Experimental review on V_{us} extraction from Kaon decays

Matteo Palutan
LNF-INFN, KLOE collaboration
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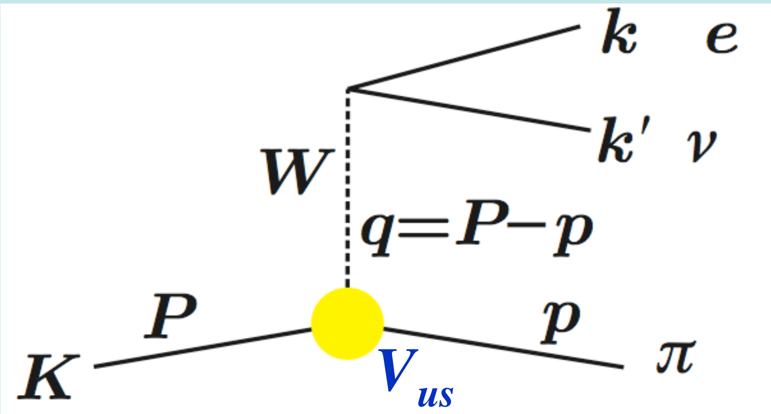

Co-starring:
BNL 865, ISTRA+, KLOE, KTeV, NA48

Combination of experimental results by M. Moulson (FlaviaNet working group)

http://www.lnf.infn.it/wg/vus

Working Group on Precise SM Tests in K Decays

- Home
- Master Formulae
- Branching Ratios
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- Acknowledgements

ISTRA+:
Oleg Yushchenko (Protvino)
Vladimir Obraztsov (Protvino)

KLOE:
Matthew Moulson (Frascati) web coordinator
Patrizia De Simone (Frascati)

KTeV:
Sasha Glazov (DESY)

NA48:
Rainer Wanke (Uni. Mainz)
Michele Veltri (Uni. Urbino)
Mauro Piccini (CERN)

Theory:
Johan Bijnens (Lund)
Vincenzo Cirigliano (Los Alamos)
Juerg Gasser (Bern)
Claudio Gatti (Frascati)
Richard Hill (FNAL)
Federico Mescia (Frascati)
Jan Stern (Orsay)

Honorary chair: **Paolo Franzini (LNF)** Coordinators: **Mario Antonelli (LNF)** and **Gino Isidori (LNF)**

K_{l3} decays, V_{us} and CKM unitarity

At present, most precise test of CKM unitarity is from 1st row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta$$

- $0^+ \to 0^+$ β decays: $2|V_{ud}|dV_{ud} \sim 0.0005$
- K_{l3} decays: $2|V_{us}|dV_{us} \sim 0.0010$

→ **2002** (2004 PDG)	Old Kl3 data give Δ = 0.0031(15): 2σ unitarity violation
2003	BNL 865 measures higher BR for K^+e3: V_{us} consistent with unitarity
2004-2006	KTeV, KLOE, ISTRA+, NA48 measure BRs, lifetimes, form factor slopes:

- Much higher statistics
- Radiative corrections carefully taken into account
- Proper reporting of correlations between measurements

2005 CKM WG1 report	Δ = 0.0002(14): unitarity to better than 1σ

Includes many (but not all) of the last developments

This talk	**Update with all recent measurements (even if preliminary)**

V_{us} from K_{l3} decay rates

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{Kl}(\lambda) (1+2\Delta_K^{SU(2)}+2\Delta_{Kl}^{EM})$$

with $K = K^+, K^0$; $l = e, \mu$ and C_K^2 = 1/2 for K^+, 1 for K^0

Inputs from theory:

S_{EW} Universal short distance EW correction (1.0232)

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element at zero momentum transfer (t=0)

$\Delta_K^{SU(2)}$ Form factor correction for strong SU(2) breaking

Δ_{Kl}^{EM} Long distance EM effects

Inputs from experiment:

$\Gamma(K_{l3(\gamma)})$ **Branching ratios** with well determined treatment of radiative decays; **lifetimes**

$I_{Kl}(\lambda)$ Phase space integral: λs parameterize form factor dependence on *t* :

K_{e3} : only λ_+ (or λ_+' λ_+'')

$K_{\mu3}$: need λ_+ and λ_0

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: $K^{\pm}$

3) Form factor slopes and phase space

4) Extraction of $|V_{us}|\,f_{+}(0)$ (and V_{us})

K_L branching ratios from KTeV

5 ratios of main BRs from independent samples of 10^5-10^6 events collected with a single trigger

KTeV, PRD 70 (2004)

2-track ratios

BR($K_{\mu3}$)/ BR(K_{e3}) = 0.6640(26)

BR($\pi^+\pi^-\pi^0$)/ BR(K_{e3}) = 0.3078(18)

BR($\pi^+\pi^-$)/ BR(K_{e3}) = 0.004856(28)

Neutral ratio

BR($2\pi^0$)/BR($3\pi^0$) = 0.004446(25)

Mixed ratio

BR($3\pi^0$)/ BR(K_{e3}) = 0.4782(55)

- 6 decays = 99.93% of K_L width, KTeV combines ratios to extract BRs
- Fit to BRs uses the 5 ratios (correlations available)

K_L branching ratios from NA48

1) K_L beam, 2-track sample, 80×10^6 events (6×10^6 signal)

NA48, PLB 602 (2004)

BR(K_{e3})/ BR(2 track) = 0.4978(35)

- NA48 evaluates BR(K_{e3}) from BR(2 track) = 1.0048-BR$(3\pi^0)_{\mathrm{KTeV}}$
- The measured ratio used in fit to BRs

2) Measurement of $\Gamma(K_L \to 3\pi^0)/\ \Gamma(K_S \to 2\pi^0)$ with same number of K_L and K_S produced on target, $2\text{-}6\times10^5$ signal events; use $\Gamma(K_S \to 2\pi^0)$ to extract $\Gamma(K_L \to 3\pi^0)$

NA48, 2004 preliminary

BR($K_L \to 3\pi^0$)/ τ_L = 3.795(58) MHz

K_L branching ratios and lifetime from KLOE

1) Absolute BRs: K_L decays tagged by $K_S \to \pi^+\pi^-$

13×10^6 tagged, 10^5 -10^6 signal

KLOE, PLB 632 (2006)

$BR^{(0)}(K_{e3})$ = 0.4049(21)

$BR^{(0)}(K_{\mu3})$ = 0.2726(16)

$BR^{(0)}(3\pi^0)$ = 0.2018(24)

$BR^{(0)}(\pi^+\pi^-\pi^0)$ = 0.1276(15)

at $\tau_L^{(0)}$ = 51.54 ns, with
dBR/BR = 0.67 $d\tau_L/\tau_L$
(geometrical acceptance)

Correlations available

KLOE results: set ΣBR(i)=1 and solve for τ_L
Fit to BRs: use unconstrained BRs with dependence on τ_L

2) Lifetime: measurement from an independent sample of 15×10^6 $K_L \to \pi^0\pi^0\pi^0$ events

uniform reconstruction eff. over $0.4\tau_L$

KLOE, PLB 626 (2005)

τ_L = 50.92(30) ns

Fit to K_L BR and lifetime measurements

18 input measurements:

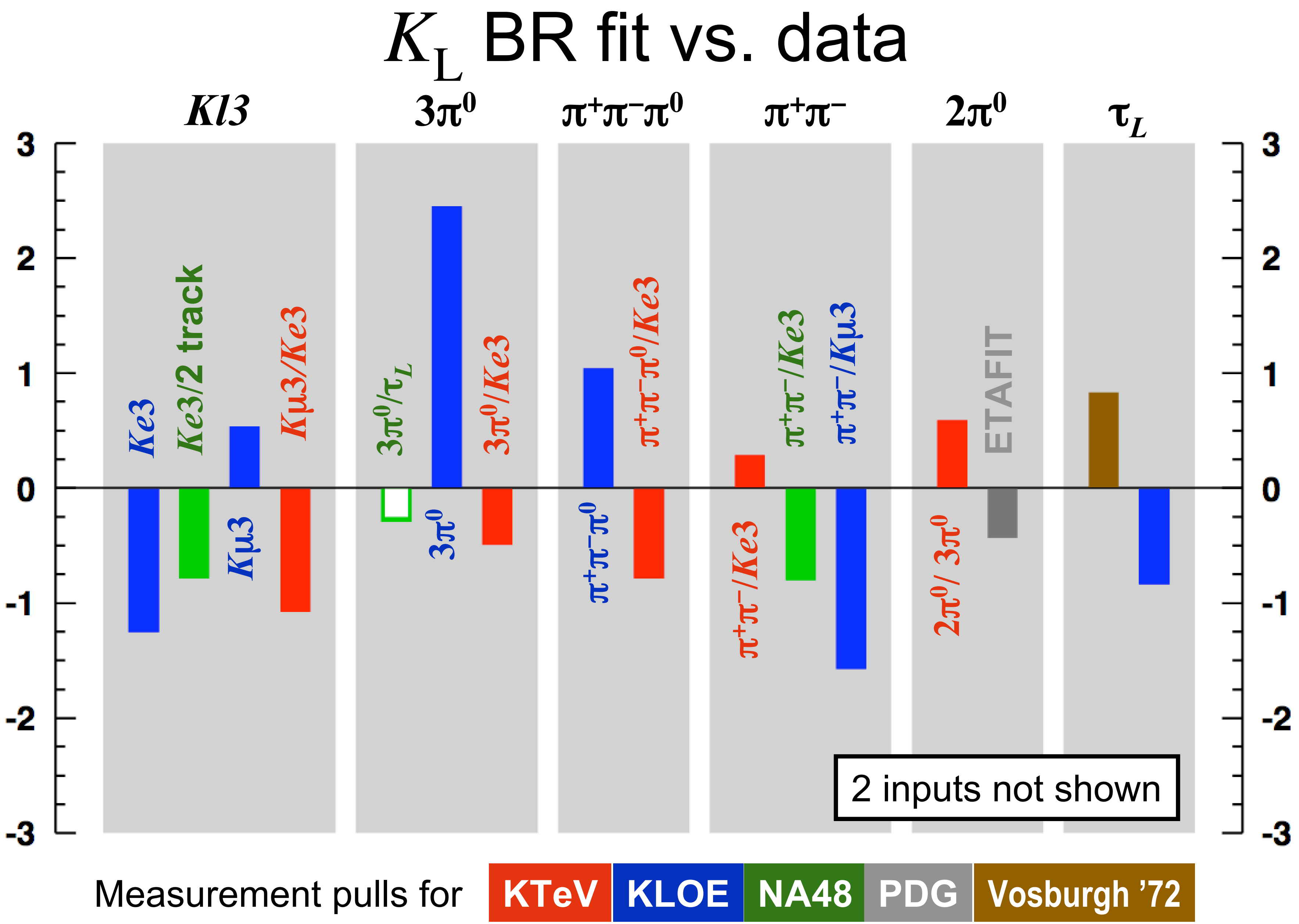
- **5 KTeV** ratios
- **NA48** K_{e3} /2t and $\Gamma(3\pi^0)$
- **4 KLOE** BRs
- **KLOE**, **NA48** $\pi^+\pi^-/K_{l3}$
- **KLOE**, **NA48** $\gamma\gamma/3\pi^0$
- **PDG** ETAFIT for $\pi^+\pi^-/\pi^0\pi^0$
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh** '72 τ_L

1 constraint: ΣBR=1

FlaviaNet fit

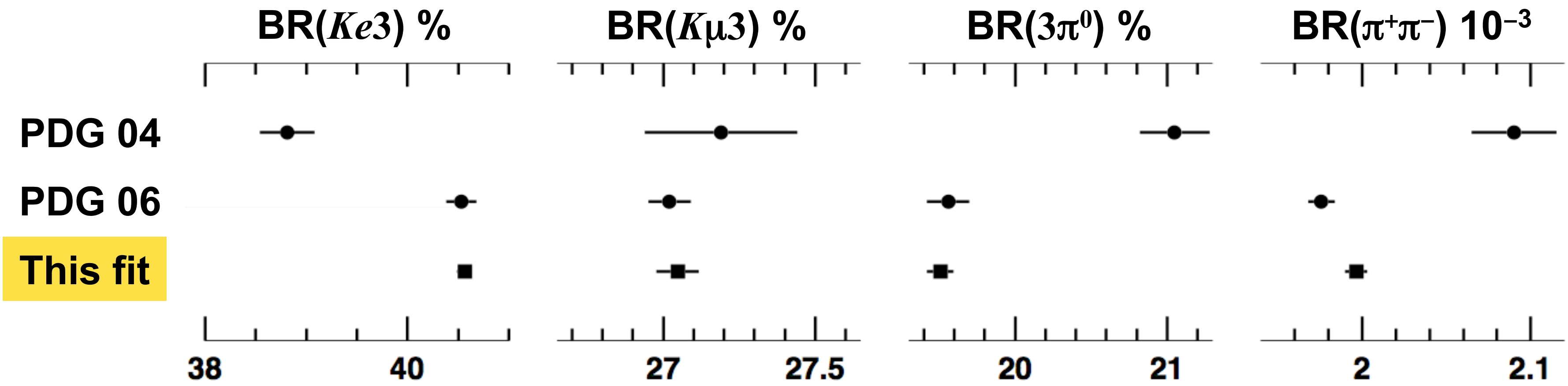
Parameter	Value	S
BR($Ke3$)	0.40563(74)	1.1
BR($K\mu3$)	0.27047(71)	1.1
BR($3\pi^0$)	0.19507(86)	1.2
BR($\pi^+\pi^-\pi^0$)	0.12542(57)	1.1
BR($\pi^+\pi^-$)	$1.9966(67)\times10^{-3}$	1.1
BR($2\pi^0$)	$8.644(42)\times10^{-4}$	1.3
BR($\gamma\gamma$)	$5.470(40)\times10^{-4}$	1.1
τ_L	51.173(200) ns	1.1

χ^2/ **ndf = 20.2/11** (Prob = 4.3%)



KL BR fit vs. data
Kl3
3π0
π+π−π0
π+π−
2π0
τL
Ke3
Ke3/2 track
Kμ3
Kμ3/Ke3
3π0/τL
3π0
3π0/Ke3
π+π−π0
π+π−π0/Ke3
π+π−/Ke3
π+π−/Ke3
π+π−/Kμ3
2π0/ 3π0
ETAFIT
2 inputs not shown
3
2
1
0
-1
-2
-3
Measurement pulls for
KTeV
KLOE
NA48
PDG
Vosburgh '72

Evolution of K_L BRs



Differences between *FlaviaNet* and PDG '06 are minor

Since PDG '04:

- Proper radiative corrections, especially for *Ke3*
- Exclusion of old measurements

5% decrease of $R_{\mu e}$: from 0.701(9) to 0.6668(24)

BR($K_S \to \pi e \nu$) and K_S lifetime

1) BRs from KLOE tagged K_S beam, 1.2×10^8 events

20% of full data sample

KLOE **PLB 632 (2006)**	**BR($K_S \to \pi e \nu$)/BR($K_S \to \pi^+\pi^-$) = 10.19(13) × 10^{-4}**
KLOE **EPJC 48 (2006)**	**BR($K_S \to \pi^+\pi^-$) /BR($K_S \to \pi^0\pi^0$) = 2.2459(54)** Averaged with KLOE '02

These two measurements completely determine main K_S BRs

$$\mathrm{BR}(K_S \to \pi e \nu) = 7.046(91) \times 10^{-4}$$

2) Lifetime from fit to CP parameters, does not assume CPT

PDG '06

$$\tau_S = 0.08958(5)\ \mathrm{ns}$$

Constrained by **NA48 '02** and **KTeV '03** values

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: $K^{\pm}$

3) Form factor slopes and phase space

4) Extraction of $|V_{us}| f_+(0)$ (and V_{us})

New results on $K^{\pm}_{l3}$ BRs

1) NA48 measures $BR(K^{\pm}_{l3})/BR(\pi^{\pm}\pi^{0})$ using K^{+} K^{-} simultaneous beams: 30k K^{-}_{l3}, 50k K^{+}_{l3}, statistics dominated

NA48/2 **EPJC 50 (2007)**	$BR(K^{\pm}_{e3})/BR(\pi^{\pm}\pi^{0}) = 0.2470(9)(4)$ $BR(K^{\pm}_{\mu3})/BR(\pi^{\pm}\pi^{0}) = 0.1637(6)(3)$

updated this conference

2) ISTRA+ measures $BR(K^{-}_{e3})/BR(\pi^{-}\pi^{0})$: 2.2×10^{6} K^{-}_{e3}, systematics dominated

ISTRA+ **arXiv: 0704.2052**	$BR(K^{-}_{e3})/BR(\pi^{-}\pi^{0}) = 0.2449(4)(14)$

3) KLOE measures absolute $BR(K^{\pm}_{e3})$ and $BR(K^{\pm}_{\mu3})$, tagging with $K^{\pm}\rightarrow\mu^{\pm}\nu$ and $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}$: 8 measurements in total, each with 10^{5}

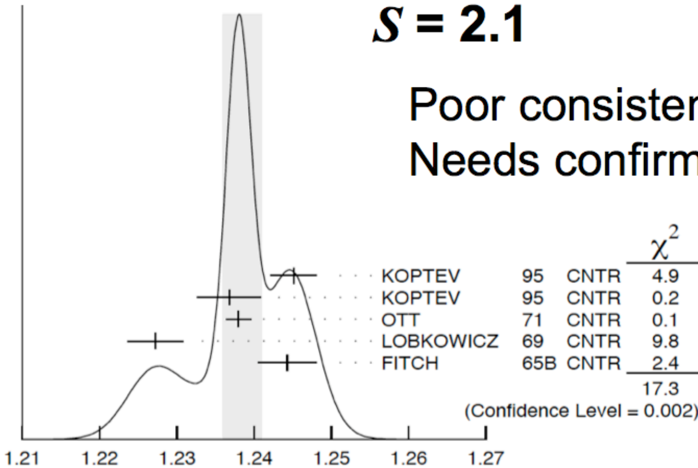
KLOE **updated** **preliminary**	$BR^{(0)}(K^{\pm}_{e3}) = 4.965(52)\%$ $BR^{(0)}(K^{\pm}_{\mu3}) = 3.233(39)\%$	at $\tau_{\pm}^{(0)} = 12.385$ ns, with $dBR/BR = -0.5 d\tau_{\pm}/\tau_{\pm}$

$K^{\pm}$ lifetime from KLOE

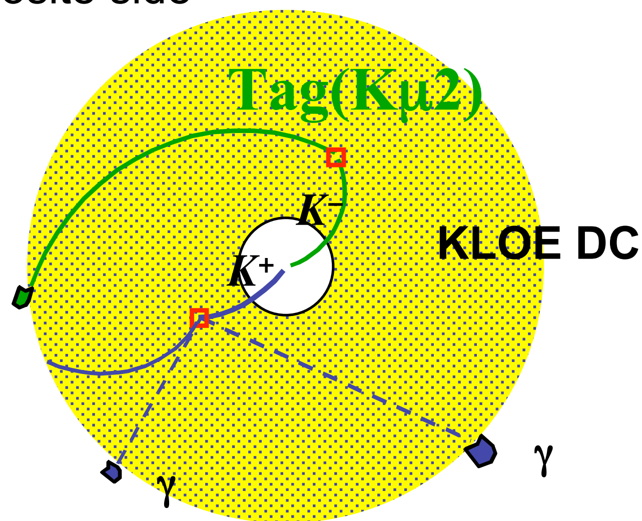
PDG average

12.385(25) ns
S = 2.1

Poor consistency
Needs confirmation



- Tag events with $K^{\pm} \to \mu\nu$ decay
- Identify a kaon decay on the opposite side



KLOE preliminary, 2 different methods:

$\tau_{\pm}$ from the K decay length, using tagged vertices in DC

$\tau_{\pm}$= 12.367(44)(65) ns

$\tau_{\pm}$ from the K decay time, using γ from $K^{\pm} \to \pi^{\pm}\pi^0$ decays

$\tau_{\pm}$= 12.391(49)(25) ns

Combined result (ρ = 0.34): **$\tau_{\pm}$= 12.384(48) ns**

Fit to $K^{\pm}$ BR and lifetime measurements

Not possible to fit only new $K^{\pm}$ data (unlike K_L)

Only $Kl3$, $Kl3/\pi\pi^0$ and $K\mu2$ have been measured recently

- $Kl3$ and $\pi\pi^0$ highly correlated in fit (-0.64, -0.79 for $Ke3$, $K\mu3$)
- New measurement of $\pi\pi^0$ is crucial

For channels like $\pi\pi^0$ and $\pi^+\pi^+\pi^-$, fit rests heavily on Chiang '72

- No radiative corrections
- 6 BRs constrained by ΣBR = 1, correlations unavailable

Compared to PDG '06, *FlaviaNet* fit:

- Uses new results from NA48/2, ISTRA+ and KLOE
- Does not use BR($\pi^0\pi^0 e\nu$) as a free parameter

Fit results to $K^{\pm}$ BR and lifetime

31 input measurements:

- **5 older τ values in PDG**
- **2 KLOE τ**
- **KLOE BR($\mu\nu$)**
- **KLOE $Ke3$, $K\mu3$ BRs**
- **ISTRA+ $K_{e3}/\pi\pi^0$**
- **NA48/2 $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 K_{e3}/Kdal**
- **6 Chiang '72 BRs**
- **3 old $\pi\pi^0/\mu\nu$**
- **2 old $Ke3$/2 body**
- **3 $K\mu3/Ke3$ (2 old)**
- **2 old + 1 KLOE results on 3π**

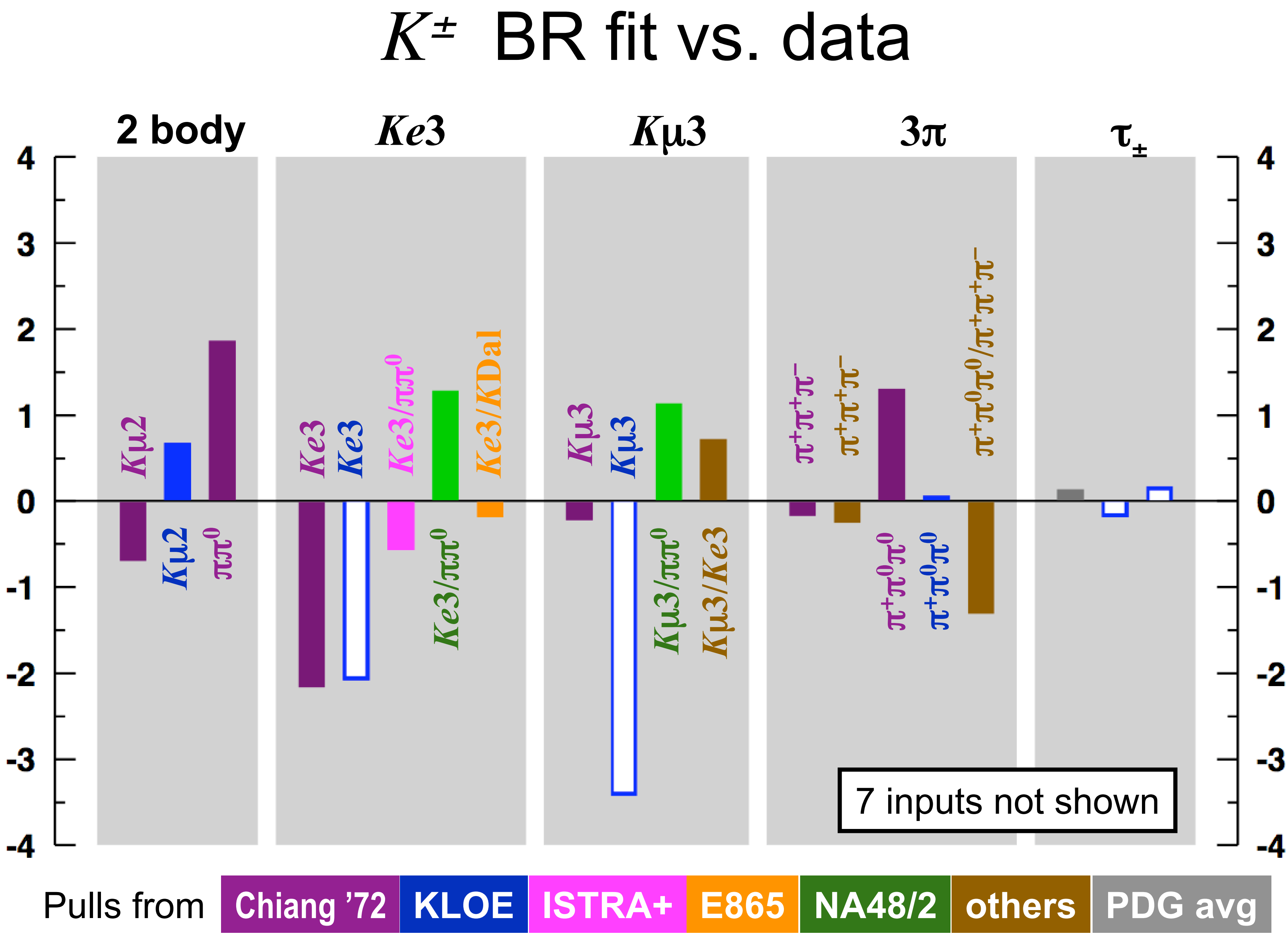
1 constraint: ΣBR=1

FlaviaNet fit

Parameter	Value	S
BR($\mu\nu$)	63.545(132)%	1.2
BR($\pi\pi^0$)	20.656(100)%	1.3
BR($\pi\pi\pi$)	5.5962(303)%	
BR($Ke3$)	5.0758(290)%	1.3
BR($K\mu3$)	3.3656(280)%	1.7
BR($\pi\pi^0\pi^0$)	1.7614(226)%	1.1
$\tau_{\pm}$	12.3840(193) ns	1.7

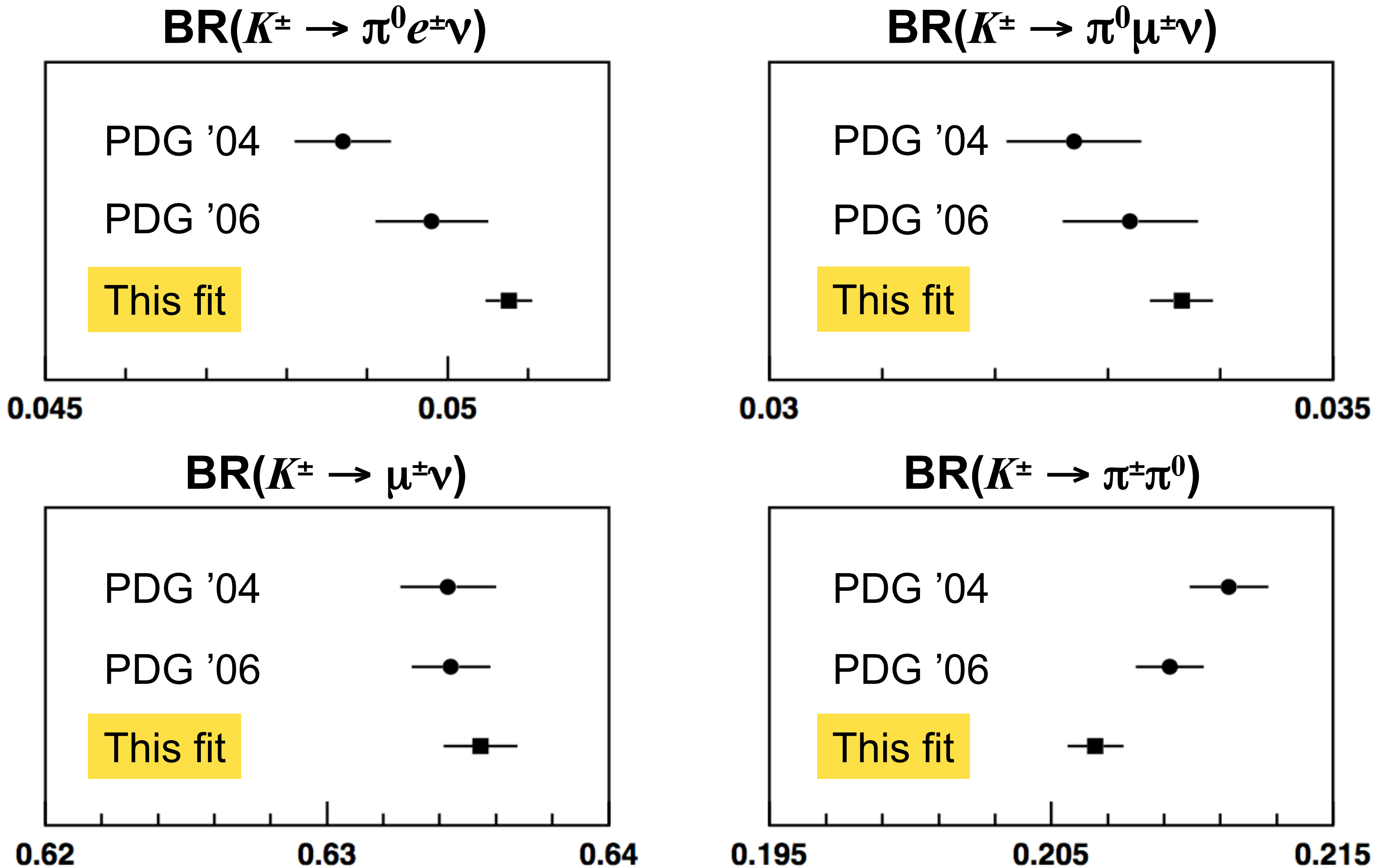
χ^2/ ndf = 52/25 (Prob = 0.11%)

Improves to χ^2/ ndf = 35/21 (2.7%)
with no changes to central values or errors, if 5 older $\tau_{\pm}$ measurements replaced by PDG avg (with S=2.1)



K± BR fit vs. data
2 body
Ke3
Kμ3
3π
τ±
Kμ2
Kμ2
ππ0
Ke3
Ke3
Ke3/ππ0
Ke3/ππ0
Ke3/KDal
Kμ3
Kμ3
Kμ3/ππ0
Kμ3/Ke3
π+π+π−
π+π+π−
π+π0π0
π+π0π0
π+π0π0/π+π+π−
7 inputs not shown
Pulls from
Chiang '72
KLOE
ISTRA+
E865
NA48/2
others
PDG avg

Evolution of $K^{\pm}$ BRs



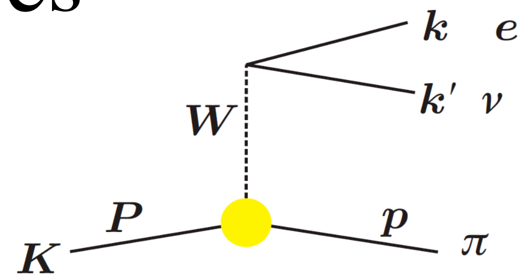
1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: $K^{\pm}$

3) Form factor slopes and phase space

4) Extraction of $|V_{\mathrm{us}}| f_+(0)$ (and V_{us})

K_{l3} form-factor slopes



Hadronic matrix element:

$$\langle\pi|\, J_\alpha\, |K\rangle = f(0) \times [\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha]$$

$f_-(t)$ term only important for $K_{\mu3}$.

For $K_{\mu3}$, use $f_+(t)$ and $\quad f_0(t) = f_+(t) + \dfrac{t}{m_K^2 - m_{\pi+}^2} f_-(t)$

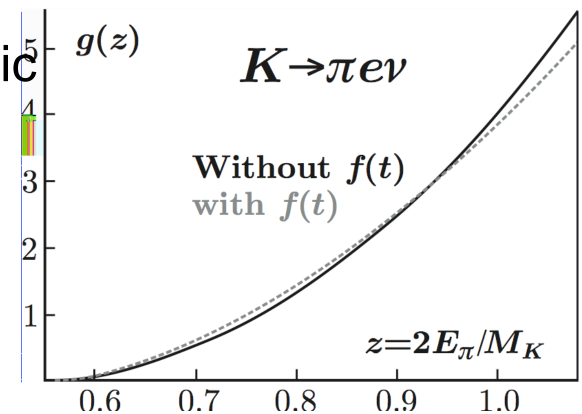
For V_{us}, need integral over phase space of squared matrix element

Expand form factor:

Linear: $\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0}\,[t/m_{\pi+}^2]$

Quadratic: $\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\,[t/m_{\pi+}^2] + 1/2\,\lambda''_{+,0}\,[t/m_{\pi+}^2]^2$

Fits to t-distribution give poor sensitivity to quadratic terms



Polar fit:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

$$\lambda' = (m_{\pi+}/M)^2$$

$$\lambda'' = 2\lambda'^2$$

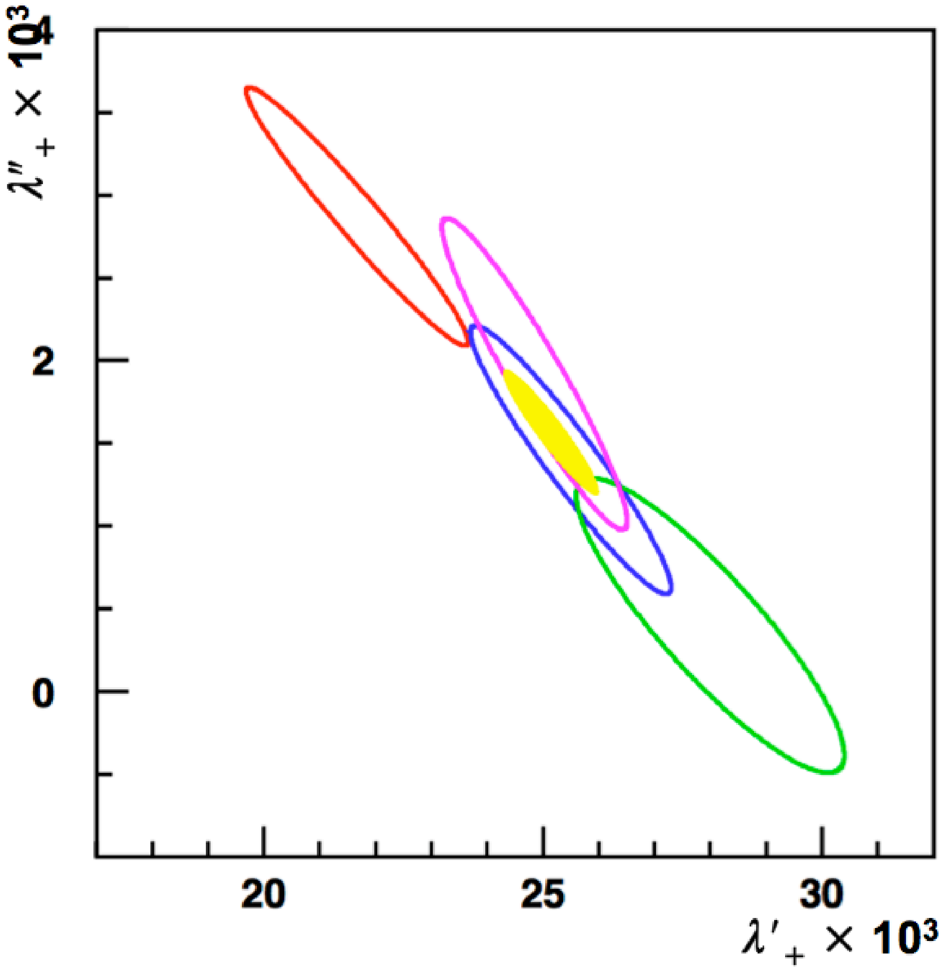
Current data on *Ke3* form-factor slopes

	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	analysis
ISTRA+ **PLB 589 (2004)**	24.9 ± 1.7	1.9 ± 0.9	0.9×10^6 $K^- e3$ $\rho(E_e, E_\pi)$
KTeV **PRD 70 (2004)**	21.7 ± 2.0	2.9 ± 0.8	1.9×10^6 $K_L e3$ $t^\pi_\perp$
NA48 **PLB 604 (2004)**	28.0 ± 2.4	0.4 ± 0.9	5.6×10^6 $K_L e3$ $\rho(t_{low}, t_{high})$
KLOE **PLB 632 (2006)**	25.5 ± 1.8	1.4 ± 0.8	2.0×10^6 $K_L e3$ t from K_S

* ISTRA+ results are rescaled by $(m_{\pi+}/m_{\pi0})^2$

Ke3 slopes: quadratic fits

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** *FlaviaNet* fit



Slope parameters × 10^3

$\lambda'_+ = 25.15 \pm 0.87$

$\lambda''_+ = 1.57 \pm 0.38$

$\rho(\lambda'_+, \lambda''_+) = -0.941$

χ^2/ ndf = 5.3/6 (51%)

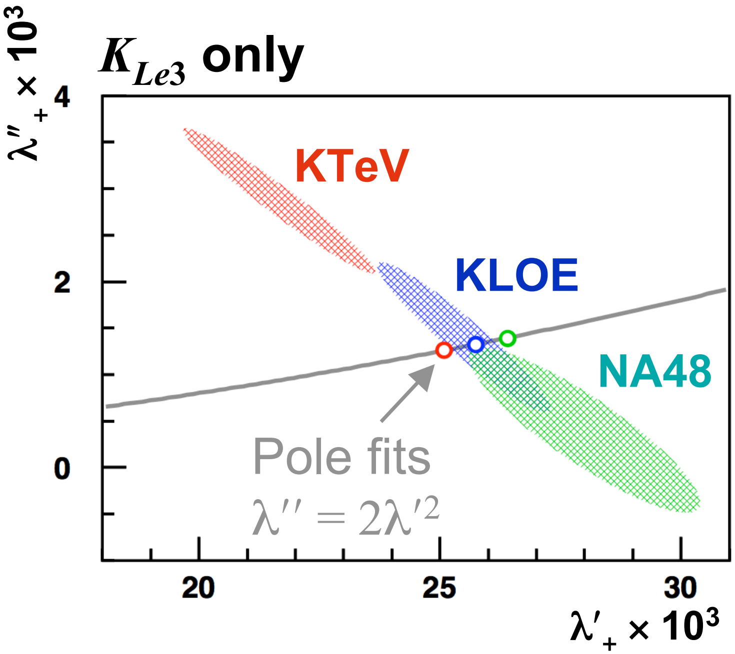
Excellent compatibility

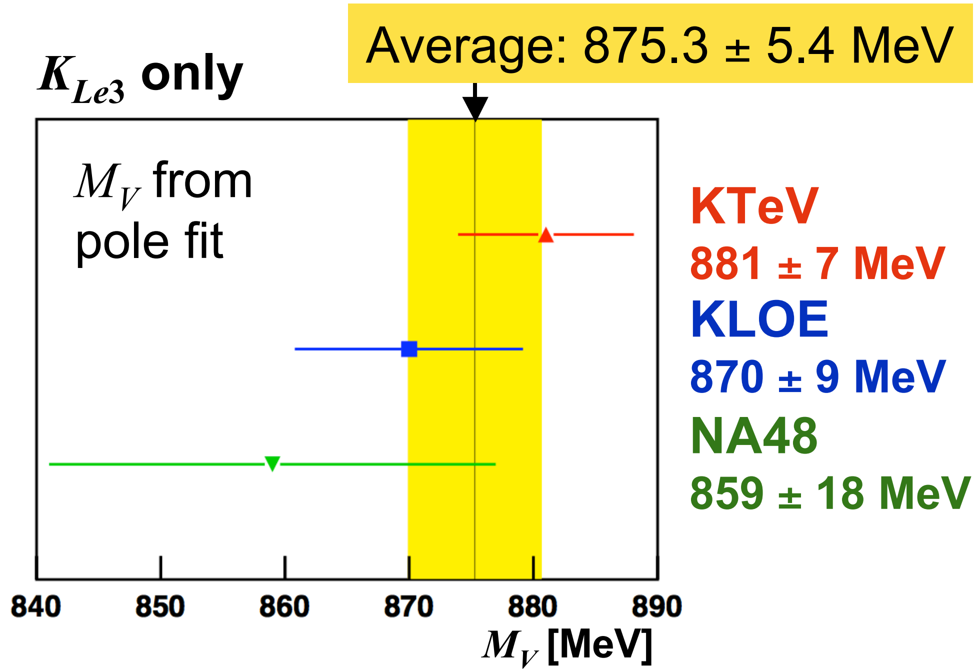
Significance of $\lambda''_+ \sim 4\sigma$

$I(K^0e3)$= 0.15465(21)

$I(K^+e3)$= 0.15901(22)

K_{e3} slopes: Quadratic vs. pole fits





K_{Le3} data	$P(\chi^2)$ fit Quad	K_{Le3} integral Quad	$P(\chi^2)$ fit Pole	K_{Le3} integral Pole	Difference
KTeV	54%	0.15378(51)	43%	0.15449(25)	**+0.46%**
KLOE	92%	0.15472(42)	92%	0.15489(33)	**+0.11%**
K_{Le3} avg (no ISTRA+)		0.15456(29)		0.15469(19)	**+0.09%**

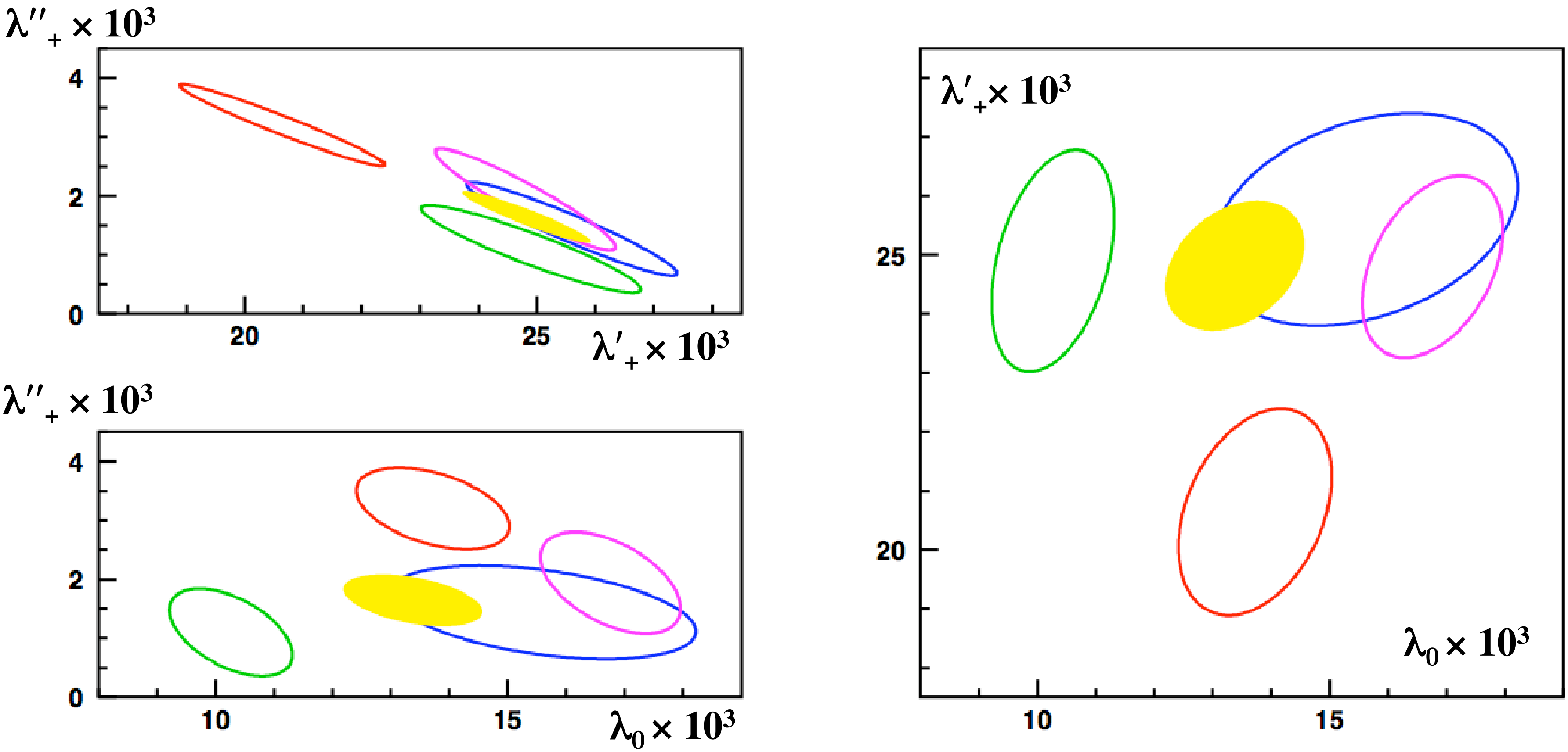
Current data on $K\mu3$ form-factor slopes

	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$	analysis
ISTRA+ **PLB 581 (2004)**	**23.0 ± 6.4**	**2.3 ± 2.3**	**17.1 ± 2.2**	0.9×10^6 $K^-\mu3$ $\rho(E_\mu,E_\pi)$
KTeV **PRD 70 (2004)**	**17.0 ± 3.7**	**4.4 ± 1.5**	**12.8 ± 1.8**	1.5×10^6 $K_L\mu3$ $\rho(t^\mu_\perp,M_{\pi\mu})$
NA48 **PLB 647 (2007)**	**20.5 ± 3.3**	**2.6 ± 1.4**	**9.5 ± 1.4**	2.3×10^6 $K_L\mu3$ $\rho(E_\mu,E_\pi)_{low}$
KLOE **preliminary**	**25.6 ± 1.8**	**1.4 ± 0.8**	**15 6± 2.6**	1.8×10^6 $K_L\mu3$ E_ν spectrum + K_Le3

* ISTRA+ results are rescaled by $(m_{\pi+}/m_{\pi0})^2$

Fit to *Kl3* form factor slopes

Slopes from KTeV KLOE ISTRA+ NA48 *FlaviaNet* fit



$$\chi^2/\ \text{ndf} = 53/13\ (\text{Prob} = 10^{-6})$$

Kl3 form factor slopes: fit results

Although compatibility poor, no a priori reason to exclude any result

Inconsistency parameterized by scale factors for fit results

Slopes × 10^3:

λ'_+ = **24.82 ± 1.10** **S = 1.4**

λ''_+ = **1.64 ± 0.44** **S = 1.3**

λ_0 = **13.38 ± 1.19** **S = 1.9**

χ^2/ ndf = 53/13 (10^{-6})

Correlation coefficients:

	λ'_+	λ_0
λ''_+	**–0.95**	**–0.43**
λ'_+		**+0.32**

Phase space integrals	
$I(K^0e3)$	**0.15454(29)**
$I(K^+e3)$	**0.15889(30)**
$I(K^0\mu3)$	**0.10209(31)**
$I(K^+\mu3)$	**0.10504(32)**

These results used to evaluate $|V_{us}|\, f_+(0)$ for all modes

Can we rely on our determination of $I(K_{l3})$?

1) Previous fit, all experiments

Slope parameters × 10^3			
λ'_+	=	24.82 ± 1.10	S=1.4
λ''_+	=	1.64 ± 0.44	S=1.3
$\boldsymbol{\lambda_0}$	**=**	$\mathbf{13.38 \pm 1.19}$	**S=1.9**
χ^2/ndf	=	53/13 (10^{-6})	

Integrals	
$I(K^0e3)$	0.15454(29)
$I(K^+e3)$	0.15889(30)
$I(K^0\mu3)$	0.10209(31)
$I(K^+\mu3)$	0.10504(32)

2) All experiments but NA48 $K_{\mu3}$

Slope parameters × 10^3		
λ'_+	=	24.95 ± 0.83
λ''_+	=	1.59 ± 0.36
$\boldsymbol{\lambda_0}$	**=**	$\mathbf{16.01 \pm 0.79}$
χ^2/ndf	=	12.2/10 (27.1%)

Integrals	
$I(K^0e3)$	0.15457(21)
$I(K^+e3)$	0.15892(21)
$I(K^0\mu3)$	0.10268(20)
$I(K^+\mu3)$	0.10565(21)

$\Delta[I(K\mu3)] = 0.6\%$! … but $Ke3$+$K\mu3$ average gives $\Delta[V_{us}f_+(0)] = -0.08\%$

Can we observe λ_0''?

Determination of $f_+(0)$ with ChPT calculations at $O(p^6)$, would benefit from the measurement of scalar form factor curvature (Bijnens and Talavera, 2003):

$$2C_{12}^r + C_{34}^r \sim \lambda_0' \qquad C_{12}^r \sim \lambda_0''$$

[C_{ij}^r = LEC's at order $O(p^6)$]

Unfortunately, for $K\mu3$ there are two FF and 4 parameters → high correlations → big errors. If we compute errors on λ_0'' from the (no experimental resolution) we get:

$$\rho(\lambda_0', \lambda_0'') = -0.999$$

$d\lambda_0'' \sim 20/\sqrt{N}$	$d\lambda_0'' \sim 2.5/\sqrt{N}$
Fit to λ_+', λ_+'', λ_0', λ_0''	Fit to λ_0', λ_0'' (λ_+', λ_+'' fixed)
impossible	**25M** events for $d\lambda_0'' = 0.5\times10^{-3}$

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: $K^{\pm}$

3) Form factor slopes and phase space

4) Extraction of $|V_{\mathrm{us}}|\, f_+(0)$ (and V_{us})

SU(2) and EM corrections

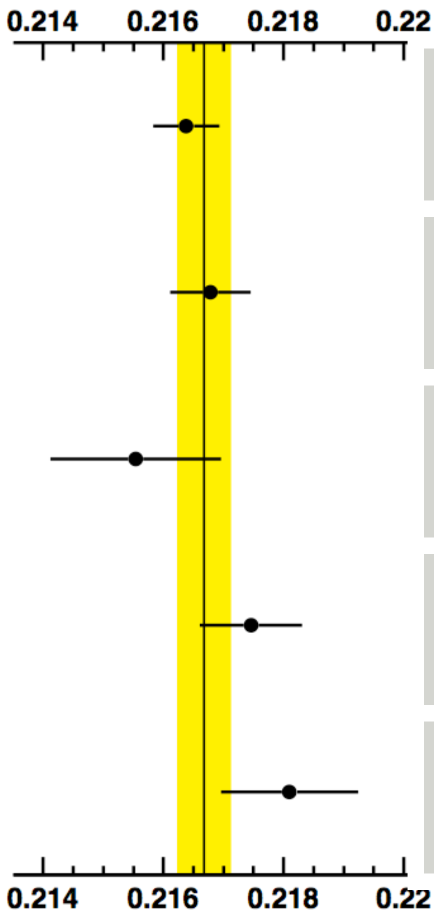
	$\Delta^{SU(2)}$	Δ^{EM}		
	ChPT Cirigliano et al. 02	***ChPT*** Cirigliano et al.02	***ChPT*** Neufeld	***hadr. model*** Andre *04*
K^0e3	0	+0.52(10)%	+0.57(15)%	+0.65(15)%
$K^0\mu3$	0		+0.80(15)%	+0.95(15)%
K^+e3	+2.31(22)%	+0.03(10)%	+0.08(15)%	
$K^+\mu3$	+2.31(22)%		–0.12(15)%	

values used to extract $|V_{us}|f_+(0)$

- Δ^{EM} for full phase space: all measurements assumed fully inclusive
- Different estimates of Δ^{EM} agree within the quoted errors
- First estimate of $\Delta^{EM}(K^+\mu3)$ from Neufeld removes dominant contribution to the error on $|V_{us}|\, f_+(0)$ for this mode

$|V_{us}|f_+(0)$ from K_{l3} data

$|V_{us}|\, f_+(0)$



Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	0.21638(55)	0.25	0.09	0.19	0.10	0.10
$K_L \mu 3$	0.21678(67)	0.31	0.10	0.18	0.15	0.15
$K_S e3$	0.21554(142)	0.66	0.65	0.03	0.10	0.10
$K^{\pm} e3$	0.21746(85)	0.39	0.29	0.09	0.24	0.09
$K^{\pm} \mu 3$	0.21810(114)	0.52	0.42	0.09	0.26	0.15

Average: $|V_{us}|\, f_+(0)$ = 0.21668(45) **χ^2/ndf = 2.74/4 (60%)**

$|V_{us}|f_+(0)$: $K^\pm$ vs $K_{L,S}$

Fit 5 modes with separate values of $|V_{us}|\,f_+(0)$ for $K^\pm$ and $K_{L,S}$ modes

- Using results of overall fit to form-factor slopes
- With $SU(2)$ corrections for $K^\pm$ modes [$\Delta^{SU(2)}{}_{\text{th}}$ = 2.31(22)%]

$K^\pm$ modes		$K_{L,S}$ modes
$\|V_{us}\|\,f_+(0)$ = 0.21756(84)	⟷	$\|V_{us}\|\,f_+(0)$ = 0.21641(50)

1.25σ difference

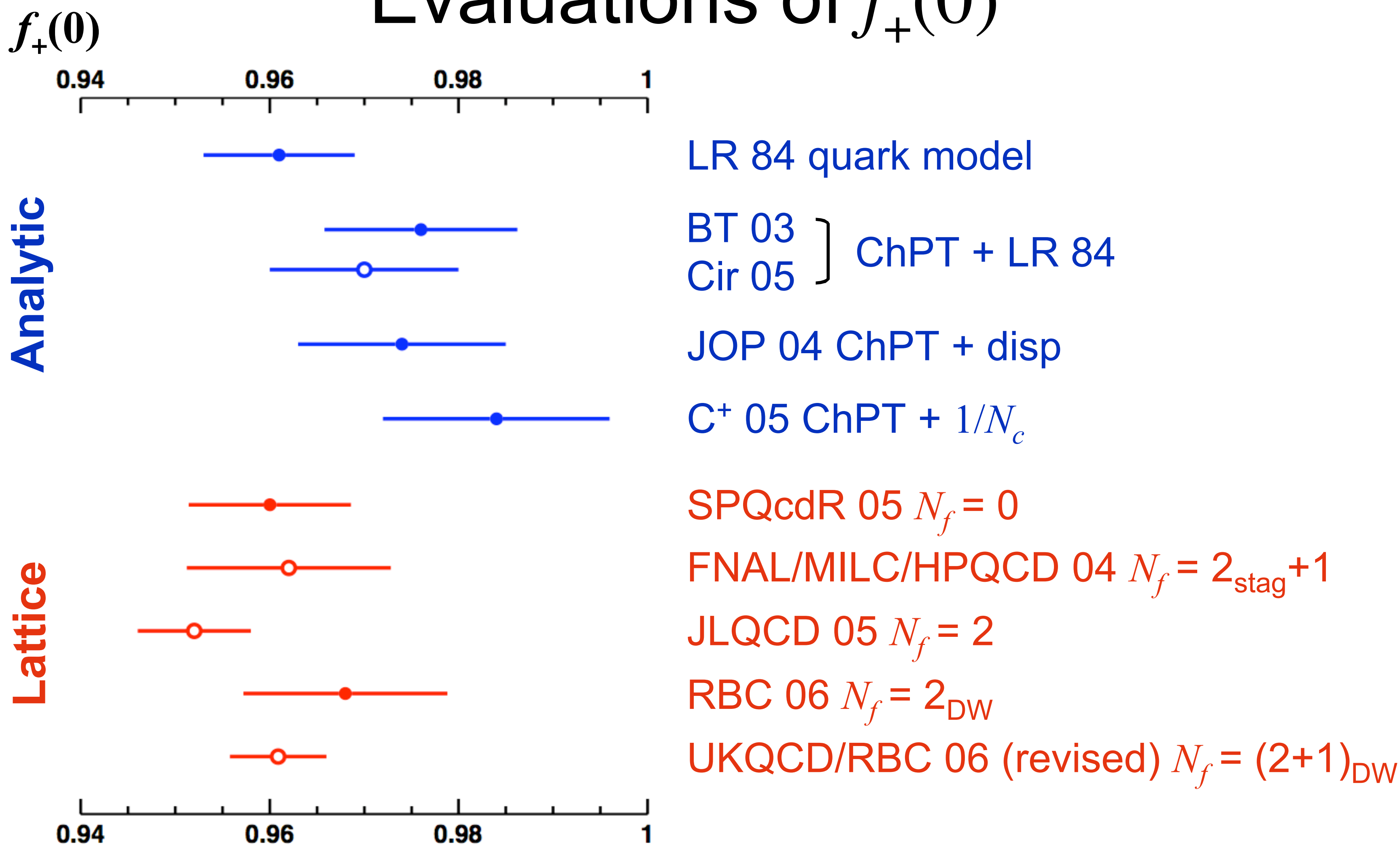
χ^2/ndf = 1.18/3 (76%) ρ = 0.11

When fit performed without $SU(2)$ corrections for $K^\pm$ modes, obtain an experimental value for $\Delta^{SU(2)}$

$K^\pm$ modes, no $SU(2)$
$|V_{us}|\,f_+(0)$ = 0.22259(71)

$\Delta^{SU(2)}{}_{\text{exp}}$ = 2.86(38)%

Evaluations of $f_+(0)$



Leutwyler & Roos estimate (LR 84) still widely used: $f_+(0)$ **= 0.961(8)**

Lattice evaluations generally agree well with this value

V_{us} from K_{l3} data and CKM unitarity

K_{l3} average: $|V_{us}|\,f_+(0)$ = 0.21668(45)

-0.1% respect to CKM06 and PDG06

Leutwyler & Roos '84

$f_+(0)$ = 0.961(8)

Conventional choice for value of $f_+(0)$ until a definitive evaluation becomes available

K_{l3} average: $|V_{us}|$ = 0.2255(19)

Marciano & Sirlin '06

$|V_{ud}|$ = 0.97377(27)

Average from $0^+ \to 0^+$ β decays with recent evaluation of EW radiative corrections

$V_{ud}^2 + V_{us}^2 - 1 = -0.0009(10)$

Compatibility with unitarity -0.9σ

-1σ if V_{ud} updated to 0.97372

V_{us}/V_{ud} from BR($K^\pm \to \mu^\pm \nu$)

Marciano '04

$$\frac{\Gamma(K^\pm \to \mu^\pm \nu(\gamma))}{\Gamma(\pi^\pm \to \mu^\pm \nu(\gamma))} = \frac{|V_{us}|^2\, f_K^2\, m_K\, (1 - m_\mu^2/m_K^2)^2}{|V_{ud}|^2\, f_\pi^2\, m_\pi\, (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Uncertainty from SD virtual corrections

MILC '06 preliminary

f_K/f_π **= 1.208(2)($^{+7}_{-14}$)**

$N_f = (2+1)_{stag}$

Cancellation of lattice-scale uncertainties

KLOE
PLB 636 (2006)

BR($K^+ \to \mu^+\nu(\gamma)$) = 0.6366(17)

Uses $K^- \to \mu^-\nu$ to tag 2-body K decays

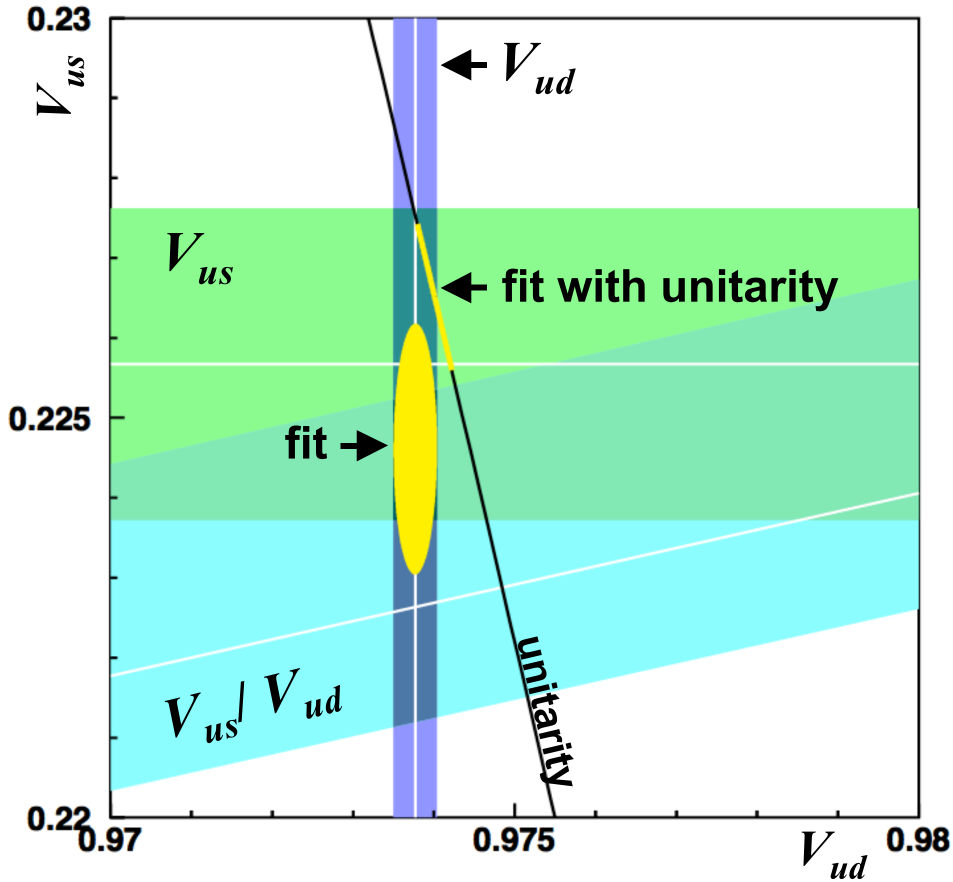
Counts $K^+ \to \mu^+\nu$ from decay-momentum spectrum

Use KLOE BR($K^+ \to \mu^+\nu(\gamma)$) instead of value from BR/lifetime fit:
Error slightly larger, but radiative contribution under better control

V_{us}/V_{ud} **= 0.2286($^{+27}_{-15}$)**

V_{ud}, V_{us} & V_{us}/V_{ud}

$f_+(0)$ from LR 84
$|V_{us}|$ = 0.2255(19) from $Kl3$



Fit results, no constraint:

V_{ud} = 0.97377(27)
V_{us} = 0.2245(16)
χ^2/ndf = 0.75/1 (39%)

Fit results, unitarity constraint:

V_{ud} = 0.97403(22)
V_{us} = 0.2264(9)
χ^2/ndf = 3.13/2 (21%)

Agreement with unitarity at 1.3σ

Summary and outlook

$Kl3$ average: $|V_{us}|\, f_+(0)$ = 0.21668(45)

Prob(χ^2) for fits:

$|V_{us}|\, f_+(0)$ 5-mode avg: 60% (good!)

K_L BR,τ fit:	**4.3%**	(acceptable)
$K^\pm$ BR,τ fit:	**0.11% → 2.7%**	(marginal)
Form-factor slopes:	**10^{-6}**	(terrible!)

- New $K^\pm$ rate measurements being finalized; still missing $\pi\pi^0$ mode
- K^0 vs $K^\pm$ modes fit within the calculated $\Delta SU(2)$ correction
- Experimental uncertainty on $|V_{us}|\, f_+(0)$ at 0.2% level
- Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$
- With $f_+(0)$ = 0.961(8), first-row unitarity test satisfied at ~1σ level

Additional information

$K_L \to \pi^+\pi^-$

New measurements of $K_L \to \pi^+\pi^-(\gamma)$ also useful in global fit

KTeV
PRD 70 (2004)

$BR(\pi^+\pi^-/Ke3) = 4.856(29) \times 10^{-3}$
1 of 5 ratios in K_L BR analysis
Contribution from direct emission (DE) negligible

KLOE
PLB 638 (2006)

$BR(\pi^+\pi^-/K\mu3) = 7.275(68) \times 10^{-3}$
Fully inclusive of DE component

NA48
PLB 645 (2007)

$BR(\pi^+\pi^-/Ke3) = 4.826(27) \times 10^{-3}$
Residual DE contribution of 0.19% subtracted

For consistency and to better satisfy Σ BR = 1 in global fit,
DE contribution of **1.52(7)%*** added to **KTeV** and **NA48** results

* From E731 ’93, KTeV ’01 and KTeV ’06 $K_L \to \pi^+\pi^-\gamma$ results

Fit to K_L BRs, τ: Comparison to PDG

Parameter	This fit Value	This fit S	PDG 2006 Value	PDG 2006 S
$Ke3$	0.40563(74)	1.1	0.4053(15)	2.1
$K\mu3$	0.27047(71)	1.1	0.2702(7)	
$3\pi^0$	0.19507(86)	1.2	0.1956(14)	1.9
$\pi^+\pi^-\pi^0$	0.12542(57)	1.1	0.1256(5)	
$\pi^+\pi^-$	$1.9966(67) \times 10^{-3}$	1.1	$1.976(8) \times 10^{-3}$	
$2\pi^0$	$8.644(42) \times 10^{-4}$	1.3	$8.69(4) \times 10^{-4}$	1.1
$\gamma\gamma$	$5.470(40) \times 10^{-4}$	1.1	$5.48(5) \times 10^{-4}$	1.2
τ	51.173(200) ns	1.1	51.14(21) ns	
	18 measurements		17 measurements	
	χ^2/ndf = 20.2/11 (4.3%)		χ^2/ndf = 14.8/10 (14.0%)	

Fit to $K^{\pm}$ BRs, τ: Comparison to PDG

Parameter	This fit Value	*S*	PDG 2006 Value	*S*
$\mu\nu$	63.545(132)%	1.2	63.44(14)%	1.2
$\pi\pi^0$	20.656(100)%	1.3	20.92(12)%	1.1
$\pi\pi\pi$	5.5962(303)%		5.590(31)%	1.1
$Ke3$	5.0758(290)%	1.3	4.98(7)%	1.3
$K\mu3$	3.3656(280)%	1.7	3.32(6)%	1.2
$\pi\pi^0\pi^0$	1.7614(226)%	1.1	1.757(24)%	1.1
$\pi^0\pi^0 e\nu$	Not in fit		$2.2(4) \times 10^{-5}$	
τ	12.3840(193) ns	1.7	12.385(24) ns	2.1
	31 measurements		26 measurements	
	χ^2/ndf = 52/25 (0.11%)		χ^2/ndf = 30.0/19 (5.2%)	

Beyond quadratic and pole fits

Hill, PRD 74 (2006):

Power series expansion based on analyticity of form factors
Constraints from crossing symmetry, e.g., bounds on $f_+(t)$ from $\tau \rightarrow K\pi\nu$ data
Rigorous estimate of error from truncation of series expansion

KTeV
PRD 74 (2006)

Refit $Ke3$ data using Hill parameterization
$I(K^0e3)$ = **0.15392(48)$_{\rm exp}$(6)$_{\rm th}$**

Bernard et al., PLB 638 (2006):

Dispersion relation for ln $f_0(t)$ subtracted at t = 0 and $t = m_K{}^2 - m_\pi{}^2$, giving:

$$f_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}\left(\ln C - G(t)\right)\right]$$

$G(t)$ evaluated using $K\pi$ scattering data and given as a polynomial

1 fit parameter: ln C = ln $f_0(m_K{}^2 - m_\pi{}^2)$
Value of ln C a test for right-handed quark currents

NA48
PLB 647 (2007)

From new analysis of $K\mu3$ form-factor slopes
ln C = 0.1438(138)

$|V_{us}| f_+(0)$: $K_{L,S}$ vs. $K^\pm$

Using separate fit results for form-factor slopes:

$K^\pm$ only, $e3$ and $\mu 3$:

ISTRA+ $e3$, $\mu 3$

λ'_+	=	24.80 ± 1.54
λ''_+	=	1.94 ± 0.86
λ_0	=	16.76 ± 1.20
χ^2/ndf	=	0.100/2 (95%)
$I(Ke3)$	=	0.15910(32)
$I(K\mu 3)$	=	0.10595(30)

K_L only, $e3$ and $\mu 3$:

KTeV avg, KLOE avg, NA48 $e3$, $\mu 3$

λ'_+	=	23.73 ± 1.74 (S=1.7)
λ''_+	=	2.07 ± 0.65 (S=1.6)
λ_0	=	11.49 ± 1.41 (S=1.8)
χ^2/ndf	=	30.4/8 (0.02%)
$I(Ke3)$	=	0.15428(48)
$I(K\mu 3)$	=	0.10156(43)

With $SU(2)$ corrections for $K^\pm$ modes:

$|V_{us}| f_+(0)$ = **0.21729(84)**
χ^2/ndf = 0.026/1 (87%)

$|V_{us}| f_+(0)$ = **0.21670(57)**
χ^2/ndf = 2.01/2 (37%)

With NO $SU(2)$ corrections for $K^\pm$ modes:

$|V_{us}| f_+(0)$ = **0.22231(71)**

$\Delta^{SU(2)}_{\text{exp}}$ = **2.59(41)%**

Results without NA48 $K\mu3$ slopes

Slope parameters × 10^3		
λ'_+	=	**24.95 ± 0.83**
λ''_+	=	**1.59 ± 0.36**
λ_0	=	**16.01 ± 0.79**
χ^2/ndf	=	12.2/10 (27.1%)

Integrals	
$I(K^0e3)$	0.15457(21)
$I(K^+e3)$	0.15892(21)
$I(K^0\mu3)$	0.10268(20)
$I(K^+\mu3)$	0.10565(21)

$\lvert V_{us}\rvert f_+(0)$	
K_Le3	**0.21636(53)**
$K_L\mu3$	**0.21616(61)**
K_Se3	**0.21552(142)**
K^+e3	**0.21744(84)**
$K^-\mu3$	**0.21748(111)**

Kl3 average
$\lvert V_{us}\rvert f_+(0)$ = 0.21651(42)
χ^2/ndf = 2.19/4 (70%)

$K^+ - K^0$ diff.: 1.3σ
$\Delta^{SU(2)}_{\mathrm{exp}}$ = 2.89(38)%

$r_{\mu e}$ = 0.9990(46)

$\lvert V_{us}\rvert$ = 0.2253(19)
Unitarity test: –1.0σ

Lepton universality with *Kl3* data

For each state of kaon charge, we evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\text{ obs}}}{[|V_{us}|\,f_+(0)]^2_{e3,\text{ obs}}} = \frac{(G_F^{\mu})^2}{(G_F^{e})^2}$$

$K^{\pm}$ modes

$r_{\mu e}$ = 1.0059(87)

$K_{L,S}$ modes

$r_{\mu e}$ = 1.0039(56)

Using 2004 BRs*
$r_{\mu e}$ = 1.019(13)

Using 2004 BRs*
$r_{\mu e}$ = 1.054(15)

Average
(incl. ρ = 0.12)
$r_{\mu e}$ = 1.0045(50)

Compare sensitivity from $\pi \to l\nu$ decays:

$(r_{e\mu})_{\pi l2}$ = 0.9966(30)
see Erler, Ramsey-Musolf '06

*Assuming current values for form-factor slopes and Δ^{EM}