Theory on Non-Leptonic and Radiative Kaon Decays

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- Introduction: Motivation

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- Introduction: Theoretical Framework

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- Non-Leptonic Kaon Decays

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- Non-Leptonic Kaon Decays
- Radiative Kaon Decays
  - $K \to \pi\gamma^* \to \pi l^+ l^-$: Status and Comments
  - Some Selected Topics

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  - Some Selected Topics
- Conclusions

Introduction: Motivation

Non-Leptonic and Radiative Kaon Decays: Goals

★ Measure Direct CP-Violation in Kaon Decays •

Experimentally well known in $K \to \pi\pi$: KTeV and NA48

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = (1.63 \pm 0.23) \cdot 10^{-3}.$$

★ Either confirm or show deviations from the SM •

★ Understanding strong-weak dynamics interplay •

Introduction: Theoretical Framework

Perturbative

Renormalization

Group techniques

& matching

at heavy quark

mass thresholds

Short – Distance information

★ Various choices of schemes, regulators, operator basis, $\cdots$

$$\Gamma_{\Delta S=1}(C_i, \nu, \cdots) = \sum_i C_i \, Q_i$$

★ NLO in NDR, HV

- A. Buras et al.
- M. Ciuchini et al.

M_W → M_b → M_c → μ

Under control: model independent √

Introduction: Theoretical Framework

Standard Model $\Delta S = 1$ operators

$Q_2 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha] \, [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta]$ •

In the presence of QCD interactions: Q_1, Q_3, Q_4, Q_5 and

$Q_6 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\beta] \sum_q [\bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha]$ •

In the presence of EW interactions (non-leptonic decays): Q_7, Q_9, Q_{10} and

$Q_8 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\beta] \sum_q \frac{3}{2} e_q [\bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha]$ •

In the presence of EW interactions (radiative decays):

$Q_{7V(A)} = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha] \, [\bar{l} \gamma_\mu (\gamma_5) l]$ •

(Magnetic dipole operators Q_{11} and Q_{12} are very suppressed)

Non-leptonic: $K \to \pi\pi$ and $K \to \pi\pi\pi$ •

Radiative : $K^0 \to \gamma\gamma$, $K^0 \to l^+ l^-$, $K \to \pi l^+ l^-$, $K \to \pi\gamma\gamma$, $K \to \pi\pi\gamma, \cdots$ (and Dalitz pair conversion) •

Introduction: Theoretical Framework

ChPT describes interactions between pions, kaons, photons, $\cdots$

Matching: QCD in the Non-Perturbative regime

Lot of work and advances: Large N_c Methods, Lattice QCD

$$\underline{\mathrm{Non-Perturbative}} \quad \left\{ \begin{array}{c} \bigstar\,\mathrm{Different\ degrees\ of} \\ \mathrm{freedom,\ operator\ basis} \\ \bigstar\,\mathrm{Regulator\ schemes}\ \cdots \\ \underline{\mathbf{\Gamma}_{\mathrm{ChPT}}(g_i, \mu, \cdots)} \end{array} \right. \quad \Downarrow$$

$$\underline{\mathrm{Long-\ Distance\ Part}}$$

$\mu < M_c$

?

ChPT

Introduction: Theoretical Framework

Chiral Perturbation Theory (ChPT) is the effective field theory of the Standard Model at low energies •

S. Weinberg; J. Gasser, H. Leutwyler

Most general Lagrangian with symmetries and relevant degrees of freedom ➩ In general, predictions depend on unknown couplings •

★ Relate different decays with same unknowns •

★ If chiral loops finite and no unknown counterterm ➩ parameter free predictions (e.g., $K_S \to \gamma\gamma$ or $K_L \to \pi^0\gamma\gamma$ at p^4) •

Long list of works applying ChPT to Kaon decays •

Introduction: Theoretical Framework

$\Delta S = 0$: f_π and meson masses at LO; $\tilde{C}$, L_i at NLO, (Gasser, Leutwyler,$\cdots$)

$\Delta S = 1$ at LO: G_E, G_8, G_8' and G_{27} in SU(3) [See Gérard, Smith, Trine for U(3)]

$$\begin{aligned}
\mathcal{L}^{(2)}_{|\Delta S|=1} &= C F_0^6 e^2\, G_E\, \mathrm{tr}(\Delta_{32} u^\dagger Q u) + C F_0^4\, [G_8\, \mathrm{tr}(\Delta_{32} u_\mu u^\mu) \\
&+ G_8'\, \mathrm{tr}(\Delta_{32}\chi_+) + G_{27}\, t^{ij,kl} \mathrm{tr}(\Delta_{ij} u_\mu) \mathrm{tr}(\Delta_{kl} u^\mu)] + \mathrm{h.c.}\,;
\end{aligned}$$

$$\begin{aligned}
u_\mu &\equiv i u^\dagger (D_\mu U)\, u^\dagger = u_\mu^\dagger\,, & \left[U \equiv u^2 = \exp\left(i\sqrt{2}\,\Phi/F_0\right)\right]\,, \\
\Delta_{ij} &= u \lambda_{ij} u^\dagger, & (\lambda_{ij})_{ab} \equiv \delta_{ia}\delta_{jb}\,, \\
\chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u\,, & [\chi = \mathrm{diag}(m_u, m_d, m_s)]\,.
\end{aligned}$$

Φ collects π, Kaon and η fields •

$\Delta S = 1$ at NLO: N_i (Ecker, Kambor, Wyler; Kambor, Missimer, Wyler; Esposito-Farèse; Ecker, Pich, de Rafael; $\cdots$)

$K \to \pi\pi$ and ε'_K: Status

★ ChPT fully at NLO including isospin breaking $\surd$

★ Rôle of final state interactions clarified $\surd$

Kambor, Missimer, Wyler; Bijnens,Pallante, J.P.; Cirigliano, Donoghue, Holstein; Pich, Pallante, Scimemi; Cirigliano, Ecker, Isidori, Müller, Neufeld, Pich; $\cdots$

$\Delta I = 1/2$ rule

Fit to data (Bijnens, Dhonte, Persson): ($G_8 = G_{27} = 1$ at large N_c)

$$\mathrm{Re}\, G_8 \quad = \quad (7.0 \pm 0.6) \left(\frac{87\mathrm{MeV}}{F_0}\right)^4 ; G_{27} = (0.50 \pm 0.06) \left(\frac{87\mathrm{MeV}}{F_0}\right)^4 .$$

★ Advances in calculation of Re G_8 and G_{27} at NLO in $1/N_c$ •

$\Delta I = 1/2$ rule within 40 % $\surd$ Bijnens, J.P. ; Hambye, Peris, de Rafael

$K \to \pi\pi$ and ε'_K: Status

ε'_K fully at NLO in ChPT,

$$\mathrm{Re}\,(\varepsilon'_K/\varepsilon_K) \;\simeq\; -\left[(1.9 \pm 0.5)\,\mathrm{Im}\,G_8 + (0.34 \pm 0.15)\,\mathrm{Im}\,(e^2 G_E)\right].$$

$\mathrm{Im}\,G_8$ and $\mathrm{Im}\,(e^2 G_E)$ are proportional to CP-violating phase $\mathrm{Im}\tau \equiv -\mathrm{Im}\,\frac{\lambda_t}{\lambda_u}$

$$(\lambda_i \equiv V_{id} V^*_{is})$$

Contribution $\Delta S = 1$ NLO counterterms $\mathrm{Im}\,\tilde{K}_i$ estimated to be negligible •

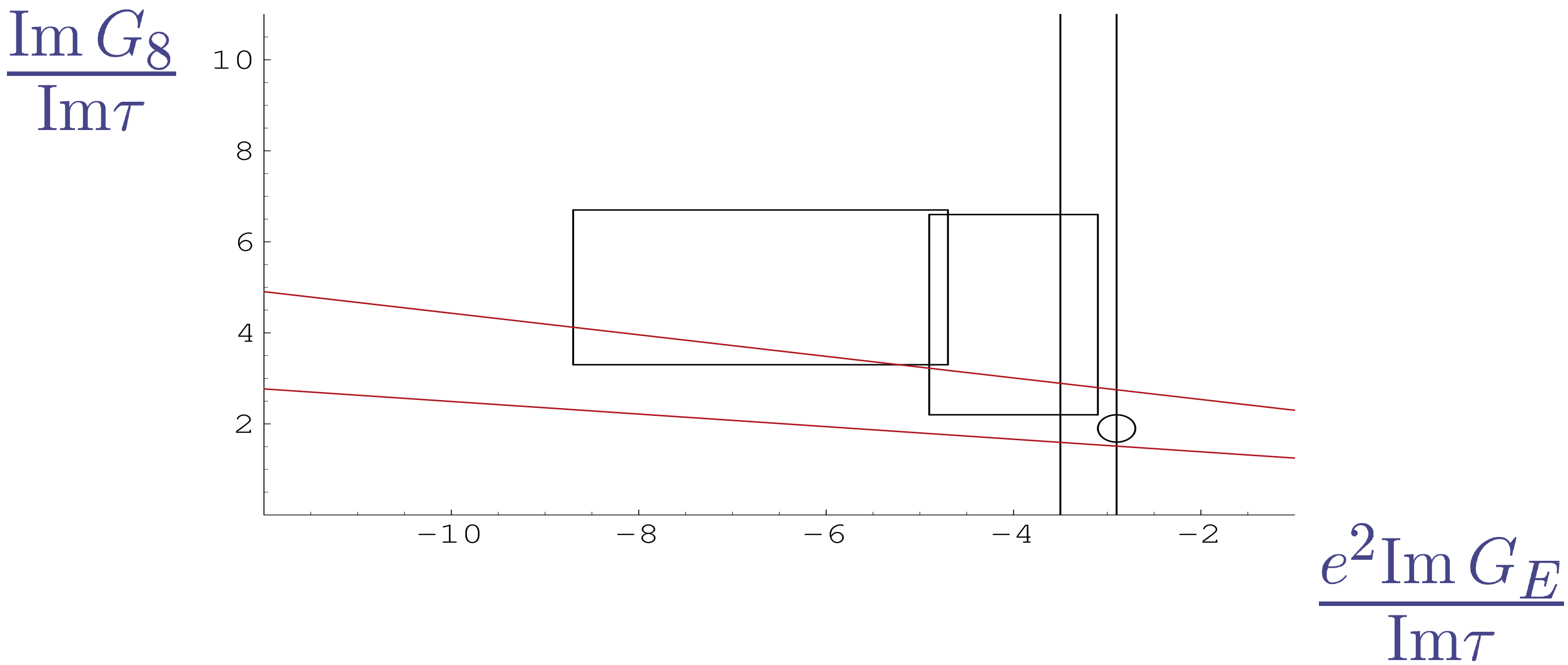
★ Recently, lot of work calculating analytically $\mathrm{Im}\,(e^2\,G_E)$ •

(Cirigliano, Donoghue, Holstein, Maltman; Bijnens, Gámiz, J.P.; Friot, Greynat, Knecht, Peris, de Rafael; Narison)

★ and $\mathrm{Im}\,G_8$ at NLO in $1/N_c$ •

(Bijnens, J.P.; Hambye et al; Hambye, Peris, de Rafael)

ε'_K Status: Theory vs Experiment



★ Large uncertainties in input parameters :

quark condensate $(20\%)^2$, L_5 (30 %) •

$K \to 3\pi$: Dalitz Plot Slopes

CP-Conserving $K \to 3\pi$ ChPT fully at NLO •

Kambor, Missimer, Wyler;
Bijnens, Dhonte, Persson (including isospin breaking);
Gámiz, J.P., Scimemi

Dalitz Plot Slopes

$$\frac{|A_{K^+\to 3\pi}(s_1, s_2, s_3)|^2}{|A_{K^+\to 3\pi}(s_{th}, s_{th}, s_{th})|^2} = 1 + g\, y + h\, y^2 + k\, x^2 + \mathcal{O}(yx^2, y^3).$$

($x = (s_1 - s_2)/m_{\pi^+}^2$ and $y = (s_3 - s_{th})/m_{\pi^+}^2$ are Dalitz variables)

★ Prediction for $K^+ \to \pi^+\pi^+\pi^-$ and $K^+ \to \pi^0\pi^0\pi^+$ slopes in nice agreement with recent NA48/2, ISTRA+ and KLOE results •

$K^+ \to 3\pi$: CP-Violating Asymmetries

Direct CP-Violating Asymmetries in Dalitz plot slope g of $K^+ \to 3\pi$ amplitude •

$$\Delta g_C \equiv \frac{g[K^+ \to \pi^+\pi^+\pi^-] - g[K^- \to \pi^-\pi^-\pi^+]}{g[K^+ \to \pi^+\pi^+\pi^-] + g[K^- \to \pi^-\pi^-\pi^+]}$$

$$\Delta g_N \equiv \frac{g[K^+ \to \pi^0\pi^0\pi^+] - g[K^- \to \pi^0\pi^0\pi^-]}{g[K^+ \to \pi^0\pi^0\pi^+] + g[K^- \to \pi^0\pi^0\pi^-]} .$$

Start at $\mathcal{O}(p^2)$ in ChPT. [Analogous asymmetries for decay rates Γ.]

A lot of previous work: LO plus various NLO estimates.

B.Grinstein, et al. • A. Bel'kov, et al. • G. D'Ambrosio, et al. •
E. Shabalin • [SUSY Contributions: G. Isidori, et al. •]

E. Gámiz, I. Scimemi, J.P. ⇨ First full NLO results in ChPT !

$K^+ \to 3\pi$: CP-Violating Asymmetries

Fully at NLO in ChPT, we get

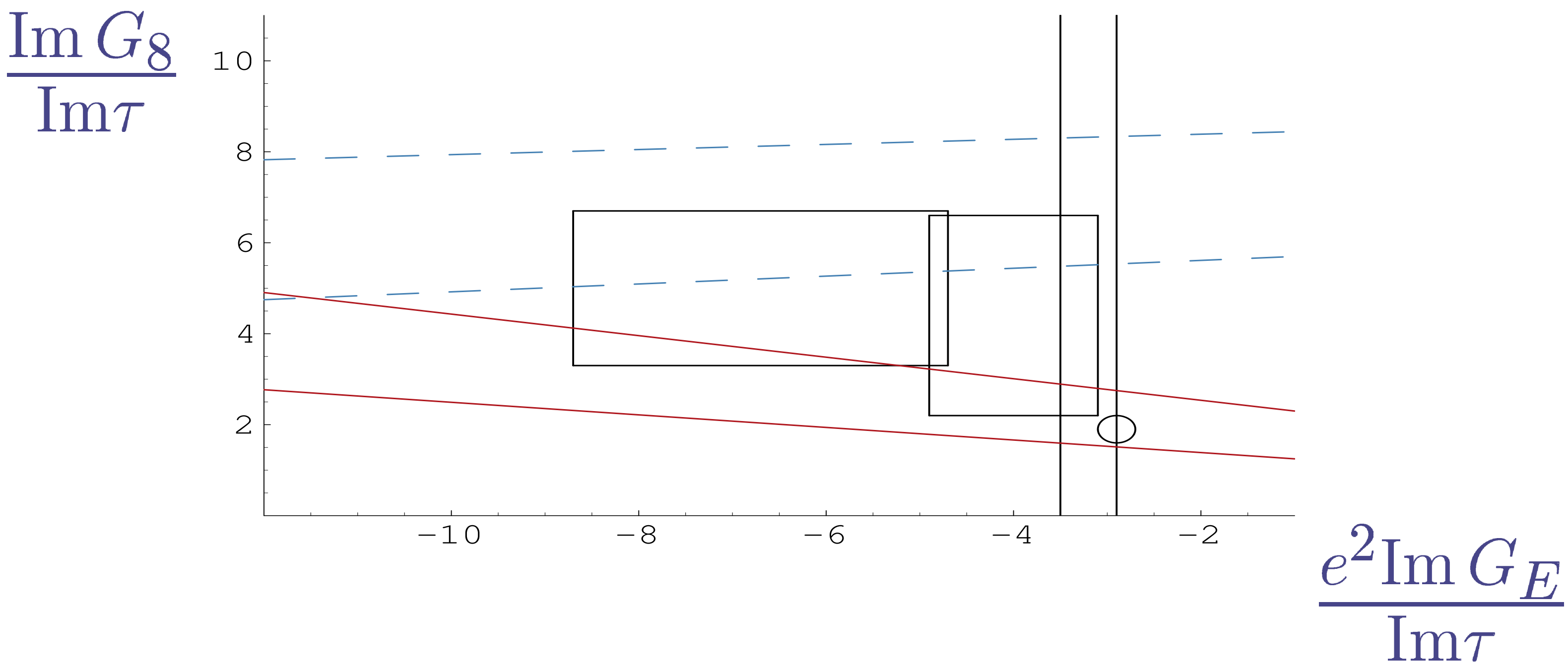
$$\frac{\Delta g_C}{10^{-2}} \simeq (0.7 \pm 0.1)\,\mathrm{Im}G_8 - (0.07 \pm 0.02)\,\mathrm{Im}(e^2 G_E)$$
$$+ (4.3 \pm 1.6)\,\mathrm{Im}\tilde{K}_2 - (18.1 \pm 2.2)\,\mathrm{Im}\tilde{K}_3\,,$$
$$\frac{\Delta g_N}{10^{-2}} \simeq -\Big[(0.04 \pm 0.08)\,\mathrm{Im}G_8 + (0.05 \pm 0.02)\,\mathrm{Im}(e^2 G_E)$$
$$+ (3.7 \pm 1.1)\,\mathrm{Im}\tilde{K}_2 + (26.3 \pm 3.6)\,\mathrm{Im}\tilde{K}_3\Big]\,.$$

Using $\mathrm{Im}\,G_8$, $\mathrm{Im}\,(e^2 G_E)$ from J. Bijnens, E. Gámiz, J.P.

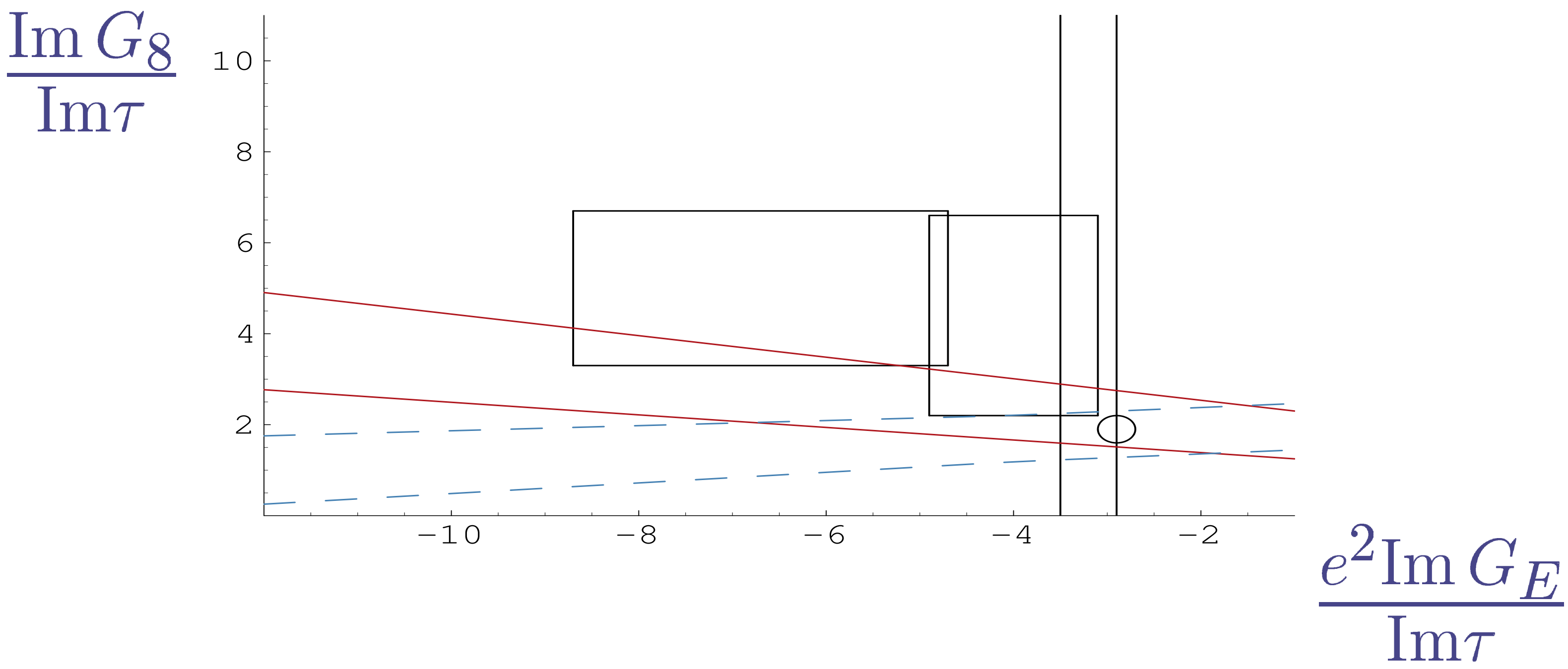
$$\text{and} \quad \frac{\mathrm{Im}\,\tilde{K}_i}{\mathrm{Re}\,\tilde{K}_i} \simeq \frac{\mathrm{Im}\,G_8}{\mathrm{Re}\,G_8} \simeq \frac{\mathrm{Im}G_8}{\mathrm{Re}G_8} \simeq (0.9 \pm 0.3)\,\mathrm{Im}\tau\,,$$
$$\tau \equiv -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\,.$$

Assumption: Ratio dominated by same dynamics at LO and NLO

ε'_K vs Δg_C: $\Delta g_C \sim -3.5 \times 10^{-5}$



ε'_K vs Δg_C: $\Delta g_C \sim -1 \times 10^{-5}$



$K^+ \to 3\pi$: CP-Violating Asymmetries

In the Standard Model,

$$\Delta g_C = -(2.4 \pm 1.2) \times 10^{-5}$$

Other input values (large N_c, $\cdots$) are within errors
Rest of asymmetries are extremely sensitive to unknown counterterms

First experimental results already appeared:

Preliminary 2003+2004 NA48/2

$$\Delta g_C = -(1.3 \pm 2.3) \times 10^{-4} \quad \Delta g_N = (2.1 \pm 1.9) \times 10^{-4}$$

(Calculations ⇨ Interesting uncertainty $\sim 0.3 \times 10^{-4}$)

$K \to \pi\gamma^* \to \pi l^+ l^-$

As an example: status and comments

ChPT LO and NLO dominant effects analysis done $\surd$

and typically, unknown couplings appear •

CP-conserving $K^+ \to \pi^+ l^+ l^-$ and $K_S \to \pi^0 l^+ l^-$ measured $\surd$

Dominated by long-distance $K \to \pi\gamma^* \to \pi l^+ l^-$ •

One coupling governs form factor at LO in ChPT •

$K^+ \to \pi^+ l^+ l^-$: ω_+ $[\mathcal{O}(N_c)]$ Ecker, Pich, de Rafael

or a_+ $[\mathcal{O}(N_c)]$ Ecker, D' Ambrosio, Isidori, Portolés

Included leading p^6 improves fit •

$K \to \pi\gamma^* \to \pi l^+ l^-$

Measured at BNL: $\omega_{+,e} = 1.49 \pm 0.02 \quad [sign(G_8)\, a_{+,e} = -(0.59 \pm 0.01)]$ •

$K_S \to \pi^0 l^+ l^-$:

Analogously, one coupling ($\mathcal{O}(1)$ in $1/N_c$) governs form factor •

NA48/1 measured: $\omega_{S,e} = [2.53^{+0.56}_{-0.45}]\,[-(1.87^{+0.56}_{-0.45})] \quad [|a_{S,e}| = 1.12^{+0.29}_{-0.23}]$ •

HyperCP and NA48/1: compatible results for muons $\surd$

$K_L \to \pi^0 e^+ e^-$: CP-violating

Small CP-conserving $K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-$ $\surd$

Buchalla, D' Ambrosio, Isidori; Isidori, Smith, Unterdorfer

$K \to \pi\gamma^* \to \pi l^+ l^-$

$$Br(K_L \to \pi^0 e^+ e^-)|_{CPV} = \left[(2.38 \pm 0.10) \, [\hat{y}_{7A}^2 + (\hat{y}_{7V} + M_{6V})^2] \left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right)^2 + (3.91 \pm 0.08) \left(\omega_{S,e} - \frac{1}{3}\right) (\hat{y}_{7V} + M_{6V}) \left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right) + (3.41 \pm 0.03) \left(\omega_{S,e} - \frac{1}{3}\right)^2 \right] \times 10^{-12}$$

$$\hat{y}_{7V,7A} \equiv \frac{y_{7V,7A}}{\alpha(M_Z)} \; ; \quad \frac{M_{6V}}{\hat{y}_{7V}} \equiv \frac{y_6 < Q_6 >}{y_{7V} < Q_{7V} >} = -(0.19 \pm 0.11) \, B_{6V} \; ;$$

$$< Q_6 > |_{\mathrm{FAC}}(\nu) = 32 \frac{\langle \overline{q} q \rangle^2 (\nu)}{F_0^6} \, [2C_{63}^r - C_{65}^r] \, (M_\rho) \; ;$$

ChPT counterterm in K^0 EM charge radius   Bijnens, Talavera

$2C_{63}^r - C_{65}^r = (1.8 \pm 0.7) \, 10^{-5} \, (F_0/87 \, \mathrm{MeV})^2$; KTeV, NA48, $K_L \to \pi^+ \pi^- e^+ e^-$

Four-quark effective action calculation   Bruno, J.P.

$2C_{63}^r - C_{65}^r = (2.2 \pm 1.1) \, 10^{-5} \, (F_0/87\mathrm{MeV})^2$

Prediction of interference term sign: Status

Gilman, Wise 1980:

$sign(G_8)\, a_S^{Q_{7V}} \simeq 0.12 > 0$ ⇨ Constructive

Buchalla, Isidori, D' Ambrosio 2003:

Q_{7V} VMD dominance in a_S and large non-VMD in a_+

$sign(G_8\, a_S^{\mathrm{VMD},Q_{7V}}) = -sign(G_8\, a_+^{\mathrm{VMD},Q_{7V}}) > 0$ ⇨ Constructive

Friot, Greynat, de Rafael 2004:

Single pole large N_c inspired approximation fit (K^* and ρ)

$\omega_+ = 1.4 \pm 0.6 > 1/3;$ $(sign(G_8)a_+ = -(0.5 \pm 0.3) < 0)$

$\omega_S = -(2.1 \pm 0.2) < 1/3;$ $(sign(G_8)a_S = -(1.2 \pm 0.1) < 0)$

⇨ Destructive (sign erratum in interference term formula)

$K \to \pi\gamma^* \to \pi l^+ l^-$

Bruno, J.P. 1993 (updated L_9):

Four-quark effective action model at NLO in $1/N_c$: $Q_1, \cdots, Q_6, Q_{7V}$

$$\omega_+ = 1.9 \pm 0.8 > 1/3; \qquad (sign(G_8)a_+ = -(0.8 \pm 0.4) < 0)$$

$$\omega_S = 1.7^{+1.3}_{-0.6} > 1/3; \qquad (sign(G_8)a_S = 0.7^{+0.7}_{-0.3} > 0)$$ ⇨ Constructive

Constructive vs Destructive:

Experimentally from muon forward-backward asymmetry •

Mescia, Smith, Trine; Donoghue, Gabbiani

Both, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ necesary to disentangle new physics from SM (and long-distance from short-distance) •

Mescia, Smith, Trine

Comments on Selected Topics

Recent developments:

- $K^+ \to \pi^+\pi^0\gamma$:

Direct electric emision - Bremßtrahlung interference

Ecker, Pich, de Rafael; Bijnens, Ecker, Pich; Ecker, Neufeld, Pich; D'Ambrosio, Isidori; D'Ambrosio, Cappiello

NA48/2: First time interference is measured ➩ Destructive

Theory expectations: Constructive ?

Again, sign of unknown coupling ? ➩ More work is needed !

See talk by D' Ambrosio at FLAVIAnet Kaon Mini-Workshop •

Comments on Selected Topics

- Interesting recent work on $U_A(1)$ anomaly effects in radiative modes using U(3) ChPT
Gérard, Smith, Trine

Better understanding of $K_L \to \gamma\gamma$ and $K_L \to \gamma l^+ l^-$

Important for $K_L \to \mu^+\mu^-$ dispersive $\gamma\gamma$ contribution

And could give sizeable effects in $K_S \to \pi^0\gamma\gamma$ and $K^+ \to \pi^+\gamma\gamma$

More experimental input on these rare decays is very welcome

Conclusions

Goal: Obtain short-distance (CP-violating phases) information and/or understand strong-weak dynamics interplay •

➯ Need theoretical effort to predict unknown couplings •

Quoted recent analytic work in that direction is based in/or related to large N_c hadronic Ansatzs

Catà, Golterman, Knecht, Peris, de Rafael; Cirigliano, Ecker, Eidemüller, Kaiser, Portolés, Pich, Rosell, Sanz-Cillero; Bijnens, Gámiz, Lipartia, J.P., $\cdots$

Lattice proposals on rare kaon decays appeared

Isidori, Martinelli, Turchetti

Lattice progress on non-leptonic kaon decays: See Sachrajda's talk

Conclusions

➾ Complementary and necesary:
Use different modes to disentagle SM from new physics (and long-distance from short-distance) •

- ε'_K vs $K^+ \to 3\pi$ CP-violating asymetries
- $K_L \to \pi^0 e^+ e^-$ vs $K_L \to \pi^0 \mu^+ \mu^-$
- ε'_K vs $K_L \to \pi^0 e^+ e^-$
- $\cdots$

Conclusions

With expected theory and experimental effort

⇨ Non-leptonic and radiative kaon decays

will continue to give very nice and interesting results •