KAON '07, Frascati, May 21 2007

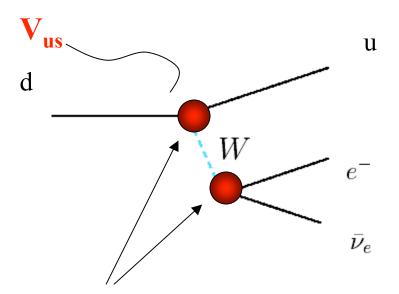
Precision tests of the Standard Model with KI3 decays

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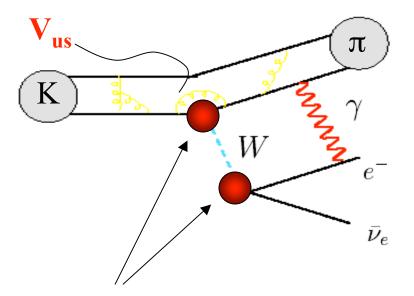


What are we after:



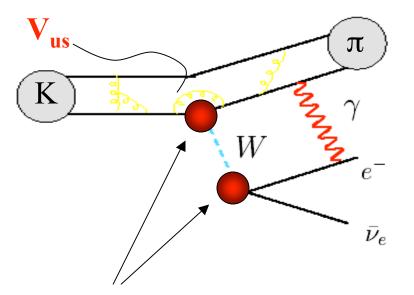
Probe nature of weak vertices

What are we after:



Probe nature of weak vertices through (despite?) hadronic decay

What are we after:



- Probe nature of weak vertices through (despite?) hadronic decay
- Th. tools: ChPT, matching techniques (1/N_C, ...), Lattice QCD
- I will discuss KI3 decays as probes of: (1) lepton universality;
 (2) V_{us} and CKM unitarity; (3) ratios of light quark masses.

Outline

- KI3 master formula and overview of current status in ChPT
- Precision SM tests with KI3 decays:
 - EM corr. \rightarrow lepton universality
 - (EM +) IB corr. \rightarrow quark mass ratios
 - (EM + IB +) SU(3) corr. \rightarrow V_{us} and CKM unitarity

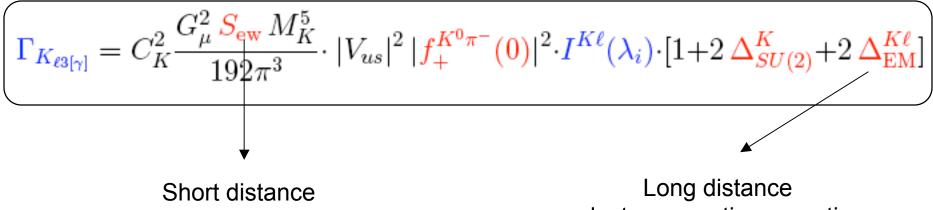
Conclusions

KI3 master formula and overview of current status in ChPT

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\left[\Gamma_{K_{\ell^{3}[\gamma]}} = C_{K}^{2} \frac{G_{\mu}^{2} S_{\text{ew}} M_{K}^{5}}{192\pi^{3}} \cdot |V_{us}|^{2} |f_{+}^{K^{0}\pi^{-}}(0)|^{2} \cdot I^{K\ell}(\lambda_{i}) \cdot [1 + 2\Delta_{SU(2)}^{K} + 2\Delta_{\text{EM}}^{K\ell}]\right]$$

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$



electroweak correction:

electromagnetic correction

$$\frac{S_{\text{ew}}}{S_{\text{ew}}} = 1 + \frac{2\alpha}{\pi} \left(1 + \frac{\alpha_s}{4\pi} \right) \log \frac{M_Z}{M_{\rho}} + O\left(\frac{\alpha \alpha_s}{\pi^2}\right) = 1.0232$$

Sirlin '82

$$K = \{K^{+}, K^{0}\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K_{\ell^{3}[\gamma]}} = C_{K}^{2} \frac{G_{\mu}^{2} S_{ew} M_{K}^{5}}{192\pi^{3}} \cdot |V_{us}|^{2} |f_{+}^{K^{0}\pi^{-}}(0)|^{2} \cdot I^{K\ell}(\lambda_{i}) \cdot [1 + 2\Delta_{SU(2)}^{K} + 2\Delta_{EM}^{K\ell}]$$

$$t = (p_{K} - p_{\pi})^{2}$$

$$\langle \pi^{-}(p_{\pi}) | \bar{s}\gamma_{\mu} u | K^{0}(p_{K}) \rangle = f_{+}^{K^{0}\pi^{-}}(t) (p_{K} + p_{\pi})_{\mu} + f_{-}^{K^{0}\pi^{-}}(t) (p_{K} - p_{\pi})_{\mu}$$

$$K = \{K^{+}, K^{0}\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K_{\ell 3[\gamma]}} = C_{K}^{2} \frac{G_{\mu}^{2} S_{ew} M_{K}^{5}}{192\pi^{3}} \cdot |V_{us}|^{2} |f_{+}^{K^{0}\pi^{-}}(0)|^{2} \cdot I^{K\ell}(\lambda_{i}) \cdot [1 + 2\Delta_{SU(2)}^{K} + 2\Delta_{EM}^{K\ell}]$$

$$t = (p_{K} - p_{\pi})^{2}$$

$$f_{+,0}(t) = f_{+}(0) \left(1 + \lambda_{+,0} \frac{t}{M_{\pi}^{2}} + \lambda_{+,0}'' \frac{t^{2}}{M_{\pi}^{4}} + \dots\right)$$

$$f_{0}(t) = f_{+}(t) + \frac{t}{M_{K}^{2} - M_{\pi}^{2}} f_{-}(t)$$

 $K = \{K^{+}, K^{0}\} \quad \ell = \{e, \mu\}$ $\Gamma_{K_{\ell^{3}[\gamma]}} = C_{K}^{2} \frac{G_{\mu}^{2} S_{ew} M_{K}^{5}}{192\pi^{3}} \cdot |V_{us}|^{2} |f_{+}^{K^{0}\pi^{-}}(0)|^{2} \cdot I^{K\ell}(\lambda_{i}) \cdot [1 + 2\Delta_{SU(2)}^{K} + 2\Delta_{EM}^{K\ell}]$ $t = (p_{K} - p_{\pi})^{2}$

$$\Delta_{SU(2)}^{K} \equiv \frac{f_{+}^{K^{+}\pi^{0}}(0)}{f_{+}^{K^{0}\pi^{-}}(0)} - 1$$

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

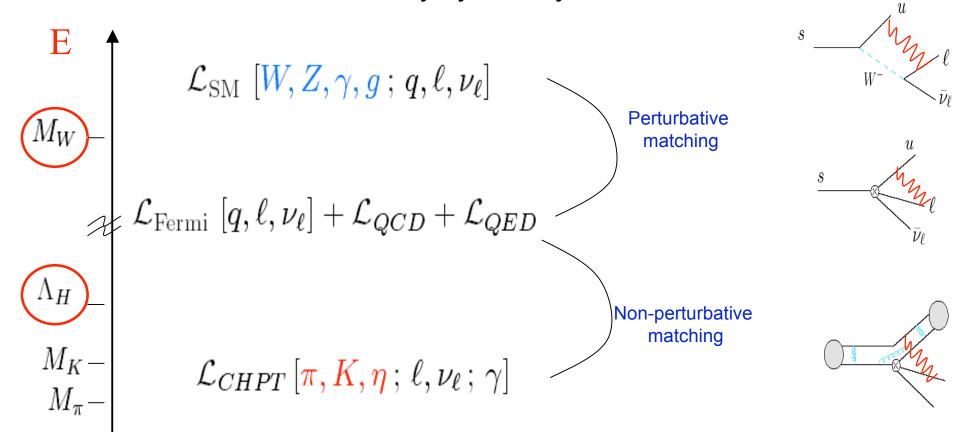
$$\left[\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_{\mu}^2 S_{\text{ew}} M_K^5}{192\pi^3} \cdot |V_{us}|^2 |f_+^{K^0 \pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\Delta_{SU(2)}^K + 2\Delta_{\text{EM}}^{K\ell}]\right]$$

$$\begin{pmatrix} \Gamma_{i} \& I^{K'}(\lambda_{i}) & \rightarrow & \text{Accessible by experiment} \\ S_{ew} & \Delta_{SU(2)}^{K} & \Delta_{EM}^{K\ell} & f_{+}^{K^{0}\pi^{-}}(0) & \rightarrow & \text{Accessible by theory} \end{pmatrix}$$

Chiral Perturbation Theory provides the framework to organize the theoretical analysis

Kaons and Chiral Perturbation Theory

- **Special role of** π, K, η : GB of S χ SB \rightarrow lightest hadrons
- Effective theory: integrate out heavy states → local interactions dictated by symmetry considerations

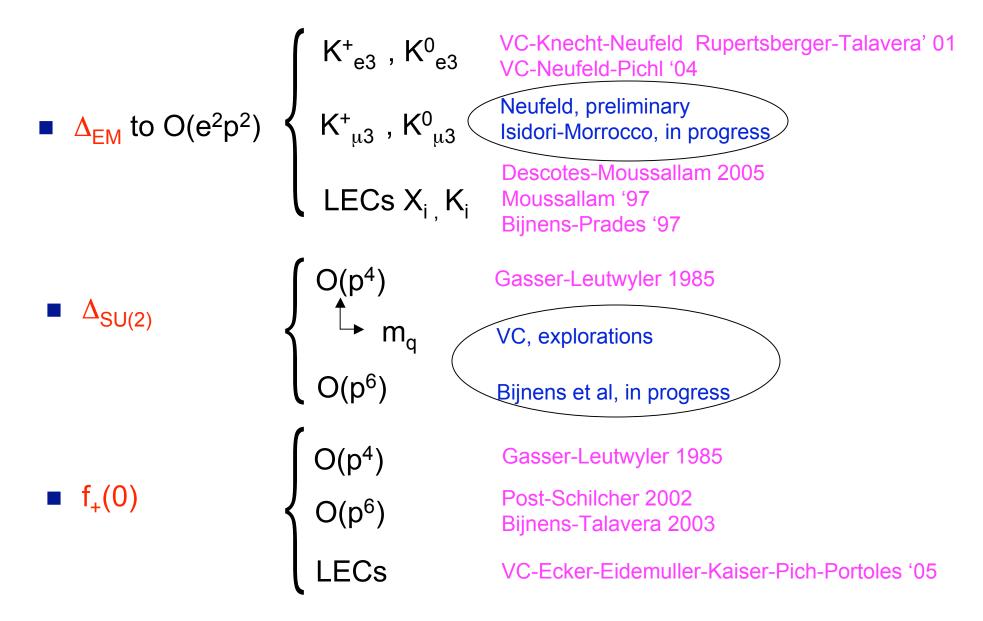


In the chiral EFT the amplitudes are systematically expanded in:

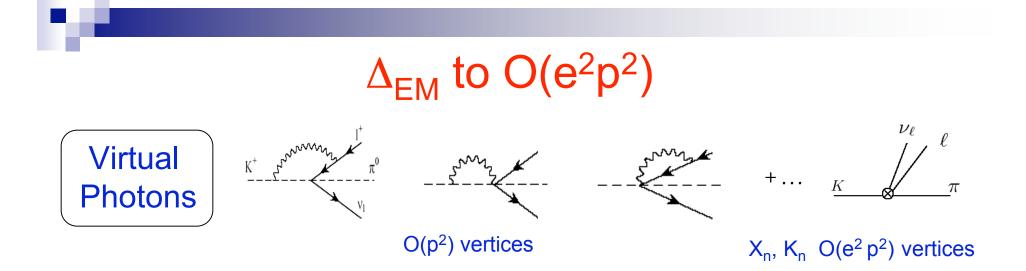
momenta [GB nature], m_{quark} + ew couplings $\int p^{2} \sim \frac{p_{ext}^{2}}{\Lambda_{H}^{2}} \approx \frac{m_{P}^{2}}{\Lambda_{H}^{2}} \qquad m_{q} \sim m_{P}^{2} \sim p^{2} \qquad G_{F}, e$ $\Lambda_{H} \sim 1 \, GeV$

- To a given order: loops (leading IR singularities)
 - "contact" terms, LECs (UV div.+ finite part, reflecting short distance physics)

Status of $K \rightarrow \pi \ell \nu$ in ChPT



EM corrections and lepton universality



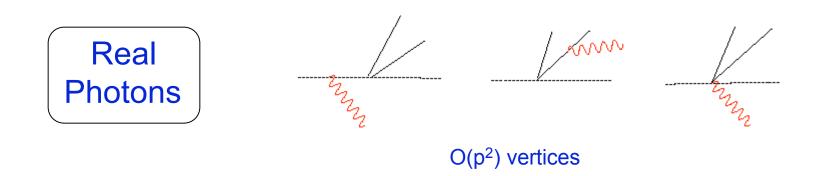
Formal matching in terms of quark currents correlators

$$\tilde{X}_i = \int_0^\infty \, dQ^2 \, W(Q^2) \, \Pi_{QCD}(Q^2) \qquad \mbox{Moussallam '97} \mbox{Descotes and Moussallam '05}$$

Saturate Π_{QCD} (Q²) with resonance interpolators (~ large N_C)

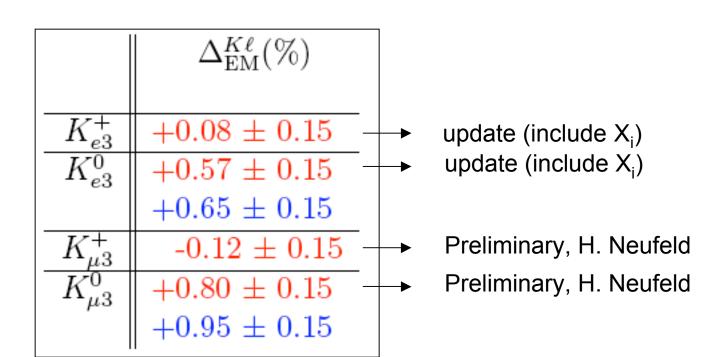
Matching result consistent with naïve dimensional analysis:

$$10^3 X_1: 0 \pm 6.7 \rightarrow -3.7$$
 $10^3 X_6^{\text{phys}}: 16 \pm 8 \rightarrow 10.4$



- Chiral power counting requires to use O(p²) amplitudes, equivalent to Low's theorem with constant form-factors
- According to experimental prescription, we quote results of the fully inclusive integration over 4-body phase space

Results:



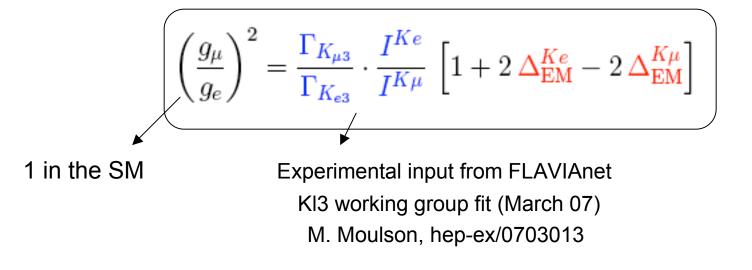
RED: ChPT to O(e² p²)
→ generous uncertainty to account for neglected higher order effects

BLUE: Andre'04 [KTeV]

- \rightarrow non-constant form factors
- \rightarrow hard UV cutoff in loops

Larger effect in K⁰ decay, as expected on account of Coulomb FSI

First application: lepton universality



From neutral K decays (more precise than charged modes):

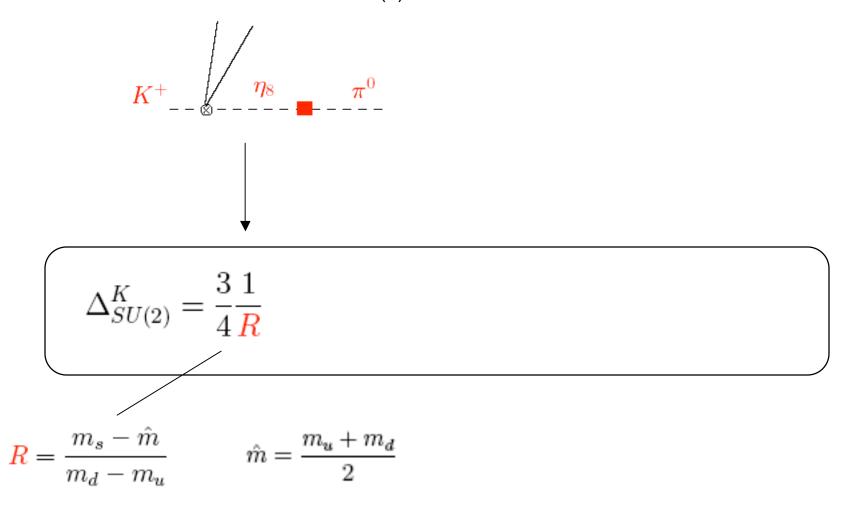
 $|g_{\mu}/g_{e}| = 1.0024 \pm 0.0027$ ~ 0.0005 from theory Approaching the limit from $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$:

 $|g_{\mu}/g_{e}| = 1.0017 \pm 0.0015$

SU(2) breaking and ratios of light quark masses

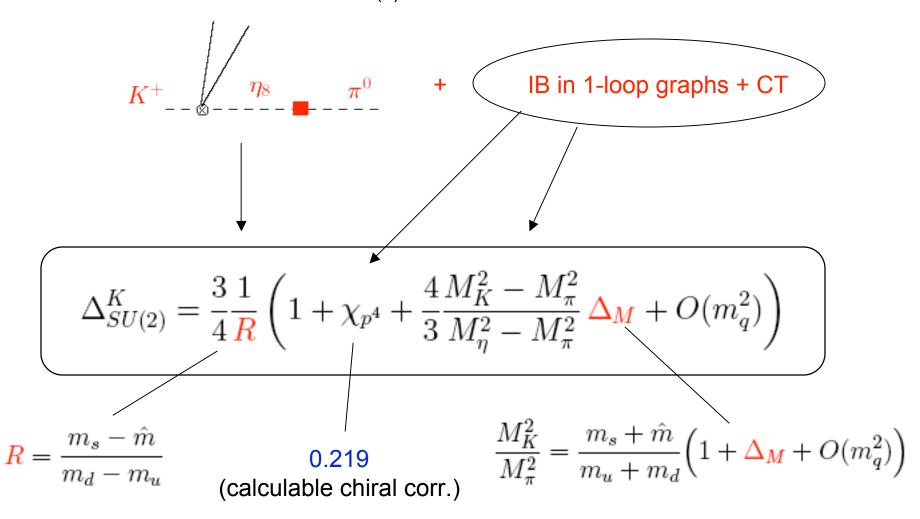
SU(2) breaking in K₁₃ and quark masses

• ChPT to O(p⁴) relates $\Delta_{SU(2)}$ to ratios of quark masses

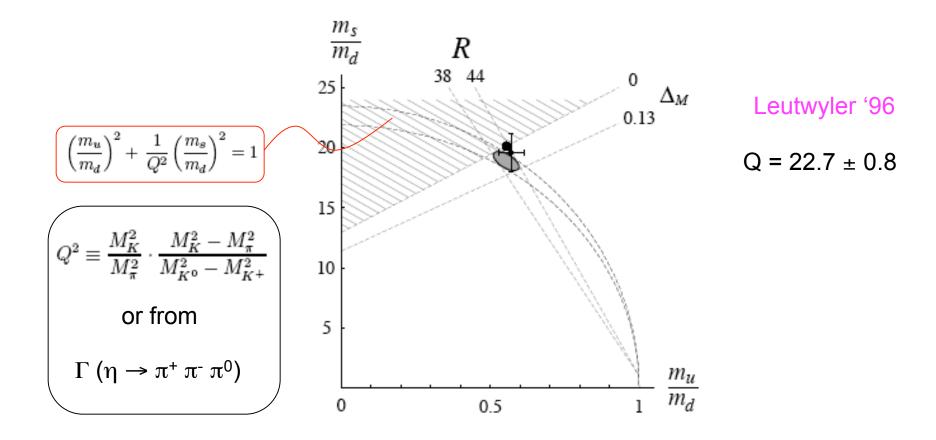


SU(2) breaking in K_{I3} and quark masses

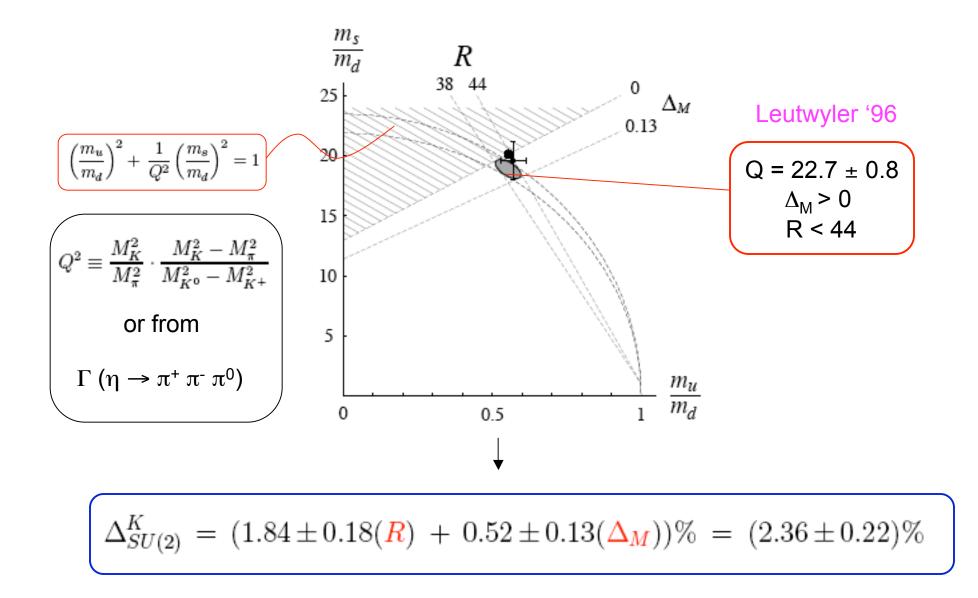
• ChPT to O(p⁴) relates $\Delta_{SU(2)}$ to ratios of quark masses



Standard analysis: input from quark mass ratios \rightarrow predict $\Delta_{SU(2)}$:

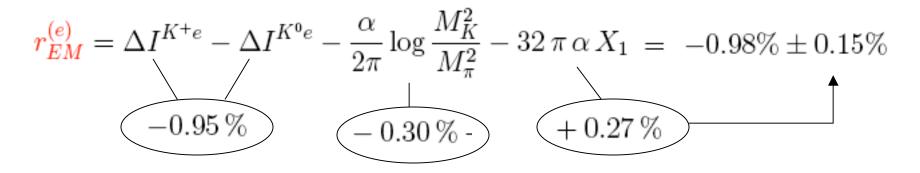


Standard analysis: input from quark mass ratios \rightarrow predict $\Delta_{SU(2)}$:



- On the other hand, data and EM corrections are becoming precise enough to allow for a phenomenological determination of Δ_{SU(2)}
- Focus on K_{e3} modes:

$$\Delta_{SU(2)}^{K} = \frac{\Gamma_{K_{e3}^{+}}}{\Gamma_{K_{e3}^{0}}} \cdot \frac{I^{K^{0}e}}{I^{K^{+}e}} \left(\frac{M_{K^{0}}}{M_{K^{+}}}\right)^{5} - \frac{1}{2} - \frac{1}{2}r_{EM}^{(e)}$$

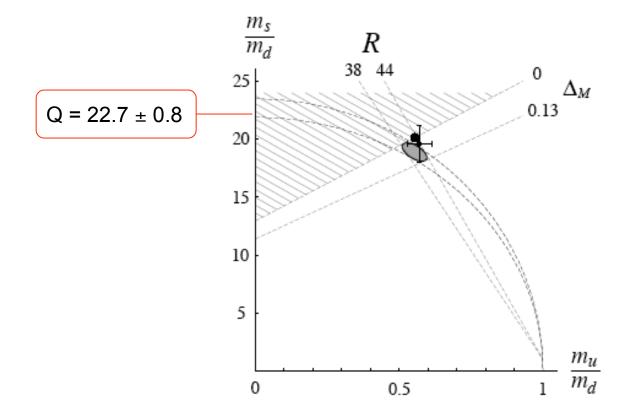


$$\Delta_{SU(2)}^{K}\Big|_{\text{pheno}} = (3.24 \pm 0.43)\%$$

2 σ disagreement
 with "standard"
 th. prediction

■ What are the implications of $\Delta_{SU(2)} \approx 3.3\%$? [vs $\Delta_{SU(2)} \approx 2.4\%$]

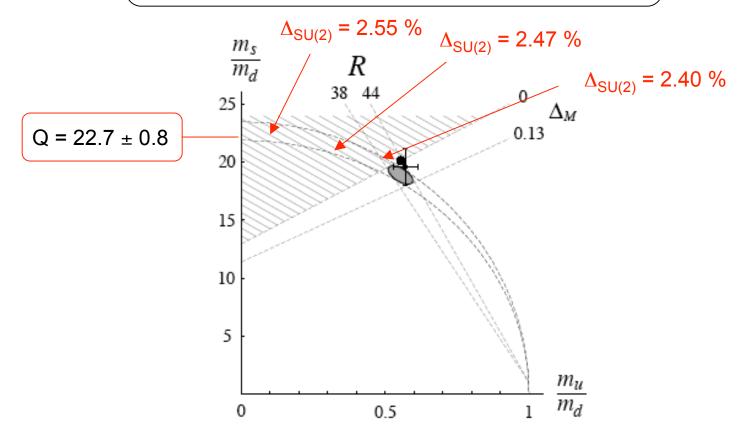
$$\left(\Delta_{SU(2)}^{K} = \frac{3}{4} \frac{1}{R} \left(1 + \chi_{p^{4}} + \frac{4}{3} \frac{M_{K}^{2} - M_{\pi}^{2}}{M_{\eta}^{2} - M_{\pi}^{2}} \Delta_{M} + O(m_{q}^{2})\right)\right)$$



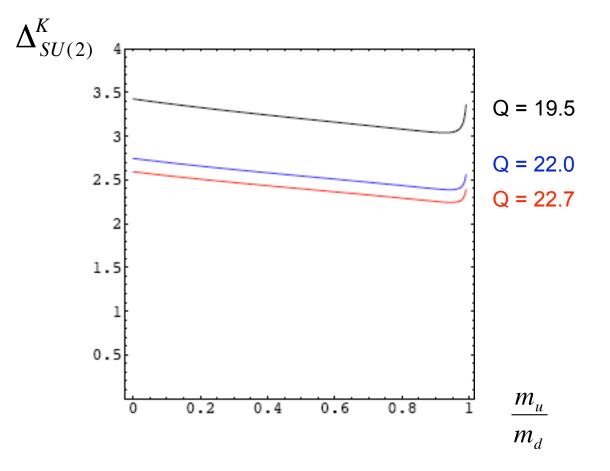
- What are the implications of $\Delta_{SU(2)} \approx 3.3\%$? [vs $\Delta_{SU(2)} \approx 2.4\%$]
- $\Delta_{SU(2)} \approx 3.3\%$ is not consistent with Q=22.7± 0.8

($\leftarrow \Delta_{SU(2)}$ nearly constant along fixed-Q ellipses !!)

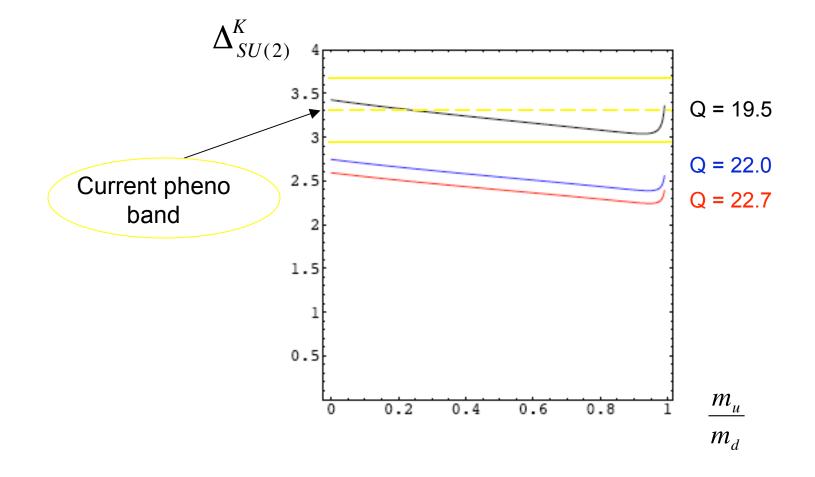
$$\Delta_{SU(2)}^{K} = \frac{3}{4} \frac{1}{R} \left(1 + \chi_{p^4} + \frac{4}{3} \frac{M_K^2 - M_\pi^2}{M_\eta^2 - M_\pi^2} \,\Delta_M + O(m_q^2) \right)$$



• $\Delta_{SU(2)} \approx 3.3\%$ suggests smaller values of Q

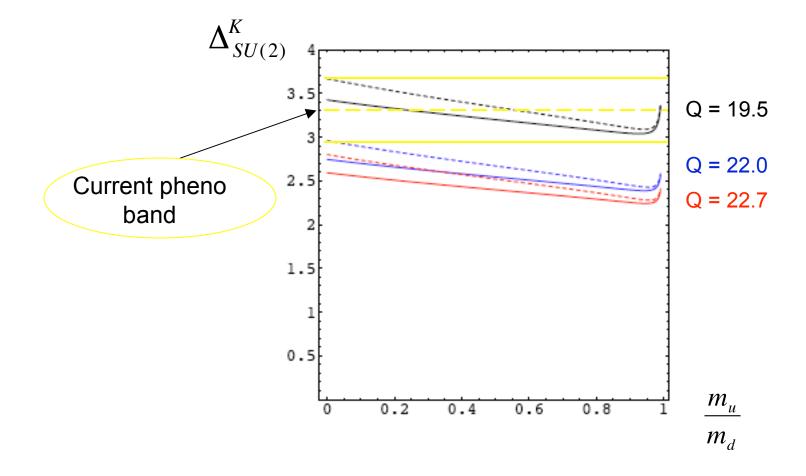


• $\Delta_{SU(2)} \approx 3.3\%$ suggests smaller values of Q



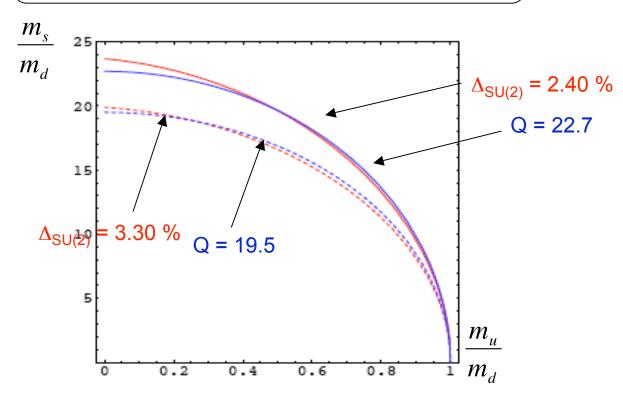
• $\Delta_{SU(2)} \approx 3.3\%$ suggests smaller values of Q

• This remains true even allowing for χ -corrections O(m²) ~ 0.3 O(m)



• Another way to look at this: ellipses parameterized by $\Delta_{SU(2)}$

$$\Delta_{SU(2)}^{K} = \frac{3}{4} \frac{1}{R} \left(1 + \chi_{p^4} + \frac{4}{3} \frac{M_K^2 - M_\pi^2}{M_\eta^2 - M_\pi^2} \Delta_M + O(m_q^2) \right)$$



■ "∆_{SU(2)}" constraint is almost degenerate with "Q" constraint → hard to use it to pin down precisely the quark mass ratios (!) In summary, assuming that EM corrections are OK (all large logs identified and included), the current tension points to:

1) If Q ~ 22 is robust and chiral corrections are of "normal" size \Rightarrow inconsistency in the KI3 data

2) If Q ~ 22 is robust and data is OK \Rightarrow

anomalously large chiral corrections

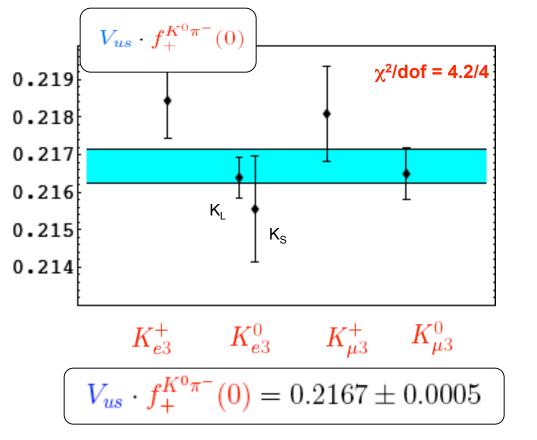
3) If data is OK and chiral corrections are of "normal" size \Rightarrow lower values of Q (< 20)

Work on chiral corrections and new data analyses is underway: we will soon be able to discriminate these possibilities

SU(3) breaking and V_{us}

$$\Delta_{\mathsf{EM}} + \Delta_{\mathsf{SU}(2)} + \exp. \, \mathsf{data} \rightarrow f_+(0) \, V_{\mathsf{us}}$$

$$V_{us} = \frac{C_K^{-1}}{f_+^{K^0 \pi^-}(0)} \left[\frac{192\pi^3 \cdot \Gamma_{K_{\ell 3}[\gamma]}}{G_\mu^2 S_{\mathsf{ew}} M_K^5 \cdot I^{K\ell}} \right]^{1/2} \cdot \frac{1}{1 + \Delta_{SU(2)}^K + \Delta_{\mathsf{EM}}^{K\ell}}$$



New results from KTeV, KLOE, NA48, ISTRA as of March 2007

I use FLAVIAnet fit (M. Moulson) hep-ex/0703013

Dominated by K⁰ modes

2.0

SU(3) breaking in
$$f_{+}^{K\pi}(0)$$

Ademollo-Gatto:

$$f_{+}^{K^{0}\pi^{-}}(0) = 1 + O(m_{s} - m_{d})^{2}$$
SU(3)_v
Chiral
Expansion:

$$f_{+}^{K^{0}\pi^{-}}(0) = 1 + f_{p^{4}} + f_{p^{6}} + \dots$$
O(m_{q}) $O(m_{q}^{2}$)
SU(3)_L × SU(3)_R

$$f_{p^{4}} \sim \frac{(m_{s} - m_{u})^{2}}{m_{s}}$$
UV finite one loop
diagrams in EFT:

$$f_{p^{4}} = -0.0227$$

SU(3) breaking in
$$f_{+}^{K\pi}(0)$$

Ademollo-Gatto:

$$f_{+}^{K^{0}\pi^{-}}(0) = 1 + O(m_{s} - m_{d})^{2}$$
SU(3)_v
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Expansion:

$$f_{+}^{K^{0}\pi^{-}}(0) = 1 + f_{p^{4}} + f_{p^{6}} + \dots$$
SU(3)_L x SU(3)_R
SU(3)_L x SU(3)_R
Up to two-loop graphs in EFT:

$$f_{p^{4}} \sim \frac{(m_{s} - m_{u})^{2}}{m_{s}}$$
Up to two-loop graphs in EFT:

$$f_{p^{4}} \sim \frac{(m_{s} - m_{u})^{2}}{m_{s}}$$
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diagrams in EFT:

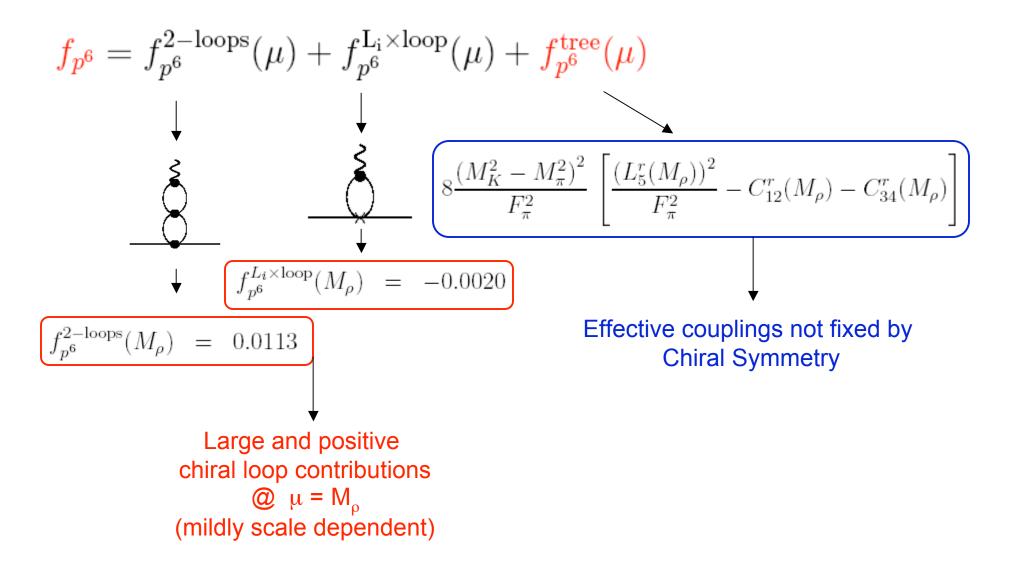
$$f_{p^{4}} = -0.0227$$

$$f_{+}^{W} = -0.016 \pm 0.008$$

$$f_{+}^{W^{0}\pi^{-}}(0) = 0.961 \pm 0.008$$

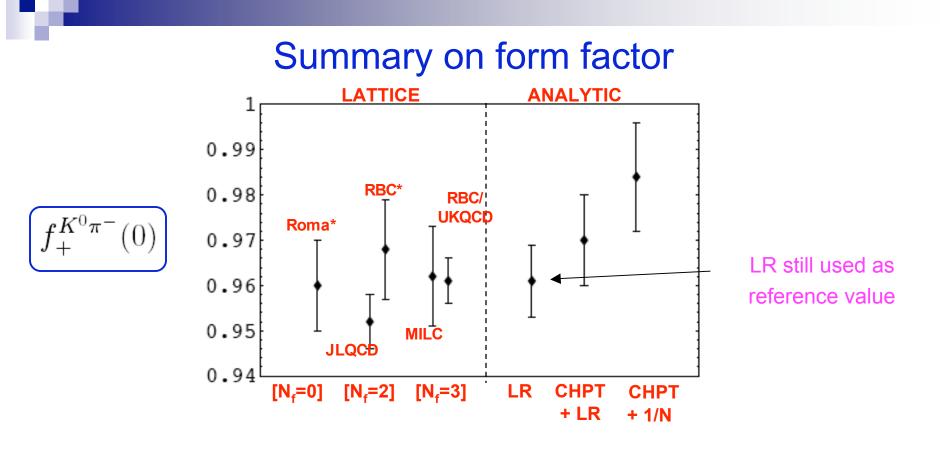
Analytic calculation of $f_+(0)$ to $O(p^6)$

Post-Schilcher '02, Bijnens-Talavera '03



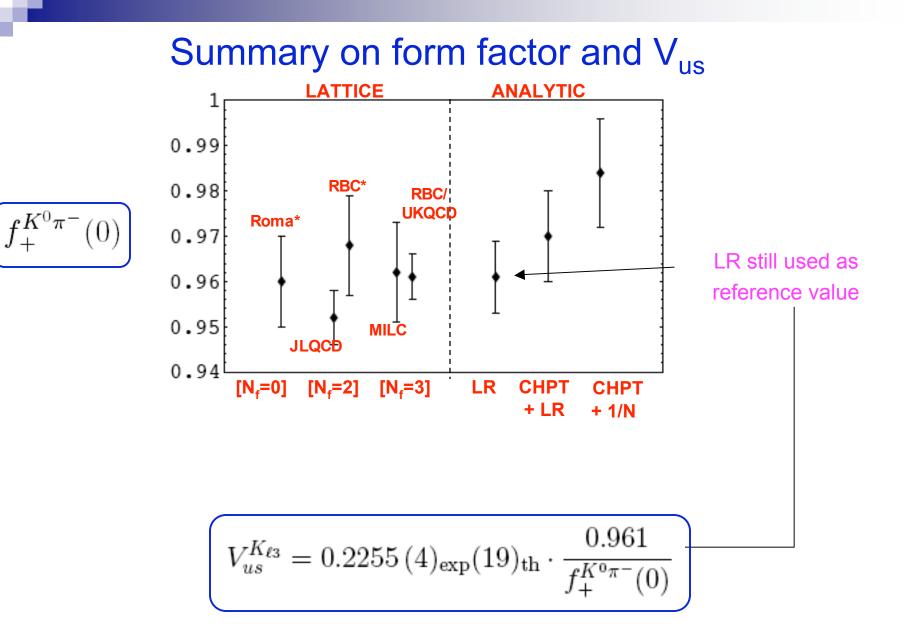
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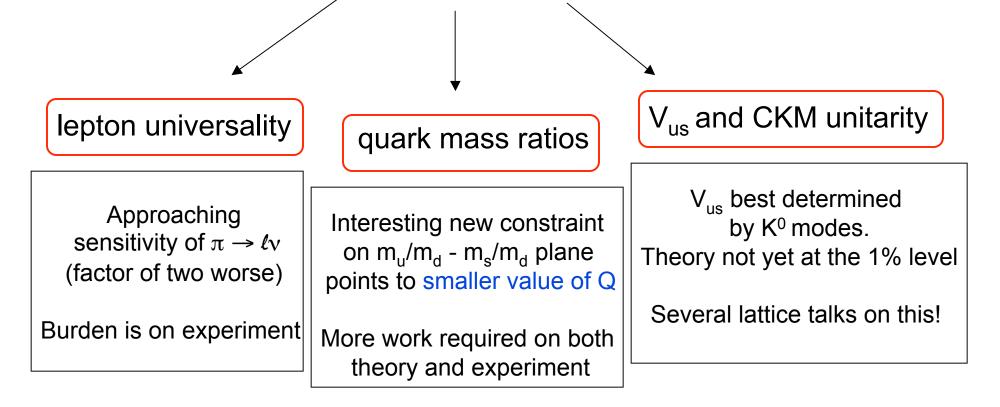
The dust hasn't settled yet... [see lattice talks]

- Inclusion of chiral logs increases analytic estimates over LR
- Key issue: understand role of (χ-logs)² both in chiral extrapolation of lattice data and in analytic estimates



Summary

- KI3 decays allow us to test different aspects of the SM
- Theoretical input: EM, SU(2), SU(3) corrections

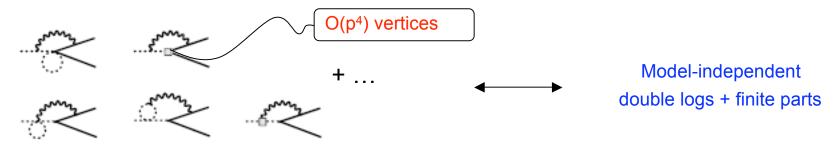


Additional slides

VC & I. Rosell, in progress

$$\begin{aligned} \mathsf{R}_{e/\mu} &= \Gamma(\mathsf{P} \rightarrow \mathsf{ev}) / \Gamma(\mathsf{P} \rightarrow \mu v) \text{ to } \mathsf{O}(\mathsf{e}^2 \mathsf{p}^4) \text{ in } \mathsf{ChP1} \\ P &= \pi, K \\ \hline R_{e/\mu}^{(P)} &= \frac{m_e^2}{m_\mu^2} \left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2} \right)^2 \times \left[1 + \Delta_{e^2 p^2}^{(P)} + \Delta_{e^2 p^4}^{(P)} + \ldots \right] \end{aligned}$$

Up to two loop graphs with virtual photons, one loop with real photons



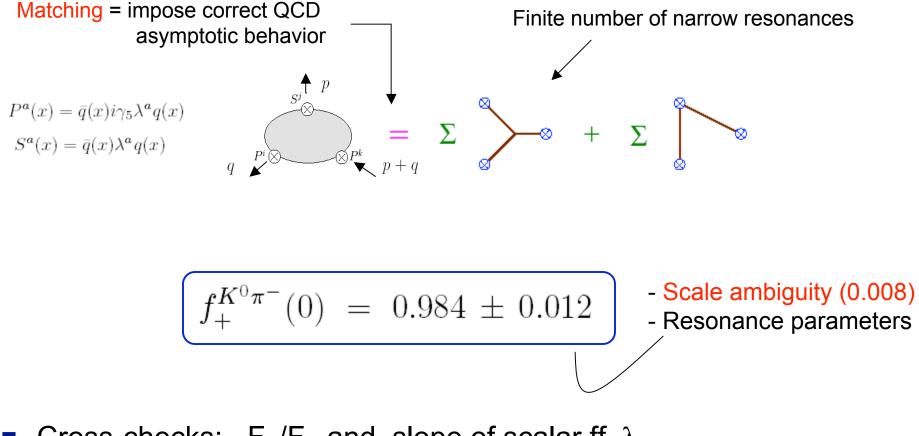
• LEC determined by matching with meromorphic approximations for $\Pi \sim \langle 0 | VA | \pi \rangle$, $\langle 0 | VV | \pi \rangle$ (~ large N_C)

$$\int d^d q \, K(q) \, \Pi_{\text{QCD}}(q) = \int d^d q \, K(q) \, \Pi_{\text{ChPT}}(q) + T_{\text{LEC}}$$

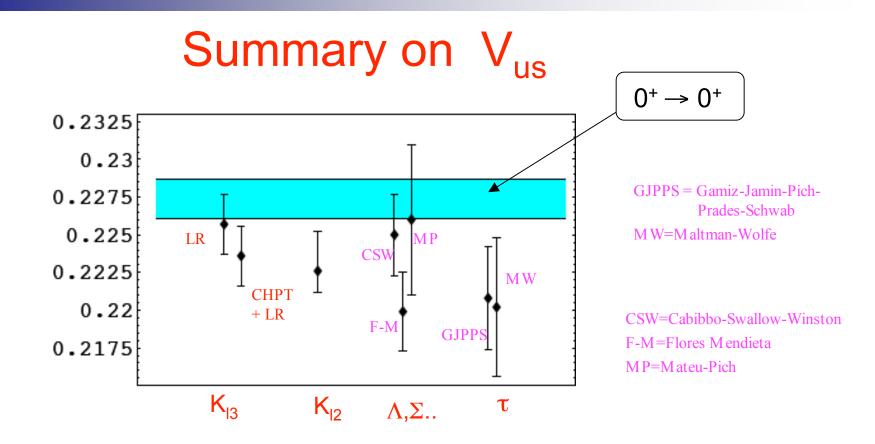
ChPT + truncated large N_C

(Cirigliano-Ecker-Eidemuller-Kaiser-Pich-Portoles 2005)

Obtain effective couplings by large-N inspired matching procedure:



• Cross-checks: F_K/F_{π} and slope of scalar ff λ_0



- At the moment K decays provide best determination of V_{us}
- Meaningful unitarity test will need to await for final value of f₊(0)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -(1.0 \pm 0.6_{V_{ud}} \pm 0.9_{V_{us}}) \cdot 10^{-3}$$