$K \to \pi$ semileptonic form factor $f_+^{K\pi}(0)$ on the lattice with 2+1 flavor Domain Wall Fermions

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Frascati

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UKQCD-RBC collaboration

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Collaboration

RBC-UKQCD collaboration

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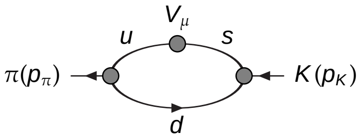
Introduction

- high precision computation of $f_0^{K\pi}(0) = f_+^{K\pi}(0)$ from first principles



$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle \qquad = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$

$$(q^2 = (p_K - p_\pi)^2)$$

- motivation: determine $|V_{us}|$ from $\Gamma \propto |V_{us} f_0(0)|^2$
- control all systematic errors
- several previous calculations $\rightarrow$ cf. the talk by Takashi Kaneko, (Tuesday, Session 1 on $|V_{us}|$ and $|V_{ud}|$)
- currently $\frac{\delta f_0^{K\pi}(0)}{f_0^{K\pi}(0)}|_{\mathrm{latt}} \approx 1\%$

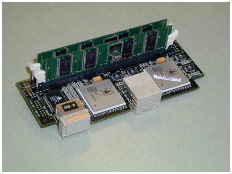
Matrix elements in lattice QCD

- Correlation functions in terms of **Euclidean** path integral

  $\langle \mathcal{O}[\bar\psi,\psi,A]\rangle = \frac{1}{Z}\int D\bar\psi D\psi DA\, \mathcal{O}(\bar\psi,\psi,A)\, e^{-S_G(U)-S_q(\bar\psi,\psi,U)}$

  Ground state matrix elements for large Euclidean times
- discretisation – space time lattice as regulator – regulator π/a

  Statistical sampling of PI with QCDOC-computer by UKQCD/RBC



- from first principles:
  tune bare parameters (coupling and quark masses)
  - lattice spacing: $a^{-1} = \frac{m_\rho^{\rm exp}}{am_V}$
  - quark masses: $\frac{am_H}{am_V} = \frac{m_H^{\rm exp}}{m_V^{\rm exp}}$ $(H = \pi, K, D, \ldots)$

Fighting to control systematic errors

- statistical
- number of dynamical flavors: $N_f = 2, 2+1$
- quark mass

  extrapolation in the quark mass guided by effective theories (χPT)
- discretisation errors (cut-off effects)
  $\rightarrow$ systematic estimation by scaling study
- finite volume errors

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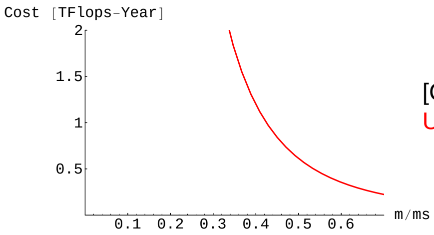

[Clark, Plot: Sachrajda]
Ukawa's Berlin Wall 2001

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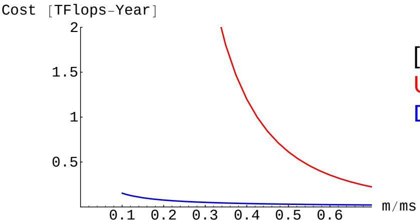

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Del Debbio et al. 2006

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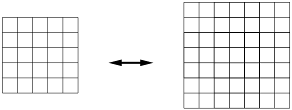
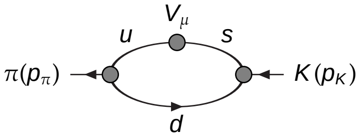
  lattice-box $\approx$ 2fm
  - negligible for $m_\pi L > 3 - 4$
  - systematic estimation: 

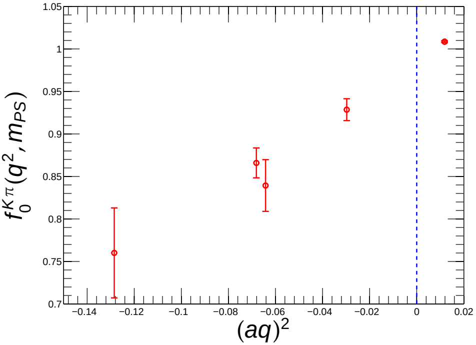
Calculation in practice

- procedure to extract $\langle \pi(p_\pi)|V_\mu|K(p_K)\rangle$ by [Becirevic et al.] (later used by [Okamoto et al.], [Tsutsui et al.], [Dawson et al.], [Antonio et al.])
- behaviour of 3-pt functions at large Euclidean time



$$\sum_{\vec{x},\vec{y}} \langle O_\pi(\vec{y}, t_\pi) V_4(\vec{x}, t) O_K(\vec{0}, 0)\rangle \overset{\text{large } t, (t_\pi - t)}{=} \frac{1}{4 m_K m_\pi} Z_P^\pi Z_P^K \langle \pi(0)|V_4|K(0)\rangle e^{-m_K t} e^{-m_\pi (t_\pi - t)}$$

Bare lattice results:



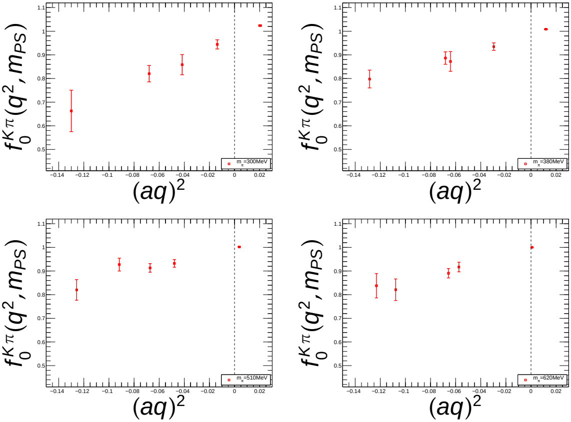
$$\langle K(0)|V_\mu|\pi(0)\rangle$$
$$\langle K(\tfrac{2\pi}{L})|V_\mu|\pi(0)\rangle$$
$$\langle K(0)|V_\mu|\pi(\tfrac{2\pi}{L})\rangle$$
$$\langle K(\sqrt{2}\tfrac{2\pi}{L})|V_\mu|\pi(0)\rangle$$
$$\langle K(0)|V_\mu|\pi(\sqrt{2}\tfrac{2\pi}{L})\rangle$$

RBC-UKQCD calculation - tackling the systematics

RBC-UKQCD configurations [Allton et al.]

• statistical	$\propto$ runtime, try to optimize analysis
• # of dynamical flavors	2+1 dynamical flavors of Domain Wall Fermions
• pion masses	$m_\pi \approx 300, 390, 520, 620$MeV
• discretisation errors	scaling study under way, currently $a^{-1} \approx 1.6$GeV
• finite volume errors	2 volumes: 1.9 and 2.8fm box-size; for 2.8fm always $m_\pi L \geq 4.5$

Data at four quark masses



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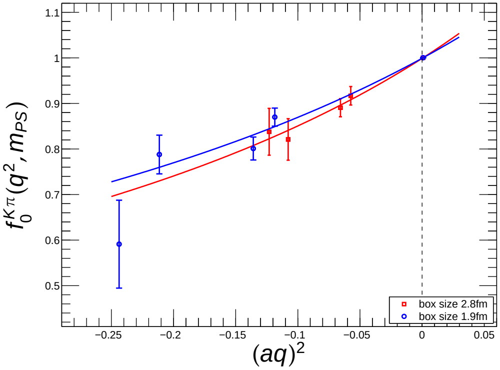
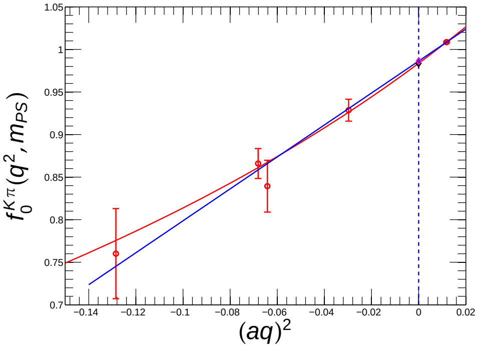
Checking finite volume errors



Remaining major systematics:



- Interpolation in q^2
  (recent development: twisted boundary conditions - lattice computations directly at $q^2 = 0$ [Boyle et al.])
- chiral extrapolation
  lattice results at unphysical Pion masses ($m_\pi > 300$MeV)

q^2-interpolation

1) first interpolate $f_0(q^2, m_{PS})$ in q^2 to $f_+(0, m_{PS}) = f_0(0, m_{PS})$

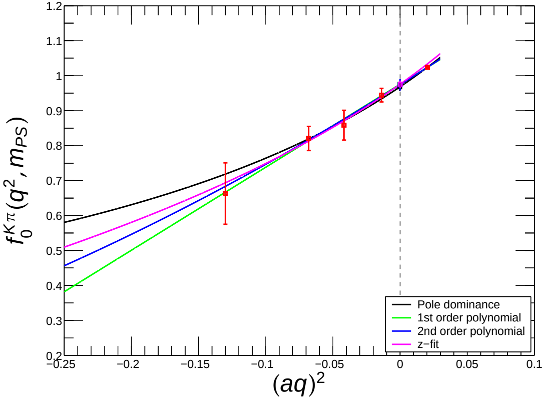
Compare various ansätze

- pole ansatz: $f_0(q^2, m_{PS}) = \frac{f_0(0, m_{PS})}{1 - q^2/M^2}$
- 1st order pol.: $f_0(q^2, m_{PS}) = f_0(0, m_{PS}) + a_1 q^2$
- 2nd order pol.: $f_0(q^2, m_{PS}) = f_0(0, m_{PS}) + a_1 q^2 + a_2 q^4$
- z-fit: 'optimised' taylor expansion using analyticity arguments [Hill 2006]

$$f_0(q^2, m_{PS}) = \frac{1}{\phi} \sum_{k=0}^{\infty} a_k z(t)^k$$

Estimate systematics based on the discrepancy between the fits

q^2-interpolation



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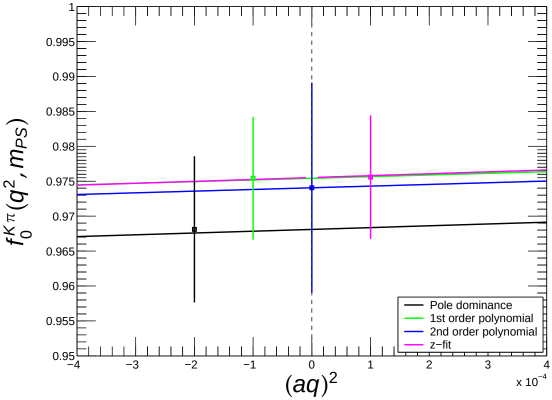

Estimate systematics based on the discrepancy between the fits

Chiral extrapolation

2) then extrapolation to the physical quark mass (chiral extrapolation)

$$f_+(0, m_{PS}) = 1 + \underbrace{f_{p^4}(m_{PS})}_{\substack{\text{analytic} \\ \text{[Gasser \& Leutwyler]}}} + \underbrace{f_{p^6}(m_{PS}, \text{LECs})}_{\substack{\text{[Post \& Schilcher]} \\ \text{[Bijnens \& Talavera]}}} + \ldots$$

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$$f_+(0, m_{PS}) - \left(1 + f_{p^4}(m_{PS})\right) = f_{p^6}(m_{PS}, \text{LECs}) + \cdots \equiv \Delta f(m_{PS})$$

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- 20 – 30% error on $\Delta f(m_{PS})$ roughly yields 1% error on $f_+(0, m_{PS})$

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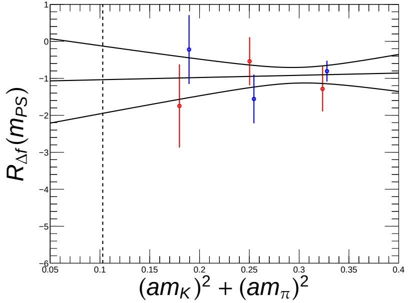
- chiral extrapolation of $\Delta f(m_{PS})$
- $20 - 30\%$ error on $\Delta f(m_{PS})$ roughly yields 1% error on $f_+(0, m_{PS})$
- Ademollo-Gatto: SU(3) breaking starts at order $(m_K^2 - m_\pi^2)^2$
  $\rightarrow$ divide out $(m_K^2 - m_\pi^2)^2$

  ansatz for chiral extrapolation then:

$$R_{\Delta f} = \frac{\Delta f(m_{PS})}{(m_K^2 - m_\pi^2)^2} = A + B(m_K^2 + m_\pi^2)$$

Pion masses 380-620MeV (status CKM 2006)

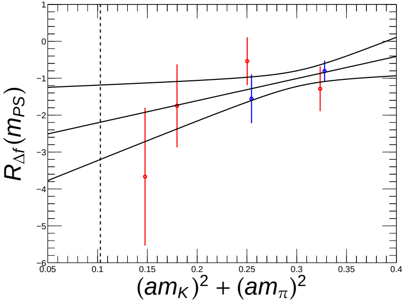


- 1.9fm-lattice
  2.8fm-lattice
- dividing by $(m_K^2 - m_\pi^2)^2 \longrightarrow$ data flat

Chiral extrapolation - independent

PRELIMINARY

Pion masses **300**-620MeV (status now)



wait for larger stats on the lightest data point

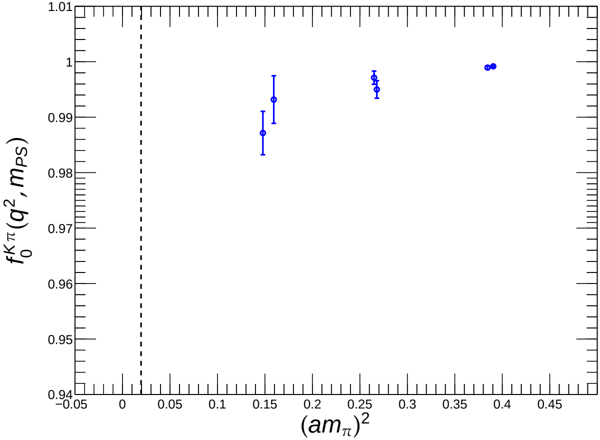
Combined fit

3) Combine q^2 and m_{PS} dependence into one fit, e.g. assuming pole dominance

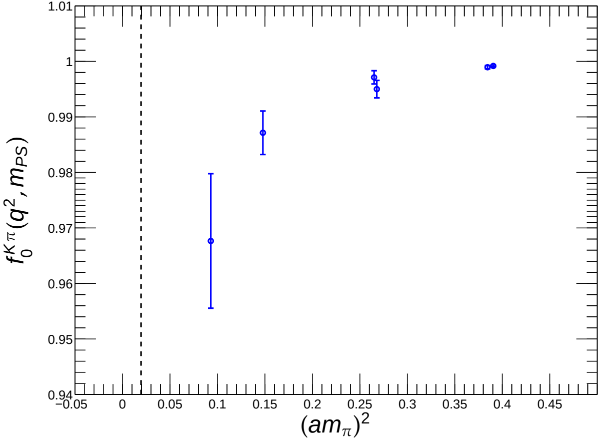
$$f_0(q^2) = \frac{1 + f_{p^4}(m_{PS}) + (m_K^2 - m_\pi^2)^2(A_1 + A_2(m_K^2 + m_\pi^2))}{1 - \frac{q^2}{M_0 + M_1(m_K^2 + m_\pi^2)}}$$

- check data in independent fits first
- simultaneous fit over all q^2 and m_{PS} data points yields smallest error on final result

Status of data as of CKM 2006

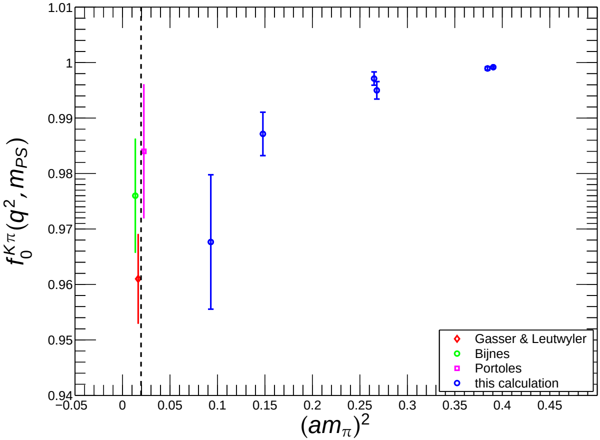


Status now (pure lattice data, no chiral extrapolation)



am_l	0.005	0.01	0.02	0.03
$16^3 \times 32 \times 16$	-	-	0.9950(16)	0.99917(16)
$24^3 \times 64 \times 16$	0.968(12)	0.9871(39)	0.9971(12)	0.99895(31)

Results PRELIMINARY



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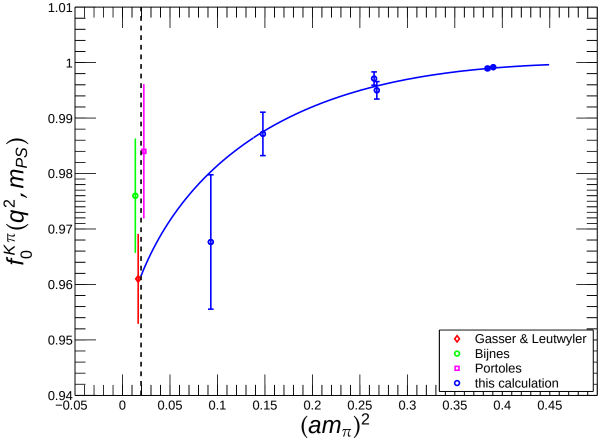

Result

- best estimate (CKM 2006 - 300MeV not yet included):

$$\Delta f = -0.0161(46)_{\text{stat}}(15)_{\chi}(16)_{q^2}(7)_{\text{cut-off}} \rightarrow f_0(0) = 0.9609(51)$$

- together with PDG2006: $|f_0(0)V_{us}| = 0.2169(9)$
- $$|V_{us}| = 0.2257(9)_{\text{exp}}(12)_{f_0(0)}$$
- Unitarity of CKM matrix:

  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$ where

  PDG: $\delta_{\text{PDG}} = 0.0008(10)$

  Our estimate: $\delta_{\text{lattice}} = 0.00076(62)$

Outlook

- full QCD simulation
- currently increasing statistics on $m_\pi = 300$MeV data point
- study of discretisation errors under way
- new technique (”twisted boundary conditions”) removes systematic due to q^2-interpolation
  - exploratory study [Boyle et al.]
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