Review on Bell-Steinberger relation

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- Based also on work with KLOE collaboration and Gino Isidori

Outline

- Motivation
  - The CPT symmetry
- Bell-Steinberger: history
- Bell-Steinberger: update
- Conclusions

CPT symmetry

- Hermiticity of the Hamiltonian (probabilty conservation), QFT
- Locality
- Lorentz invariance

$\Longrightarrow$ CPT conservation

CPT violated at the Plank scale

- Quantum gravity may lead to CPT violation
- The low energy limit not known
- Interesting probe

$$|M_K - M_{\bar{K}}| < 10^{-18} M_K$$

CPT violation: Non-locality?

- Non-locality is enough? Barenboim,Lykken

Add to the Dirac lagrangian

$$\mathbf{S} = \frac{i\eta}{\pi} \int d^3x \int dt dt' \, \bar{\psi}(t, \mathbf{x}) \, \frac{1}{t - t'} \, \psi(t', \mathbf{x}).$$

Run into trouble with causality

We want still keep states that go from an initial state to a final state in a S-matrix approach

CPT violation: Break Lorentz invariance

Change the coefficient of the square of the magnetic field in the Lagrangian of quantum electrodynamics:

$$\vec{B}^2 \rightarrow (1+\epsilon)\vec{B}^2$$

This will cause the velocity of light c, given by $c^2 = 1+\epsilon$ to differ from the maximum velocity of particles, which remains equal to one.

$$\mathcal{L}^{eff}_{SU(3)\times SU(2)\times U(1)} = \tfrac{1}{2} i\overline{\psi}\Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \overline{\psi} M \psi$$

Spurions break Lorenz sym. Coleman-Glashow,Kostelecky et al.

$$\Gamma^\nu \equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu}$$

QM mechanics must be valid even if $C\not{P}T$ in the K's mass matrix

$$i\frac{d}{dt}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = [M - i\Gamma/2]\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix}$$

For any superposition of K_S, K_L mass and width eigenstates

$$|\Psi\rangle = a|K_S\rangle + b|K_L\rangle$$

$$\sum_\Gamma \mid \langle\Gamma \mid T \mid \Psi\rangle \mid^2 = -\frac{d}{d\tau} \mid \Psi \mid^2$$

Bell Steinberger relations

Terms proportional to $\mid a \mid^2$ and $\mid b \mid^2$

$$\Gamma_L = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_L \rangle \mid^2 \qquad \Gamma_S = \sum_\Gamma \int d\Gamma \mid \langle \Gamma \mid T \mid K_S \rangle \mid^2$$

Mixed terms, proportional to $ab^* \Longrightarrow$

$$-i(M_L^* - M_S) \overbrace{\langle K_L \mid K_S \rangle}^{\frac{2\text{Re}(\tilde{\epsilon})}{1+|\tilde{\epsilon}|^2}} = \sum_\Gamma \int d\Gamma \overbrace{\left(\langle \Gamma \mid T \mid K_L \rangle\right)^* \langle \Gamma \mid T \mid K_S \rangle}^{\alpha_f} .$$

$$\left(\frac{\Gamma_S+\Gamma_L}{2}-i\Delta m\right)\frac{2\mathrm{Re}(\tilde{\epsilon})}{1+\mid\tilde{\epsilon}\mid^2}=\sum_{\Gamma}\int d\Gamma\left(\langle\Gamma\mid T\mid K_L\rangle\right)^*\langle\Gamma\mid T\mid K_S\rangle$$

Using the Schwartz inequality

$$\left|\frac{\Gamma_S+\Gamma_L}{2}-i\Delta m\right|\frac{2\mathrm{Re}(\tilde{\epsilon})}{1+\mid\tilde{\epsilon}\mid^2}\leq\sqrt{\Gamma_L\Gamma_S}\implies\frac{2\mathrm{Re}\tilde{\epsilon}}{1+\mid\tilde{\epsilon}\mid^2}\leq 2.9\times 10^{-2}$$

$\implies$ BS relation among $\Gamma_L, \Gamma_S, \Delta m, p, q$ and α_f

BS assume $\langle K_L\mid K_S\rangle=\frac{\mid p\mid^2-\mid q\mid^2}{\mid p\mid^2-\mid q\mid^2}\stackrel{BS}{=}\frac{2\mathrm{Re}(\tilde{\epsilon})}{1+\mid\tilde{\epsilon}\mid^2}$

$C\not{P}T$ in the K's mass matrix

Diagonalize $\begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$

$$K_{S,L} = \frac{1}{\sqrt{2\left(1 + |\epsilon_{S,L}|^2\right)}} \left[(1 + \epsilon_{S,L})\, K^0 + (1 - \epsilon_{S,L})\, \bar{K}^0 \right]$$

$$\begin{aligned} \epsilon_{S,L} &= \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &= \epsilon_M \mp \Delta \end{aligned}$$

$$\epsilon_M \equiv |\epsilon_M| e^{i\varphi_{SW}} \qquad \tan\varphi_{SW} = \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$$

$C\not{P}T$ in semileptonic decays

$$
\begin{aligned}
A(K^0 \to l^+ \nu \pi^-) &= a + b = a(1-y) \\
A(K^0 \to l^- \nu \pi^+) &= c + d = a^*(x_+ - x_-)^* \\
A(\bar{K}^0 \to l^- \nu \pi^+) &= a^* - b^* = a^*(1+y)^* \\
A(\bar{K}^0 \to l^+ \nu \pi^-) &= c^* - d^* = a(x_+ + x_-)
\end{aligned}
$$

$b, d \quad (y, x_-) \qquad C\not{P}\not{T}$ $\qquad c, d \quad (x_+, x_-) \qquad \Delta S = -\Delta Q$

$$
A_{S,L} = \frac{\Gamma^{l^+}_{S,L} - \Gamma^{l^-}_{S,L}}{\Gamma^{l^+}_{S,L} + \Gamma^{l^-}_{S,L}} = 2\Re(\epsilon_{S,L}) + 2\Re\left(\frac{b}{a}\right) \mp 2\Re\left(\frac{d^*}{a}\right)
$$

- $A_S - A_L = 4(\Re(\Delta) + \Re(x_-))$ $\qquad A_S + A_L = 4(\Re(\epsilon_M) - \Re(y))$

$C\not{P}T$ in $K \to \pi\pi$

$$A(K^0 \to \pi\pi(I)) \equiv (A_I + B_I)e^{i\delta_I}$$
$$A(\bar{K}^0 \to \pi\pi(I)) \equiv (A_I^* - B_I^*)e^{i\delta_I}$$

- B_I is $C\not{P}T$ as $\left(\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}} \quad \eta_{00}=|\eta_{00}|e^{i\phi_{00}}\right)$

$$\phi_{+-} - \phi_{00} = 0.2 \pm 0.4^o \qquad \text{KTEV,NA48}$$

Bell Steinberger relations: CPLEAR,KTeV,NA48,KLOE

$$\left[\frac{\Gamma_S+\Gamma_L}{\Gamma_S-\Gamma_L}+i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2}-i\Im(\Delta)\right]=\frac{1}{\Gamma_S-\Gamma_L}\sum_f \overbrace{A_L(f)A_S^*(f)}^{(\alpha_f)}$$

Also a new analysis by Gino+KLOE and me

Actual SM expectations for Bell Steinberger relations, KLOE+Gino

Channel	$B(K_S)$	$B(K_L)$	$10^5\ \alpha_f^{\mathrm{SM}}$
$\pi^+\pi^-(\gamma)$	0.69	2.1×10^{-3}	$110.8 + 105.1i$
$\pi^0\pi^0$	0.31	9.3×10^{-3}	$49.2 + 46.6i$
$\pi^\pm e^\mp \nu$	6.7×10^{-4}	0.39	$0.22 + 0.00i$
$\pi^\pm \mu^\mp \nu$	4.7×10^{-4}	0.27	$0.17 + 0.00i$
$\pi^0\pi^0\pi^0$	1.9×10^{-9}	0.21	$0.06 + 0.06i$
$\pi^+\pi^-\pi^0$	2.7×10^{-7}	0.12	$0.04 + 0.04i$
$\pi^+\pi^-\gamma_{\mathrm{DE}}$	10^{-5}	10^{-5}	< 0.01

α_f determinations for Bell Steinberger relations

$$\alpha_{\pi^+\pi^-} = \big((1.115 \pm 0.015) + i(1.055 \pm 0.015)\big) \times 10^{-3}$$
$$\alpha_{\pi^0\pi^0} = \big((0.489 \pm 0.007) + i(0.468 \pm 0.007)\big) \times 10^{-3}$$

$\alpha_{\pi\pi\pi}$ from CPLEAR, NA48, KLOE

Time dependent studies: CPLEAR

CPLEAR Study of tagged $K^0(\bar{K}^0)$

$$\frac{\Gamma(K^0(t) \to f) - \Gamma(\bar{K}^0(t) \to f)}{\Gamma(K^0(t) \to f) + \Gamma(\bar{K}^0(t) \to f)}$$

$$\left[\Re\left(A_L^f A_S^{f*}\right)\cos(\Delta m t) + \Im\left(A_L^f A_S^{f*}\right)\sin(\Delta m t)\right]$$

$$\Longrightarrow \Delta m, \qquad A(K_{L,S} \to \pi^+\pi^-\pi^0)\,, \qquad A(K_{L,S} \to \pi^0\pi^0\pi^0)\,,$$

$$A(K_{L,S} \to \pi l \nu)$$

CPLEAR determination of semileptonic $\alpha_{\pi l \nu}$

$$\sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle = 2\Gamma(K_L \to \pi\ell\nu)\left(\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\right.$$

$$= 2\Gamma(K_L \to \pi\ell\nu)\left((A_S + A_L)/4 - i(\Im(x_+) + \Im(\Delta))\right)$$

Time asymm. allow CPLEAR to obtain $\alpha_{\pi l \nu}$ from

	value	Correlation coefficients			
$\Re(\Delta)$	$(3.0 \pm 3.4) \times 10^{-4}$	1			
$\Im(\Delta)$	$(-1.5 \pm 2.3) \times 10^{-2}$	0.44	1		
$\Re(x_-)$	$(0.2 \pm 1.3) \times 10^{-2}$	-0.56	-0.97	1	
$\Im(x_+)$	$(1.2 \pm 2.2) \times 10^{-2}$	-0.60	-0.91	0.96	1

KLOE determination of semileptonic $\alpha_{\pi l \nu}$

KLOE adds the measurement of $A_S - A_L = 4[\Re(\delta) + \Re(x_-)] = (-2 \pm 10) \times 10^{-3}$. The results, referred to as the $K_{\ell 3}$ average, are given in

	value	Correlation coefficients				
$\Re(\Delta)$	$(3.4 \pm 2.8) \times 10^{-4}$	1				
$\Im(\Delta)$	$(-1.0 \pm 0.7) \times 10^{-2}$	-0.27	1			
$\Re(x_-)$	$(-0.07 \pm 0.25) \times 10^{-2}$	-0.23	-0.58	1		
$\Im(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	-0.35	-0.12	0.57	1	
$A_S + A_L$	$(0.5 \pm 1.0) \times 10^{-2}$	-0.12	-0.62	0.99	0.54	1

Bell-Steinberger determination

Unitarity allows a determination of $\Re(\epsilon)$ not requiring CPT

$$\Re(\epsilon) = (164.9 \pm 2.5) \times 10^{-5}, \qquad \Im(\Delta) = (2.4 \pm 5.0) \times 10^{-5}.$$

$$\Delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] \,.$$

$$-5.3 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\overline{K}^0} < 6.3 \times 10^{-19} \text{ GeV} \quad \text{at 95 \% CL}\,.$$

improving CPLEAR $|m_{K^0} - m_{\overline{K}^0}| < 12.7 \times 10^{-19}$ GeV at 90% CL

Conclusions

- We are seeing the determination of the unitarity of the row of the CKM matrix at 0.2% fighting for small possible NP contributions; HERE we do not even know how large is NP contributions: could be very large..
- Remember CP lesson (not theoretically predicted)
- Scaling argument correct?