, of new physics model with genera scalar, spinor, and vector field content and arbitrary interactions. **[de Blas et al, 1711.10391]** **[X-X. Li et al., 2307.10380]**

➢ **One-loop**:

- Diagrammatic methods: MatchMakerEFT using QGRAF&FORM **[A. Carmona et al, 2112.10787]**
- Functional methods: Matchete based on Covariant Derivative Expansion(CDE) **[J. F.-Martín et al, 2212.04510]**
- Matching to dim-6 is automatic, but higher order is not available.

➢ **On-shell matching development**:

- The off-shell matching approach at high dimensional faces challenges such as an excessive number of operators and difficulties in simplification
- On-shell approach avoid the problem of operators reduction **[XL&Zhou, 2309.10851]** **[M. Chala et al, 2411.12798]**

➢ **Two-loop**:

- Partial results obtained based on the CDE method **[J. F.-Martínet al, 2311.13630]** **[J. F.-Martínet al, 2412.12270]**

Why Renormalizarion of SMEFT

- EFT is renormalizable order by order in $1/\Lambda$.

"Non-renormalizable theories… are as renormalizable as renormalizable theories", Weinberg '2009

- Scale separation: coefficients are matched to BSM at scale Λ, but are probed at much lower scales

- Collider observables span a wide range of energies, therefore running and mixing effects of the theory are needed.

1. DY ~ TeV
2. Top physics ~ m_{top}
3. Higgs physics ~ m_H
4. LEP & EWPO ~m_Z
5. Flavour ~ m_b

- For some processes, the RG-induced effects offer comparable/stronger bound compared to the tree-level constraints

[Aoude et al, 2212.05067] [Heinrich and Lang, 2409.19578]
[Di Noi, Gröber ,2312.11327][R. Bartocci et al, 2412.09674]

Renormalizarion of SMEFT

RG mixing

$$\frac{dC_i(\mu)}{d\log\mu} = \frac{\alpha}{\pi}\gamma_{ij}C_j(\mu)$$

Anomalous dimension matrix

- Important for scale uncertainty.
- assumptions about some coefficients being zero at low scales are not valid

➢ One-loop:

- **Diagrammatic methods:**

  Anomalous dimensions are known for all operators up to dim-7.

  [Chankow&Plucien, 9306333] [Manohar&Trott, 1308.2627][R. Boughezal et al,2408.15378]

- **Functional methods:**

  Master formula for bosonic dimension-six operators using super-heat-kernel expansion.

  [Buchalla et al, 1904.07840]

➢ Beyond one-loop:

- Two- and three-loop anomalous dimensions of **CP-violating gluonic operator**. **[de Vries et al,1907.04923]**
- Two-loop **dim-5** **anomalous dimensions** is completed recently **[A. Ibarra et al, 2411.08011]**
- Two-loop RGE for **dim-6** **operators** is in progress **[L. Born et al, 2410.07320] [G. Duhr et al, 2503.01954]**

RG accuracy in SMEFT

	Dim-5	Dim-6	Dim-7	Dim-8
One-loop	✓	✓	✓	✓
Two-loop	✓	✓	✗	✗

- The dim-8 anomalous dimension matrix (ADM) has not been fully obtained
- The difficulty in calculating the dim-8 ADM:
  1. Very large operator set
  2. Time-consuming reduction process

(for the same reason, the UV dictionary remains incomplete)

- Positivity will help to understand dim-8 ADM

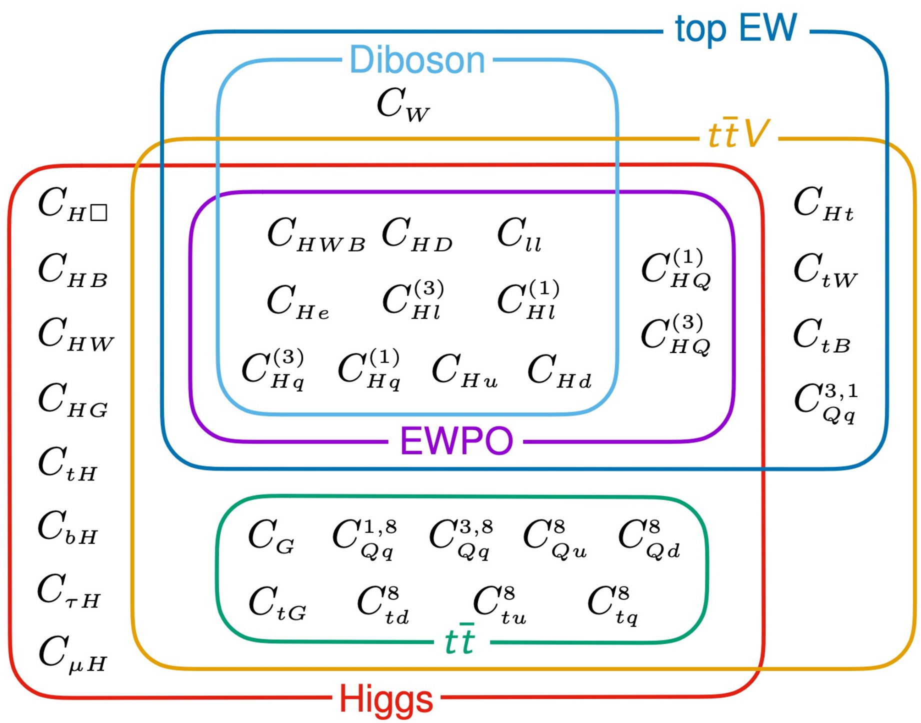
Global analysis of SMEFT

- Wilson coefficients in SMEFT are **highly correlated** and only **global analysis** can give meaningful results.
  1. One operator influences different observables
  2. One observable can be influenced by many operators
- Datasets: EWPOs, diboson, Higgs, Top, ...
- The Global-fit likelihood:

$$-2\log\mathcal{L} = \frac{1}{n_{\rm dat}}\sum_{i,j=1}^{n_{\rm dat}}\left(\sigma_{i,\rm SMEFT}(c)-\sigma_{i,\rm exp}\right)(\mathrm{cov}^{-1})_{ij}\left(\sigma_{j,\rm SMEFT}(c)-\sigma_{j,\rm exp}\right)$$

$\mathrm{cov} = \mathrm{cov}_{\rm th} + \mathrm{cov}_{\rm exp}$

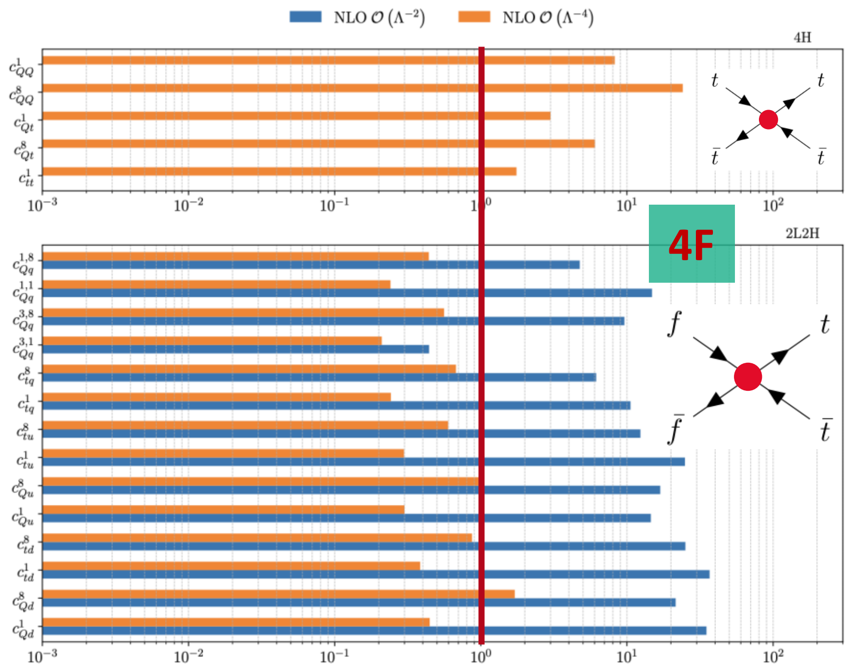
$\mathcal{O}(1/\Lambda^2)$ or $\mathcal{O}(1/\Lambda^4)$

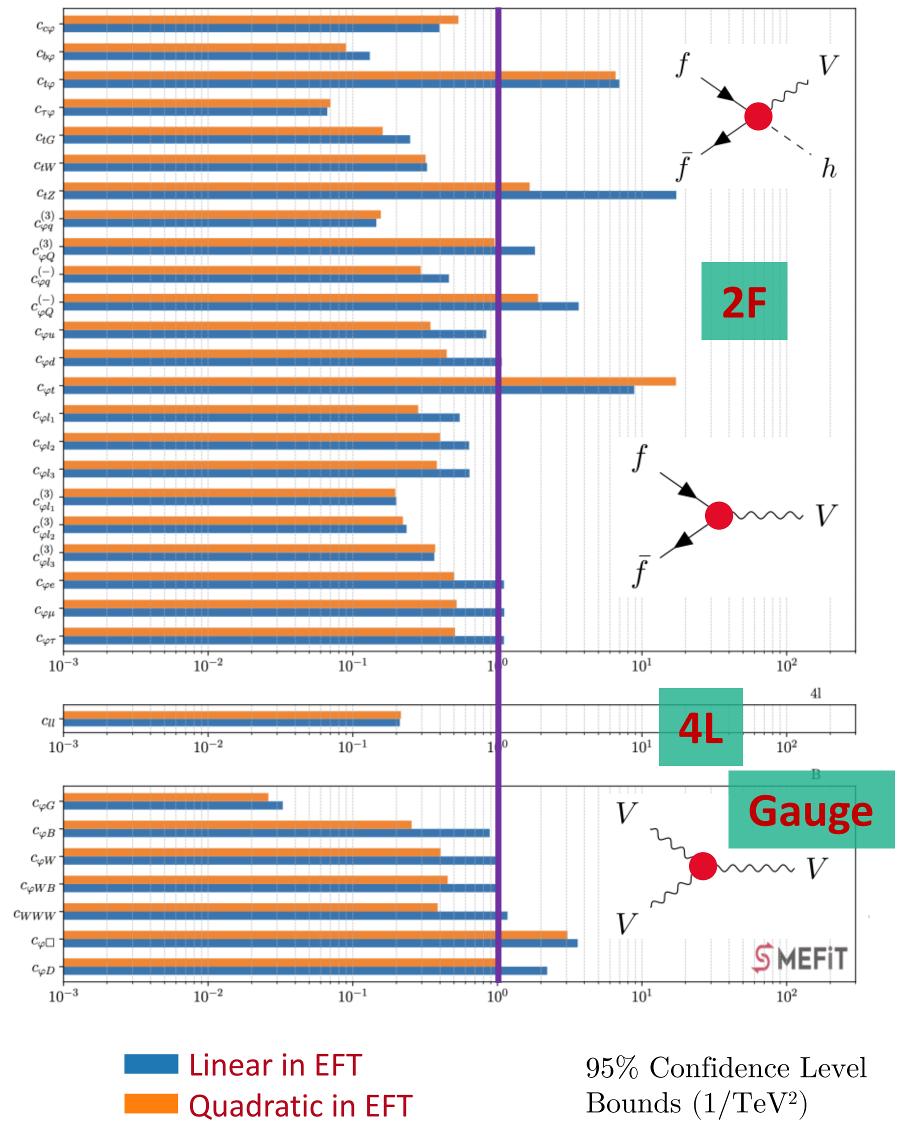


operator sets contributing to the individual datasets at LO

[J. Ellis et al., 2012.02779]

Global analysis of SMEFT





- **Results from SMEFit3.0**

  [E.Celada et al, 2404.12809]

- **Most Wilson coefficients have bounds below 1 for Λ= 1 TeV**

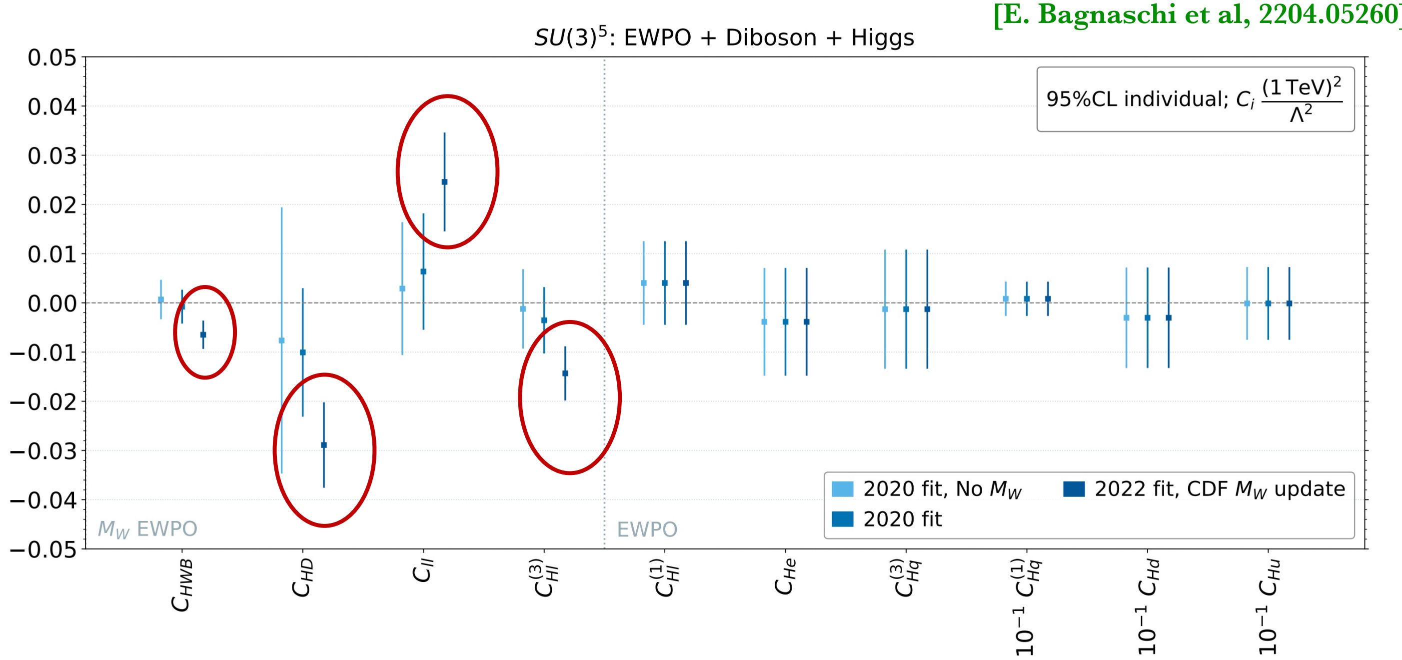
- **Yet no clear evidence for NP**

Inverse problem: inferring UV

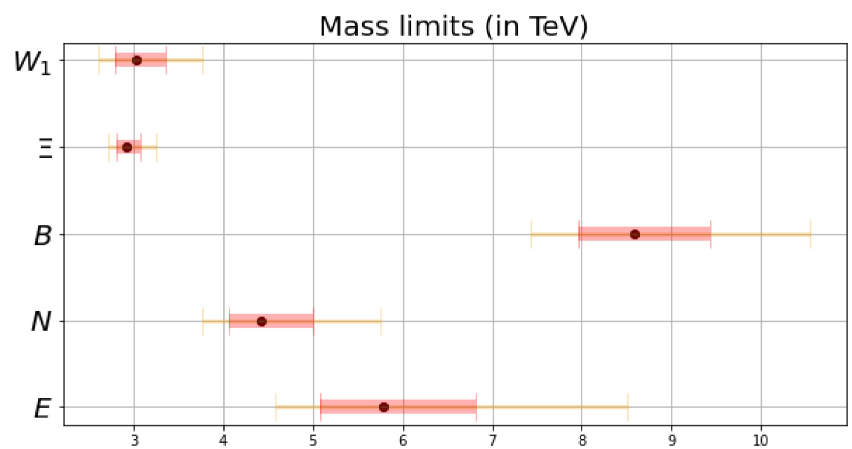
CDF measurement of m_W

[CDF Collaboration, 22’]

[E. Bagnaschi et al, 2204.05260]



Model	C_{HD}	C_{ll}	$C_{Hl}^{(3)}$	$C_{Hl}^{(1)}$	C_{He}	$C_{H\Box}$	$C_{\tau H}$	C_{tH}	C_{bH}
S_1		-1							
Σ			$\frac{1}{16}$	$\frac{3}{16}$			$\frac{y_\tau}{4}$		
Σ_1			$\frac{1}{16}$	$-\frac{3}{16}$			$\frac{y_\tau}{8}$		
N			$-\frac{1}{4}$	$\frac{1}{4}$					
E			$-\frac{1}{4}$	$-\frac{1}{4}$			$\frac{y_\tau}{2}$		
B_1	1					$-\frac{1}{2}$	$-\frac{y_\tau}{2}$	$-\frac{y_t}{2}$	$-\frac{y_b}{2}$
B	-2						$-y_\tau$	$-y_t$	$-y_b$
Ξ	$-2\left(\frac{1}{M_\Xi}\right)^2$					$\frac{1}{2}\left(\frac{1}{M_\Xi}\right)^2$	$y_\tau\left(\frac{1}{M_\Xi}\right)^2$	$y_t\left(\frac{1}{M_\Xi}\right)^2$	$y_b\left(\frac{1}{M_\Xi}\right)^2$
W_1	$-\frac{1}{4}$					$-\frac{1}{8}$	$-\frac{y_\tau}{8}$	$-\frac{y_t}{8}$	$-\frac{y_b}{8}$
W	$\frac{1}{2}$					$-\frac{1}{2}$	$-y_\tau$	$-y_t$	$-y_b$



Inverse problem: inferring UV

Likelihood in coeffs.

Log L(c)

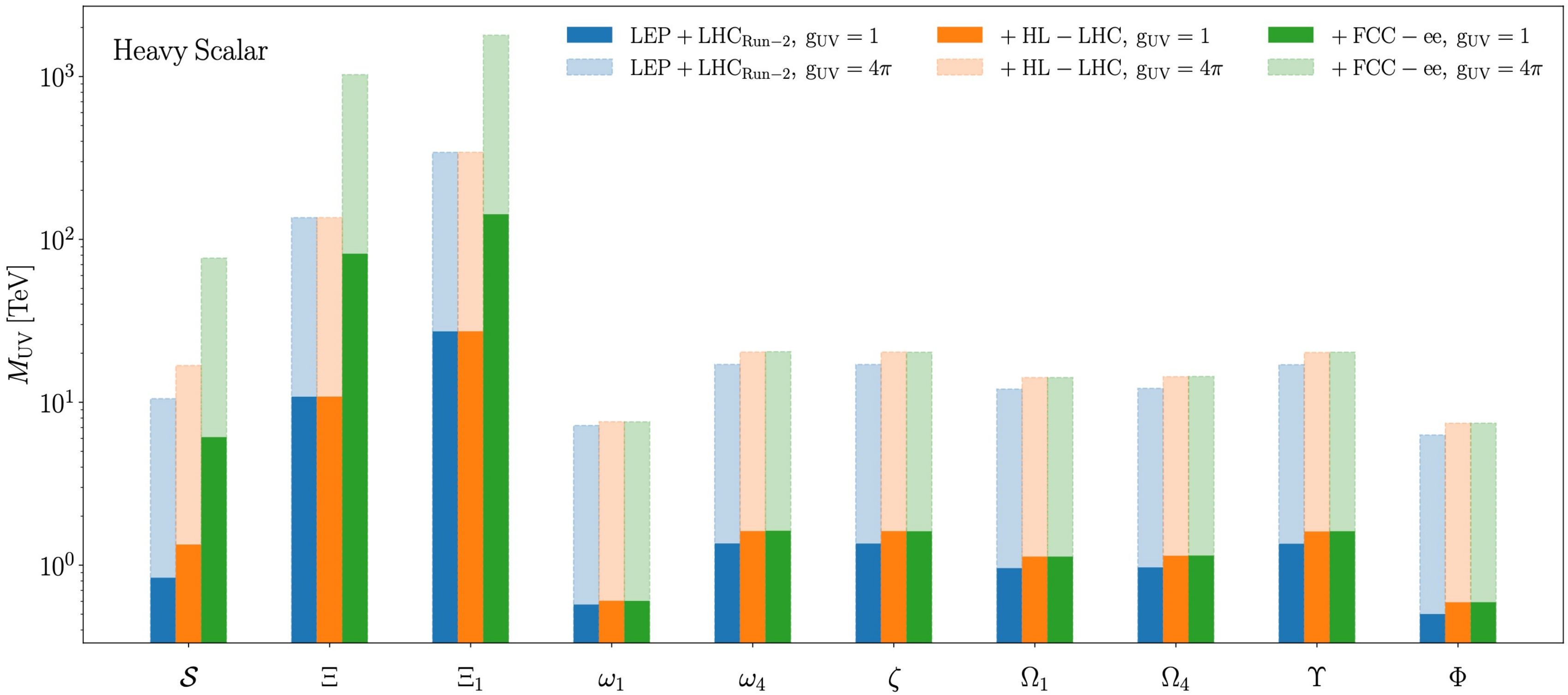
$$\frac{c_i}{\Lambda^2} = f(g_{UV}, m_{UV})$$

Likelihood in UV couplings

Log L(g_{UV}, m_{UV})

- Less parameters
- Stronger correlations
- Model dependent

Sensitivity to the BSM scale **[E.Celada et al, 2404.12809]**



Outline

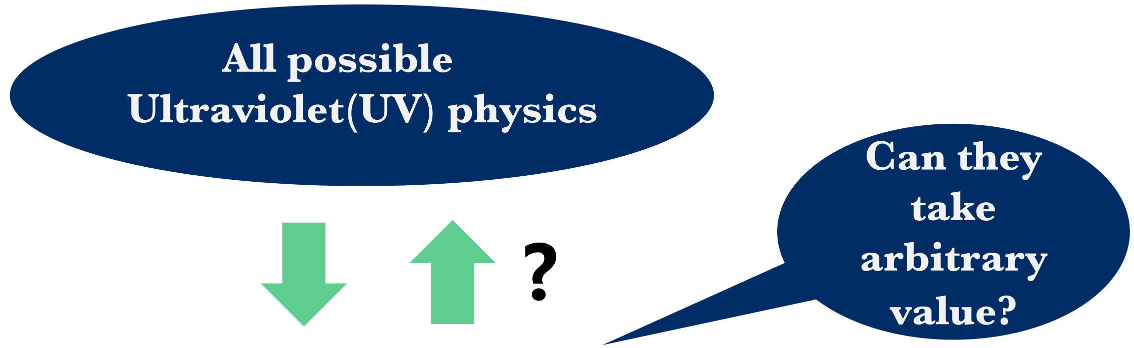
Review on the SMEFT

- Introduction
- Global fit and inferring UV
- RGE running and mixing

Positivity bounds

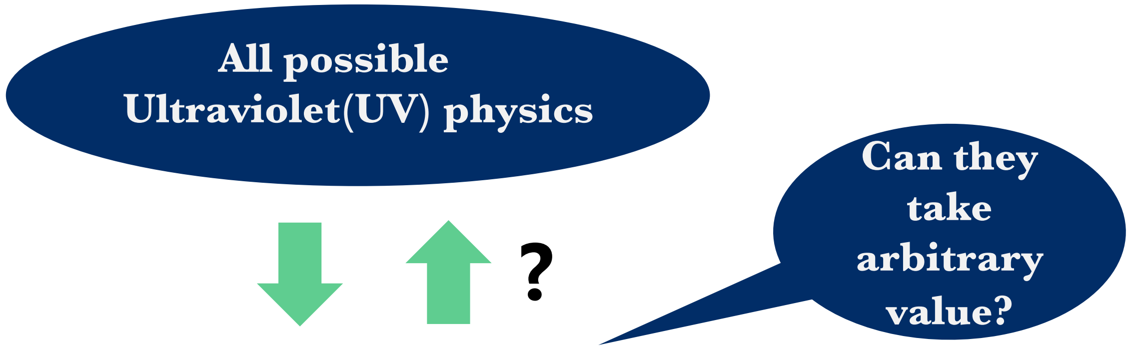
- Convex cone approach
- Pheno. of the positivity and inverse problem
- Constraints on anomalous dimension matrix

Summary



SMEFT : $\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \cdots$

Introdution: Positivity Bounds



SMEFT : $\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \cdots$

If UV physics satisfied **causality**, **unitarity**, **Lorentz** symmetry, **crossing** symmetry…

$$\begin{cases} \sum_i a_i C_i \geq 0 \\ \ldots \end{cases}$$

Positivity bounds are a set of inequalities that constrain Wilson Coeffs.

All possible Ultraviolet(UV) physics

?

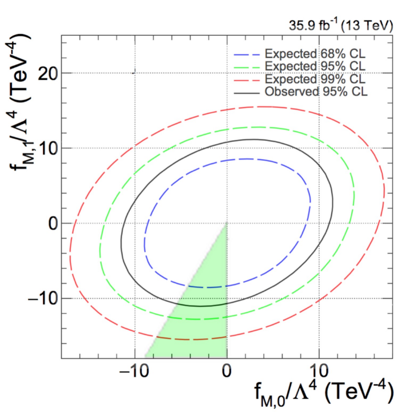
Can they take arbitrary value?

SMEFT :
$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \cdots$$

If UV physics satisfied **causality**, **unitarity**, **Lorentz** symmetry, **crossing** symmetry…

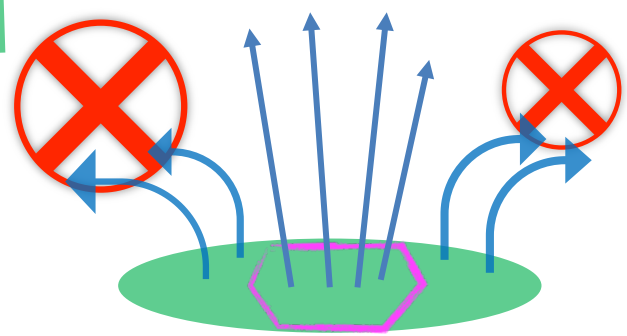
$$\begin{cases} \sum_i a_i C_i \geq 0 \\ \ldots \end{cases}$$

Positivity bounds are a set of inequalities that constrain Wilson Coeffs.



RG ADM
(zeros, signs)

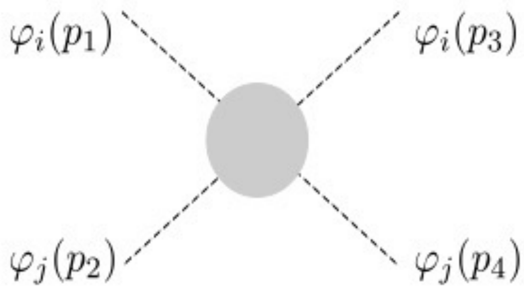
Many UV models



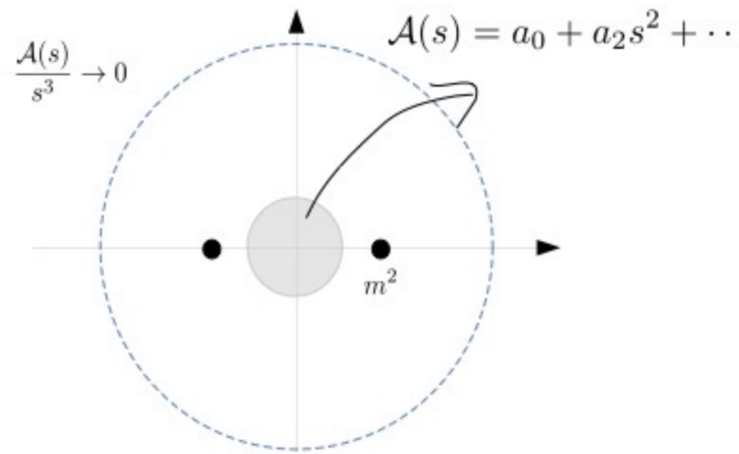
Introdution: Positivity Bounds

- 2-to-2 tree forward amplitude (spin-0): **[A. Adams, et al. JHEP 10 (2006) 014]**

$$|i\rangle + |j\rangle \to |i\rangle + |j\rangle$$



$$\mathcal{A}(s) \equiv \mathcal{A}(s, t=0)$$



$$0 = \sum \mathrm{res} \frac{\mathcal{A}(s)}{s^3} = a_2 - \frac{1}{\pi} \int s \frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

- Apply it to SMEFT, the leading energy dependence only, s^2, comes from Dim-8 operators

- To extract dim-8 effect, we consider: $$M \equiv \frac{d^2 \mathcal{A}(s,0)}{ds^2}$$

Introdution: Positivity Bounds

Istituto Nazionale di Fisica Nucleare

General 2-to-2 scattering of massless scalars at tree-level:

$$\hat{\mathcal{M}}(\hat{s},t) = \sum_{n+m>0} c_{n,m}\hat{s}^n t^m$$

- $t=0$
- $c_n, n \geq 2$
- One-field

Higher-dimension coef.

[N. Arkani-Hamed, et al. 2012.15849]
[S.D. Chowdhury, et al, 2112.11755]
[B. Bellazzini. et al. 2304.02550]

- $t \neq 0$

Beyond the forward limit

[A. Tolley et al., 2011.02400]
[S. C-Huot. et al. 2011.02957]
[S. C-Huot. et al. 2201.06602]

Formal theory or gravity EFT

- $t=0$
- c_2
- Multi-field

We focus

Bounds for multi-field

[T. Trott, 2011.10058]
[Zhang&Zhou 2005.03047]
[M. McCullough et al, 2312.03834]

SMEFT

- **Elastic approach**

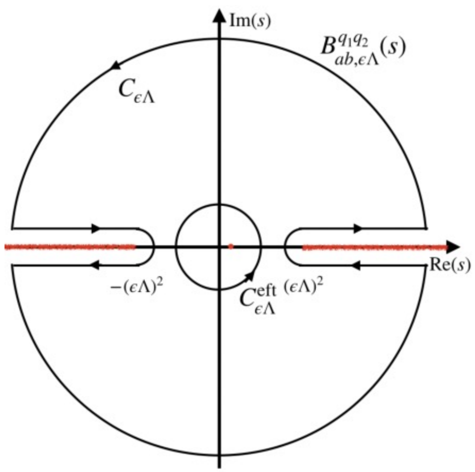
Superposition elastic: $M(|u\rangle + |v\rangle \to |u\rangle + |v\rangle) = u^i v^j u^{k*} v^{l*} M^{ijkl} \geq 0$

with $|u\rangle = u^i|i\rangle, |v\rangle = v^j|j\rangle$

- **Convex cone approach(this talk focus)**

Positivity Bounds

Optical theorem: $\text{Disc}M\,(s) = i\sum_X \int d\Pi_X M_{ij\to X}\,(s+i\epsilon)\, M^*_{kl\to X}(s+i\epsilon)$



[Q. Bi, et al, JHEP 06(2019) 137]

$$\frac{d^2}{ds^2}\mathcal{A}_{ij\to kl}(s,t=0)$$

$$= \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} ds \frac{\mathcal{A}_{ij\to X}(s,\Pi_X)\mathcal{A}^*_{kl\to X}(s,\Pi_X)}{\pi s^3} + (j \leftrightarrow l)$$

$X =$ BSM states

$\epsilon \leq 1$

s -> u crossing

Master formula:

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds}{\pi s^3} m_X^{ij} m_X^{kl*} + (j \leftrightarrow l)$$

Forward scattering amp, at low energy (calcxulable in EFT), represented by Wilson coeff.

Amplitude of SM -> X

Convex cone approach

[Zhang&Zhou 2005.03047]

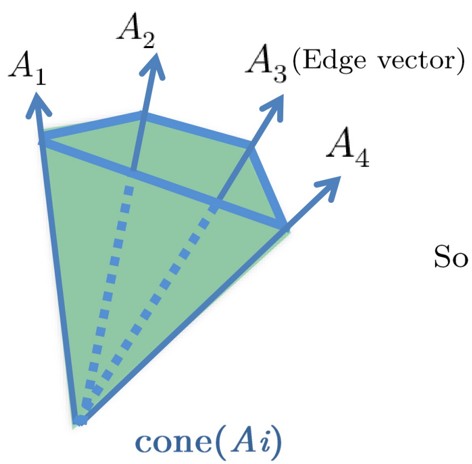
Master formula:

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds}{\pi s^3} m_X^{ij} m_X^{kl*} + (j \leftrightarrow l)$$

M^{ijkl} **is positive linear combinations of**

$${m_X}^{ij}{m_X}^{kl} + {m_X}^{il}{m_X}^{kj}$$

1. M^{ijkl} is a convex cone



So

Any vector inside cone can always be written as **positive** linear combinations of Ai

Positivity bounds arise as boundary of cone!

[Q. Bi, et al, JHEP 06(2019) 137] [B. Fuks, et al, CPC 45 (2021) 2, 023108]

Edge vectors -> Extremal rays (ER)

2. X couple to i and j , X (UV) belong to the direct product space of i and j

$$\mathbf{r}_i \otimes \mathbf{r}_j = \mathbf{X}_1 \oplus \mathbf{X}_2 \oplus \dots$$

$$M^{ijkl} = \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu}{\pi} \frac{m_X{}^{ij} m_X{}^{kl}}{(\mu - M^2/2)^3} + (j \leftrightarrow l)$$

$$M(ij \to X^\alpha) = \langle X|\mathcal{M}|\mathbf{X}_r\rangle C^{r,\alpha}_{i,j}$$

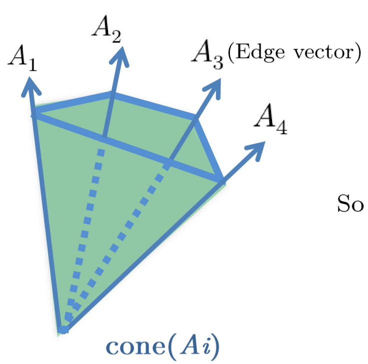
C is the CG coefficients for the direct sum decomposition of $\mathbf{r}_i \otimes \mathbf{r}_j$

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \sum_{X \ in \ \mathbf{X}_r} \frac{|\langle X|\mathcal{M}|\mathbf{X}_r\rangle|^2}{\pi(\mu - M^2/2)^3} P_r^{i(j|k|l)}$$

The only necessary information is

Projector: $P_r^{i(j|k|l)} \equiv \Sigma_\alpha C^{r,\alpha}_{i,j} \left(C^{r,\alpha}_{k,l}\right)^*$

M^{ijkl} is a convex cone: $\text{cone}\left(\left\{P_r^{i(j|k|l)}\right\}\right)$



So Edge vectors → Extremal rays (ER) → 1-particle extension

- **UV state *X* lives in the irrep r**

Positivity bounds in the SMEFT

INFN Istituto Nazionale di Fisica Nucleare

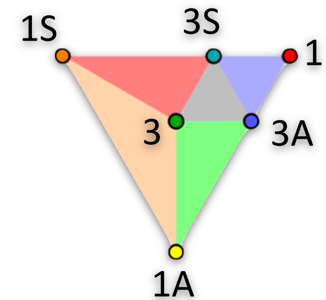
4-Higgs operators $\mathbf{2 \otimes 2 = 1 \oplus 3}$

[Remmen&Rodd, 1908.09845]
[Zhang&Zhou 2005.03047]

$$O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi],$$
$$O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi],$$
$$O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi].$$

$$F_{S,0} \geq 0\,,$$
$$F_{S,0} + F_{S,2} \geq 0\,,$$
$$F_{S,0} + F_{S,1} + F_{S,2} \geq 0\,.$$

Triangular cone



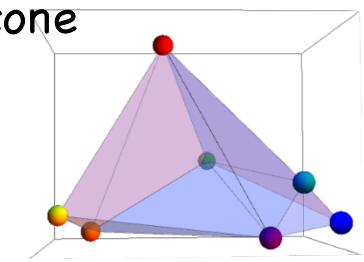
4-W operators $\mathbf{3 \otimes 3 = 1 \oplus 3 \oplus 5}$

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$

[Q. Bi, et al, 1902.08977]
[K. Yamashita, et al, 2009.04490]

$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 0,$$
$$F_{T,2} + 8F_{T,10} \geq 0,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.$$

6-facet 4D cone



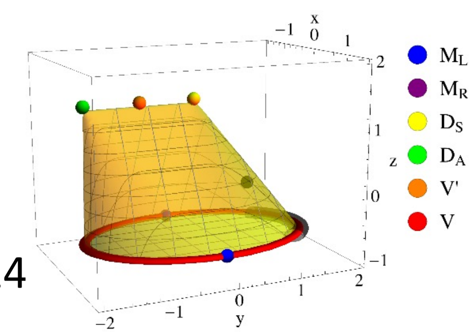
4-electron operators

[B. Fuks, et al, 2009.02212]

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$

$$C_1 \leq 0, C_3 \geq 0, C_4 \leq 0,$$
$$2\sqrt{C_1 C_4} \geq C_2$$
$$2\sqrt{C_1 C_4} \geq -(C_2 + C_3)$$

4D "circular cone"

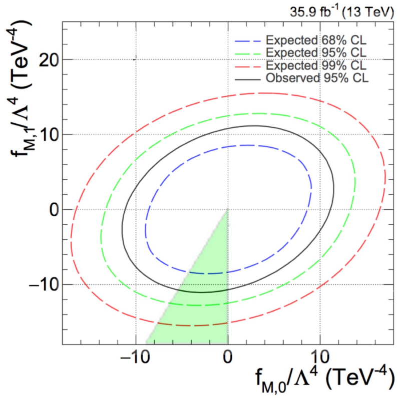


More bounds see: [C. Zhang, 2112.11665] [XL&Chala, arXiv: 2309.16611]

Pheno study in collider

- Since positivity imposes constraints on dim-8 coeffs., we should identify observables that are sensitive to dim-8 operators.

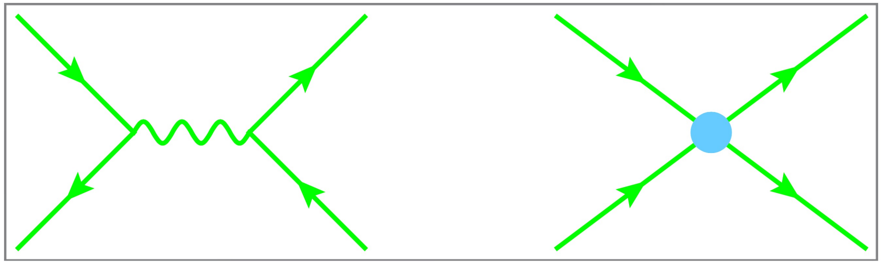
1. ZZ and Zγ production
   [Bellazzini&Riva, 1806.09640]
2. Quartic gauge-boson coupling (aQGC)
   [Q. Bi, et al. 1902.08977][K. Yamashita et al, 2009.04490]
3. e+e- to γ γ
   [J. Gu, et al. 2011.03055]
4. Higher angular coeffs. in DY
   [S.Alioli, et al. 2003.11615] [XL et al. 2204.13121]
5. e+e- to e+e-
   [B. Fuks, et al, 2009.02212]



- In the ideal case, we are able to infer UV information from convex cone viewpoint based on pheno. studies

e+e- scattering

[B. Fuks, et al, 2009.02212]



$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
~~$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)\ ,$~~

In ee → ee, C_5 does not give an independent contribution:

$$\vec{C}^{(8)} = (C_1, C_2, C_3, C_4)$$

UV states and interactions

Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

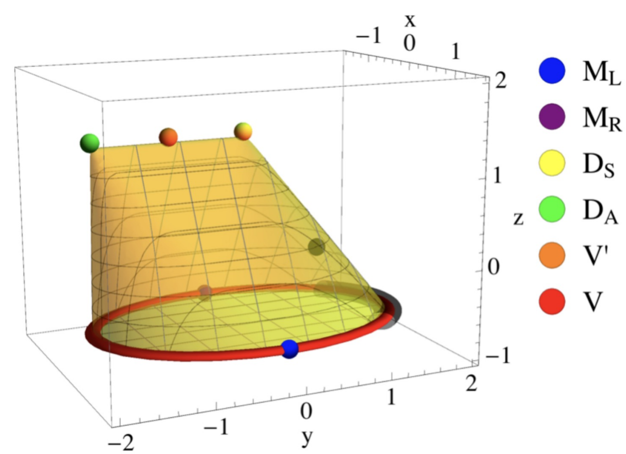
$$\mathcal{L}_{\text{int}} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\Big(\bar{L}\gamma^\mu L + \kappa_i\bar{e}\gamma^\mu e\Big)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

Projectors:

$$\vec{c}_D^{\,(8)} = (0,0,1,0),$$
$$\vec{c}_{M_L}^{\,(8)} = (0,0,0,-1),$$
$$\vec{c}_{M_R}^{\,(8)} = (-1,0,0,0),$$
$$\vec{c}_{V'}^{\,(8)} = (0,0,-1,2),$$
$$\vec{c}_{V(\kappa)}^{\,(8)} = (-\kappa^2/2, -\kappa, 0, -1/2).$$

bounds

$$C_1 \le 0,$$
$$C_4 + C_5 \le 0,$$
$$C_5 \le 0,$$
$$C_3 \ge 0,$$
$$2\sqrt{C_1(C_4+C_5)} \ge C_2,$$
$$2\sqrt{C_1(C_4+C_5)} \ge -(C_2+C_3).$$

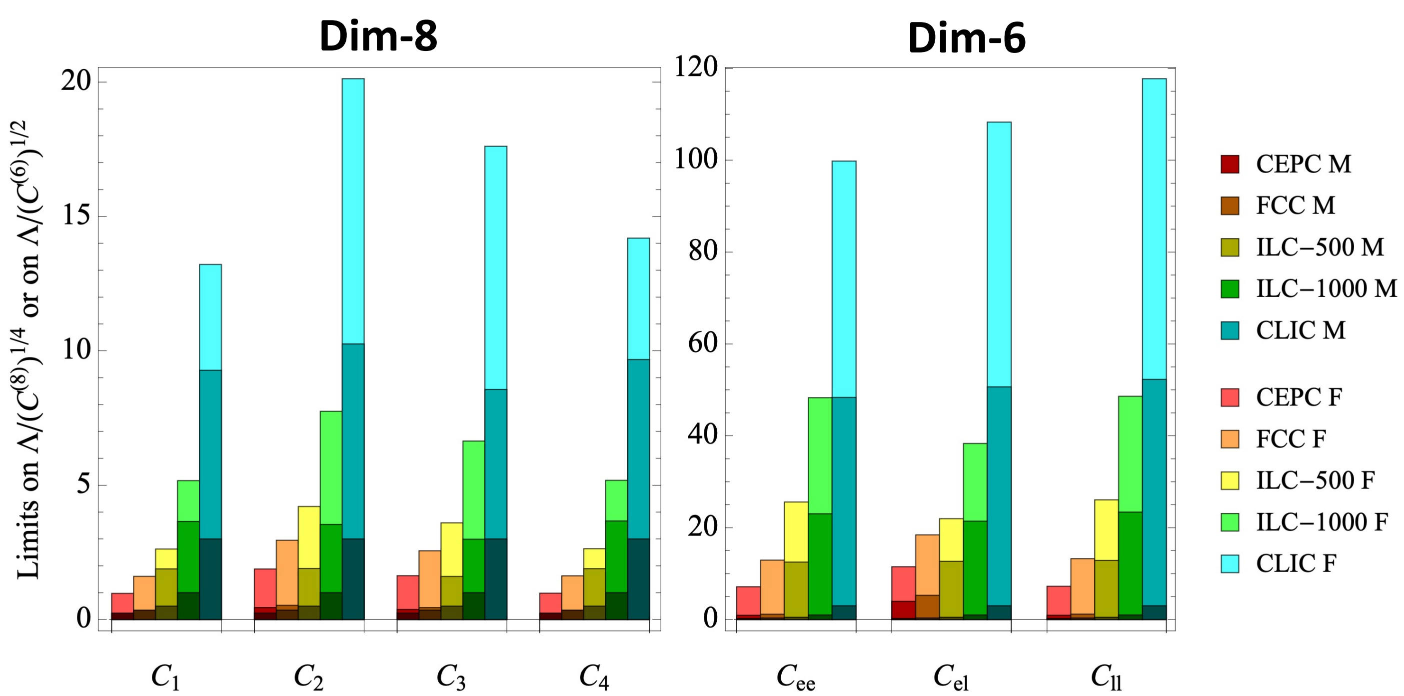


e+e- scattering at future lepton collider

Different future collider operation runs

Scenario	Beam polarization $P(e^-, e^+)$	Runs (luminosity @ energy), [ab^{-1}] @ [GeV] 1	2	3	4
CEPC	None	2.6@161	5.6@240		
FCC-ee	None	10@161	5@240	0.2@350	1.5@365
ILC-500	(−80%, 30%)	0.9@250	0.135@350	1.6@500	
	(80%, −30%)	0.9@250	0.045@350	1.6@500	
ILC-1000	(−80%, 30%)	0.9@250	0.135@350	1.6@500	1.25@1000
	(80%, −30%)	0.9@250	0.045@350	1.6@500	1.25@1000
CLIC	(−80%, 0%)	0.5@380	2@1500	4@3000	
	(80%, 0%)	0.5@380	0.5@1500	1@3000	

Limits on the new physics characterization scale, (the darkest color the largest center-of-mass energy of each collider project)



Inverse problem: dim-6 case

➢ Assume D-type scalar extension, $g_D = 0.8$, $M_D = 2$ TeV

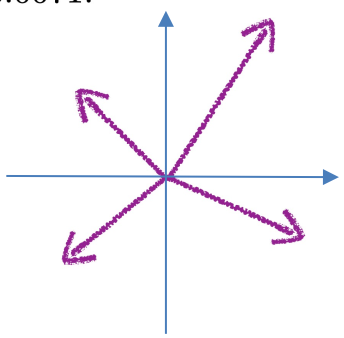
Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\text{int}} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\left(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\right)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

➢ What can we observe at ILC(1 TeV run)?
global fit result →

$$C_{ee} = 0 \pm 0.0024, \qquad C_{el} = -0.08 \pm 0.0035,$$
$$C_{ll} = 0 \pm 0.0023,$$
$$C_1 = 0 \pm 0.0074, \qquad C_2 = 0 \pm 0.0077,$$
$$C_3 = 0.04 \pm 0.020, \qquad C_4 = 0 \pm 0.0071.$$

➢ What to conclude at dim-6?



- Individual case: SM is extended by one-particle

$$-C_{el} = \frac{g_D^2}{2M_D^2} \longrightarrow M_D/g_D \in [2.45, 2.56] \text{ TeV}$$

Great!
But we can not exclude other possibilities

- Marginalized case: SM is extended by more particles

$$-C_{el} = \frac{g_D^2}{2M_D^2} - \frac{g_{V'}^2}{M_{V'}^2} = 0.08 \pm 0.0035 \text{ TeV}^{-2}$$

useless

Excluding UV states in dim-8

- What can we conclude at dim-8?

$$\vec{C}^{(8)} = \sum_X w_X \vec{c}_X^{(8)}, \qquad \text{with } w_X = \textstyle\sum_i w_{Xi} \geq 0$$

$$\vec{c}_D^{(8)} = (0,0,1,0),$$
$$\vec{c}_{M_L}^{(8)} = (0,0,0,-1),$$
$$\vec{c}_{M_R}^{(8)} = (-1,0,0,0),$$
$$\vec{c}_{V'}^{(8)} = (0,0,-1,2),$$
$$\vec{c}_{V(\kappa)}^{(8)} = (-\kappa^2/2,-\kappa,0,-1/2).$$

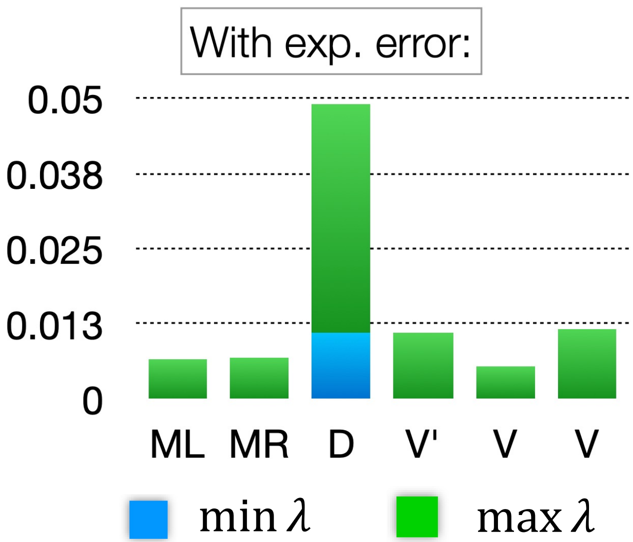
- Strategy to exclude UV:

$$\lambda_{\max} \equiv \max_{\lambda} \left[\vec{C}^{(8)} - \lambda \vec{c}_{X'}^{(8)} \in \mathcal{C};\ \chi^2(\vec{C},\vec{C}_0) \leq \chi_c^2\right]$$

The maximal prob. that X' doesn't break positivity and explains data

$C_{ee} = 0 \pm 0.0024$, $C_{el} = -0.08 \pm 0.0035$,
$C_{ll} = 0 \pm 0.0023$,
$C_1 = 0 \pm 0.0074$, $C_2 = 0 \pm 0.0077$,
$C_3 = 0.04 \pm 0.020$, $C_4 = 0 \pm 0.0071$.

The upper bounds on $\lambda_{\max} \geq \frac{g_i^2}{M_i^4}$



X	$\vec{c}_X^{(8)}$	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
M_L	$(0,0,0,-1)$	0.0067	≥ 3.5 TeV
M_R	$(-1,0,0,0)$	0.0069	≥ 3.5 TeV
V (with $\kappa = 1$)	$(-1/2,-1,0,-1/2)$	0.0055	≥ 3.7 TeV
V (with $\kappa = -1$)	$(-1/2,1,0,-1/2)$	0.0116	≥ 3.0 TeV
V'	$(0,-1,2,0)$	0.0109	≥ 3.1 TeV

$M_D/\sqrt{g_D} \in [2.1, 3.1]\text{TeV}$ **Great!**

not only obtain limit, but also exclude other particles

Excluding UV states in dim-8

Remarks

1. Dim-6 measurement, solving inverse problem can correctly obtain $M_D/g_D \in [2.45, 2.56]$ TeV by assuming one-particle extension

2. But we cannot solve the inverse problem when considering extensions with more particles, as their contributions cancel each other out

3. Dim-8 measurement would universally exclude all alternative models, independent of any model assumptions.

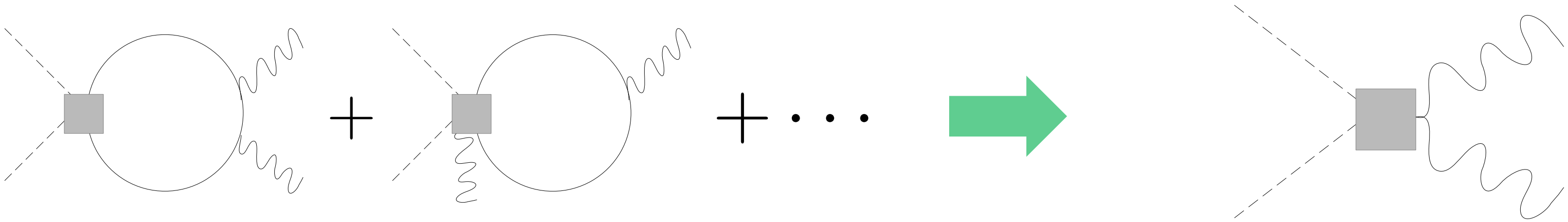
4. It can be understood that in dim-8 space, each UV state contributes positively and will not be canceled out

Observation: There are zeros in mixing of specific operators of different classes

$$\mathcal{O}^{(1)}_{e^2\phi^2D^3} = i(\bar{e}\gamma^\mu D^\nu e)(D_{(\mu}D_{\nu)}\phi^\dagger\phi) + \text{h.c.}$$

$$\mathcal{O}^{(2)}_{e^2\phi^2D^3} = i(\bar{e}\gamma^\mu D^\nu e)(\phi^\dagger D_{(\mu}D_{\nu)}\phi) + \text{h.c.}$$

$$\Rightarrow \quad \mathcal{O}^{(1)}_{B^2\phi^2D^2} = (D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho}B^\rho_\nu$$



$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2D^3} - \mathcal{O}^{(2)}_{e^2\phi^2D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3}} \not\rightarrow \mathcal{O}^{(1)}_{B^2\phi^2D^2}$$

- It is not so clear how to anticipate them, not even with amplitude methods
- It can be understood from the positivity perspective.

Constraints on ADM

- Positivity perspective, we have: $-c^{(1)}_{B^2\phi^2D^2} \geq 0\,.$ $\quad -c^{(1)}_{e^2\phi^2D^3} - c^{(2)}_{e^2\phi^2D^3} \geq 0\,.$

- Running of $c^{(1)}_{B^2\phi^2D^2}$ is positive! ➡ $\dot{c}^{(1)}_{B^2\phi^2D^2} \geq 0$ [M. Chala, 2301.09995]

- The positivity should be satisfied in any UV

- The beta function can only be linear in the coefficients combinations:

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = \alpha\left(c^{(1)}_{e^2\phi^2D^2} + c^{(2)}_{e^2\phi^2D^2}\right) \quad \text{with } \alpha < 0$$

$$\Rightarrow \quad \underbrace{\mathcal{O}^{(1)}_{e^2\phi^2D^3} - \mathcal{O}^{(2)}_{e^2\phi^2D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3}} \not\to \mathcal{O}^{(1)}_{B^2\phi^2D^2}$$

We not only determine the zero value, and but also fix the sign of α !

In general:

- From positivity, some tree-level O_i obey $c_i \geq 0$
- If O_i involves fields not present in O_j or C_j not constrained by positivity, then $\gamma_{ij} = 0$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	+	+	+	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	+	+	+	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	+	+	+	×	×	$\boxed{0}$	−	$\boxed{0}$	−	−	0	0	$\boxed{0}$	−
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	−	0	0	$\boxed{0}$	−
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	$\boxed{0}$	−
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	0	0	0	$\boxed{0}$	−
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	−	−	−	×	×

with SU(3), see [XL&Chala, 2309.16611]

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Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

with SU(3), see **[XL&Chala, 2309.16611]**

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$\boxed{-\frac{4\lvert Y\rvert^2}{3}}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

with SU(3), see **[XL&Chala, 2309.16611]**

Constraints on ADM

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{g^2-\lvert Y\rvert^2}$	$\times$	$\boxed{0}$	$\boxed{-\frac{4\lvert Y\rvert^2}{3}}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

with SU(3), see **[XL&Chala, 2309.16611]**

Summary

- SMEFT is a useful tool for precise physics study in searching for NP, significant advancements have been made in both phenomenology and theory.

- Positivity structures arise at the dim-8 level in EFT coefficient space, as a consequence of axiomatic QFT principles.

- Positivity application in SMEFT is a lively field, but the phenomenological relevance of dim-8 physics still to be fully understood

- The positivity bounds can be used to study inverse problem, infer UV information based on the geometry perspective

- Positivity bounds on dimension-8 interactions restrict different aspects of (certain) their anomalous dimensions (zeros, signs, inequalities)

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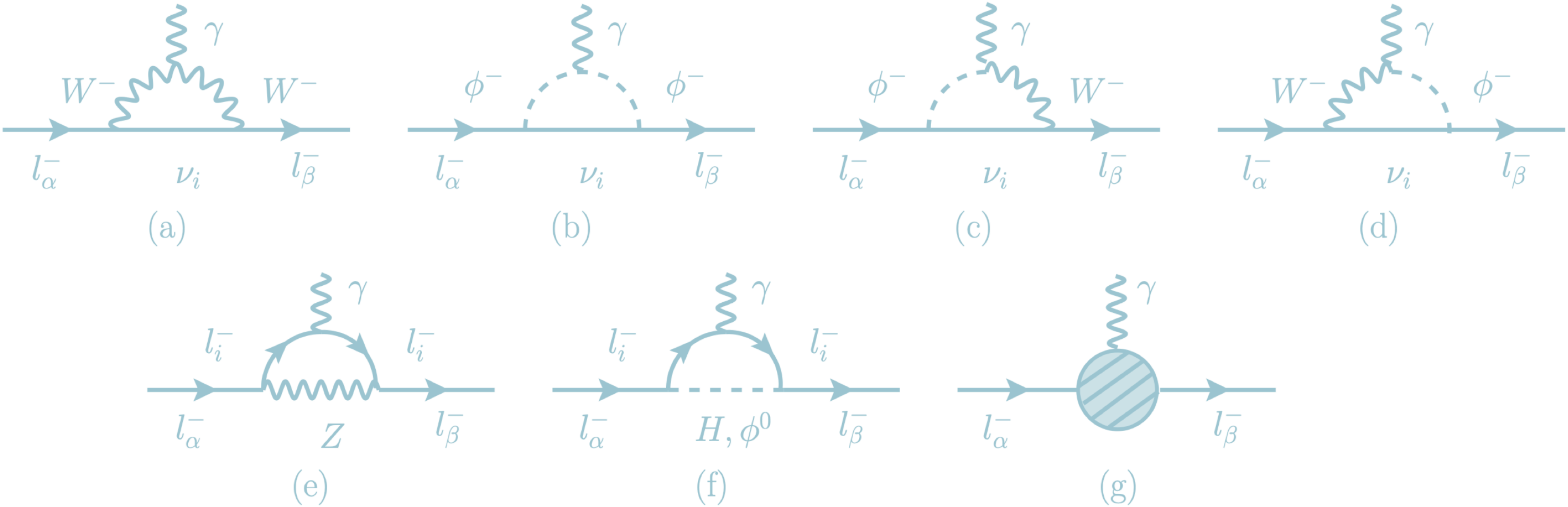
Thanks for your attention

References

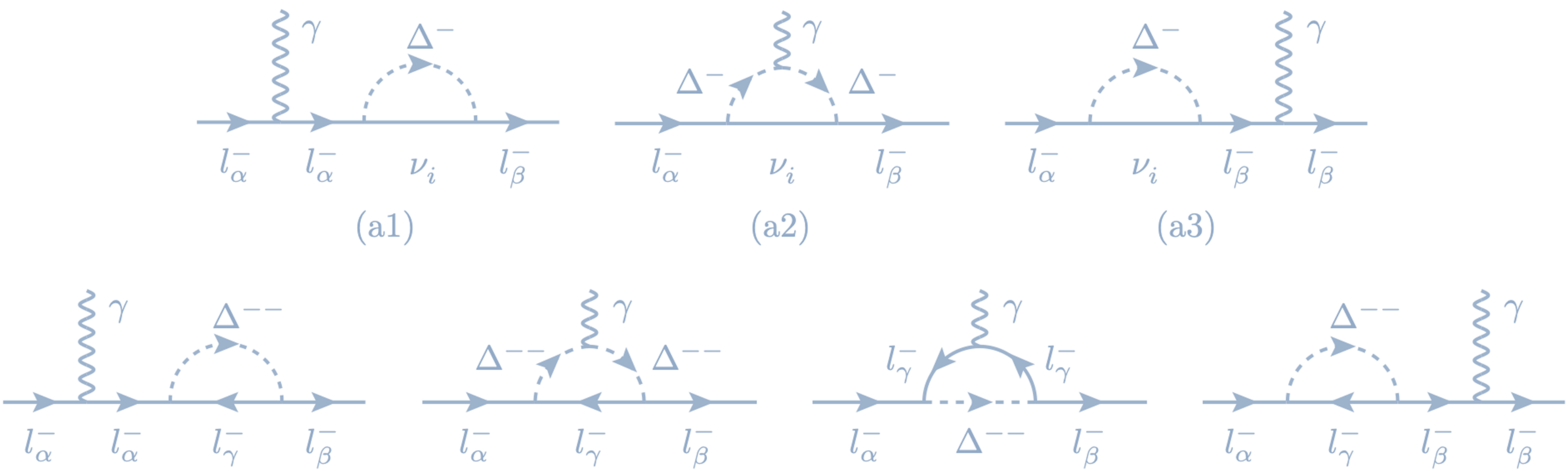
This talk collects the following references:

- Cen Zhang's talk @ Positivity and the Bootstrap Workshop "Positivity bounds in SMEFT and the inverse problem", 2021 online
- Mikael Chala's talk @ 5th NPKI workshop "Understanding the quantum structure of the dimension-8 SMEFT", 2023, Busan



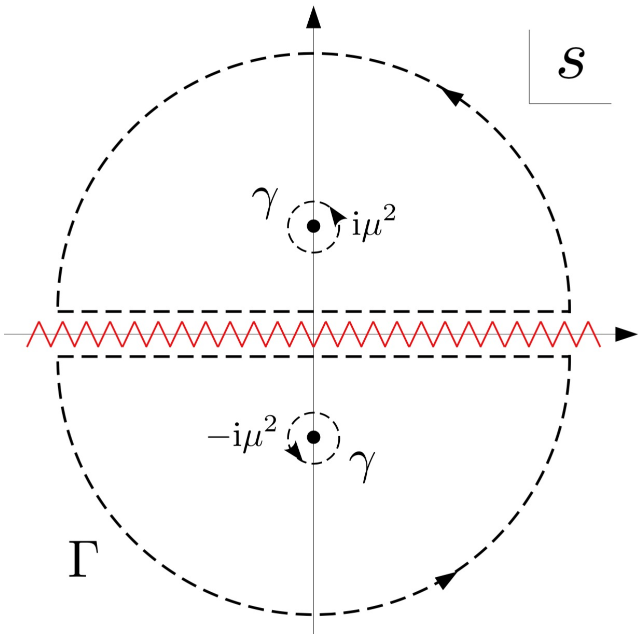
γ
W⁻ W⁻
ϕ⁻ ϕ⁻
ϕ⁻ W⁻
W⁻ ϕ⁻
l_α^- ν_i l_β^-
(a)
(b)
(c)
(d)
l_i^- l_i^-
Z
H, ϕ^0
(e)
(f)
(g)

Backup



γ
Δ^-
l_α^- ν_i l_β^-
(a1)
(a2)
(a3)
Δ^{--}
l_γ^-

Constraints on ADM



$\mathcal{A}(s) \sim -(\beta_0 + \beta_2 s^2 + \beta_4 s^4 + \cdots) \log \frac{s}{\Lambda^2}$

Consider the simplified EFT with only U(1) symmetry

$$\Sigma(\mu) \equiv \frac{1}{2\pi i} \int_\gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} = \frac{1}{2\pi i} \int_\Gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3},$$

$$= \frac{1}{4\mu^4}\left[\beta_0 - 3\beta_2\mu^4 + 5\beta_4\mu^8 + \cdots\right] \geq 0$$

higher g_1 higher μ

for $\mu \leq \mu'$ and $g_1 \leq g_1'$

$$\frac{4}{3}\Sigma(\mu) \approx -\beta_2 \geq 0$$

$\gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)}$

1. We have $\gamma_{ij} c_j^{(8)} \leq 0$ provided the γ' vanishes
2. The c_i's own anomalous dimension will not be constrained
3. If $c_j^{(8)}$ is not constrained, $\gamma_{ij} = 0$

Testing the SM

- If all dim-8 coefficients are consistent with 0, all states can be excluded to above certain scales

    - Not possible at dim-6

$$\begin{aligned} &\text{maximum} \quad \lambda \\ &\text{subject to} \quad \vec{C} - \lambda \vec{C}_k \in \mathbf{C} \\ &\qquad\quad \text{and} \quad \chi^2\left(\vec{C}, \vec{C}_{\text{EXP}}\right) \le \chi^2_c \end{aligned}$$

X	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
D	0.0076	≥ 3.4 TeV
M_L	0.0053	≥ 3.7 TeV
M_R	0.0054	≥ 3.7 TeV
V'	0.0056	≥ 3.7 TeV
V (with $\kappa = 1$)	0.0041	≥ 4.0 TeV
V (with $\kappa = -1$)	0.0041	≥ 4.0 TeV