RADIATIVE AND SEMILEPTONIC KAON DECAYS IN CHPT

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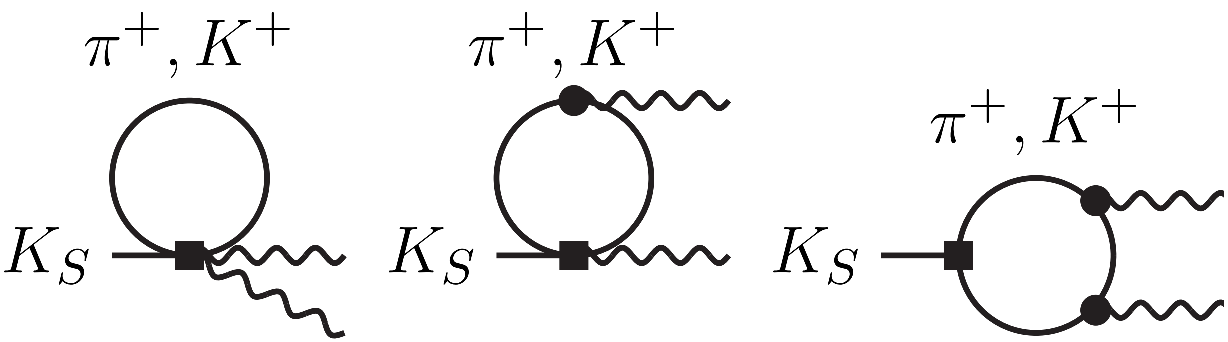
Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

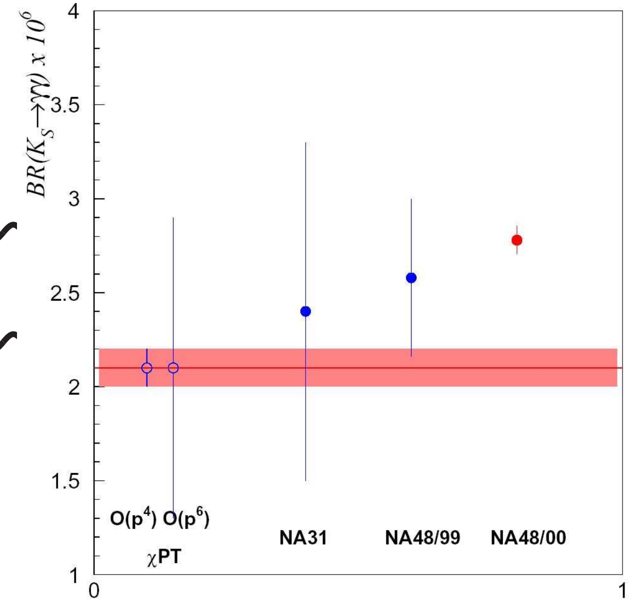
Overview

- A few comments about $K_{S,L} \to \gamma\gamma$ and $K_{S,L} \to \pi^0\gamma\gamma$
- Semileptonic Decays
- Radiative Semileptonic Decays
- Note: 2nd Eurodaphne report: hep-ph/9411311
- Note: tests of the anomaly including sign

$K_S \to \gamma\gamma$

Well predicted by CHPT at order p^4 from Goity, D'Ambrosio, Espriu





Prediction was: BR = $2.1 \cdot 10^{-6}$

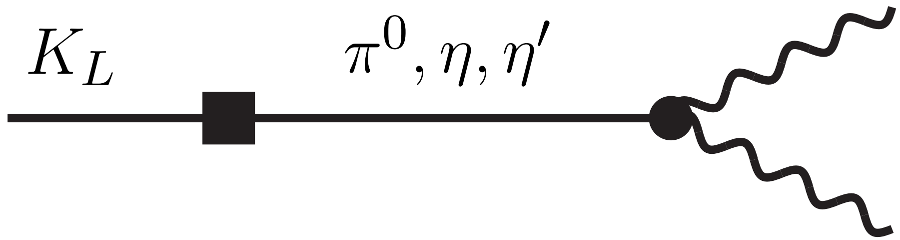
NA48: $2.78(6)(4) \cdot 10^{-6}$ (PLB 551 2003)

No full p^6 calculation exists, FSI effects estimated

Some other rare decays

- $K_L \to \gamma\gamma$
  Needs work: main contribution is full of cancellations:

  difficult



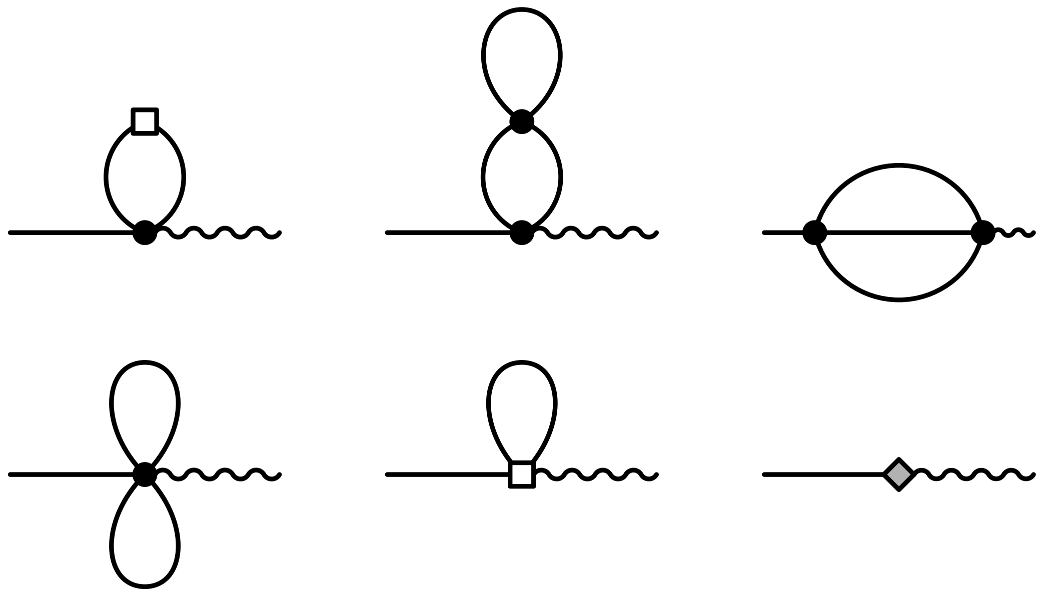
- $K_L \to \pi^0\gamma\gamma$
  $K_L \to \pi^0\gamma\gamma$ OK predicted by CHPT
  Main succes: events must be at high $m_{\gamma\gamma}$
  Rate still a problem
- $K_S \to \pi^0\gamma\gamma$
  Similar problems as in $K_L \to \gamma\gamma$
- Ecker, Pich, de Rafael, D'Ambrosio,...

Semileptonic Decays

- $K \to \ell\nu$: known to order p^6
- $K \to \pi\ell\nu$: known to order p^6, isospin breaking at p^6 preliminary
- $K \to \pi\pi\ell\nu$: F, G and H known to p^6, R only to p^4.
- $K \to \pi\pi\pi\ell\nu$: known to p^2

$K_{\ell 2}$

Input for determining F_K.



Diagrams:

Amoros,JB,Talavera NPB 2000

Main use: determining L_5^r

Typical convergence:

$$\frac{F_K}{F_\pi} = 1.22 = 11. + 0.162 + 0.058$$

$$\frac{F_\pi}{F_0} = 1 + 0.135 - 0.075$$

$K_{\ell 3}$

- H. Leutwyler and M. Roos, Z.Phys.C25:91,1984.
- J. Gasser and H. Leutwyler, Nucl.Phys.B250:517-538,1985.
- J. Bijnens and P. Talavera, hep-ph/0303103, Nucl. Phys. B669 (2003) 341-362
- V. Cirigliano et al., hep-ph/0110153, Eur.Phys.J.C23:121-133,2002.
- J. Bijnens and K. Ghorbani, to be published.

$K_{\ell 3}$ Definitions

$$K^+_{\ell 3}: \quad K^+(p) \rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$
$$K^0_{\ell 3}: \quad K^0(p) \rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$$K^+_{\ell 3}: \quad T = \frac{G_F}{\sqrt{2}} V^\star_{us} \ell^\mu F_\mu{}^+(p',p)$$
$$\ell^\mu = \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell)$$
$$F_\mu{}^+(p',p) = < \pi^0(p') \mid V_\mu^{4-i5}(0) \mid K^+(p) >$$
$$= \frac{1}{\sqrt{2}}[(p'+p)_\mu f_+^{K^+\pi^0}(t) + (p-p')_\mu f_-^{K^+\pi^0}(t)]$$

Isospin: $f_+^{K^0\pi^-}(t) = f_+^{K^+\pi^0}(t) = f_+(t)$

$$f_-^{K^0\pi^-}(t) = f_-^{K^+\pi^0}(t) = f_-(t)$$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

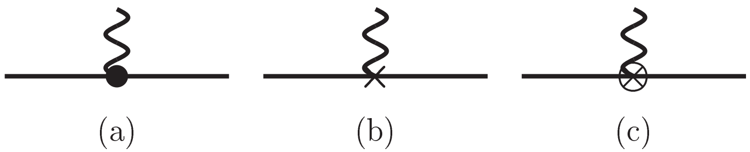
Old Usual parametrization:

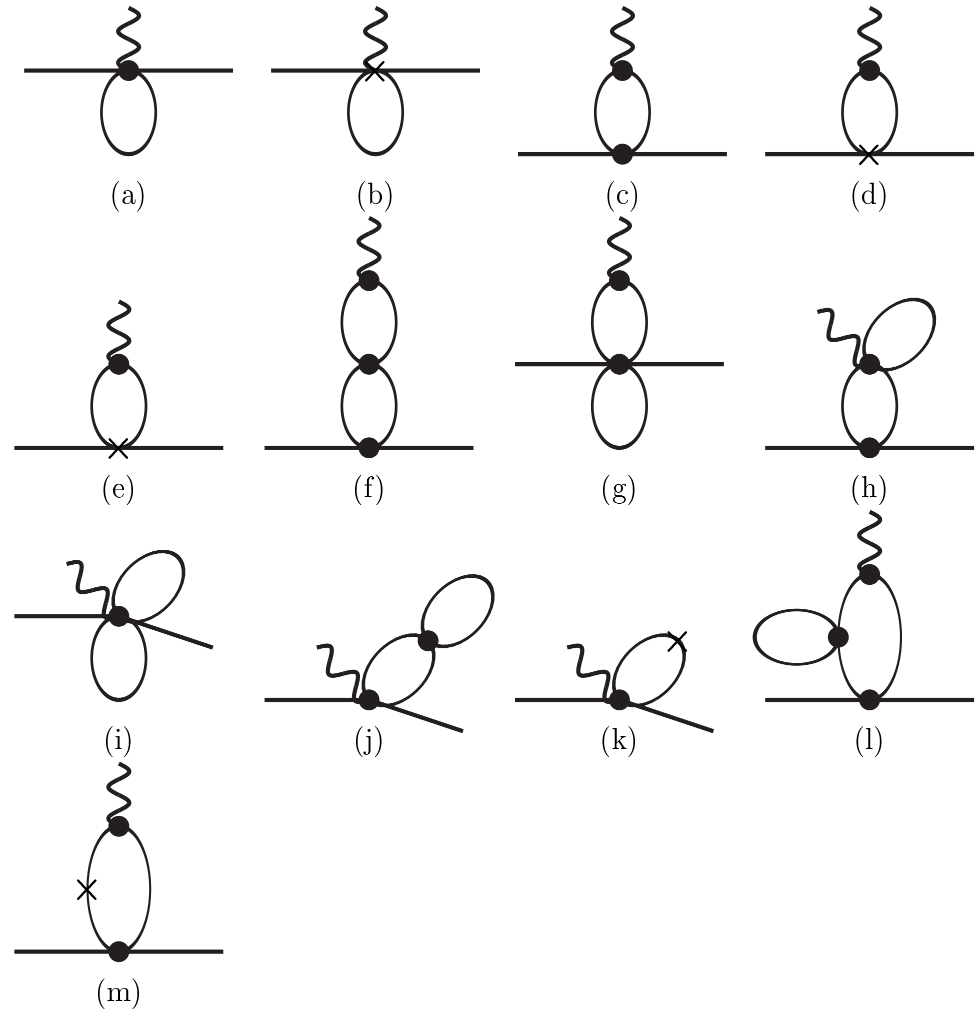
$$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

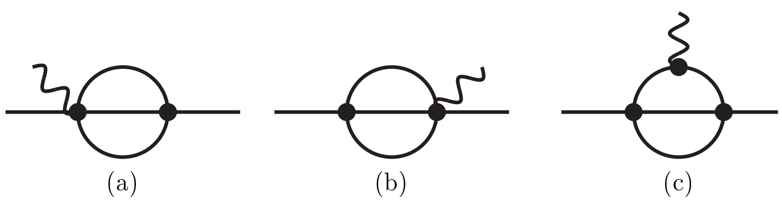
$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leutwyler-Roos: see later
  - Radiative corrections: Cirigliano et al
- Know experimentally $f_+(0)$

$K_{\ell 3}$ Diagrams







● : p^2 vertex

$\times$: p^4 vertex

$\otimes$: p^6 vertex

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

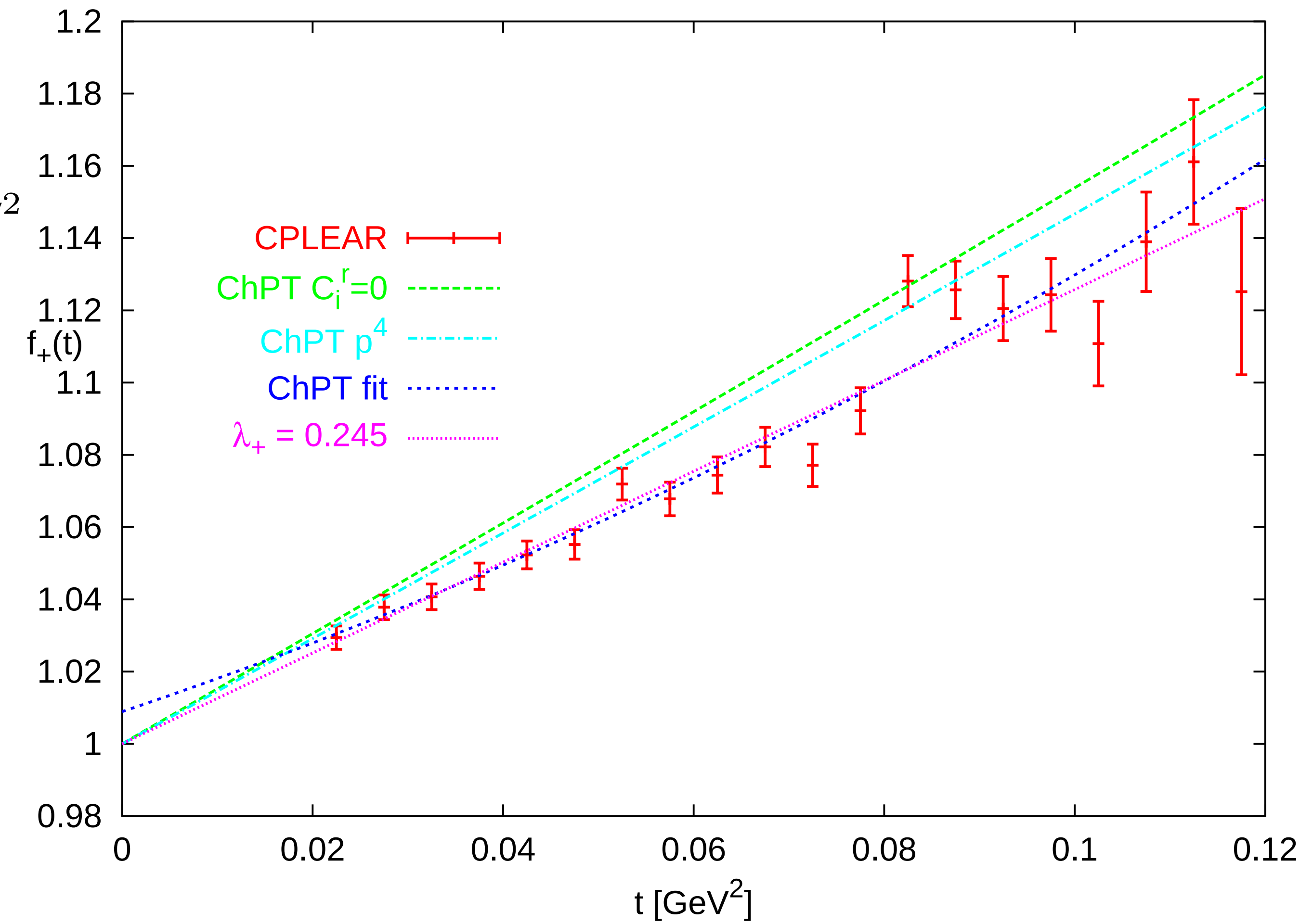
ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} = -(4.7 \pm 0.5)\ 10^{-5}$ GeV^2

$(c_+ = 3.2$ GeV$^{-4})$

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) &= 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
&\quad +8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
&\quad -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

Experiment

Form Factor comparison

KTeV [PRD 70(2004)]

K^0_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 4σ level

$K^0_{\mu 3}$ quadraqtic fit: $\lambda_0 = (13.72 \pm 1.31)\ 10^{-3}$

Slopes consistent for K^0_{e3} and $K^0_{\mu 3}$

ISTRA+ [PLB 581(2004), PLB589(2004)]

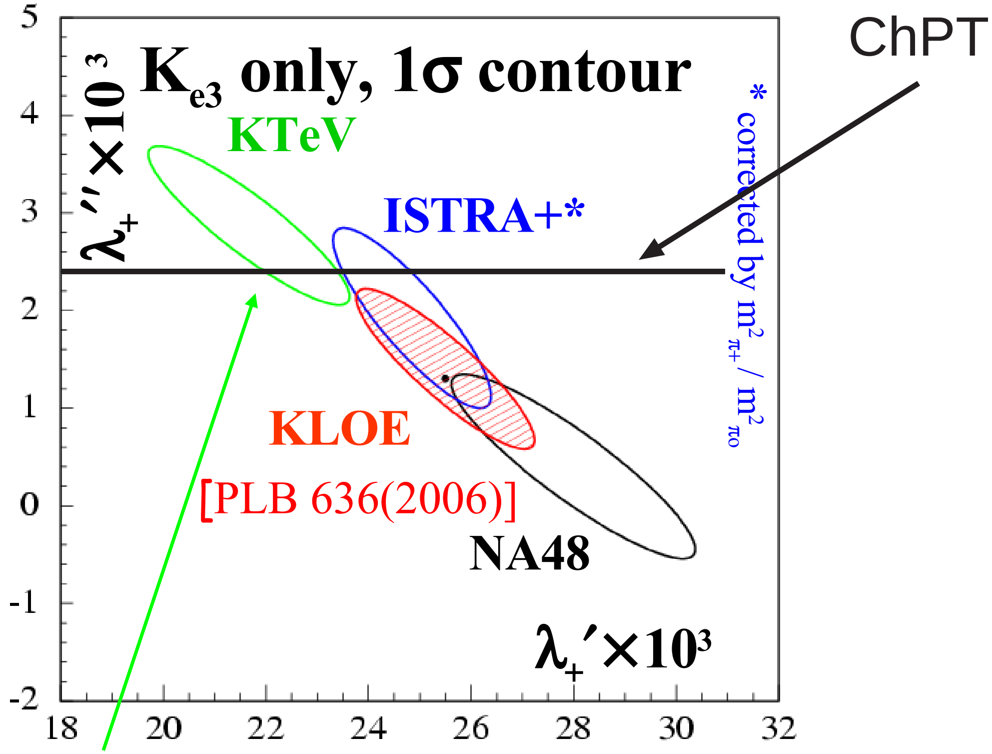
K^-_{e3} quadratic fit: $\lambda''_+ \neq 0$ @ 2σ level

$K^-_{\mu 3}$ quadratic fit: $\lambda_0 = (17.11 \pm 2.31)\ 10^{-3}$

NA48 [PLB 604(2004), HEP2005 289]

K^0_{e3}: No evidence for quadratic term

$K^0_{\mu 3}$ linear fit: $\lambda_0 = (12.0\ \pm 1.7)\ 10^{-3}$



$\lambda''_+ \to -1\%$ phase space integral

$\to +0.5\%$ for V_{us}

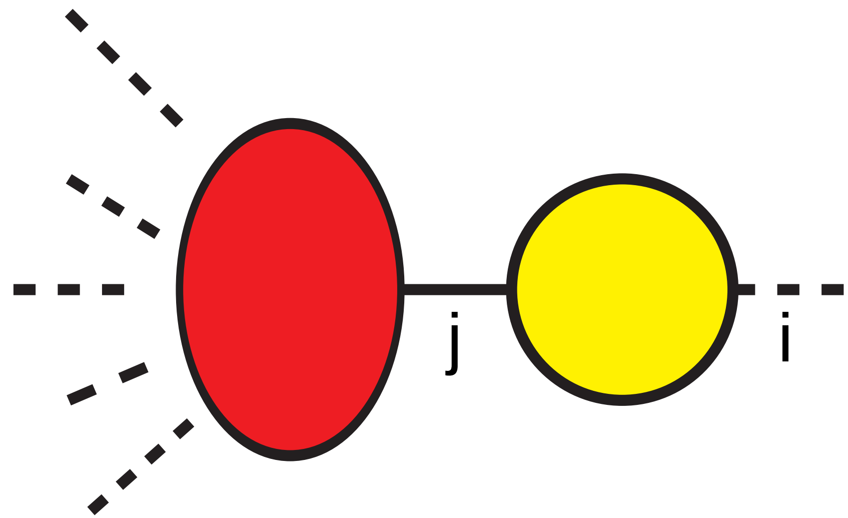
A WG on Precise SM tests in K decays M.Antonelli LNF-INFN – FLAVIANET meeting Barcelona

New for isospin breaking

Take LSZ into account properly

Amoros, JB, Talavera, 2001

Matrix element:



$$\mathcal{A}_{i_1\ldots i_n} = \left(\frac{(-i)^n}{\sqrt{Z_{i_1}\ldots Z_{i_n}}}\right) \prod_{i=1}^{n} \lim_{k_i^2\to m_i^2}(k_i^2-m_i^2)\, G_{i_1\ldots i_n}(k_1,\ldots,k_n)$$

Isospin Breaking

$$
\begin{aligned}
\mathcal{A}_3 &= \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
&+ \left[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left(Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \right. \\
&+ \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left(\frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
&\left. - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right]
\end{aligned}
$$

Isospin Breaking

$$
\begin{aligned}
\mathcal{A}_3 \;=\;& \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
& + \Bigg[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left(Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \\
& + \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left(\frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
& - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \Bigg]
\end{aligned}
$$

Compute all diagrams, produce numerical programs, . . .

Numerical results for $f_+(0)$

VERY PRELIMINARY

Decay	p^2	p^4	pure 2-loop	L_i^r at p^6	C_i^r	total
		Iso conserving calculation				
K^0	1	-0.02266	0.01130	0.00332	???	0.99196
K^+	1	-0.02276	0.01104	0.00320	???	0.99154
		$m_u/m_d = 0.45$				
K^0	1	-0.02310	0.01131	0.00325	???	0.99146
K^+	1.02465	-0.01741	0.00379	0.00648	???	1.01751
ratio	1.02465	1.0311				1.0262
		$m_u/m_d = 0.58$				
K^0	1	-0.02299	0.01124	0.00325	???	0.99150
K^+	1.01702	-0.01897	0.00657	0.00551	???	1.01013
ratio	1.0170	1.0215				1.0188

Some Comments

- K^0 only again on $C_{12}^r + C_{34}^r$
  Ademollo-Gatto + Callan-Treiman as in isoconserving case but $(m_s - m_u)^2$
- Not checked yet whether C_i^r in K^+ decay can be determined
- p^6 lowers the isospin breaking
- p^6 fit has $m_u/m_d = 0.45$ and not 0.58
- $0.58 \to 0.52$ from p^6 and $0.52 \to 0.45$ from violation of Dashen's theorem
- Very preliminary numerics: p^6 correction about -0.5% but with changed m_u/m_d, total effect $+0.5\%$ compared to Leutwyler-Roos

$K_{\ell 4}$

$$\begin{aligned} K^+(p) &\rightarrow \pi^+(p_+)\pi^-(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,, \\ K^+(p) &\rightarrow \pi^0(p_+)\pi^0(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,, \\ K^0(p) &\rightarrow \pi^-(p_+)\pi^0(p_-)\ell^+(p_\ell)\nu_\ell(p_\nu)\,. \end{aligned}$$

Kinematical variables for hadronic system: t, u, s_π, s_ℓ

$$T^{+-} = \frac{G_F}{\sqrt{2}} V_{us}^* \overline{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_\ell)(V^\mu - A^\mu)\,,$$

$$\begin{aligned} V_\mu &= -\frac{H}{m_K^3}\epsilon_{\mu\nu\rho\sigma}(p_\ell + p_\nu)^\nu(p_+ + p_-)^\rho(p_+ - p_-)^\sigma\,, \\ A_\mu &= -\frac{i}{m_K}\left[(p_+ + p_-)_\mu\, F + (p_+ - p_-)_\mu\, G + (p_\ell + p_\nu)_\mu\, R\right]\,. \end{aligned}$$

$K_{\ell 4}$

$$T^{+-} = \frac{T^{-0}}{\sqrt{2}} + T^{00}$$

T^{-0} is anti-symmetric under $t \leftrightarrow u$

T^{00} is symmetric.

Lowest order: Weinberg: $F = G = \dfrac{m_K}{\sqrt{2}F_\pi}$,

Order p^4: JB 1990, Riggenbach et al. 1991

Could fit data with reasonable corrections:
Determine $L_i^r\ i = 1, 2, 3$

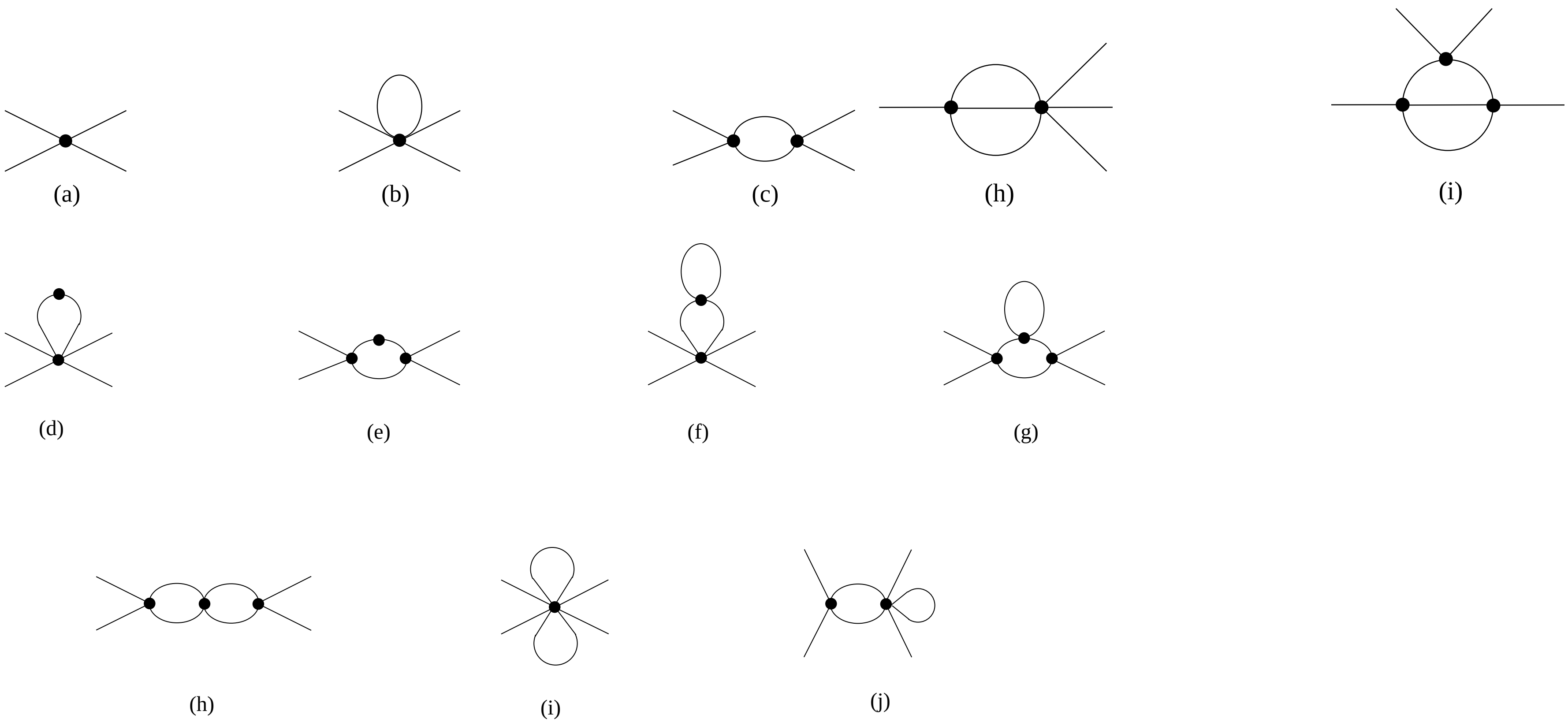
Dispersive estimate of p^6 corrections:
JB, Colangelo, Gasser 1994

$K_{\ell 4}$

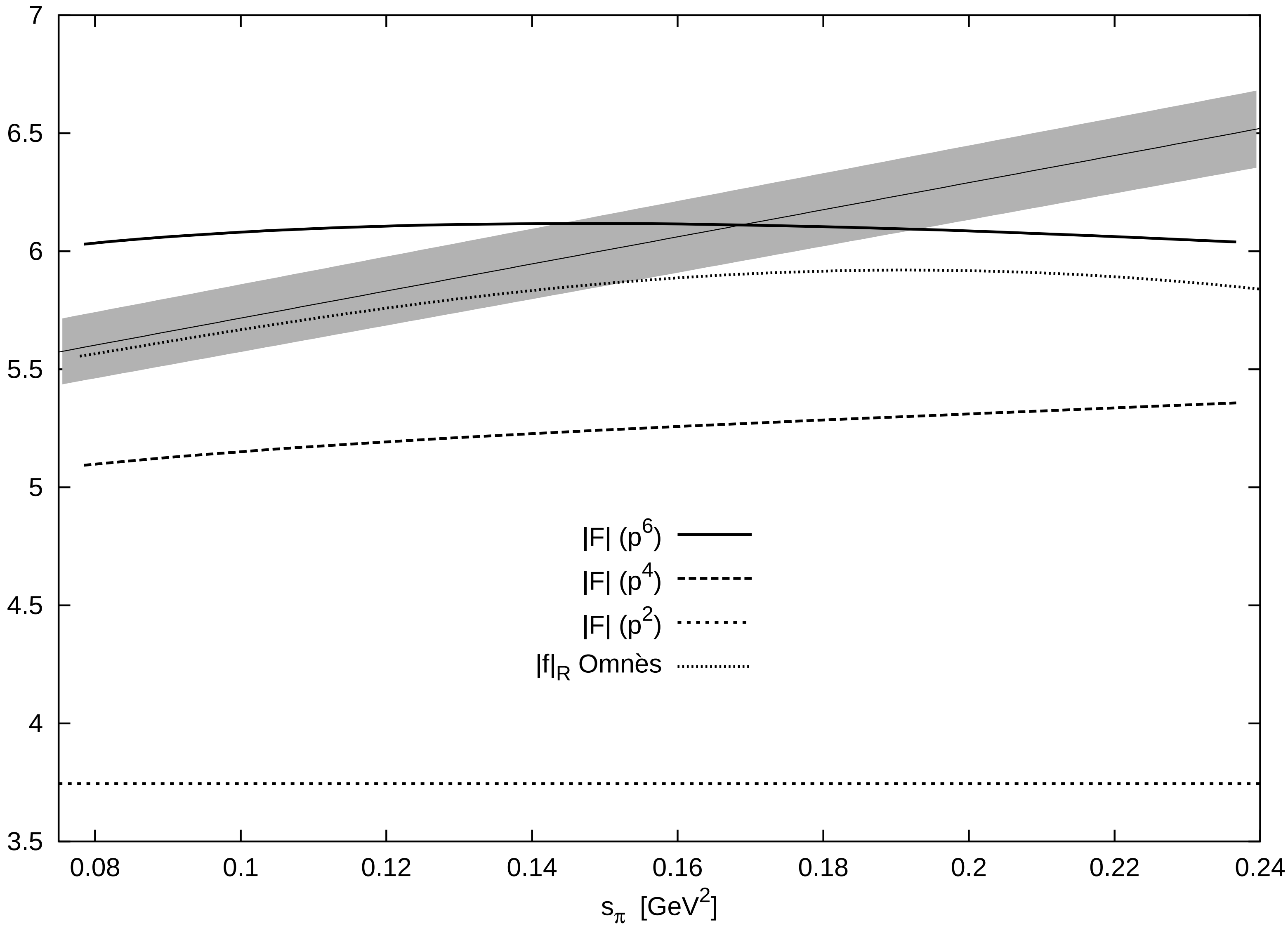
Parametrization for experiment: Amoros,JB, 1999

Full p^6 calculation: Amoros, JB, Talavera 2000

Ametller,JB,Bramon,Cornet 1993 (H only)

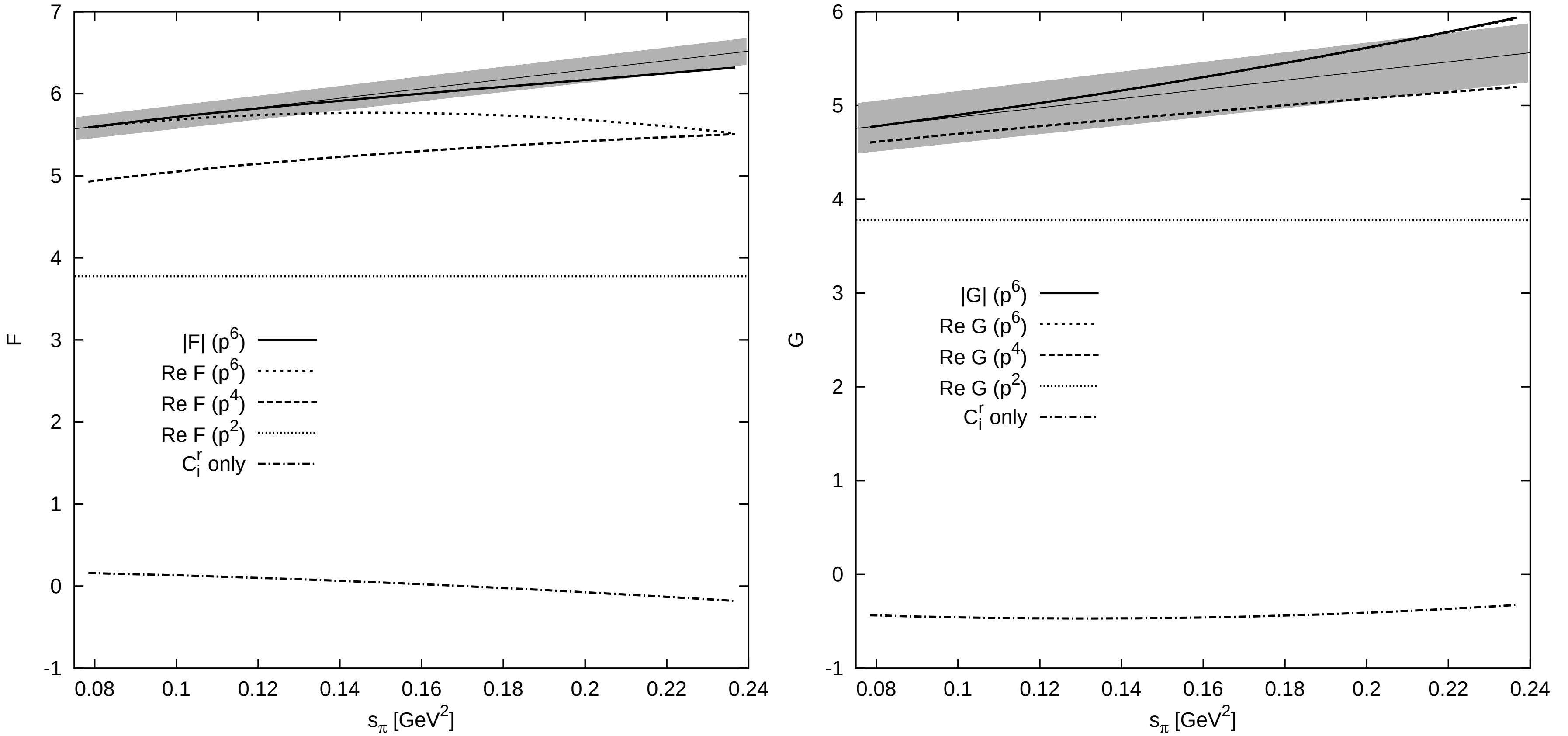


$K_{\ell 4}$



7
6.5
6
5.5
5
4.5
4
3.5
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
s_π [GeV²]
|F| (p⁶)
|F| (p⁴)
|F| (p²)
|f|_R Omnès

$K_{\ell 4}$



F
G
s_π [GeV^2]
s_π [GeV^2]
|F| (p^6)
Re F (p^6)
Re F (p^4)
Re F (p^2)
C_i^r only
|G| (p^6)
Re G (p^6)
Re G (p^4)
Re G (p^2)
C_i^r only

General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit
- Looking forward to newest data and new fit

K_{e5}

	branching ratio
$K^+ \longrightarrow \pi^+\pi^-\pi^0 e^+\nu_e$	$3 \cdot 10^{-12}$
$K^+ \longrightarrow \pi^0\pi^0\pi^0 e^+\nu_e$	$2.5 \cdot 10^{-12}$
$K^0 \longrightarrow \pi^0\pi^0\pi^- e^+\nu_e$	$12 \cdot 10^{-12}$
$K^0 \longrightarrow \pi^+\pi^-\pi^- e^+\nu_e$	$33 \cdot 10^{-12}$

S. Blaser, Phys.Lett.B345:287-290,1995 hep-ph/9410368

$K_{\ell 2\gamma}$

$$K^+(p) \to l^+(p_l)\nu_l(p_\nu)\gamma(q) \qquad [K_{l2\gamma}]$$

$$\begin{aligned}
T &= -iG_F e V_{us}^\star \epsilon_\mu^\star \{F_K L^\mu - H^{\mu\nu} l_\nu\} \\
L^\mu &= m_l \bar{u}(p_\nu)(1+\gamma_5)\left(\frac{2p^\mu}{2pq} - \frac{2p_l^\mu + \not{q}\gamma^\mu}{2p_l q}\right) v(p_l) \\
l^\mu &= \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_l) \\
H^{\mu\nu} &= iV(W^2)\epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta - A(W^2)(qWg^{\mu\nu} - W^\mu q^\nu) \\
W^\mu &= (p-q)^\mu = (p_l + p_\nu)^\mu .
\end{aligned}$$

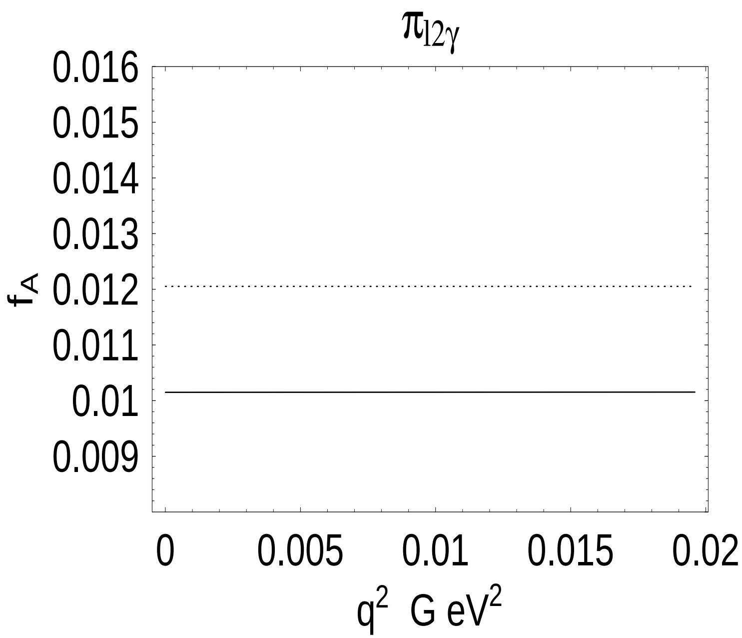
L_μ: IB or inner Bremsstrahlung part

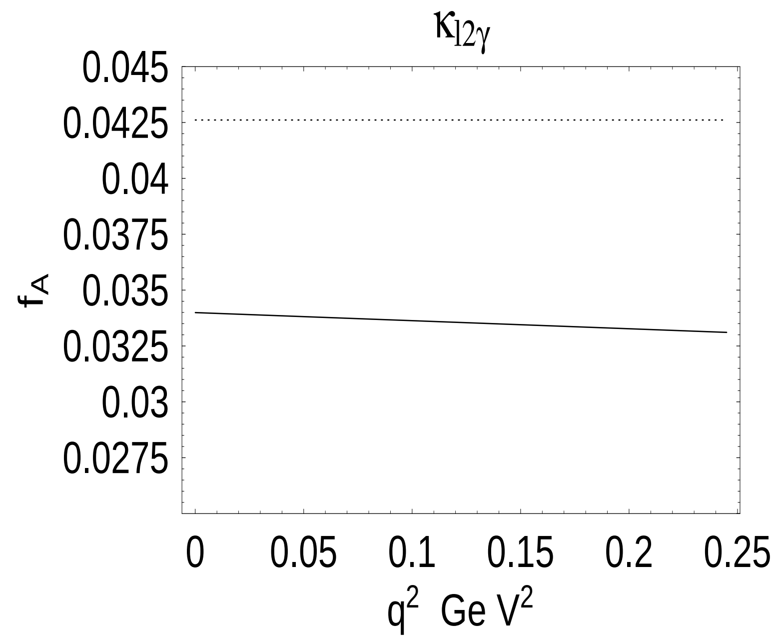
V and A: SD or structure dependent part, starts at p^4

V: anomaly at p^4, known to p^6: Ametller,JB,Bramon,Cornet 1993

A: p^4 JB,Ecker,Gasser 1993, p^6 Geng,Ho, Wu 2004

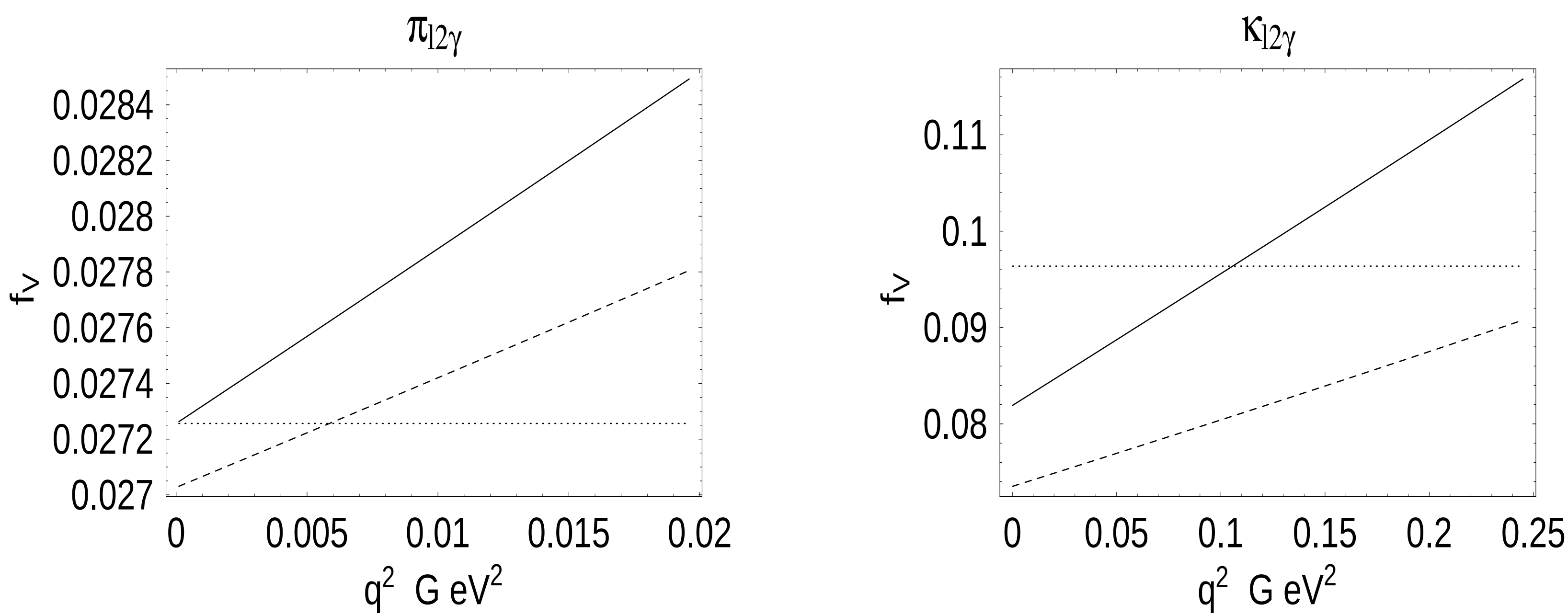
$K_{\ell 2\gamma}$





From Geng,Ho, Wu 2004

$K_{\ell 2\gamma}$



From Geng,Ho, Wu 2004

dotted: p^4

solid p^6 C_i^W from VMD, dashed p^6 C_i^W from CQM

$K_{\ell 3\gamma}$

p^2: Fearing, Fischbach, Smith 1970 IB only

p^4: JB, Ecker, Gasser, 1993

p^6: Axial form-factors fully known

p^6: Vector form-factors: approximately known

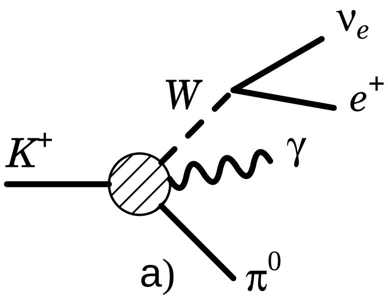
Gasser,Kubis,Paver,Verbeni hep-ph/0412130: $K_{Le\nu\gamma}$

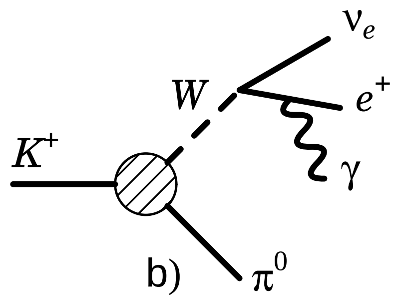
Müller,Kubis,Meißner hep-ph/0607151: T-odd correlations

Kubis,Müller,Gasser,Schmid hep-ph/0611366: $K^+_{e\nu\gamma}$

Approximately known: structure functions smooth
cuts: p-wave or far away: approximate by polynomials

$K_{\ell 3\gamma}$





Remainder is from Kubis et al. 2006

$$
\begin{aligned}
T(K^+_{e3\gamma}) &= \frac{G_F}{\sqrt{2}}\, e\, V^*_{us}\, \epsilon^\mu(q)^* \Big[\big(V_{\mu\nu} - A_{\mu\nu}\big)\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, v(p_e) \\
&+ \frac{F_\nu}{2 p_e q}\, \bar{u}(p_\nu)\, \gamma^\nu\, (1-\gamma_5)\, \big(m_e - \not{p}_e - \not{q}\big)\, \gamma_\mu\, v(p_e) \Big]
\end{aligned}
$$

$K_{\ell 3\gamma}$

$$A_{\mu\nu} = \frac{i}{\sqrt{2}}\left[\epsilon_{\mu\nu\rho\sigma}\left(A_1\, p'^{\rho} q^{\sigma} + A_2\, q^{\rho} W^{\sigma}\right) + \epsilon_{\mu\lambda\rho\sigma}\, p'^{\lambda} q^{\rho} W^{\sigma}\left(\frac{A_3}{M_K^2 - W^2} W_\nu + A_4\, p'_\nu\right)\right],$$

$$V_{\mu\nu} = V_{\mu\nu}^{IB} + V_{\mu\nu}^{SD}$$

$V_{\mu\nu}^{SD}$ has again 4 structure function V_i

$V_{\mu\nu}^{IB}$: IB part, mainly determined by Low's theorem and from the $K_{\ell 3}$ form-factors

$$R\left(E_\gamma^{\rm cut},\, \theta_{e\gamma}^{\rm cut}\right) = \frac{\Gamma\left(K^{\pm}_{e3\gamma}, E^*_\gamma > E_\gamma^{\rm cut}, \theta^*_{e\gamma} > \theta^{\rm cut}_{e\gamma}\right)}{\Gamma\left(K^{\pm}_{e3}\right)},$$

Many uncertainties drop out

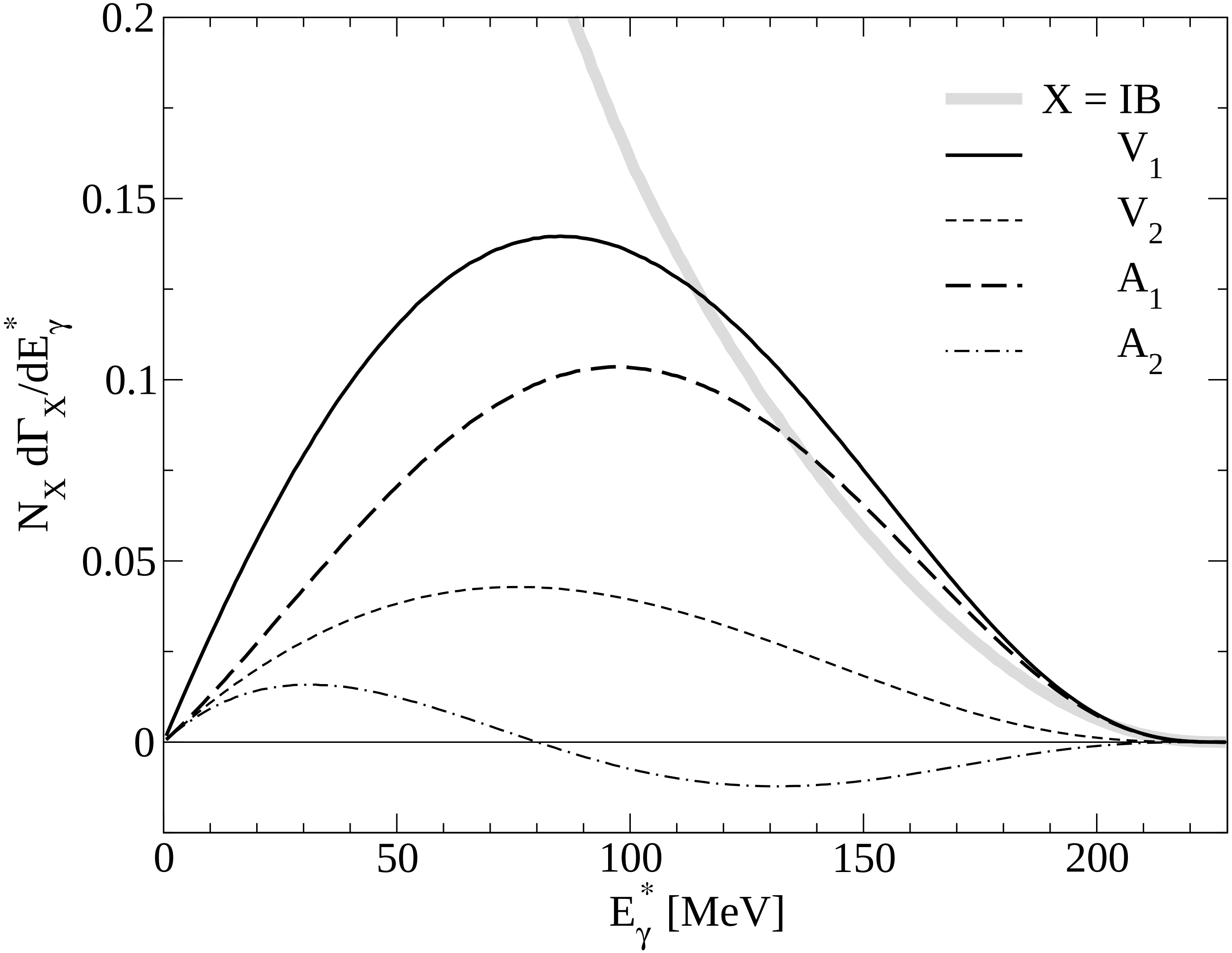
$K_{\ell 3\gamma}$

$$R\left(\bar\lambda_+,\bar\lambda''_+\right)=R(1,0)\left\{1+c_1\ \left(\bar\lambda_+-1\right)+c_2\ \left(\bar\lambda_+-1\right)^2+c_3\,\bar\lambda''_+ +\ldots\right\}$$

$R^{\rm IB}$ accordingly (with expansion coefficients $c_i^{\rm IB}$)

$E_\gamma^{\rm cut}$	$\theta_{e\gamma}^{\rm cut}$	$R^{\rm IB}\cdot 10^2$	$R\cdot 10^2$	$c_1\cdot 10^3$	$c_2\cdot 10^4$	$c_3\cdot 10^4$
30 MeV	20°	0.640	0.633 ± 0.002	12.5 ± 0.4	-5.4 ± 0.3	16.9 ± 0.4
30 MeV	10°	0.925	0.918 ± 0.002	11.1 ± 0.3	-4.7 ± 0.2	15.0 ± 0.3
10 MeV	20°	1.211	1.204 ± 0.002	7.5 ± 0.2	-3.2 ± 0.2	10.1 ± 0.2
10 MeV	10°	1.792	1.785 ± 0.002	6.7 ± 0.2	-2.8 ± 0.1	9.0 ± 0.1
10 MeV	$26^\circ-53^\circ$	0.554	0.553 ± 0.001	5.7 ± 0.1	-2.4 ± 0.1	7.5 ± 0.1

$K_{\ell 3\gamma}$



$$\frac{d\Gamma}{dE^*_\gamma} = \frac{d\Gamma_{\rm IB}}{dE^*_\gamma} + \sum_{i=1}^{4}\left(\langle V_i\rangle\,\frac{d\Gamma_{V_i}}{dE^*_\gamma} + \langle A_i\rangle\,\frac{d\Gamma_{A_i}}{dE^*_\gamma}\right) + \mathcal{O}\Big(|T^{\rm SD}|^2,\,\Delta V_i,\,\Delta A_i\Big)$$

Anomaly

The anomaly including the sign can be tested in many of the decays I talked about, they are all known to p^6.

Conclusions

Chiral Perturbation Theory has played and will play a major role in Kaon physics and especially in the decays talked about here.

Recent results:

- Precise predictions for K_{e3}: isospin breaking at p^6
- Radiative decay: Precise predictions for the ratio R and predictions for the dependence on form factors