Local Realism vs Quantum Mechanics with Entangled Neutral Kaons

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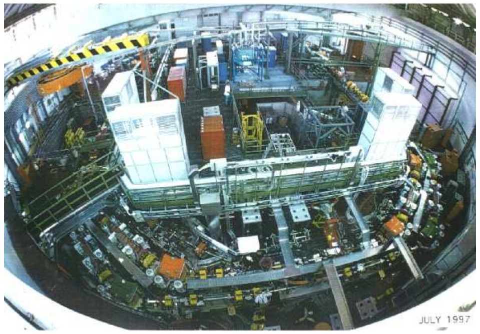

OUTLINE

- Local Realism and Bell's Theorem
- Requirements for a Genuine Bell–Type Test and Loopholes
- Neutral Kaons: Bases and Measurement Procedures
- Entangled $K^0\bar{K}^0$ Pairs
- A Classification of Bell's Inequalities
- A Review on Bell–Type Tests with Neutral Kaons
- Conclusions

[1] *Bell's inequality tests with meson–antimeson pairs*,
A. Bramon, R. Escribano and G. G., Found. Phys. **36**, 563 (2006) [quant-ph/0501069].

[2] *Entanglement, Bell inequalities and decoherence in particle physics*,
R. A. Bertlmann, Lect. Notes Phys. **689**, 1 (2006) [quant-ph/0410028].

[3] *The Einstein, Podolsky and Rosen paradox in atomic, nuclear and particle physics*,
A. Afriat and F. Selleri (Plenum Press, New York, 1998).

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Local Realism and Bell's Theorem

[A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935)]

THE PHILOSOPHY OF REALISM

The existence of quantum world is objective and independent of observation. Any measurement performed on a quantum system produces an outcome with a definite and predetermined value (HIDDEN–VARIABLES)

LOCALITY ASSUMPTION

Physical phenomena in a space–time region cannot be affected by events occurring in space–like separated regions (RELATIVISTIC CAUSALITY)

BELL'S THEOREM... [J. Bell, Physics **1**, 195 (1964)]

Some statistical predictions of QM for entangled systems are incompatible with the consequences of realism and locality

...WITH INEQUALITIES

Constraints among the observables (probabilities or expectation values) deduced from the hypoteses of realism and locality

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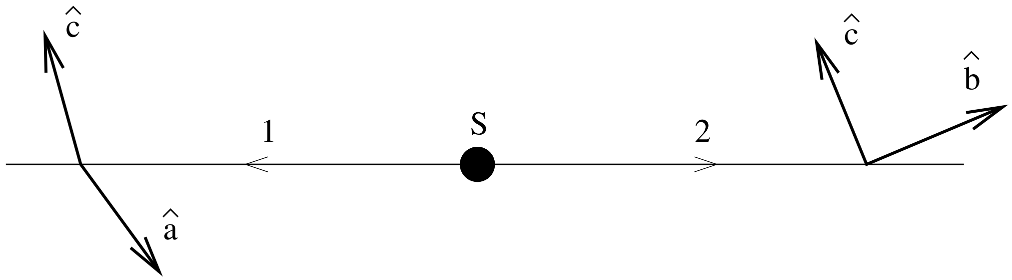
[D. Bohm, *Quantum Theory* (Prentice Hall, Englewood Cliffs, N. J., 1951)]

An example of ENTANGLED state : the EPR–Bohm spin singlet state



$$|J=0, J_z=0\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\}$$

- A source with total angular momentum zero decays, at rest, into two spin 1/2 particles which fly apart with opposite momenta
- A quantum correlated and non–interacting system composed by two spatially separated entities
- The state of the global system is not a tensorial product of superposition of states of the two subsystems $\Longleftrightarrow$ the global system is the single, indivisible quantum (each one of the two particles cannot be represented by a state vector) $\Longrightarrow$ non–factorizable joint probabilities, $P_{\rm QM}[\sigma_a, \sigma_b] = [1 - \sigma_a \sigma_b \cos\theta_{a,b}]/4$.

Wigner's inequality [E. P. Wigner, Am. J. Phys. **38**, 1005 (1970)]:

$$P_{\rm LR}[\sigma_a =\uparrow, \sigma_b =\downarrow] \leq P_{\rm LR}[\sigma_a =\uparrow, \sigma_c =\uparrow] + P_{\rm LR}[\sigma_c =\uparrow, \sigma_b =\downarrow]$$

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Requirements for a Genuine Bell–Type Test and Loopholes

(1) An entangled state is used;
(2) Alternative and mutually exclusive measurements are chosen at will and performed on both members of that state (Reality);
(3) Each single measurement has dichotomic outcomes;
(4) The measurement processes on the two members of the state must be space–like separated from each other (Locality).

- Experiments confronting QM vs LR performed with entangled (atomic cascade and optical) photons and ions.
- Agreement with QM, violation of *non genuine* Bell's inequalities: Supplementary Assumptions (not implicit in LR, plausible but not testable) required when interpreting the data
- No experiment violated *genuine* Bell's inequalities: No loophole–free test
- Detection Loophole: real detection efficiencies are lower than the theoretical thresholds required for the QM violation of genuine Bell's inequalities ($\Longrightarrow$ normalizations to the sample of detected events: fair sampling)
- Locality Loophole: joint measurements are not space–like separated ($\Longrightarrow$ slower–than–light communications are allowed)

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Neutral Kaons: Bases and Measurement Procedures

$\Longrightarrow$ Extensions to other sectors of Physics? In Particle Physics: $K^0\bar{K}^0$, $B^0\bar{B}^0$,...

- Entangled $K^0\bar{K}^0$ states copiously produced in $\phi(1020)$ decays and $p\bar{p}$ annihilation processes at rest.
- The strong nature of kaon hadronic interactions should contribute to close the detection loophole, since it enhances the efficiencies to detect the products of kaon decays and kaon interactions with ordinary matter.
- Kaons produced in $\phi \to K^0\bar{K}^0$ or $p\bar{p} \to K^0\bar{K}^0$ fly apart from each other at relativistic velocities and can fulfill the condition of space–like separation. However, the locality loophole could be closed only with equipments able to prepare, actively and very rapidly, the alternative kaon measurement settings.
- For spin 1/2 particles one can measure the spin projection along *any* space direction. Dichotomic measurements on neutral kaons reduce to *only two* kinds:
  - Strangeness: $K^0/\bar{K}^0$, via strong $K^0N/\bar{K}^0N$ interactions in absorber materials. Low efficiency for thin absorbers
  - Lifetime: K_L/K_S, via free space prompt (K_S) and late (K_L) weak kaon decays. Quite efficient due to the smallness of Γ_L/Γ_S and ϵ

  The kaon regeneration and strangeness oscillations phenomena provide the number of measurement bases (3 or 4) required for Bell–type tests.

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Bases in quasi–spin space

Analogy with spin–1/2 particles (and any two–level quantum system or qubit):

$$|K^0\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix} \text{ “spin up”} \qquad |\bar{K}^0\rangle \equiv \begin{pmatrix}0\\1\end{pmatrix} \text{ “spin down”}$$

STRANGENESS BASIS: $\{K^0, \bar{K}^0\}$ (strangeness conserving interactions)

GENERIC BASIS $\{K_\alpha, K_\alpha^\perp\}$ along the quasi–spin axis α:

$$|K_\alpha\rangle = \alpha|K^0\rangle + \bar{\alpha}|\bar{K}^0\rangle$$
$$|K_\alpha^\perp\rangle = -\bar{\alpha}^*|K^0\rangle + \alpha^*|\bar{K}^0\rangle$$

$$\langle K_\alpha|K_\alpha\rangle = \langle K_\alpha^\perp|K_\alpha^\perp\rangle = |\alpha|^2 + |\bar{\alpha}|^2 = 1 \qquad \langle K_\alpha|K_\alpha^\perp\rangle = 0$$

FREE–SPACE BASIS: $\{K_S, K_L\}$ (free–space time evolution)

$$|K_{S,L}(\tau)\rangle = e^{-i\lambda_{S,L}\tau}|K_{S,L}\rangle \qquad \lambda_{S,L} = m_{S,L} - i\Gamma_{S,L}/2$$

$$|K_{S,L}\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left\{(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle\right\}$$

$$\langle K_L|K_S\rangle = \langle K_S|K_L\rangle = 2\,\mathrm{Re}\,\epsilon/(1+|\epsilon|^2) \simeq 3.2\times 10^{-3}$$

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INSIDE–MATTER BASIS: $\{K'_S, K'_L\}$ (inside matter time propagation)

[A. Di Domenico, Nucl. Phys. **B 450**, 293 (1995)]

$$|K'_{S,L}(\tau)\rangle = e^{-i\lambda'_{S,L}\tau}|K'_{S,L}\rangle$$

$$\lambda'_{S,L} = \frac{\lambda_S + \lambda_L}{2} - \frac{\pi\nu}{m_K}(f_0 + \bar{f}_0) \pm \frac{\lambda_S - \lambda_L}{2}\sqrt{1+4\rho^2}$$

Decreasing $K'_{S,L}$ intensities with τ [Im $\lambda'_{S,L}$]: weak decays and *absorption* via kaon–nucleon strong interactions [$\mathrm{Im}(f_0 + \bar{f}_0)$]

$$|K'_S\rangle = \frac{1}{\sqrt{1+|r\bar{\rho}|^2}}\left[|K^0\rangle + r\bar{\rho}|\bar{K}^0\rangle\right]$$

$$|K'_L\rangle = \frac{1}{\sqrt{1+|r(\bar{\rho})^{-1}|^2}}\left[|K^0\rangle - r(\bar{\rho})^{-1}|\bar{K}^0\rangle\right]$$

$$\langle K'_S|K'_L\rangle = \langle K'_L|K'_S\rangle^* = \frac{1-|r|^2(\bar{\rho}^*/\bar{\rho})}{\sqrt{1+|r\bar{\rho}|^2}\sqrt{1+|r/\bar{\rho}|^2}}$$

$$r \equiv \frac{1-\epsilon}{1+\epsilon} \qquad \rho \equiv \frac{\pi\nu}{m_K}\frac{f_0 - \bar{f}_0}{\lambda_S - \lambda_L} \qquad \bar{\rho} \equiv \sqrt{1+4\rho^2} + 2\rho$$

$f_0 \neq \bar{f}_0$: "rotations" in quasi–spin space and $K_S \leftrightarrow K_L$ transitions. The medium is a *regenerator* for surviving kaons. Usual regenerators: $\rho \simeq \mathrm{Re}\,\rho \sim 10^{-2}$, $\bar{\rho} \simeq \bar{\rho}^*$.

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Active vs Passive Measurements

[A. Bramon, G. G. and B. C. Hiesmayr, Phys. Rev. **A 69**, 062111 (2004)]

- Active Measurement: at the measurement time the experimenter, exerting his/her free will, either places a slab of material (strangeness) or allows for free–space kaon propagation (lifetime). One measurement excludes the other.

- Passive Measurement: one exploits the quantum dynamics of kaon decays in free space.
  - Lifetime: neglecting CP violation ($\epsilon = 0$), $K_S \to \pi\pi$ and $K_L \to \pi\pi\pi$
  - Strangeness: assuming the $\Delta Q = \Delta S$ rule, $K^0 \to \pi^- l^+ \nu_l$ and $\bar{K}^0 \to \pi^+ l^- \bar{\nu}_l$ $(l = e, \mu)$

  No control on the time when the measurement occurs, nor on the basis in which the measurement is performed.

Experiments performed with passive measurements prevents the derivation of genuine Bell's inequalities [A. Bramon, R. Escribano and G. G., J. Mod. Opt. **52**, 1681 (2005) [quant-ph/0410122]; L. Kasday, in *Foundations of Quantum Mechanics*, B. d'Espagnat ed. (New York, Academic Press, 1971) p. 195. Proceedings of the International School of Physics 'Enrico Fermi', Course IL].

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Entangled $K^0\bar{K}^0$ Pairs

MAXIMAL ENTANGLEMENT

From $\phi \to K^0\bar{K}^0$ one starts at $\tau = 0$ with an initial $J^{PC} = 1^{--}$ state:

$$\begin{aligned}|\phi(0)\rangle &= \frac{1}{\sqrt{2}}\left\{|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r\right\} \\ &= \frac{1}{\sqrt{2}}\frac{1+|\epsilon|^2}{1-\epsilon^2}\left\{|K_L\rangle_l|K_S\rangle_r - |K_S\rangle_l|K_L\rangle_r\right\}\end{aligned}$$

NON–MAXIMAL ENTANGLEMENT

Two–time state:

$$|\phi(\tau_l,\tau_r)\rangle = \frac{e^{-(\Gamma_L\,\tau_l+\Gamma_S\,\tau_r)/2}}{\sqrt{2}}\left\{|K_L\rangle_l|K_S\rangle_r - e^{i\Delta m(\tau_l-\tau_r)}e^{\Delta\Gamma(\tau_l-\tau_r)/2}|K_S\rangle_l|K_L\rangle_r\right\}$$

$$\begin{aligned}|\phi(\tau_l,\tau_r)\rangle &= \frac{1}{2\sqrt{2}}e^{-(\Gamma_L\,\tau_l+\Gamma_S\,\tau_r)/2} \\ &\times\left\{\left(1-e^{i\Delta m(\tau_l-\tau_r)}e^{\Delta\Gamma(\tau_l-\tau_r)/2}\right)\left[|K^0\rangle_l|K^0\rangle_r - |\bar{K}^0\rangle_l|\bar{K}^0\rangle_r\right]\right. \\ &\left.+\left(1+e^{i\Delta m(\tau_l-\tau_r)}e^{\Delta\Gamma(\tau_l-\tau_r)/2}\right)\left[|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r\right]\right\}\end{aligned}$$

$$\Delta m = m_L - m_S \qquad \Delta\Gamma = \Gamma_L - \Gamma_S$$

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[A. Bramon and G. G., Phys. Rev. Lett. **88**, 040403 (2002); **89**, 160401 (2002)]

Thin kaon regenerator ($\Delta t << \tau_S$) along the right kaon beam

$$|K_S\rangle_r \rightarrow |K_S\rangle_r + \eta|K_L\rangle_r \qquad \eta = i\frac{\pi\nu}{m_K}(f_0 - \bar{f}_0)\Delta\tau$$

$$|K_L\rangle_r \rightarrow |K_L\rangle_r + \eta|K_S\rangle_r$$

$$|\phi_\eta\rangle = \frac{1}{\sqrt{2}}\frac{1+|\epsilon|^2}{1-\epsilon^2}\{|K_S\rangle_l|K_L\rangle_r - |K_L\rangle_l|K_S\rangle_r + \eta|K_S\rangle_l|K_S\rangle_r - \eta|K_L\rangle_l|K_L\rangle_r\}$$

Free evolution up to $\tau_S << T << \tau_L$ and normalizing to undecayed pairs

$$\boxed{|\Phi\rangle = \frac{1}{\sqrt{N}}\{|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle + R\,|K_L\rangle|K_L\rangle + R'\,|K_S\rangle|K_S\rangle\}}$$

$$N = 2 + |R|^2 + |R'|^2 - 2(\langle K_L|K_S\rangle)^2[1 - \mathrm{Re}(R^* R')]$$

$$R = -\eta\, \exp\{[i(m_S - m_L) + (\Gamma_S - \Gamma_L)/2]\,T\} \xrightarrow[T\simeq 10\,\tau_S]{} \mathcal{O}(1)$$

$$R' = -\eta^2/R \xrightarrow[T\simeq 10\,\tau_S]{} \mathcal{O}(10^{-6})$$

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A Classification of Bell's Inequalities

$$\text{Genuine BI} \xrightarrow{\text{Supplementary Assumptions}} \text{Non–Genuine BI}$$

Clauser–Horne

Clauser–Horne–Shimony–Holt

Wigner

Clauser–Horne Inequality

Interpretation with Local Hidden–Variables: restore *locality* and *realism* in quantum world by additional, unobservable, deterministic or stochastic variables

Realism
$$P(K_\alpha, K_\beta) = \int d\lambda\, \rho(\lambda)\, p(K_\alpha, K_\beta|\lambda) \qquad \int d\lambda\, \rho(\lambda) = 1$$

Locality Condition
$$p(K_\alpha, K_\beta|\lambda) = p(K_\alpha, *|\lambda)\, p(*, K_\beta|\lambda)$$

$$p(K_\alpha, *|\lambda) \equiv p(K_\alpha, K_\gamma|\lambda) + p(K_\alpha, K_\gamma^\perp|\lambda) + p(K_\alpha, U_\gamma|\lambda) \quad \forall\, \{|K_\gamma\rangle, |K_\gamma^\perp\rangle\}$$

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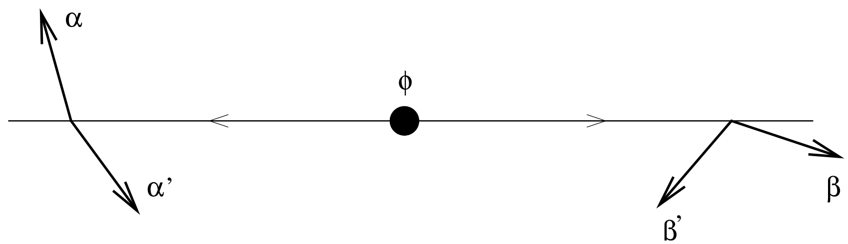
Mathematical Lemma: [J. F. Clauser and M. A. Horne, Phys. Rev. **D 10**, 526 (1974)]

$$0 \leq x_1,\, x_2,\, x_3,\, x_4 \leq 1 \quad \rightarrow \quad -1 \leq x_1 x_2 - x_1 x_4 + x_3 x_2 + x_3 x_4 - x_3 - x_2 \leq 0$$

$$x_1 = p(K_\alpha, *|\lambda), \quad x_2 = p(*, K_\beta|\lambda), \quad x_3 = p(K_{\alpha'}, *|\lambda), \quad x_4 = p(*, K_{\beta'}|\lambda)$$



CLAUSER–HORNE INEQUALITY

$$S \equiv P(K_\alpha, K_\beta) - P(K_\alpha, K_{\beta'}) + P(K_{\alpha'}, K_\beta) + P(K_{\alpha'}, K_{\beta'}) - P(K_{\alpha'}, *) - P(*, K_\beta)$$

$$-1 \leq S \leq 0$$

$$P(K_{\alpha'}, *) = P(K_{\alpha'}, K_\beta) + P(K_{\alpha'}, K_\beta^\perp) + P(K_{\alpha'}, U_\beta)$$

$$P(*, K_\beta) = P(K_\alpha, K_\beta) + P(K_\alpha^\perp, K_\beta) + P(U_\alpha, K_\beta)$$

EBERHARD INEQUALITY

$$P(K_{\alpha'}, K_{\beta'}) \leq P(K_{\alpha'}, K_\beta^\perp) + P(K_\alpha^\perp, K_\beta) + P(K_\alpha, K_{\beta'}) + P(K_{\alpha'}, U_\beta) + P(U_\alpha, K_\beta)$$

[P. H. Eberhard, Phys. Rev. **A 47**, R747 (1993)]

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Clauser–Horne and Eberhard inequalities hardly provide feasible tests of LR vs QM: too high thresholds for the detection efficiencies to attain violations by QM $[P(K_\alpha, K_\beta) \propto \eta_\alpha \, \eta_\beta]$

Supplementary Assumptions beyond Realism and Locality required to obtain testable, but non–genuine, inequalities

Clauser–Horne–Shimony–Holt Inequality

[J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969)]

$$E(\alpha, \beta) = \int d\lambda \, \rho(\lambda) A(\alpha|\lambda) B(\beta|\lambda)$$

Deterministic hidden–variables: $A(\alpha|\lambda) = +1 \ (K_\alpha), \ \ -1 \ (K_\alpha^\perp), \ \ 0 \ (U_\alpha)$

Stochastic hidden–variables: $-1 \leq A(\alpha|\lambda) \leq +1$

$$E(\alpha, \beta) = P(Y_\alpha, Y_\beta) + P(N_\alpha, N_\beta) - P(Y_\alpha, N_\beta) - P(N_\alpha, Y_\beta)$$

$$\begin{aligned}
P(Y_\alpha, Y_\beta) &= P(K_\alpha, K_\beta) \\
P(N_\alpha, N_\beta) &= P(K_\alpha^\perp, K_\beta^\perp) + P(K_\alpha^\perp, U_\beta) + P(U_\alpha, K_\beta^\perp) + P(U_\alpha, U_\beta) \\
P(Y_\alpha, N_\beta) &= P(K_\alpha, K_\beta^\perp) + P(K_\alpha, U_\beta) \\
P(N_\alpha, Y_\beta) &= P(K_\alpha^\perp, K_\beta) + P(U_\alpha, K_\beta)
\end{aligned}$$

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Supplementary Assumption 1 $\Longrightarrow$ Fair Sampling: the set of measured events represents an undistorted sample of the whole set of states emitted by the source $\Longleftrightarrow$ all detection efficiencies $= 1$

CLAUSER–HORNE–SHIMONY–HOLT INEQUALITY

$$|E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta')| \leq 2$$

$$E(\alpha, \beta) = P(K_\alpha, K_\beta) + P(K_\alpha^\perp, K_\beta^\perp) - P(K_\alpha, K_\beta^\perp) - P(K_\alpha^\perp, K_\beta) \qquad \eta_\alpha = \eta_\beta = 1$$

WIGNER INEQUALITY WITH 4 SETTINGS

$$P(K_{\alpha'}, K_{\beta'}) \leq P(K_{\alpha'}, K_\beta^\perp) + P(K_\alpha^\perp, K_\beta) + P(K_\alpha, K_{\beta'})$$

Supplementary Assumption 2 $\Longrightarrow$ Deterministic Local Realistic Theories: in addition require $P(K_\alpha^\perp, K_\beta) = 0$. For maximal entanglement that means perfect anticorrelation: $K_\alpha \equiv K_\beta^\perp$

WIGNER INEQUALITY WITH 3 SETTINGS

$$P(K_{\alpha'}, K_{\beta'}) \leq P(K_{\alpha'}, K_\alpha) + P(K_\alpha, K_{\beta'})$$

[E. P. Wigner, Am. J. Phys. **38**, 1005 (1970)]

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A Review on Bell–Type Tests with Neutral Kaons

Quasi–spin Measurement Axes: $\alpha, \beta, \alpha', \beta'$

- Observable bases: $\{K^0, \bar{K}^0\}$ or $\{K_S, K_L\}$
- Detection times: τ_l, τ_r
- Regeneration: η_l, η_r

I. Assuming Fair Sampling and Determinism

3 settings Wigner inequality with maximally entangled state

$$|\phi(\tau,\tau)\rangle = \frac{e^{-(\Gamma_L+\Gamma_S)\tau}}{\sqrt{2}} \left\{ |K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle \right\}$$

Uchiyama [F. Uchiyama, Phys. Lett. **A 231**, 295 (1997)]

$$P(K_S,K^0) \le P(K_S,K_1) + P(K_1,K^0) \quad \xrightarrow[\text{QM}]{} \quad \mathrm{Re}\,\epsilon \le |\epsilon|^2 \ \ \text{violated}$$

- $\tau_l = \tau_r \equiv \tau \to 0$
- passive measurement in the unphysical CP basis $\{K_1, K_2\}$
- gedanken experiment

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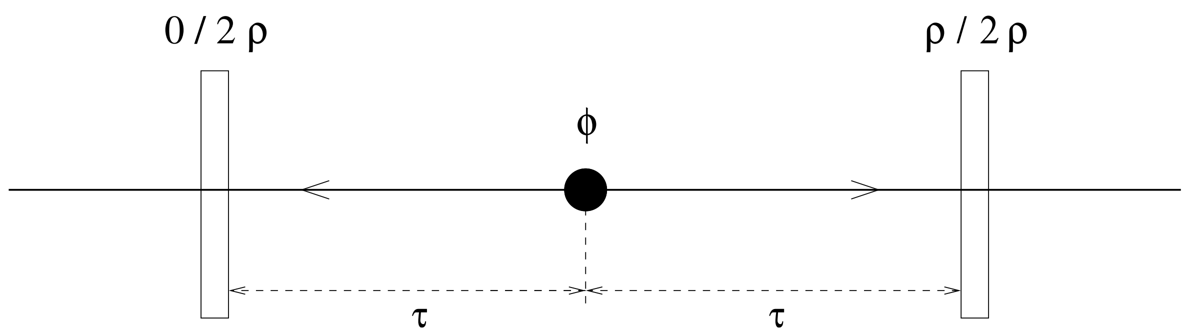
Bramon–Nowakowski [A. Bramon and M. Nowakowski, Phys. Rev. Lett. **83**, 1 (1999)]

Strangeness measurements and thin regenerators



$$|K'_S\rangle = |K_S\rangle + \eta |K_L\rangle \qquad |K'_L\rangle = |K_L\rangle + \eta |K_S\rangle$$

$$\eta \equiv i\,(\lambda_S - \lambda_L)\,\rho\Delta\tau = -\left(i\Delta m + \frac{1}{2}\Delta\Gamma\right)\rho\Delta\tau$$

$$P(K^0,0;\bar{K}^0,\rho) \leq P(K^0,0;\bar{K}^0,2\rho) + P(\bar{K}^0,2\rho;\bar{K}^0,\rho) \quad \xrightarrow[\text{QM}]{} \quad \mathrm{Re}\,\eta \leq 0$$

$$P(K^0,0;\bar{K}^0,\rho) \leq P(K^0,0;K^0,2\rho) + P(K^0,2\rho;\bar{K}^0,\rho) \quad \xrightarrow[\text{QM}]{} \quad \mathrm{Re}\,\eta \geq 0$$

✦ Small $|\eta| \simeq 10^{-3} \div 10^{-2} \Longrightarrow$ violations of the inequalities $\lesssim$ %

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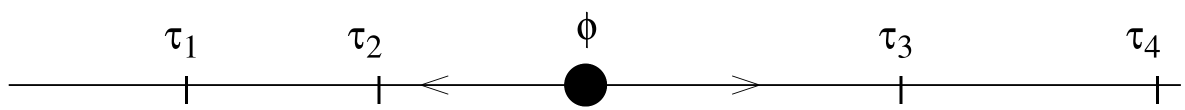
II. Assuming Fair Sampling

4 settings Wigner inequality $\Longleftrightarrow$ Clauser–Horne–Shimony–Holt inequality

Ghirardi et al.

[G. C. Ghirardi, R. Grassi and T. Webern, in *Proceedings of the Workshop on Physics and Detectors for DaΦne*, edited by G. Pancheri, p. 261 (INFN, LNF, 1991)]

Strangeness measurements at 4 different times



$$|E(\tau_1,\tau_3) - E(\tau_1,\tau_4) + E(\tau_2,\tau_3) + E(\tau_2,\tau_4)| \leq 2$$

$$E(\tau_l,\tau_r) = P(Y,\tau_l;Y,\tau_r) + P(N,\tau_l;N,\tau_r) - P(Y,\tau_l;N,\tau_r) - P(N,\tau_l;Y,\tau_r)$$

Y (Yes) and N (No) answer the question whether a $\bar{K}^0$ is detected

$$E_{\rm QM}(\tau_l,\tau_r) = -\exp\left\{-(\Gamma_L+\Gamma_S)(\tau_l+\tau_r)/2\right\}\cos[\Delta m\,(\tau_l-\tau_r)] \qquad \eta=\bar{\eta}=1$$

✦ Never violated by QM! because strangeness oscillations proceed too slowly and cannot compete with the more rapid kaon decays: $\Delta m/(\Gamma_L+\Gamma_S) \simeq 0.5 < 1.0$

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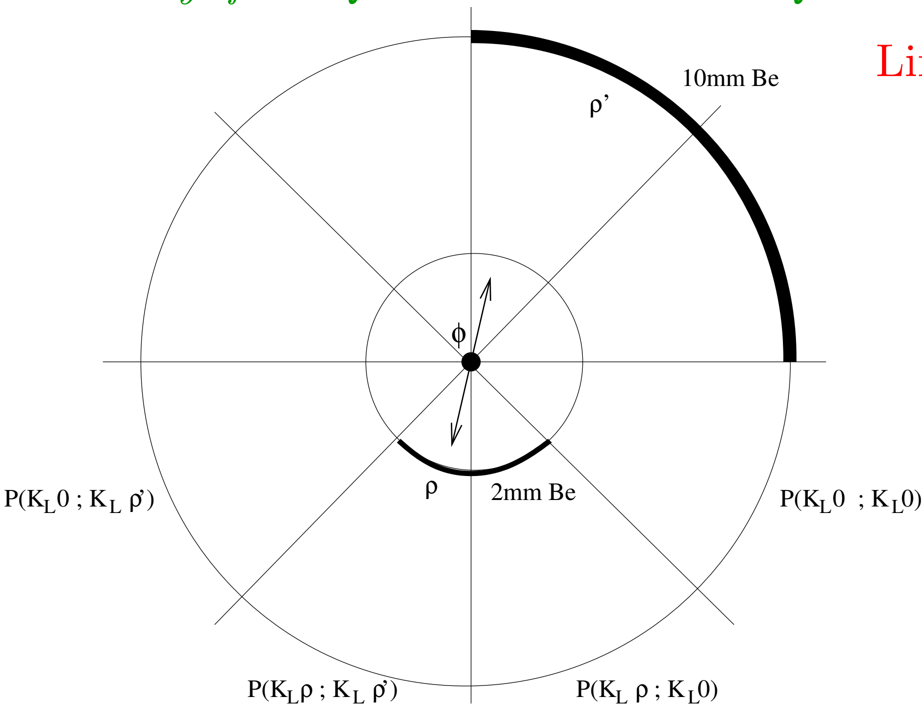
Eberhard and Go–Di Domenico [P. H. Eberhard, Nucl. Phys. **B 398**, 155 (1993); A. Go and A. Di Domenico, mini–workshop on *Neutral kaon interferometry at a Φ–Factory: from Quantum Mechanics to Quantum Gravity*, LNF (Italy), March 24, 2006]

Lifetime measurements and regenerators



$$P(K_L, \rho; K_L, \rho') \leq P(K_L, \rho; K_L, 0) + P(K_L, 0; K_L, 0) + P(K_L, 0; K_L, \rho')$$

- Violated by QM due to constructive interference effect between the two regeneration processes
- Suggestion by Go and Di Domenico for an experiment at Daphne

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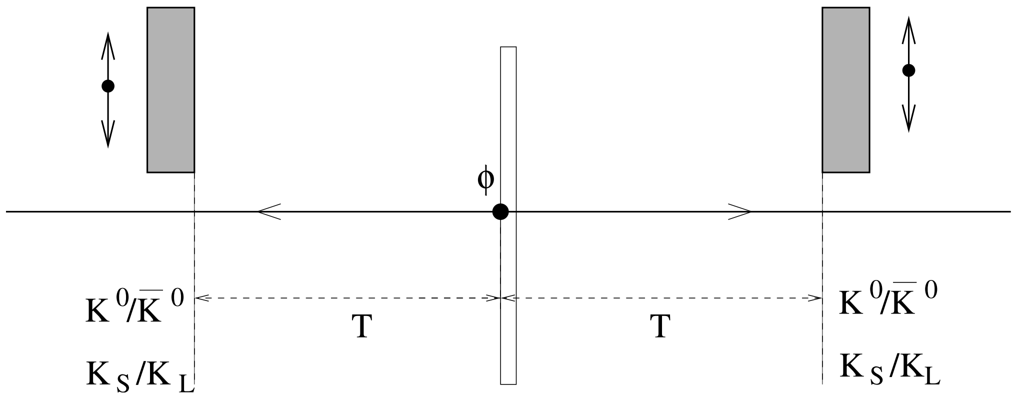
Bramon–Garbarino [A. Bramon and G. G., Phys. Rev. Lett. **88**, 040403 (2002)]

Lifetime and Strangeness measurements and thin regenerators



$$|\Phi\rangle = \frac{1}{\sqrt{2+|R|^2}}\left\{|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle + R\,|K_L\rangle|K_L\rangle\right\}$$

$$R = -\eta\ \exp\left\{[i(m_S - m_L) + (\Gamma_S - \Gamma_L)/2]\,T\right\} \xrightarrow[T\simeq 10\,\tau_S]{} \mathcal{O}(1)$$

$$P(K_S,\bar{K}^0) \le P(K_S,K_S) + P(K^0,K_L) + P(\bar{K}^0,\bar{K}^0) \xrightarrow[\rm QM]{} \frac{2-{\rm Re}\,R+\frac{1}{4}|R|^2}{2+|R|^2} \le 1$$

$$P(\bar{K}^0,K_S) \le P(\bar{K}^0,\bar{K}^0) + P(K_L,K^0) + P(K_S,K_S) \xrightarrow[\rm QM]{} \frac{2+{\rm Re}\,R+\frac{1}{4}|R|^2}{2+|R|^2} \le 1$$

✦ One of the two is violated if $|{\rm Re}\,R| \ge 3|R|^2/4$. Maximum violation (1.14) for $|{\rm Re}\,R| = 0.56$, ${\rm Im}\,R = 0$ (1.55 mm Be regenerator, $T \simeq 11.1\,\tau_S$)

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III. An attempt of Genuine Test

[A. Bramon and G. G., Phys. Rev. Lett. **89**, 160401 (2002); A. Bramon, R. Escribano and G. G., Found. Phys. **36**, 563 (2006)]

Ideal case (neglect K_S/K_L misidentifications)

Hardy's non–locality without inequalities [L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993)]

Hardy's state: $R = -1$ $$|\Phi_H\rangle = \frac{1}{\sqrt{3}}\{|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle - |K_L\rangle|K_L\rangle\}$$

$$\begin{aligned} P_{\rm QM}(K^0, \bar{K}^0) &= \eta\,\bar{\eta}/12 \\ P_{\rm QM}(K^0, K_L) &= 0 \\ P_{\rm QM}(K_L, \bar{K}^0) &= 0 \\ P_{\rm QM}(K_S, K_S) &= 0 \end{aligned}$$

CANNOT BE REPRODUCED BY LOCAL REALISM

$$P_{\rm LR}(K_S, K_S) \geq P_{\rm LR}(K^0, \bar{K}^0) = \eta\,\bar{\eta}/12 > 0$$

✦ Perfect K_S vs K_L discrimination $\Longleftrightarrow$ test of LR vs QM even for infinitesimal strangeness detection efficiencies

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Realistic case

- Account for the unavoidable misidentifications between K_S and K_L states: $\langle K_S | K_L \rangle \neq 0$, $\Gamma_L/\Gamma_S \simeq 1/579$
- Lifetime: $\pi\pi$ decays in $[T, T+5.82\,\tau_S]$ identify K_S's — otherwise K_L's

$$
\begin{aligned}
P_S &= 1 - BR(K_S \to \pi\pi)\exp(-5.82) - BR(K_S \to \pi l \nu_l) = 0.99594 \\
P_L &= 1 - BR(K_L \to \pi\pi)[1-\exp(-5.82/579)] = 0.99997
\end{aligned}
$$

$$
\begin{aligned}
P_{\rm QM}(K^0, \bar{K}^0) &= \eta\bar{\eta}/12 \\
P_{\rm QM}(K^0, K_L) &= 6.77 \times 10^{-4}\, \eta\, \eta_\tau \\
P_{\rm QM}(K_L, \bar{K}^0) &= 6.77 \times 10^{-4}\, \bar{\eta}\, \eta_\tau \\
P_{\rm QM}(K_S, K_S) &= 1.19 \times 10^{-5}\, \eta_\tau^2
\end{aligned}
$$

- Hardy's proof without inequalities no more applicable
  $\Longrightarrow$ Derive a contradiction between LR and QM through a Bell's inequality

Eberhard inequality [P. H. Eberhard, Phys. Rev. **A 47**, R747 (1993)]

$$
H \equiv \frac{P(K^0, \bar{K}^0)}{P(K^0, K_L) + P(K_S, K_S) + P(K_L, \bar{K}^0) + P(K^0, U_{\rm Lif}) + P(U_{\rm Lif}, \bar{K}^0)} \leq 1
$$

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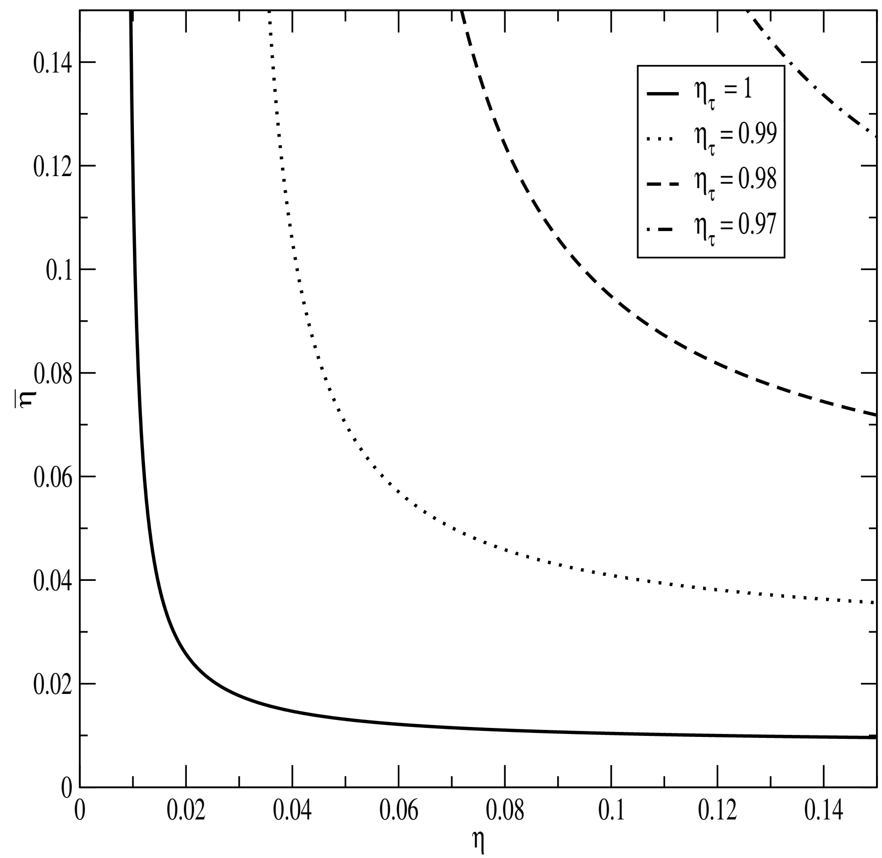
- $H_{\rm LR} \leq 1 \Longleftrightarrow$ *Genuine* Bell's inequality
- $H_{\rm QM}^{P_L=P_S=\eta_\tau=1} \to \infty \;\; \forall\, \eta, \bar{\eta} \neq 0$
- Fair sampling assumption: $H_{\rm QM}^{\eta=\bar{\eta}=\eta_\tau=1} \simeq 60$
- $H_{\rm QM}^{\eta_\tau=1} > 1$ if $\eta = \bar{\eta} > 0.023$, $\eta = \bar{\eta}/2 > 0.017$ ...



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Conclusions

- Important to extend tests of LR vs QM to new sectors of Physics $\Longrightarrow$ Entangled Neutral Kaons from ϕ decays are promising

- Various forms of Bell's inequalities can be deduced. The most appropriate proposals require kaon regenerators

- LOCALITY LOOPHOLE very hard to close with neutral kaons

- A test free from the DETECTION LOOPHOLE requires almost ideal Lifetime measurements and a few % Strangeness detection efficiencies

- MORE FEASIBLE PROPOSALS, incorporating supplementary assumptions, predict violations which are detectable at Daphne

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