Lattice Progress on ε´/ε

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For the RBC and UKQCD Collaborations

Kaon 2007
INFN Frascati
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- Introduction and reminder about quenched ε´/ε results
- Progress with 2+1 flavor dynamical QCD simulations
- Chiral perturbation theory and lattice data for m_π and f_π
- The denominator of ε´/ε: B_K and ε
- Preliminary results for $\Delta S = 1$ matrix elements for ε´/ε
- Conclusions

CP Violation in the Kaon System

- Two amplitudes determine ϵ and ϵ'

$$\eta_{+-} = \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \epsilon + \epsilon' \qquad \eta_{00} = \frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_S^0 \to \pi^0\pi^0)} = \epsilon - 2\epsilon'$$

- SM: $\overline{K}^0 - K^0$ mixing via $Q^{(\Delta S=2)} = (\bar{s}_\alpha d_\alpha)_{V-A}\,(\bar{s}_\beta d_\beta)_{V-A}$ defines B_K as;

$$\boxed{\langle \overline{K}^0 | Q^{(\Delta S=2)}(\mu) | K^0 | \rangle} \equiv \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$

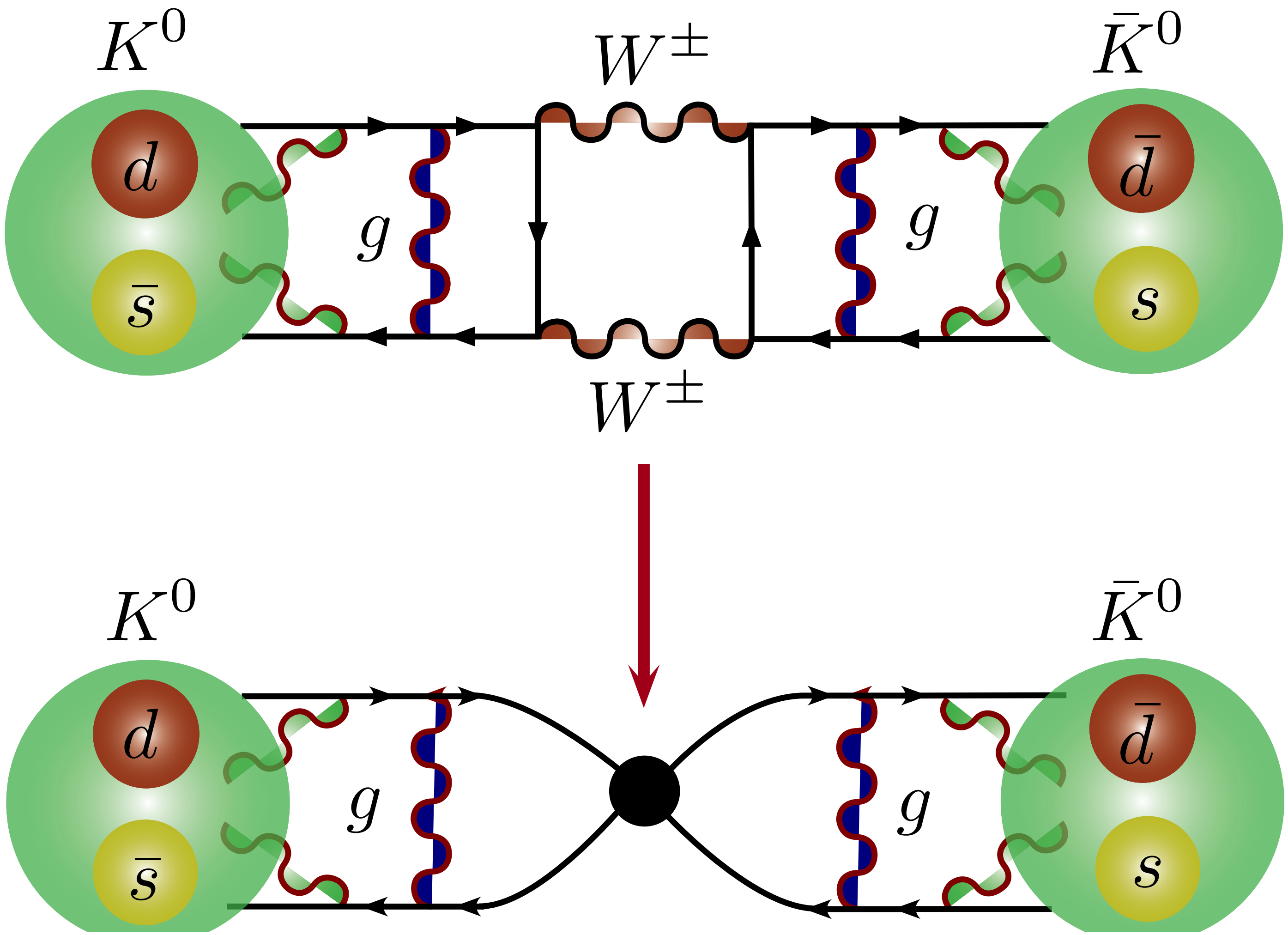
- RGI parameter $\hat{B}_K \equiv B_K(\mu)\left[\alpha_s^{(3)}(\mu)\right]^{-2/9}\left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3\right]$ relates SM and ϵ

$$\epsilon = \hat{B}_K\, \mathrm{Im}\lambda_t \frac{G_F^2 f_K^2 m_K M_W^2}{12\sqrt{2}\pi^2 \Delta M_K} \{\mathrm{Re}\lambda_c\left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right] - \mathrm{Re}\lambda_t\, \eta_2 S_0(x_t)\} \exp(i\pi/4)$$

- Defining $\boxed{A(K^0 \to \pi\pi(I))} \equiv A_I e^{(i\delta_I)}$, $P_2 \equiv \mathrm{Im}A_2/\mathrm{ReA}_2$, $P_0 \equiv \mathrm{Im}A_0/\mathrm{ReA}_0$:

$$\epsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\right)\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right) \qquad w \equiv \frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2} \approx 22$$

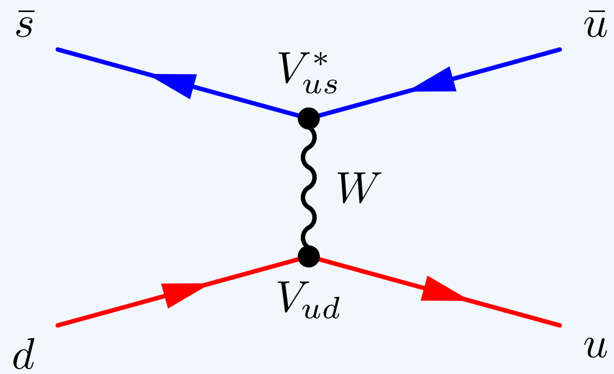
Low Energy Standard Model Diagrams for B_K



Electroweak process at high energy scales reduce to a single 4-fermion operator at low energies

Need correctly normalized value of the $Q^{\Delta S=2}$ operator in kaon states.

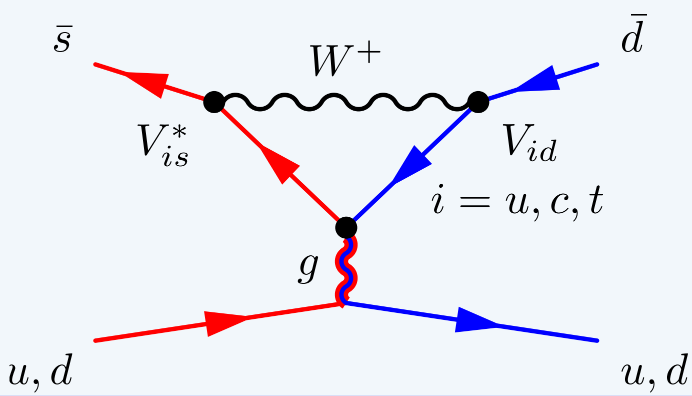
Standard Model Diagrams for ε´/ε



Exchange diagrams

$Q_1 + Q_2$

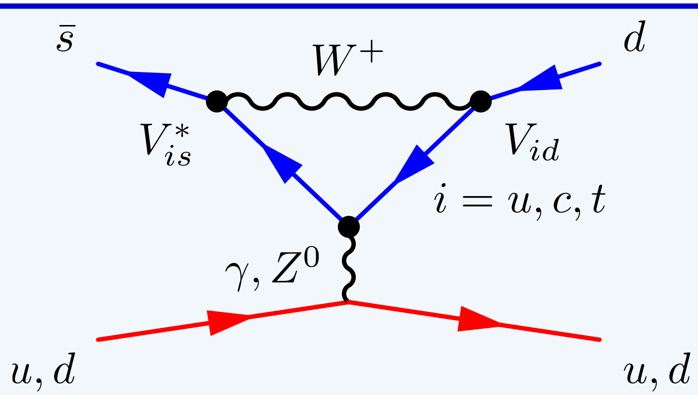
These should dominate ΔI = 1/2 rule at μ = 2 GeV



Gluonic penguins

$Q_3 + Q_4 + Q_5 + Q_6$

Q_6 should dominate ΔI = 1/2 contribution to ε´/ε at μ = 2 GeV



Electroweak penguins

$Q_7 + Q_8 + Q_9 + Q_{10}$

Q_8 should dominate ΔI = 3/2 contribution to ε´/ε at μ = 2 GeV

$K \to \pi\pi$ in 3-flavor Effective Theory

- Hamiltonian for 3-flavor effective theory: only 7 of 10 operators independent

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $K \to \pi\pi$ from lattice calculations and LO chiral perturbation theory.

Irrep	Isospin	$K^+ \to \pi^+$	$K^0 \to \pi^+\pi^-$
(27,1)	1/2, 3/2	$-\frac{4m_M^2}{f^2}\alpha^{(27,1)}$	$-\frac{4i}{f^3}m_{K^0}^2\alpha^{(27,1)}$
(8,8)	1/2, 3/2	$-\frac{12}{f^2}\alpha^{(8,8)}$	$-\frac{12i}{f^3}\alpha^{(8,8)}$
(8,1)	1/2	$\frac{4m_M^2}{f^2}\left(\alpha_1^{(8,1)} - \alpha_2^{(8,1)}\right)$	$\frac{4i}{f^3}m_{K^0}^2\alpha_1^{(8,1)}$

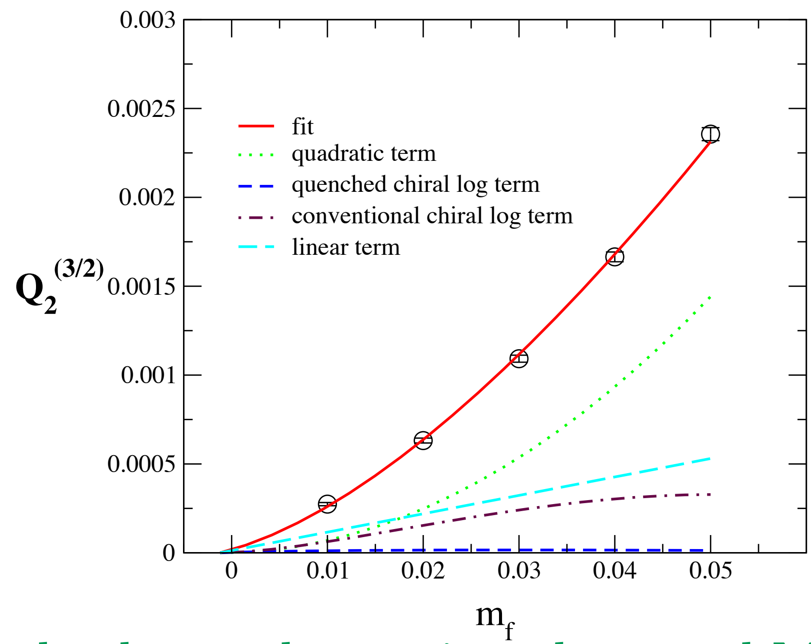
- (8,1) coefficient $\alpha_2^{(8,1)}$ is power divergent, $\mathcal{O}(1/a^2)$. Determine from $K \to |0\rangle$

Quenched Chiral Extrapolations (27,1) and (8,1)

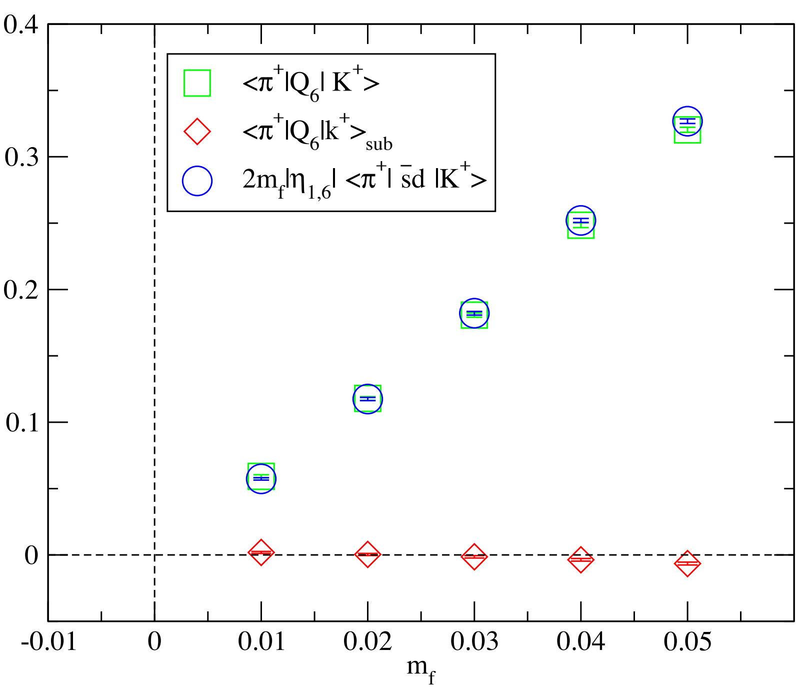
Fit with known continuum chiral logarithm for quenched theory

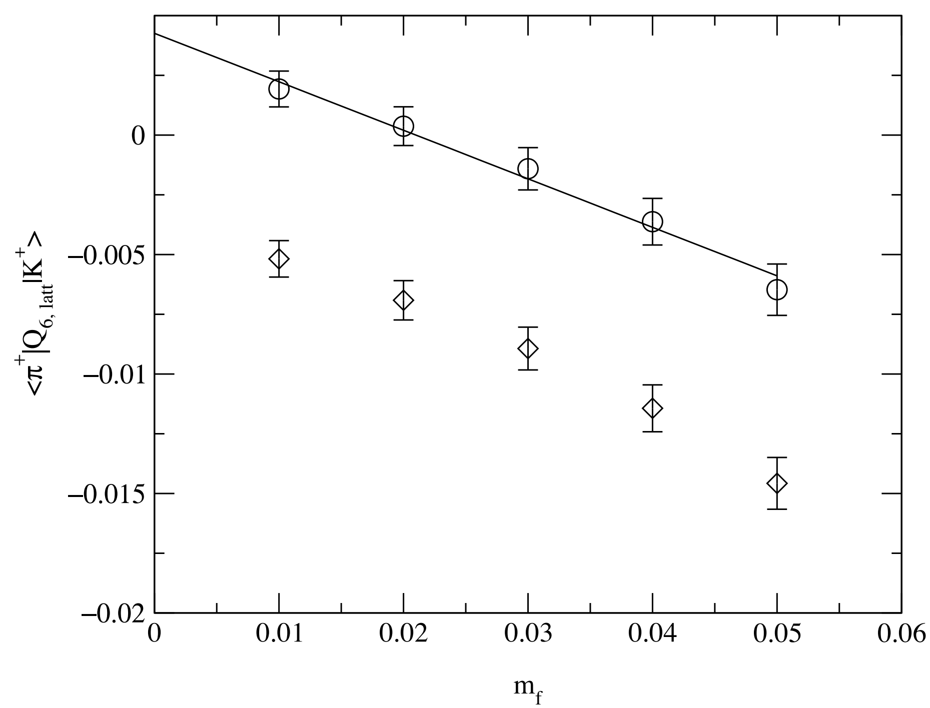
$$1 - \frac{6m_M^2}{(4\pi f)^2}\ln(m_M^2/\Lambda^2)$$

Good description of data, but
$400\ \text{MeV} \le m_{\text{PS}} \le 800\ \text{MeV}$

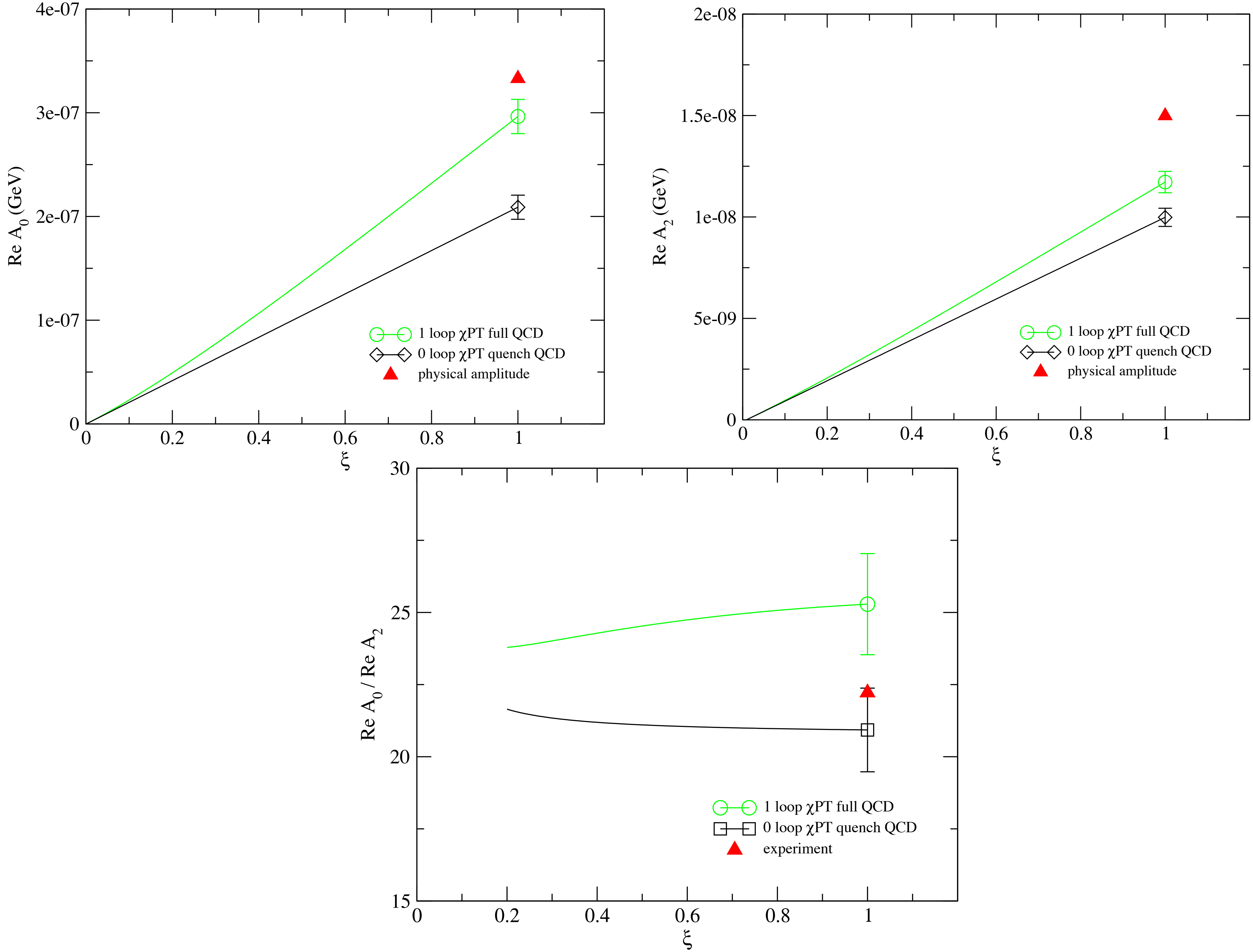


Only slope relevant in subtracted ME





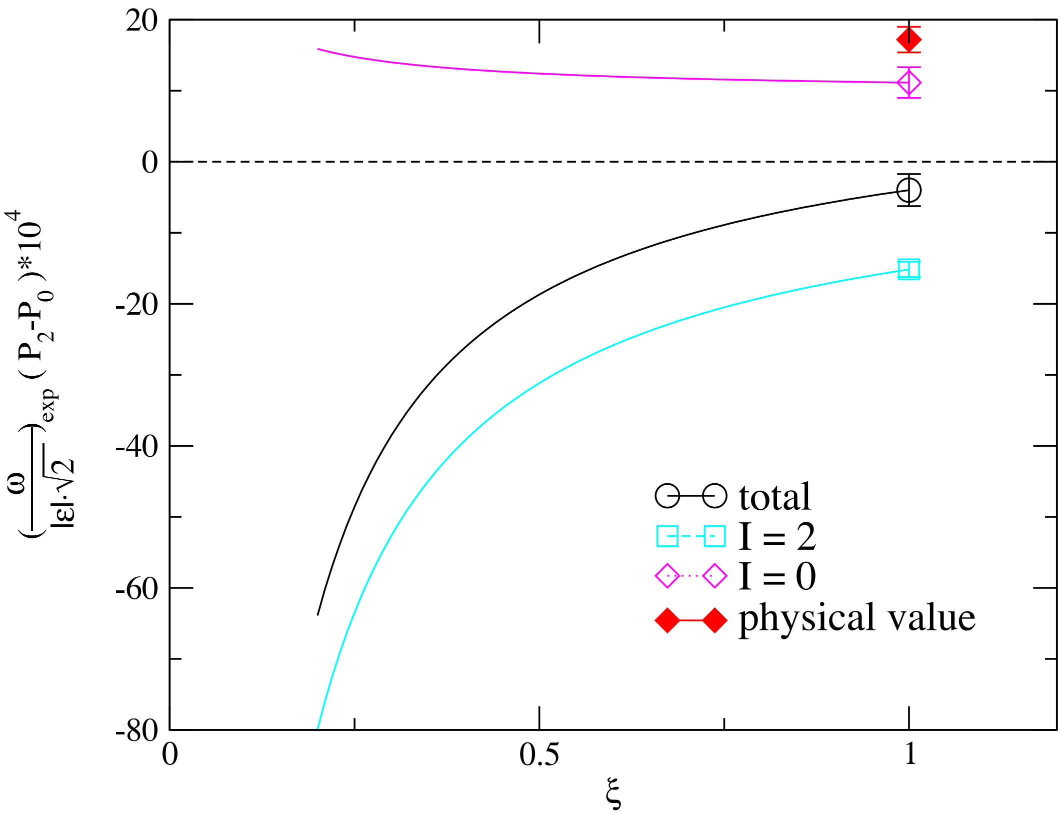
Real $K \to \pi\pi$ Amplitudes from Quenched QCD and χPT



ϵ'/ϵ from Quenched QCD and χPT

- Dominant contribution: Q_2 to Re A_2 and Re A_0, Q_6 to Im A_0, Q_8 to Im A_2.
- Contributions depend on renormalization scale GeV
- Schematic formula for ϵ'/ϵ

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \left(\frac{\omega}{\sqrt{2}|\epsilon|}\right)_{\mathrm{exp}} \left\{ \left[\frac{\alpha_{\mathrm{W}}\alpha_8}{\alpha_{\mathrm{W}}\alpha_8 + \alpha_2 m_{K^0}^2 \xi}\right]^{(3/2)} - \left[\frac{\alpha_{\mathrm{W}}\alpha_8 + \alpha_{\mathrm{S}}\alpha_6 m_{K^0}^2 \xi}{\alpha_{\mathrm{W}}\alpha_8 + \alpha_2 m_{K^0}^2 \xi}\right]^{(1/2)} \right\}$$



Achieving Accurate Kaon Physics on the Lattice

Issue	Current status
Quenched approximation	2+1 flavor DWF and ASQTAD
Chiral symmetry breaking	Staggered fermions Twisted mass Wilson fermions Domain wall fermion ✓ Overlap fermions ✓
Heavy pions	ASQTAD: one pion has $m_l = m_s/10$ DWF: correct light pions with $m_l = m_s/7$
Operator Renormalization	Non-perturbative renormalization (NPR) Schrodinger functional methods
Extrapolation to chiral limit	Chiral perturbation theory: DWF - continuum like ASQTAD - include taste breaking
Multiparticle final states	1) Avoid via ChPT 2) Use finite volume effects
More computing speed	Many sustained Teraflops currently

25,000 nodes at Brookhaven
RBRC and USDOE machines

14,000 nodes at the University of Edinburgh

Collaboration Members

RBC members:

Y. Aoki, C. Aubin, T. Blum, M. Cheng, N. Christ, S. Cohen, C. Dawson, T. Doi, K. Hashimoto, T. Ishikawa, T. Izubuchi, C. Jung, M. Li, S. Li, M. Lightman, H. Lin, M. Lin, O. Loktik, R. Mawhinney, S. Ohta, S. Sasaki, E. Scholz, A. Soni, T. Yamazaki

UKQCD members:

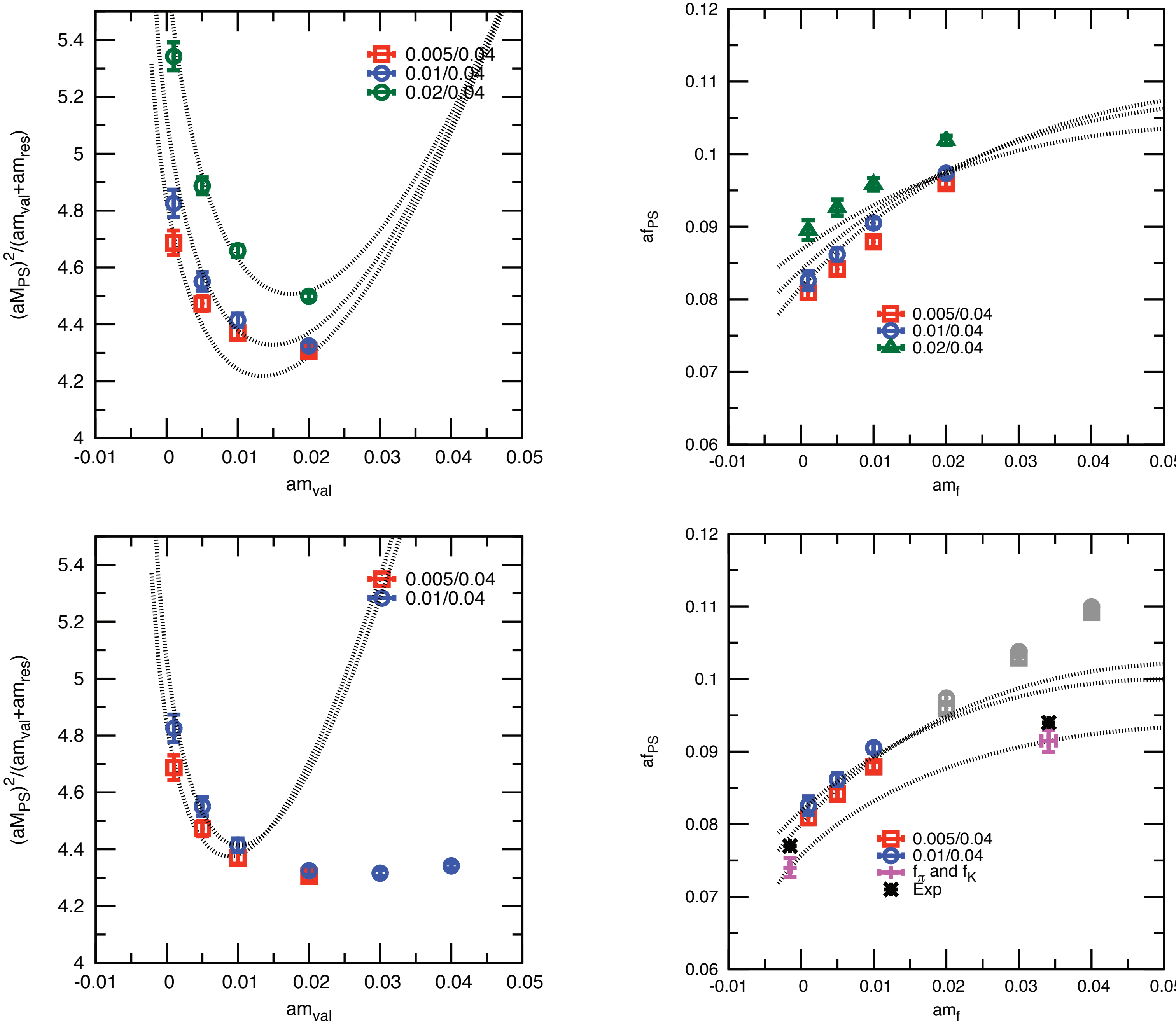
C. Allton, D. Antonio, K. Bowler, P. Boyle, M. Clark, J. Flynn, A. Hart, B. Joo, A. Juettner, A. Kennedy, R. Kenway, C. Kim, C. Maynard, J. Noaki, B. Pendleton, C. Sachrajda, A. Trivini, R. Tweedie, J. Wennekers, A. Yamaguchi, J. Zanotti

Zero Temperature Ensembles

Volume	a^{-1} (GeV)	(m_l, m_s)	m_{res}	MD time units
$16^3 \times 32 \times 12$	1.69(5)	(0.02,∞) (0.03,∞) (0.04,∞)	0.00137(5)	2680.5 3097.5 3252.5
$16^3 \times 32 \times 8$	1.8(1)	(0.02, 0.04) (0.04, 0.04)	0.0107(1)	1797.5 1797.5
$16^3 \times 32 \times 16$	1.62(4)	(0.01, 0.04) (0.02, 0.04) (0.03,0.04)	0.00308(4)	4015 4045 4020+3580
$24^3 \times 64 \times 16$	1.6-1.7	(0.005,0.04) (0.01,0.04) (0.02,0.04) (0.03,0.04)	0.0031	4500 3785 2850 2813
$32^3 \times 64 \times 16$	2.1-2.2	(0.004, 0.03) (0.006, 0.03)	≈ 0.0005	500 892

First row is with DBW2 gauge action, all others use the Iwasaki action.

Partially Quenched NLO ChPT for m_π and f_π



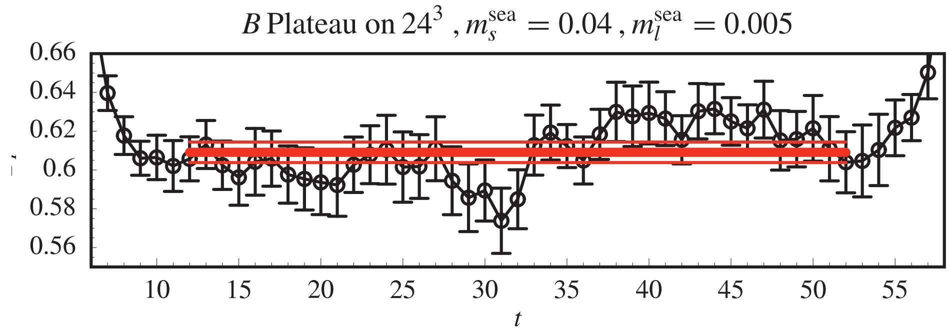
$(3\ \text{fm})^3$ volume	
m_{dyn} (MeV)	m_{val} (MeV)
550	550
400	400
320	320
	220
400	400
320	320
	220

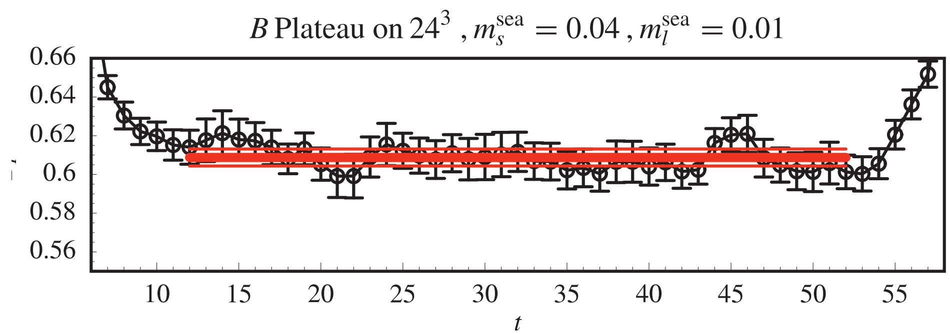
Meifeng Lin
DWF@10

B_K Plateau

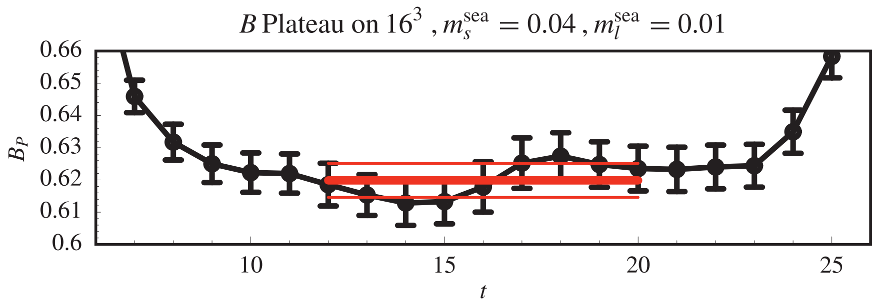
Saul Cohen

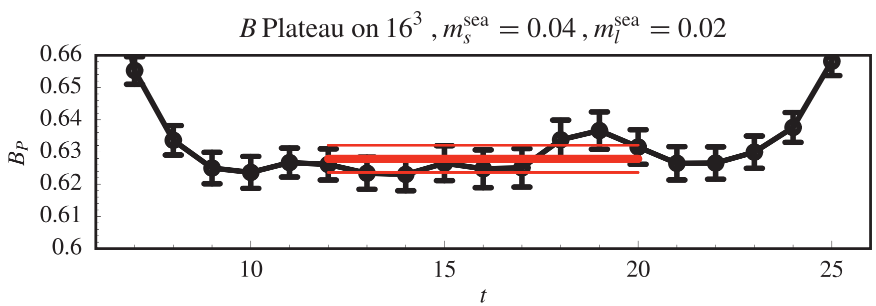
24^3 volume

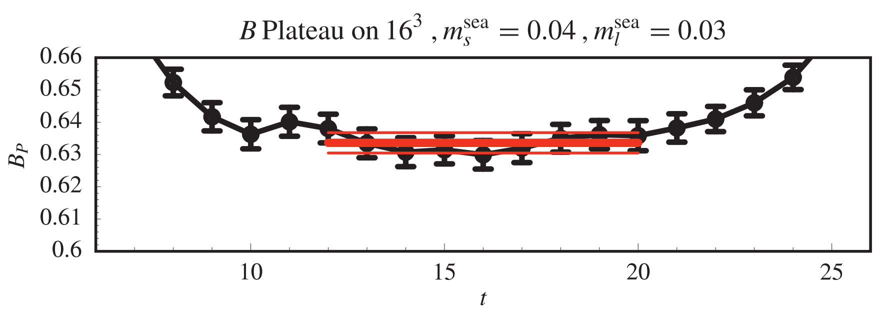




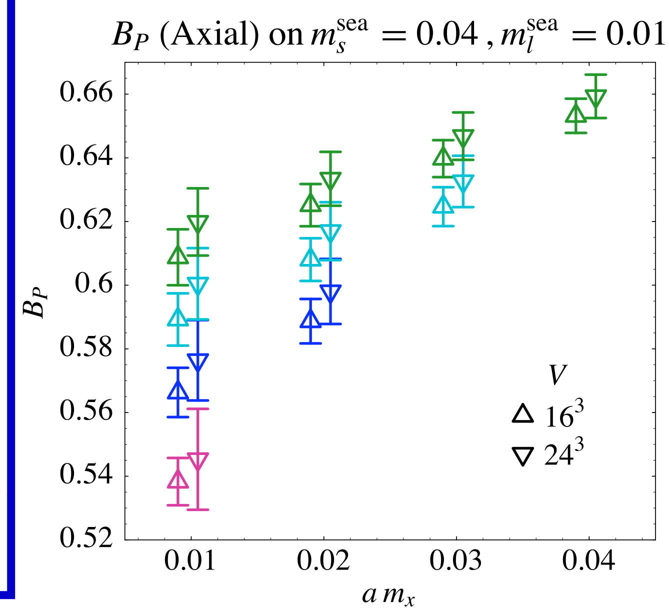
16^3 volume







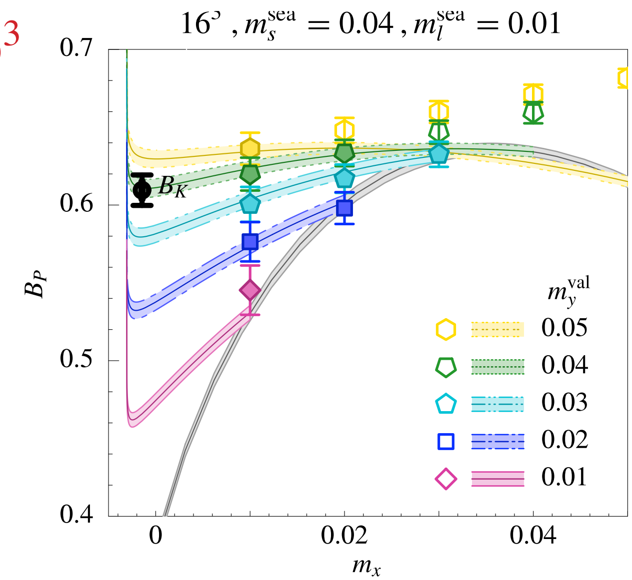
No statistically relevant difference between volumes at fixed quark mass

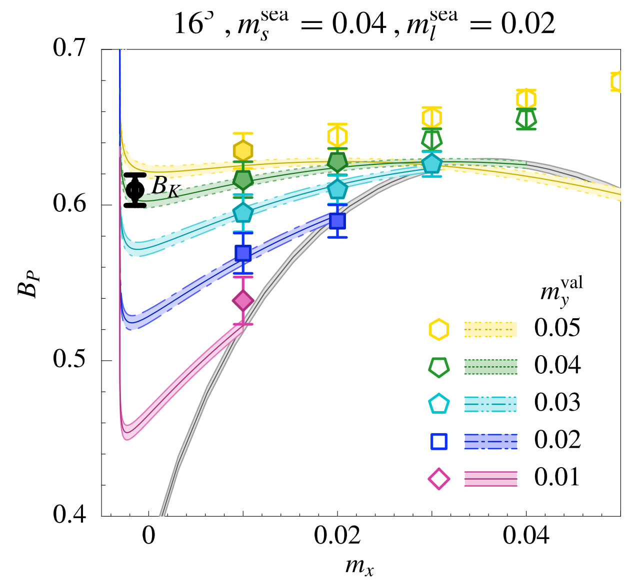


Fitting B_{PS} to NLO Partially Quenched ChPT

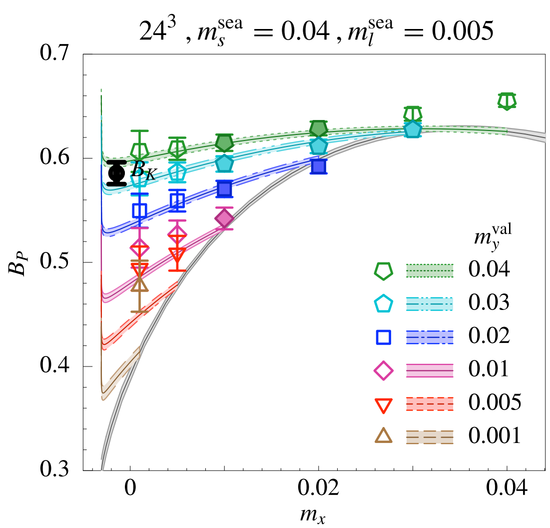
Both 16^3 and 24^3 volumes are fit to same range of masses, 400 to 750 MeV.

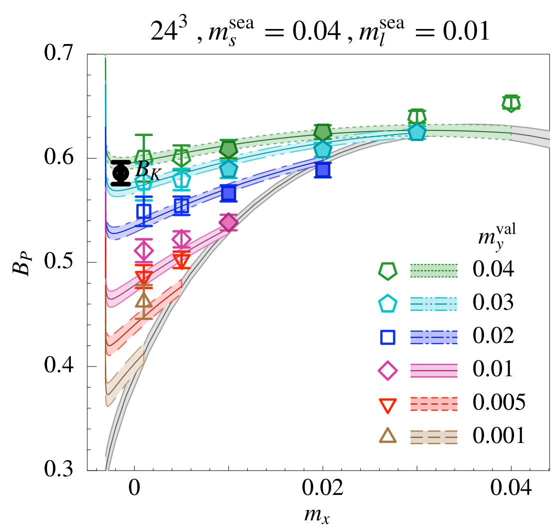
16^3





24^3



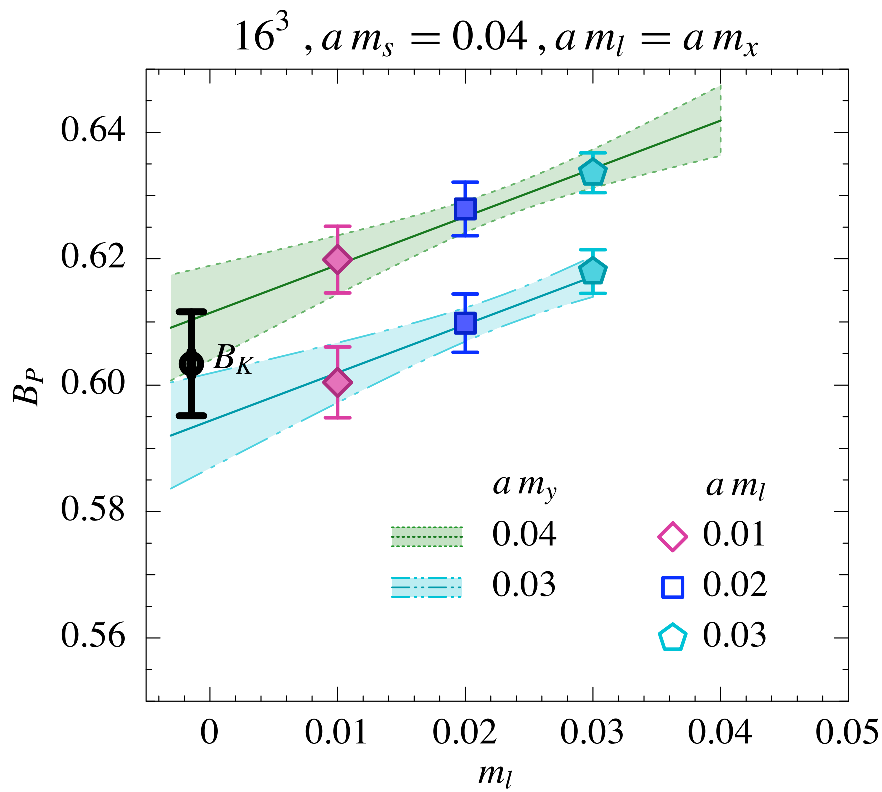


NLO formula are reasonable interpolations of our data, but fail to go through light quark mass points.

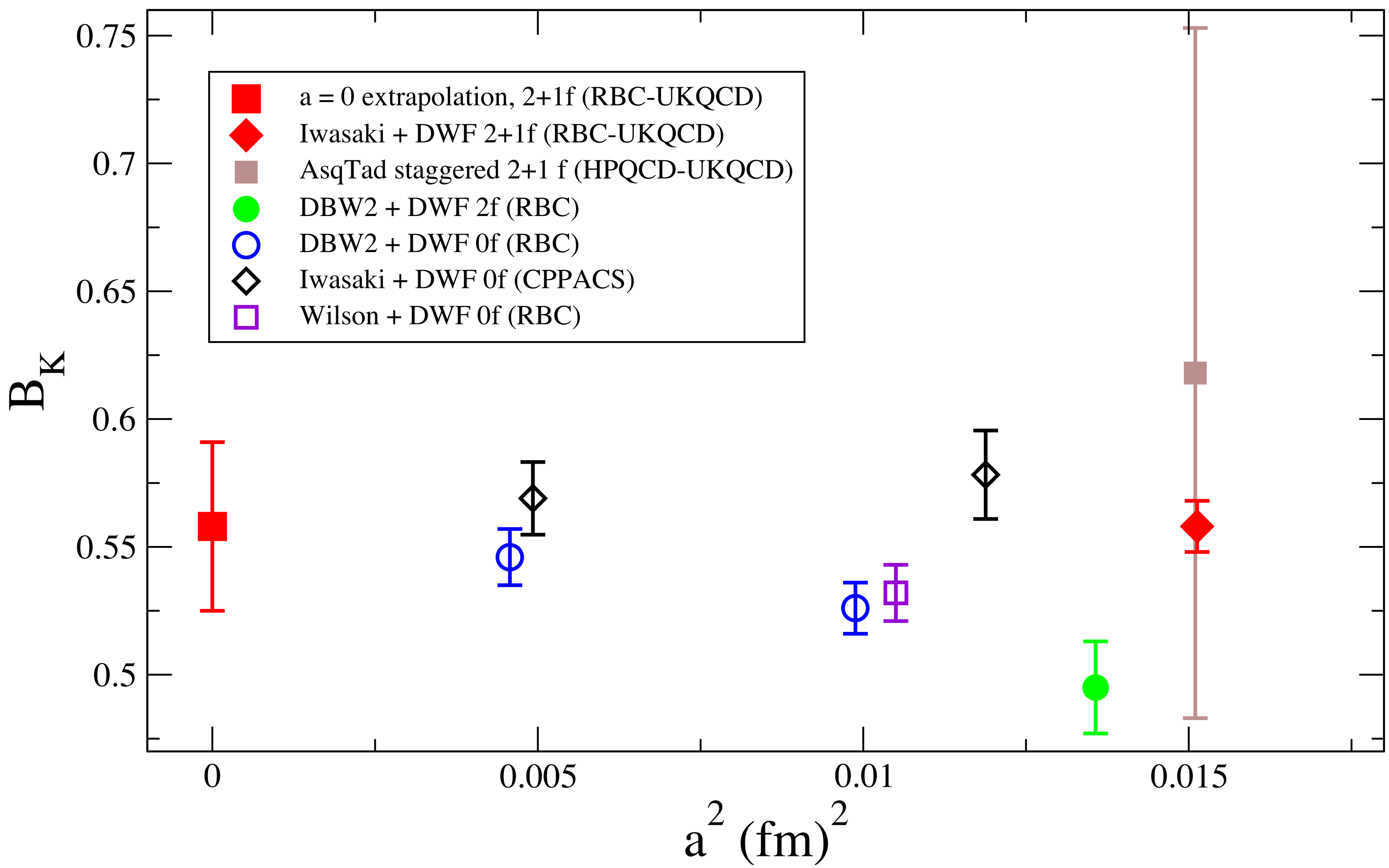
Unitary Extrapolation for B_K

Compare simple unitary extrapolation to result with partially quenched ChPT.

Get same result within statistical errors.

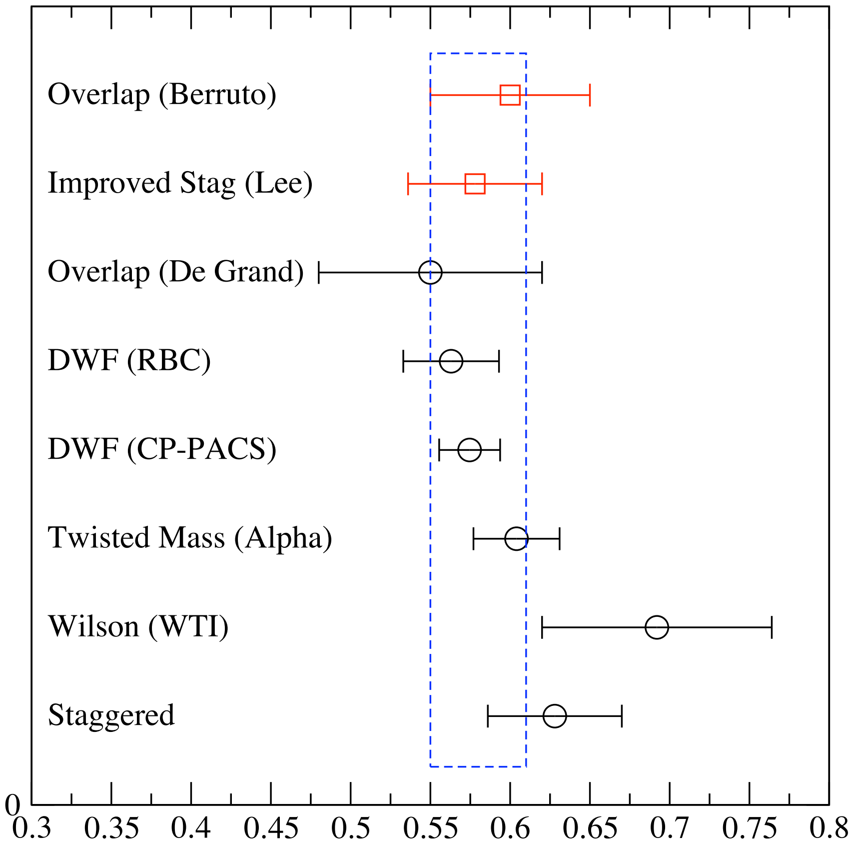


Final Result for B_K



$B_K^{(\overline{MS})}(2\text{ GeV}) = 0.557(12)(29)$ extrapolated to continuum

B_K Comparison



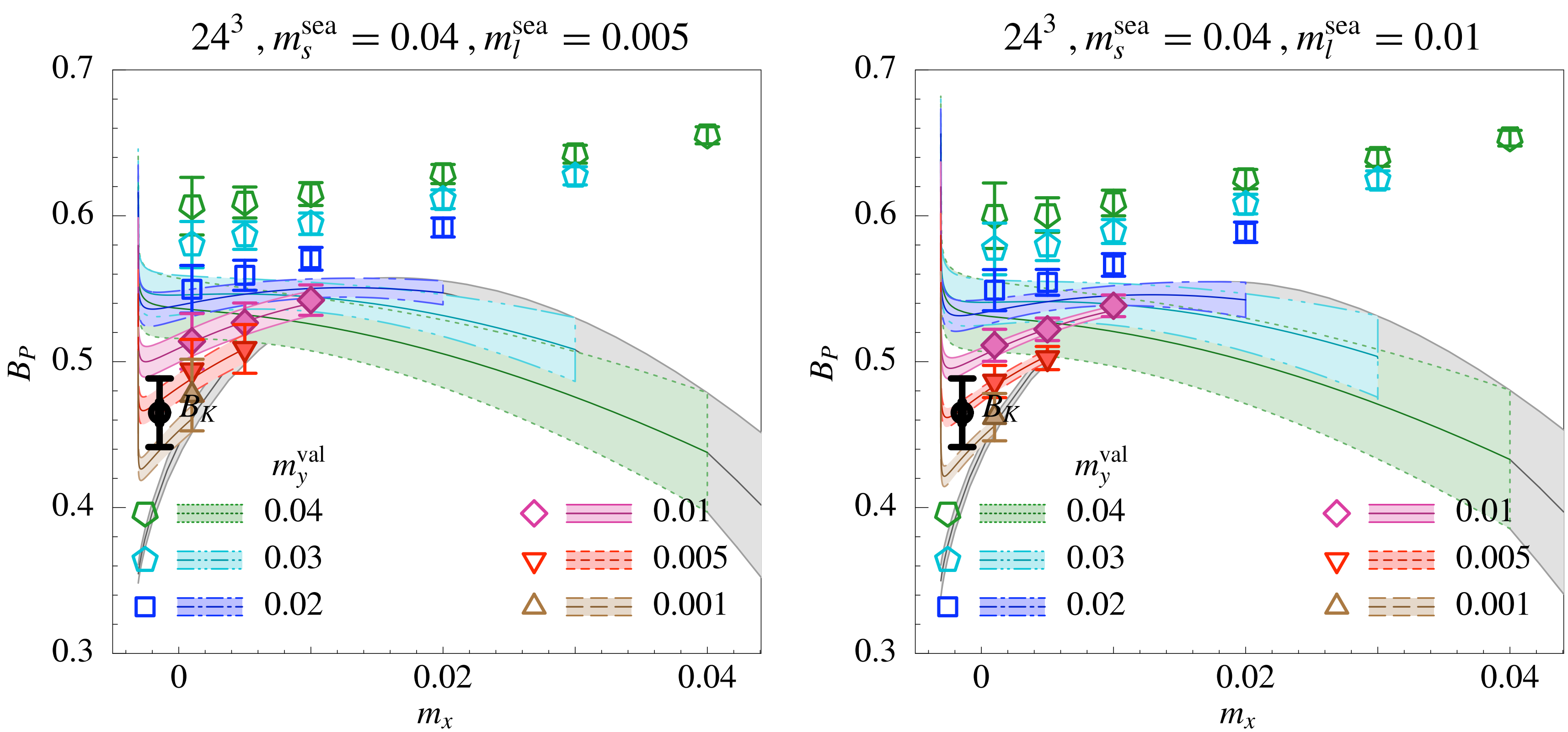
Graph from
Chris Dawson
Lattice 2005

$B_K^{(\overline{MS})}(2\text{ GeV}) = 0.557(12)(29)$ extrapolated to continuum
RBC and UKQCD Collaborations

$B_K^{(\overline{MS})}(2\text{ GeV}) = 0.58(3)(6)$ world average
Chris Dawson, Lattice 2005, PoS(LAT2005) 007

B_K in the Chiral Limit: B_0

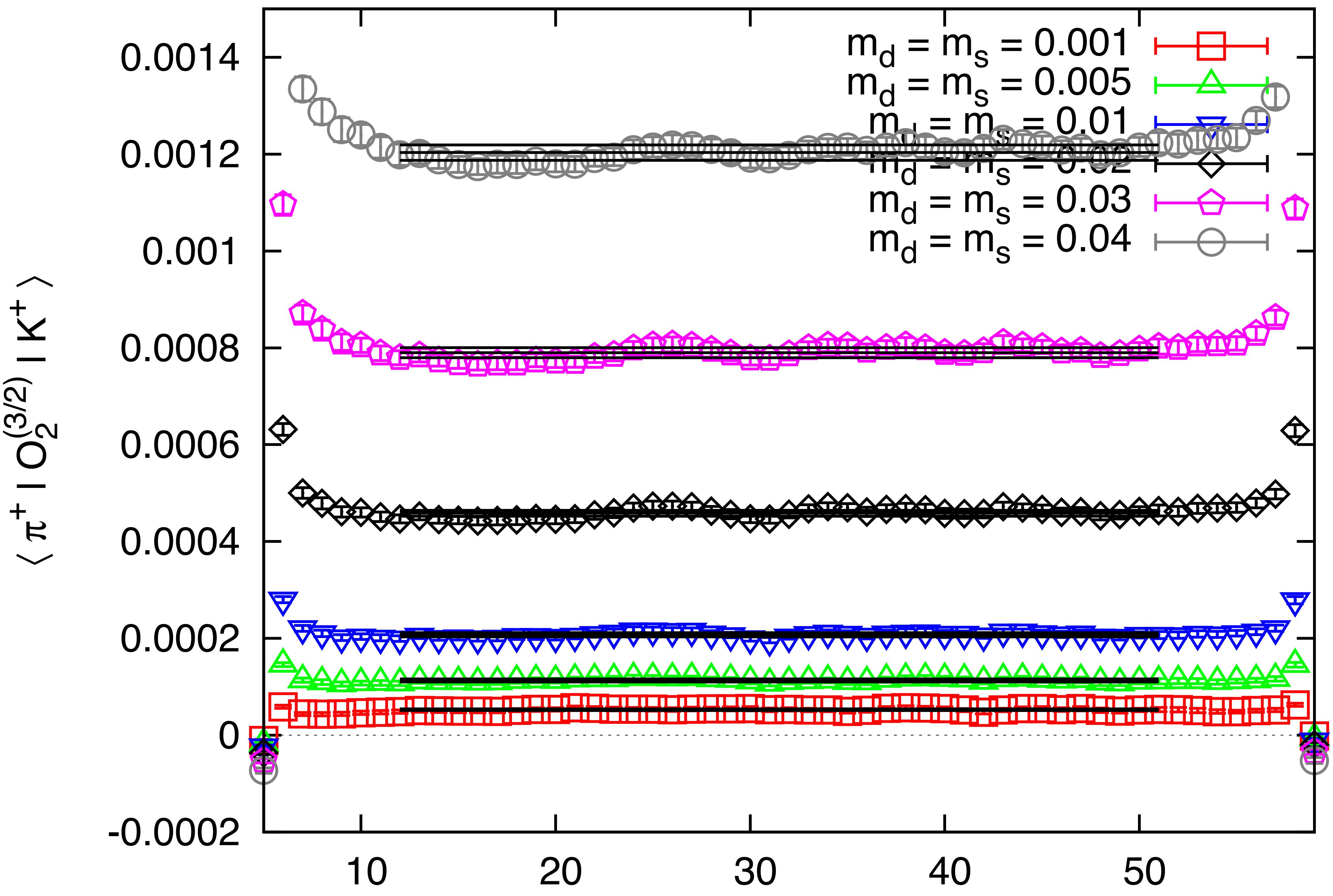
- Use data from (3 fm)^3 volume
- Only pseudoscalars with mass ≤ 400 MeV
- 12 data points used in fits
- Preliminary result: $B_0^{(\overline{MS})}(2\text{ GeV}) = 0.34(5)$



ε'/ε on 2+1 flavor, $(3\ \mathrm{fm})^3$ ensembles

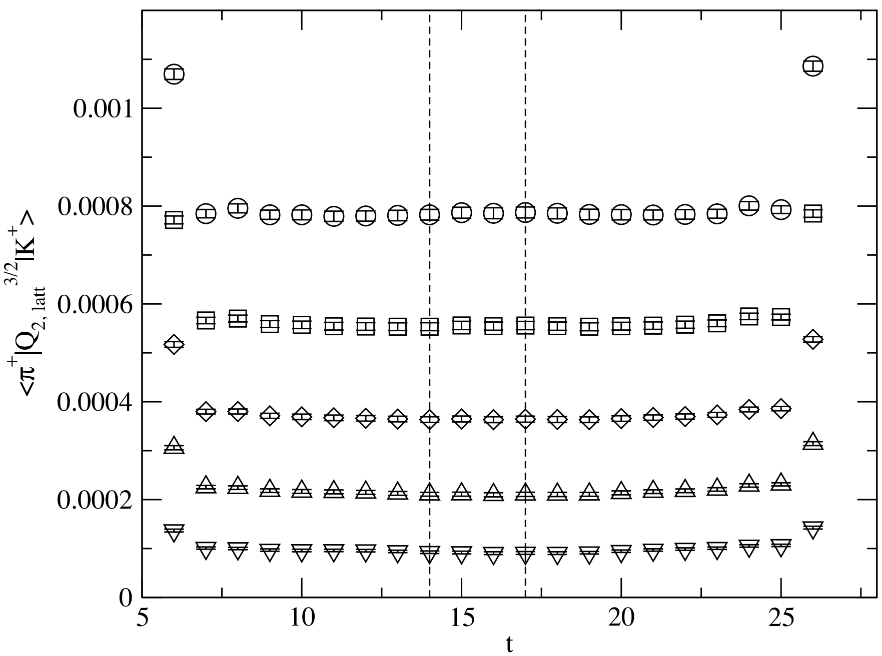
- Valence masses 0.001, 0.005, 0.01, 0.02, 0.03, 0.04 ($m_s/10$ to m_s)
- Concentrating on 0.005/0.04 and 0.01/0.04 ensembles
- Large contributions by Tom Blum, Saul Cohen, Sam Li.
- 0.005/0.04 ensemble: 40 configurations separated by 80 MD time units. 0.01/0.04 ensemble: 30 configurations separated by 80 MD time units
- Concentrating on lighter quark masses where NLO chiralperturbation theory should be reasonable.
- Coulomb gauge fixed wall sources at t = 5 and 59
- Random noise source of length 40 for pupil calculations
- 1/2 of time in wall source calculations, the other 1/2 in pupils

ΔI = 3/2 Plateau

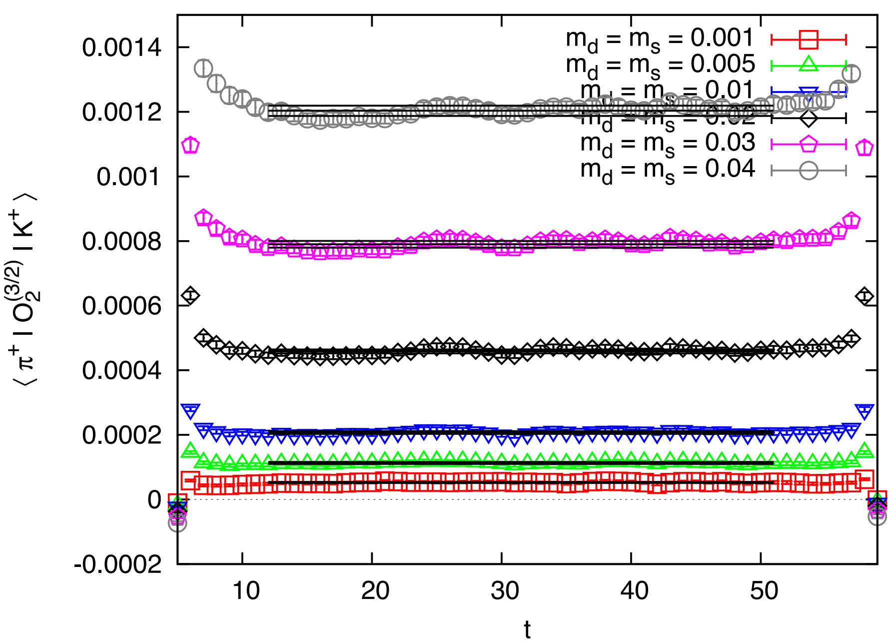


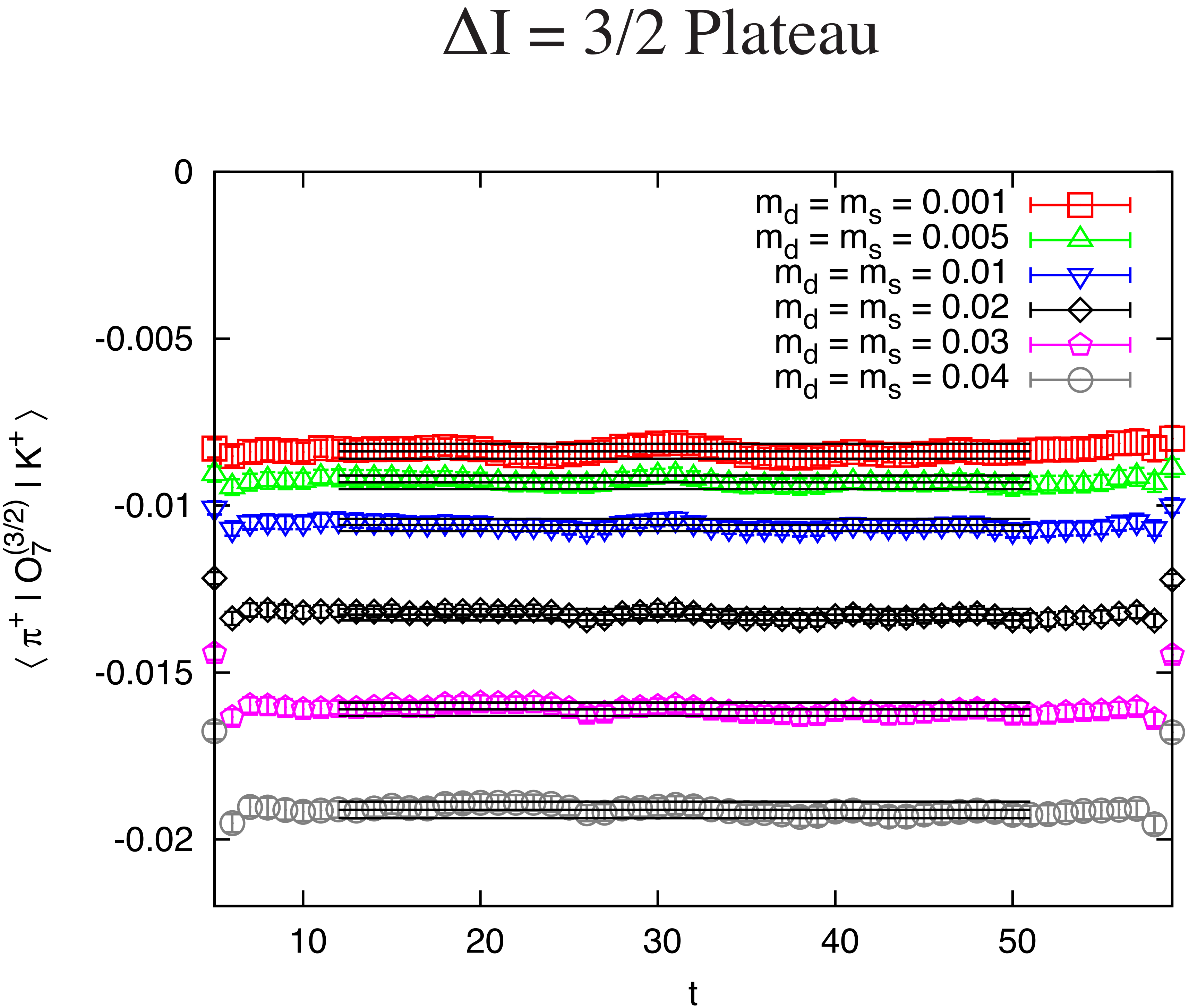
ΔI = 3/2 Plateau Comparisons

Previous Quenched



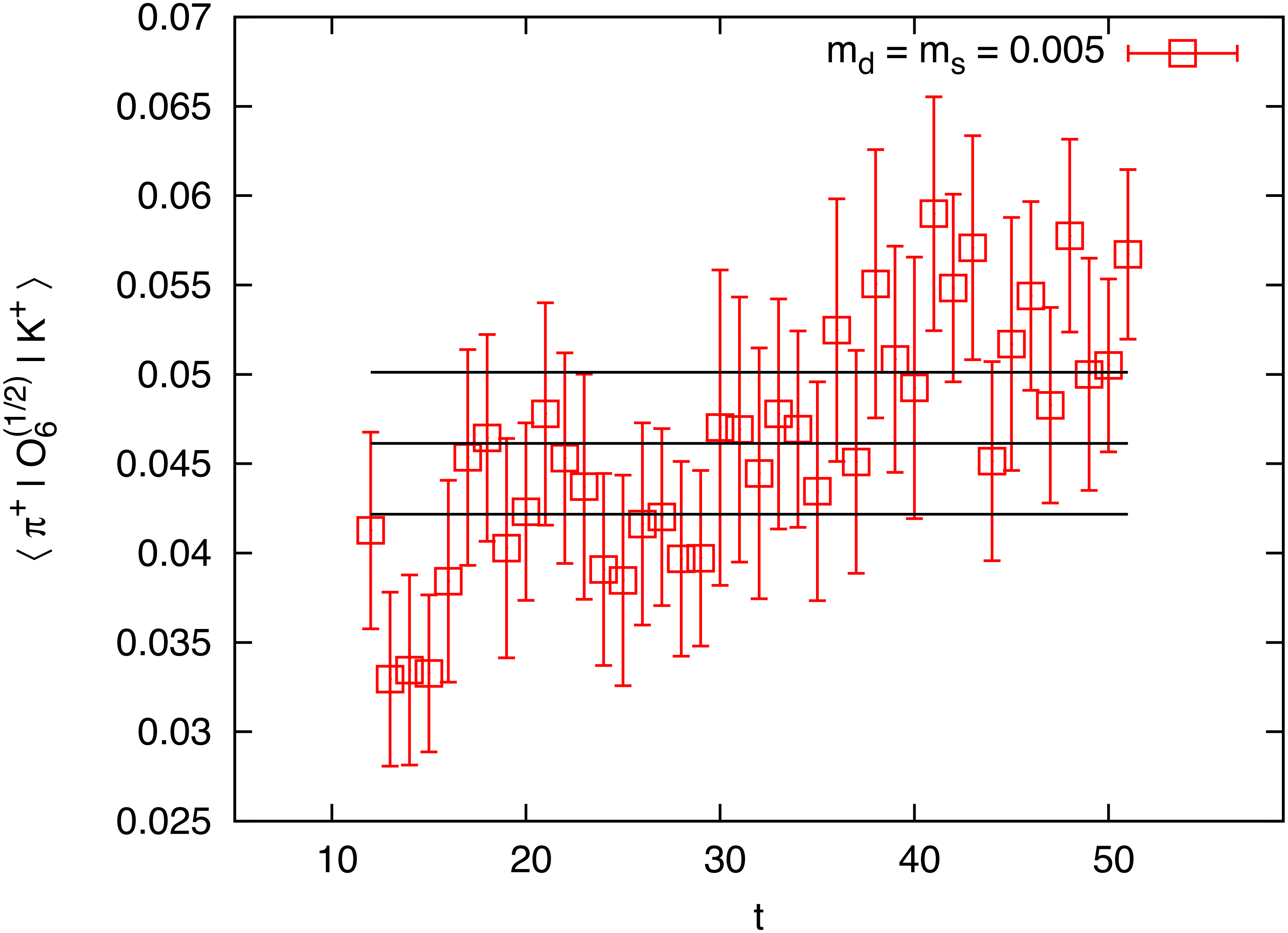
New 2+1 Flavor



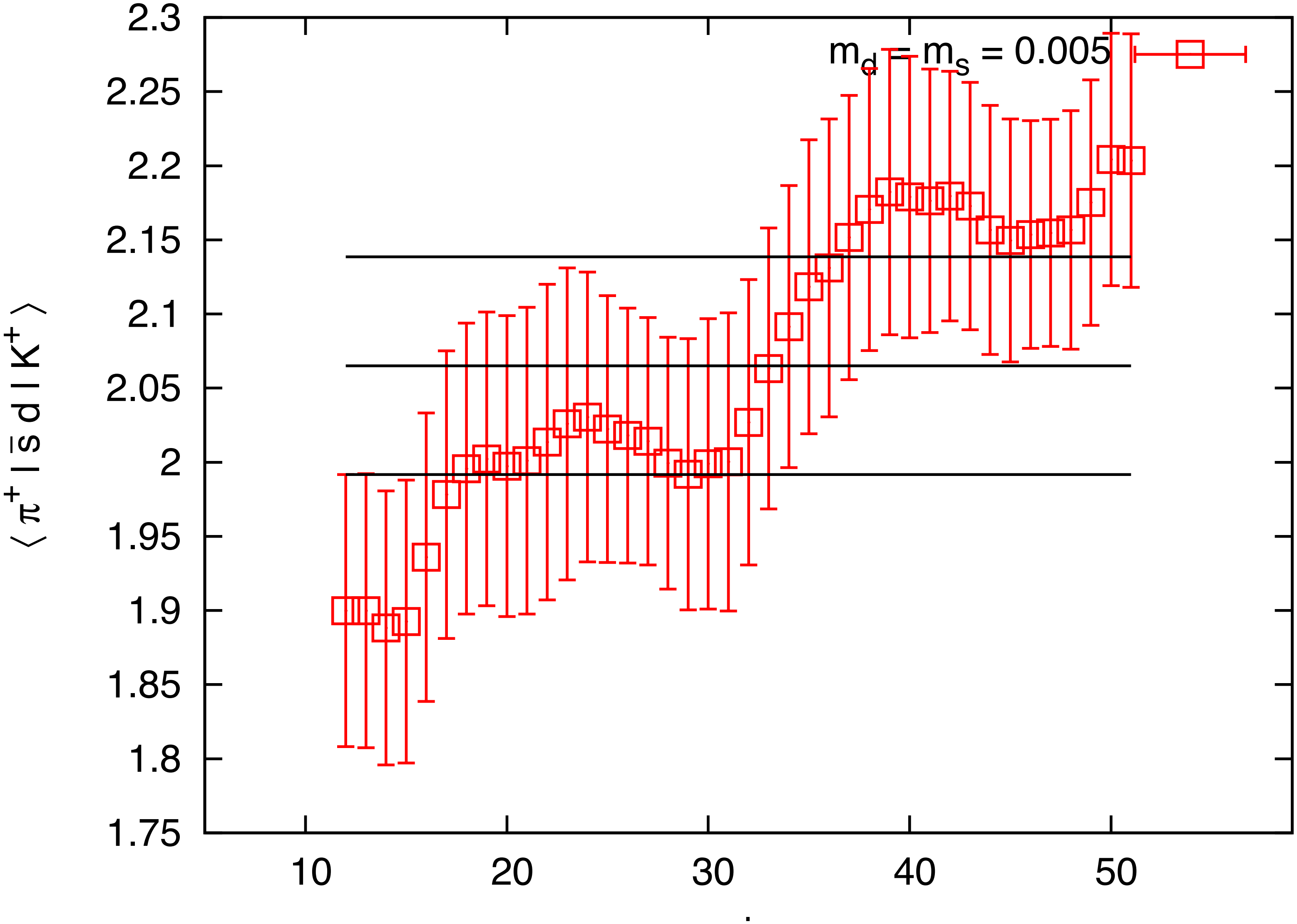


ΔI = 3/2 Plateau
0
-0.005
-0.01
-0.015
-0.02
10
20
30
40
50
t
⟨ π⁺ | O₇^(3/2) | K⁺ ⟩
m_d = m_s = 0.001
m_d = m_s = 0.005
m_d = m_s = 0.01
m_d = m_s = 0.02
m_d = m_s = 0.03
m_d = m_s = 0.04

ΔI = 1/2 Plateau - unsubtracted Q_6

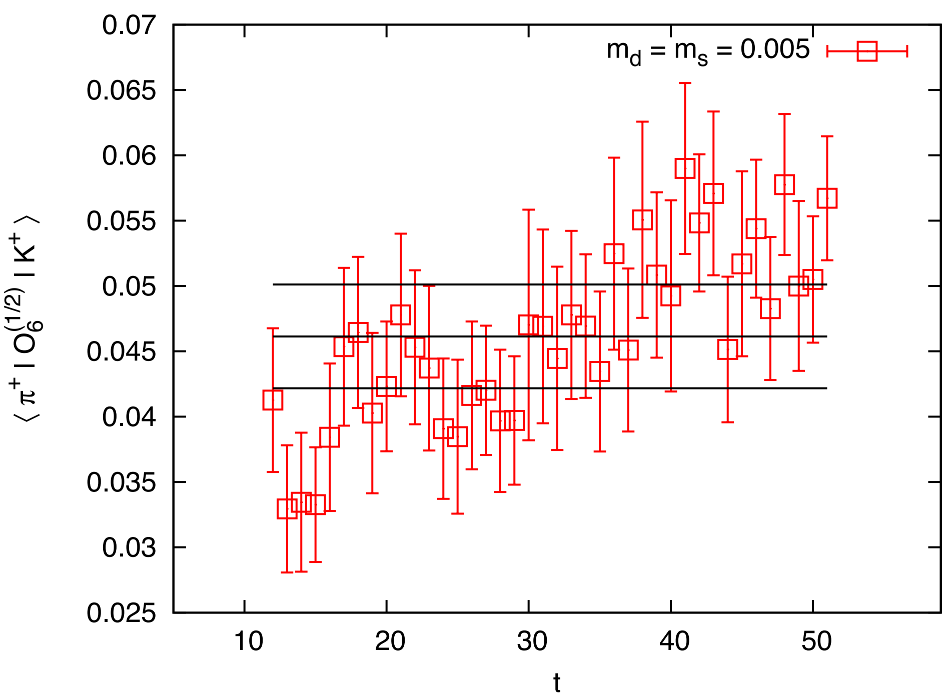


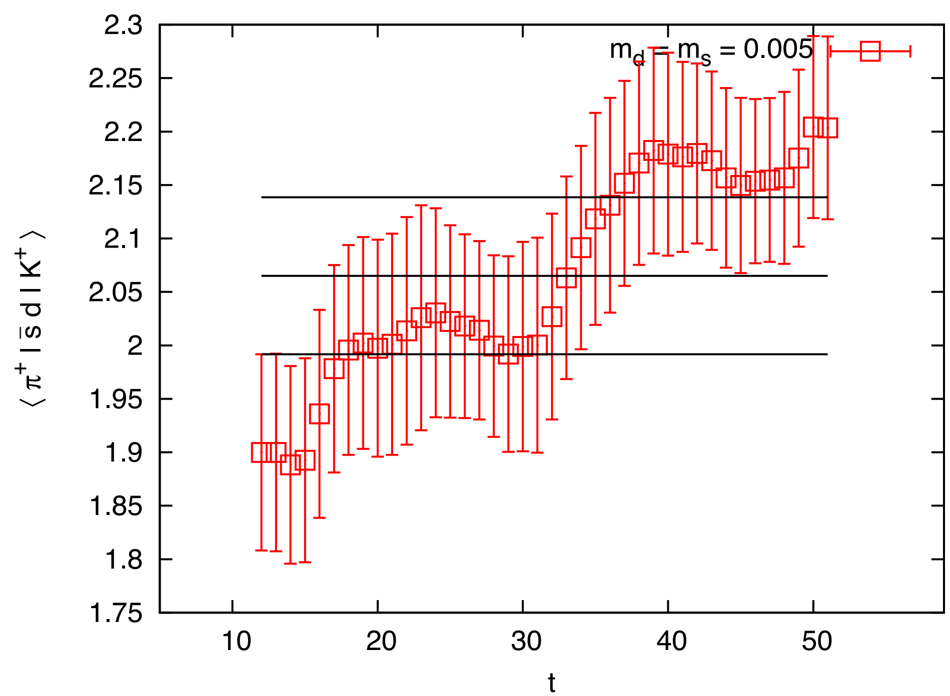
ΔI = 1/2 Plateau - $\bar{s}d$



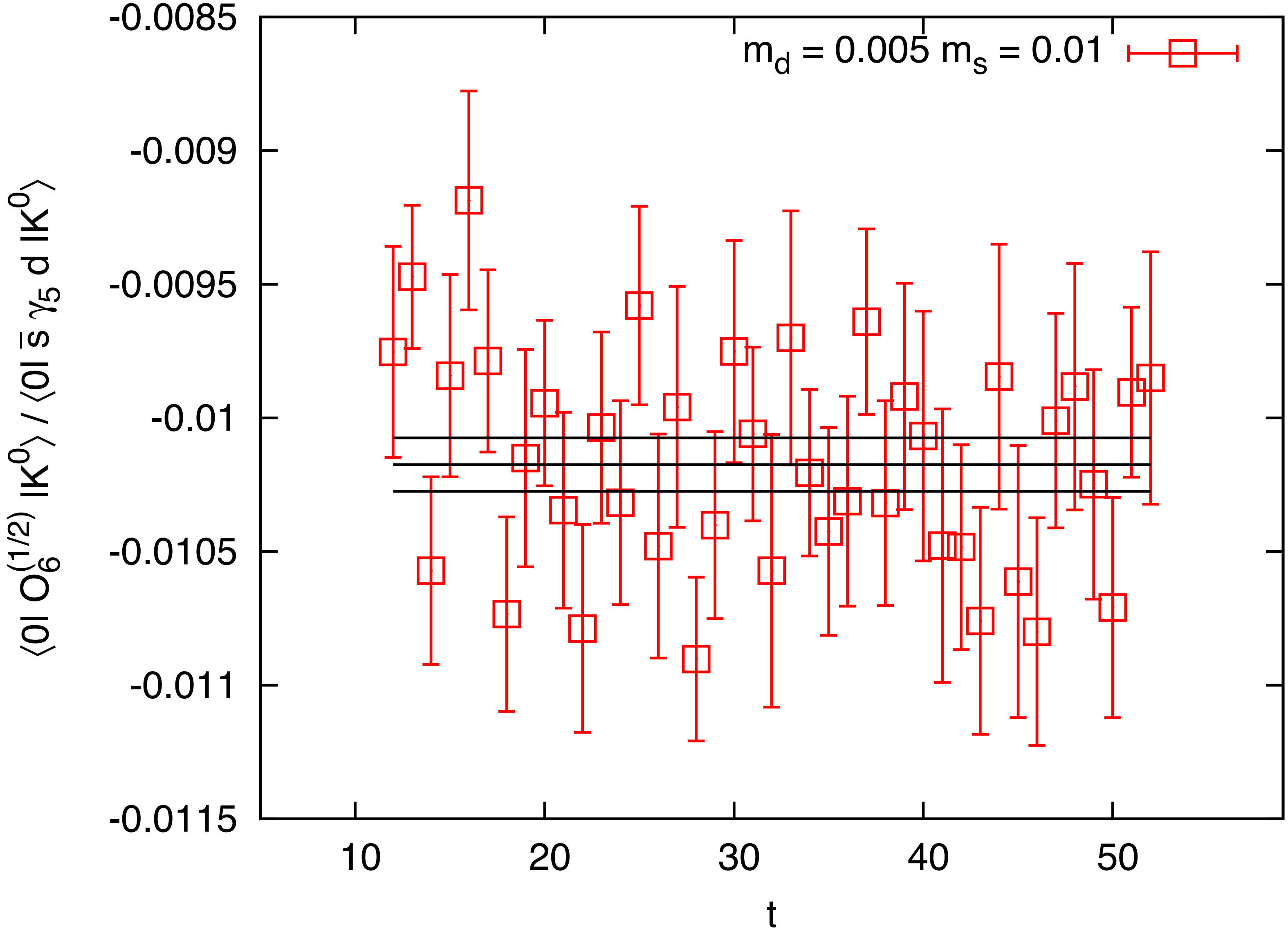
Comparing $\Delta I = 1/2$ Plateau

Common fluctuations in Q_6 and $\bar{s}d$





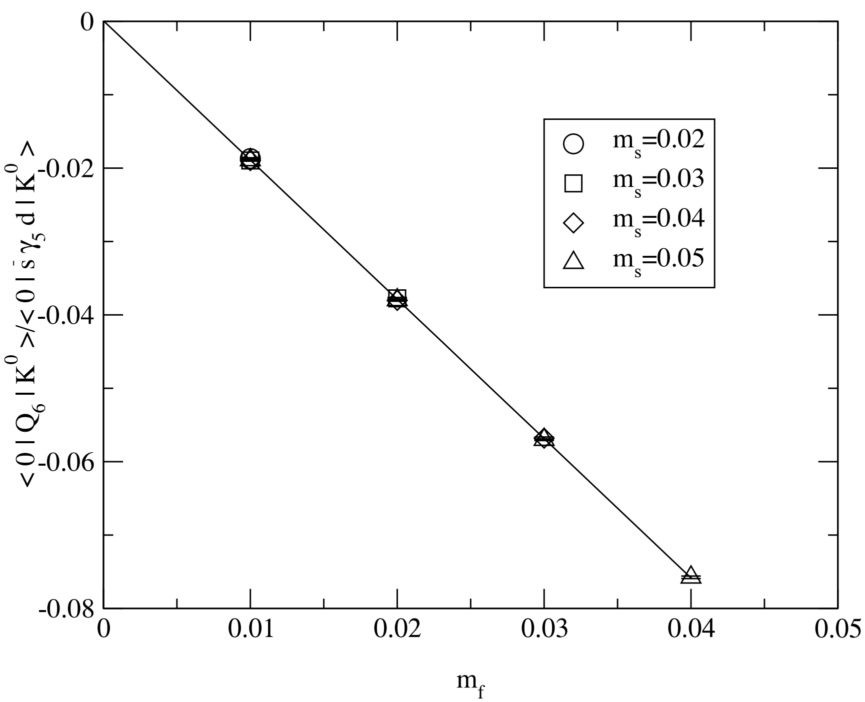
ΔI = 1/2 Subtraction



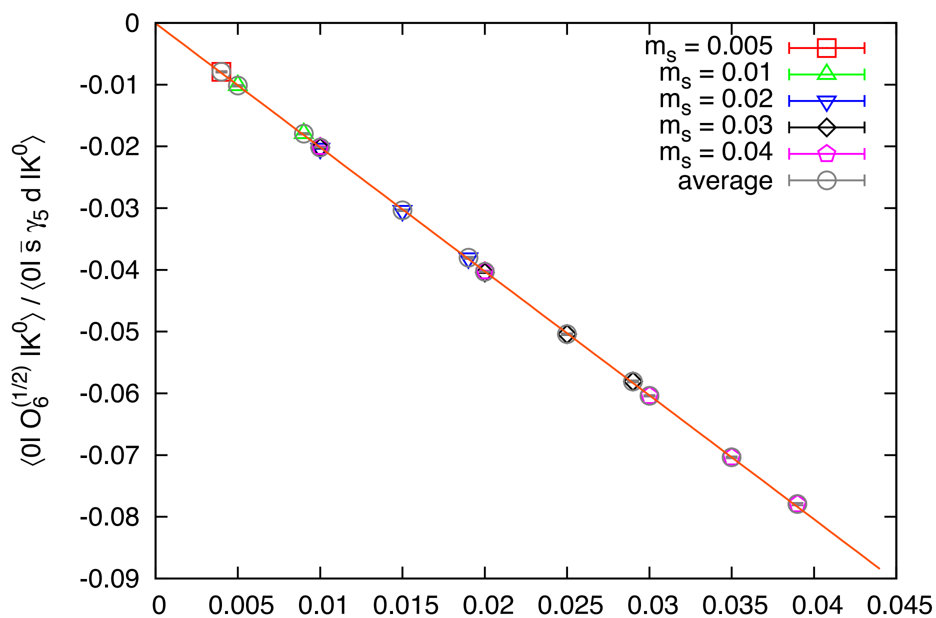
ΔI = 1/2 Subtraction Coefficient Comparisons

$$\frac{\langle 0|Q_{i,\mathrm{lat}}|K^0\rangle}{\langle 0|(\bar{s}\gamma_5 d)_{\mathrm{lat}}|K^0\rangle} = \eta_{0,i} + \eta_{1,i}\,(m_s - m_d)$$

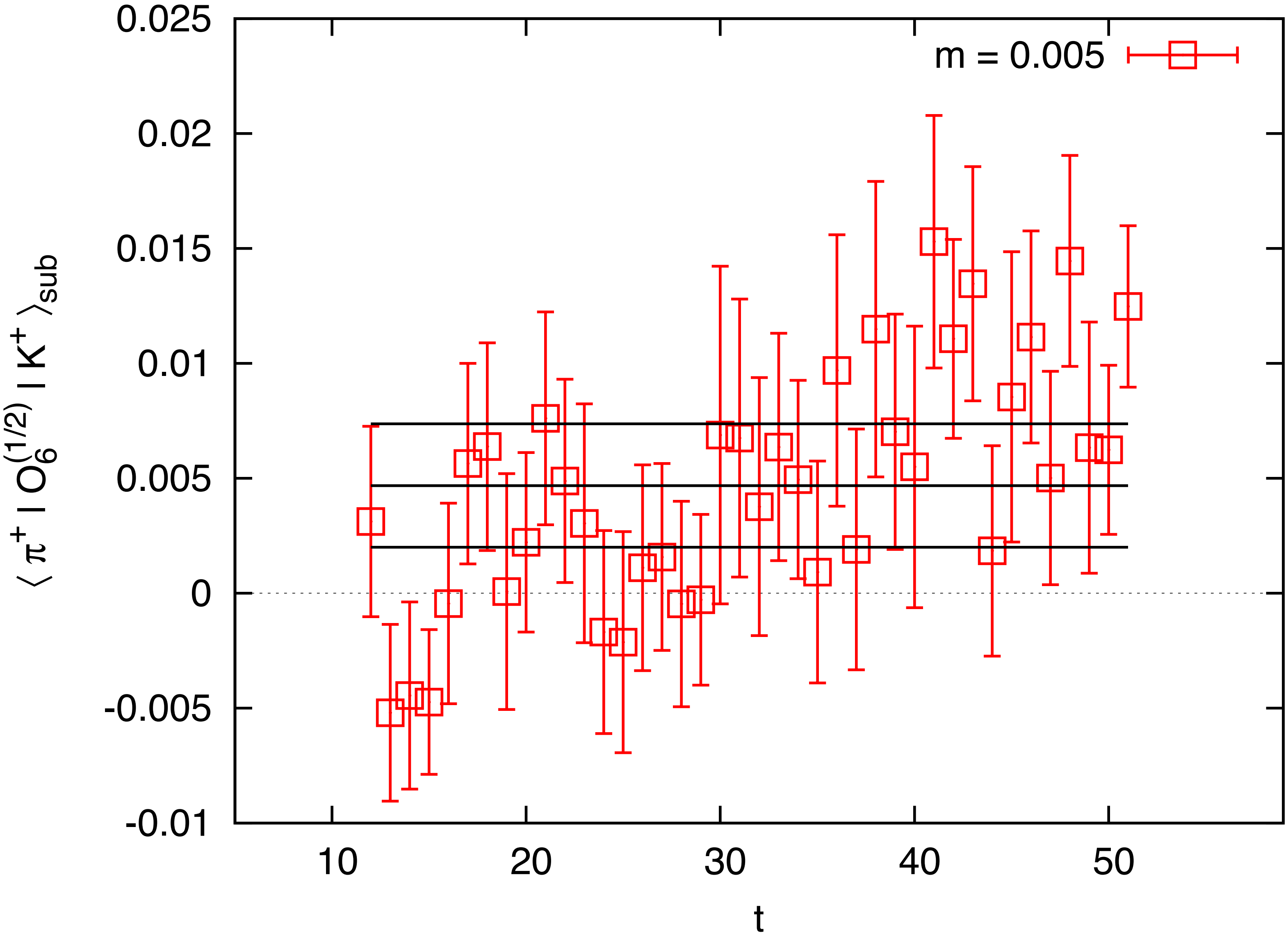
Previous Quenched



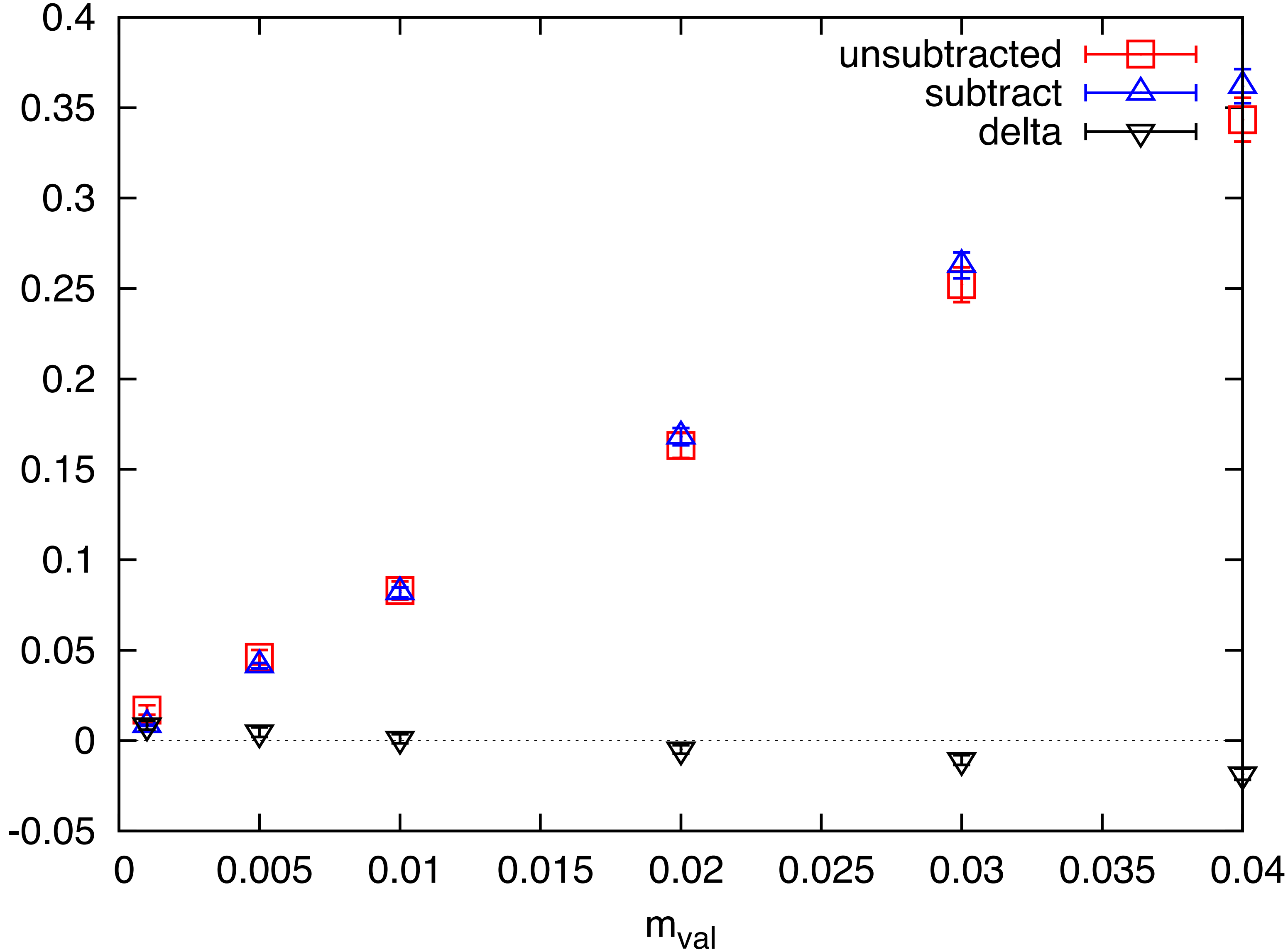
New 2+1 Flavor



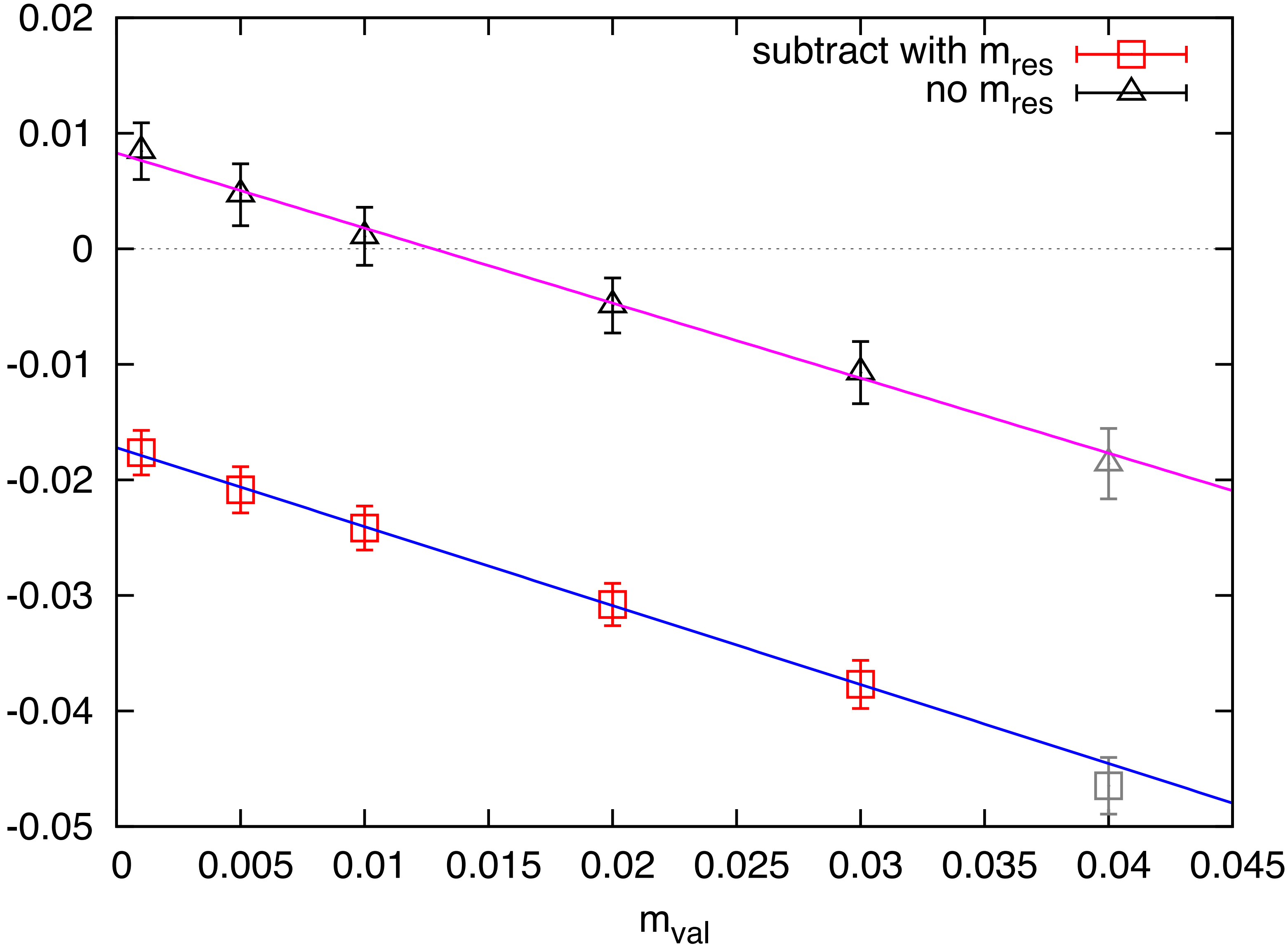
Subtracting Q_6



Subtracting Q_6



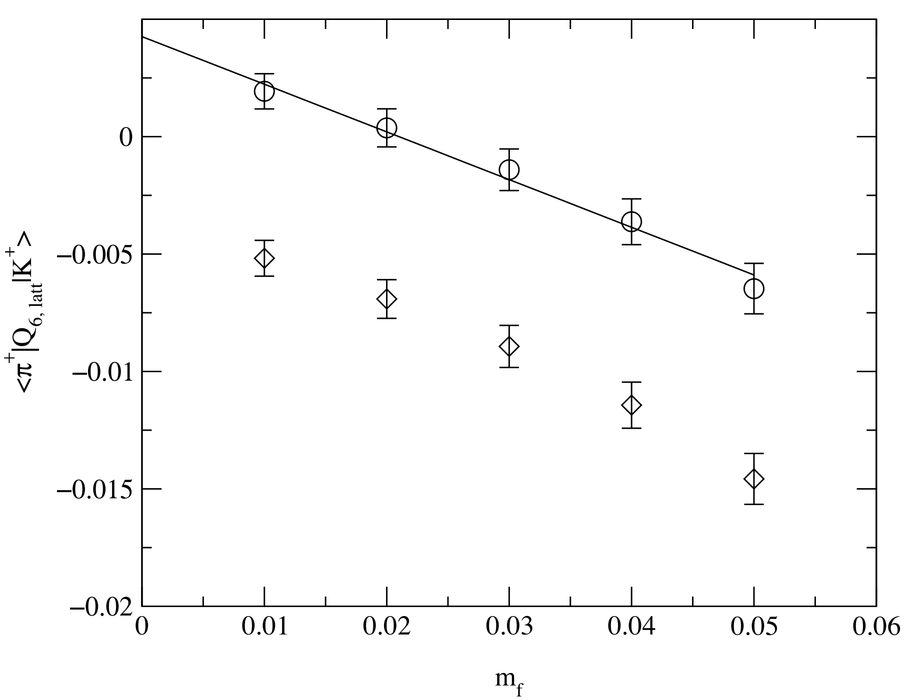
Subtracted Q_6



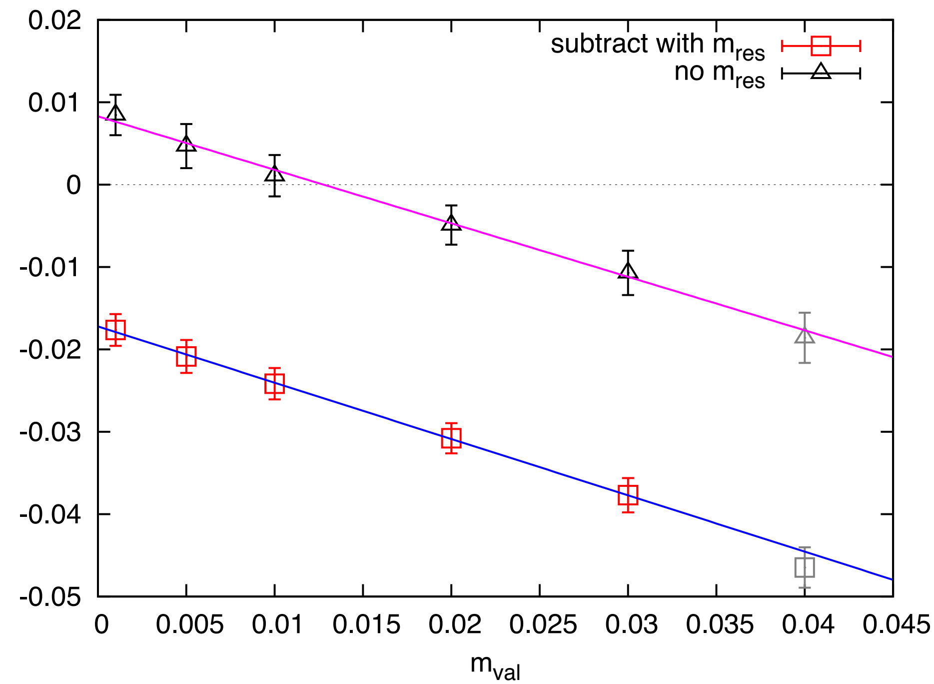
Subtracted Q_6 Comparison

$$\langle \pi^+ | Q_{i,\mathrm{lat}}^{(1/2)} | K^+ \rangle + \eta_{1,i}(m_s + m_d) \langle \pi^+ | (\bar{s}d)_{\mathrm{lat}} | K^+ \rangle$$

Previous Quenched



New 2+1 Flavor



m_{res} and $\Delta S = 1$ matrix elements

- Spurion field Ω at midpoint represents residual χSB
- Transforms as $(\bar{3},3)$ under chiral symmetry
- For low energy observables, Ω goes to m_{res}
- For divergent quantities, new parameters enter which are $O(m_{res})$
- Due to unsupressed modes in 5-d, two powers of Ω can enter with the same size as a single power of Ω
- Higher order terms are a few percent effect and can be subtracted
- Discussed by Christ and Sharpe at DWF@10 meeting at BNL

$$\langle K^+|Q_6|\pi^+\rangle \sim \left\{\frac{(m+\Omega)}{a^3} + \frac{(m+\Omega)^3}{a^3}\right\}\langle K^+|\bar{s}d|\pi^+\rangle + \frac{\Omega}{a}\langle K^+|\bar{s}\sigma\cdot Fd|\pi^+\rangle$$

Chiral Perturbation Theory and ε'/ε

- Simulations will have a fixed, dynamical strange quark mass, which may be outside range of utility of NLO ChPT
- Lightest quark mass, $m_s/10$, may need finite volume corrections added to ChPT formula.
- 2+1 flavor partially quenched ChPT being done by Aubin, Laiho and Li
- (8,8) and (27,1) operators complete. (8,1) operators are well under-way

ε´/ε Summary

- Have summarized RBC-UKQCD calculation of NLO coefficients from K -> π and K -> vacuum
- Work on K -> ππ at unphysical kinematics underway
- Lee and Sharpe are using ASQTAD staggered fermions, with smearings, to calculate B_K , B_7 and B_8. Testing to see how much smearing can help with operator mixing
- Hernandez, et. al. are working in the epsilon regime (quenched) to explore ΔI = 1/2 rule
- Lellouch, et. al. are using overlap fermions on lattices generated with 2 flavors of Wilson fermions to look at B_K and ε´/ε.

Conclusions

- 2+1 flavor DWF QCD simulations well underway
  (3 fm)^3 volumes at two lattice scales
  $m_l = m_s / 5$ on $a^{-1} = 1.6$ GeV lattices
  $m_l = m_s / 7$ on $a^{-1} = 2.1$ GeV lattices
- $\Delta S = 1$ matrix elements appear to be benefitting from large spatial volume, giving reduced statistical errors.
- From comparison of ChPT to data, we are investigating range of pseudoscalar masses where NLO ChPT is accurate to, say 10%.
- For ε'/ε, NLO fits for $\Delta I = 3/2$ amplitudes should work.
- For $\Delta I = 1/2$ amplitudes, statistical errors will likely limit NLO fits
- K -> $\pi\pi$, tests underway to get needed constant.
- Major systematic in final result likely ChPT
- Multiparticle final states a few years away...