Dispersive Representation and Shape of Kl_3 form factors.

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Paper: Phys. Lett. B638:480, 2006
(hep-ph/0603202).
+ In preparation.

International Conference on Kaon Physics

Frascati, May 21-25, 2007

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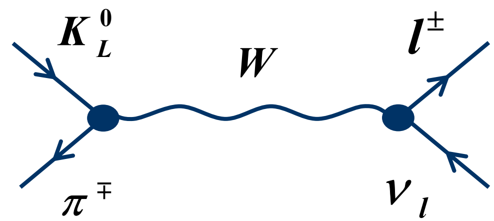
   - Branchings Ratios Br(μ/e).
   - Extraction of $|f_+(0)V_{us}|$.
   - Measurement of $\Delta_{SU(2)}$ and quark mass ratio R.

1. Introduction

- $K_L^0 \rightarrow \pi^{\mp} l^{\pm} \nu_l$



The hadronic element :

$$\left\langle \pi^-(p_\pi) \right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left| \mathrm{K}^0(\mathrm{p_K}) \right\rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

→ $f_+(t)$, $f_-(t)$: form factors

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

- We consider : - the vector form factor $f_+(t)$.
  - the scalar form factor :

$$f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

→ Normalization : - $\hat{f}_+(t) = \frac{f_+(t)}{f_+(0)}$, $\hat{f}_+(0) = 1$

- $f\;(t) = \frac{f_s(t)}{f_+(0)}$, $f\;(0) = 1$

$$\frac{\Gamma_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta^{EM}_{K^{+/0}l}\right) \left| f_+^{K^{+/0}}(0) \mathcal{V}_{us} \right|^2 I^l_{K^{+/0}} \qquad l = (e, \mu)$$

Kaon life time

Radiative Corrections

$$I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \hat{f}_+(t), f(t)\right)$$

- $\hat{f}_+(t) = \frac{f_+(t)}{f_+(0)}$, $f(t) = \frac{f_s(t)}{f_+(0)}$ form factor shapes from Dalitz plot ⇨ phase spaces integrals I^l_K , branching ratios Br(μ/e)=$\Gamma_{K\mu3}/\Gamma_{Ke3}$

- Measurement of $f_+(0)^{K^{0/+}} |\mathcal{V}_{us}|$ considering the decay rates $\Gamma(K_{e3})$, $\Gamma(K_{\mu3})$.

- Comparing $f_+(0)|\mathcal{V}_{us}|$ for K^0 and K^+, we deduce $f_+(0)^{K^0}|\mathcal{V}_{us}|$ and $\Delta_{SU(2)}$ (⇨ access to R).

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

$$\frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} = 1 + \Delta_{SU(2)}$$

2. Motivation : Callan-Treiman theorem in $K_{\mu 3}^{L}$ decays.

2.1 Theoretical knowledge : CT relation.

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = f(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

- Corrections of order m_u, m_d
  - → No chiral logarithms : $\Delta_{CT} \sim \mathcal{O}\left(m_{u,d} / 4\pi F_\pi\right)$
  - → Isospin limit $m_d = m_u$: $\Delta_{CT}^{NLO} \sim -3.5.10^{-3}$ **[Gasser & Leutwyler]**
  - → K^0 decay : no small denominators (π^0 - η mixing $\mathcal{O}\left((m_d - m_u)/m_s\right)$.
  - → K^+ decay case : enhancement by π^0 - η mixing in the final state

    ⇨ $\Delta_{CT}^{K^+} \sim few\ 10^{-2}$

- Corrections $\mathcal{O}(p^6)$ have to be evaluated : 2 LECs to be determined
  → Should not change the order of magnitude.

2.2 Test of the Standard Model.

$$C_{SM} = f(\Delta_{K\pi}) = \frac{F_K |\mathcal{V}^{us}|}{F_\pi |\mathcal{V}^{ud}| f_+(0) |\mathcal{V}^{us}|} |\mathcal{V}^{ud}| + \Delta_{CT}$$

- C is predicted in the Standard Model using the measured Br : Br(K_{l2}/π_{l2}), $\Gamma(K_{e3})$ and $|\mathcal{V}_{ud}|$.

$\Rightarrow$ $C_{SM} = 1.2440 \pm 0.0039 + \Delta_{CT}$

- $\mathcal{V}_{us}$ not needed in this prediction.

$\Rightarrow$ Relation which tests the Standard Model very accurately.

How to measure C ?

2.3 What do we know experimentally ?

- Data available from KTeV, NA48 and KLOE.

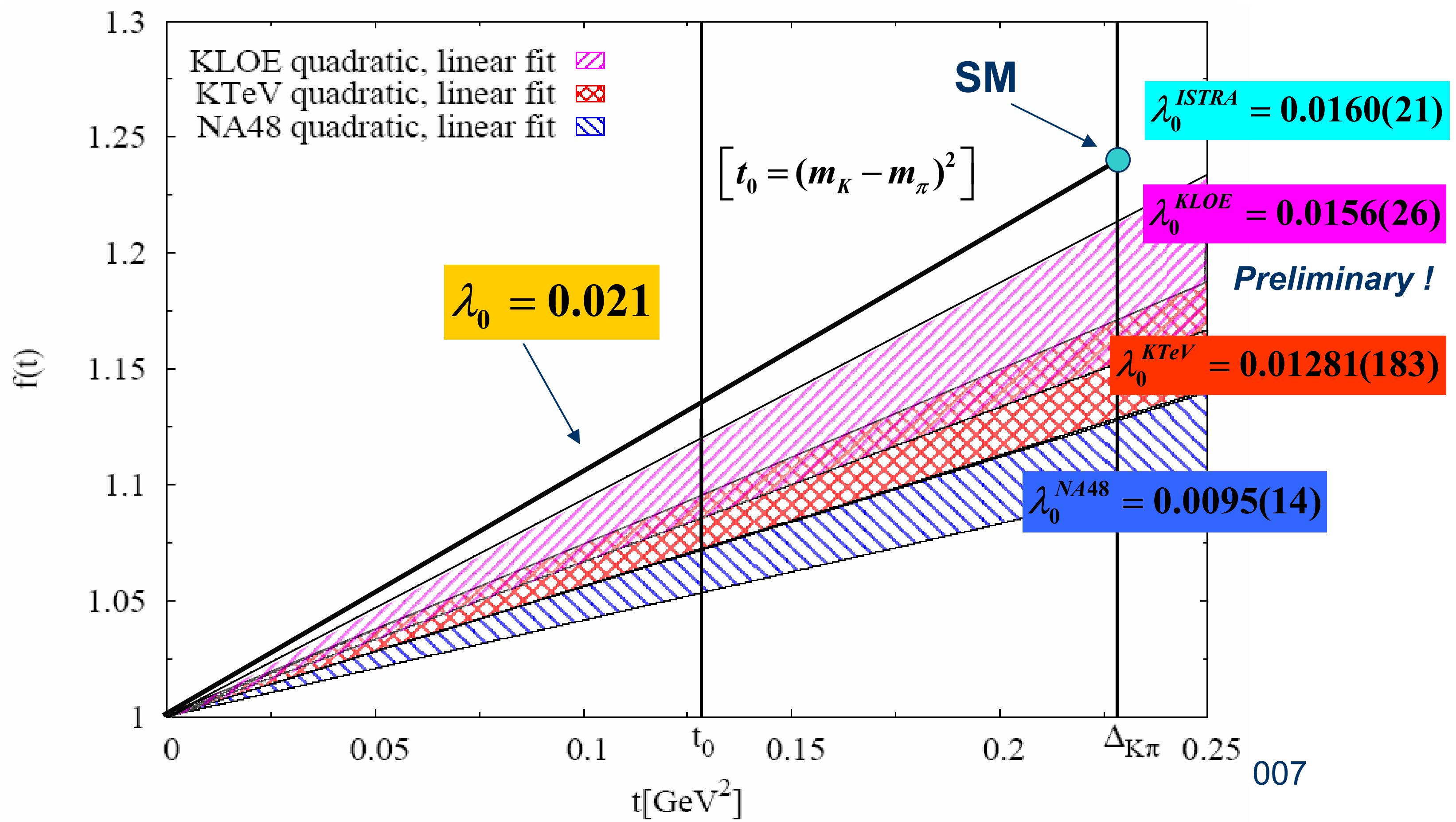
- Necessity to parameterize the 2 form factors $\hat{f}_+$ and f to fit the measured distributions.

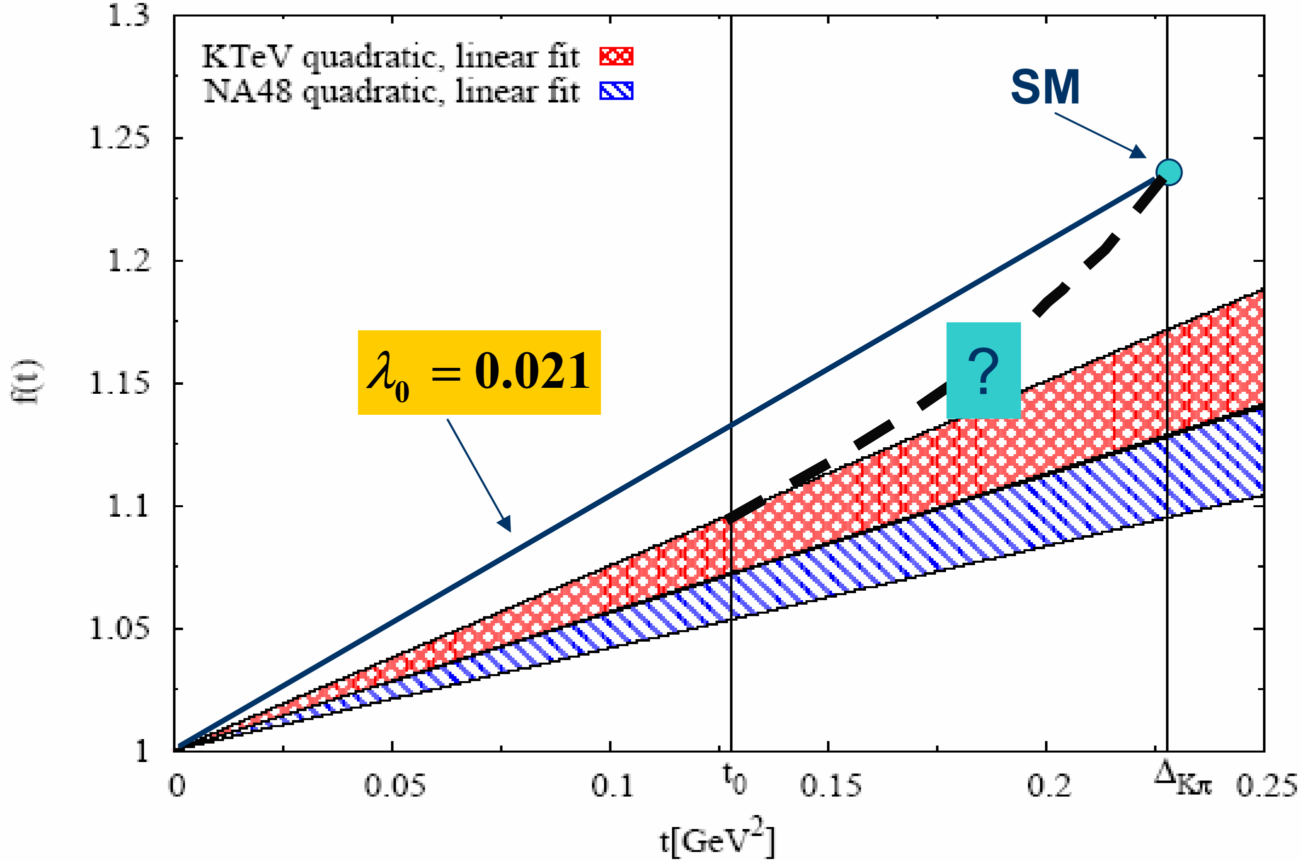
- Usually, use of linear parameterization or pole parameterization for the normalized scalar form factor f :

$$f_{lin}(t,\lambda) = 1 + \lambda \frac{t}{m_\pi^2} \qquad f_{pol}(t, m_s) = \frac{m_s^2}{m_s^2 - t}$$

Example of the linear parametrization.

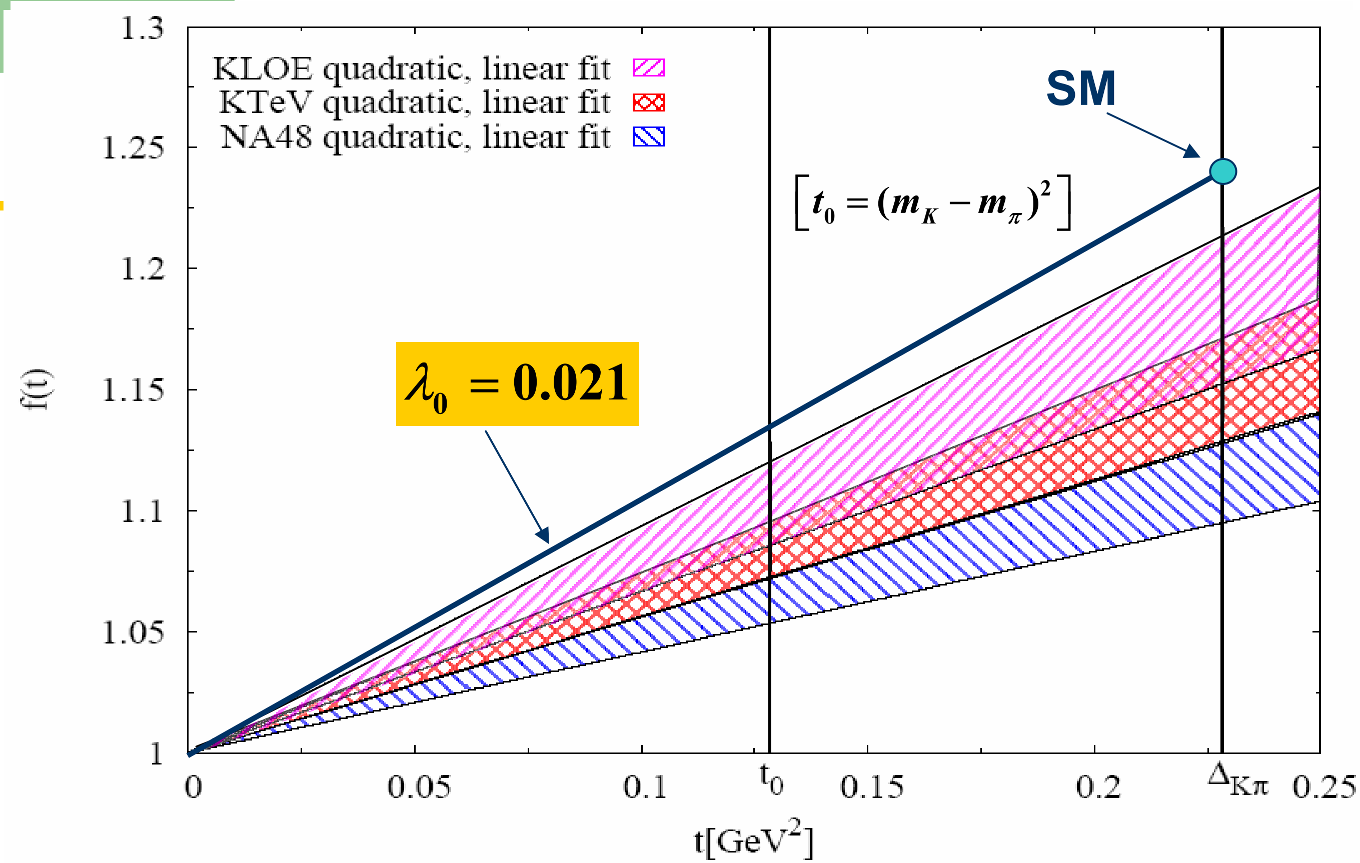
- Results of the linear fit :
  → Quadratic parametrization for $\hat{f}_+$ and linear one for f : $f(t) = 1 + \lambda_0^{exp} \frac{t}{m_\pi^2}$





- But a positive curvature exists f ''(t) > 0. Linearity in the physical region ? But curvature can not be neglected to go **up to** $\Delta_{K\pi}$!
- What is λ_0^{exp} ? ⇨ $\lambda_0^{exp} > \lambda_0 = m_\pi^2 f'(0)$
- λ_0^{exp} may depend on the fitted distribution: different weight given to different areas of the Dalitz plot. λ_0^{exp} is likely method dependent. It could explain the different results.

The linear representation is not satisfactory !



- The value of λ_0 is hard to predict precisely ⟹ SU(3) ChPT prediction (expansion in m_s).
- λ_0 is hard to measure and to predict ⟹ Consider instead C !

How to improve the parametrization to measure $C=f(\Delta_{K\pi})$?

3. A dispersive representation of the Kπ scalar and vector form factor.

Or how to use our theoretical knowledge: the Callan-Treiman Theorem.

3.1 Introduction.

- Problem : How to construct a very precise representation of f(t) between 0 and $\Delta_{K\pi}$?

- Knowledge :
  - → f(0) = 1
  - → f($\Delta_{K\pi}$) = C, Callan-Treiman point
  - → $K\pi$ scattering phase
  - → Asymptotic behaviour of the form factor : $f(s) \underset{s \to -\infty}{=} \mathcal{O}(1/s)$

- A dispersion relation with two substractions at 0 and $\Delta_{K\pi}$ for ln(f(t)) :

$$f(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

  - → ϕ(t) phase of form factor : $f(t) = |f(t)|\, e^{i\phi(t)}$

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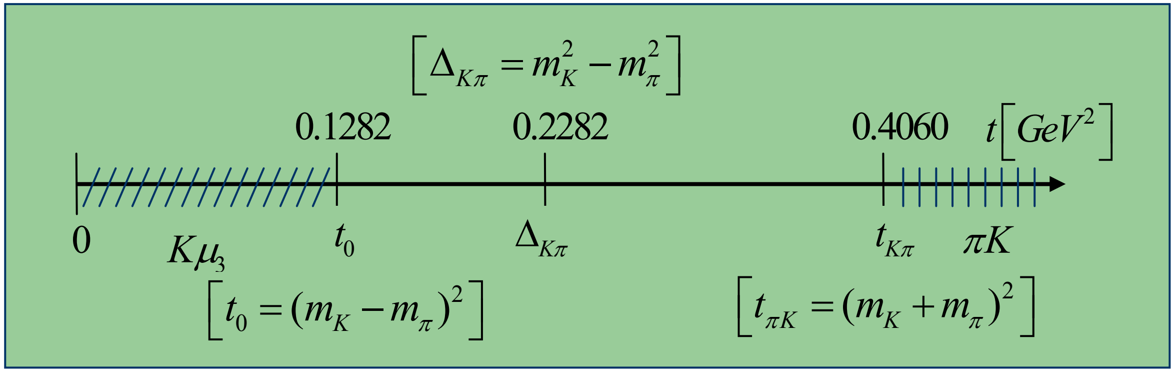


2.2 Description.

$$f(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

→ ϕ(t) phase of form factor : $f(t) = |f(t)|e^{i\phi(t)}$

- Two unknowns: ϕ(s) and lnC=ln(f(t=$\Delta_{K\pi}$))

- ϕ(s) a priori unknown but

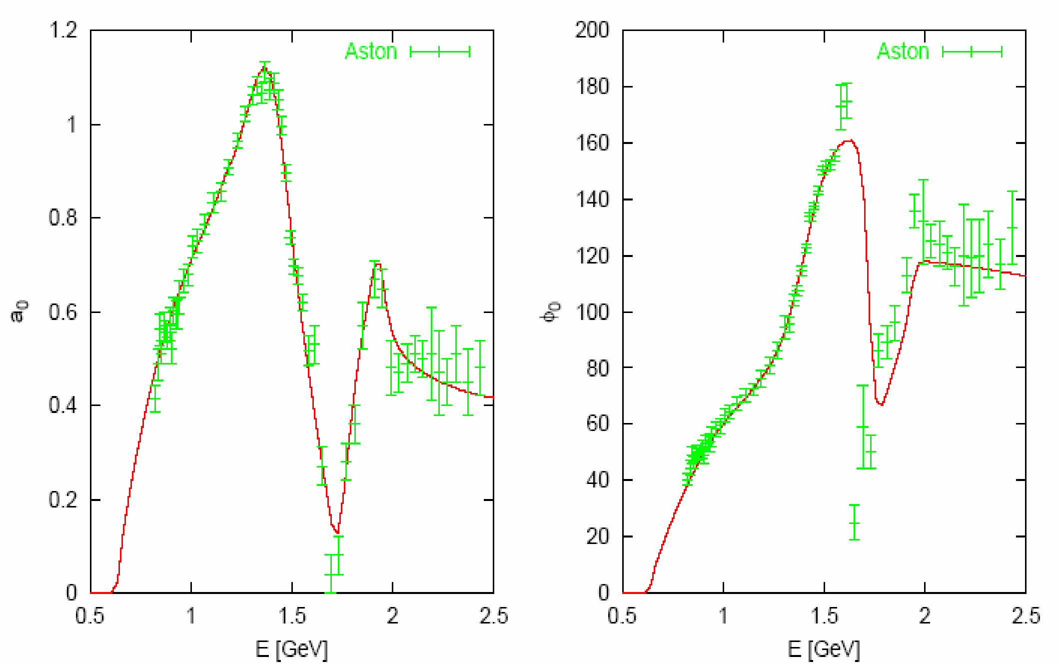
  → $f(s) \underset{s \to -\infty}{=} \mathcal{O}(1/s)$ ⟹ For large s, $\phi(s) \to \pi$. Rapid convergence of G(t).

  → Sum rule $G(-\infty) = \ln C$.

  → At « low energy » $\phi(s) = \delta^{1/2}_{K\pi}(s)$, S wave I=1/2 $K\pi$ scattering phase, well known [**Watson theorem**].

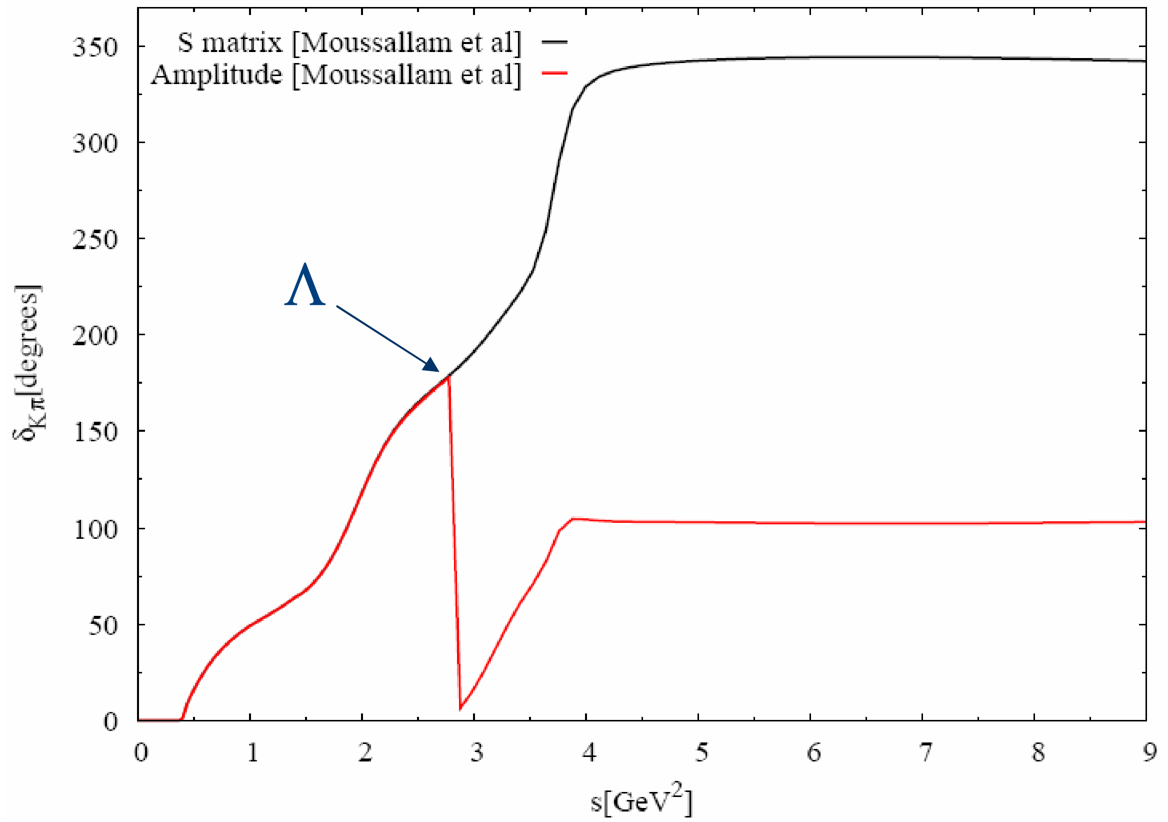
$K\pi$ scattering phase [Buettiker, Descotes, Moussallam '02]

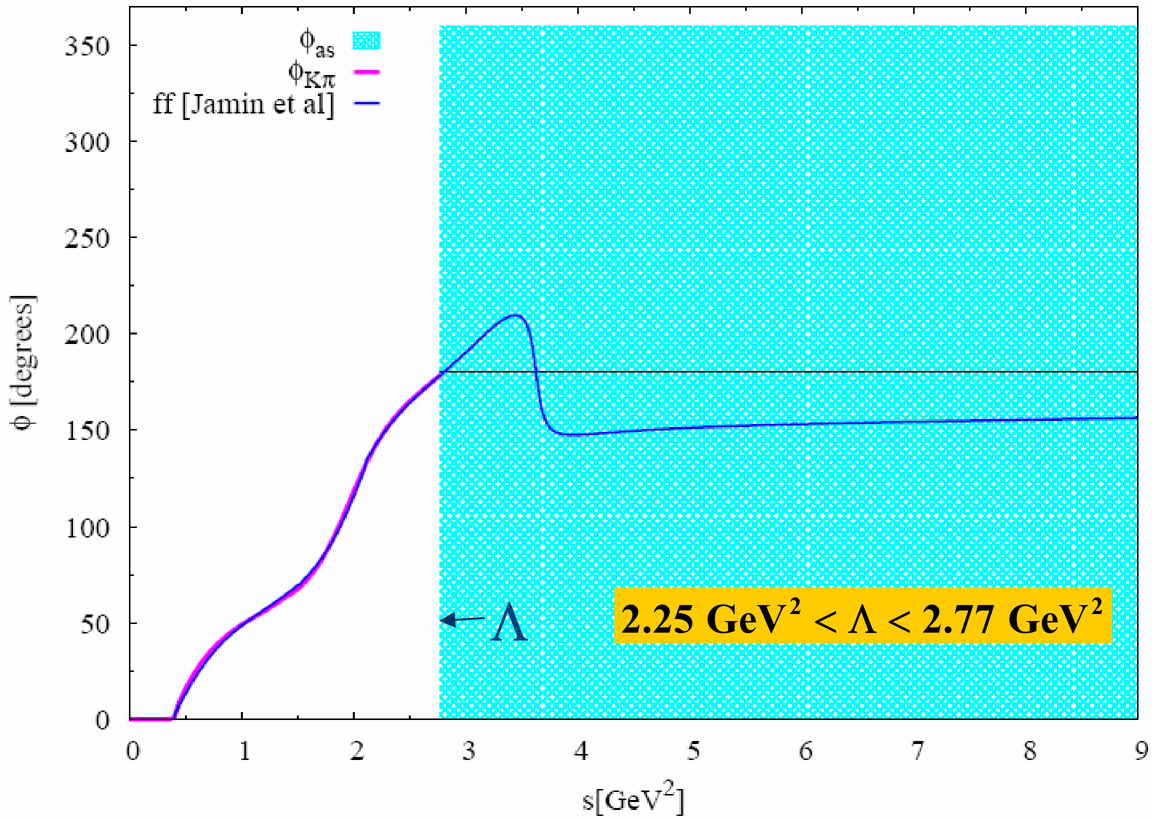
- Experimental input for $1\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$
  - $\pi K \to \pi K$
    [Eastbrook et al] &
    [Aston et al]
  - $\pi\pi \to K\bar{K}$
    [Cohen et al] &
    [Etkin et al]



- Solving Roy-Steiner equations : Analyticity, Unitarity and Crossing ⇨ set of six coupled equations in terms of 2 parameters $a_{1/2}$ and $a_{3/2}$. 4 S,P waves from $\pi K \to \pi K$; 2 S, P waves from $\pi\pi \to K\bar{K}$.
- $E > 2.5\ \mathrm{GeV}$: Regge Phenomenology.

$K\pi$ scattering phase [Buettiker, Descotes, Moussallam '02]





- Elastic up to ~1.5 GeV ⇒

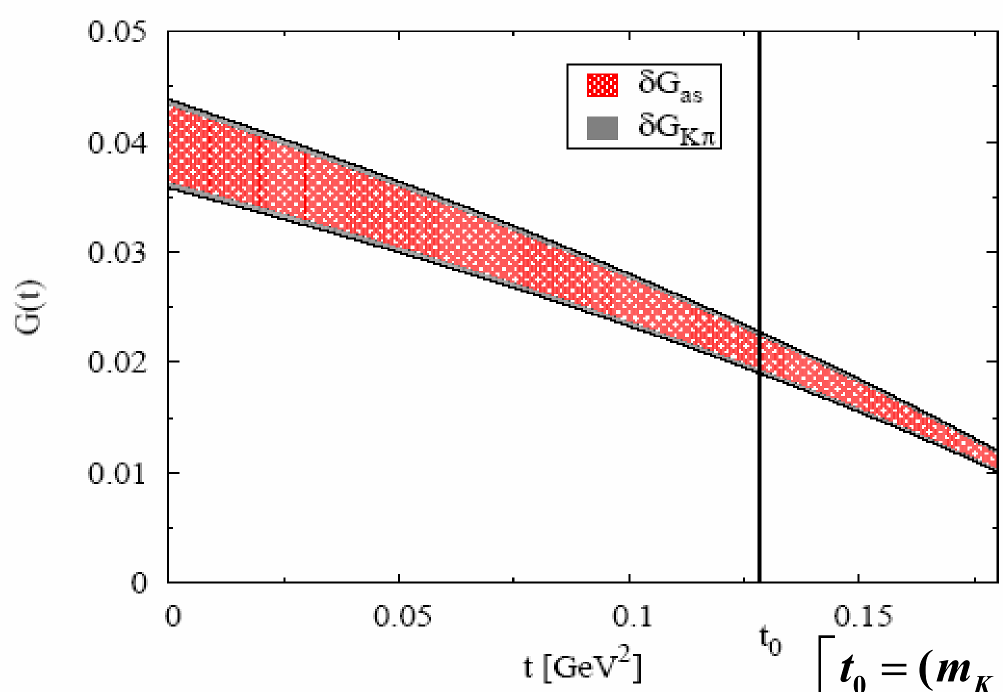
$$t < \Lambda : \ \phi(t) = \delta_{\pi,K}^{s,\frac{1}{2}}(t)$$
$$t > \Lambda : \ \phi(t) = \pi$$

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{K\pi}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

⇒

$$G(t) = G_{K\pi}(\Lambda, t) + G_{as}(\Lambda, t) \pm \delta G$$

$\int_{t_{K\pi}}^{\Lambda}$ ↗ $\int_{\Lambda}^{\infty}$ ↗



$\left[t_0 = (m_K - m_\pi)^2 \right]$

G(t) with the uncertainties added in quadrature

$$G(0) = 0.0398 \pm 0.0040$$

G(t) does not exceed 20% of the expected value of lnC

lnC~0.20

- Result not sensitive to Λ (G(0) remains stable) except for the sum rule:

$$\ln C = G(-\infty) = \frac{\Delta_{K\pi}}{\pi} \int_{t_{\pi K}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s - \Delta_{K\pi})}$$

⇨ For $2.25\ \text{GeV}^2 < \Lambda < 2.77\ \text{GeV}^2$

$$0.1335 < G(-\infty) < 0.3425$$

⇨ Very sensitive to Λ and to the unknown phase at high energy !

3.3 How to determine lnC ?

- Use the dispersive representation of f(t) in the fit of the $K_{\mu 3}$ Dalitz plot.

- To that aim, we need to parametrize the vector form factor ⇨ use also a dispersive representation for the vector form factor.

3.4 Dispersive parametrization of the Kπ vector form factor

- In the same way that for the scalar form factor, a dispersion relation with two substractions for ln($f_+(t)$) :
  2 substraction points at low energy :
  → $\hat{f}_+(0) = 1$
  → $\hat{f}_+^{'}(0) = \Lambda_+ / m_\pi^2$

$$\hat{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$ with $$H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\phi(s)}{(s-t)}$$

  → ϕ(t) phase of form factor : $\hat{f}_+(t) = \left|\hat{f}_+(t)\right| e^{i\phi(t)}$

- ϕ(s) a priori unknown but
  → $\hat{f}_+(s) \underset{s\to-\infty}{=} \mathcal{O}(1/s)$ ⟹ For large s, ϕ(s) → π. Rapid convergence of H(t).
  → At « low energy » $\phi(s) = \delta^{1/2}_{K\pi}(s)$, P wave I=1/2 Kπ scattering phase [**Watson theorem**].

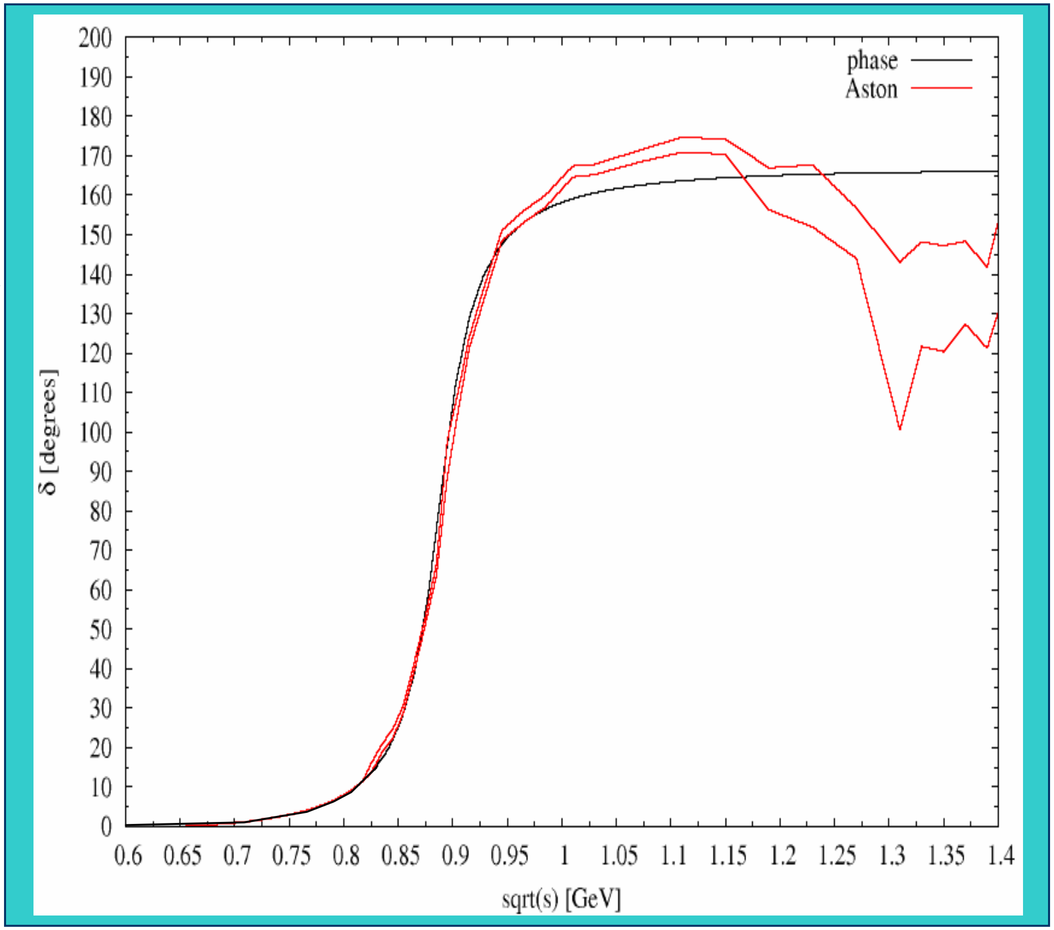
$K\pi$ scattering phase

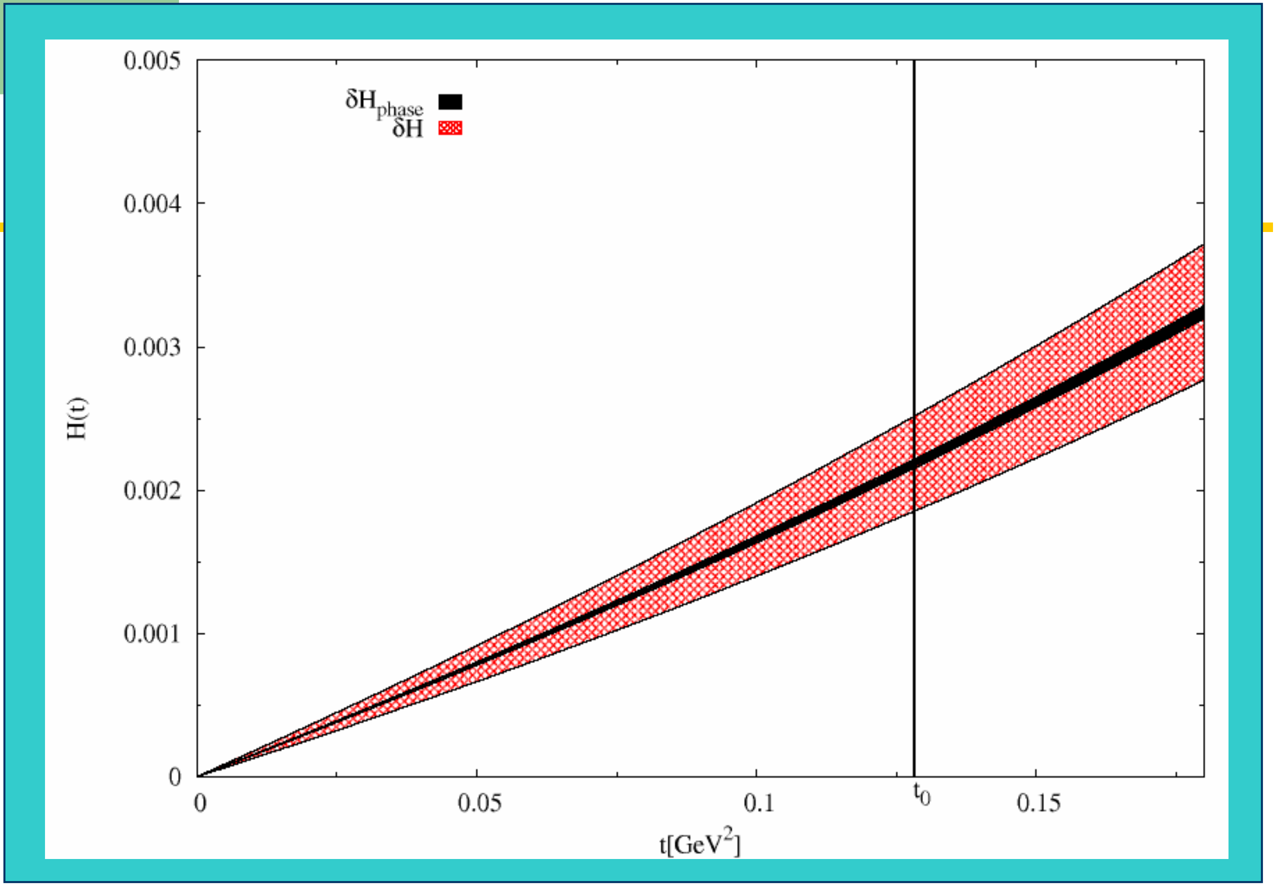
- Experimental input for $0.825\ \text{GeV} < E < 2.5\ \text{GeV}$ $\pi K \to \pi K$ **[Aston et al].**

- Extrapolation of the phase down to the threshold complicated ⇨ lack of relevant experimental inputs.

- Construction of the partial wave amplitude : Breit-Wigner (K*(892)) a la Gounaris-Sakourai (Analyticity, Unitarity and Correct threshold behavior)

- Reproduce the value of p-wave scattering length.





- Representation very close to the pole one.

- It also exists a sum rule :

$$\Lambda_+ = -H(-\infty) = -\frac{m_\pi^2}{\pi}\int_{s_{\pi K}}^{\infty}\frac{ds}{s^2}\phi(s)$$

⇨ For Λ = 2 GeV^2 ,
H(-∞) = -0.2358 ± 0.0974

4. Measurement of lnC and Λ_+ .

4.1 NA48 results.

- 1^{rst} dedicated analysis by NA48 :

$$f(t) = \exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - G(t))\right]$$ and $$\hat{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}(\lambda_+ + H(t))\right]$$

⇨ $$\begin{cases} \ln C_{exp} = 0.1438 \pm 0.0140 \\ \Lambda_+ = 0.0233 \pm 0.0009 \end{cases}$$ with $\rho(\ln C, \Lambda_+) = -0.44$

[NA48, Phys. Lett. B647:341, 2007]

- Test of consistency : compatible with the sum rules ?

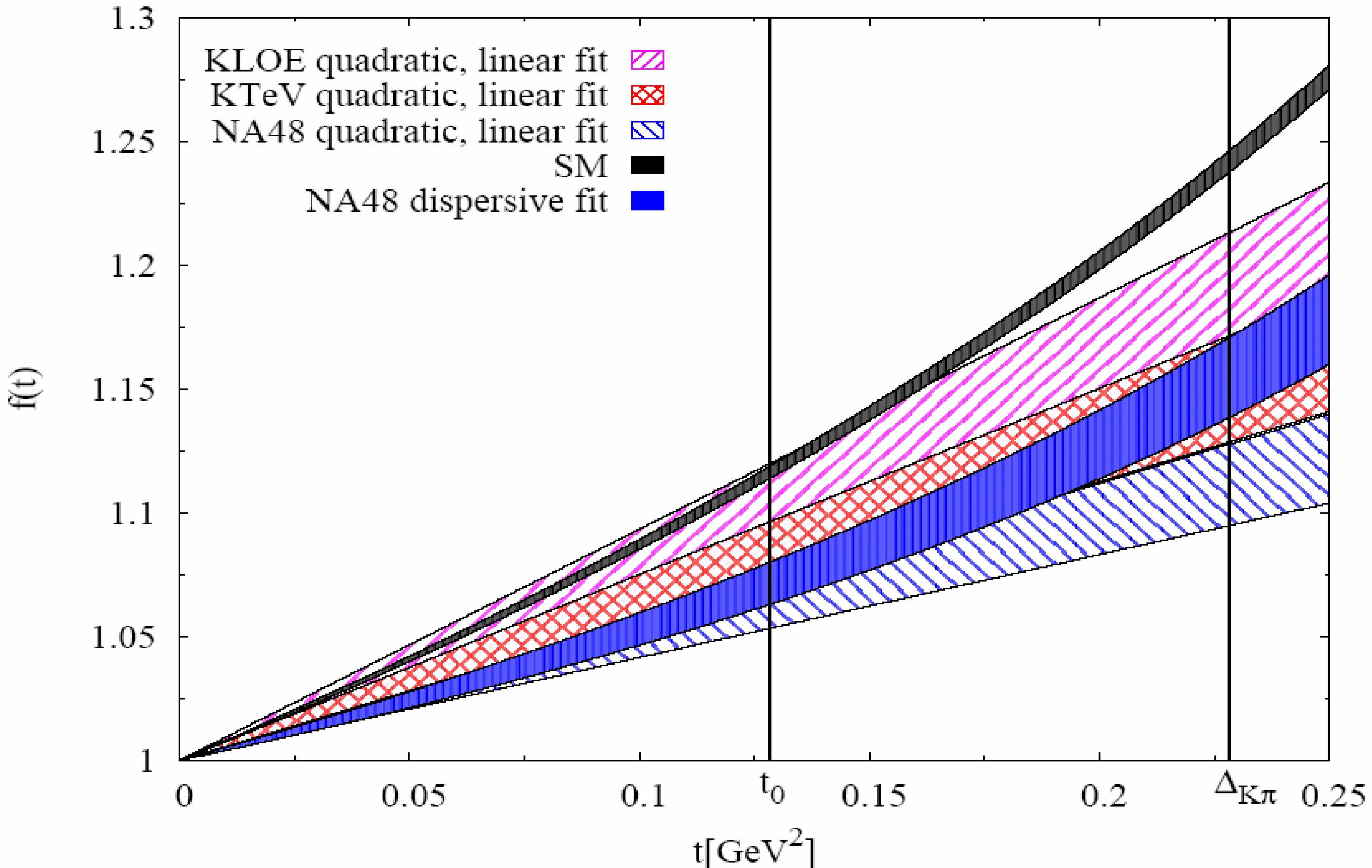
⇨ Yes, compatible within the error bars $\left(\ln C = G(-\infty), \Lambda_+ = -H(-\infty)\right)$

$0.1335 < G(-\infty) < 0.3425$ and $-0.332 < H(-\infty) < -0.1384$

The sum rules are not very stringent since they depend on the behaviour of the phases at high energy. ⇨ Indicate that the phase of f(t) after Λ drops before reaching π.

4.2 Advantages of the parametrization.

- One parameter for the scalar and vector form factor parametrization allows to determine both the slope and the curvature.



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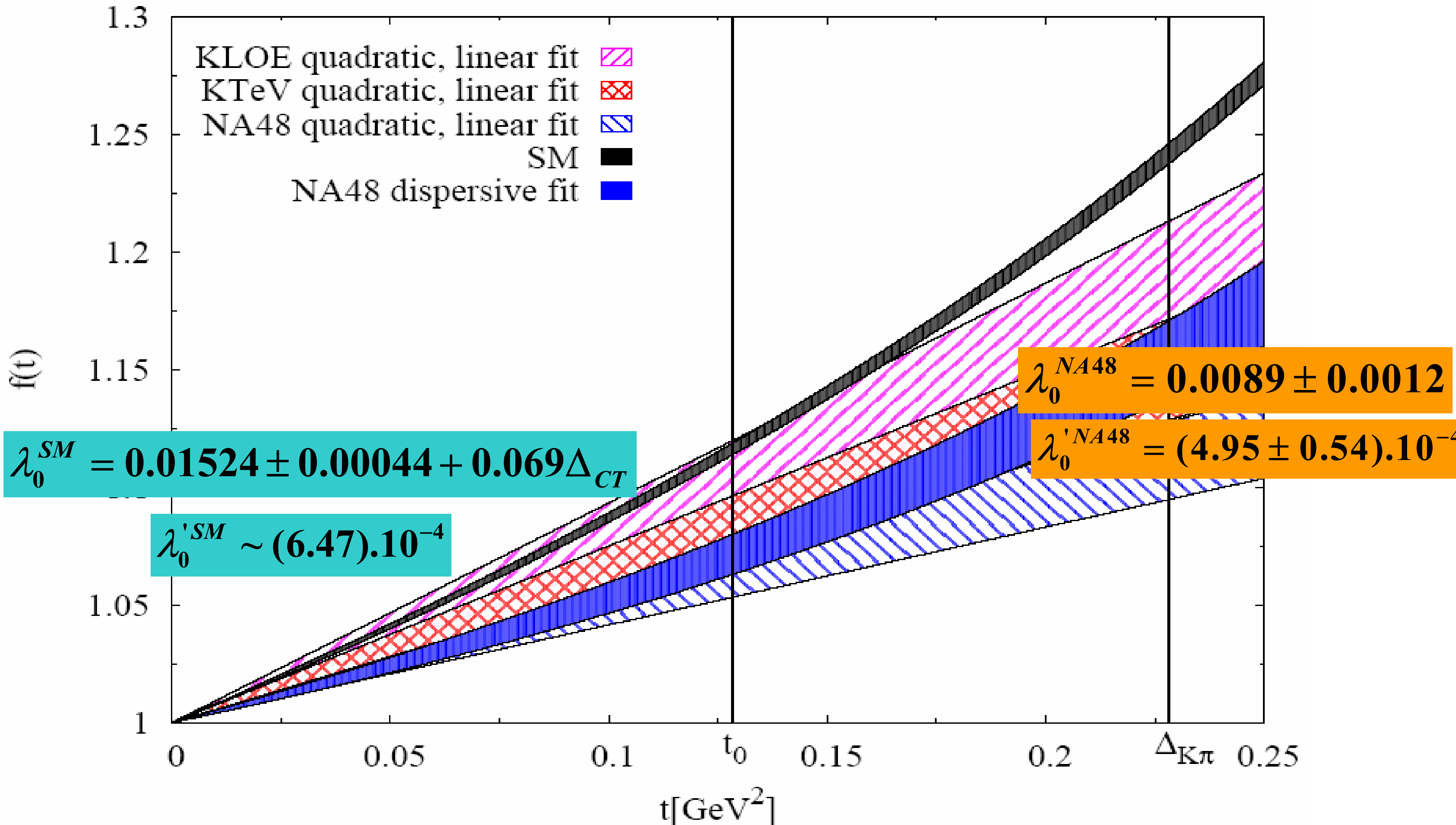
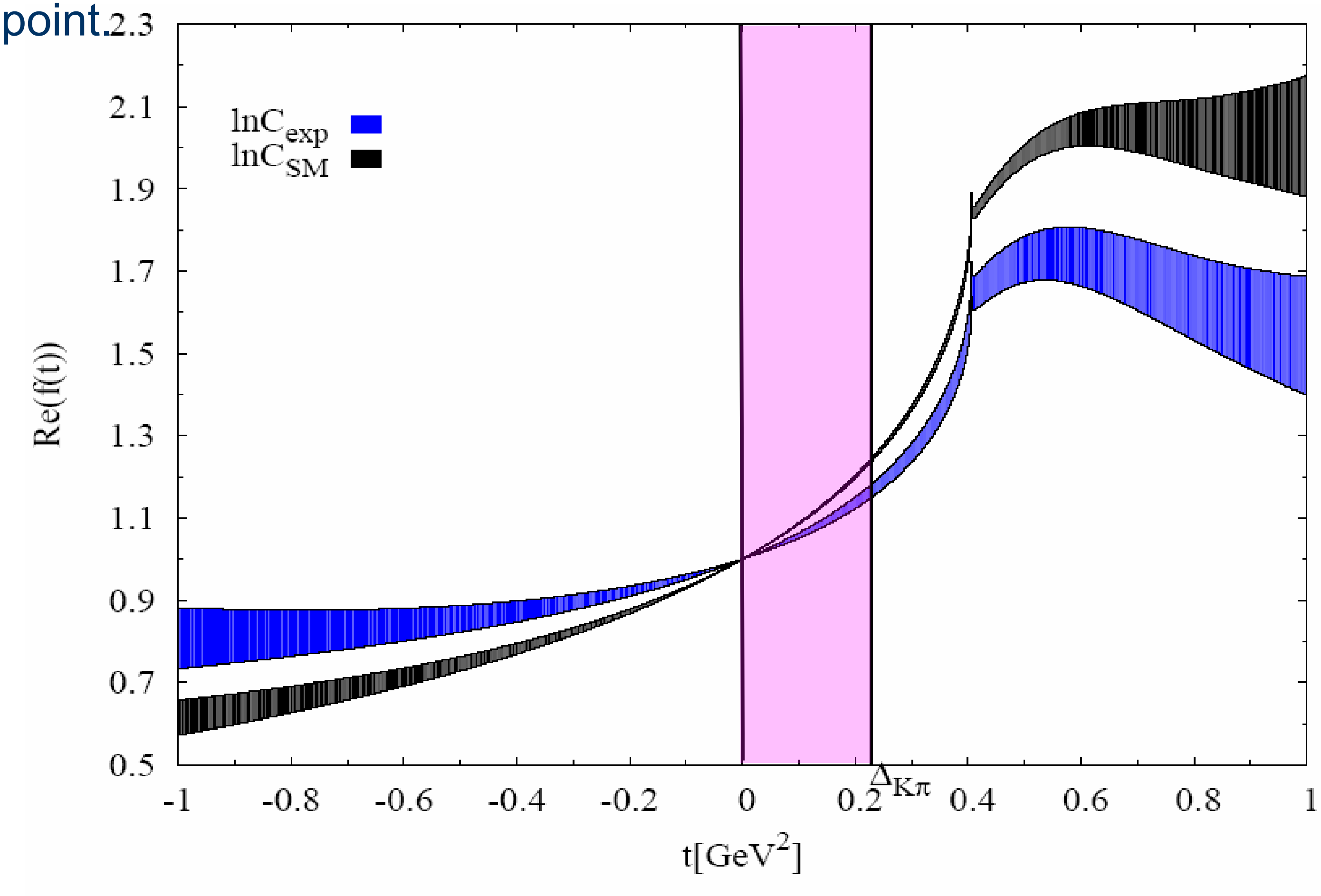


4.2 Advantages of the parametrization.

- Very precise parametrization in the physical region and up to the CT point.



- Possible test of the SM (cf Talk J.Stern)

5. Applications.

-Branching Ratios

-Extraction of $|f_+(0)V_{us}|$.

-Measurement of $\Delta SU(2)$ and quark mass ratio R.

5.1 Phase Space Integrals : I_K^l

$$\frac{\Gamma_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta_{K^{+/0}l}^{EM}\right) \left| f_+^{K^{+/0}}(0) V_{us} \right|^2 I_{K^{+/0}}^l$$

Kaon life time

EM Radiative Corrections

$$I_K^l = \int dt \frac{1}{m_K^8} \lambda^{3/2} F\left(t, \hat{f}_+(t), f(t)\right)$$

- Once we have measured $\hat{f}_+(t)$ and f(t), we can calculate the phase space integrals I_K^l :

$$F\left(t, \hat{f}_+(t), f(t)\right) = \left(1 + \frac{m_l^2}{2t}\right)\left(1 - \frac{m_l^2}{t}\right)^2 \left(\hat{f}_+^2(t) + \frac{3m_l^2\left(m_K^2 - m_\pi^2\right)}{\left(2t + m_l^2\right)\lambda} f^2(t)\right)$$

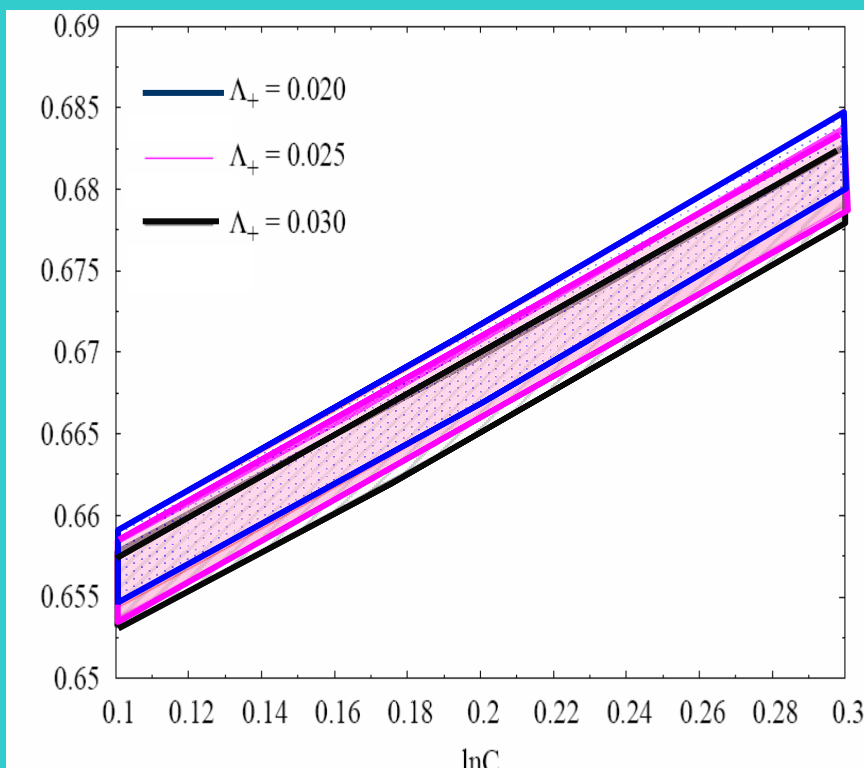
- Of course I_K^e independent of InC.

- I_K^μ dependent on Λ_+. But above all on InC.

5.2 Branching Ratios : Additional test.

$$Br(\mu/e) = \frac{\Gamma_{K^{+/0}\mu 3}}{\Gamma_{K^{+/0}e 3}} = \frac{\left(1 + 2\Delta^{EM}_{K^{+/0}\mu}\right)}{\left(1 + 2\Delta^{EM}_{K^{+/0}e}\right)} \frac{I^{\mu}_{K^{+/0}}}{I^{e}_{K^{+/0}}}$$

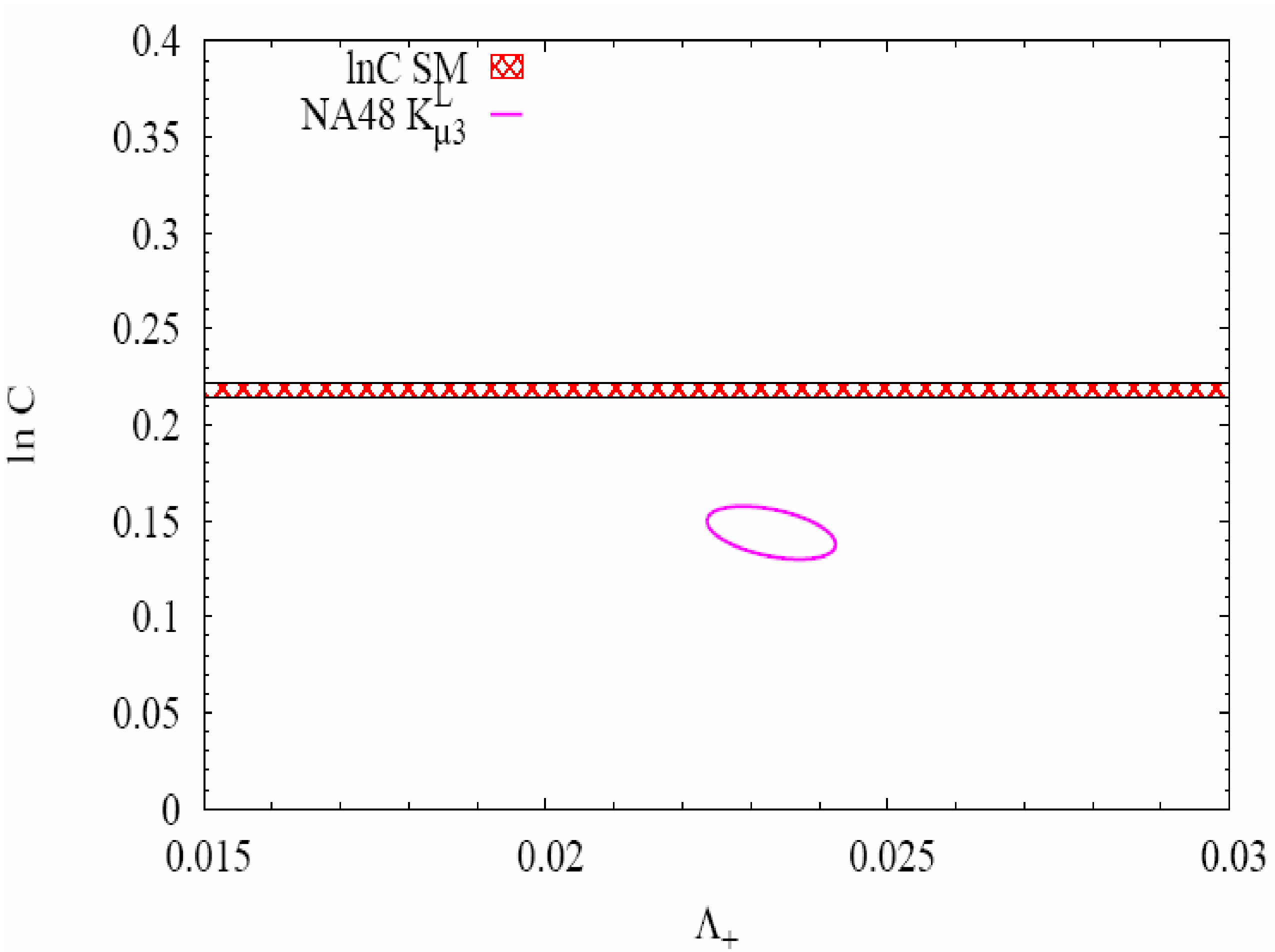
Dependence of the Br(μ/e) of K^0 in Λ_+ and lnC:



Quasi independent of Λ_+ !

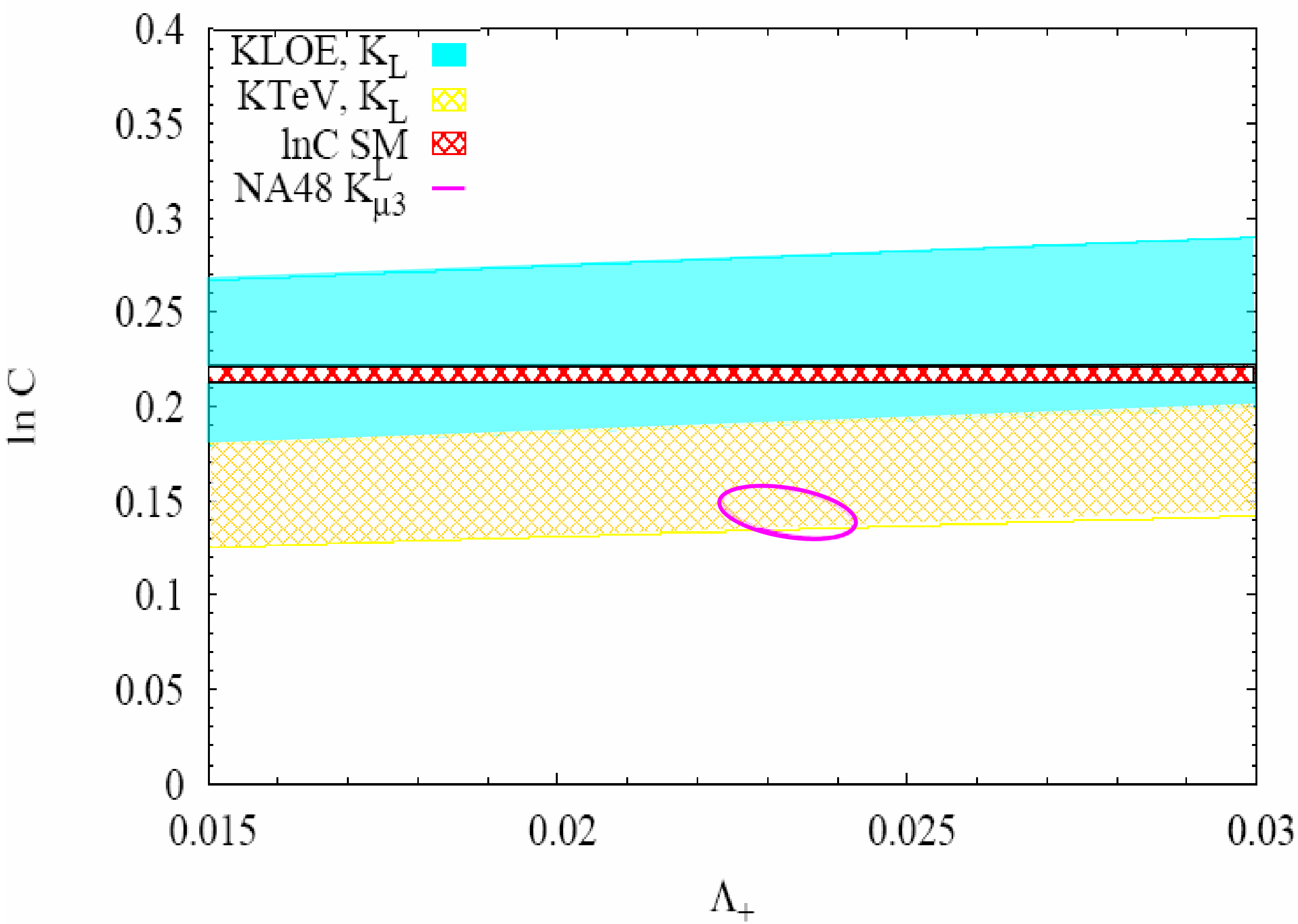
5.3 Discussion of Br(μ/e) :

- Are shown in the plane (lnC, Λ_+) the region corresponding to the Dalitz plot analysis of NA48 using the dispersive representations.



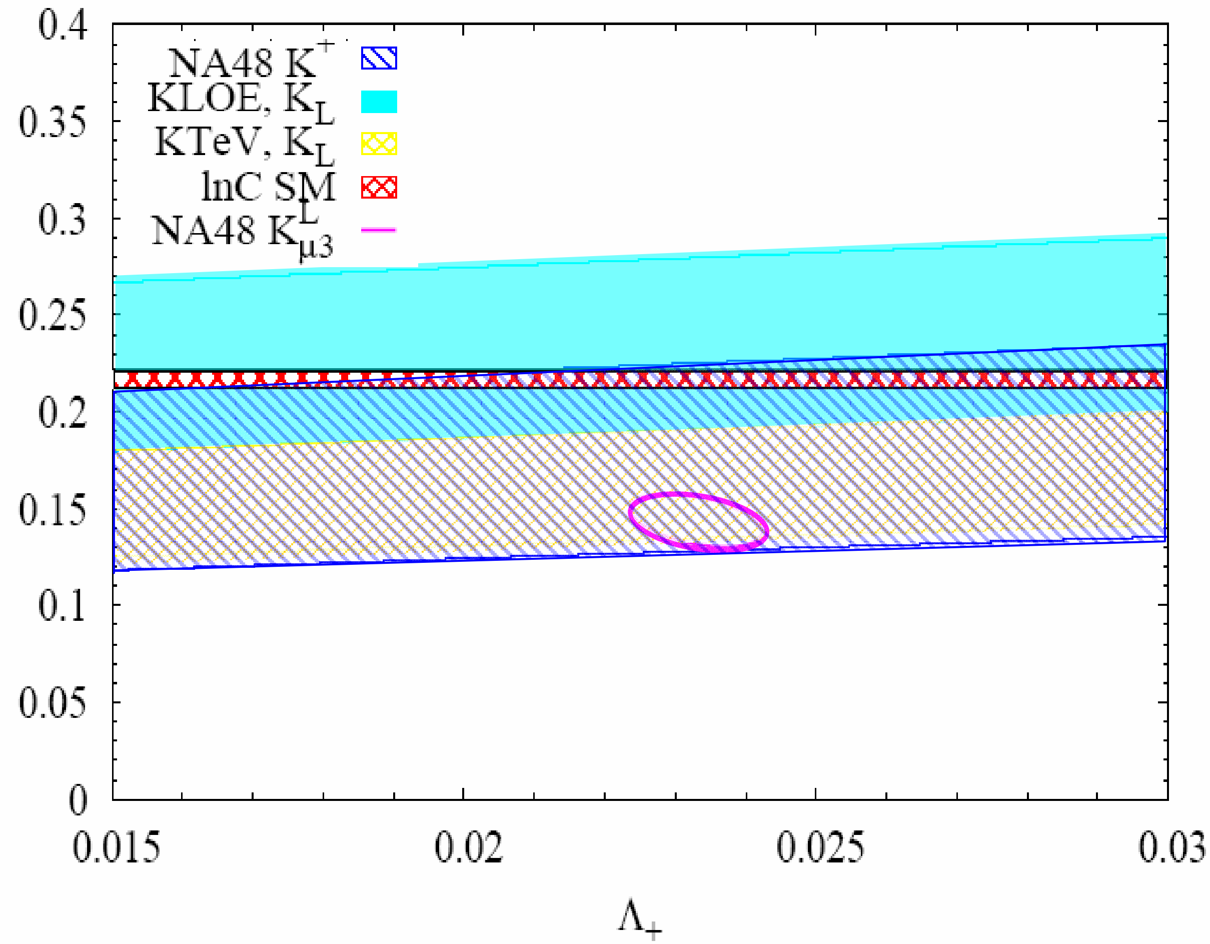
5.3 Discussion of Br(μ/e) :

- Are shown in the plane (lnC, Λ_+) the region corresponding to the Dalitz plot analysis of NA48 using the dispersive representations and the different measurements of Br(μ/e).



5.3 Discussion of Br(μ/e) :

- Are shown in the plane (lnC, Λ_+) the ellipse corresponding to the Dalitz plot analysis of NA48 using the dispersive representations and bands corresponding to different measurements of Br(μ/e).
- The region allowed by NA48 Dalitz plot analysis is consistent with KTeV K^0, NA48 K^+ Br (μ/e) measurements.
- Not quite consistent with KLOE K^L Br (μ/e) and with the SM prediction of lnC.



5.4 Extraction of $f_+^{K^{+/0}}(0)|V_{us}|$:

$$\frac{\Gamma_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta_{K^{+/0}l}^{EM}\right) \left| f_+^{K^{+/0}}(0) V_{us} \right|^2 I_{K^{+/0}}^l$$

Kaon life time

EM Radiative Corrections

$$I_K^l = \int dt \frac{1}{m_K^8} \lambda^{3/2} F\left(t, \hat{f}_+(t), f(t)\right)$$

- From the calculation of I_K^l , the measurements of τ_K, $\Gamma(K_{e3})$ or $\Gamma(K_{\mu3})$

⇨ $f_+^{K^{+/0}}(0)|V_{us}|$

- For the muons :

I_K	Inputs (τ_K, BRs, Δ_{EM})	$\mid f_+(0)V_{us} \mid_\mu^{K^0}$	$\mid f_+(0)V_{us} \mid_\mu^{K^+}$
$lnC = 0.2155(31)$ $\Lambda_+ = 0.0245$	Moulson [CKM] NA48	0.21609(60)	0.22199(112) 0.22329(95)
$lnC = 0.1438(138)$ $\Lambda_+ = 0.0233(9)$ $\rho(\Lambda_+, lnC) = -0.44$	Moulson [CKM] NA48	0.21801(68)	0.22400(118) 0.22532(102)
$\lambda_0 = 0.01330(135)$ $\lambda'_+ = 0.02484(110)$ $\lambda''_+ = 0.00161(45)$	Moulson [CKM] NA48	0.21649(70)	0.22312(119) 0.22444(103)

- Difference between K^0 and K^+ due to the isospin breaking corrections : $\Delta_{SU(2)}$ ⇨ we can evaluate it.

$$\frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} = 1 + \Delta_{SU(2)}$$

- For the muons :

I_K	Inputs (τ_K, BRs, Δ_{EM})	$\lvert f_+(0)V_{us}\rvert_\mu^{K^0}$	$\lvert f_+(0)V_{us}\rvert_\mu^{K^+}$	$\Delta SU(2)_\mu$
$lnC = 0.2155(31)$	Moulson [CKM]	0.21609(60)	0.22199(112)	0.0273(59)
$\Lambda_+ = 0.0245$	NA48		0.22329(95)	0.0332(53)
$lnC = 0.1438(138)$	Moulson [CKM]	0.21801(68)	0.22400(118)	0.0275(59)
$\Lambda_+ = 0.0233(9)$	NA48		0.22532(102)	0.0334(53)
$\rho(\Lambda_+, lnC) = -0.44$				
$\lambda_0 = 0.01330(135)$	Moulson [CKM]	0.21649(70)	0.22312(119)	0.0306(64)
$\lambda'_+ = 0.02484(110)$	NA48		0.22444(103)	0.0367(58)
$\lambda''_+ = 0.00161(45)$				

- ΔSU(2) ~ 0.03 to be compared with the prediction commonly used to extract $|f_+(0)\ V_{us}|$: $\Delta_{SU(2)}$ ~ 0.0231(22)
  ⇨ Underestimation of the isospin breaking corrections which increases artificially the extracted value of $|f_+(0)\ V_{us}|$!

- K^0 mode should be used to extract $|f_+(0)\ V_{us}|$ and K^+ mode to extract $\Delta_{SU(2)}$) !

6. Conclusion and Outlook

- Introduction of a new parametrization : a dispersive parametrization for the scalar form factor f(t) physically motivated by the Callan-Treiman Theorem.
- Linear parametrization not appropriate to go up to $\Delta_{K\pi}$:
  $\Rightarrow$ λ_0 is hard to measure and to predict !
- Give a very precise parametrization of f(t) in the region of interest.
- Uncertainties under control : two subtractions $\Rightarrow$ reduce the importance of the unknown phase at high energy.
- Allow to test the Standard Model.
- First measurement using this parametrization from NA48
  $\Rightarrow$ measurements from KLOE and KTeV awaited.
- Applications:
  - Prediction of the Br(μ/e) $\Rightarrow$ consistency between the Dalitz plot measurement and the Br measurements from K^0 and K^+.
  - Extraction from K^0 of $|f_+(0)Vus|$
  - From K+, possibility to extract and test the prediction of $\Delta_{SU(2)}$. .
- Knowning InC $\Rightarrow$ Matching with 2 loop ChPT : 2 $O(p^6)$ LECs
  $\Rightarrow$ $f_+(0)$ and thus extraction of V_{us}.

Additional Slides.

	lnC=0.2155	$\lambda_0 = 0.0150$	$\lambda_0 = 0.021$	lnC=0.1438	$\lambda_0 = 0.0089$
$I_\mu^{K^0}$	0.10245	0.10217	0.10348	0.10107	0.10085
$I_\mu^{K^+}$	0.10609	0.10602	0.10748	0.10465	0.10458

TAB. 1 – Results for $\Lambda_+ = 0.0245$

	lnC=0.2155	$\lambda_0 = 0.0150$	lnC=0.1438	$\lambda_0 = 0.0089$
$I_\mu^{K^0}$	0.10203	0.10176	0.10065	0.10045
$I_\mu^{K^+}$	0.10563	0.10556	0.10419	0.10412

TAB. 2 – Results for $\Lambda_+ = 0.0233$

1.3 Test of the Standard Model

- C_{SM} is determined without the knowledge of $\mathcal{V}^{us}$

$$C_{SM} = f(\Delta_{K\pi}) = \underbrace{\frac{F_K |\mathcal{V}^{us}|}{F_\pi |\mathcal{V}^{ud}| f_+(0) |\mathcal{V}^{us}|} |\mathcal{V}^{ud}|}_{B_{exp}} + \Delta_{CT}$$

- Experimental results :
  - → $\left(\frac{F_K}{F_\pi} \left| \frac{\mathcal{V}^{us}}{\mathcal{V}^{ud}} \right| \right) = 0.27618(48)$ [updated using recent KLOE measurement of $K_{\mu 2}$]
  - → $f_+^{K^0}(0) |\mathcal{V}^{us}| = 0.21619(55)$ Average of most recent measurements of NA48, KTeV, KLOE.
  - → $|\mathcal{V}^{ud}| = 0.97377(26)$ [Towner & Hardy] ($0^+ \to 0^+$) updated by [Marciano & Sirlin '05]

⇨ $C_{SM} = 1.2440 \pm 0.0039 + \Delta_{CT}$

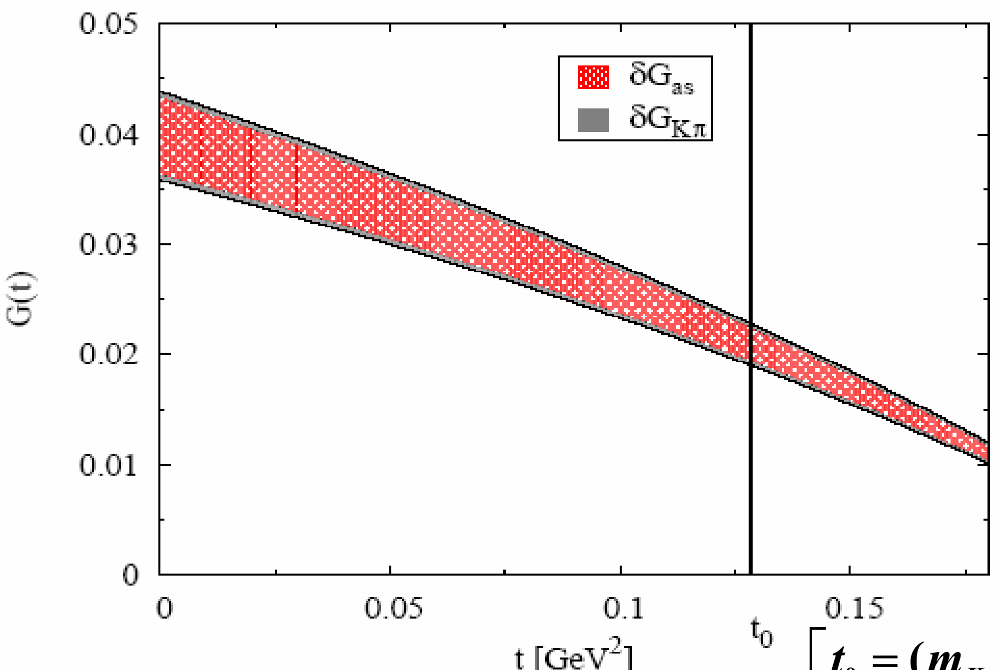
$$\ln C_{SM} = 0.2183 \pm 0.0031 + \frac{\Delta_{CT}}{B_{exp}}$$

- Estimate of the uncertainties :
  - →Conservative estimate of $\delta G_{as}(t)$: For $t>2.77$ GeV², $\phi(t) = \pi \pm \pi$

$$\delta G_{as}(\Lambda,t) = G_{as}(\Lambda,t) = \frac{\Delta_{K\pi}}{t}\ln\left(1-\frac{t}{\Lambda}\right) - \ln\left(1-\frac{\Delta_{K\pi}}{\Lambda}\right)$$

$\delta G_{as}(t)<0.0036$ in the physical region

  - →Estimate of $\delta G_{K\pi}(t)$: $\delta G_{K\pi}(t) \le 0.05 \times G_{K\pi}(t)$



$$\left[t_0 = (m_K - m_\pi)^2\right]$$

G(t) with the uncertainties added in quadrature

$$G(0) = 0.0398 \pm 0.0040$$

G(t) does not exceed 20% of the expected value of lnC

lnC~0.20

- Elastic up to ~1.4 GeV (K*(1414)) :

$$\Rightarrow \quad t < \Lambda = 2\ GeV^2 : \ \phi(t) = \delta_{\pi,K}^{p,\frac{1}{2}}(t)$$
$$t > \Lambda \quad : \ \phi(t) = \pi$$

$$H(t) = \frac{m_\pi^2\, t}{\pi} \int_{t_{K\pi}}^{\infty} \frac{ds}{s^2} \frac{\phi(s)}{(s-t)} \quad \Rightarrow \quad H(t) = H_{K\pi}(\Lambda, t) + H_{as}(\Lambda, t) \pm \delta H$$

$$H_{K\pi}: \int_{t_{K\pi}}^{\Lambda} \qquad H_{as}: \int_{\Lambda}^{\infty}$$

- Estimate of the uncertainties :
  - Conservative estimate of $\delta H_{as}(t)$: For t>2 GeV², $\phi(t) = \pi \pm \pi$

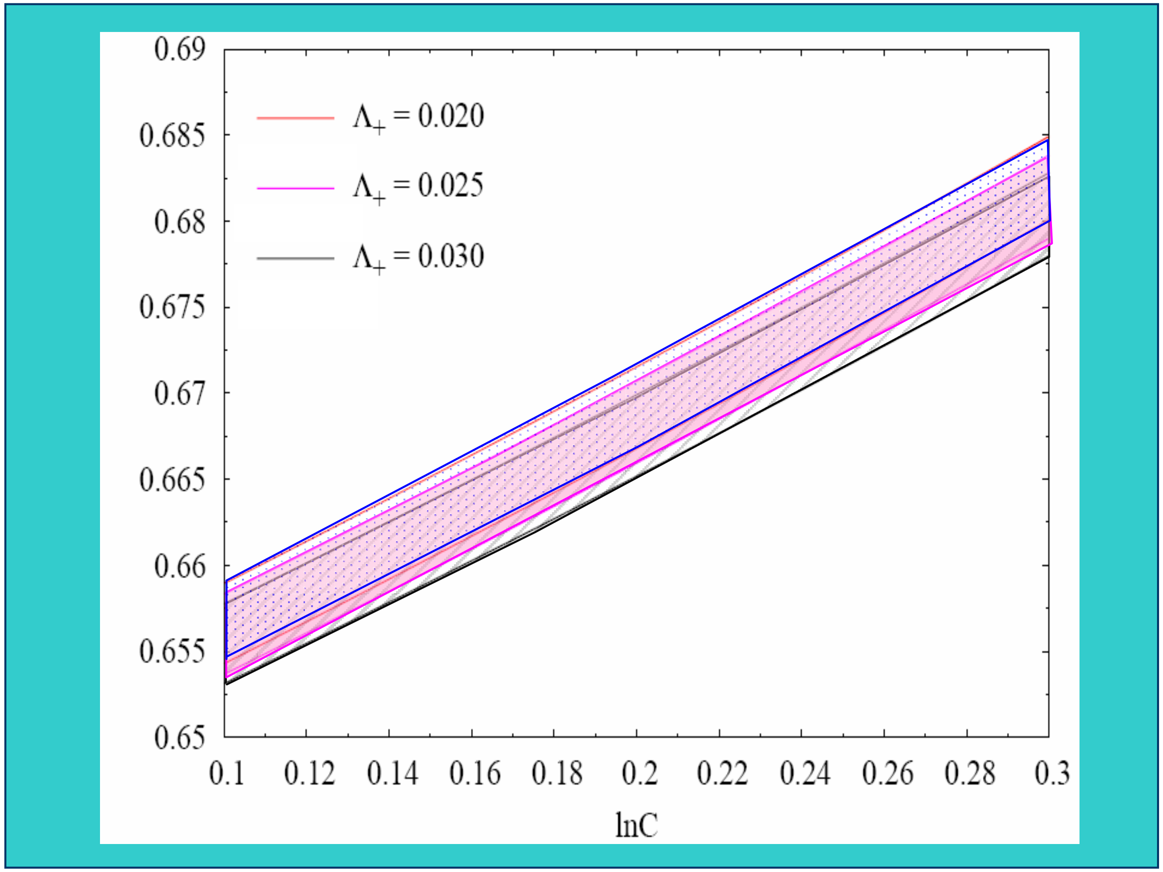
$$\delta H_{as}(\Lambda, t) = H_{as}(\Lambda, t) = -\frac{m_\pi^2}{t} \ln\left(1 - \frac{t}{\Lambda}\right) - \frac{m_\pi^2}{\Lambda}$$

$\delta H_{as}(t) < 0.0033$ in the physical region

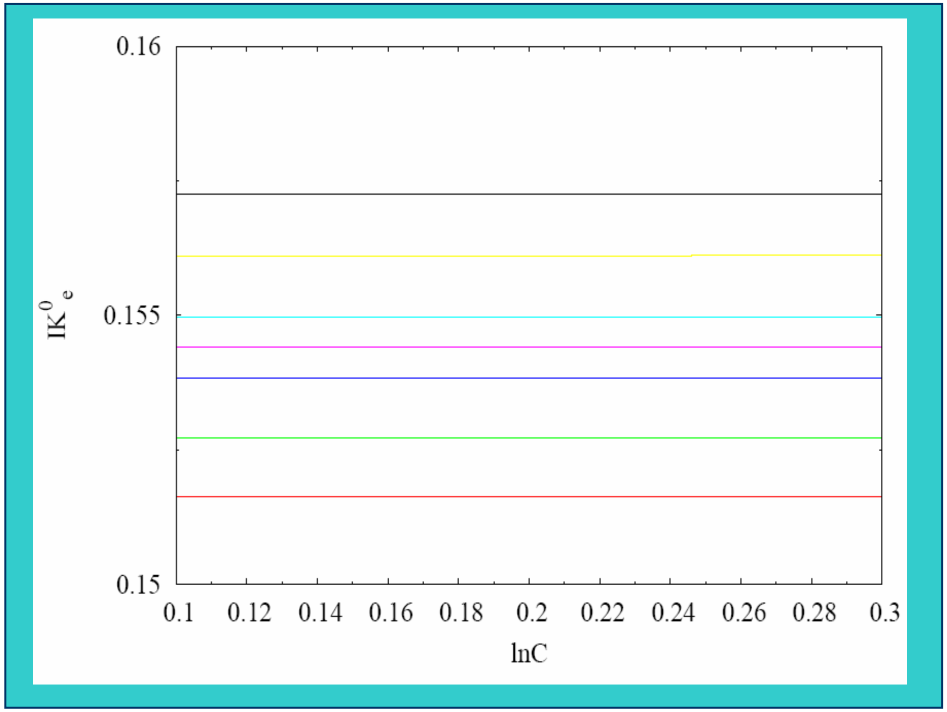
  - Estimate of $\delta H_{K\pi}(t)$: uncertainties of Aston et al.

4.2 Branching Ratios.

- Dependance of the Br of K^0 in Λ_+ and lnC:

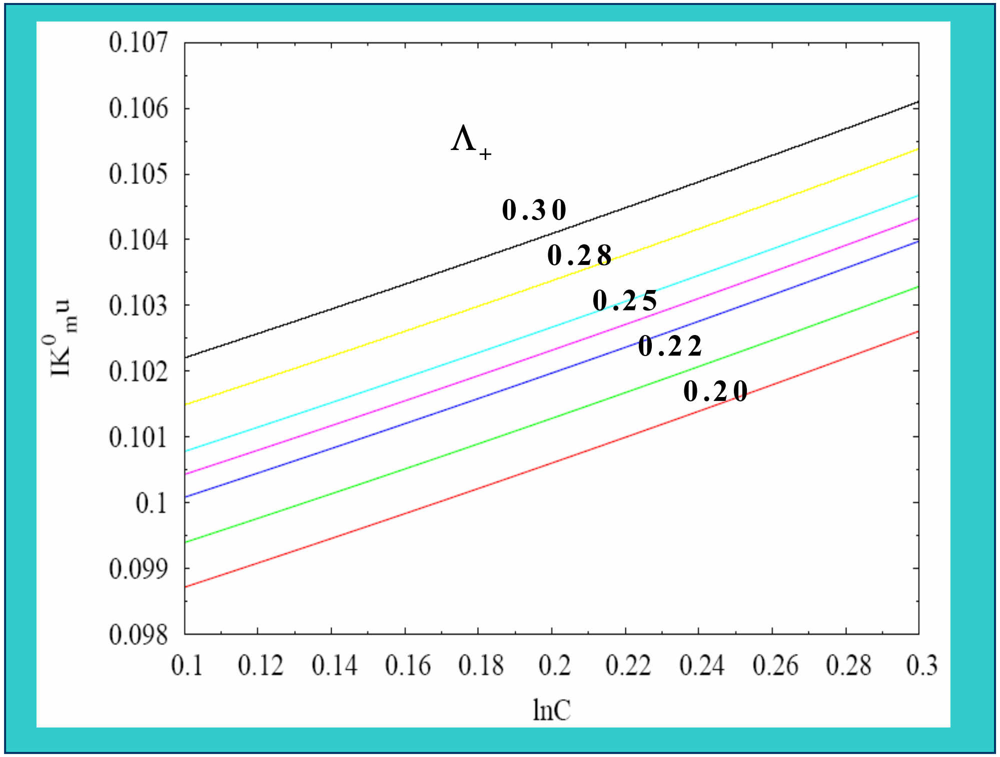


- Dependance of I_K^l to the 2 parameters lnC and Λ_+ : For example, take the K^0 case :



- $I_{K^0}^e$ independant of lnC.

- Dependance of I_K^l to the 2 parameters lnC and Λ_+ : For example, take the K^0 case :

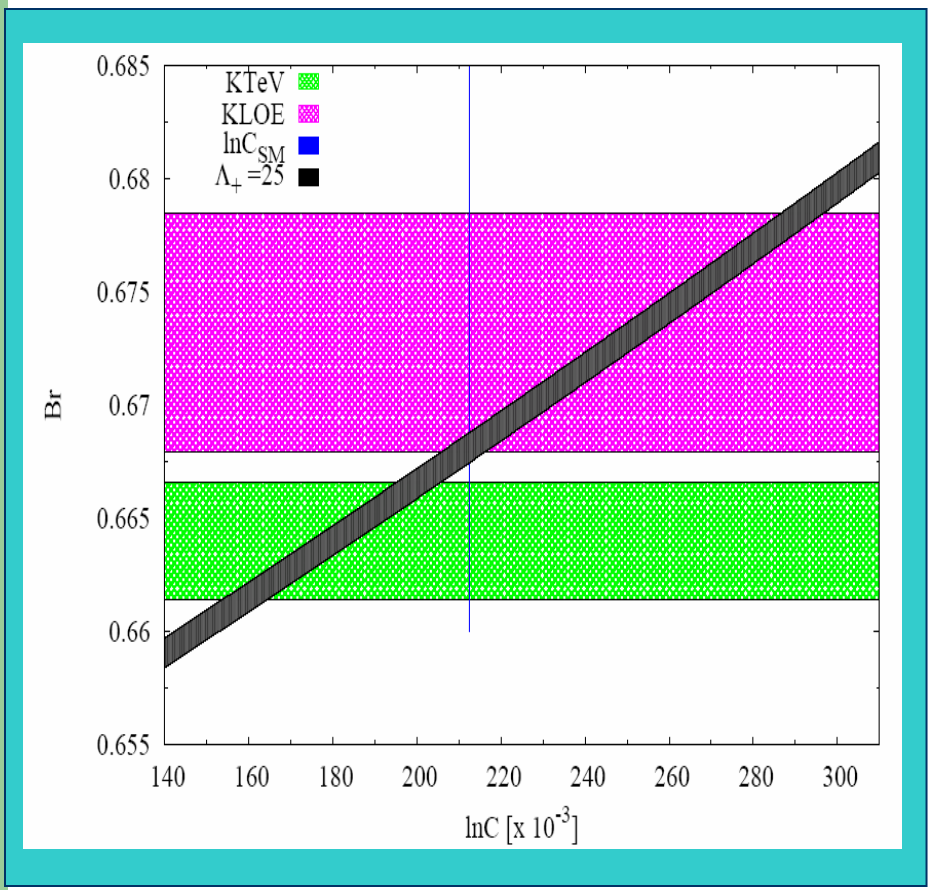


- Of course I_K^e independant of lnC.

- I_K^μ weakly dependant on $\Lambda_{+.}$.

4.2 Branching Ratios.

- Taking Λ_+ =0.025, we can compare with the measured Br :
  - For K^0 :



KLOE :

$$Br(K^0) = 0.6732 \pm 0.0053$$

$$\Rightarrow \quad 0.210 < \ln C < 0.290$$

KTeV :

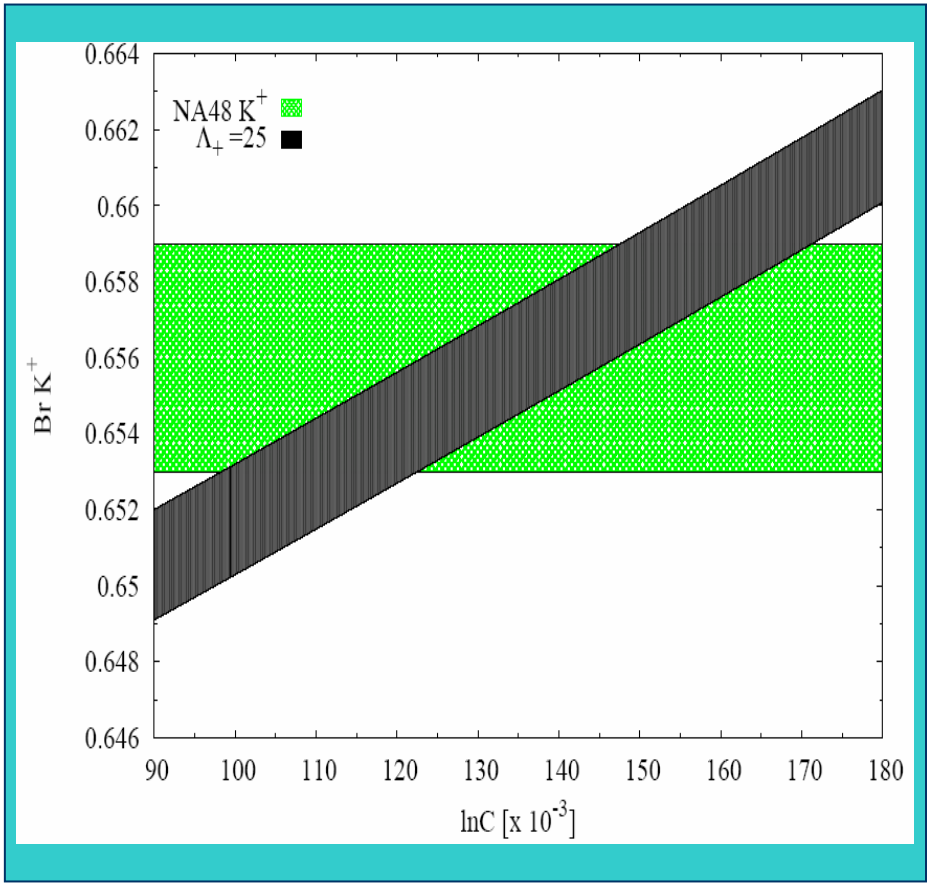
$$Br(K^0) = 0.6640 \pm 0.0026$$

$$\Rightarrow \quad 0.160 < \ln C < 0.200$$

Incompatibility between the experiments ?
Not possible to determine lnC precisely enought from Br measurements !

4.2 Branching Ratios.

- Taking $\Lambda_+ = 0.025$, we can compare with the measured Br :
  - For K^+ :



NA48 :

$$Br(K^+) = 0.656 \pm 0.003$$

Not possible to determine lnC Precisely enought as well for K^+ !