V_{us} from τ Decays

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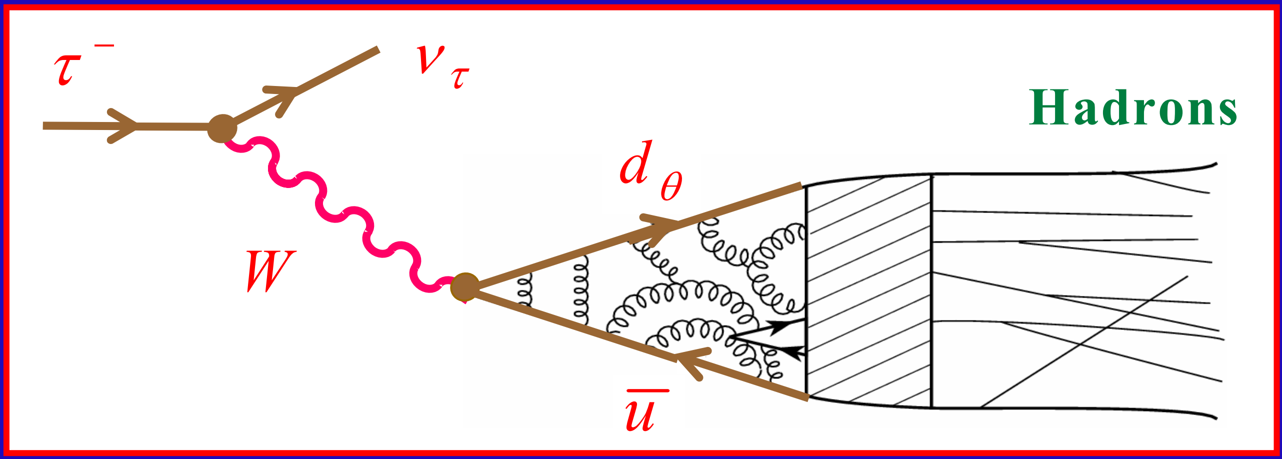
IFIC, Valencia

KAON'07

Frascati

21-25 May 2007

HADRONIC TAU DECAY

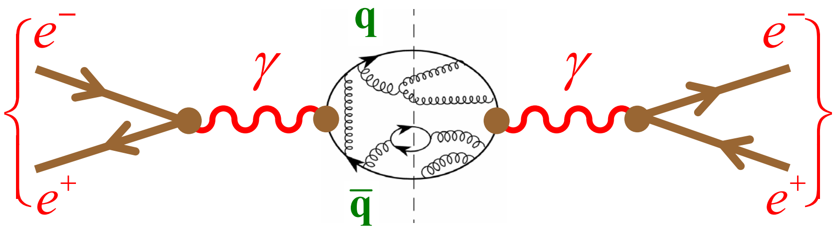


$$d_\theta = V_{ud}\; d + V_{us}\; s$$

Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\,\bar{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.639 \pm 0.011$$

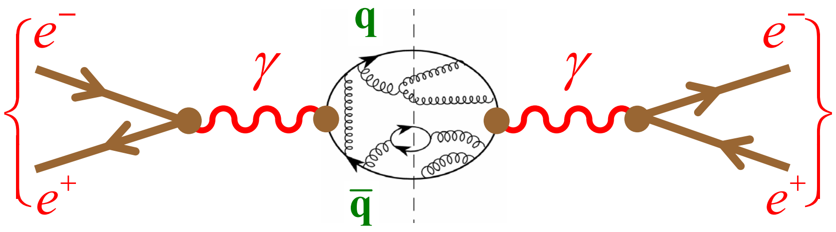
$\sigma \sim \mathrm{Im}$



$$\frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi\, \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i\int d^4x\; e^{iqx}\,\langle 0|T[J^{\mu}_{\mathrm{em}}(x)\,J^{\nu}_{\mathrm{em}}(0)]|0\rangle \;=\; \left(-g^{\mu\nu}q^2 + q^{\mu}q^{\nu}\right)\Pi_{\mathrm{em}}(q^2)$$

$\sigma \sim \mathrm{Im}$



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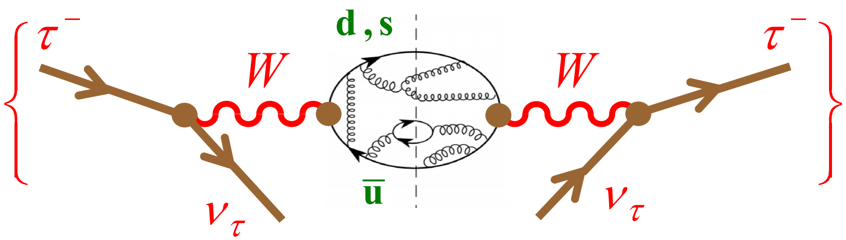
$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \mathrm{Im}$

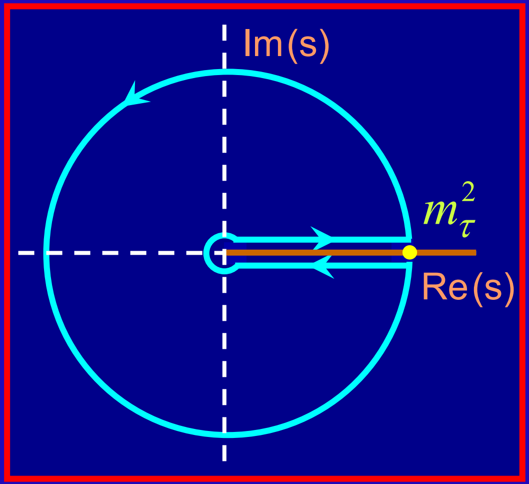


$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} dx \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2 \left[\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i \int d^4x \; e^{iqx} \langle 0 | T[J^\mu_{ij}(x) J^\nu_{ij}(0)^\dagger] | 0 \rangle = \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{ij,J}(q^2)$$

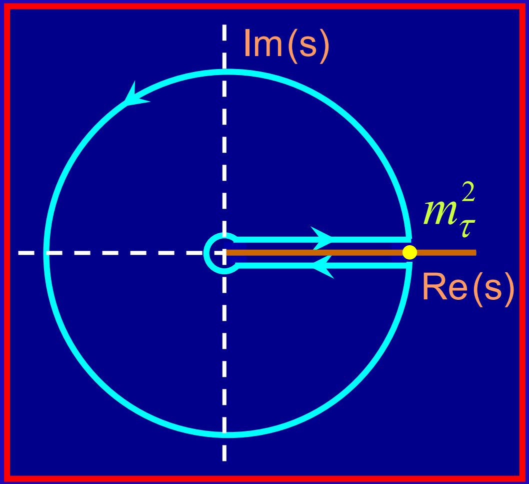
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \mathrm{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\mathrm{Im}\,\Pi^{(1)}(x m_\tau^2) + \mathrm{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

Braaten-Narison-Pich

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\, (1-x)^2 \left[(1+2x)\, \text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2) \right]$$



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$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\, \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE

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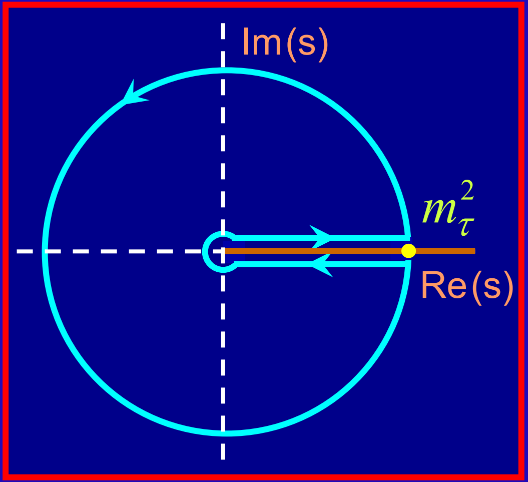

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OPE

$$R_\tau = N_C\, S_{\mathrm{EW}}\,(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\mathrm{EW}} = 1.0201\ (3)$; $\delta_{\mathrm{NP}} = -0.004 \pm 0.002$

Marciano-Sirlin, Braaten-Li, Erler

Fitted from data

$\delta_{\mathrm{P}} = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Perturbative: (m_q=0)

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n \quad ; \quad K_0 = K_1 = 1 \;,\; K_2 = 1.63982 \;,\; K_3 = 6.37101$$

$$\Rightarrow \quad \delta_{\mathrm{P}} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \cdots$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

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Power Corrections:

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$ and the moments

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

Different sensitivity to power corrections through k , l

The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{\mathrm{NP}} = -0.004 \pm 0.002$$

ALEPH, CLEO, OPAL

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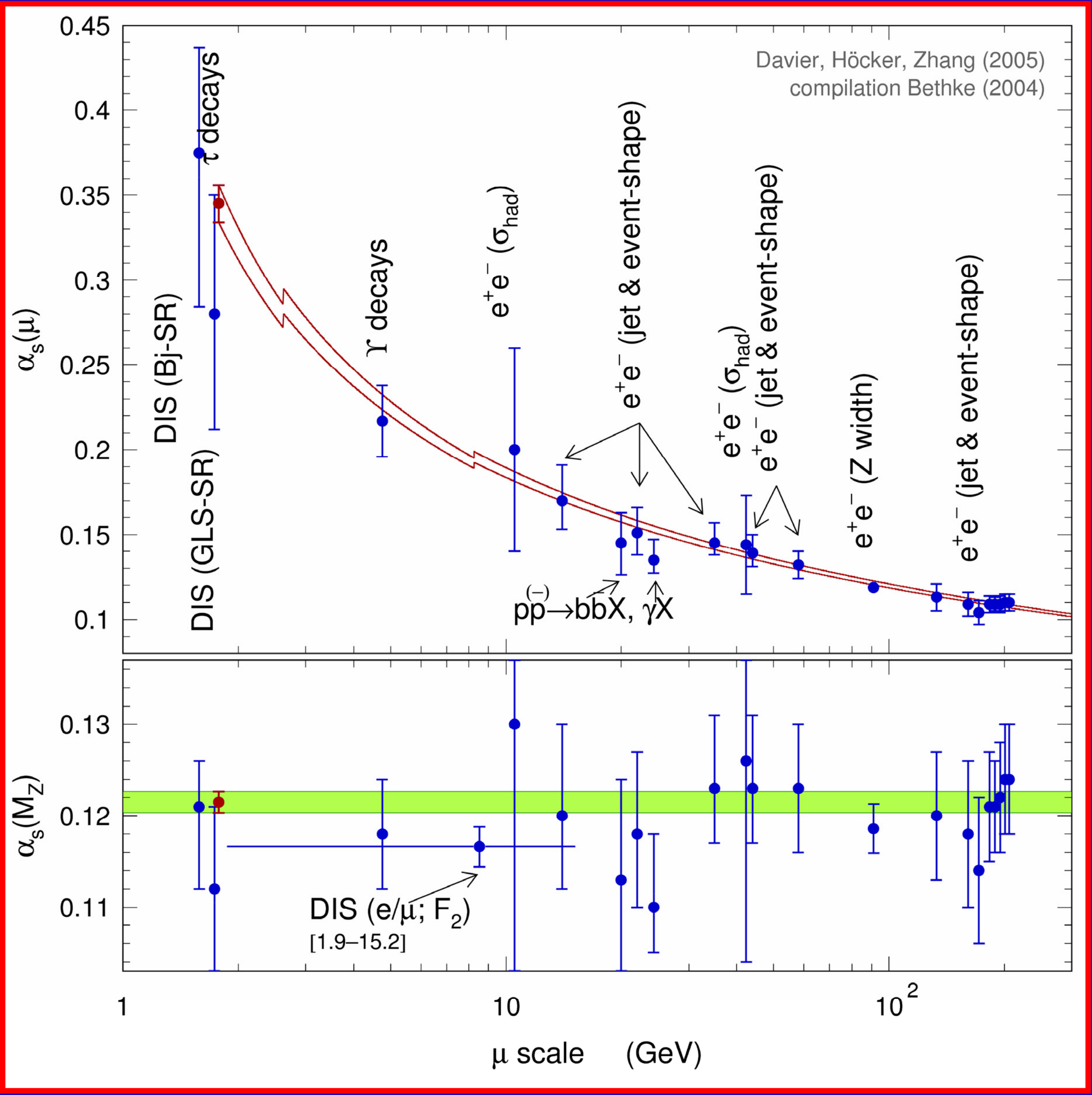
ALEPH, CLEO, OPAL

$R_{\tau,V} = 1.787 \pm 0.013 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.013 \quad ; \quad R_{\tau,V+A} = 3.482 \pm 0.014$ (ALEPH 2005)



$$\alpha_s(m_\tau^2) = 0.345 \pm 0.010$$

$$\alpha_s(M_Z^2) = 0.1215 \pm 0.0012$$

$$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1186 \pm 0.0027$$

The most precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0029 \pm 0.0010_\tau \pm 0.0027_Z$$

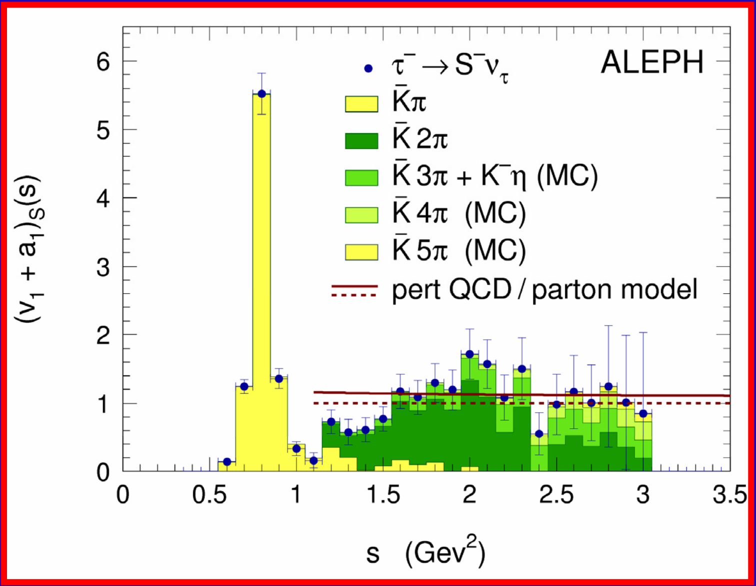
SU(3) Breaking

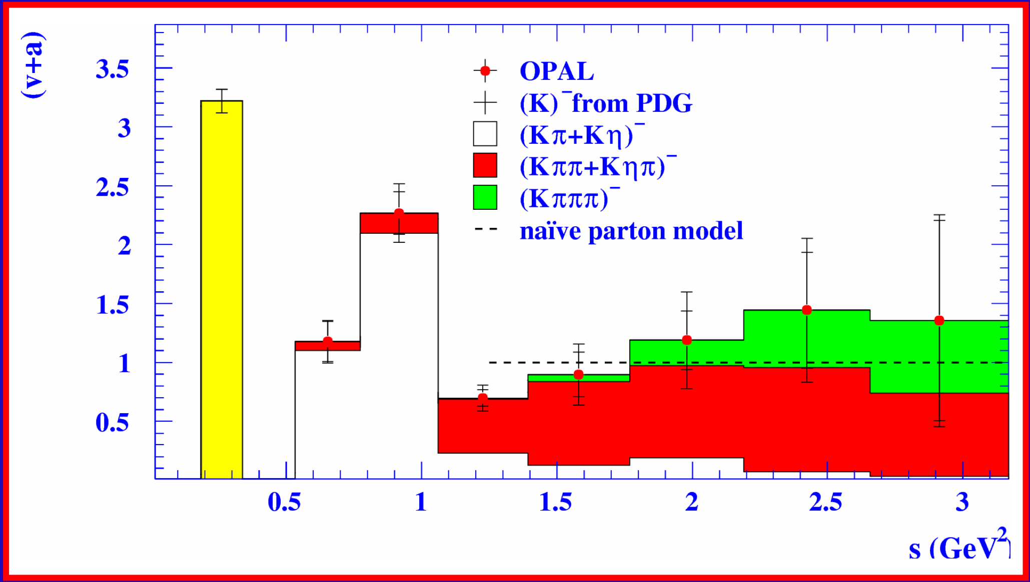
$$R_\tau^{kl} = N_C\, S_{\rm EW} \left\{ \left(|V_{ud}|^2 + |V_{us}|^2 \right) \left[1 + \delta^{kl(0)} \right] + \sum_{D\geq 2} \left[|V_{ud}|^2\, \delta_{ud}^{kl(D)} + |V_{us}|^2\, \delta_{us}^{kl(D)} \right] \right\}$$

⬇

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = N_C\, S_{\rm EW} \sum_{D\geq 2} \left[\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right]$$

Strange Spectral Function: SU(3) Breaking





(k,l)	ALEPH	OPAL
(0,0)	0.39 ± 0.14	0.26 ± 0.12
(1,0)	0.38 ± 0.08	0.28 ± 0.09
(2,0)	0.37 ± 0.05	0.30 ± 0.07
(3,0)	0.40 ± 0.04	0.33 ± 0.05
(4,0)	0.40 ± 0.04	0.34 ± 0.04

$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)$$

→ $m_s(m_\tau)$ **determination**

V_{us} and QCD uncertainties

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Known to $O(\alpha_s^3)$

- $\Delta_{kl}(\alpha_s)$ gets **longitudinal (J=0)** and **transverse (J=0+1)** contributions
- Divergent QCD series for J=0
- **Longitudinal contribution determined through data:**
  - Kaon pole (f_K) (dominant J=0 contribution)
  - Pion pole (f_π)
  - $(K\pi)_{J=0}$ (S-wave $K\pi$ scattering)
  - …
- Smaller uncertainties

	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

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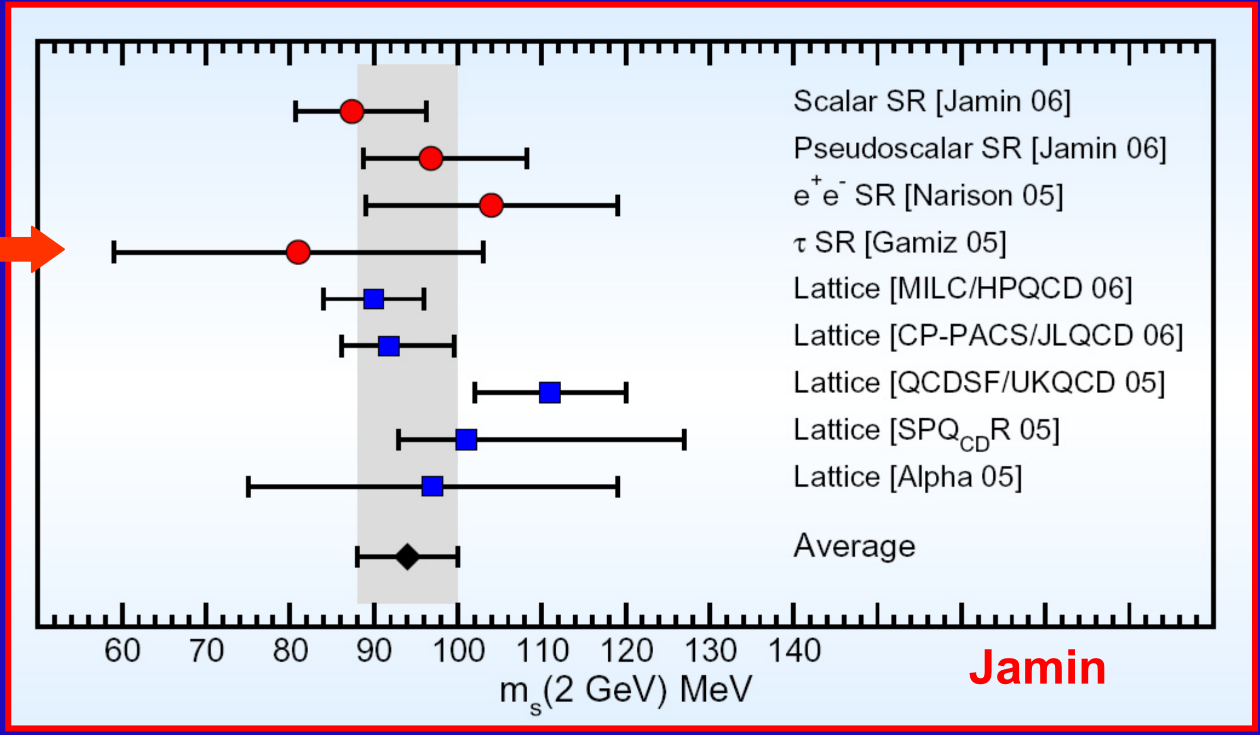
$$\delta R_{\tau,\mathrm{th}}^{00} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{9.3\,(3.4)\, m_s^2}_{m_s(2\ \mathrm{GeV}) = 0.094\,(6)} + 0.0034\,(28) = 0.240\,(32)$$

OPAL τ data



Large uncertainty from V_{us}

Strong sensitivity to V_{us}

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ data: $R^{00}_{\tau,S} = 0.1686\ (47)$

$R^{00}_{\tau,V+A} = 3.471\ (11)$

PDG 06: $|V_{ud}| = 0.97377\ (27)$

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\mathrm{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

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$\delta R_{\tau,\mathrm{th}}^{00} = 0$ ➡ $|V_{us}| = 0.215$

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Taking as input (from non τ sources) $m_s(2\ \mathrm{GeV}) = 94 \pm 6\ \ \mathrm{MeV}$:

$\delta R^{00}_{\tau,\mathrm{th}} = 0.240\ (32)$ ➡ $$|V_{us}| = 0.2220 \pm 0.0031_{\mathrm{exp}} \pm 0.0011_{\mathrm{th}}$$

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\text{th}}}$$

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K_{l3}: $|V_{us}| = 0.2234 \pm 0.0024$ $(f_+(0) = 0.97 \pm 0.01)$

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

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K_{l3}: $|V_{us}| = 0.2234 \pm 0.0024$ $\quad (f_+(0) = 0.97 \pm 0.01)$

The τ could give the most precise V_{us} determination

A simultaneous m_s & V_{us} fit could be possible

However:

- Perturbative QCD corrections need to be better understood (CIPT)

$$\Delta_{00}(\alpha_s)^{L+T} = 0.753 + 0.214 + 0.065 - 0.063 + \ldots$$
$$\Delta_{10}(\alpha_s)^{L+T} = 0.912 + 0.334 + 0.192 + 0.069 + \ldots$$
$$\Delta_{20}(\alpha_s)^{L+T} = 1.055 + 0.451 + 0.330 + 0.232 + \ldots$$
$$\Delta_{30}(\alpha_s)^{L+T} = 1.190 + 0.571 + 0.484 + 0.432 + \ldots$$
$$\Delta_{40}(\alpha_s)^{L+T} = 1.324 + 0.697 + 0.657 + 0.676 + \ldots$$

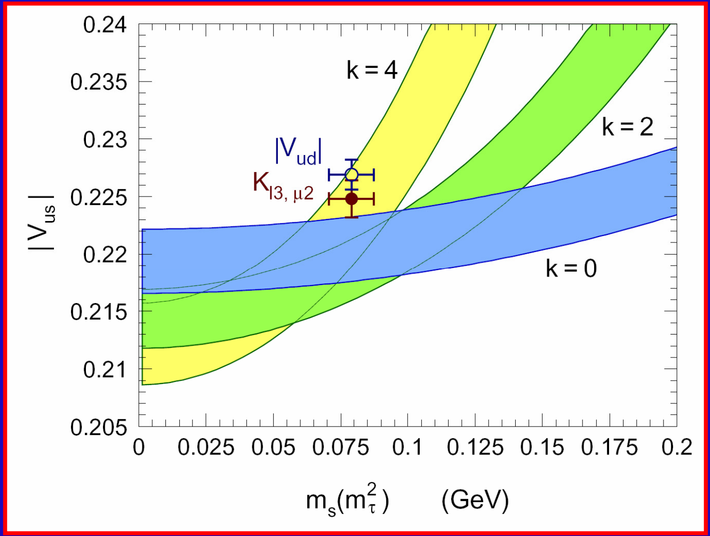
Sizeable theoretical uncertainties

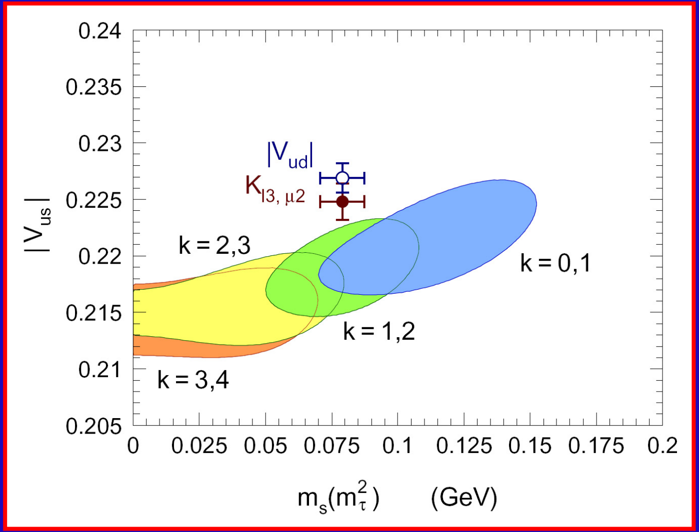
Resummations, pinched weights (Maltman & Wolfe), …

- **Not enough sensitivity with present data**

Large correlations. Low statistics. Missing decay modes …

ALEPH





Taking $V_{us} = 0.2225\,(21)$:

Chen et al '01 , J=0 included

(k,l)	m_s (MeV)	σ_{m_s} (MeV) exp.	$\lvert V_{us}\rvert$	α_s	$\langle m_s \bar{s}s\rangle$	trunc.	R-scale	th.
(0,0)	132	26	13	2	4	9	9	14
(1,0)	120	13	9	3	4	10	11	16
(2,0)	117	10	7	3	6	14	14	21
(3,0)	117	9	8	2	8	19	16	27
(4,0)	103	7	5	3	9	20	19	29

$$m_s(m_\tau) = \left(120\,^{+21}_{-26}\right)\ \text{MeV}$$

$$m_s(2\,\text{GeV}) = \left(116\,^{+20}_{-25}\right)\ \text{MeV}$$

Gámiz et al '03 , J=0 excluded

Moment	$m_s(m_\tau)$ [MeV]
(0,0)	192 ± 72
(1,0)	164 ± 31
(2,0)	137 ± 20
(3,0)	115 ± 17
(4,0)	100 ± 17

$$m_s(m_\tau) = (122 \pm 17)\ \text{MeV}$$

$$m_s(2\,\text{GeV}) = (117 \pm 17)\ \text{MeV}$$

- Strong k dependence with ALEPH data (m_s decreases with increasing k)

Spectral function underestimated at large invariant masses

→ Missing events / modes ($K\pi\pi$, $K\pi\pi\pi$, …)

- Much better behaviour with OPAL data:

Gámiz et al '05 , J=0 excluded

(0,0) → $V_{us} = 0.2208\ (34)$ →

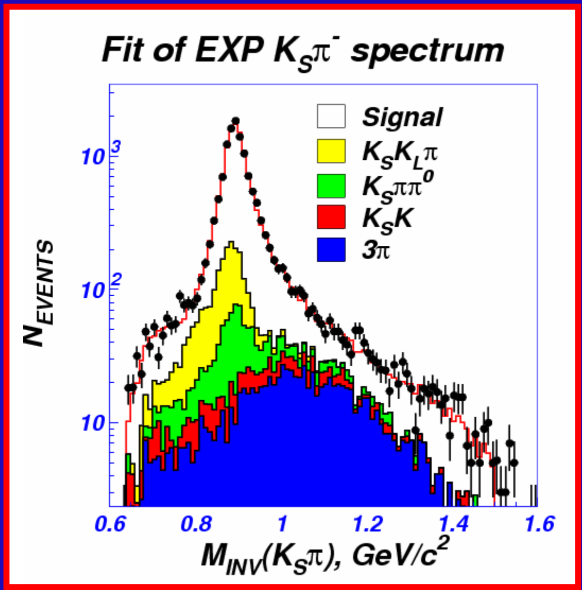
Moment	$m_s(m_\tau)$ [MeV]
(2,0)	89 ± 39
(3,0)	84 ± 27
(4,0)	78 ± 22

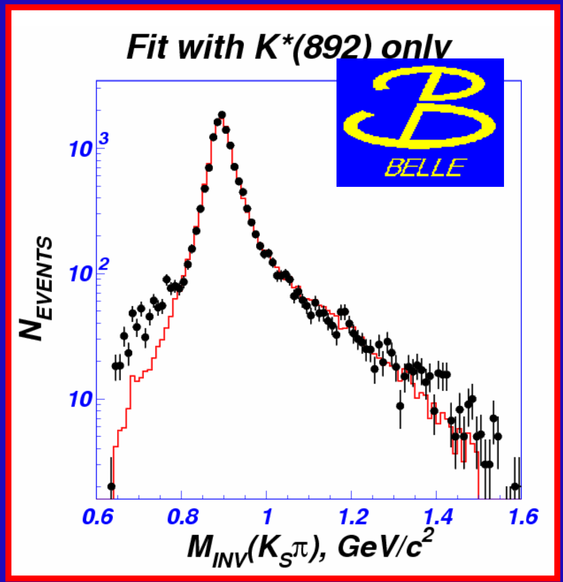
$$m_s(m_\tau) = (84 \pm 23)\ \mathrm{MeV} \qquad , \qquad m_s(2\,\mathrm{GeV}) = (81 \pm 22)\ \mathrm{MeV}$$

- $\tau \to K\nu$ from $K \to \mu\nu$ + OPAL: $V_{us} = 0.2220\ (33)$

Huge number of $\tau^+\tau^-$ events at the B Factories

B. Schwartz

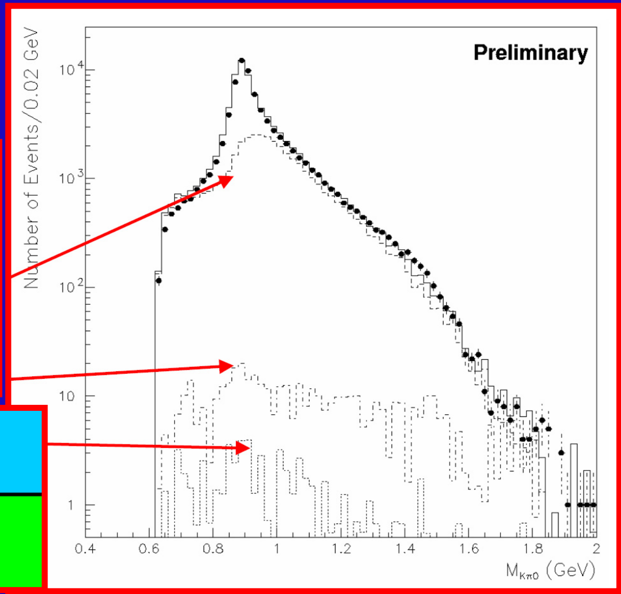




Ongoing data analysis

$\mathrm{Br}(\tau\to K_S\pi\nu_\tau) = (0.391 \pm 0.004_{stat} \pm 0.014_{syst})\ \%$

I.M. Nugent



BABAR

Preliminary Br. $K^0\pi^-$
$(4.39 \pm 0.03 \pm 0.21)\times 10^{-3}$

SUMMARY

The τ could give the most precise V_{us} determination

- From LEP data (low statistics) one gets:

$$|V_{us}| = 0.2220 \pm 0.0031_{\rm exp} \pm 0.0011_{\rm th}$$

- Accuracy similar already to K_{l3}: $|V_{us}| = 0.2234 \pm 0.0024$ $[f_+(0) = 0.97 \pm 0.01]$

Interesting challenge for the B Factories & BESIII