Theoretical progress on cusp effect and K_{e4} decays

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Content

- Introduction
- The cusp in $K \to 3\pi$ decays I
- The cusp in $K \to 3\pi$ decays II
- A comment on K_{e4} decays
- Summary

INTRODUCTION

A paradise for theoreticians

QCD, at

$$m_u = m_d \,,\ M_\pi = 139.57\,\text{MeV}\ ,\ F_\pi = 92.4\,\text{MeV}$$

No photons: $F_{\mu\nu} = 0$

A paradise

Predictions for $\pi\pi$ scattering lengths (ChPT+ROY+data):

$$a_0^0 = 0.220 \pm 0.005 \qquad a_0^2 = -0.0444 \pm 0.0010$$

Colangelo, Gasser, Leutwyler 2000

The real world

Experiments where scattering lengths can be measured:

$K \to 3\pi$	cusp	CERN/SPS NA48
$K \to \pi\pi e \nu_e$	phase	CERN/PS Geneva-Saclay
		BNL E865, CERN/SPS NA48
Pionium	lifetime	CERN/PS DIRAC
etc		

These experiments are performed in the real world, where

$$e \neq 0\,;\, G_F \neq 0\,;\, M_{\pi^+} \neq M_{\pi^0}\,;\, m_u \neq m_d\,.$$

$$\text{Paradise} \quad \overset{???}{\Longleftrightarrow} \quad \text{Real world}$$

Main part of my talk is devoted to this question

• THE CUSP IN $K \to 3\pi$ DECAYS I

Photons: virtual only

Cusp in $K^+ \to \pi^+\pi^0\pi^0$

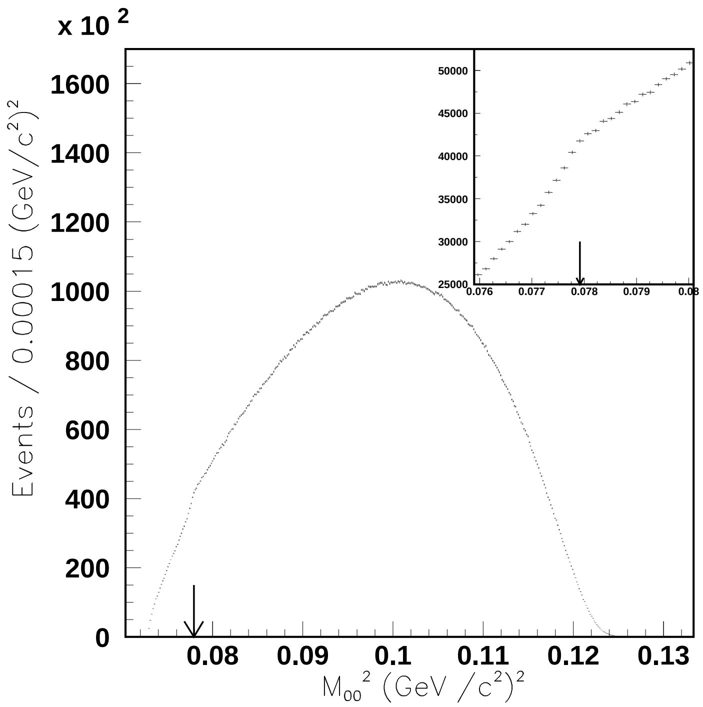
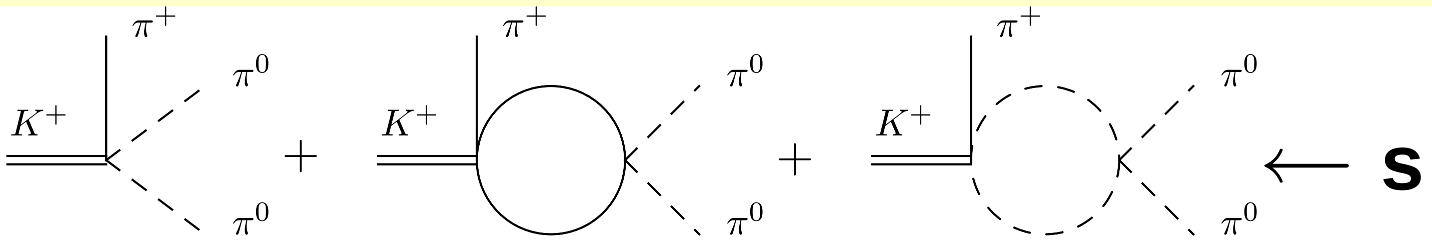


Figure from J.R. Batley et al. (NA48 collaboration) PLB 633, 173 (2006)

Reason for cusp



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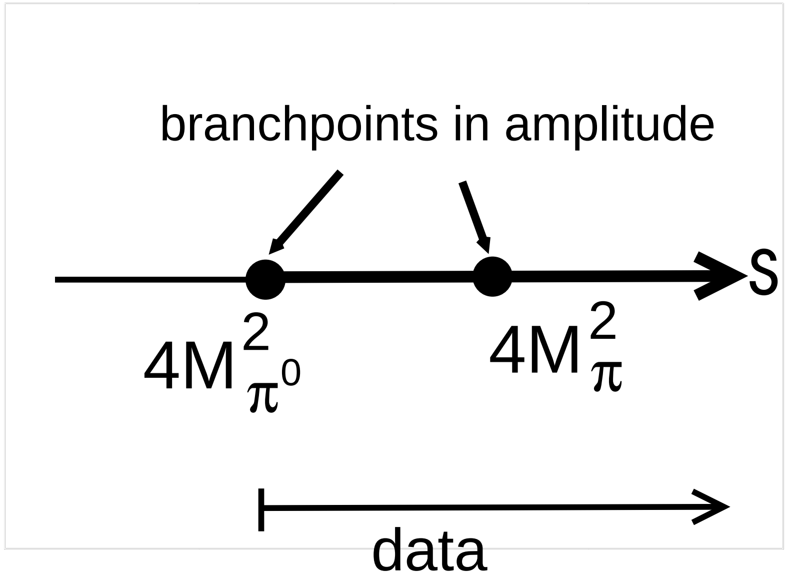


$\Rightarrow$ Singularity at $s = 4M_\pi^2$ generates cusp in differential decay rate.

$\Rightarrow$ Strength of cusp is proportional to $a_0 - a_2$.

Cabibbo 2004; Cabibbo, Isidori 2005

See also: Meissner, Muller and Steininger 1997: Cusp in $\pi\pi \to \pi\pi$

Several approaches to cusp analysis

- Cabibbo, PRL 93:121801,2004

  Cabibbo, Isidori, JHEP 0503 (2005) 021

  analyticity + unitarity + expansion in $\pi\pi$ scattering lengths
- Gámiz, Prades, Scimemi, hep-ph/0602023

  ChPT + variation of Cabibbo/Isidori approach
- Colangelo, J.G., Kubis, Rusetsky, PLB 638 (2006) 187

  Lagrangian framework: non-relativistic QFT. Advantage: Is a systematic procedure.

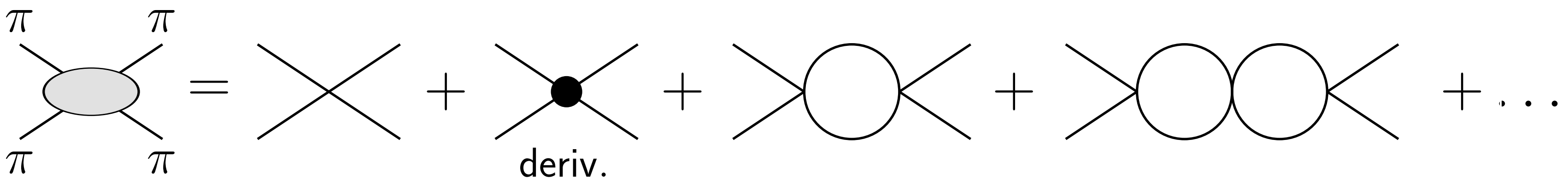
  November 2006: M. Bissegger, A. Fuhrer joined the group

NRQFT : power counting

momenta	:	$\lvert\mathbf{p}\rvert/M_\pi = \mathcal{O}(\epsilon)$
kinetic energy	:	$T = \omega(\mathbf{p}) - M_\pi = \mathcal{O}(\epsilon^2)$
in $K \to 3\pi$	:	$M_K - \sum_i M_i = \sum_i T_i = \mathcal{O}(\epsilon^2)$

where $\omega(\mathbf{p}) = \sqrt{M_\pi^2 + \mathbf{p}^2}$

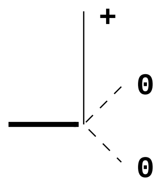
- non-relativistic region = whole decay region (and slightly beyond)
- two-fold expansion in ϵ and $\pi\pi$ scatting length a
- at given order a, ϵ, only finite number of graphs contribute
  $\Rightarrow$ power counting:



- each loop $\propto i\lvert\mathbf{p}\rvert = \mathcal{O}(\epsilon)$ suppressed

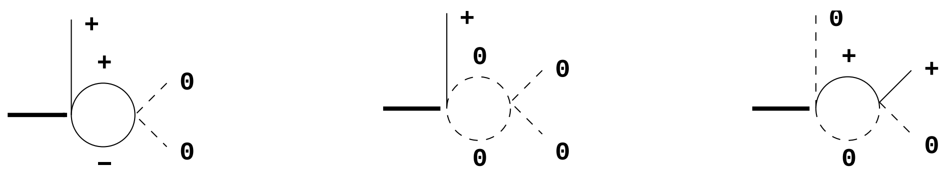
NRQFT: The graphs for $K^+ \to \pi^0\pi^0\pi^+$

Tree

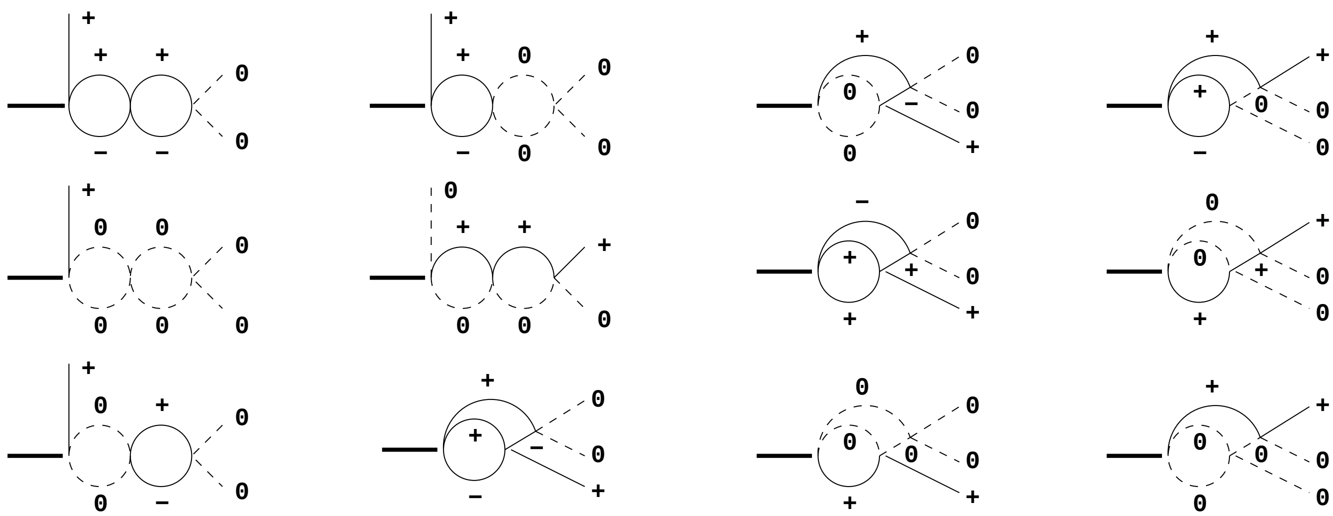


$1, \epsilon^2, \epsilon^4, \ldots$

1-loop



$a\epsilon, a\epsilon^3, a\epsilon^5, \ldots$



2-loops:

$a^2\epsilon^2, a^2\epsilon^4, \cdots$

Comparison with Cabibbo & Isidori

- CI: Analyticity + unitarity + cluster decomposition

  in addition: $\mathcal{M} = A + vB$; A, B analytic ●●

- Gámiz, Prades and Scimemi confirm the result of CI, if the same approximations are made. Analyze uncertainties.
- Here: Lagrangian framework. Analyticity, unitarity, cluster decomposition built in. ●● not true at two-loop order.
- Here: Systematic, power counting. Calculations are now performed at order $\epsilon^4, a\epsilon^5, a^2\epsilon^2$, Fortran programs are available [progress: for all four channels]
- Numerically, difference to CI is tiny in channel $K^+ \to \pi^0\pi^0\pi^+$
- Near threshold, our amplitudes for $K^+ \to 3\pi$ agree with CI. $K_L^0 \to 3\pi$ not yet checked.

THE CUSP IN $K \to 3\pi$ DECAYS II

More photons

$$\text{Paradise} \quad \overset{???}{\Longleftrightarrow} \quad \text{Real world}$$

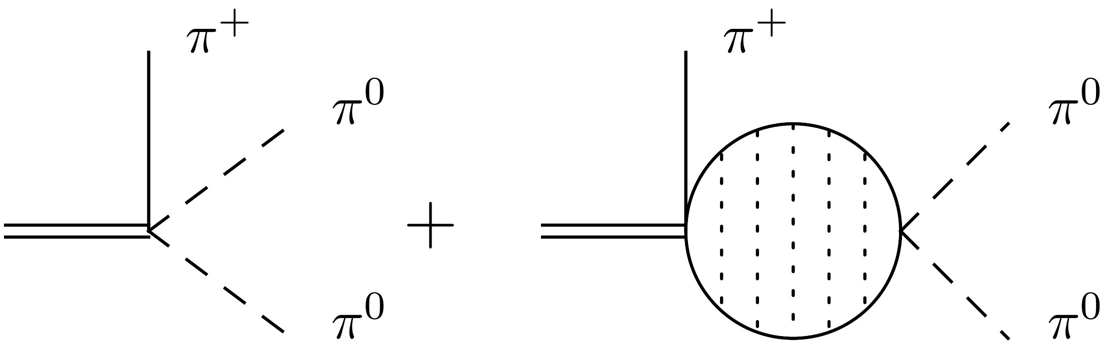
Is work in progress

In collaboration with M. Bissegger, A. Fuhrer, B. Kubis, A. Rusetsky

Pionium

Ionisation energy of ground state: 1.86 KeV

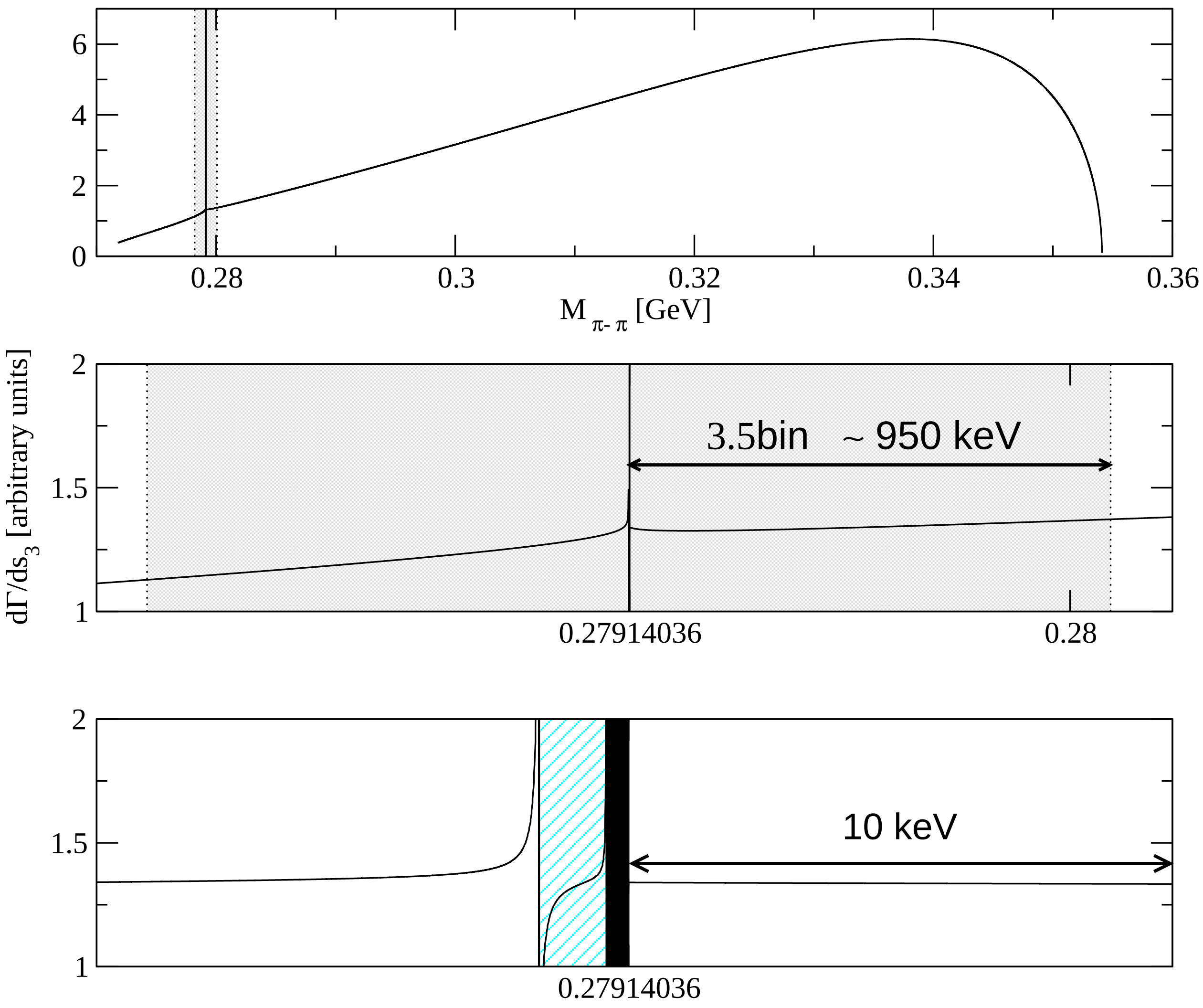
Width of ground state: $\simeq 0.2$ eV [$A_{\pi\pi} \to \pi^0\pi^0 : \; \tau = 2.9 \times 10^{-15}$ sec]



Schwinger function: generates pionium poles

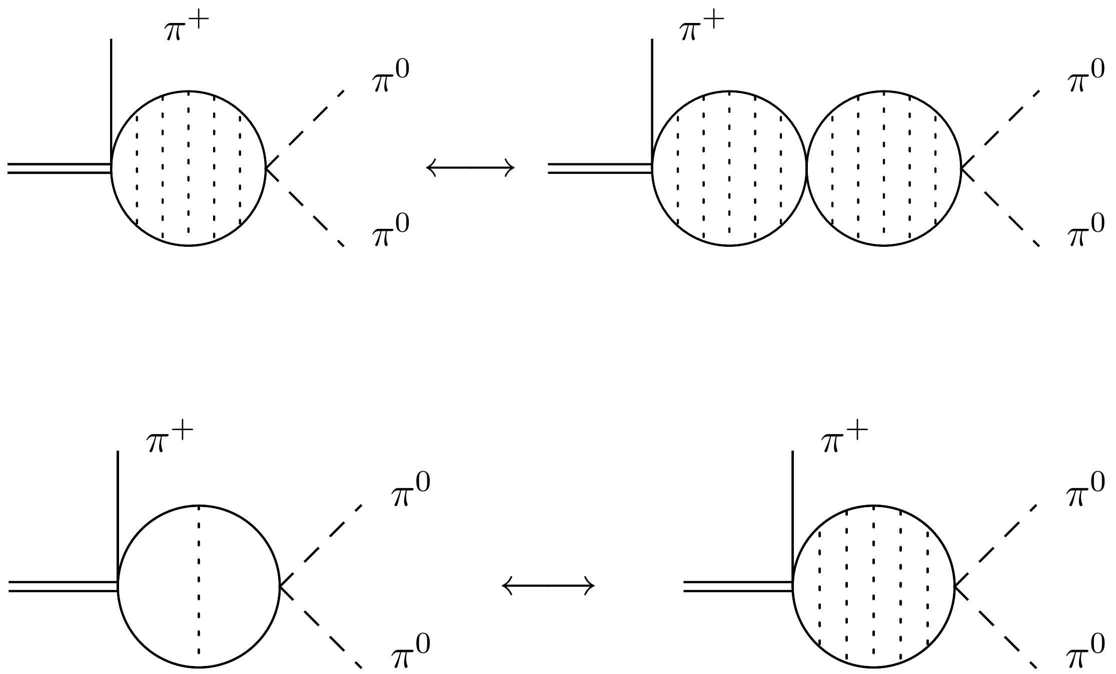
- ⇒ Analysis of NA48/2 collaboration excludes 7 bins from threshold region ($\simeq$ 2 MeV)
- ⇒ Can we include more of the bins?

The 7 bins

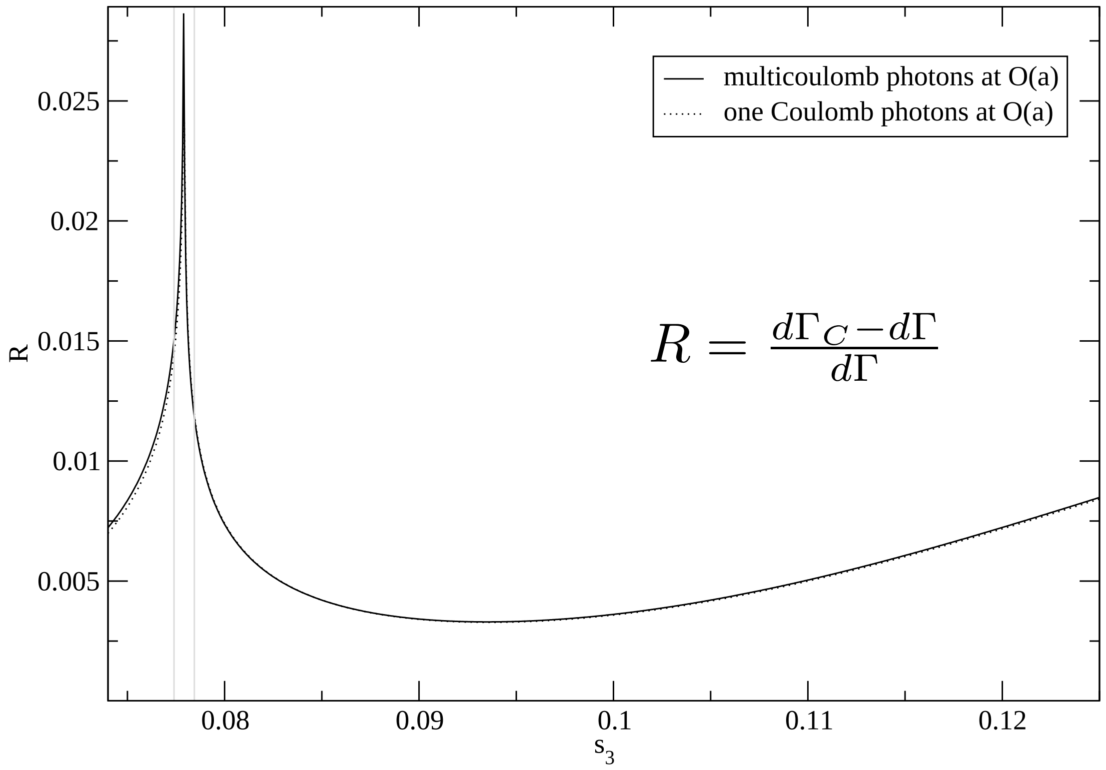


0
2
4
6
0.28
0.3
0.32
0.34
0.36
M π- π [GeV]
dΓ/ds3 [arbitrary units]
1
1.5
2
3.5bin ~ 950 keV
0.27914036
0.28
1
1.5
2
10 keV
0.27914036

How many virtual Coulomb photons needed?



Useful model: Gevorkyan, Tarasov, Voskresenskaya, 2006
Gevorkyan, Madigozhin, Tarasov, Voskresenskaya 2007
Reproduces NRQFT, if the same approximations are made

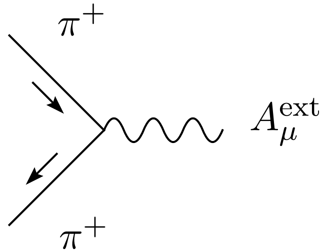


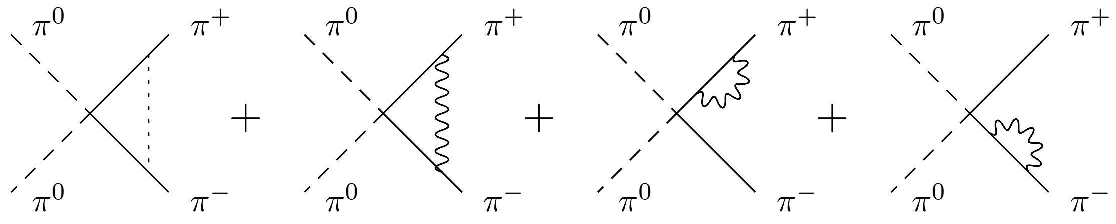
Conclusion: It suffices to include one Coulomb exchange at order a. Number of bins can be reduced, one still knows what one is doing.

Real photons

Status

- We understand by now infrared photons in NRQFT (technically quite demanding: need threshold expanded amplitudes)
- Have started to include real photons
- I cannot explain all we did - please ask privately if you are interested.





K_{e4} DECAYS

A comment

$K \to \pi\pi e\nu$ decays

The decay amplitude contains the axial current matrix element

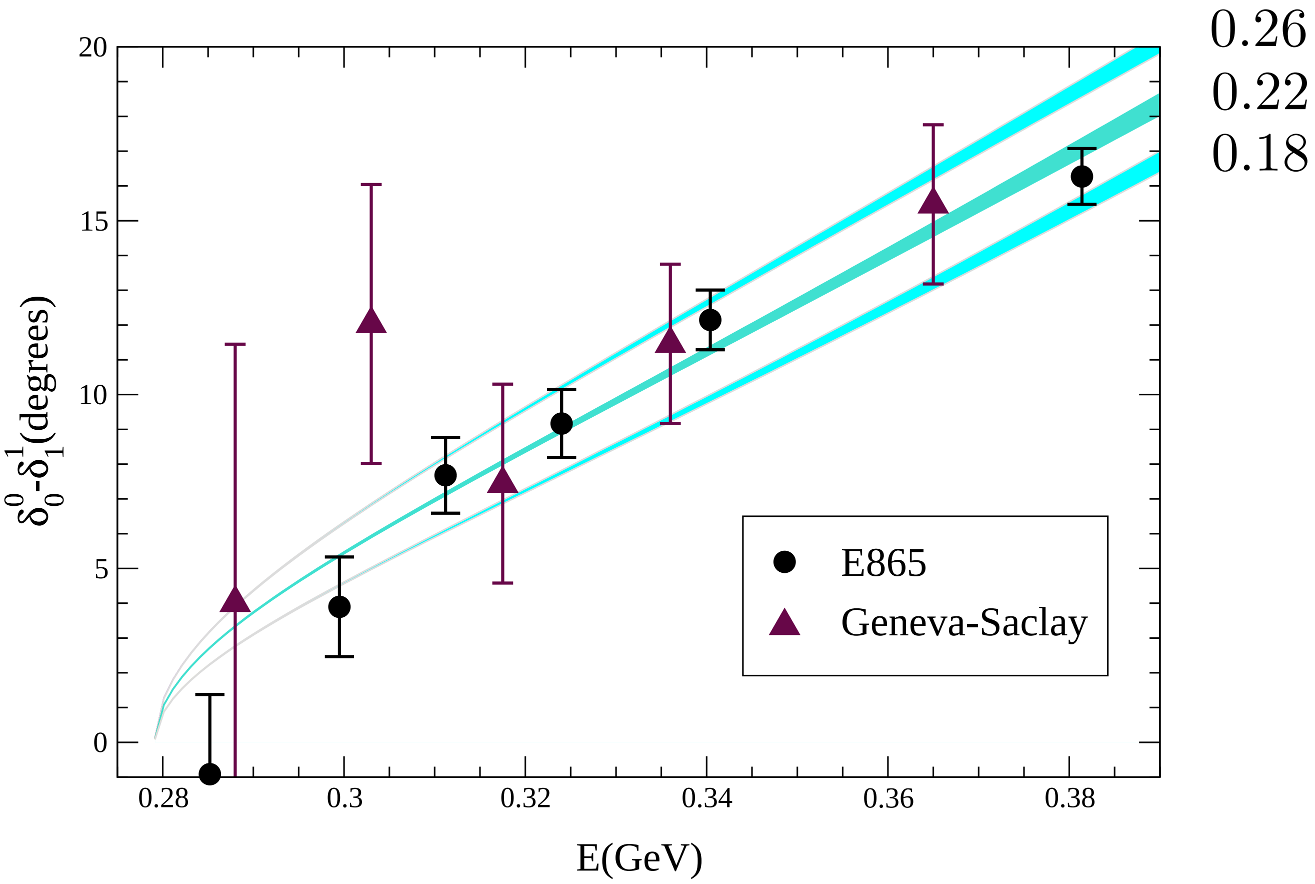
$$\langle \pi^+\pi^- | A_\mu | K^+ \rangle \quad = \quad \frac{1}{iM_K}[P_\mu F + Q_\mu G + L_\mu R]$$

with

$$F = f_s \exp i\delta_0^0 + f_p \exp i\delta_1^1 + D - \text{waves}$$

This is Watson's theorem, true in the isospin symmetry limit.
Allows one to measure the phase difference $\delta_0^0 - \delta_1^1$.

See talk by Brigitte Bloch-Devaux and by Gilberto Colangelo for details


0.26
0.22
0.18
20
15
10
5
0
0.28
0.3
0.32
0.34
0.36
0.38
E(GeV)
δ_0^0-δ_1^1(degrees)
E865
Geneva-Saclay

On radiative corrections

- Real/virtual photons:
  NA48: PHOTOS
  E865: Neveu, Scherk (1968): scalar QED, ϕ^4
- Virtual photons:
  NA48: Sommerfeld factor (Summing vertex corrections)
- M. Knecht, L. Mercolli: check on PHOTOS, work in progress

Remark: PHOTOS does not know about matrix element
$K \to \pi\pi e\nu\gamma \Rightarrow$ check is mandatory

After having taken these corrections into account:

Recent results

Preliminary results from NA48 analyzes

$a_0^0 = 0.256 \pm 0.008_{\rm stat} \pm 0.007_{\rm syst} \pm 0.018_{\rm th}$ Moriond 2007

$a_0^0 = 0.256 \pm 0.011$ Montpellier 2006

(stat.+syst. error added)

B. Bloch-Devaux

These results are in conflict with the prediction

$$a_0^0 = 0.220 \pm 0.005$$

Colangelo,Gasser,Leutwyler 2000

Generalized low energy expansion

Jan Stern, V Kaon mini workshop at CERN, Dec.12, 2006:

“ $\bar{l}_3$ becomes large and negative, signalling a strong effect of $\bar{s}s$ vacuum fluctuations. The structure of QCD vacuum is likely not the simplest one could dream of.

BUT WHY SHOULD IT BE?”

Occam’s razor

William Occam, 14th century:

“Entia non sunt multiplicanda praeter necessitatem”

What phase is measured by NA48?

NA48 measurements are performed in the real world, not in the idealized one where the predictions were made.

In the real world, form factors have a more complicated holomorphic structure than in QCD.

For illustration, I consider the scalar form factor of charged pions:

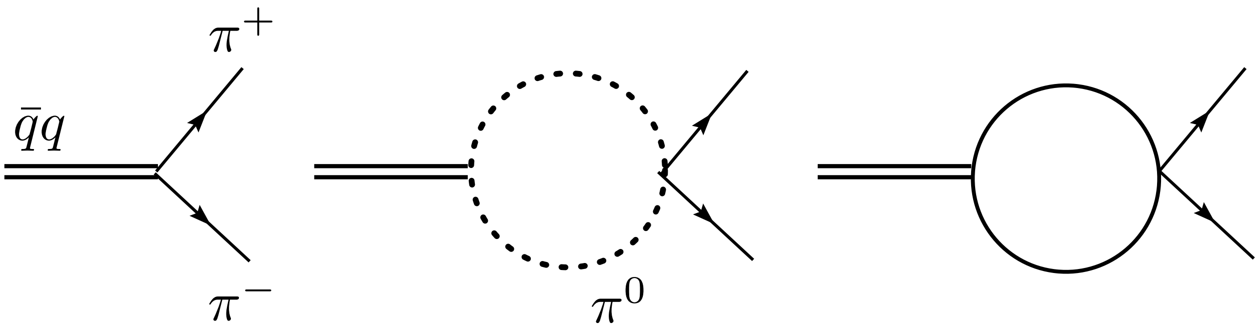
$$\langle \pi^+(p)\pi^-(p')|\bar{u}u + \bar{d}d|0\rangle = \Gamma(s)\ ;\ \ s = (p+p')^2$$

$$F(s) = \Gamma(s)/\Gamma(0)$$

Scalar form factor in QCD

$$F(s) = |F(s)| \exp i\delta_0^0(s) \; ; \; 4M_\pi^2 < s < 16M_\pi^2$$

An analogous relation holds for one of the form factors measured in K_{e4} decays.



ChPT , 1 loop

$$\delta_0^0(s) = \frac{(2s - M_\pi^2)}{32\pi F_\pi^2}\sigma_c \; , \; \sigma_c = \left(1 - \frac{4M_\pi^2}{s}\right)^{1/2}$$

Scalar form factor in the real world

Include electromagnetic effects
$\Rightarrow$ Pion masses split, loop functions behave differently
$\Rightarrow$ holomorphic properties change

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + Z\langle QUQU^\dagger\rangle + \text{counterterms}$$

$$\bar{F}(s) = |\bar{F}(s)| \exp i\delta(s)$$

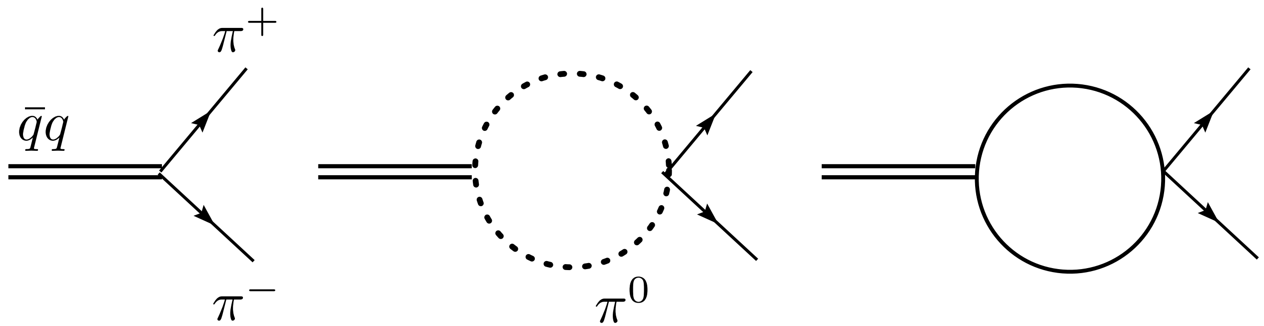
$$\delta(s) = \frac{1}{32\pi F_\pi^2}([4(M_\pi^2 - M_{\pi^0}^2) + s]\sigma_c + [s - M_{\pi^0}^2]\sigma_0)$$

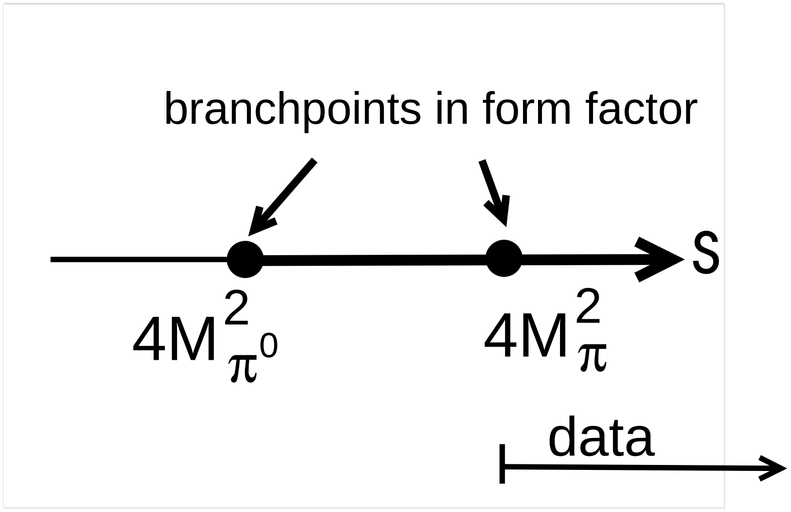
$$\sigma_0 = \left(1 - 4M_{\pi^0}^2/s\right)^{1/2}$$

δ is the phase measured in K_{e4} decays (modulo universality). It differs from δ_0^0 by about 10 mrad.

J. G., A.Rusetzky, Internal note to NA48, March 2007

See also: Gevorkyan, Sissakian, Tarasov, Torosyan, Voskresenskaya, arXiv:0704.2675 (April 2007).

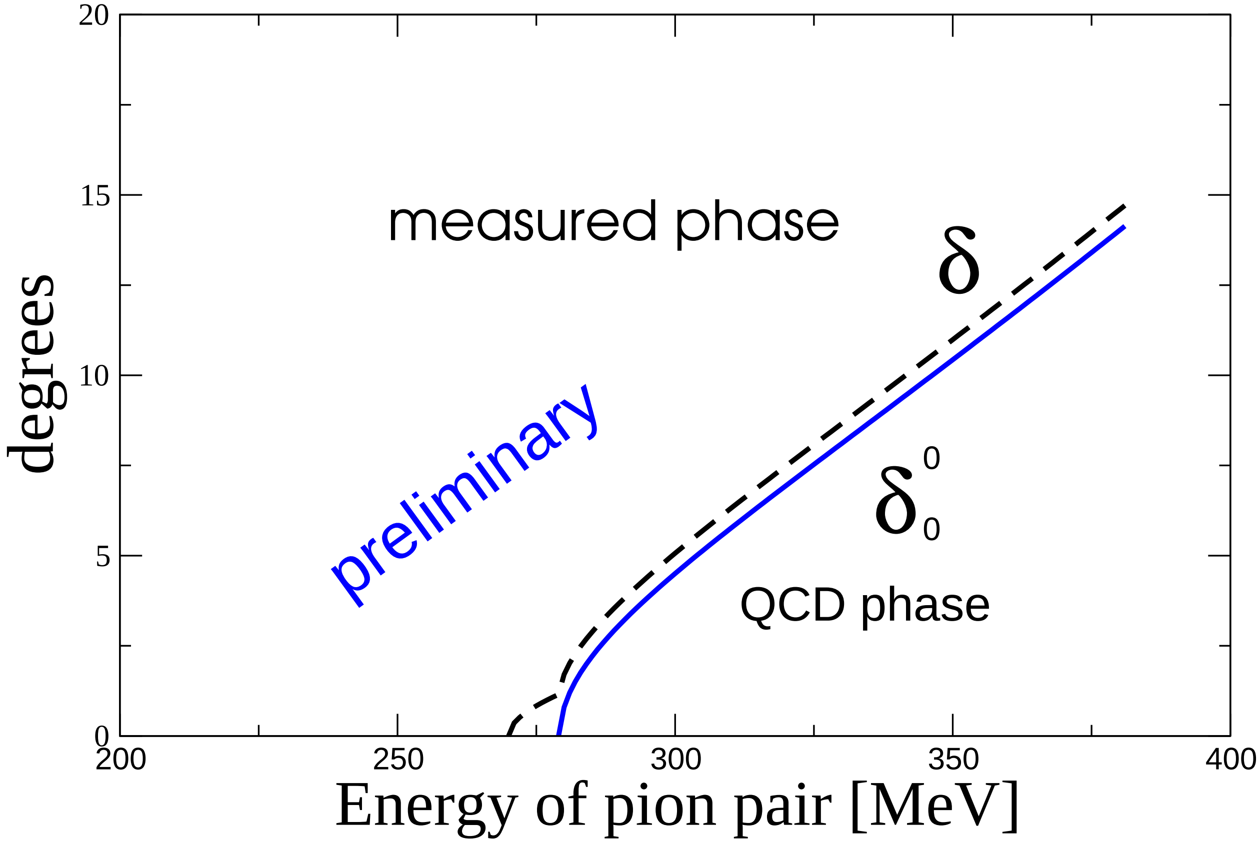




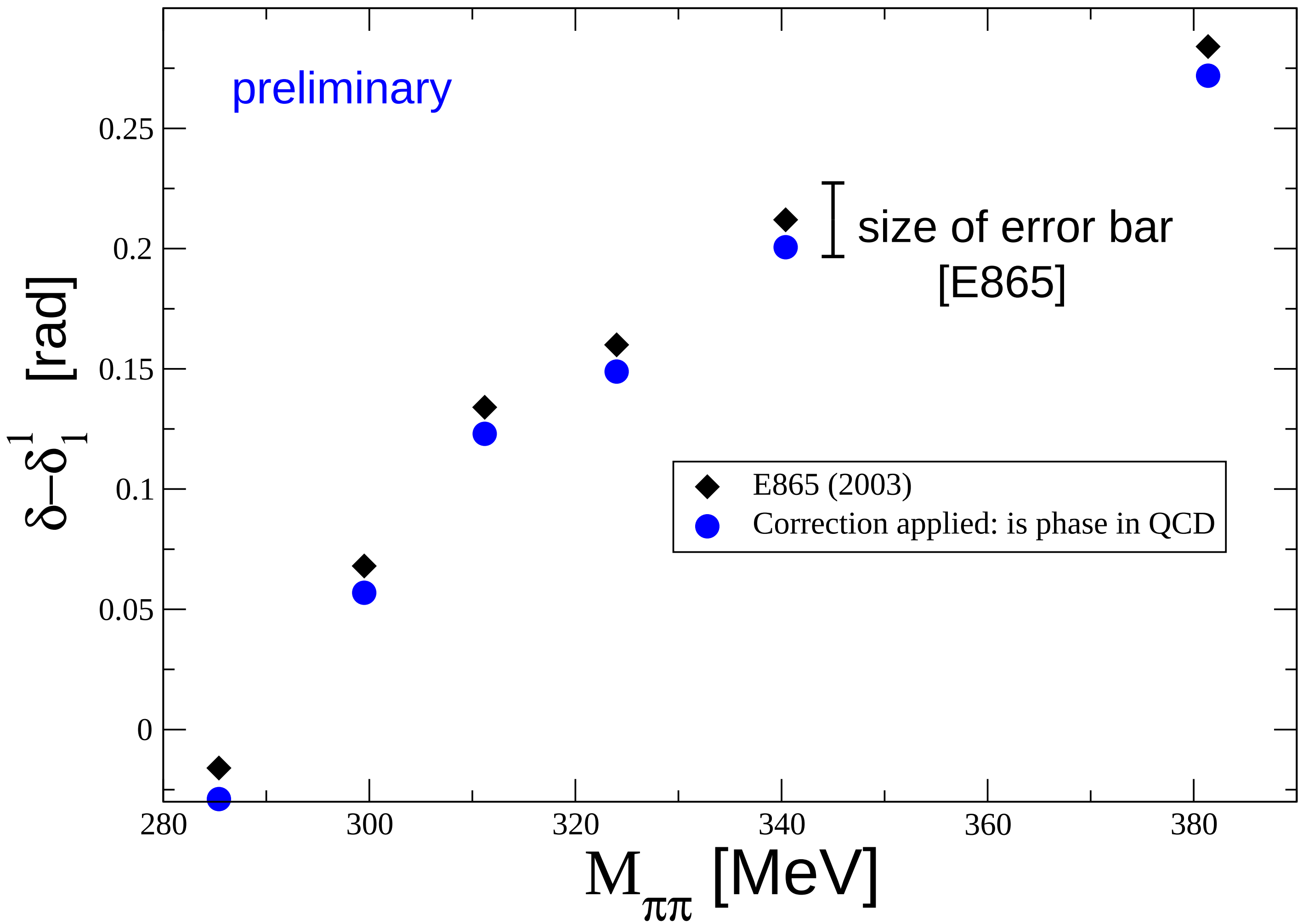
$\Rightarrow$ Singularity at $s = 4M_\pi^2$ generates cusp in phase

Compare cusp in $K \to 3\pi$; K_{l3}^0 form factor

Phase: Measured vs. QCD



Occam's razor at work



- Two-loop calculation: G. Colangelo, J.G., A. Rusetsky

E865 data

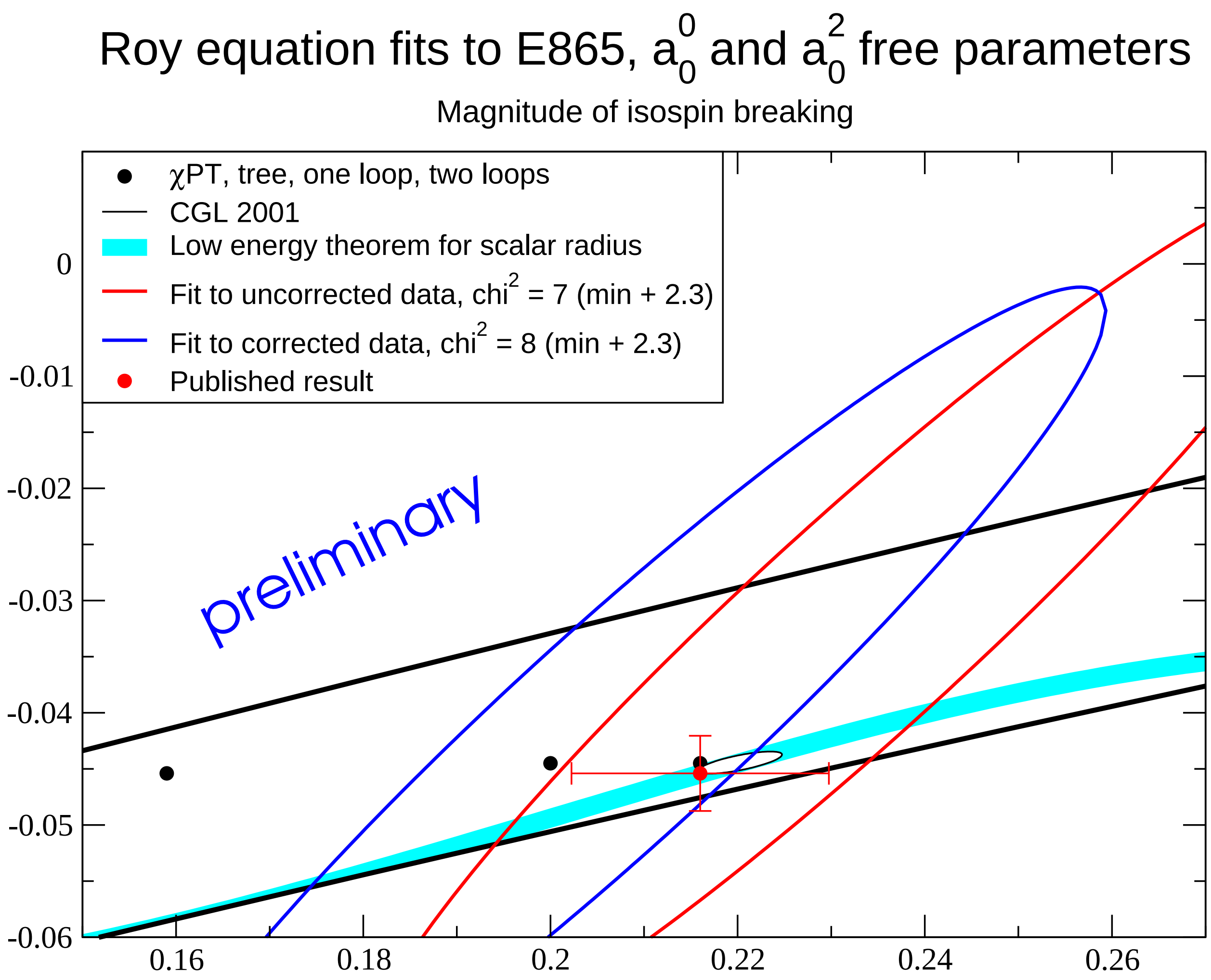


Figure: Courtesy of H. Leutwyler

Summary of progress made

$$\boxed{K \to 3\pi}$$

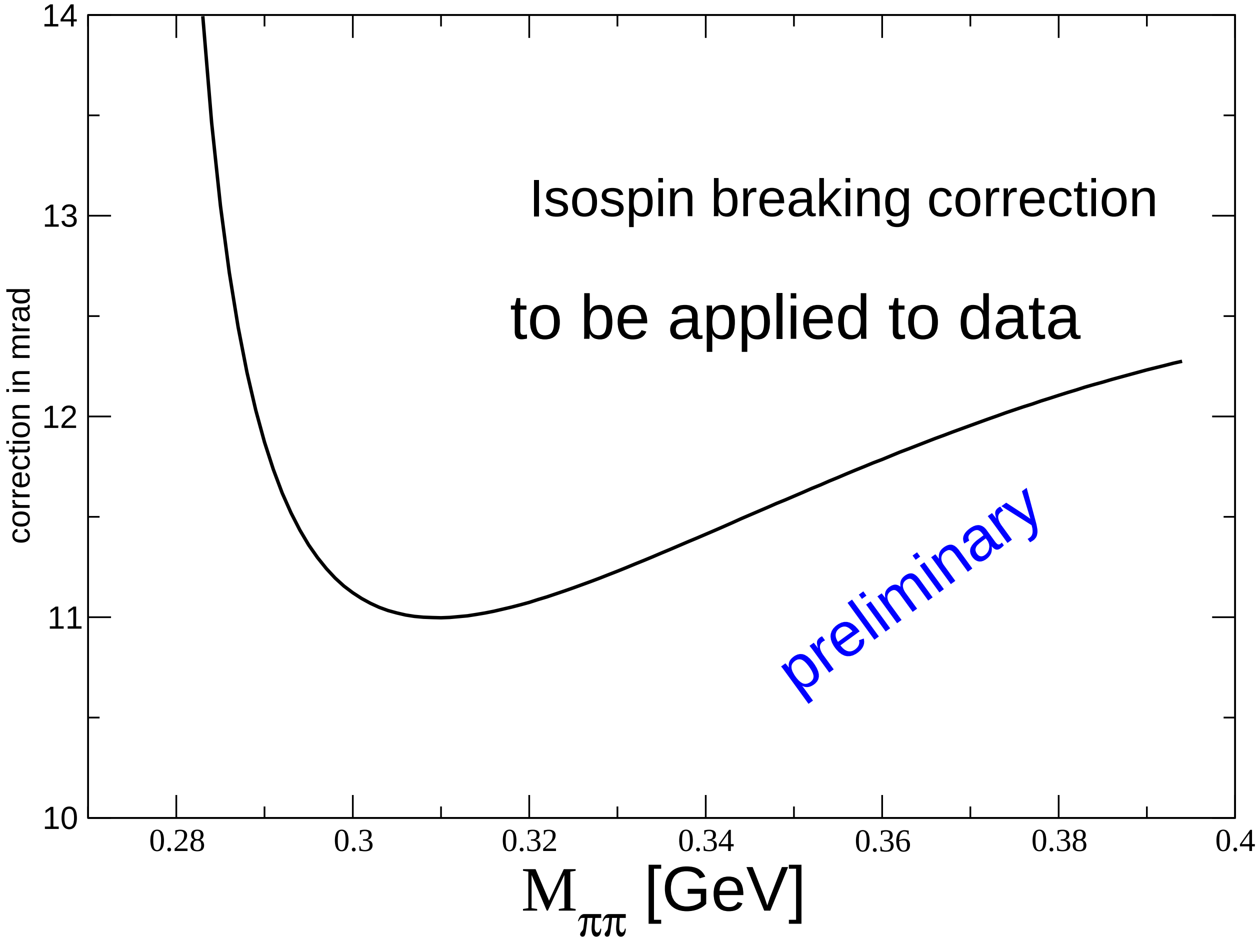
- Amplitudes calculated in effective framework, all channels. Fortran programs available.
- Virtual photons: $O(a\alpha, \alpha)$ suffices away from threshold (away needs to be specified in collaboration with data analysis).
- Real photons are under investigation

$$\boxed{K_{e4}}$$

- Substantial isospin breaking effect identified : cusp in phase. Decreases a_0^0. Was overlooked by theorists until very recently.
- Theoretical uncertainty in this correction needs to be worked out properly, making also use of recent work by
  Haefeli, Ivanov and Schmid: matching in electromagnetic sector (work in progress)

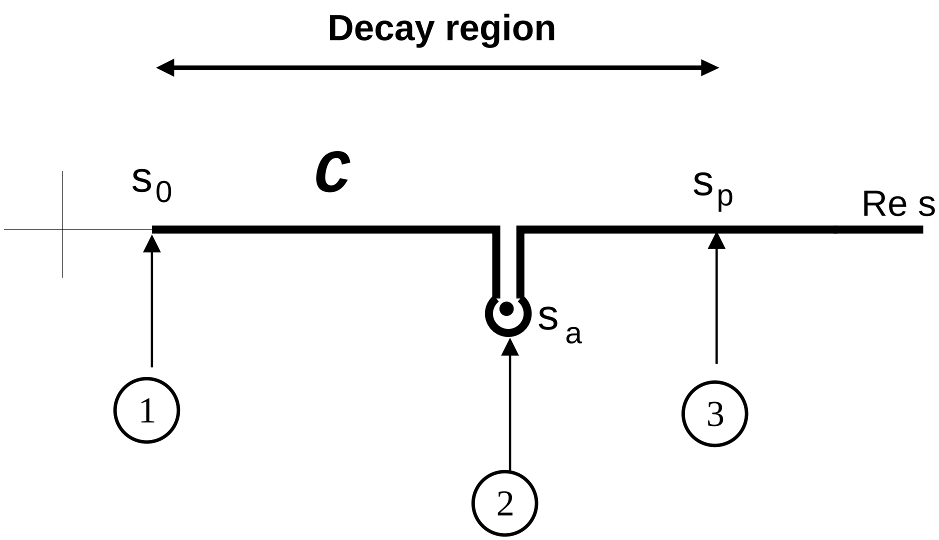
SPARES

Isospin corrections



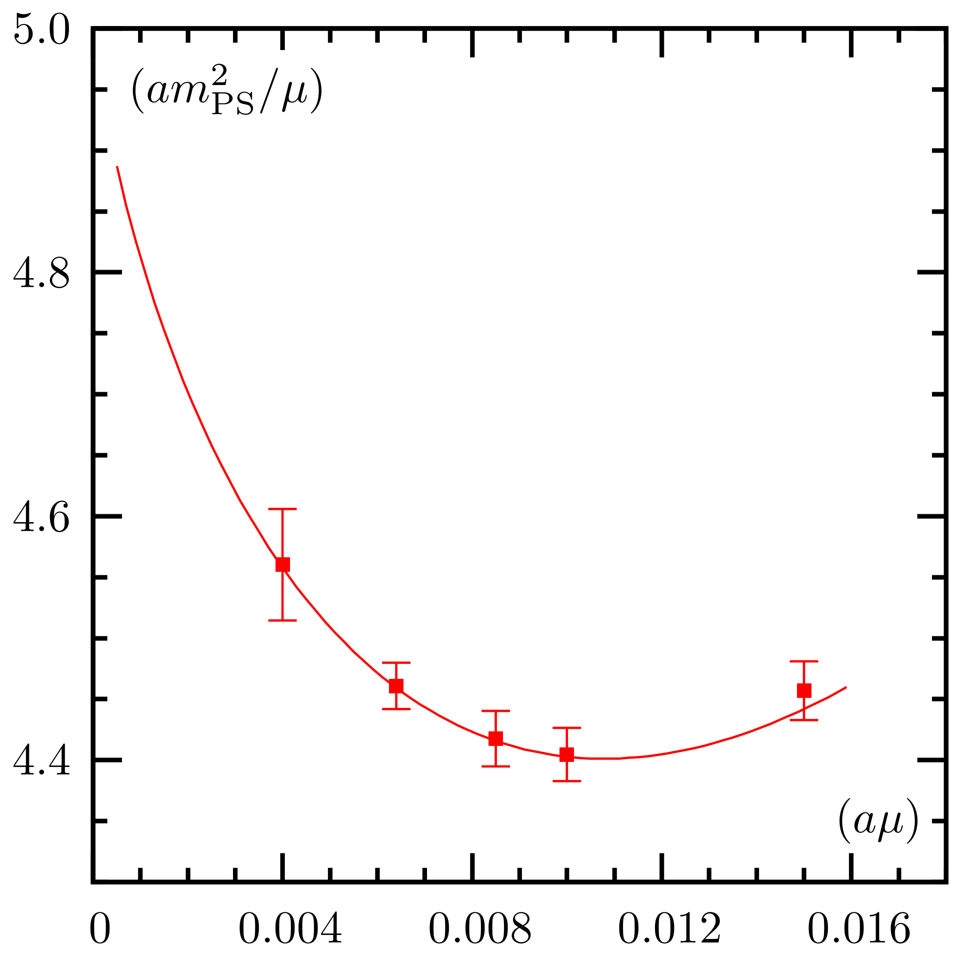
Analytic properties

$$G(s) = \frac{s}{2\pi i} {}^{\mathcal{C}}\!\!\int_{s_0}^{\infty} \frac{dx}{x} \frac{1}{x-s} A(x; M_i)$$



- anomalous threshold at s_a
- G is infinite at s_p [approaching from below]

Lattice: Twisted mass collaboration hep-lat/0701012



$$\bar{l}_3 = 3.65 \pm 0.12$$

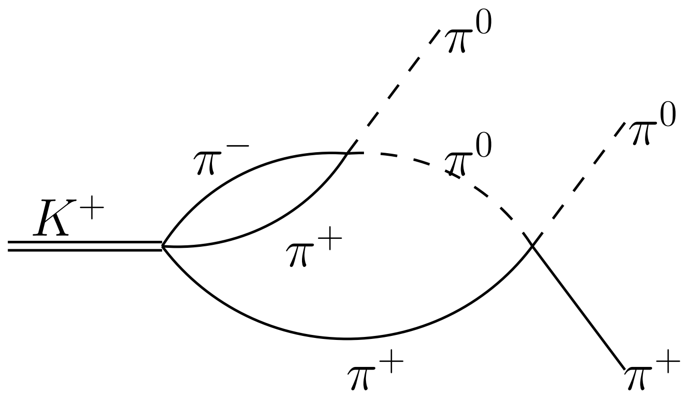
Figure from hep-lat/0701012

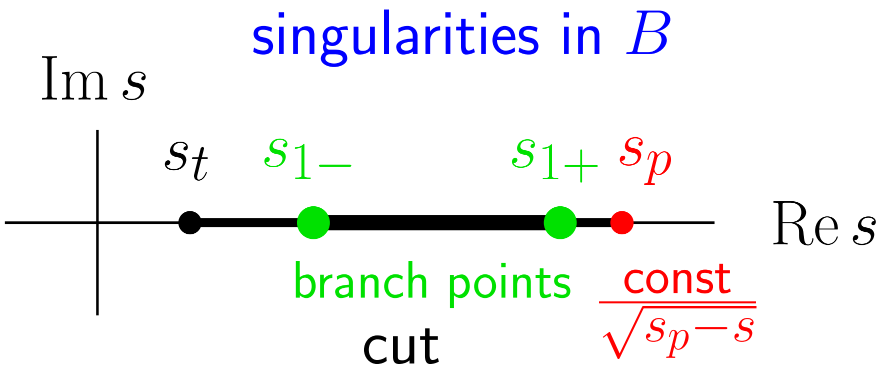
Analytic properties of amplitude

- 2-loop function can be expressed in terms of logarithms; close to threshold, equal mass in inner loops:

$$F(s) = \frac{v_{+-}(s)}{256\pi^2}\sqrt{\frac{M_K^2 - 9M_\pi^2}{M_K^2 - M_\pi^2}} + \cdots = \mathcal{O}(\epsilon^2)$$

- away from threshold, singularity structure is complicated; decomposition $\mathcal{M} = A + vB$ (A, B analytic) does not hold:





- checked against singularity structure of relativistic loop diagram