

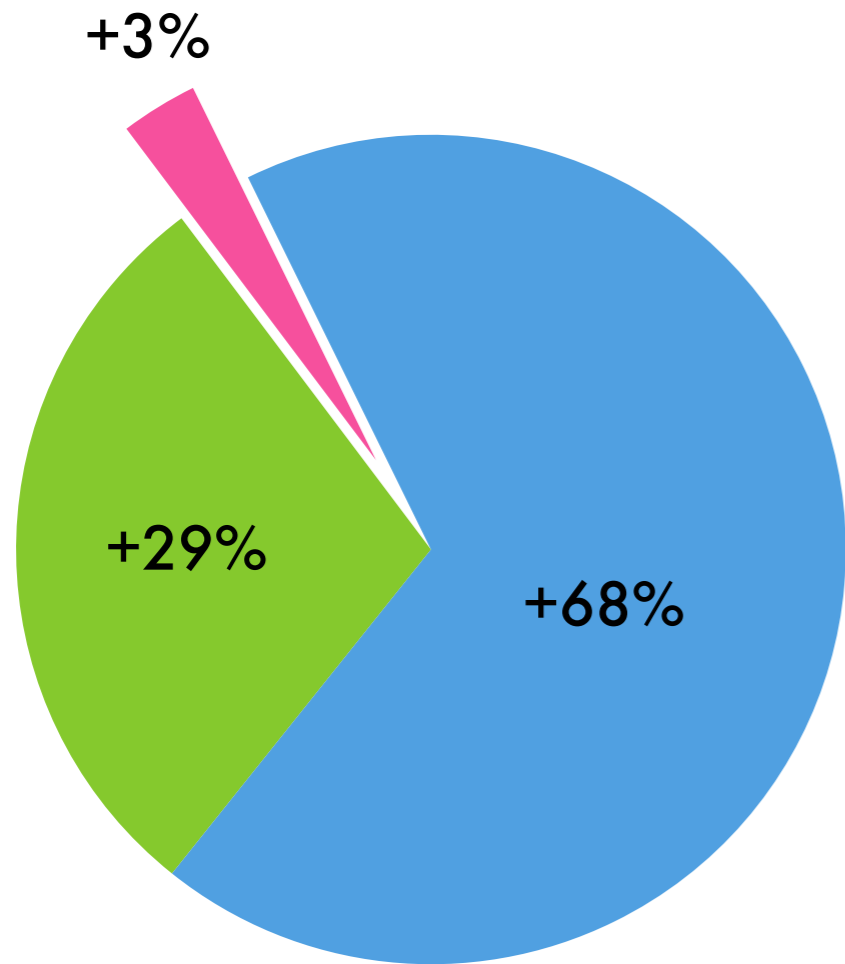
Rare K - (vs.) B -decays

Ulrich Haisch
University of Zürich

Kaon International Conference 2007,
May 21–25, 2007, Frascati, Italy

Warm-up: basic facts about $s \rightarrow d\nu\bar{\nu}$

$$\mathcal{A}_{\text{SM}}(s \rightarrow d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd} X_{\text{SM}}^q \propto \frac{m_t^2}{M_W^2} (\lambda^5 + i\lambda^5) + \frac{m_c^2}{M_W^2} \ln \frac{m_c}{M_W} \lambda + \frac{\Lambda^2}{M_W^2} \lambda$$



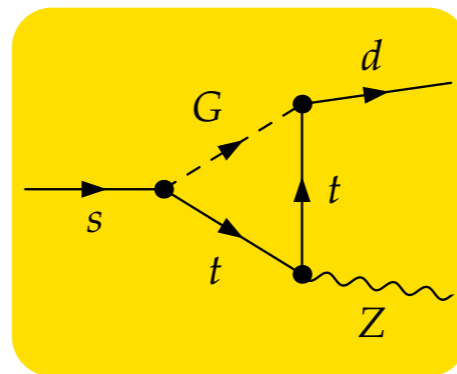
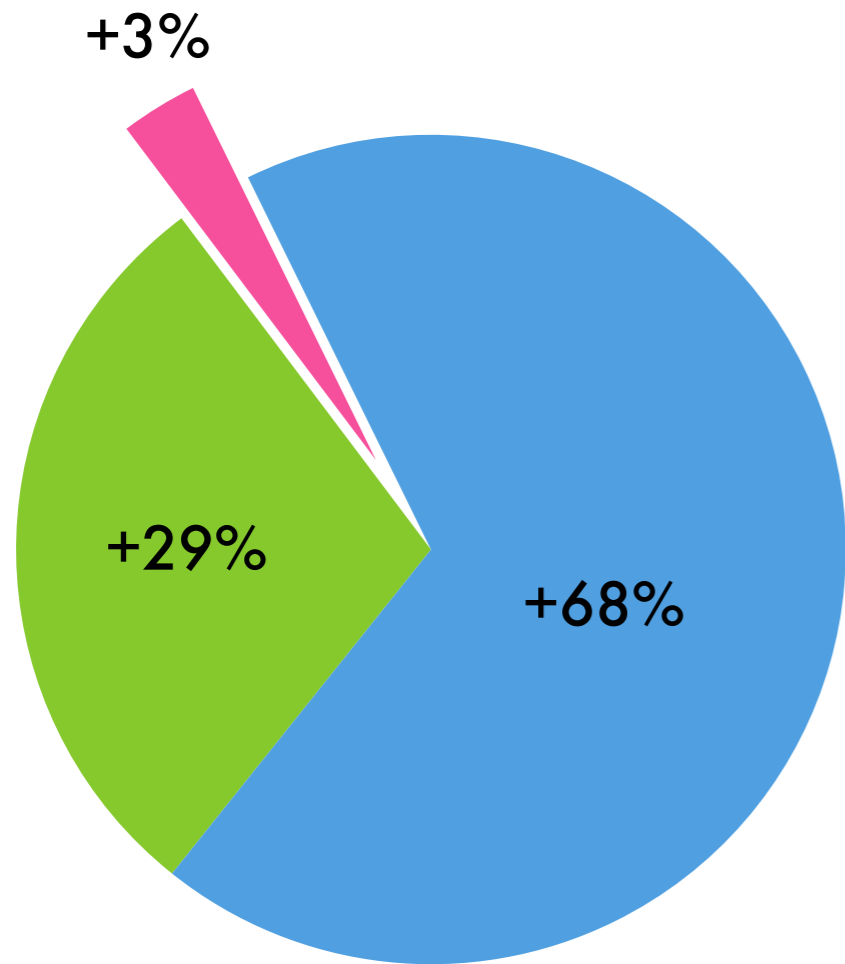
● top ● charm ● up

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

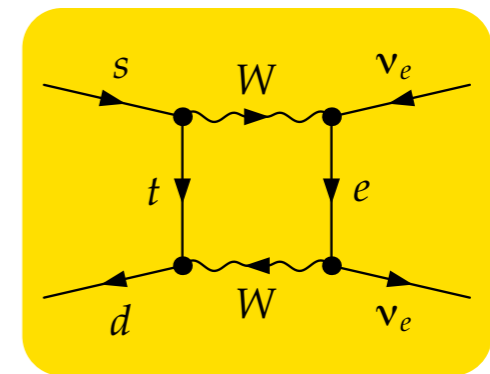
- V-A current is conserved:
large logarithms appear
only in charm sector

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$$\propto \frac{m_t^2}{M_W^2}$$



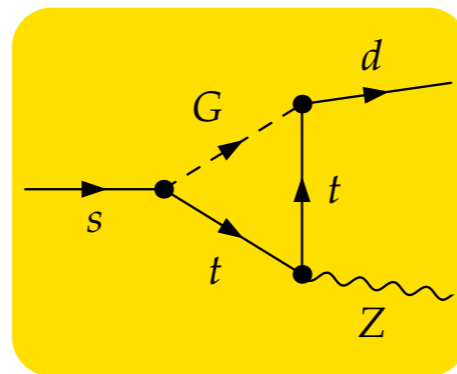
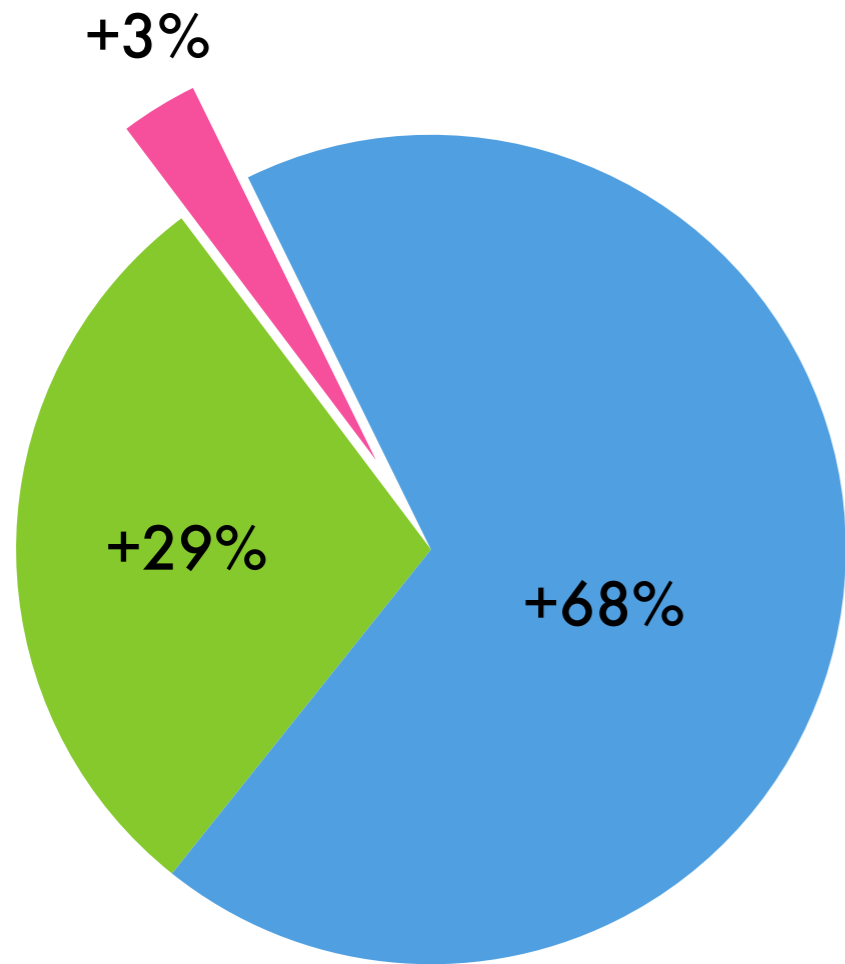
$$\propto \text{const.}$$

- $SU(2)_L$ breaking in Z-penguin amplitude leads to power-like GIM mechanism

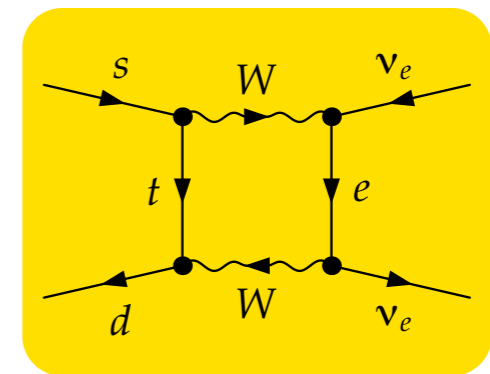
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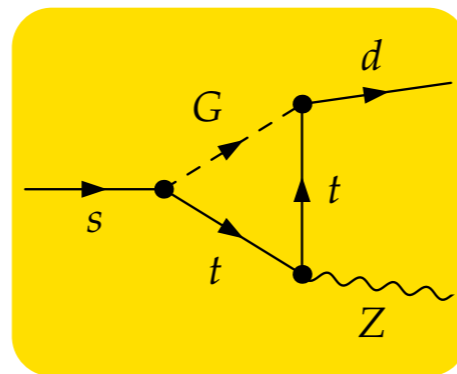
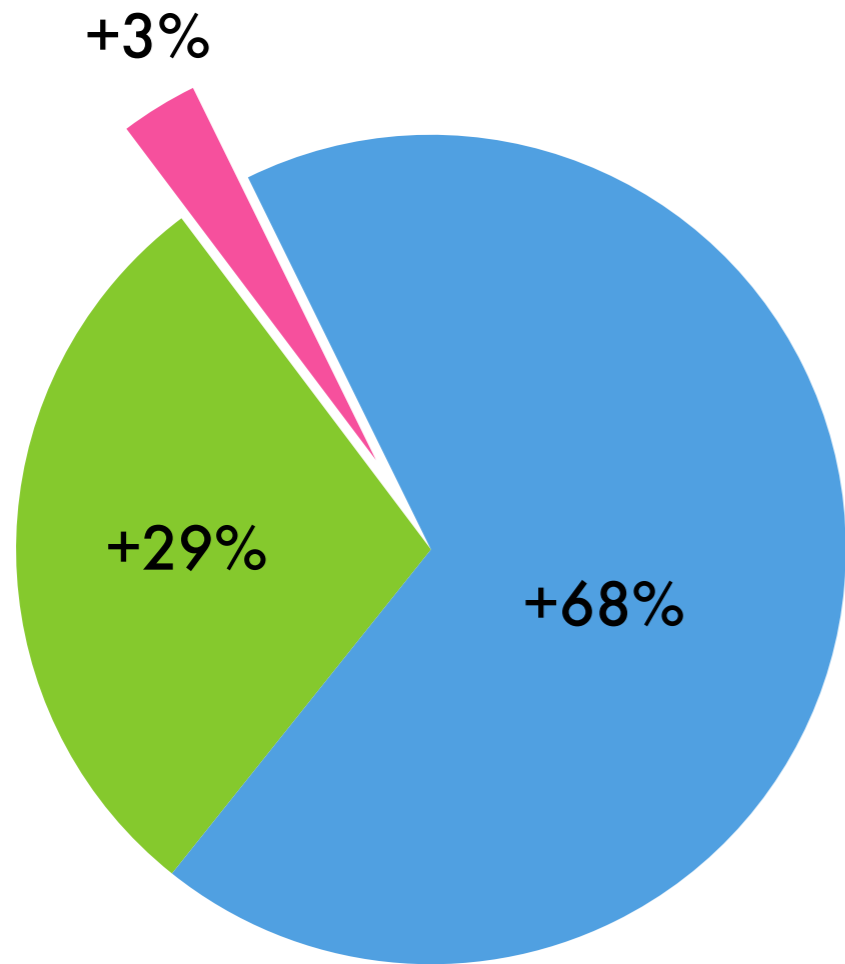
$$\propto \text{const.}$$

● top ● charm ● up

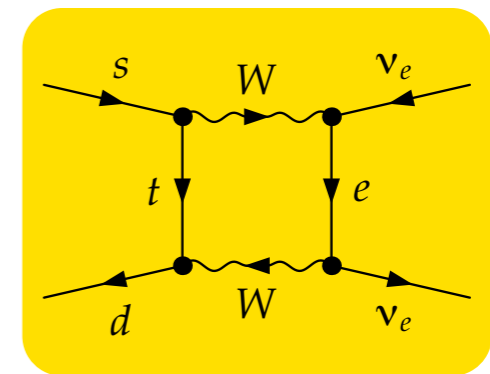
- large CP violating phase in dominant short-distance contribution due to top

Warm-up: basic facts about $s \rightarrow d\nu\bar{\nu}$

$$\mathcal{A}_{\text{SM}}(s \rightarrow d\nu\bar{\nu}) = \sum_{q=u,c,t} V_{qs}^* V_{qd} X_{\text{SM}}^q \propto \frac{m_t^2}{M_W^2} (\lambda^5 + i\lambda^5) + \frac{m_c^2}{M_W^2} \ln \frac{m_c}{M_W} \lambda + \frac{\Lambda^2}{M_W^2} \lambda$$



$$\propto \frac{m_t^2}{M_W^2}$$



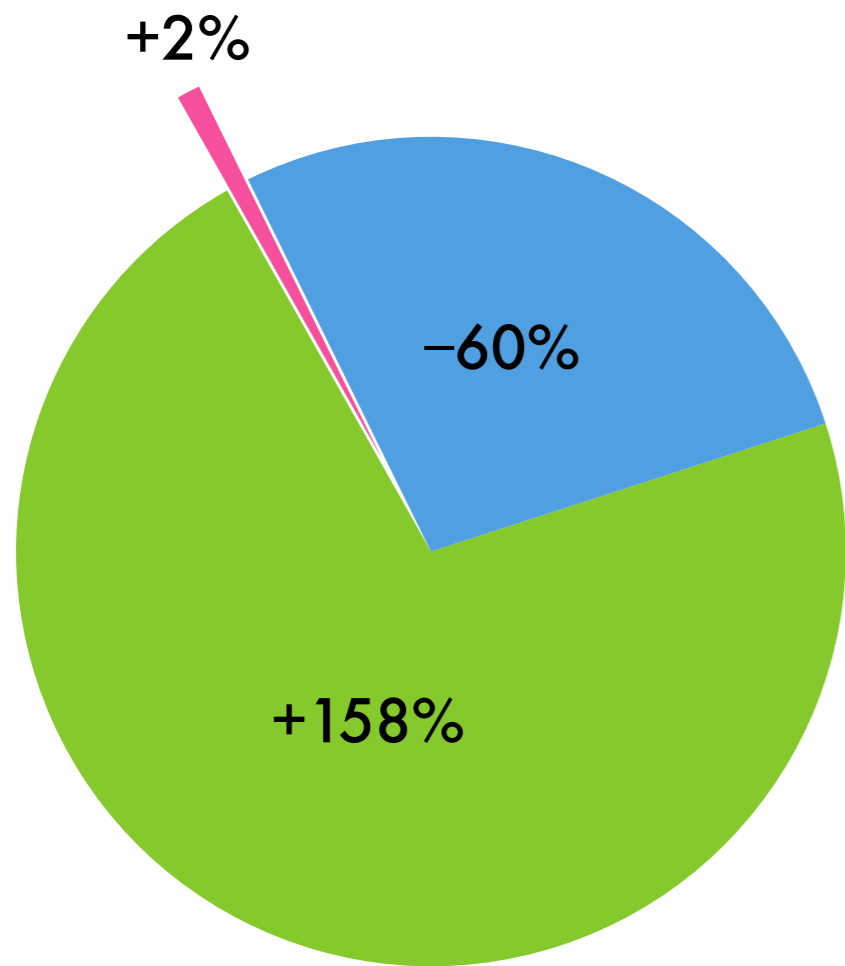
$$\propto \text{const.}$$

● top ● charm ● up

- thus: $s \rightarrow d\nu\bar{\nu}$ exceptional tool to discover non-MFV physics where hard GIM is not active

Warm-up: basic facts about $b \rightarrow s\gamma$

$$\mathcal{A}_{\text{SM}}(b \rightarrow s\gamma) = \sum_{q=u,c,t} V_{qb}^* V_{qs} K_{\text{SM}}^q \propto \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^4$$



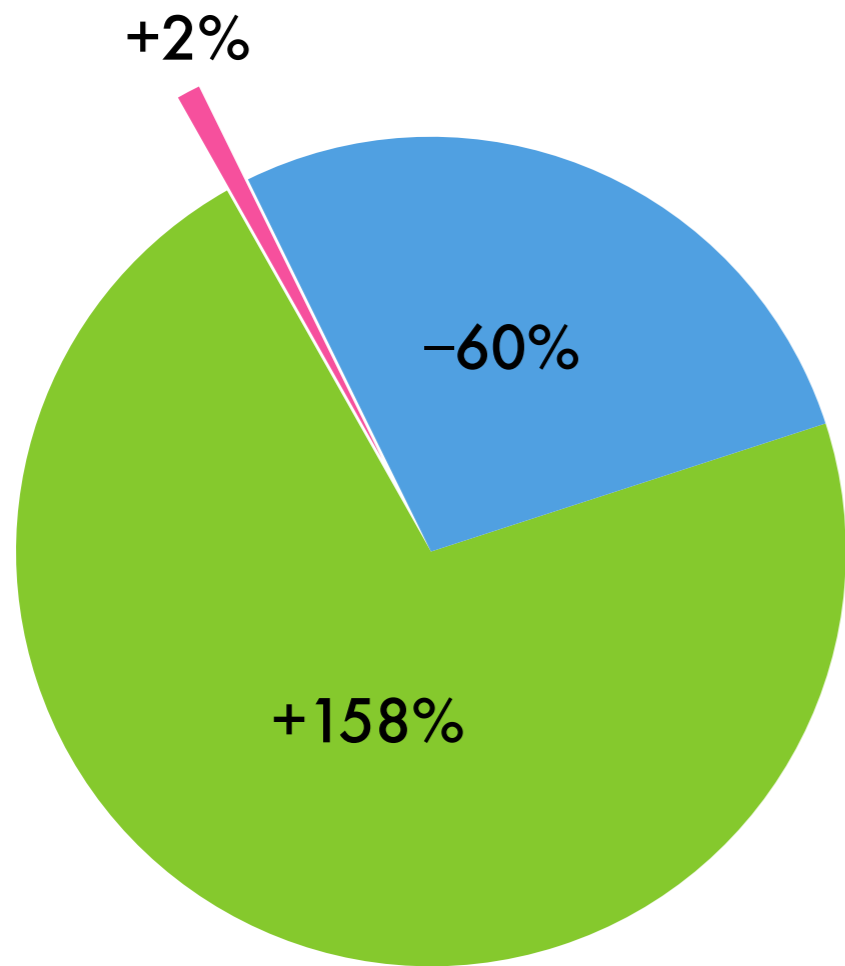
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$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}$$

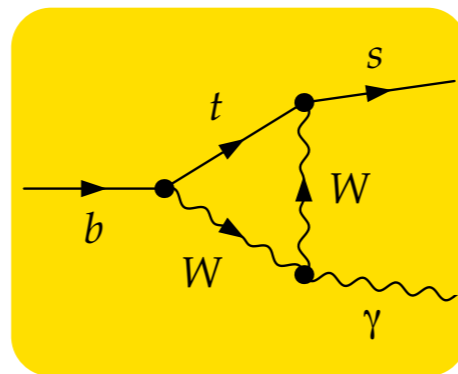
- tensor current not conserved:
 $b \rightarrow s\gamma$ depends logarithmically
on scale where Q_7 is generated

Warm-up: basic facts about $b \rightarrow s\gamma$

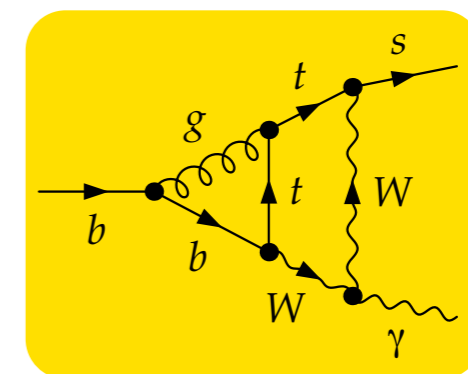
$$\mathcal{A}_{\text{SM}}(b \rightarrow s\gamma) = \sum_{q=u,c,t} V_{qb}^* V_{qs} K_{\text{SM}}^q \propto \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^4$$



● top ● charm ● up



$\propto \text{const.}$

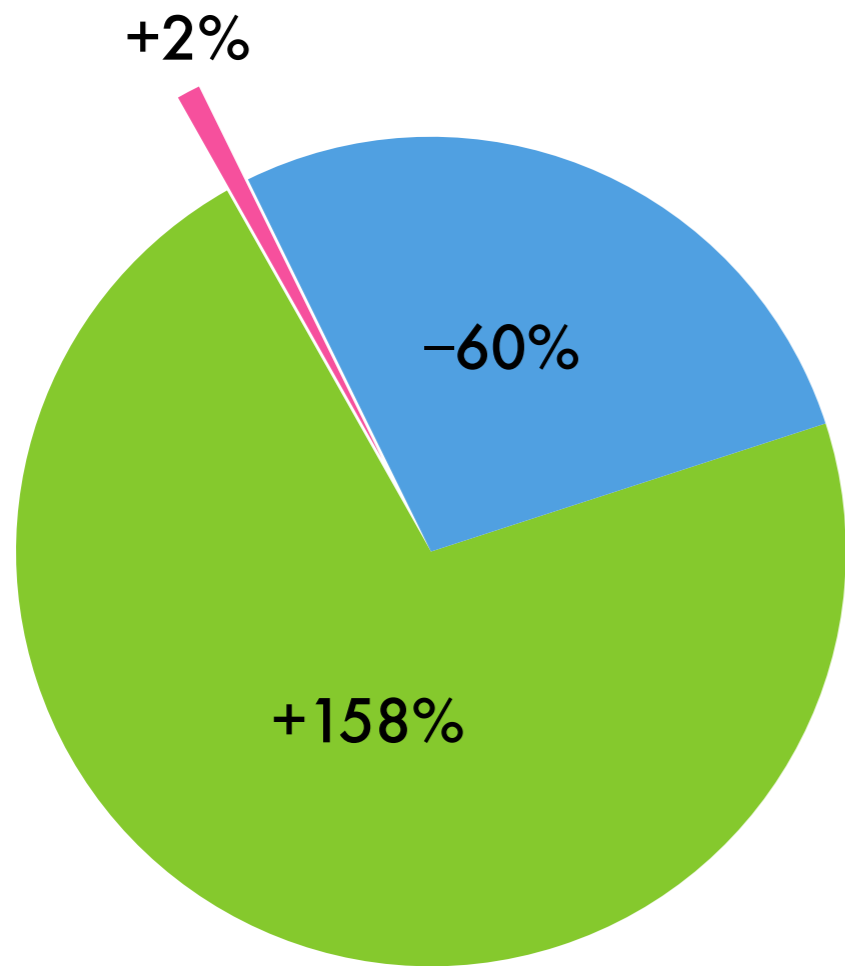


$\propto \alpha_s \ln \frac{m_b}{M_W}$

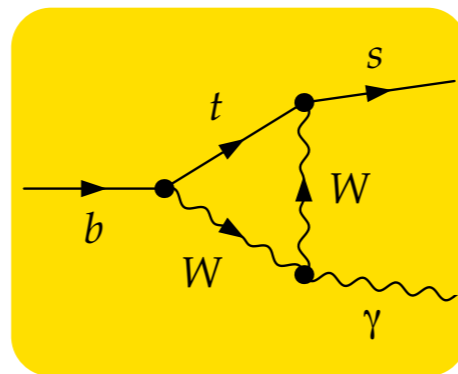
- gluonic corrections lead to mild logarithmic GIM suppression of $b \rightarrow s\gamma$ amplitude

Warm-up: basic facts about $b \rightarrow s \gamma$

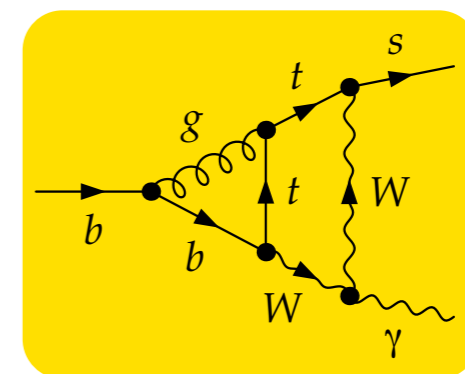
$$\mathcal{A}_{\text{SM}}(b \rightarrow s \gamma) = \sum_{q=u,c,t} V_{qb}^* V_{qs} K_{\text{SM}}^q \propto \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^4$$



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$\propto \text{const.}$

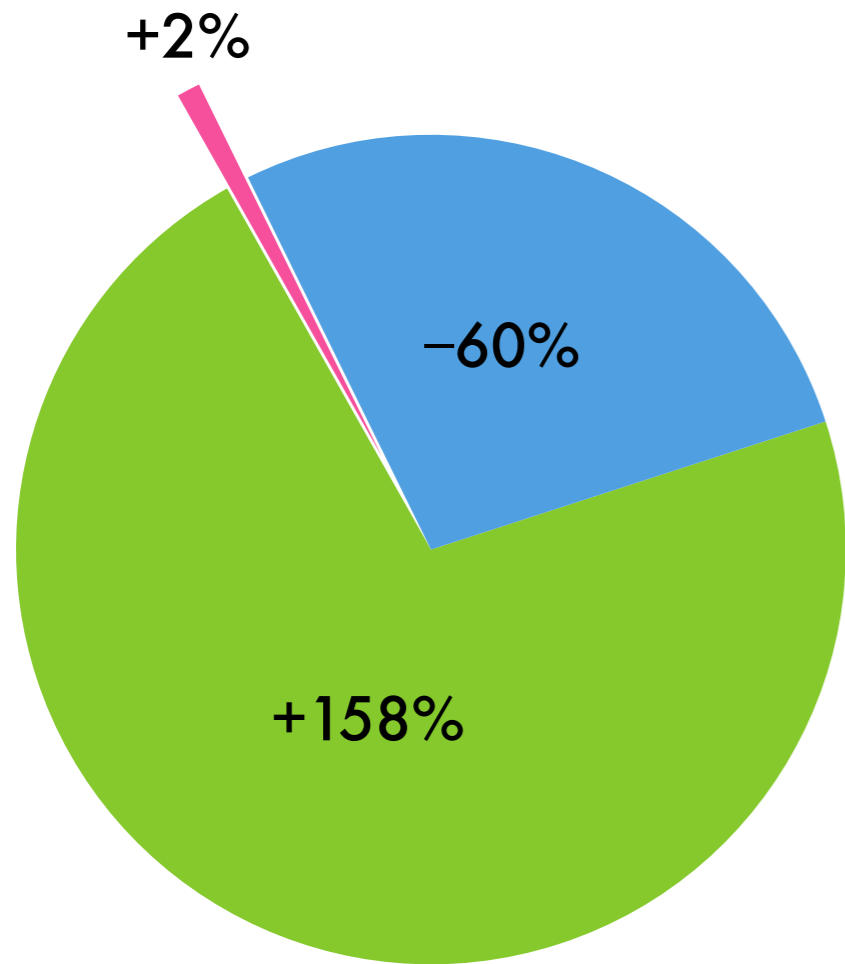


$\propto \alpha_s \ln \frac{m_b}{M_W}$

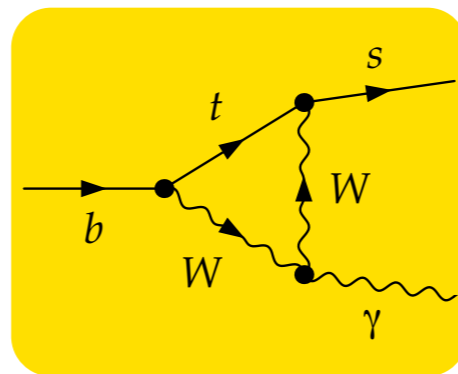
- sensitivity on high scale physics
"swamped" by renormalization group effects

Warm-up: basic facts about $b \rightarrow s\gamma$

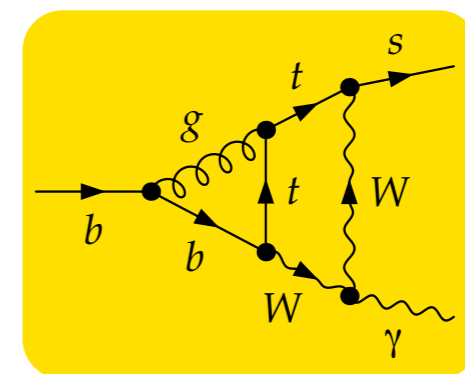
$$\mathcal{A}_{\text{SM}}(b \rightarrow s\gamma) = \sum_{q=u,c,t} V_{qb}^* V_{qs} K_{\text{SM}}^q \propto \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^2 + \ln \frac{m_b}{M_W} \lambda^4$$



● top ● charm ● up



$\propto \text{const.}$

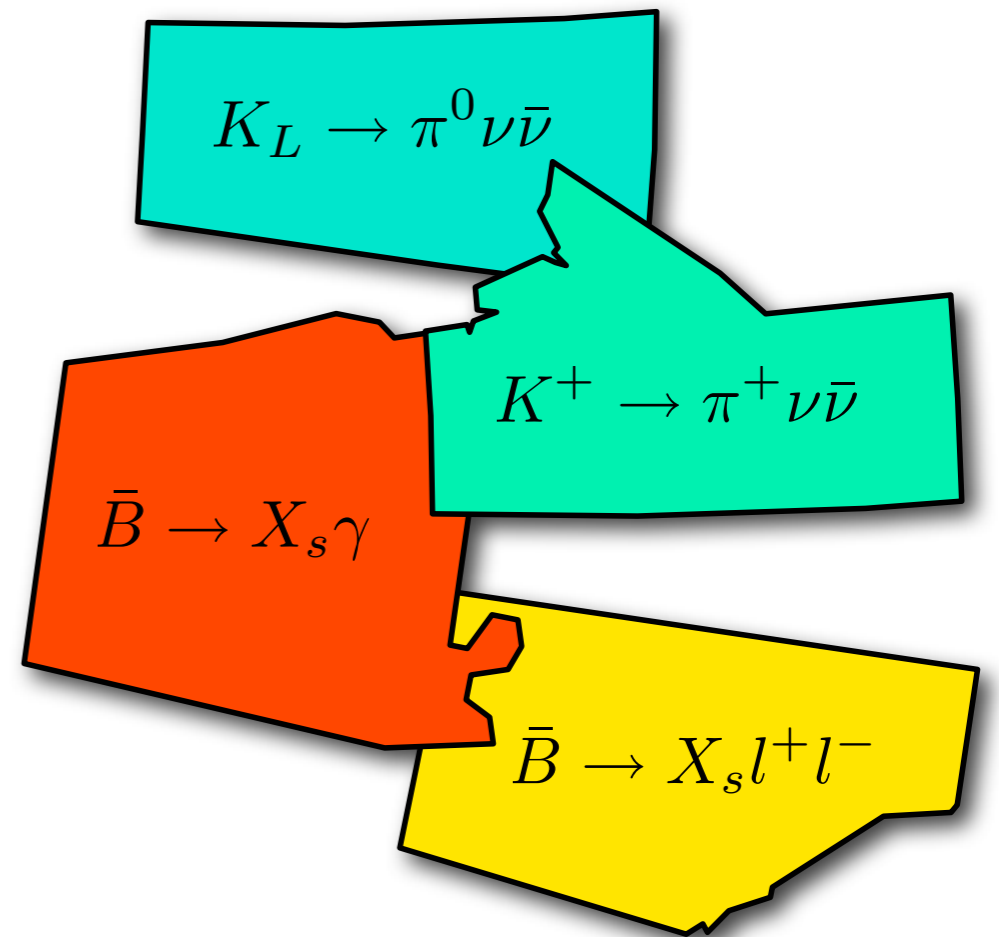


$\propto \alpha_s \ln \frac{m_b}{M_W}$

- thus: $b \rightarrow s\gamma$ wonderful QCD laboratory, providing stringent constraints on new physics

Next 23⁽⁺⁾ minutes ...

- recent progress in SM
- next theoretical obstacles
- joining forces: K - & B -decays
- no conclusions



$K \rightarrow \pi \nu \bar{\nu}$ matrix elements from K_{13} in ChPT

$$\kappa_L = \frac{G_F^2 m_K^5 \alpha^2 (M_Z)}{256 \pi^5 s_W^4} \lambda^8 (\tau_L |\lambda \times f_+^{K^0 \pi^+}(0)|^2)_{\text{exp}} \left(\begin{array}{c} r r_K \\ r_{0+} \end{array} \right)^2 \mathcal{I}_\nu^0,$$

$$\kappa_+ = \frac{G_F^2 m_K^5 \alpha^2 (M_Z)}{256 \pi^5 s_W^4} \lambda^8 \tau_+ (r_K |\lambda \times f_+^{K^0 \pi^+}(0)|^2)_{\text{exp}} \mathcal{I}_\nu^+$$

$$r_K = \frac{f_+^{K^+ \pi^+}(0)}{f_+^{K^0 \pi^+}(0)} = 1.0015 \pm 0.0007^\dagger$$

$$r_{0+} = \frac{f_+^{K^+ \pi^{0(+)}(q^2)}(q^2)}{f_+^{K^0 \pi^{+(0)}(q^2)}(q^2)} = 1.0238 \pm 0.0035^\dagger$$

$$r = \frac{f_+^{K^+ \pi^0}(q^2) f_+^{K^0 \pi^0}(q^2)}{f_+^{K^0 \pi^+}(q^2) f_+^{K^+ \pi^+}(q^2)} = 1.0000 \pm 0.0002^\dagger$$

$$\epsilon^{(2)} \propto \frac{m_u - m_d}{m_s}$$

- classic ChPT analysis of $\mathcal{O}(p^2 \epsilon^{(2)})^*$ isospin-breaking effects
very recently extended to $\mathcal{O}(p^4 \epsilon^{(2)})$ & partially $\mathcal{O}(p^6 \epsilon^{(2)})^\dagger$

* Marciano & Parsa '96

† Mescia & Smith '07

K_L & K_+ : main messages*

- overall uncertainties on $K_L \rightarrow \pi^0 \nu \bar{\nu}$ & $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ matrix elements are reduced by factor 4 & 7
- further reduction of errors possible with better data on K_{l3} slopes & K_{l3}^+ branching ratios

	$(r_{0+})_{\text{theo}}$	$(r_{0+})_{\text{exp}}^{\text{KLOE}}$	$(r_{0+})_{\text{exp}}$	τ_+	$f(0)_{K_{l3}}$	\mathcal{I}	r_K	r	future (?)
K_L	2.229 ± 0.017	2.229 ± 0.036	2.190 ± 0.018	–	77%	12%	9%	2%	± 0.013
K_+	5.168 ± 0.025	5.168 ± 0.025	5.168 ± 0.025	19%	43%	21%	17%	–	± 0.023

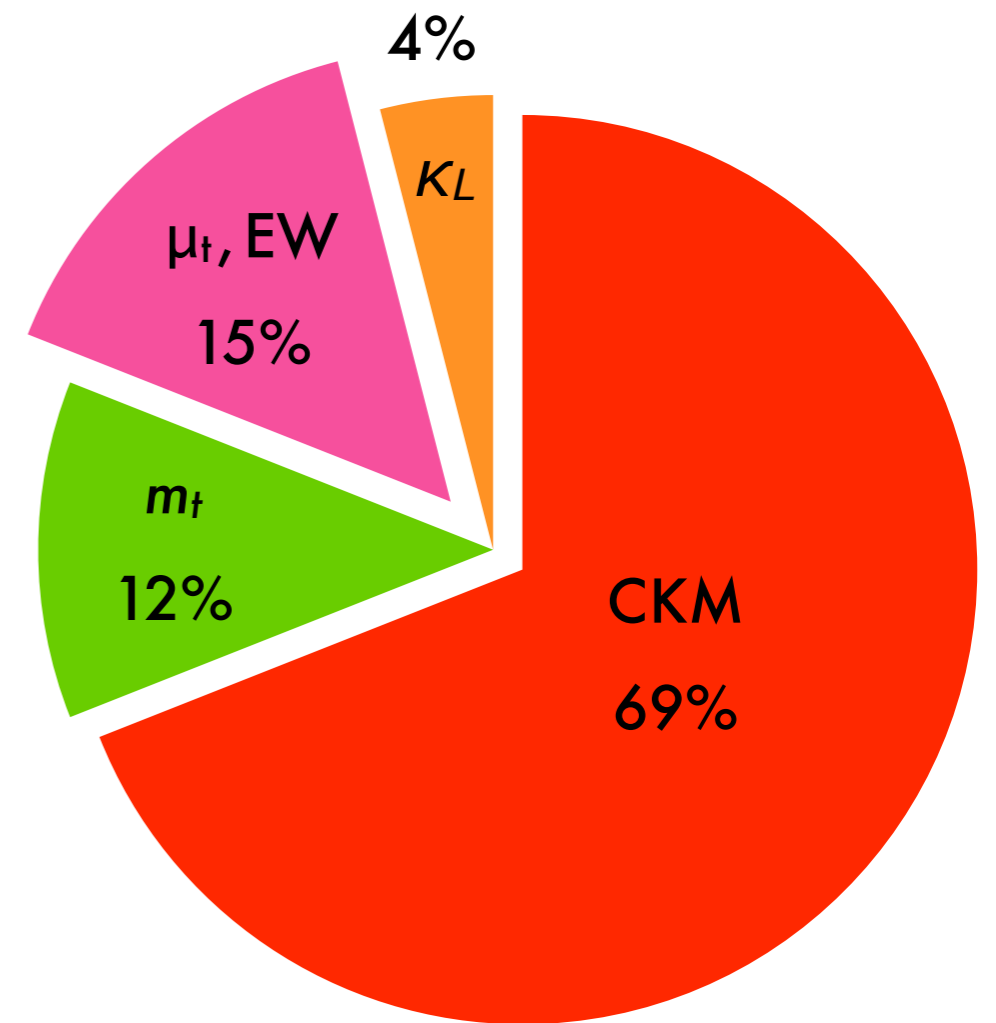
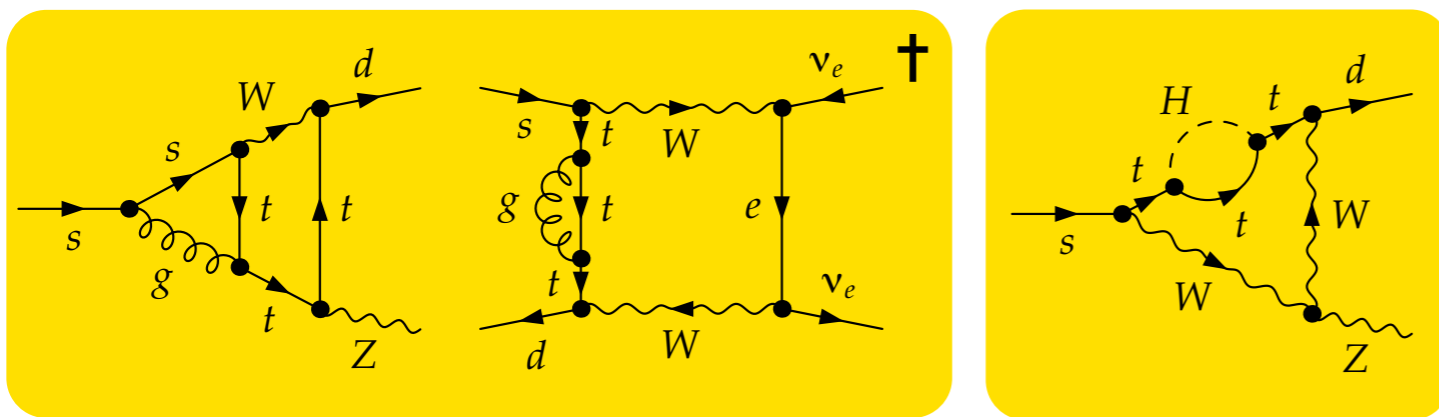
* Mescia & Smith '07

SM prediction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left[\frac{\text{Im}(V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.54 \pm 0.35) \times 10^{-11}$$

$$\kappa_L = (2.229 \pm 0.017) \times 10^{-10} \left(\frac{\lambda}{0.225} \right)^8 *$$

$$X = 1.456 \pm 0.017_{m_t} \pm 0.013_{\mu_t} \pm 0.015_{EW}^\ddagger$$



* Mescia & Smith '07

† Misiak & Urban '99, Buchalla & Buras '99

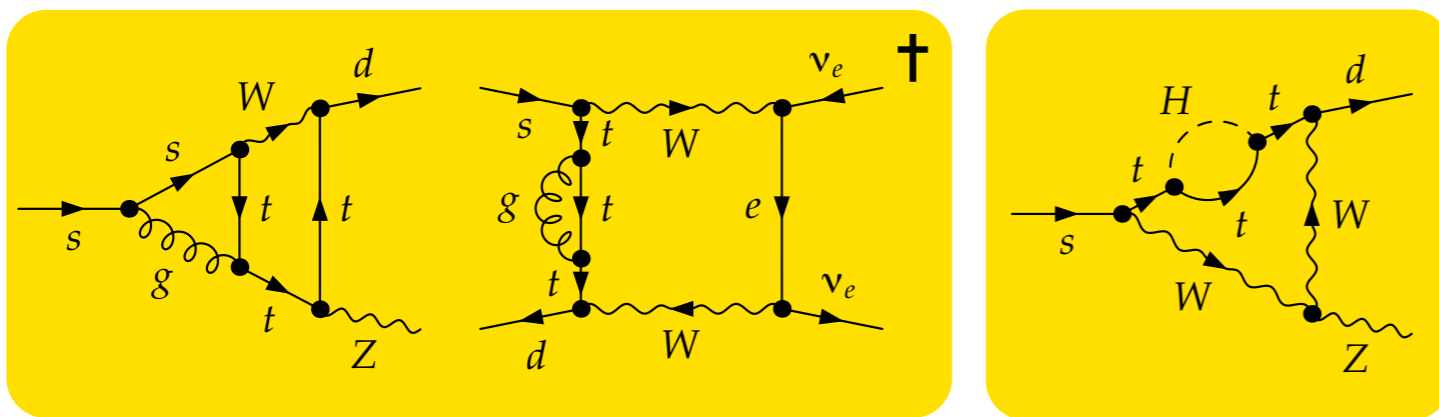
‡ Buchalla & Buras '97

SM prediction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$: upshot

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left[\frac{\text{Im}(V_{ts}^* V_{td})}{\lambda^5} X \right]^2 = (2.54 \pm 0.35) \times 10^{-11}$$

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$$X = 1.456 \pm 0.017_{m_t} \pm 0.013_{\mu_t} \pm 0.015_{\text{EW}}^\ddagger$$



- unknown NNLO & EW corrections dominate theory error of 3%
- within SM amount of CP violation could be determined with unmatched precision

* Mescia & Smith '07

† Misiak & Urban '99, Buchalla & Buras '99

‡ Buchalla & Buras '97

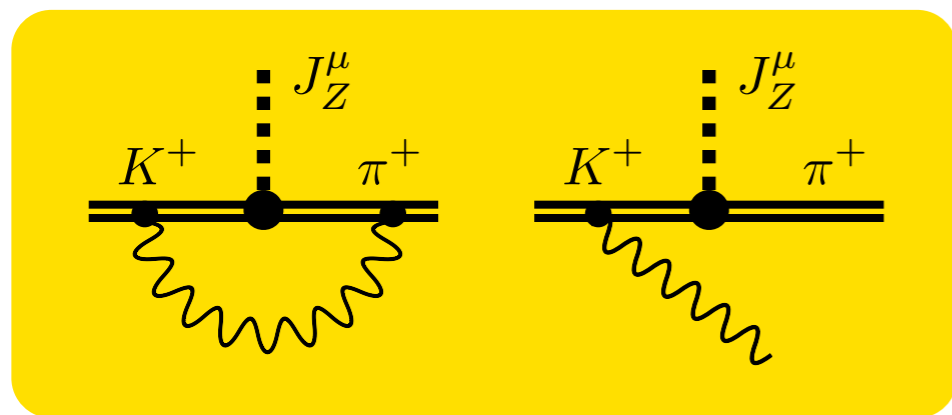
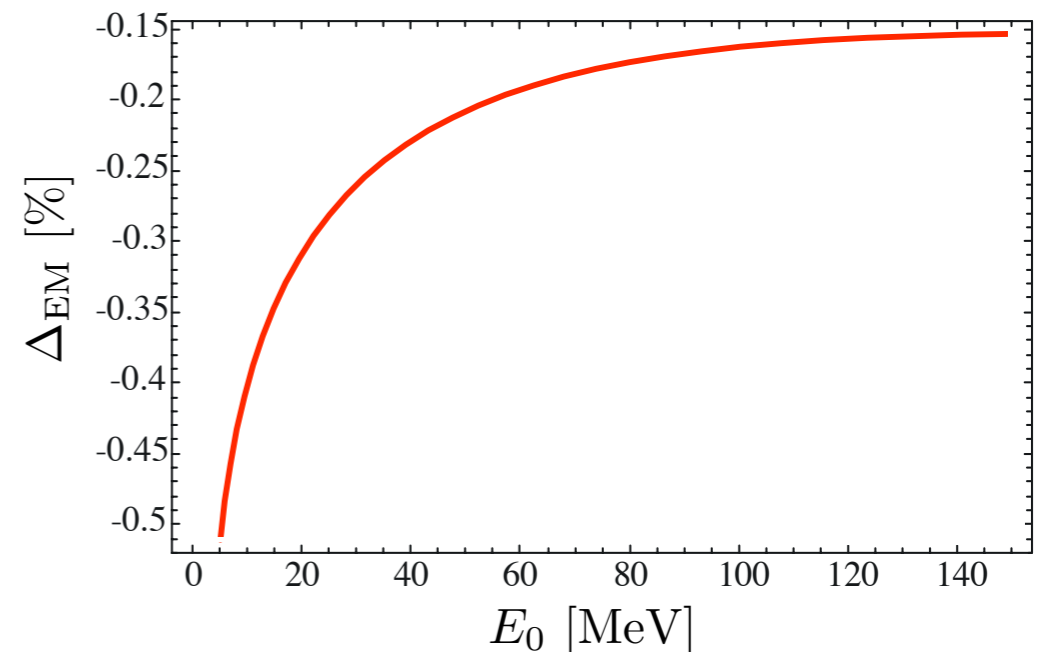
SM prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: κ_+ & Δ_{EM}

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\text{EM}}) \left| \frac{x}{\lambda^5} \right|^2$$

$$\Delta_{\text{EM}}(E_\gamma < 20 \text{ MeV}) = -0.003^*$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \text{Re}(V_{cs}^* V_{cd})(P_c + \delta P_{c,u})$$

$$\kappa_+ = (5.168 \pm 0.025) \times 10^{-10} \left(\frac{\lambda}{0.225} \right)^8^*$$



- long-distance QED corrections at $\mathcal{O}(p^2\alpha)$ in ChPT are known now

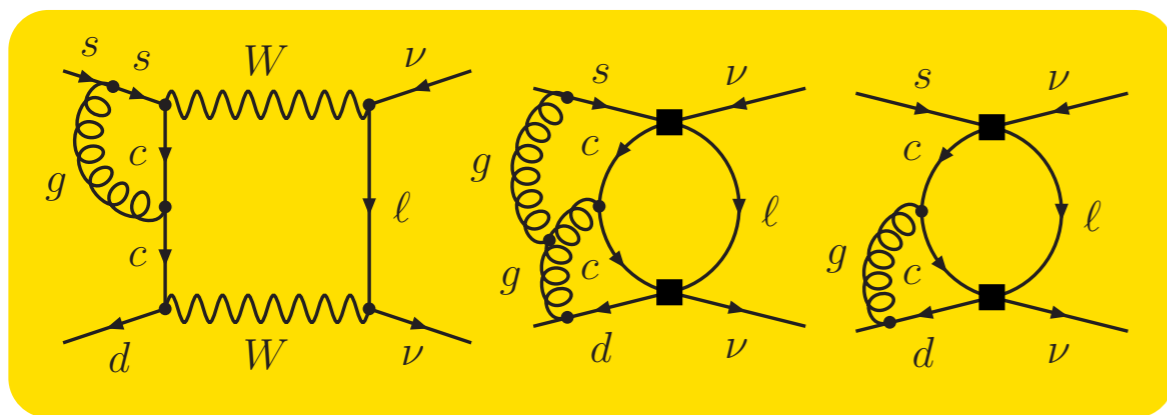
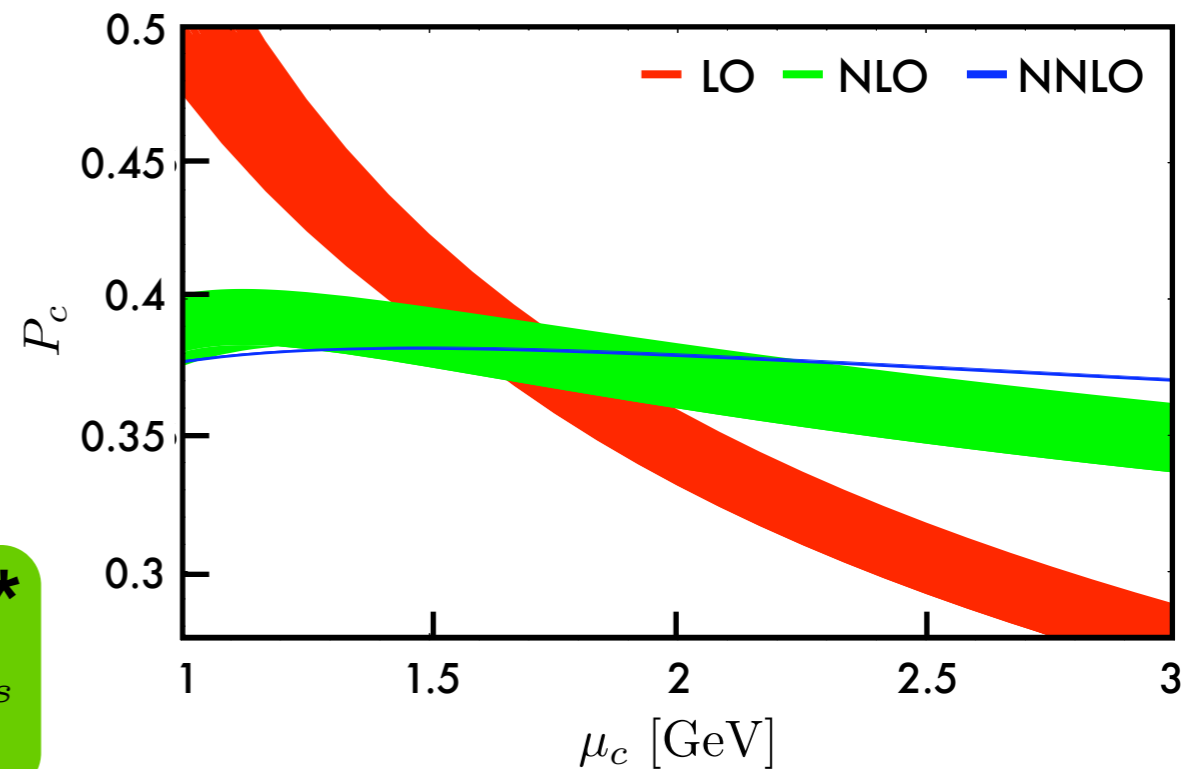
* Mescia & Smith '07

SM prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: P_c

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\text{EM}}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \text{Re}(V_{cs}^* V_{cd}) (P_c + \delta P_{c,u})$$

$$P_c = 0.374 \pm 0.009_{\text{pert}} \pm 0.031_{m_c} \pm 0.009_{\alpha_s}^*$$



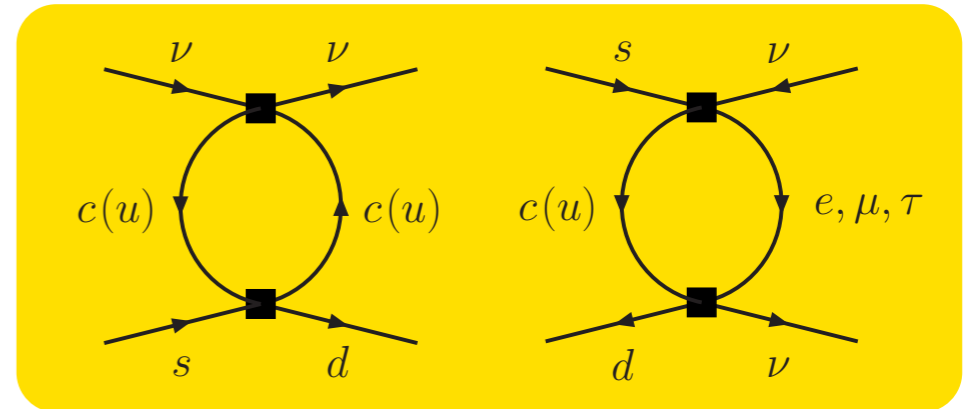
- NNLO calculation of P_c leads to reduction of theoretical error from 10% down to 2.5%

*Buras et al. '05, '06

SM prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: $\delta P_{c,u}$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\text{EM}}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \text{Re}(V_{cs}^* V_{cd}) (P_c + \delta P_{c,u})$$



$$Q_1^{(8)} = (\bar{s}_L \gamma_\mu d_L) \partial^2 (\bar{\nu}_L \gamma^\mu \nu_L),$$

$$Q_2^{(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu \overrightarrow{D}^\nu d_L) (\bar{\nu}_L \gamma^\mu \nu_L),$$

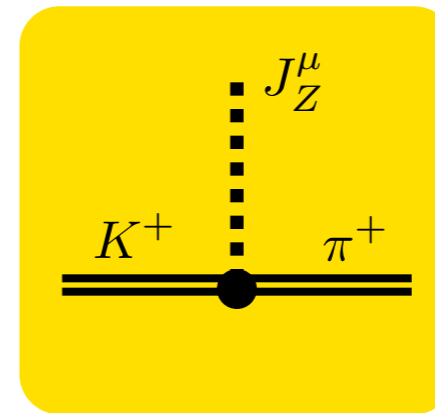
$$Q_3^{(8)} = (\bar{s}_L \overleftarrow{D}_\nu \gamma_\mu d_L) (\bar{\nu}_L (\overleftarrow{\partial}^\nu - \overrightarrow{\partial}^\nu) \gamma^\mu \nu_L)$$

- local charm effects due to dimension-eight operators naively of $\mathcal{O}(m_K^2/m_c^2) \approx 15\%$
- genuine long-distance up effects of $\mathcal{O}(\Lambda^2/m_c^2) \approx 10\%$

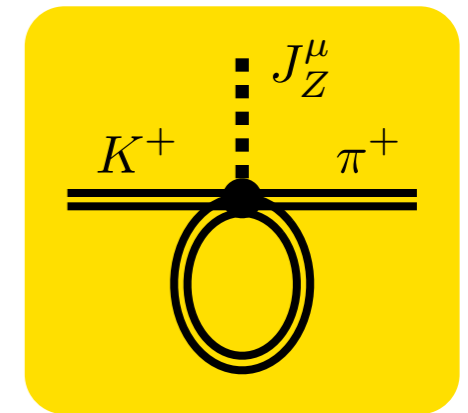
SM prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: $\delta P_{c,u}$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\text{EM}}) \left| \frac{x}{\lambda^5} \right|^2$$

$$x = V_{ts}^* V_{td} X + \lambda^4 \text{Re}(V_{cs}^* V_{cd}) (P_c + \delta P_{c,u})$$



$$O(G_F^2 p^2)$$



$$O(G_F^2 p^4)$$

$$\delta P_{c,u} = 0.04 \pm 0.02^*$$

- effects scale as $O(\pi^2 F_\pi^2 / m_c^2) \approx 5\%$ & enhance SM branching ratio by 6%

- theoretical uncertainty related to non-perturbative effects due to charm & up may be reduced further by dedicated lattice analysis[†]

*Isidori et al. '05 †Isidori et al. '05

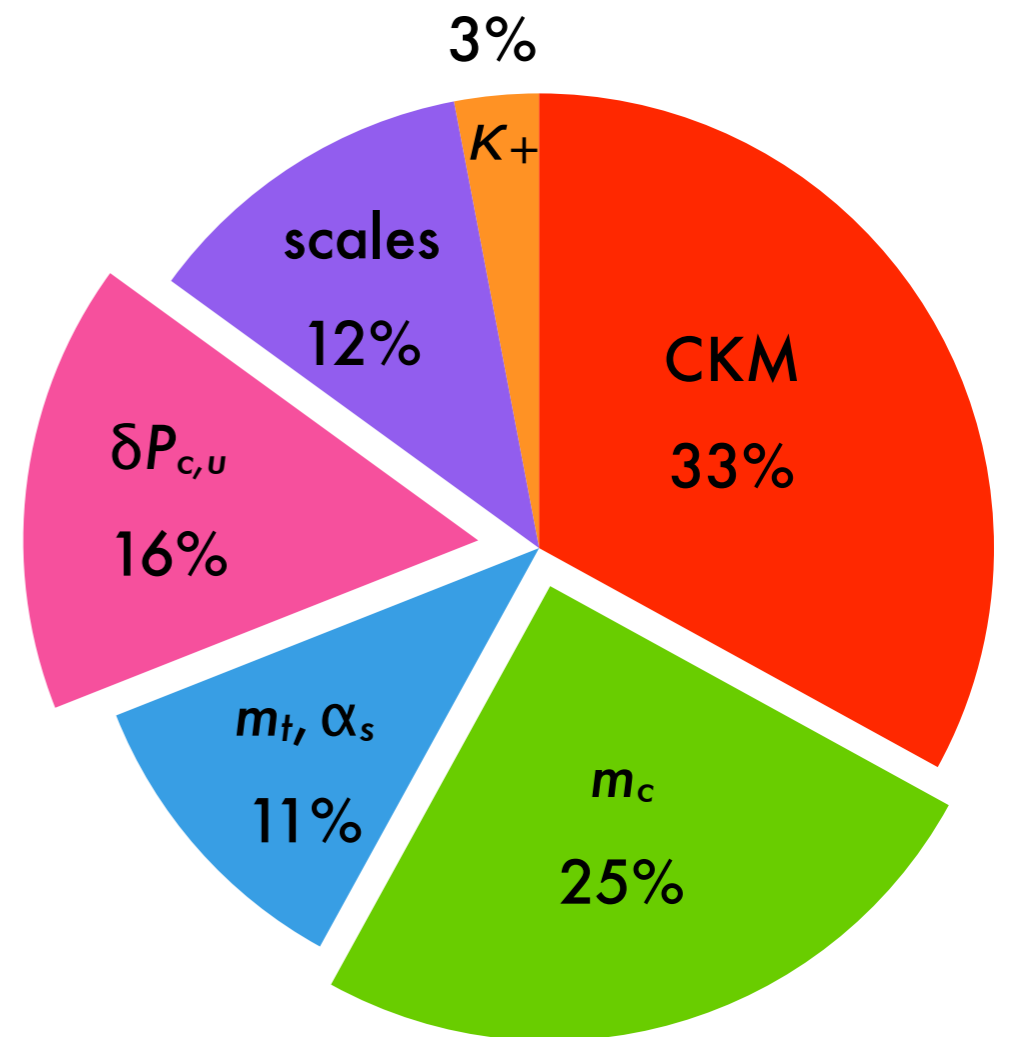
SM prediction(s) of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \{7.96 \pm 0.86, 7.90 \pm 0.67, 7.46 \pm 0.91\} \times 10^{-11}$$

$$m_c(m_c) = (1.30 \pm 0.05) \text{ GeV}$$

$$m_c(m_c) = (1.286 \pm 0.013) \text{ GeV}^*$$

$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054) \text{ GeV}^\dagger$$

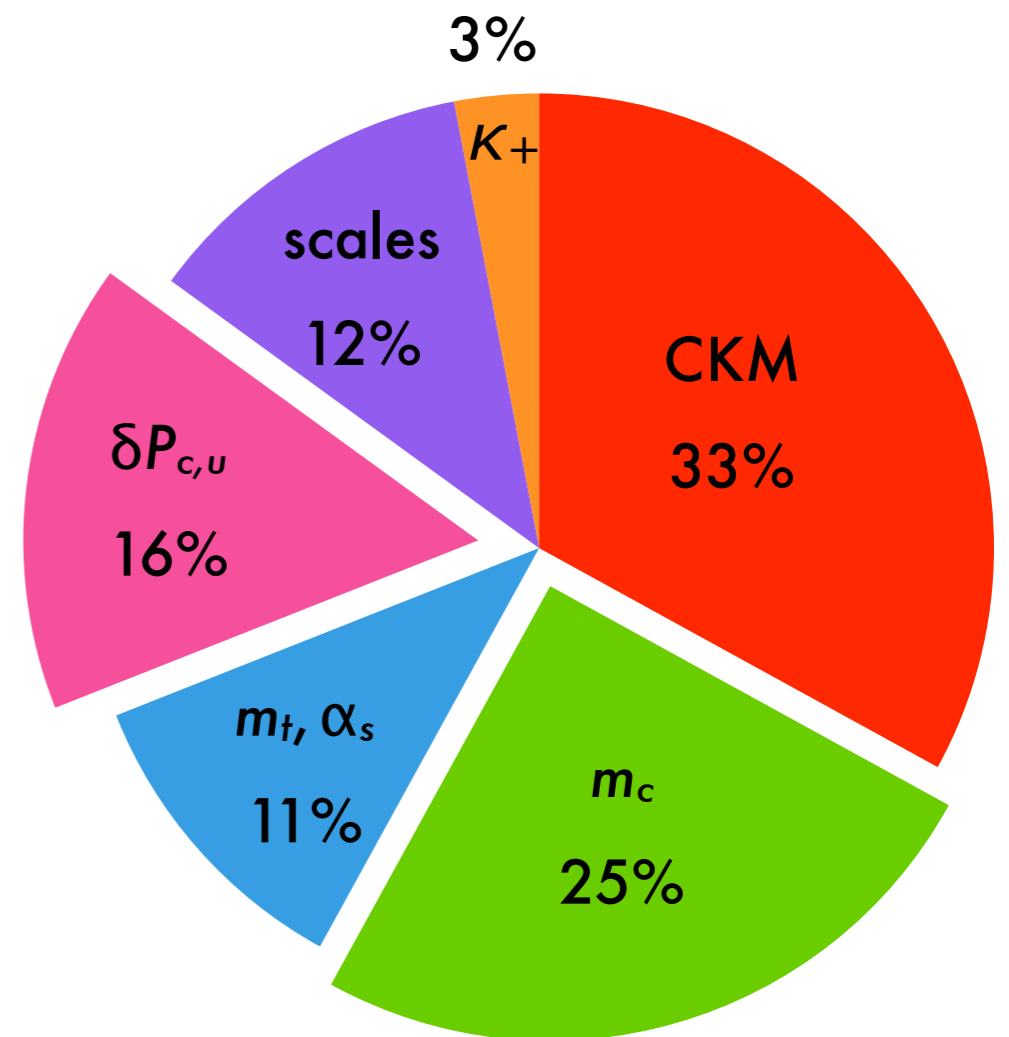


*Kühn et al. '07

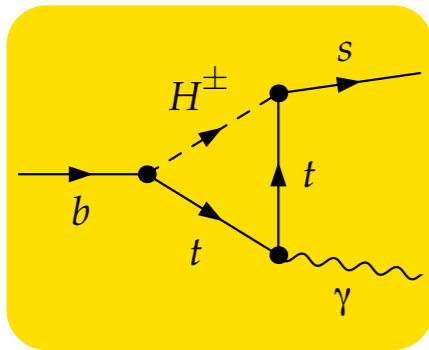
†Hoang & Manohar '05

SM prediction of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: upshot

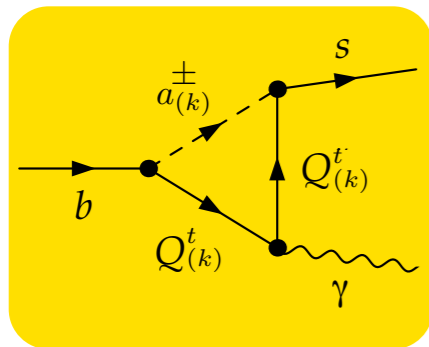
- theoretical progress in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ closely related to precision determination of charm mass
- better knowledge of long-distance effects desirable
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ new field of interesting physical applications for lattice community



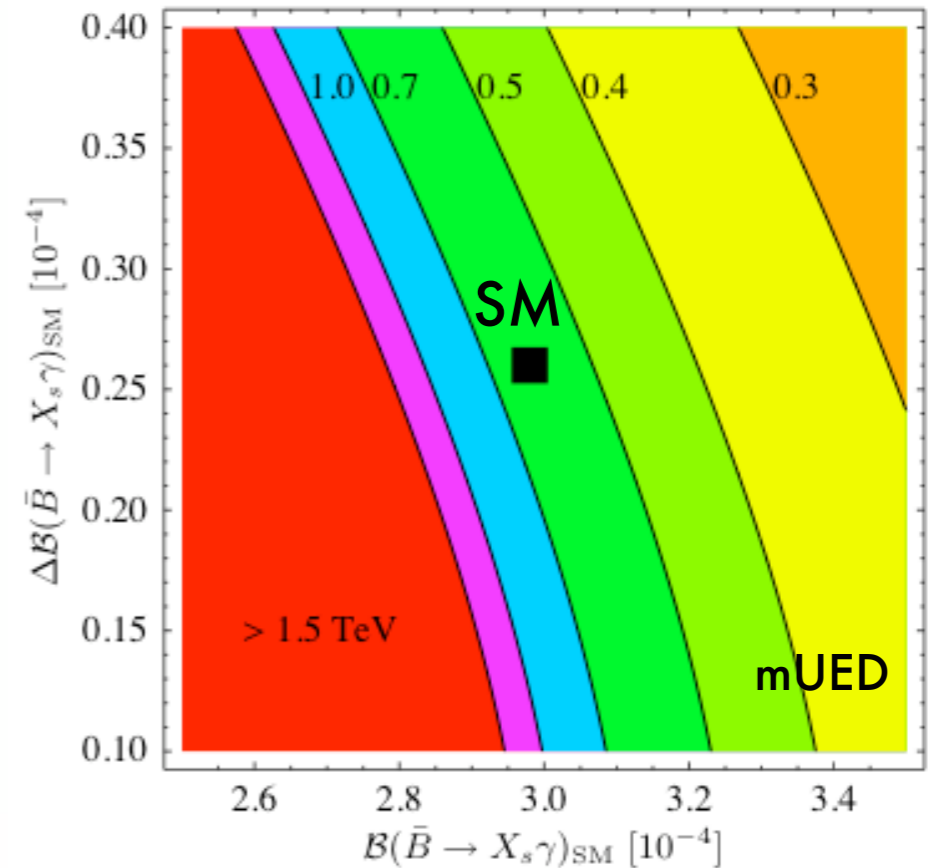
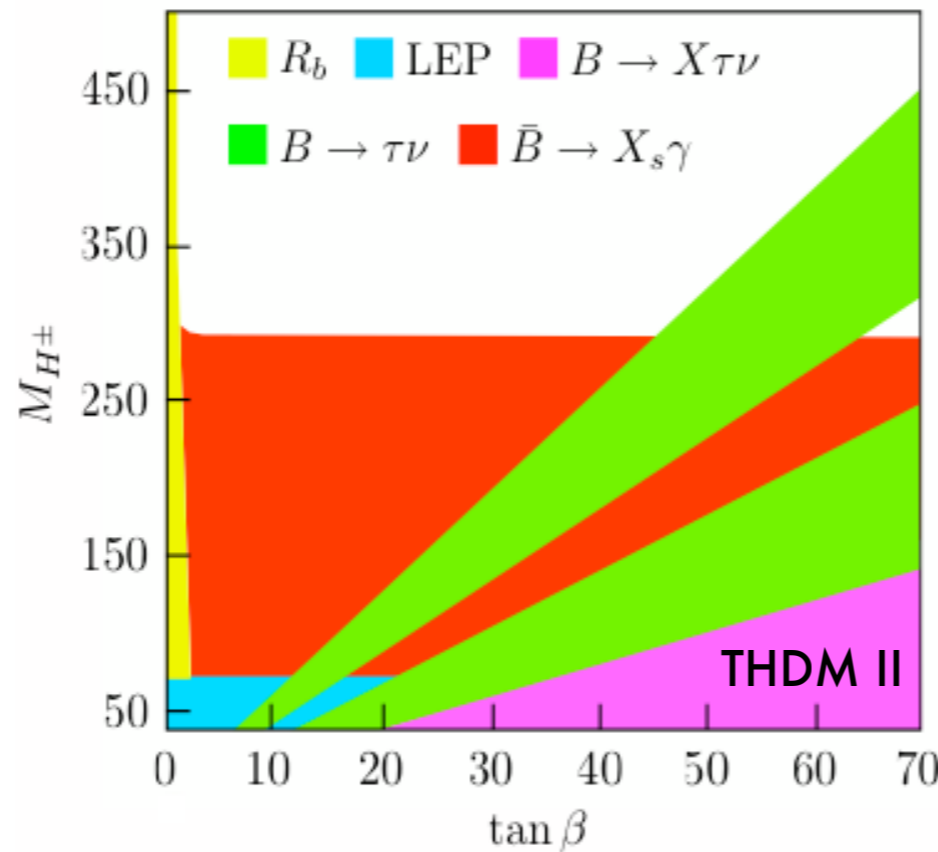
Constraints on new physics from $\bar{B} \rightarrow X_s \gamma$



THDM II



mUED

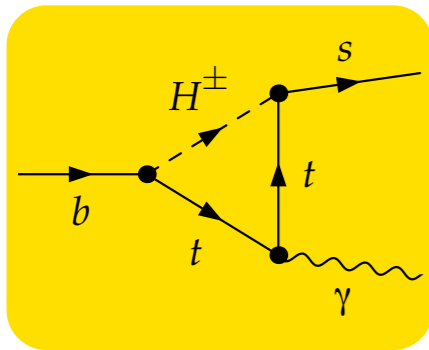


model	accuracy	effect	bound
THDM II	NLO	↑	$M_H^\pm > 295 \text{ GeV (95\% CL)}^*$
general MSSM	LO	↕	$ (\delta_{23}^d)_{LL} \lesssim 4 \times 10^{-1}, (\delta_{23}^d)_{RR} \lesssim 8 \times 10^{-1},$ $ (\delta_{23}^d)_{LR} \lesssim 6 \times 10^{-2}, (\delta_{23}^d)_{RL} \lesssim 2 \times 10^{-2}$
mUED	LO	↓	$1/R > 600 \text{ GeV (95\% CL)}^\dagger$
RS	LO	↑	$M_{KK} \gtrsim 2.4 \text{ TeV}$

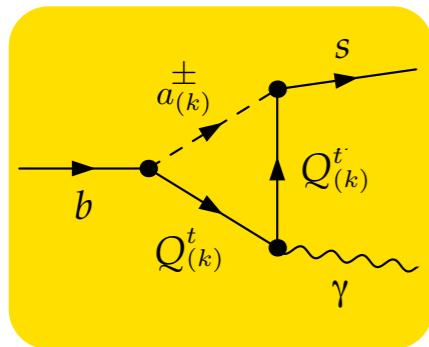
* Misiak et al. '06

† UH & Weiler '07

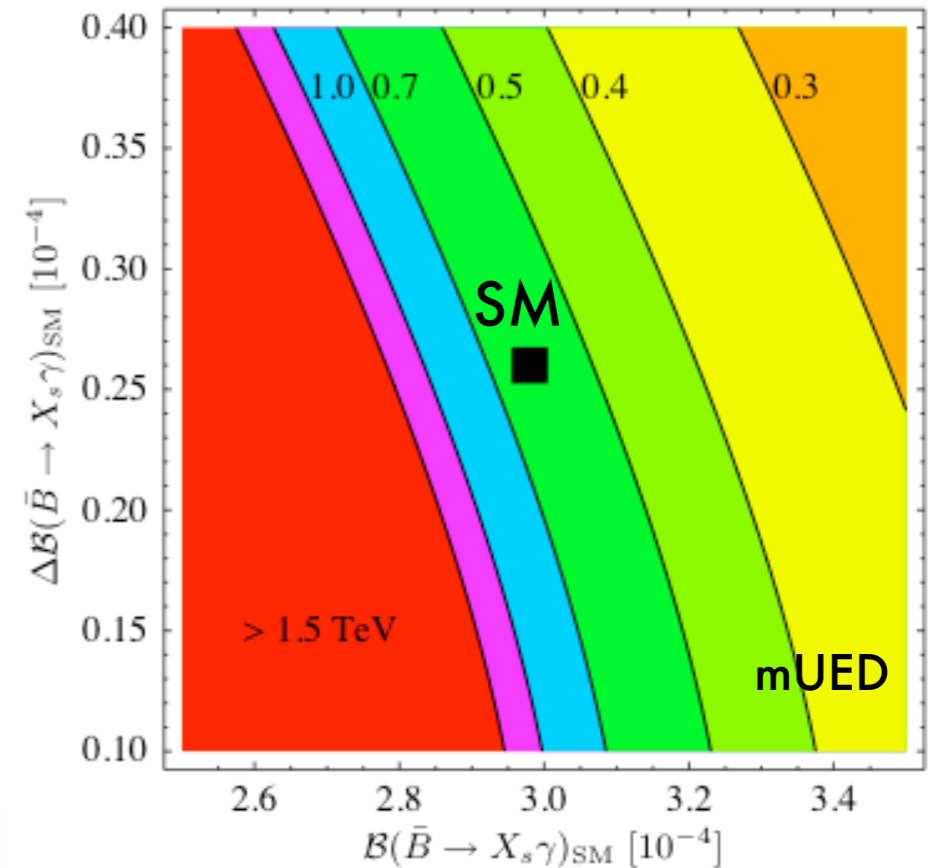
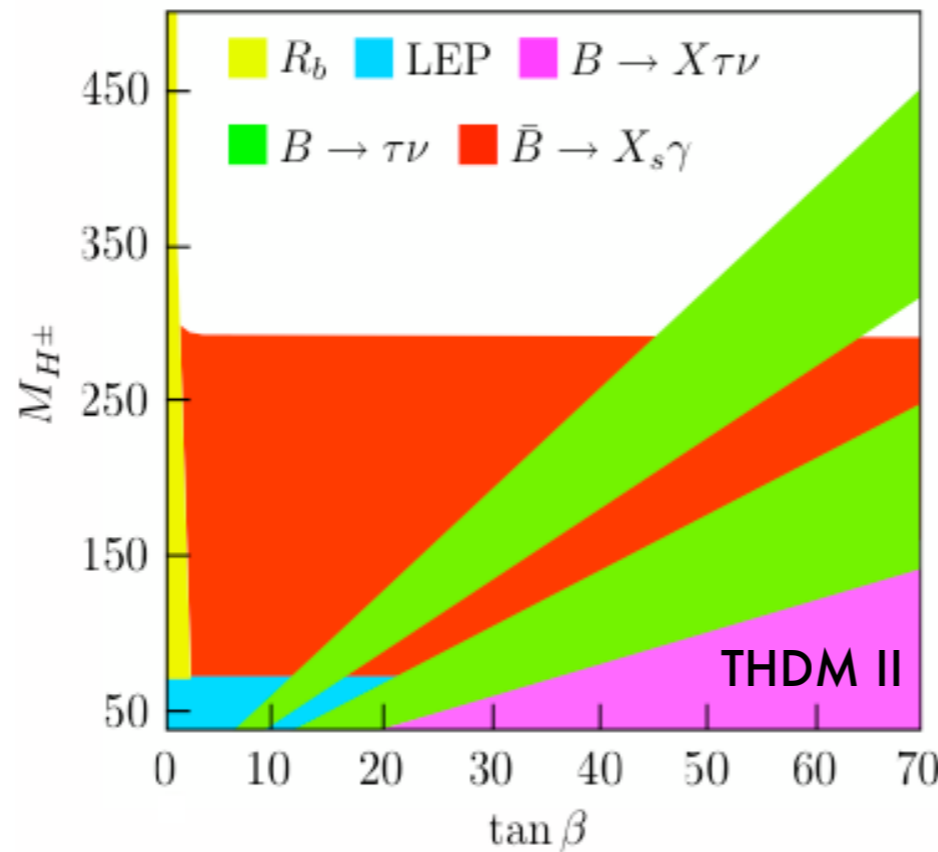
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THDM II



mUED



model	bound
THDM II	$M_{H^\pm} > 295 \text{ GeV (95\% CL)}^*$
general MSSM	$ (\delta_{23}^d)_{LL} \lesssim 4 \times 10^{-1}, (\delta_{23}^d)_{RR} \lesssim 8 \times 10^{-1},$ $ (\delta_{23}^d)_{LR} \lesssim 6 \times 10^{-2}, (\delta_{23}^d)_{RL} \lesssim 2 \times 10^{-2}$
mUED	$1/R > 600 \text{ GeV (95\% CL)}^\dagger$
RS	$M_{KK} \gtrsim 2.4 \text{ TeV}$

- constraints depend in non-negligible way on theory error in SM

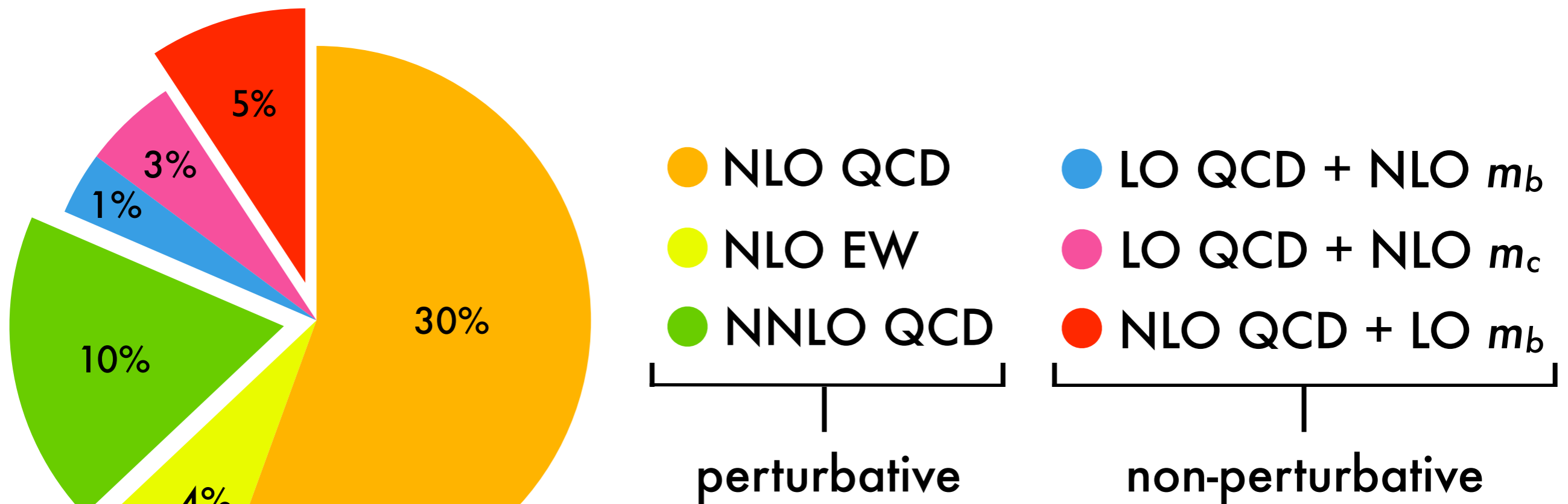
* Misiak et al. '06

† UH & Weiler '07

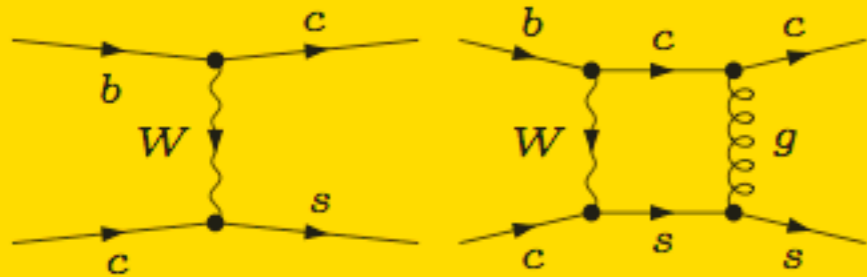
Corrections to $\bar{B} \rightarrow X_s \gamma$ beyond LO in SM

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{SM}}^{E_\gamma > 1.6 \text{ GeV}} = \mathcal{B}(\bar{B} \rightarrow X_c e \bar{\nu}) \left[\frac{\Gamma(b \rightarrow s \gamma)}{\Gamma(b \rightarrow ce \bar{\nu})} \right]_{\text{LO}} f \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)$$

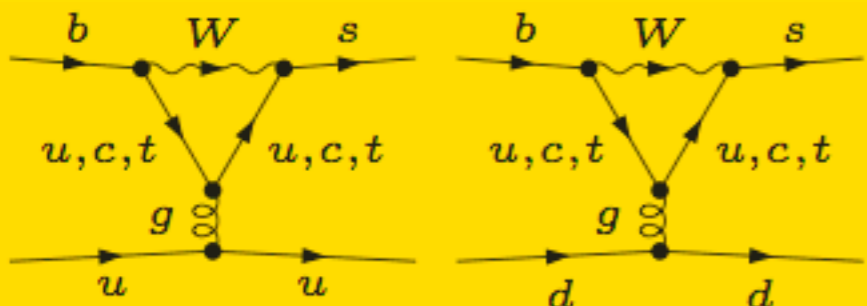
$$\times \left\{ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right) \right\}$$



Usual suspects in $\bar{B} \rightarrow X_s \gamma$



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^8 C_i(\mu) Q_i + \dots$$

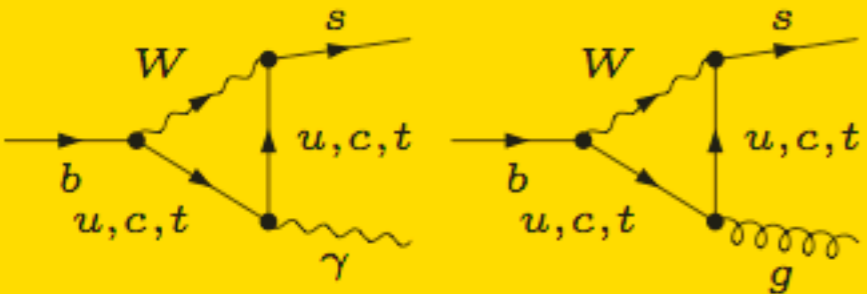


$$Q_{1,2} = (\bar{s} \Gamma_i c) (\bar{c} \Gamma'_i b)$$

$$|C_{1,2}(m_b)| \approx 1$$

$$Q_{3-6} = (\bar{s} \Gamma_i b) \sum_q (\bar{q} \Gamma'_i q)$$

$$|C_{3-6}(m_b)| < 0.07$$



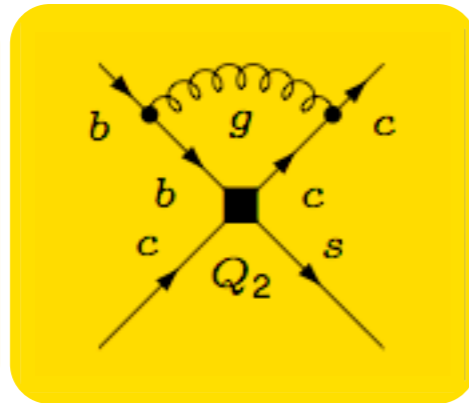
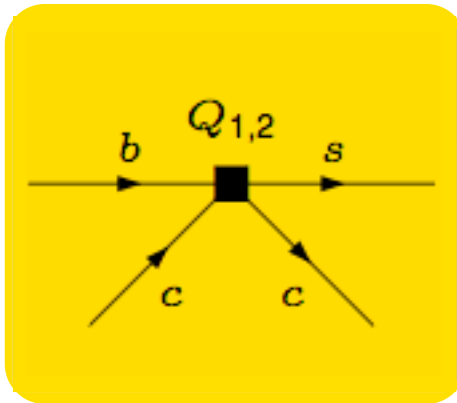
$$Q_7 = \frac{em_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$C_7(m_b) \approx -0.3$$

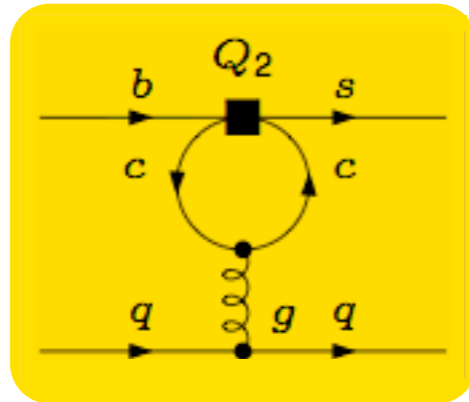
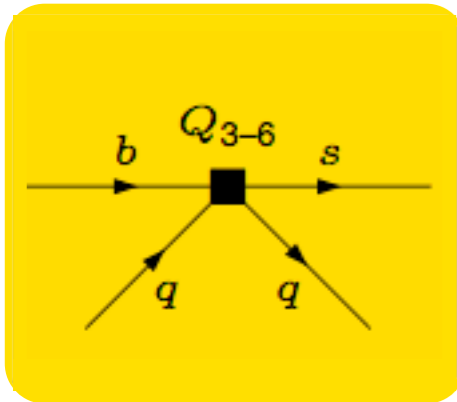
$$Q_8 = \frac{gm_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$C_8(m_b) \approx -0.15$$

Usual suspects in $\bar{B} \rightarrow X_s \gamma$



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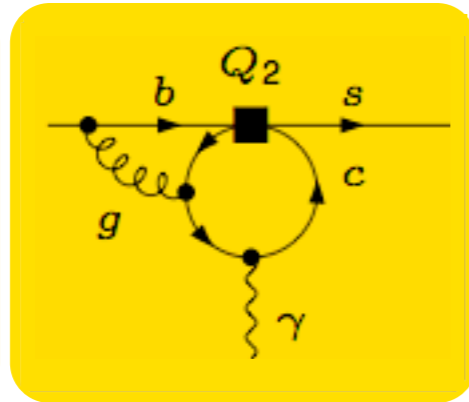
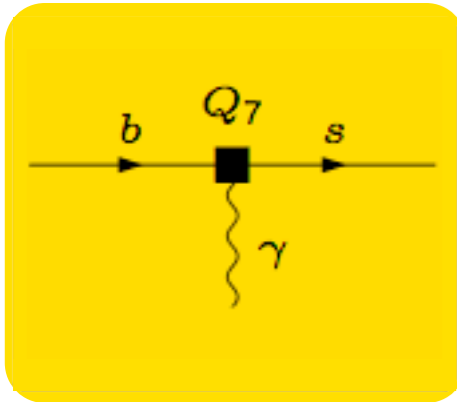


$$Q_{1,2} = (\bar{s} \Gamma_i c) (\bar{c} \Gamma'_i b)$$

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$$|C_{3-6}(m_b)| < 0.07$$



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$$Q_8 = \frac{gm_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

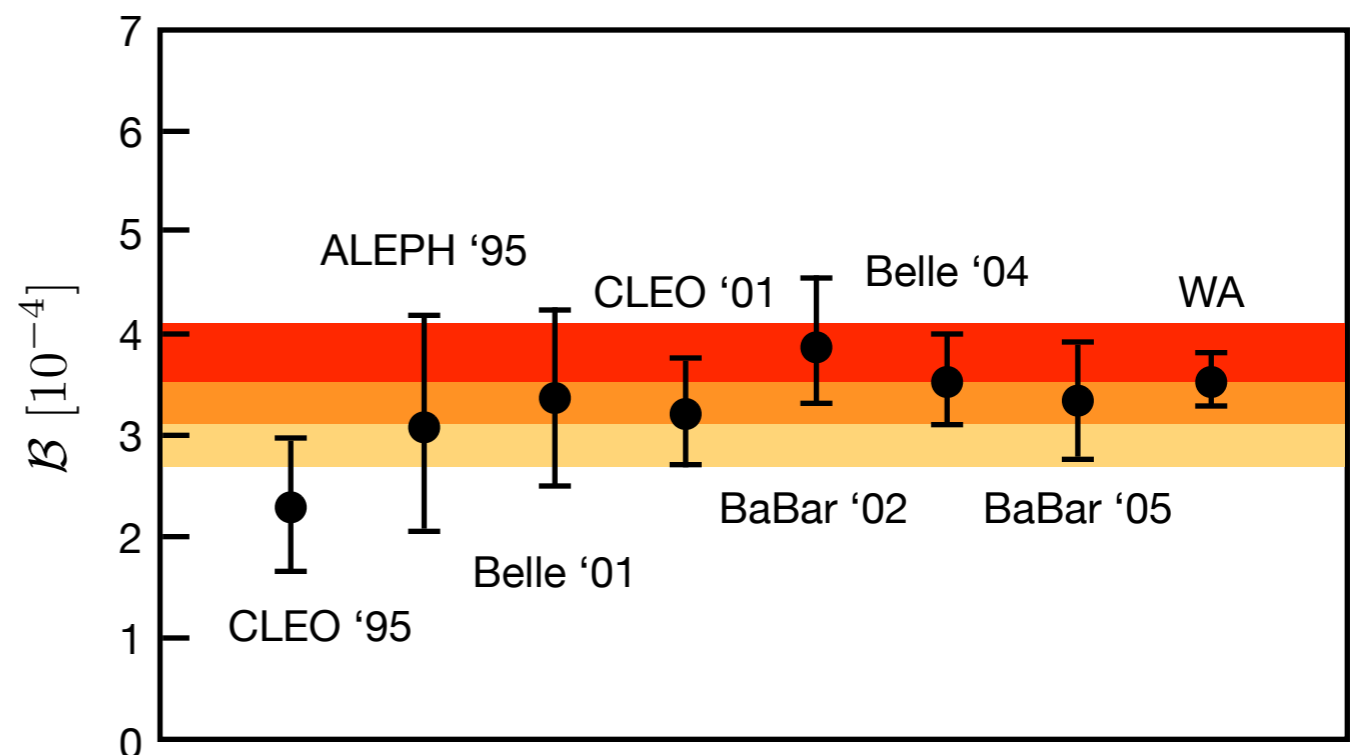
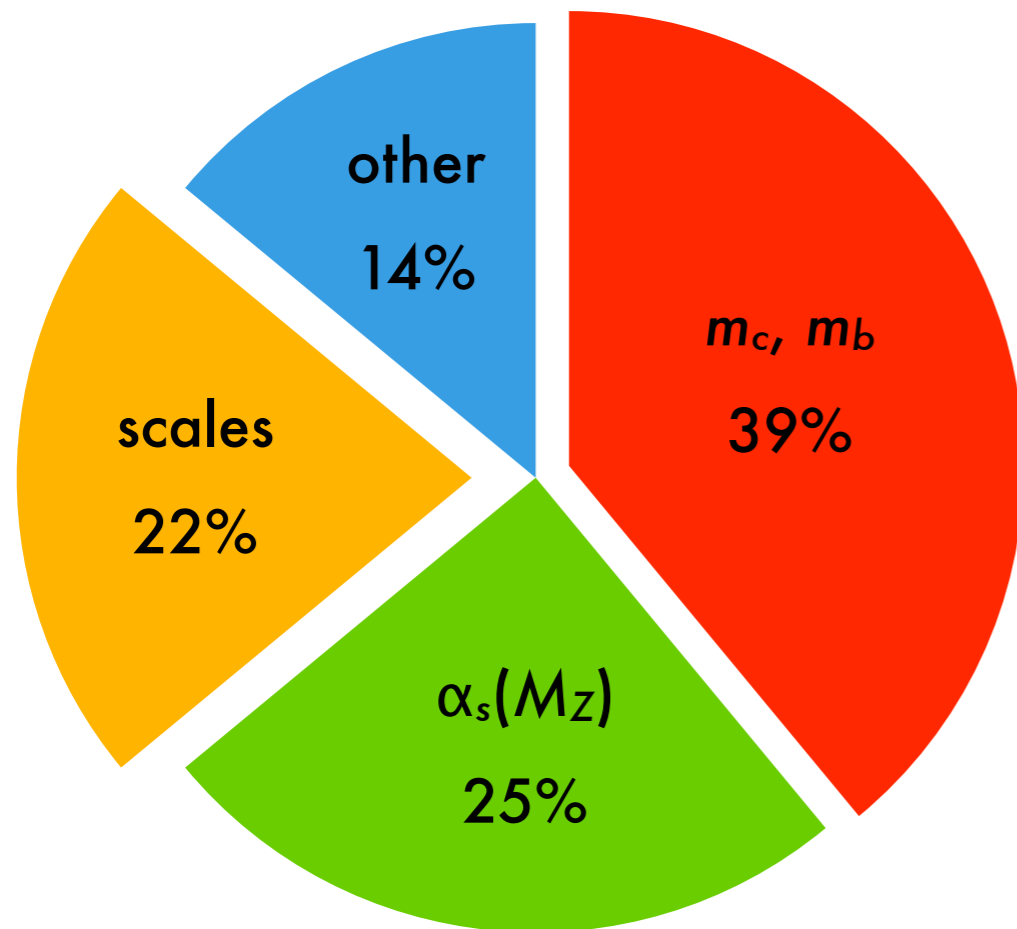
$$C_7(m_b) \approx -0.3$$

$$C_8(m_b) \approx -0.15$$

Error budget of $\bar{B} \rightarrow X_s \gamma$ at NLO in SM

$$\mathcal{B}_{\text{exp}}^{E_\gamma > 1.6 \text{ GeV}} = (3.55 \pm 0.24 \begin{smallmatrix} +0.09 \\ -0.10 \end{smallmatrix} \pm 0.03) \times 10^{-4} \quad *$$

$$\mathcal{B}_{\text{NLO}}^{E_\gamma > 1.6 \text{ GeV}} = (3.33 \pm 0.29) \times 10^{-4}, \quad m_c/m_b = 0.26$$



● $m_c/m_b = 0.22 \pm 0.04$ ($\overline{\text{MS}}$)

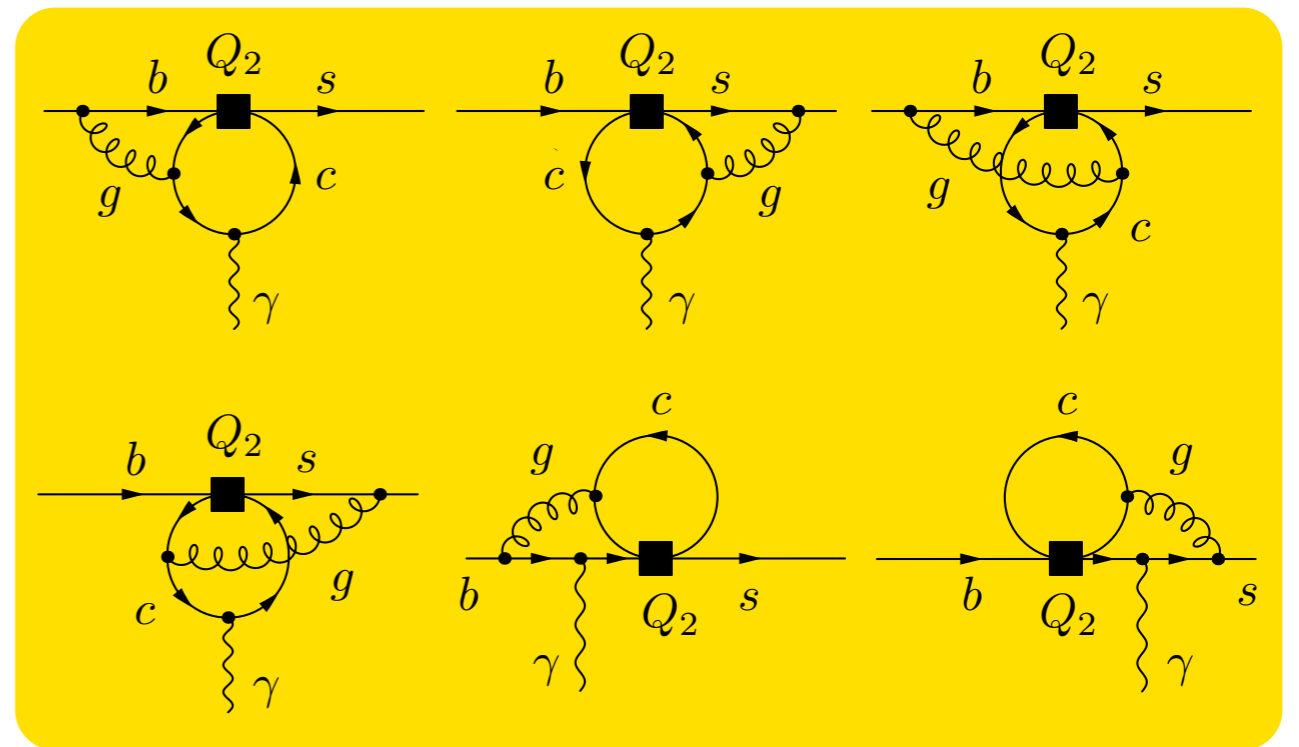
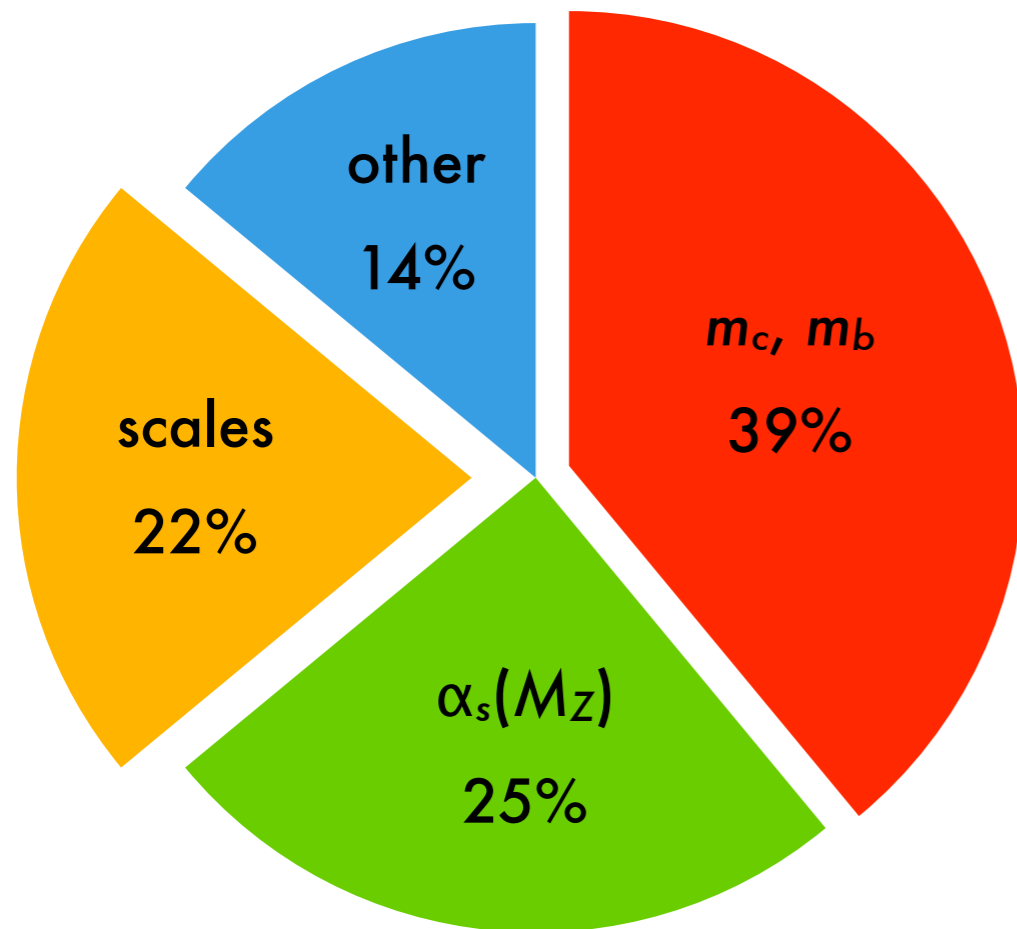
● $m_c/m_b = 0.29 \pm 0.04$ (pole)

*HFAG '06

Error budget of $\bar{B} \rightarrow X_s \gamma$ at NLO in SM

$$\mathcal{B}_{\text{exp}}^{E_\gamma > 1.6 \text{ GeV}} = (3.55 \pm 0.24 \begin{smallmatrix} +0.09 \\ -0.10 \end{smallmatrix} \pm 0.03) \times 10^{-4} \quad *$$

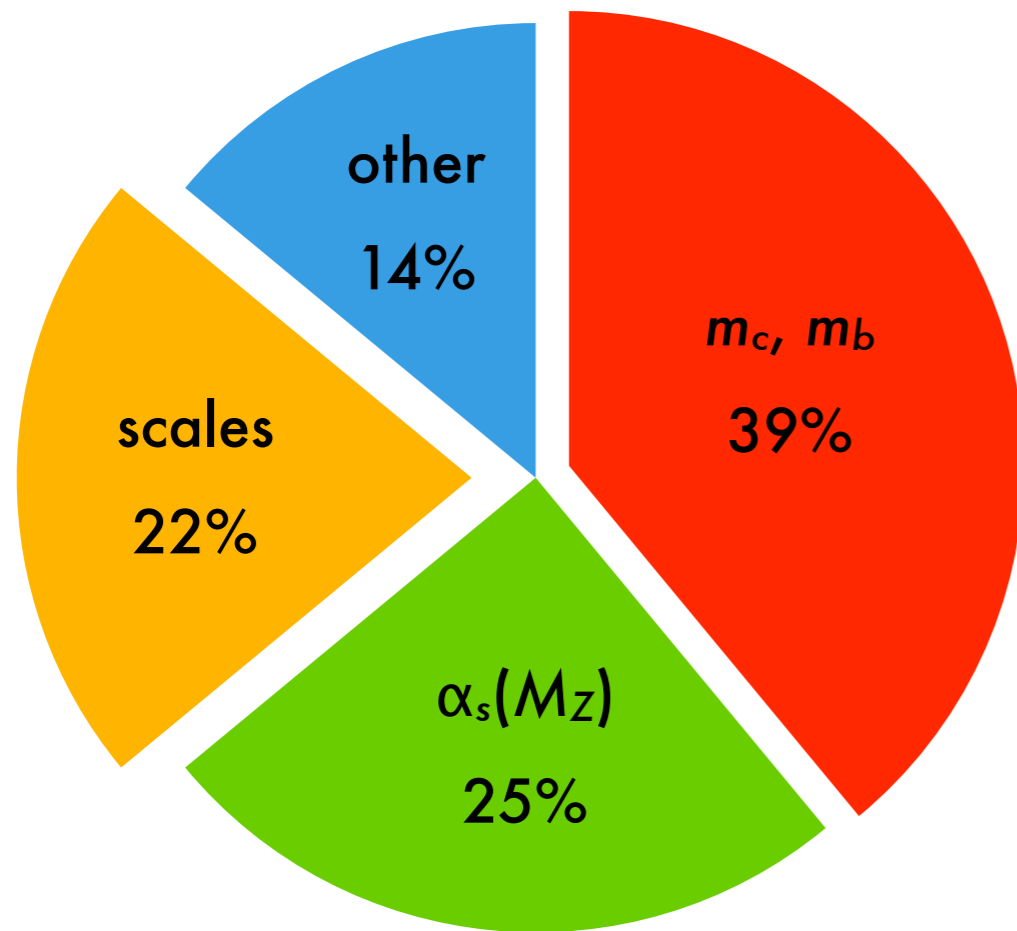
$$\mathcal{B}_{\text{NLO}}^{E_\gamma > 1.6 \text{ GeV}} = (3.33 \pm 0.29) \times 10^{-4}, \quad m_c/m_b = 0.26$$



Error budget of $\bar{B} \rightarrow X_s \gamma$ at NLO in SM

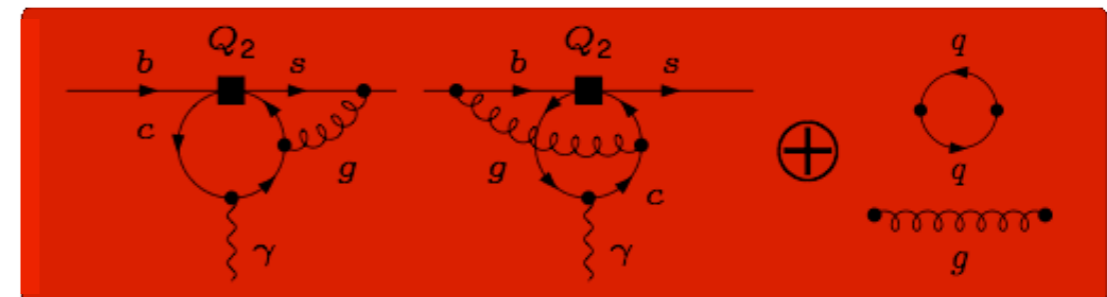
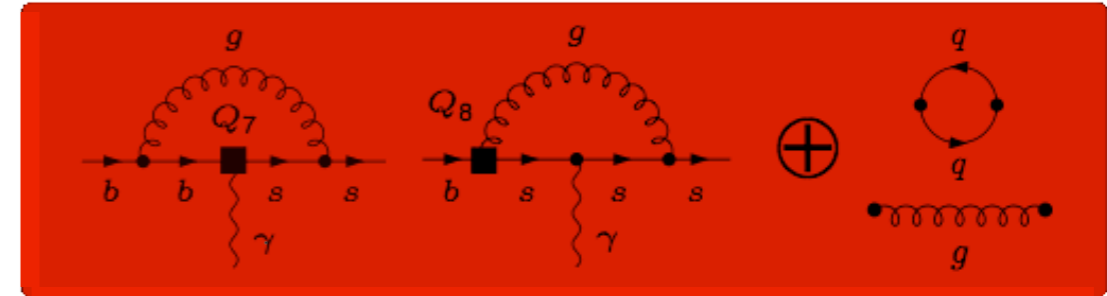
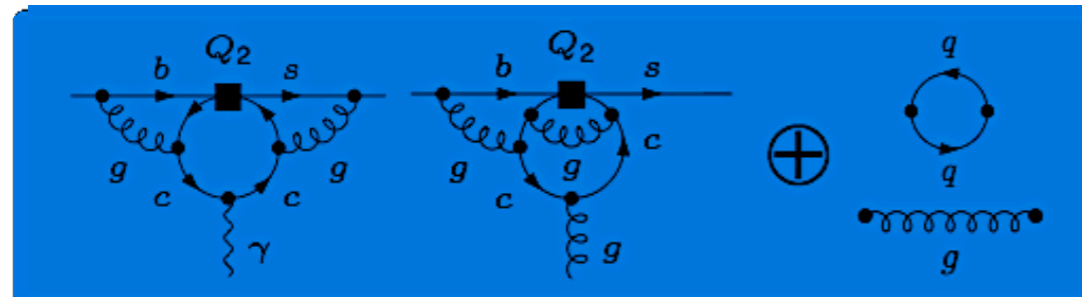
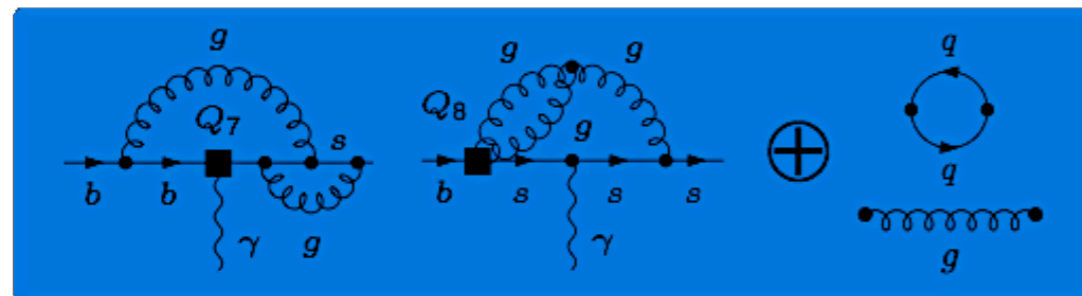
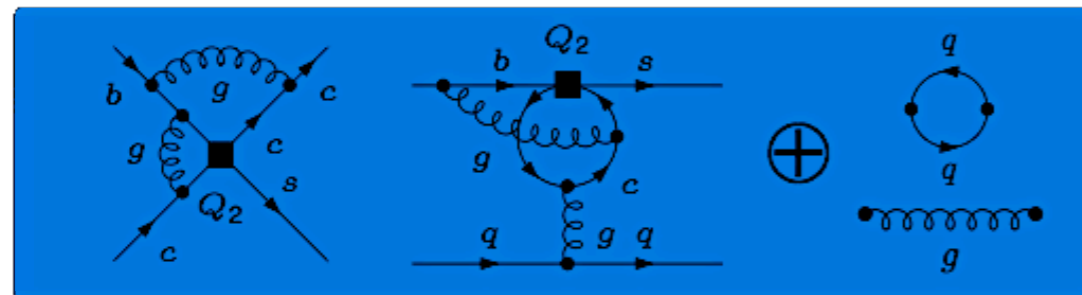
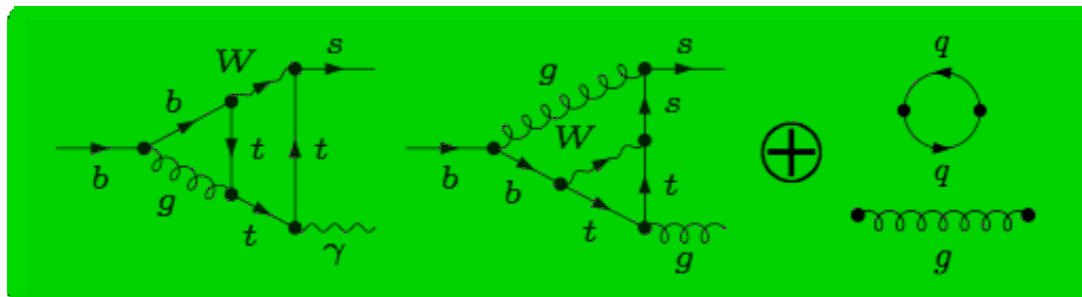
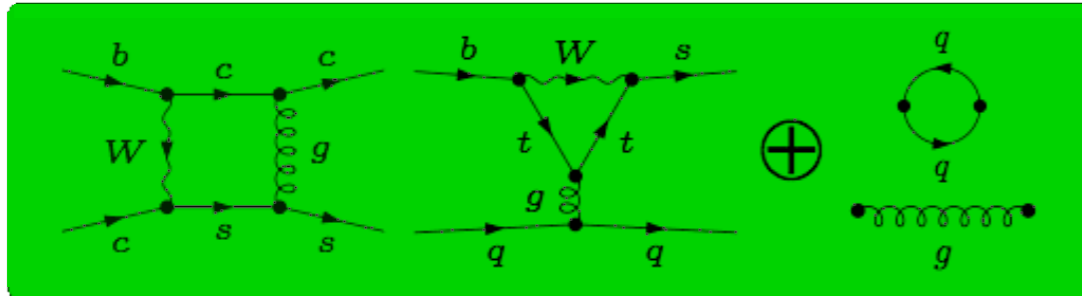
$$\mathcal{B}_{\text{exp}}^{E_\gamma > 1.6 \text{ GeV}} = (3.55 \pm 0.24 \begin{smallmatrix} +0.09 \\ -0.10 \end{smallmatrix} \pm 0.03) \times 10^{-4} \quad *$$

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- scheme ambiguity associated with charm mass, first appearing at NLO, can only be resolved by dedicated NNLO calculation

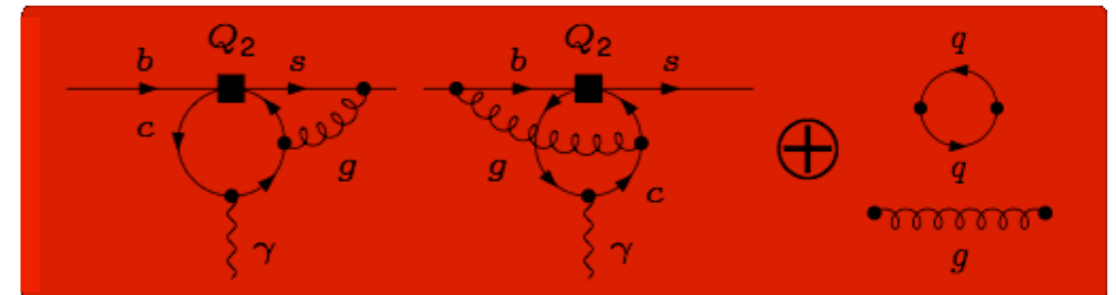
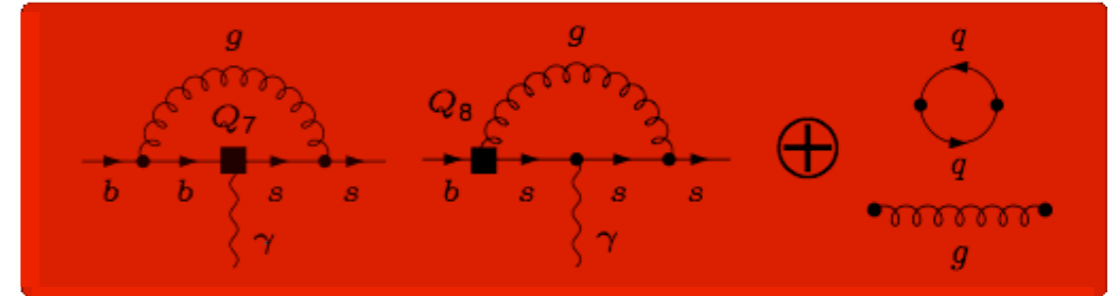
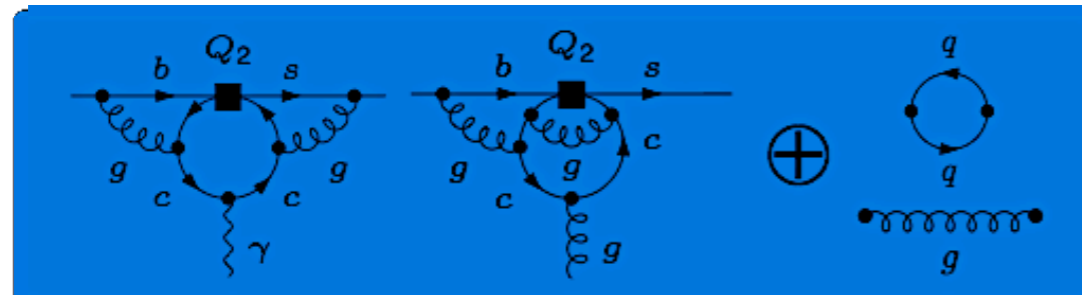
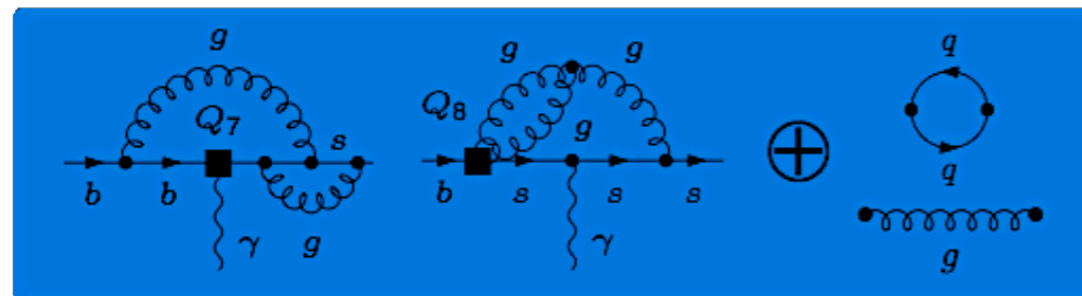
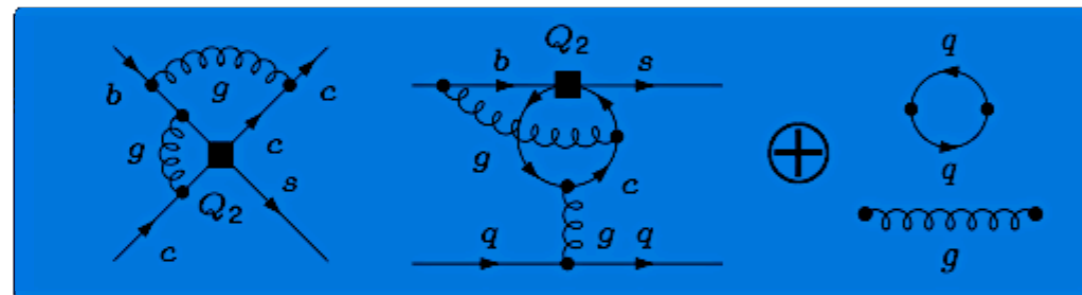
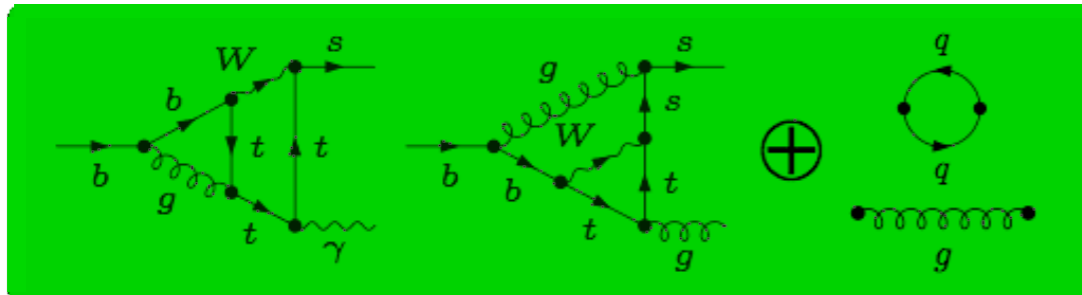
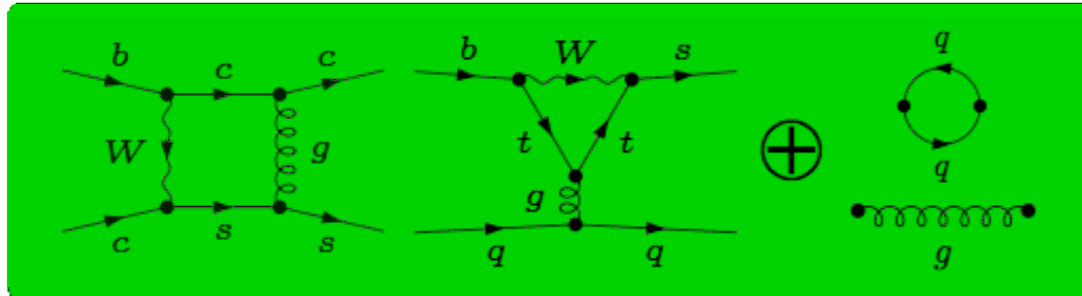
Flavor of NNLO $\bar{B} \rightarrow X_s \gamma$ SM calculation



- matching
- running
- matrix elements

- very involved task as $> 10^2$ 3-loop on-shell & $> 10^4$ 4-loop tadpole diagrams need to be computed

Flavor of NNLO $\bar{B} \rightarrow X_s \gamma$ SM calculation



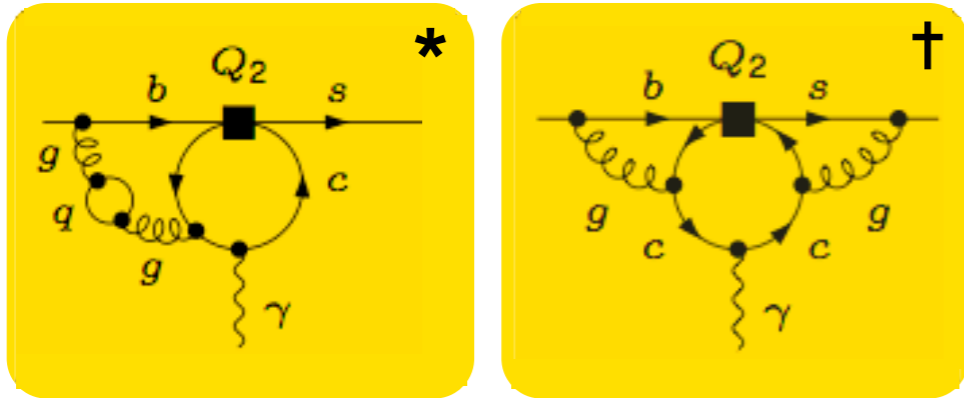
- matching*
- running†
- matrix elements‡

*Bobeth et al. '00; Misiak & Steinhauser '04

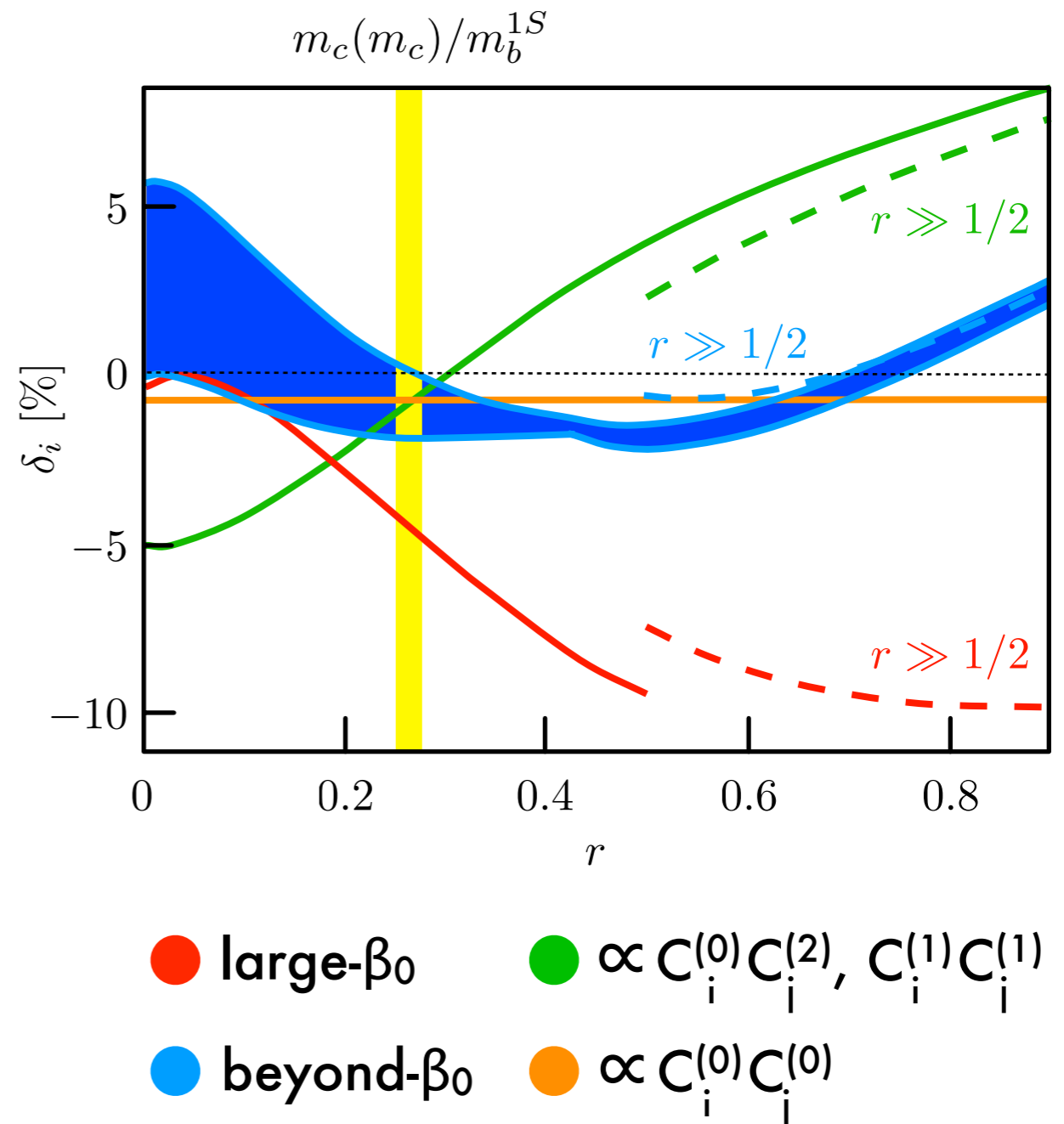
†Gorbahn & UH '04; Gorbahn et al. '05;
Czakon et al. '06

‡Bieri et al. '03; Blokland et al. '05;
Melnikov & Mitov '05; Asatrian et al. '05, '06;
Misiak & Steinhauser '06

Interpolation in charm mass for $\bar{B} \rightarrow X_s \gamma$



- (Q_2, Q_7) interference at NNLO known for arbitrary $r = m_c/m_b$ in large β_0 limit, while beyond- β_0 part only calculated for $r \gg 1/2$
- assume that β_0 piece describes full result accurately for $m_c = 0$ & perform interpolation in r



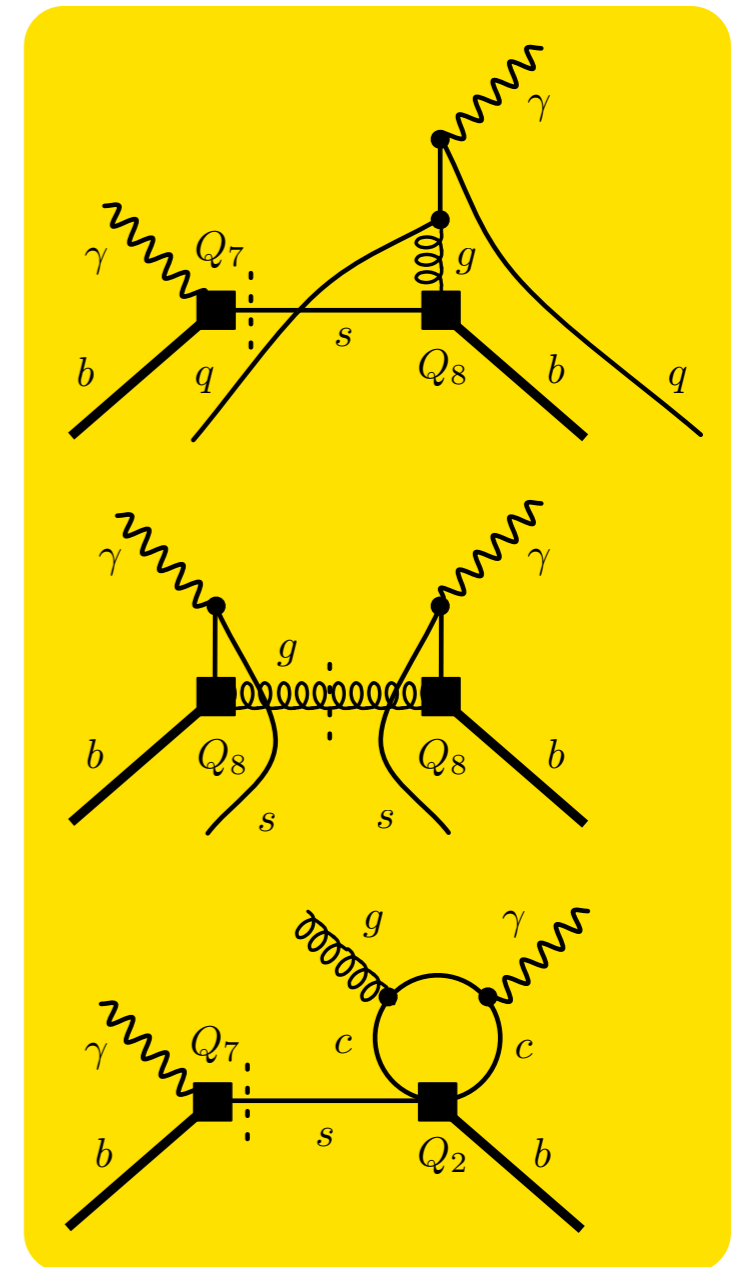
*Bieri et al. '03 †Misiak & Steinhauser '06

Non-local power corrections in $\bar{B} \rightarrow X_s \gamma$

- matrix elements of non-local operators promote corrections that scale like $\alpha_s \Lambda^3 / m_b^3$ & $\alpha_s \Lambda^2 / m_c^2$ in heavy quark to $\alpha_s \Lambda / m_b$
- part involving Q_7 & Q_8 calculated in vacuum insertion approximation:

$$\frac{\Delta \Gamma_{\text{VIA}}^{78}}{\Gamma_{77}} = -\frac{2\pi\alpha_s}{9} \sum_{q=u,d} Q_q \frac{C_8}{C_7} \frac{f_B^2 m_B}{\lambda_B^2 m_b} \approx (-1.6 \pm 1.4)\% \quad *$$

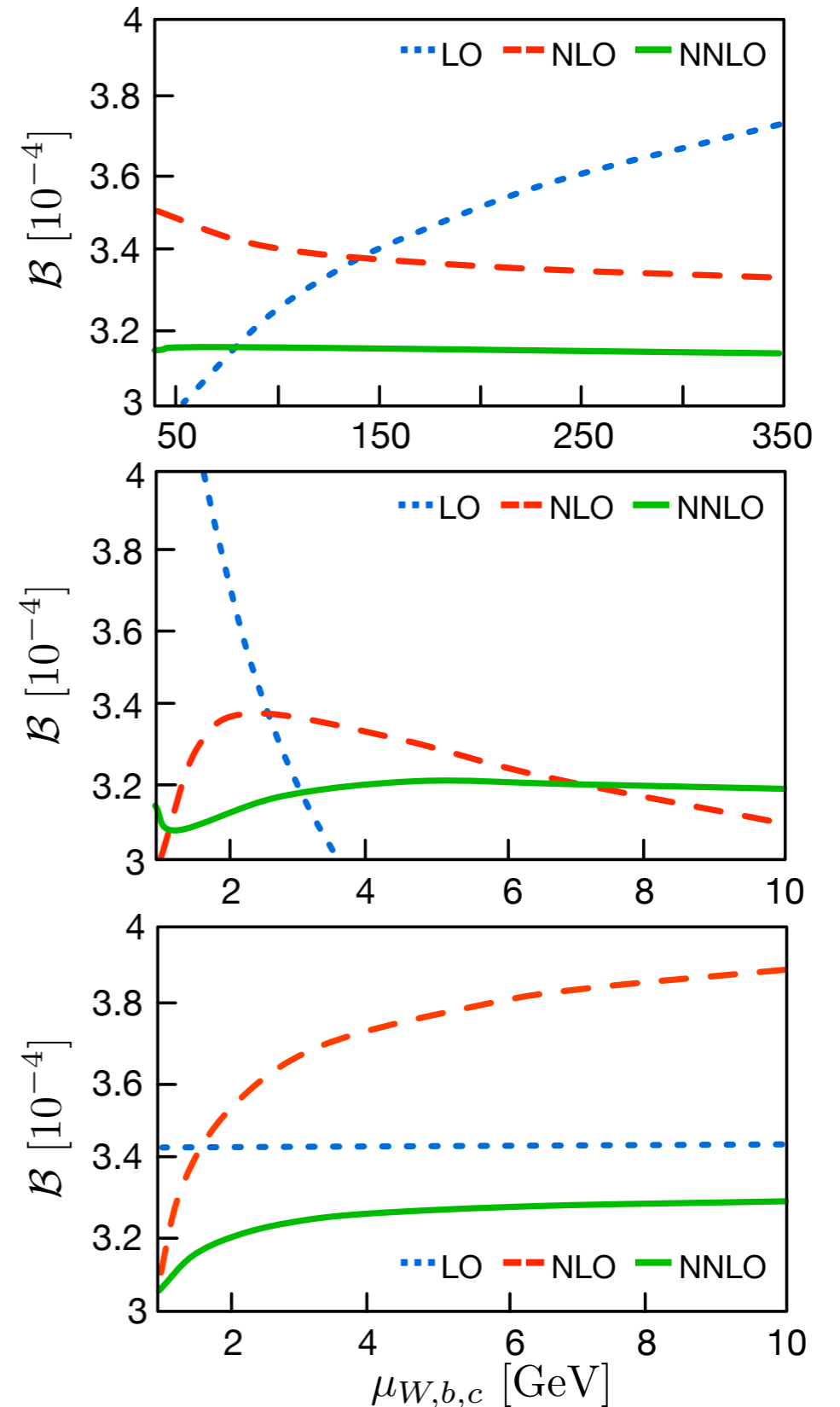
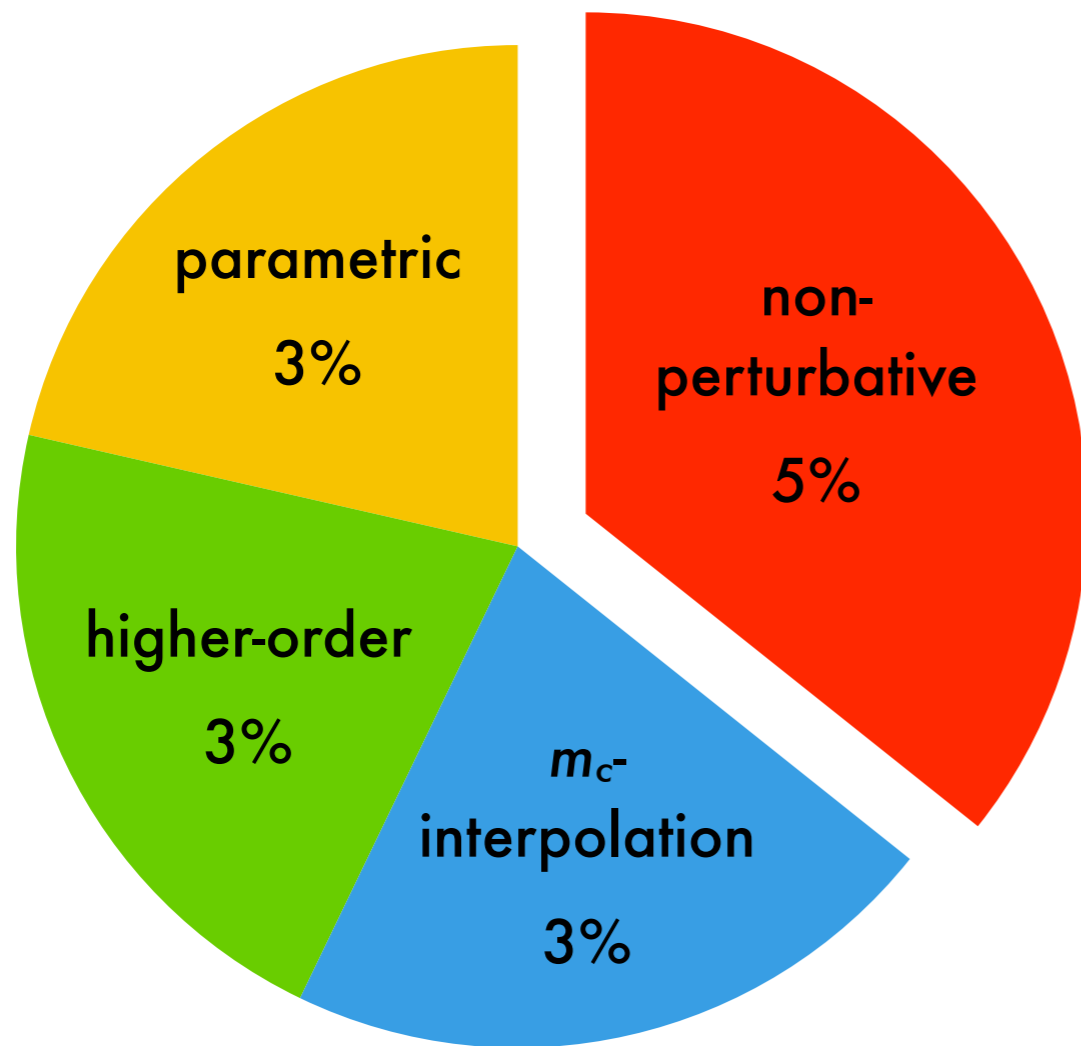
- size of power corrections difficult to estimate given present command of non-perturbative QCD on light cone



*Lee et al. '06

First NNLO estimate of $\bar{B} \rightarrow X_s \gamma$ in SM*

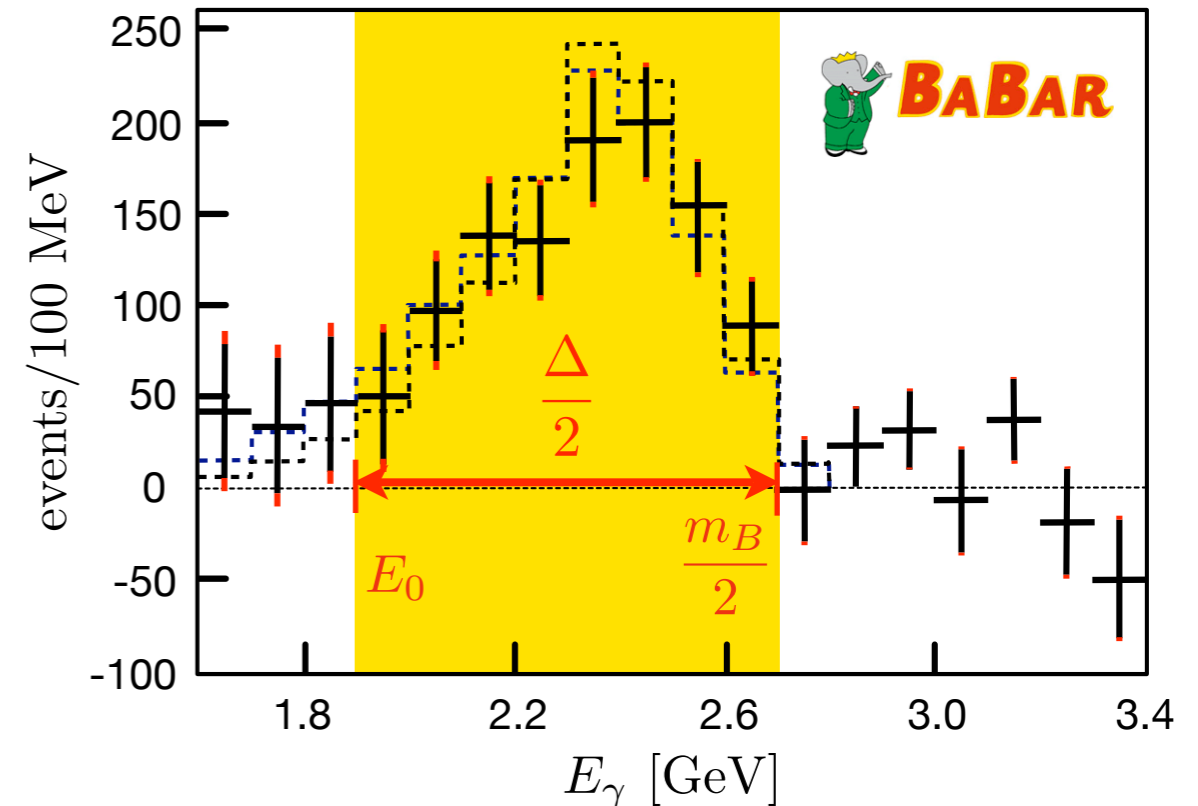
$$\mathcal{B}_{\text{NNLO}}^{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$



* Misiak et al. '06

Photon energy cut effects in $\bar{B} \rightarrow X_s \gamma$

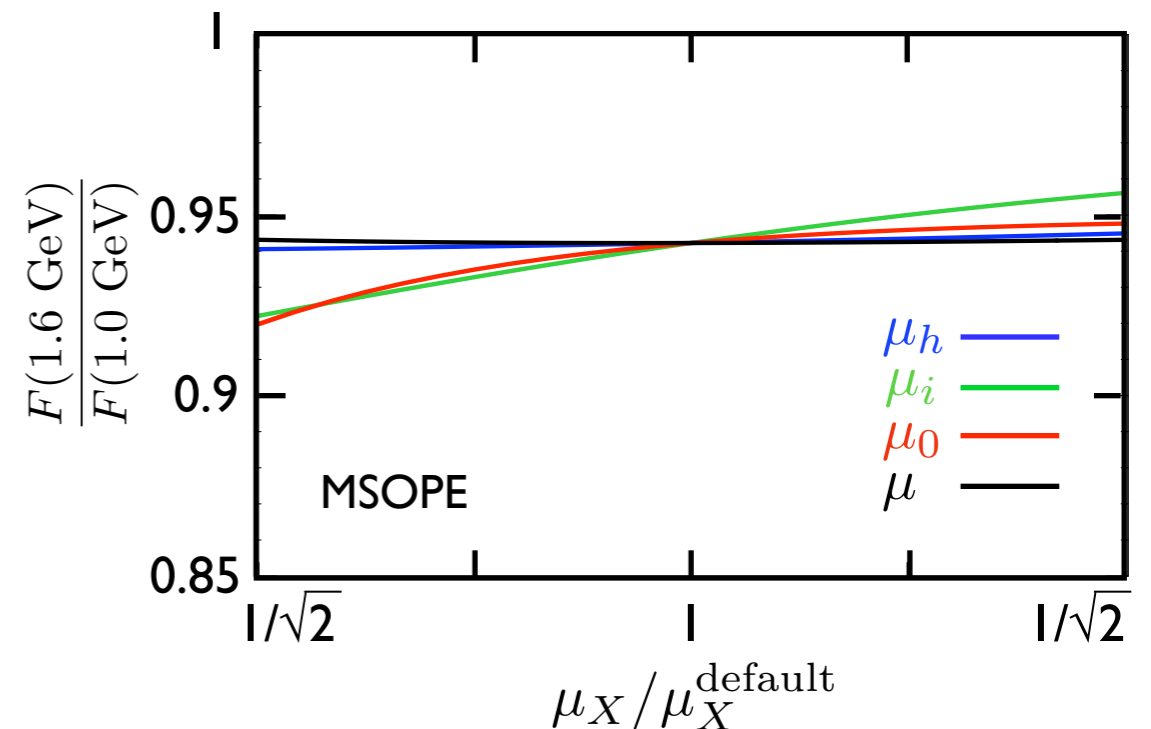
- total rate cannot be measured
- at present experimental cut of $E_0 > 1.8 \text{ GeV}$ on photon energy E_γ
- how big is rate in tail?



$$\frac{F(1.6 \text{ GeV})}{F(1.0 \text{ GeV})}_{\text{DGE}} \stackrel{*}{=} 0.98^{+0.02}_{-0.03}$$

$$\frac{F(1.6 \text{ GeV})}{F(1.0 \text{ GeV})}_{\text{OPE}} \stackrel{\dagger}{=} 0.96$$

$$\frac{F(1.6 \text{ GeV})}{F(1.0 \text{ GeV})}_{\text{MSOPE}} \stackrel{\ddagger}{=} 0.93^{+0.03}_{-0.05} \quad \begin{matrix} +0.02 \\ -0.02 \end{matrix}_{\text{pert}} \quad \begin{matrix} +0.02 \\ -0.02 \end{matrix}_{\text{hadr}} \quad \begin{matrix} +0.02 \\ -0.02 \end{matrix}_{\text{para}}$$



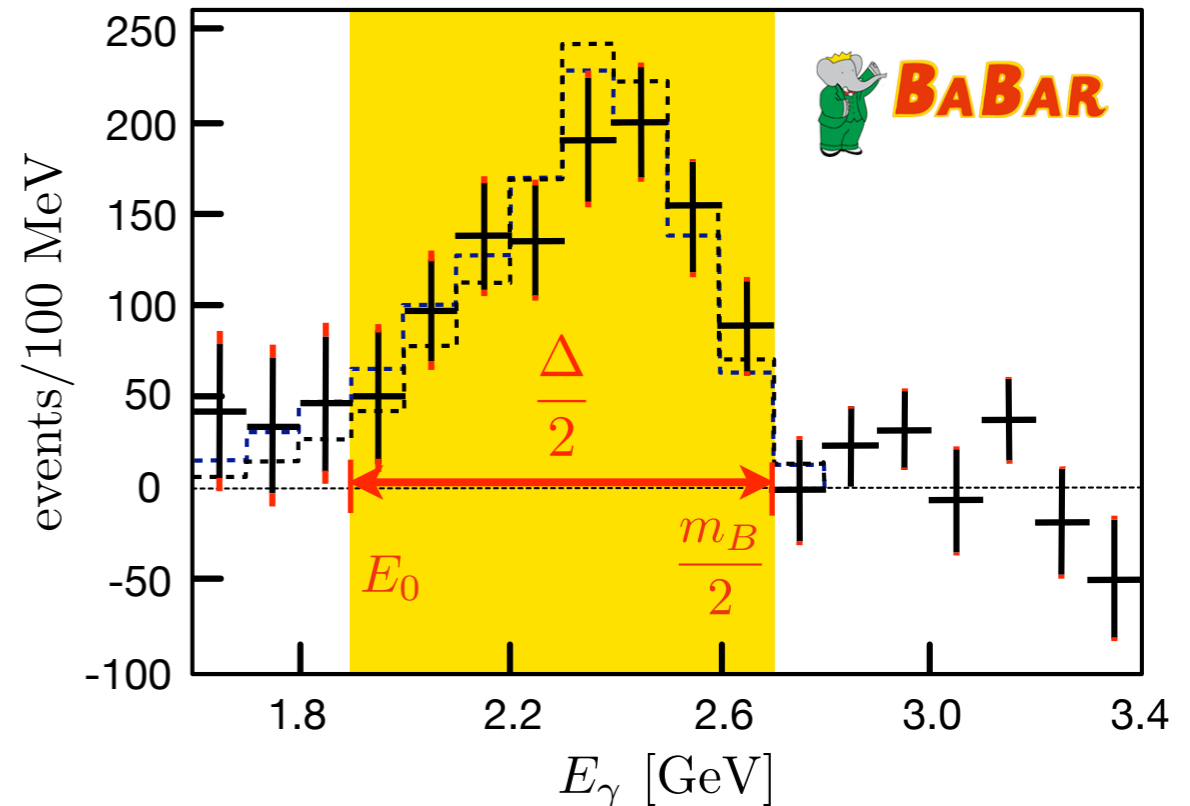
*Gardi & Andersen '06

†Misiak et al. '06

‡Becher & Neubert '06

Photon energy cut effects in $\bar{B} \rightarrow X_s \gamma$

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- to understand better if & how to precisely calculate tail of spectrum crucial

*Gardi & Andersen '06

†Misiak et al. '06

‡Becher & Neubert '06

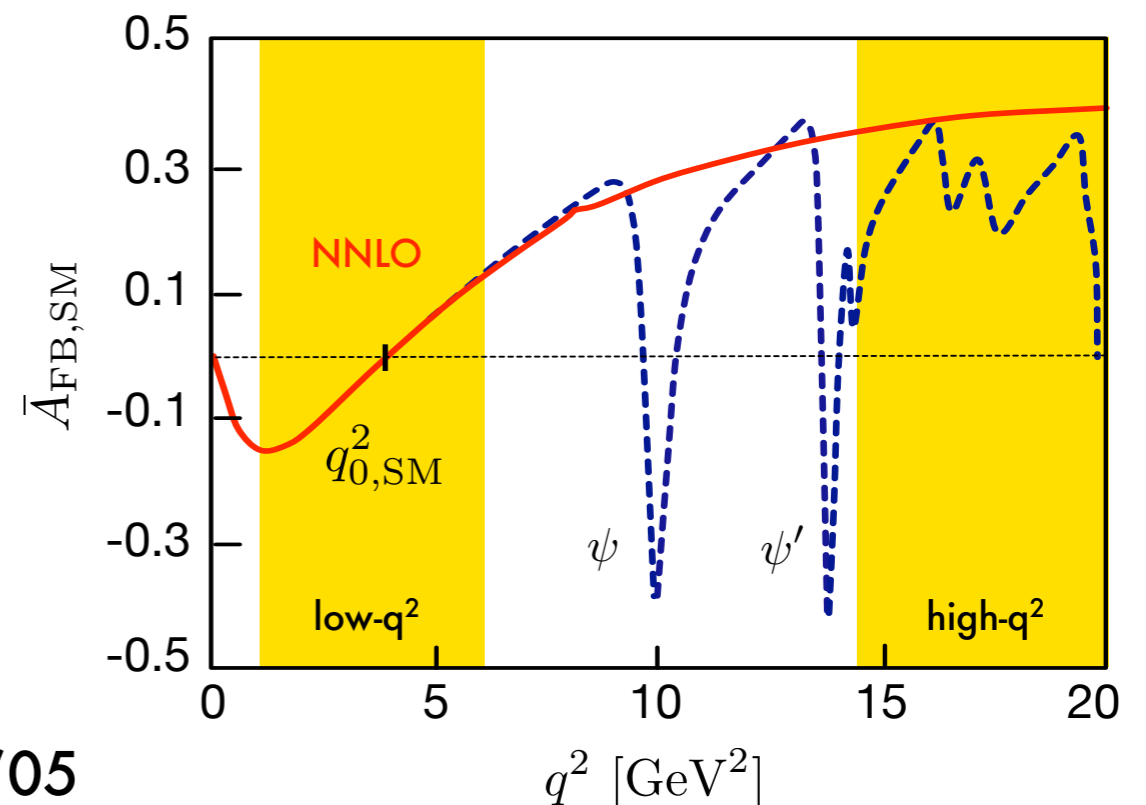
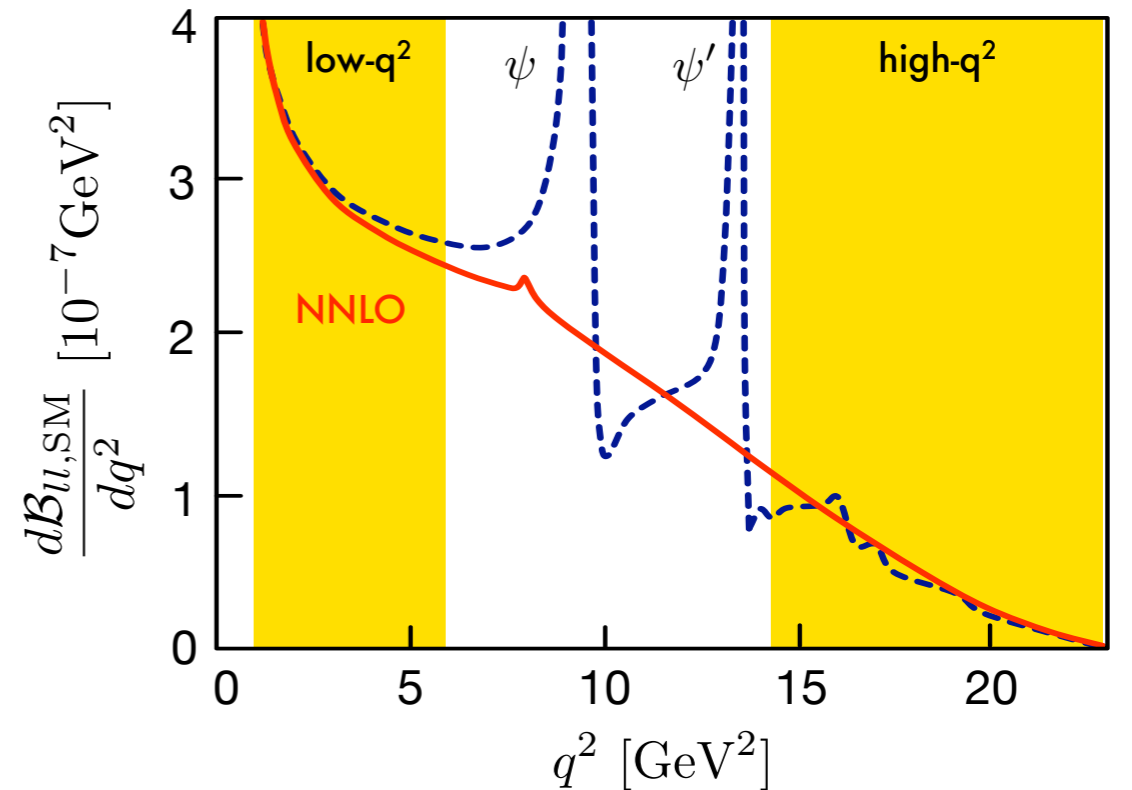
$\bar{B} \rightarrow X_s l^+ l^-$ in SM: solved problems*

- differential rate in low- q^2 region allows high precision test of SM & constraints new physics:

$$\mathcal{B}_{ll,SM}^{\text{low-}q^2} = (1.60 \pm 0.16) \times 10^{-6}$$

- zero of FB asymmetry very interesting to determine sign & magnitude of $C_7/C_9 \propto -q_0^2$:

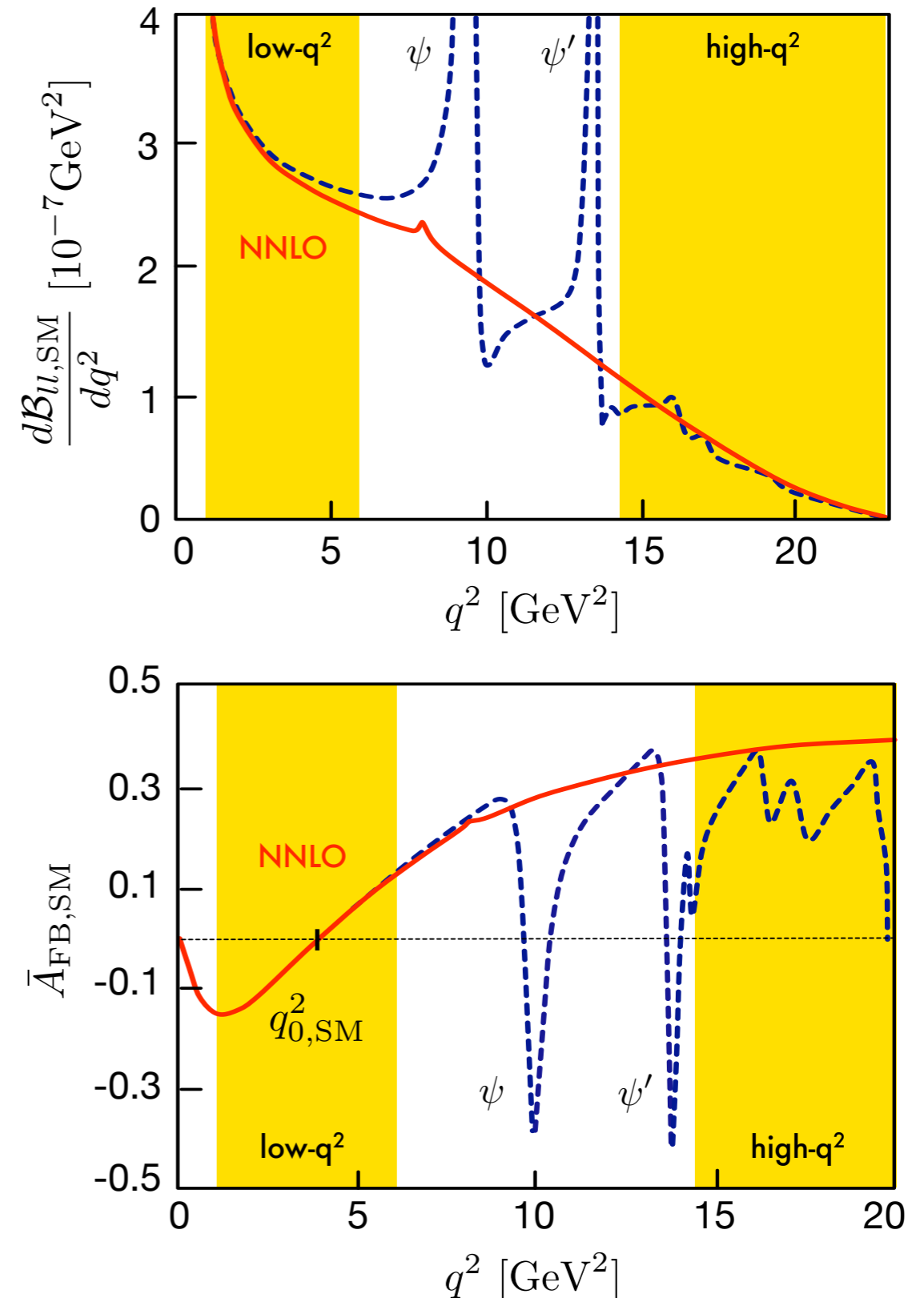
$$q_{0,SM}^2 = (3.76 \pm 0.33) \text{ GeV}^2$$



* Ghinculov et al. '03; Bobeth et al. '03; Huber et al. '05

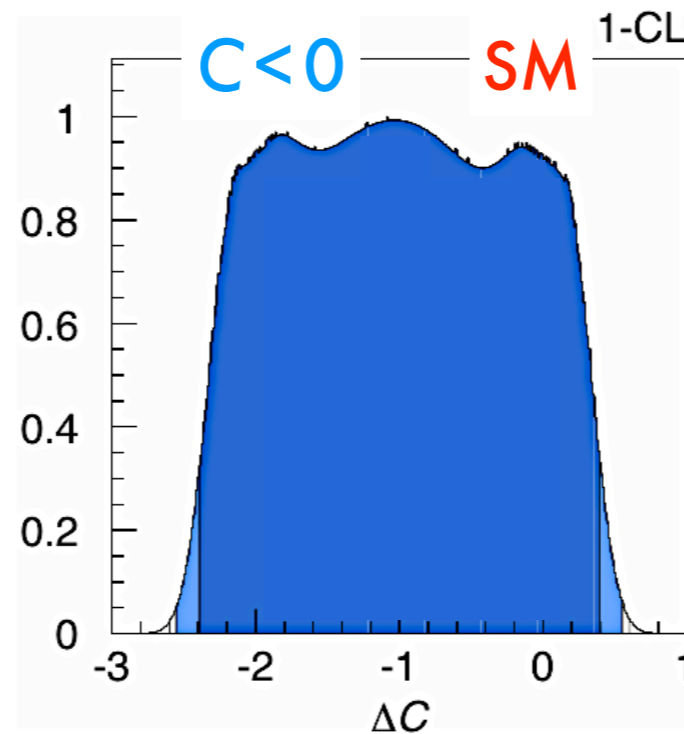
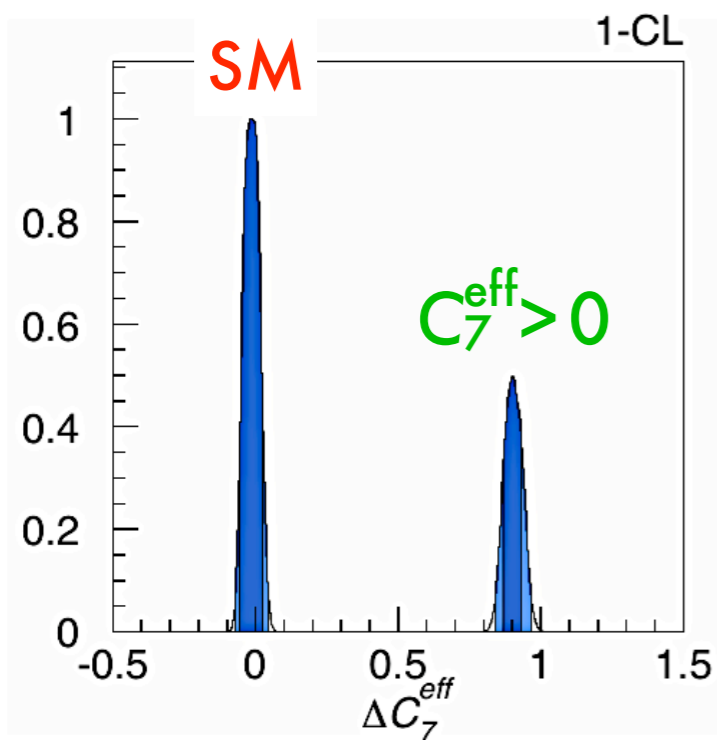
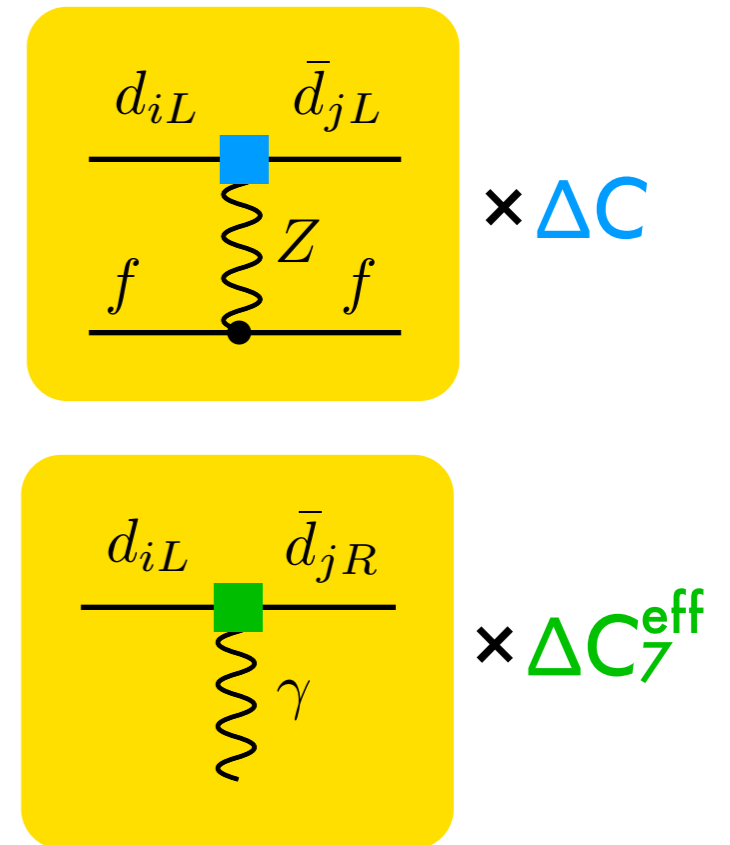
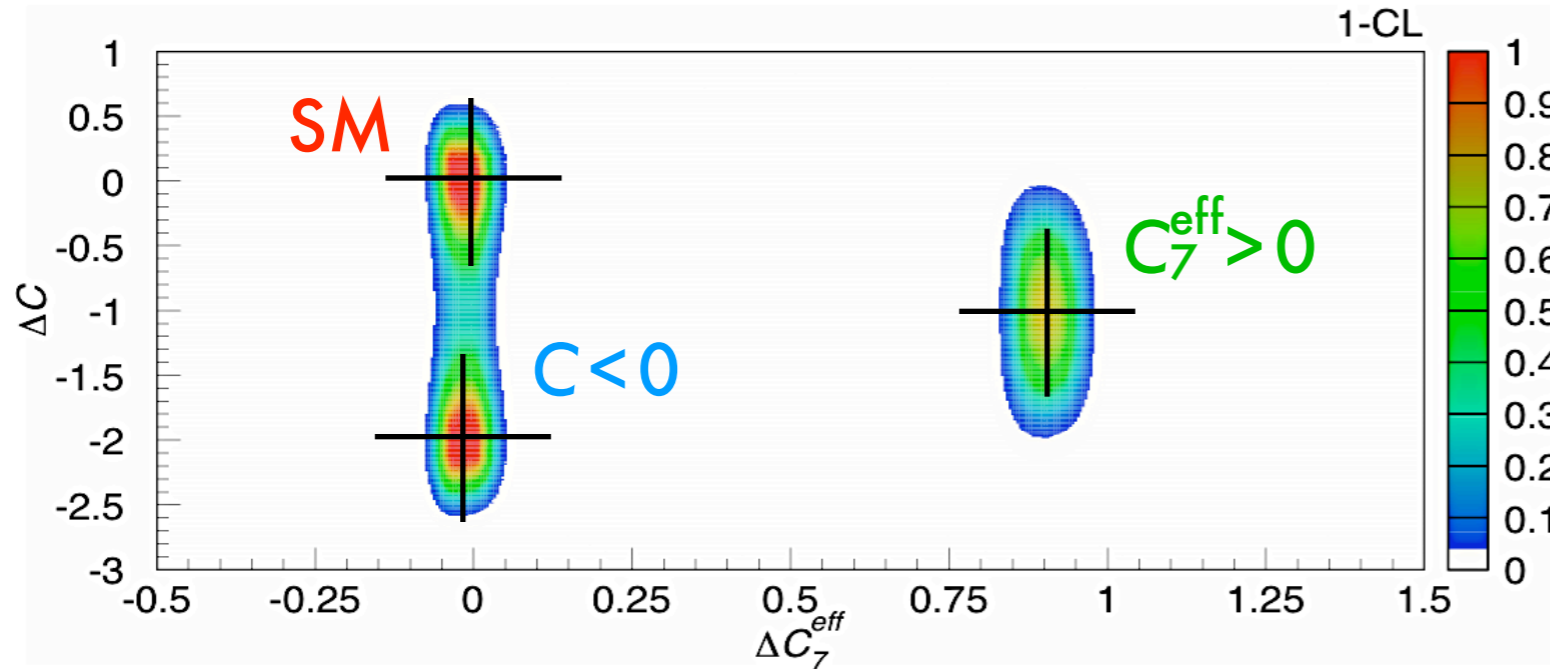
$\bar{B} \rightarrow X_s l^+ l^-$ in SM: open issues

- model-independent study of M_X -cut dependence of low- q^2 spectrum only known at NLO*
- consistent to cut out ψ, ψ', \dots & compare data with short-distance calculation?
- like in $\bar{B} \rightarrow X_s \gamma$ difficult to quantify size of effects of non-local power corrections $\alpha_s \Lambda/m_b$



*Lee et al. '06

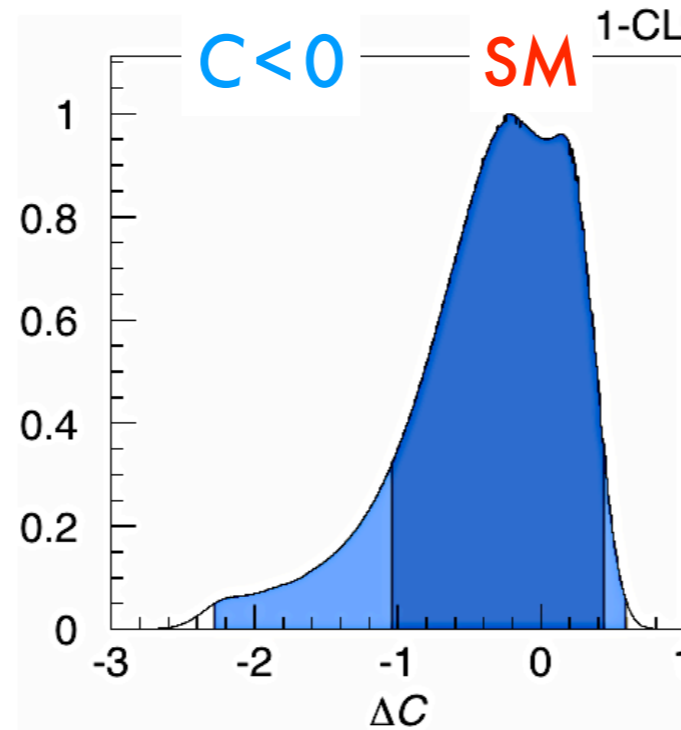
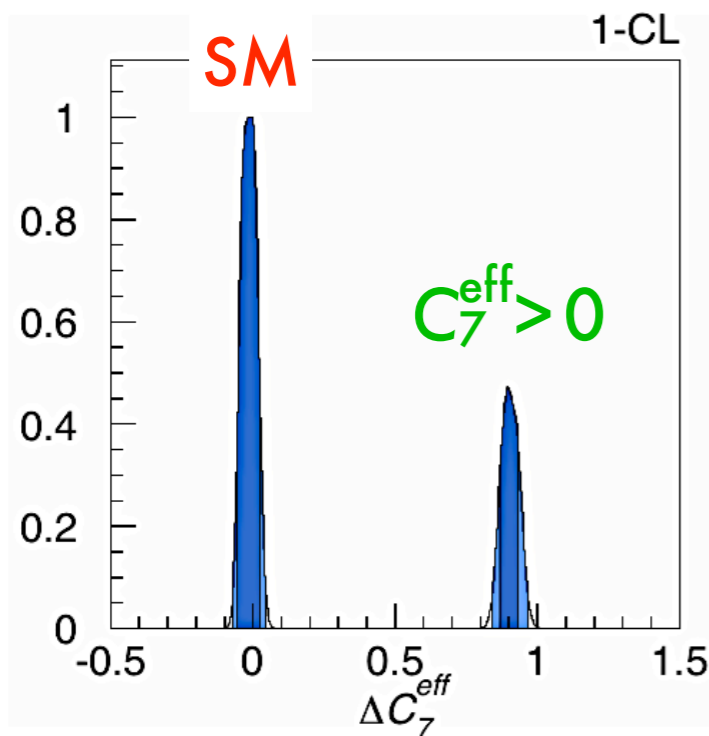
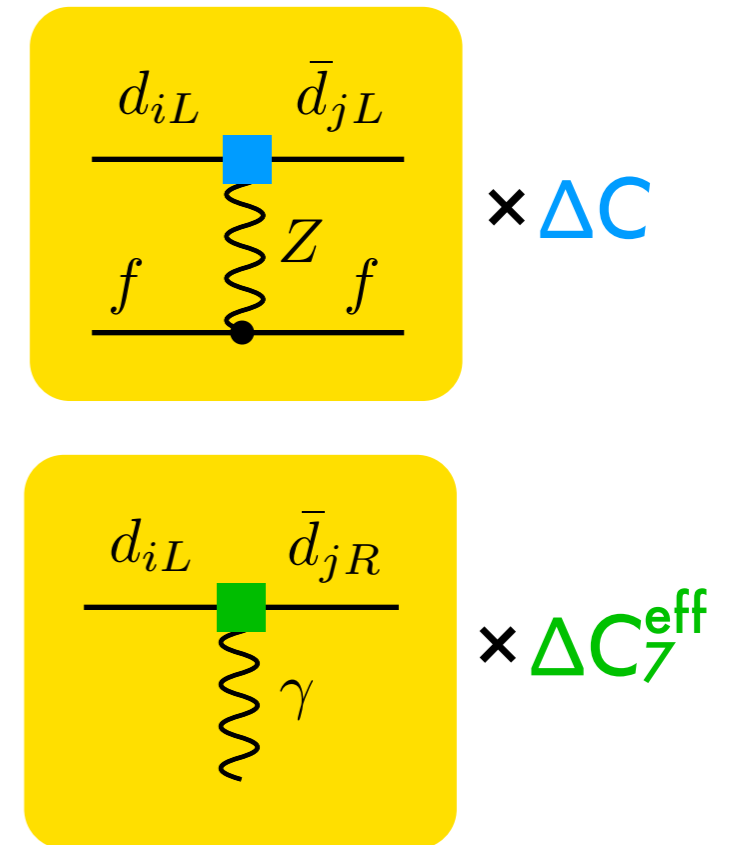
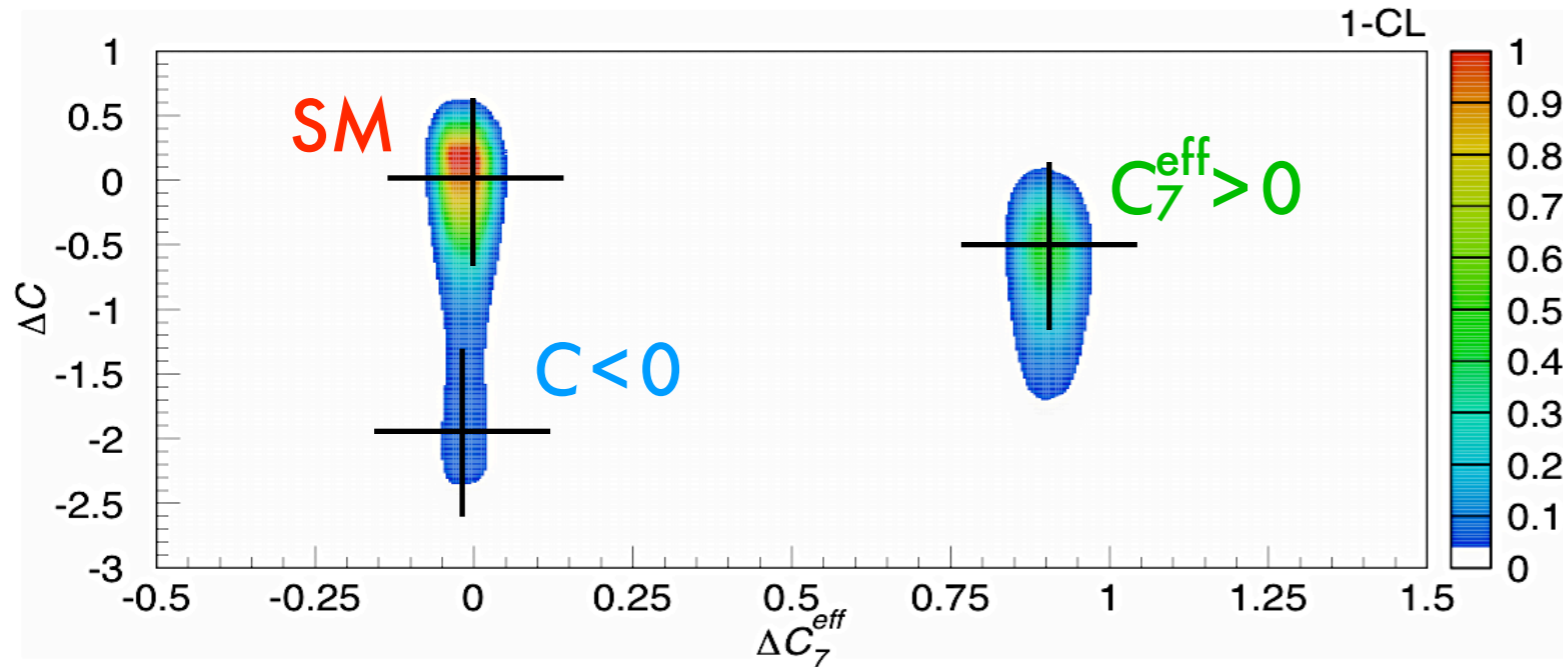
CMFV: combining $\bar{B} \rightarrow X_s \gamma$ & $\bar{B} \rightarrow X_s l^+ l^-$



- opposite sign of C_7^{eff}
disfavored by $\bar{B} \rightarrow X_s l^+ l^-$
measurements*

* Gambino et al. '04

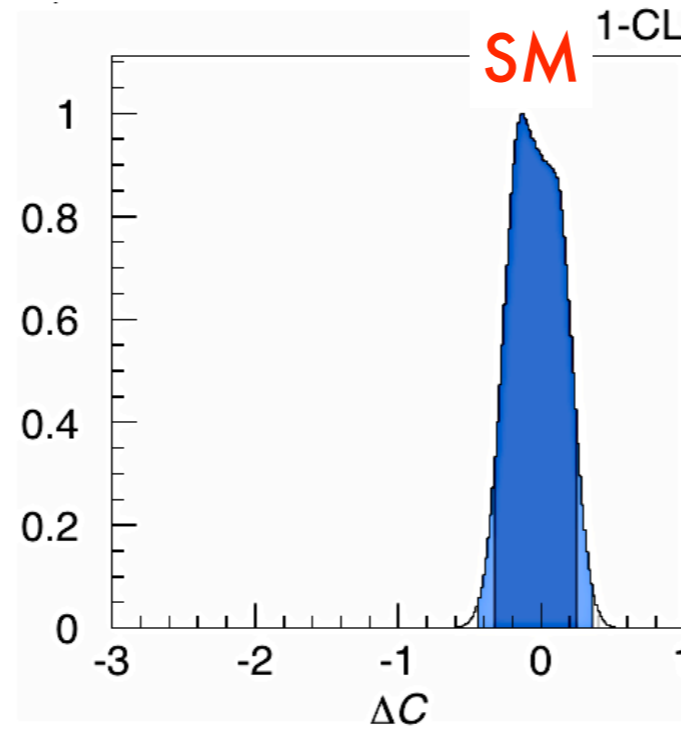
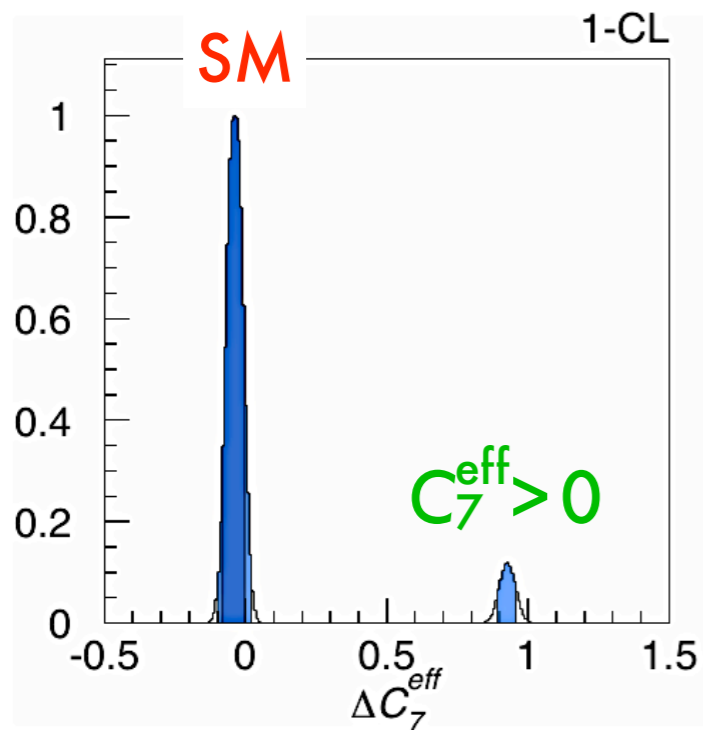
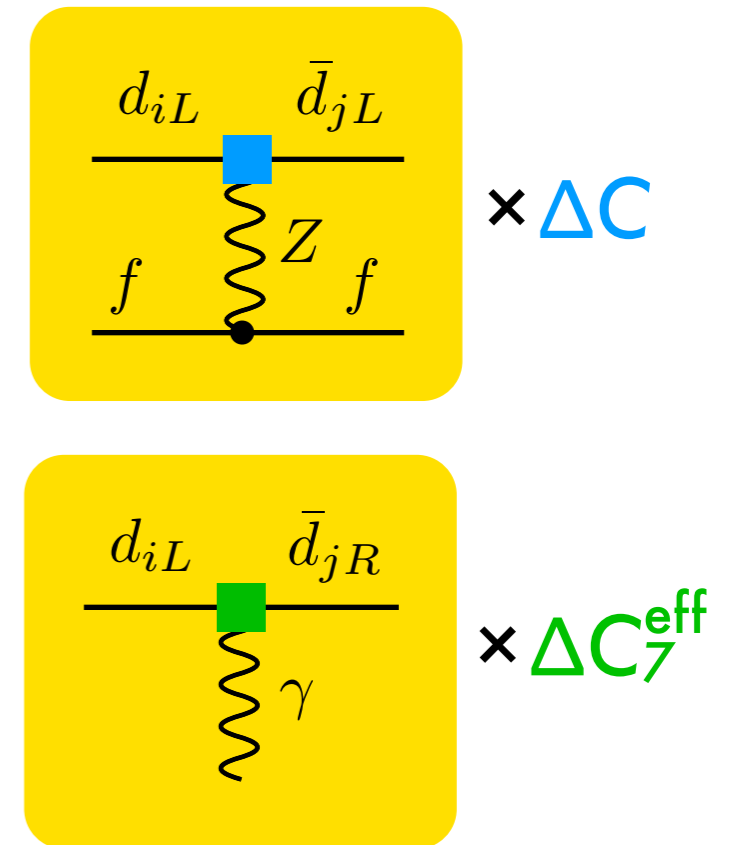
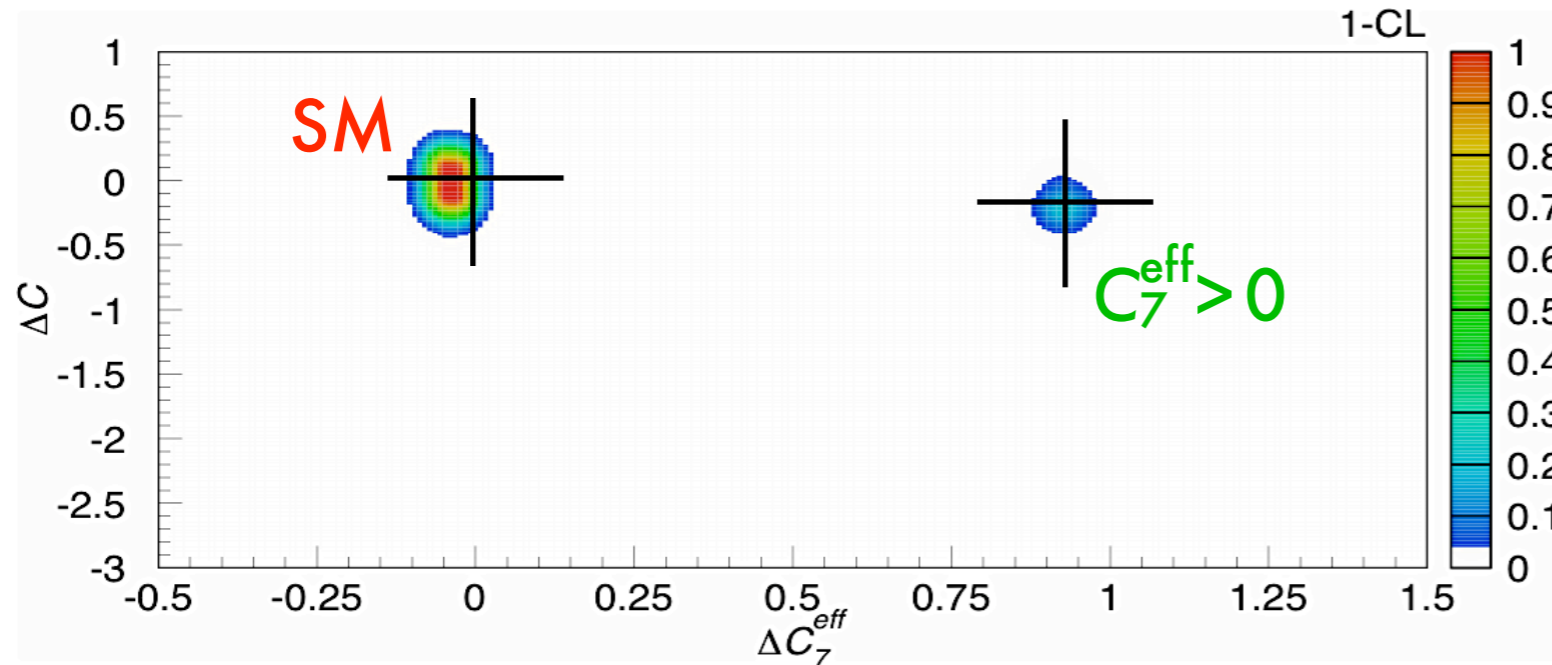
CMFV: combining $\bar{B} \rightarrow X_s \gamma, l^+ l^-$ & $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



- large destructive
- Z-penguin allowed by flavor constraints*

*Bobeth et al. '05

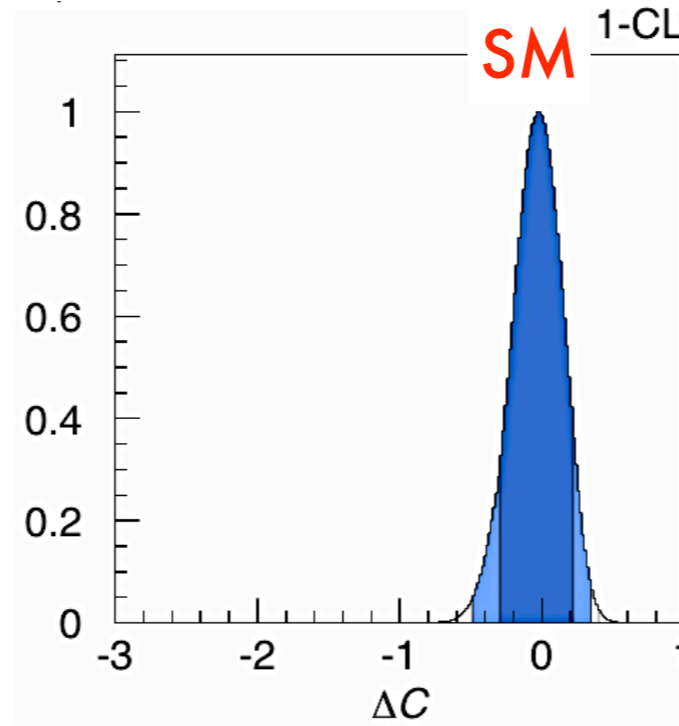
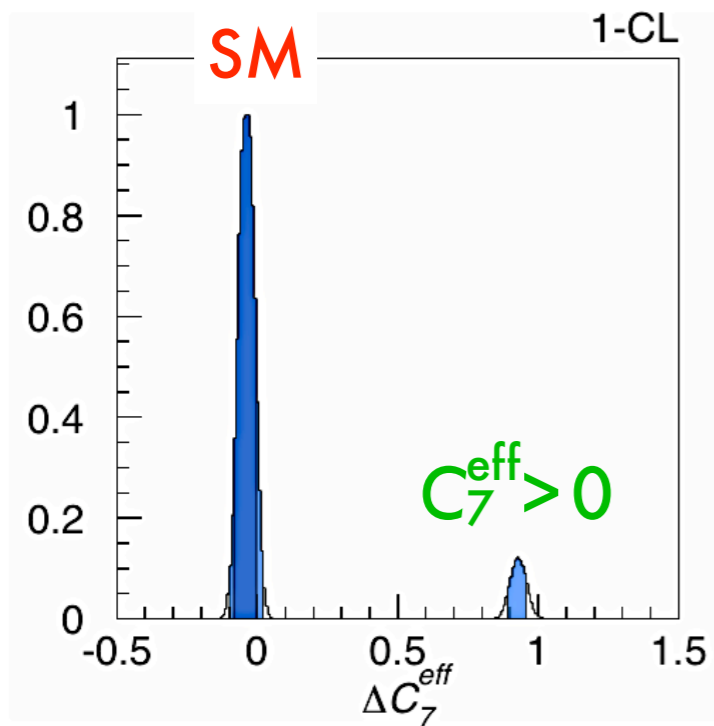
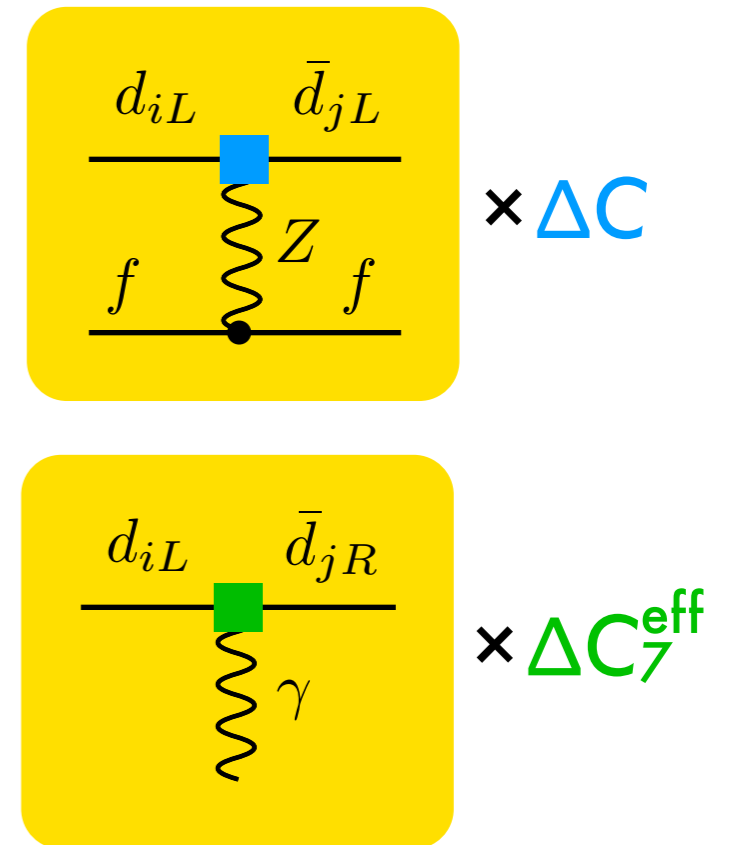
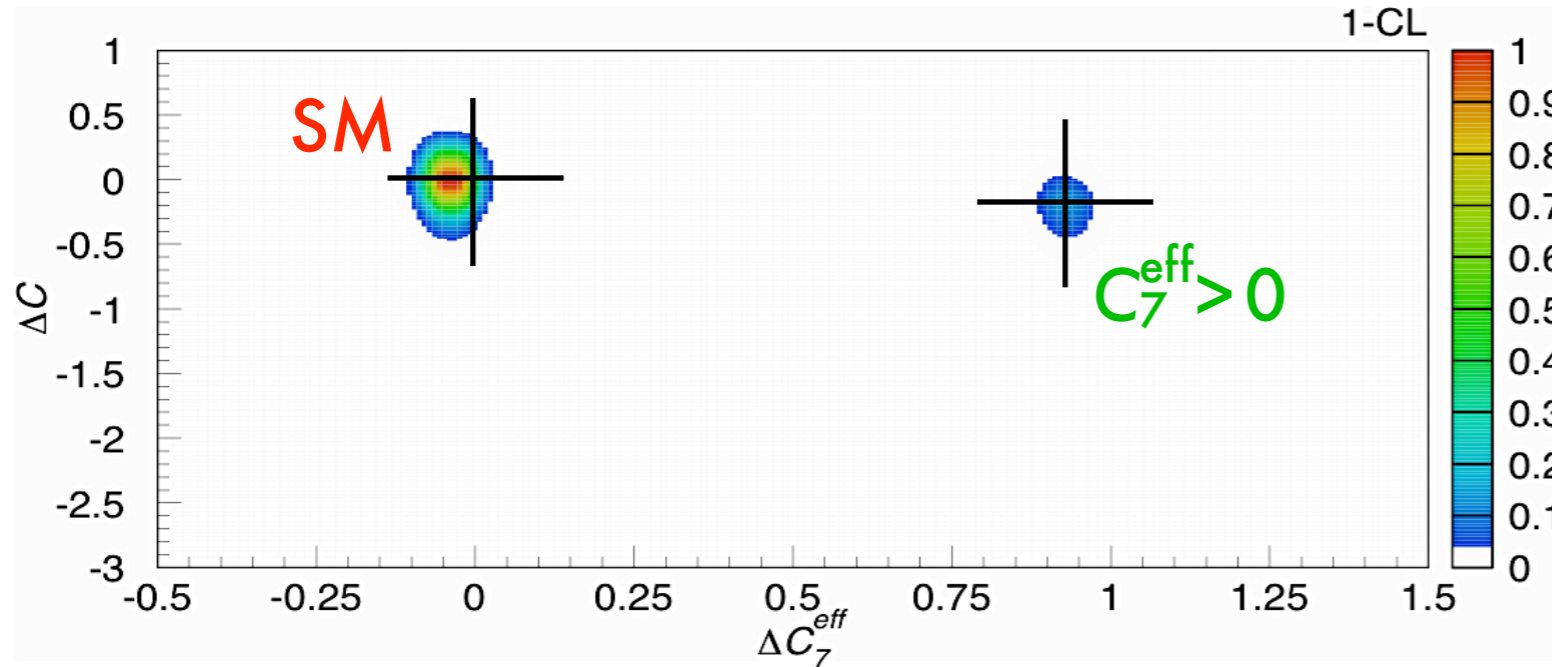
CMFV: combining $\bar{B} \rightarrow X_s \gamma, l^+ l^-$ & $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



- 2015 (?): measurement of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ close to SM excludes $\Delta C \approx -2^*$

*Bobeth et al. '05

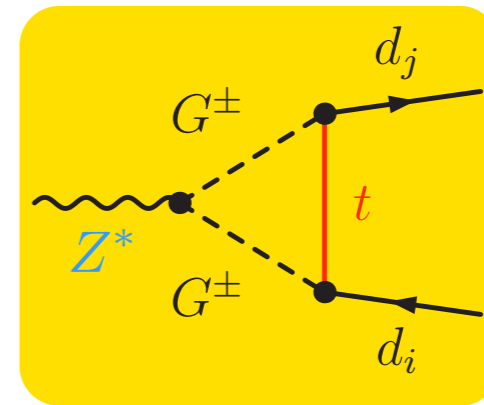
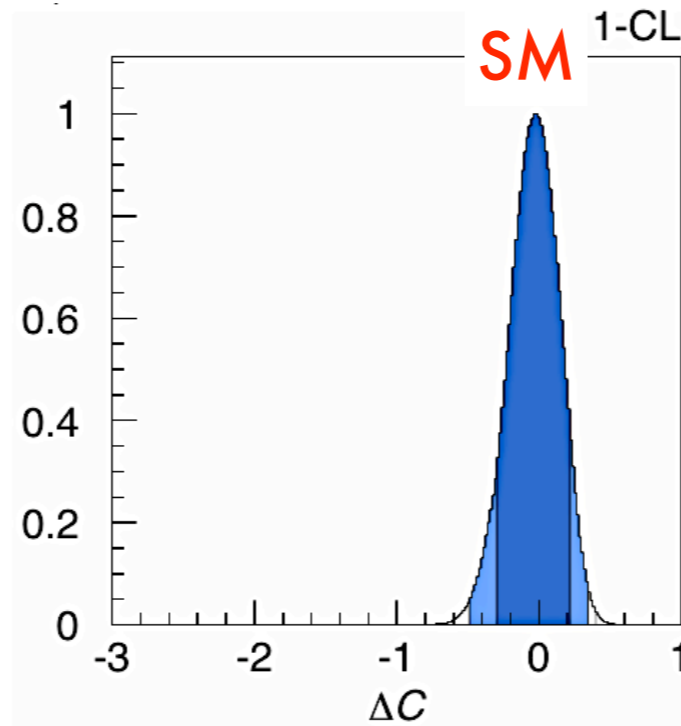
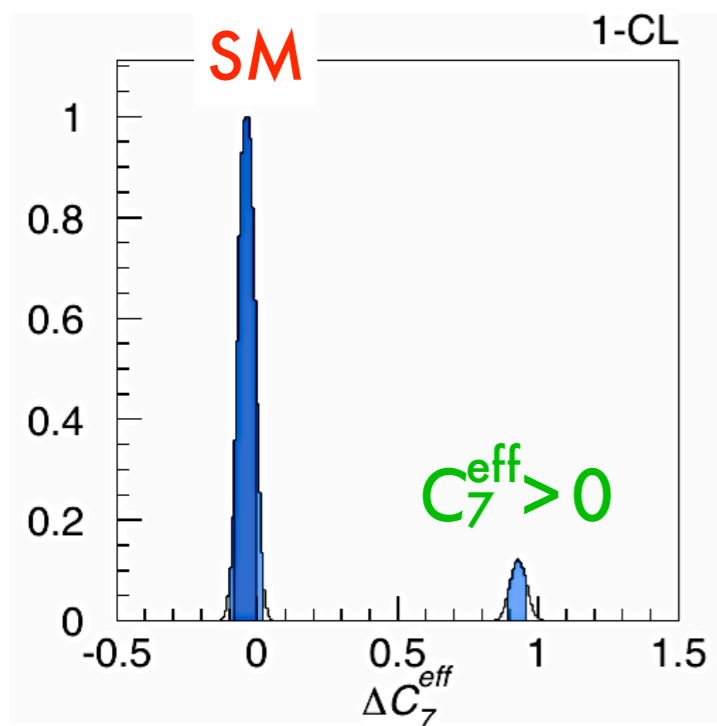
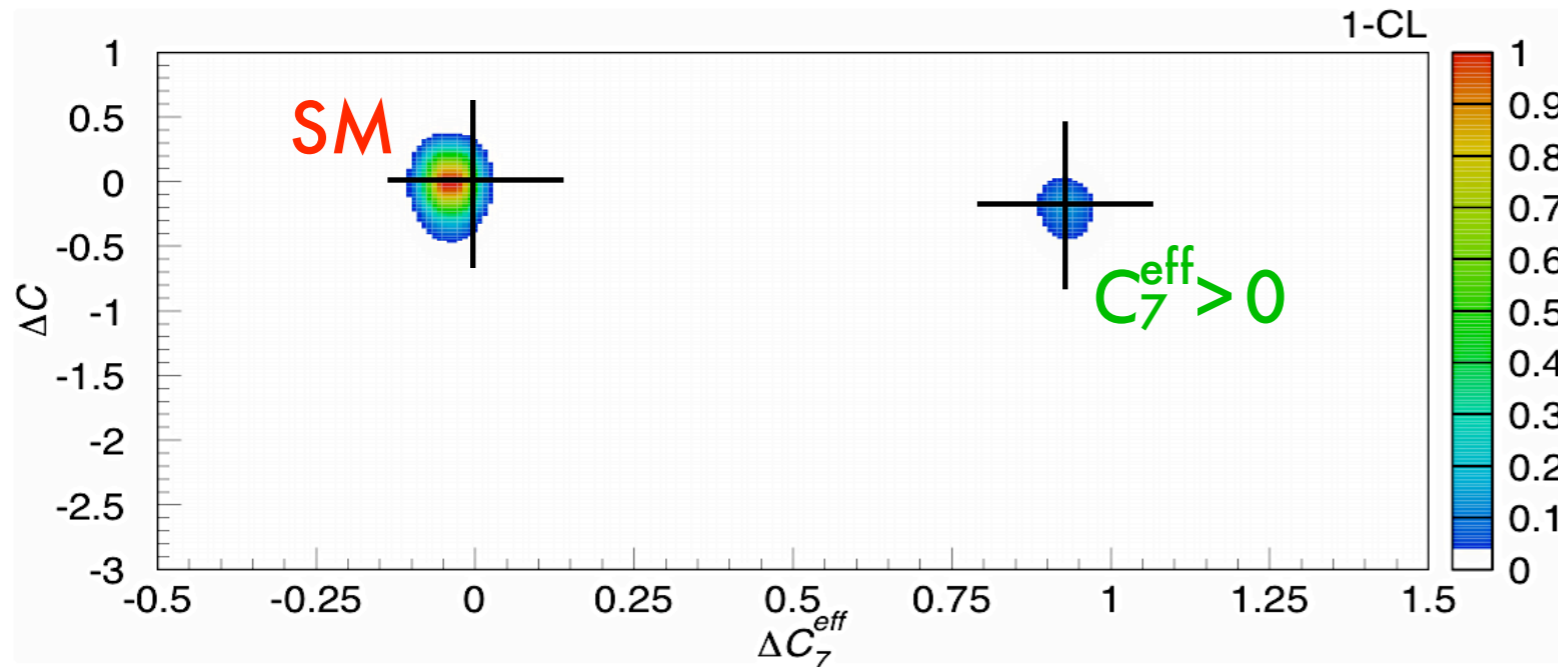
CMFV: combining $\bar{B} \rightarrow X_s \gamma, l^+ l^-$ & $Z \rightarrow b \bar{b}$



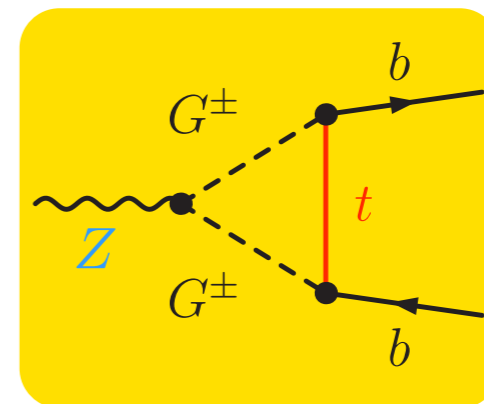
- existing $Z \rightarrow b \bar{b}$ data rule out large CMFV Z-penguin*

*UH & Weiler '07 (?)

CMFV: combining $\bar{B} \rightarrow X_s \gamma, l^+ l^-$ & $Z \rightarrow b \bar{b}$



$$\propto V_{tj}^* V_{ti} \frac{m_t^2}{M_W^2}$$

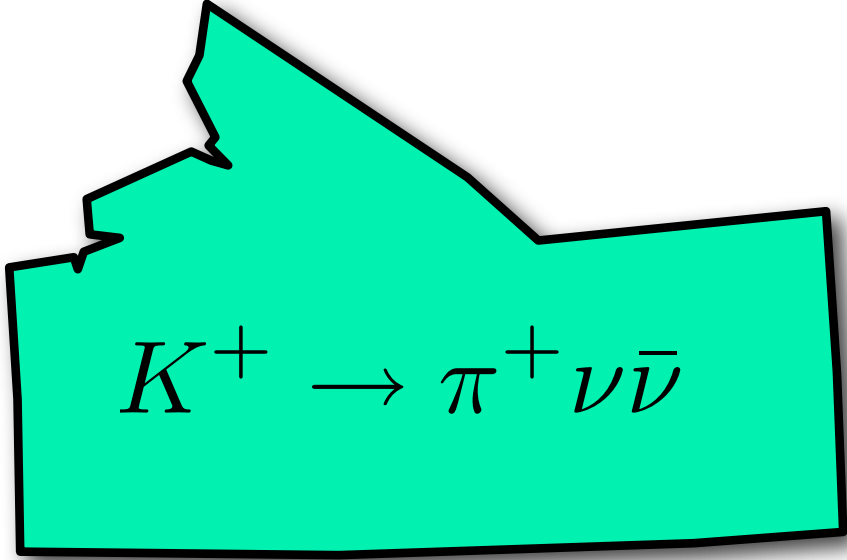



$$\propto |V_{tb}|^2 \frac{m_t^2}{M_W^2}$$

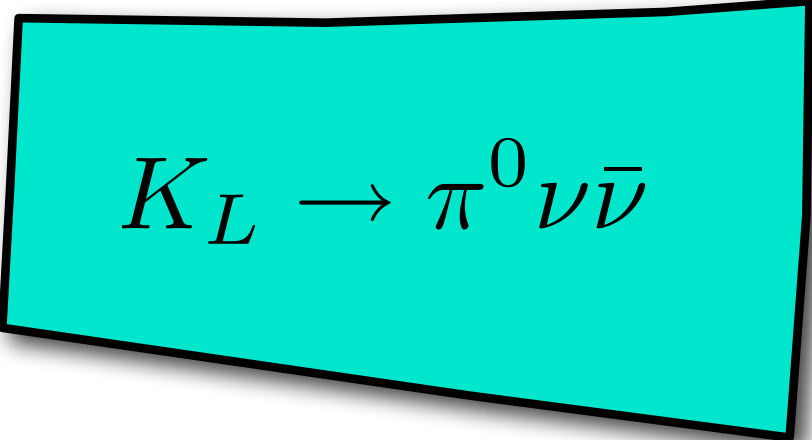
- based on observation that in CMFV $Z \rightarrow b \bar{b}$, $d_i \bar{d}_j$ are "identical"*

*UH & Weiler '07 (?)

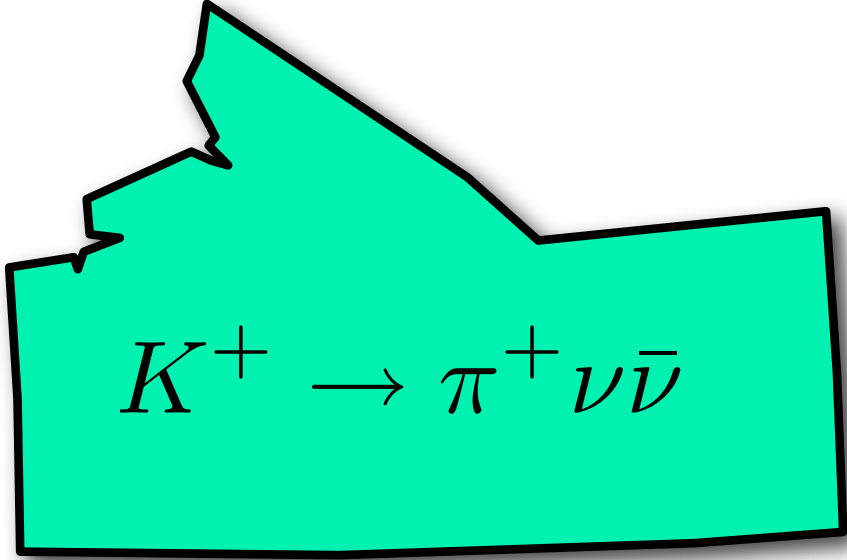
After this "upset" of rare K -decays ...



$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

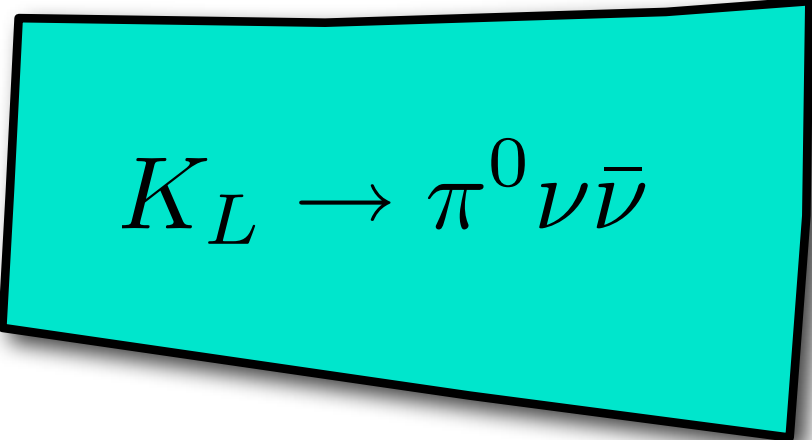

$$K_L \rightarrow \pi^0 l^+ l^-$$


$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

Cecilia & Chris will tell you now why ...


$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$


$$K_L \rightarrow \pi^0 l^+ l^-$$


$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

concerning physics beyond MFV ...

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \pi^0 l^+ l^-$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

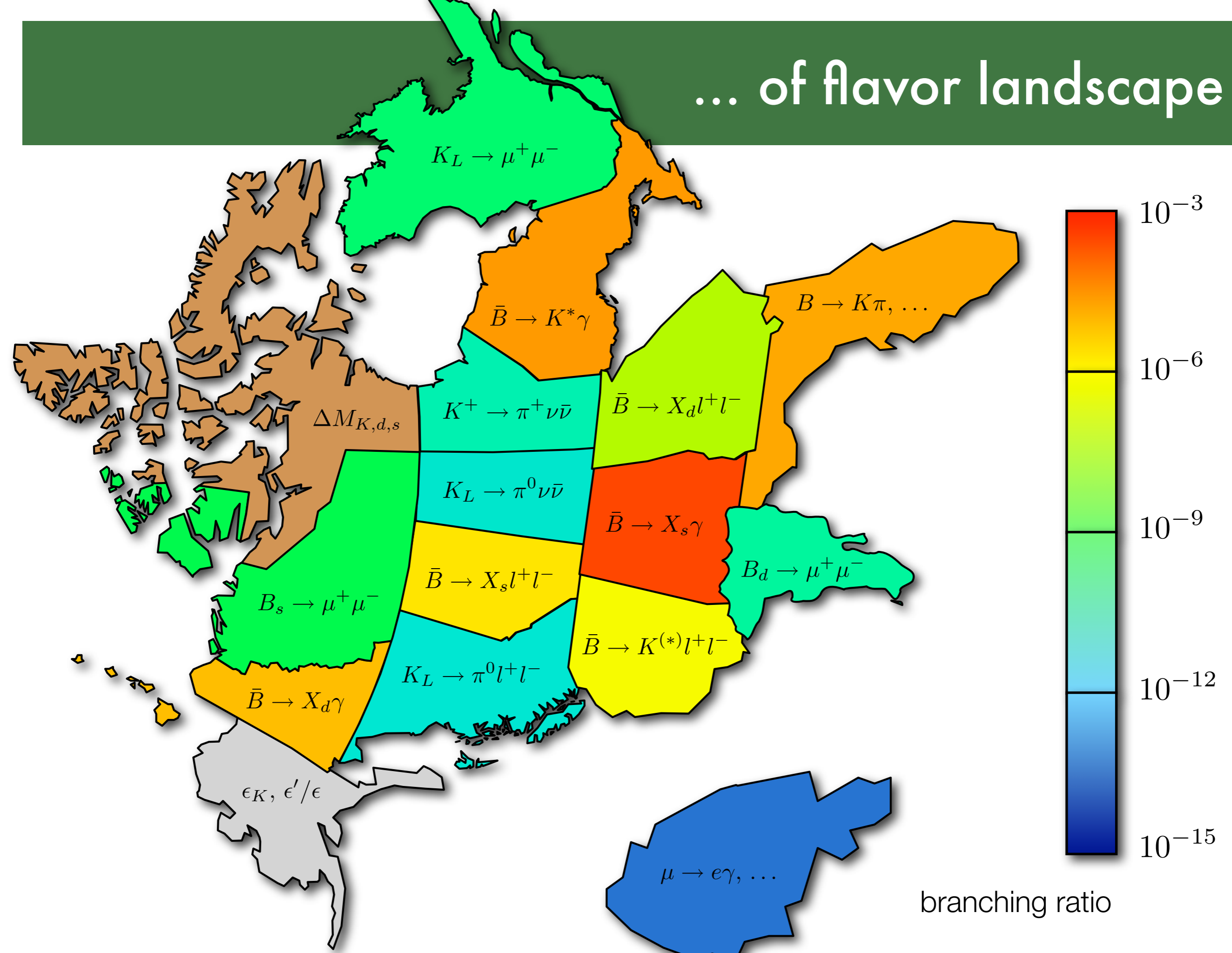
these modes are superpowers ...

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \pi^0 l^+ l^-$$

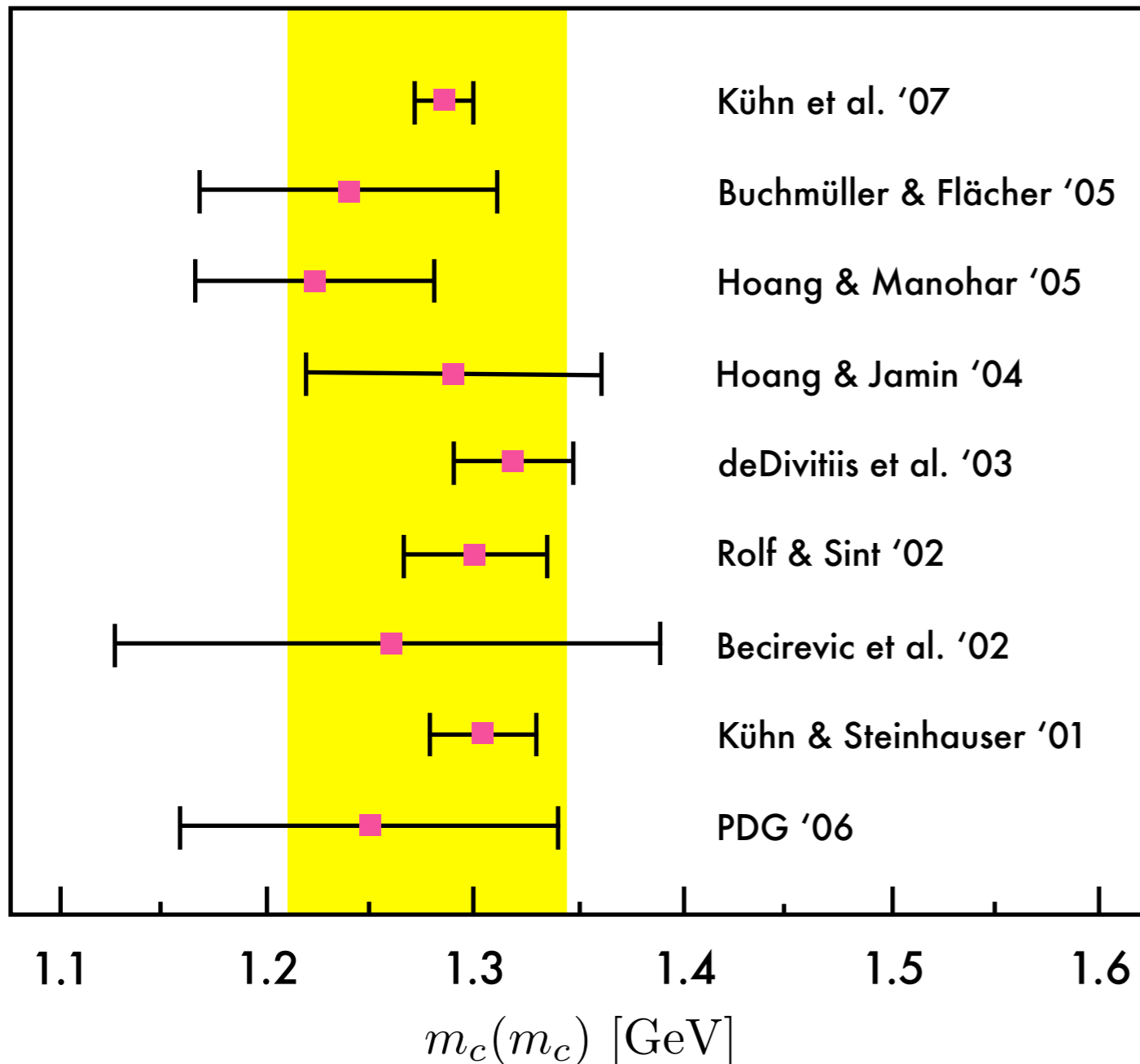
$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

... of flavor landscape



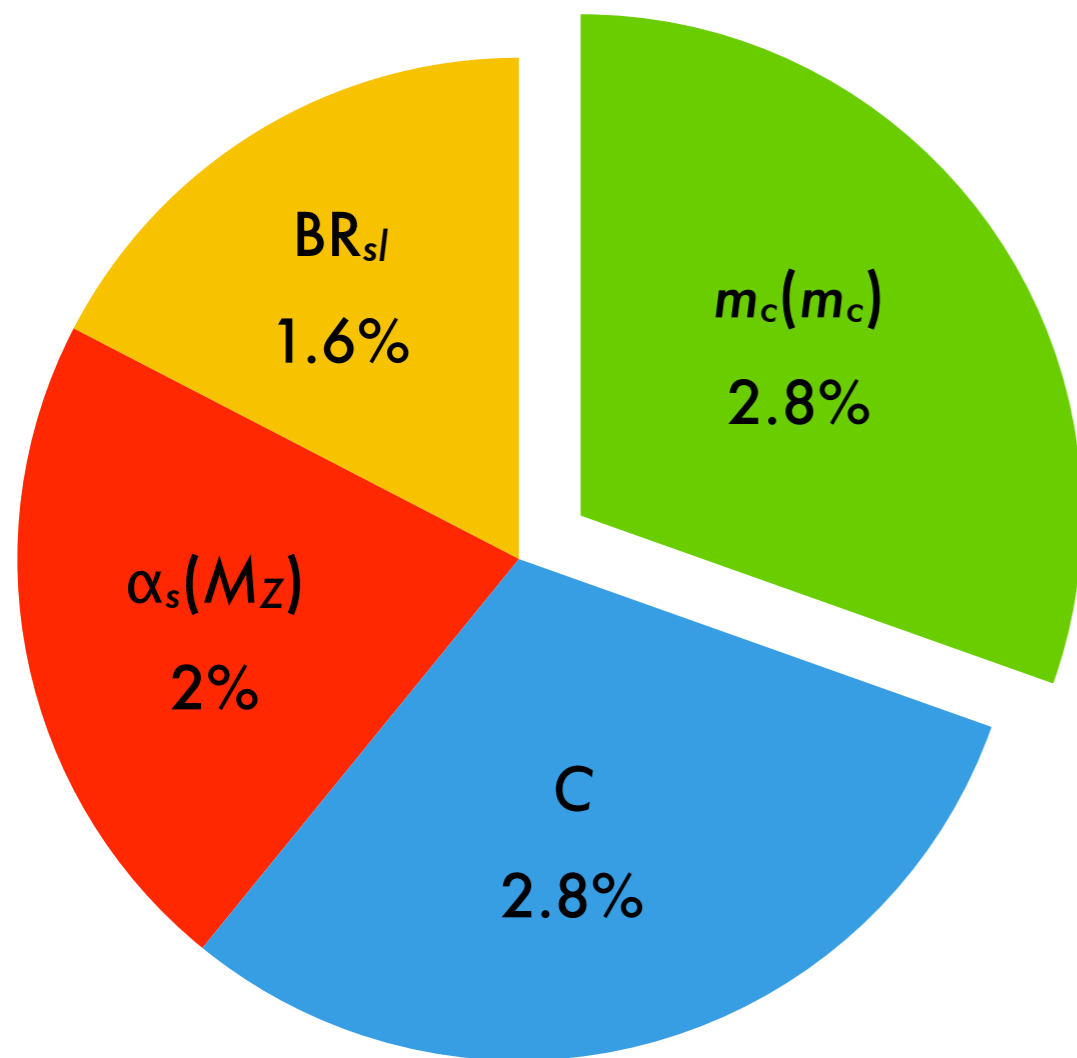
Recent determinations of charm mass

[1.21, 1.34]



$m_c(m_c)$ [GeV]	method
1.286 ± 0.013	low-momentum sum rules, N ³ LO
1.24 ± 0.07	fit to <i>B</i> -decay distribution, $\alpha_s^2\beta_0$
$1.224 \pm 0.017 \pm 0.054$	fit to <i>B</i> -decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
1.301 ± 0.034	lattice, quenched
$1.26 \pm 0.04 \pm 0.12$	lattice, quenched
1.304 ± 0.027	low-momentum sum rules, NNLO
1.25 ± 0.09	PDG 2006

Parametric errors in $\bar{B} \rightarrow X_s \gamma$ at NNLO*



$$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054) \text{ GeV}^\dagger$$

- parametric uncertainty related to charm mass already slightly smaller than estimated left over scheme ambiguity

* Misiak & Steinhauser '06

† Hoang & Manohar '05

Future (?) CKM fit from $K \rightarrow \pi \nu \bar{\nu}^*$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.0 \pm 0.8) \times 10^{-11}$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11}$$

Future (?)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 4.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.024$$

$$\sigma(\gamma) = \pm 4.7^\circ$$

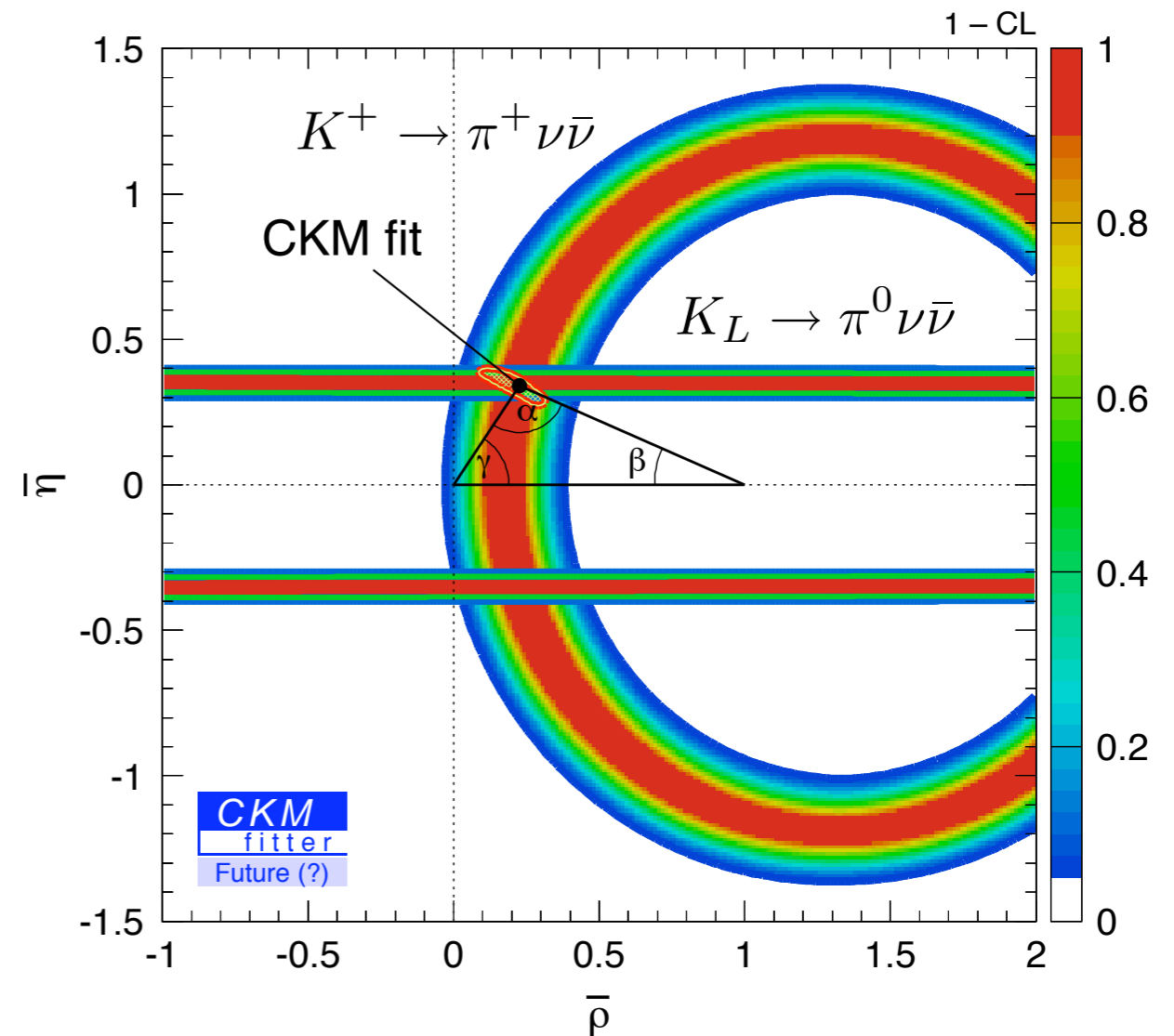
NLO
(theory error only)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

NNLO
(theory error only)



- nice CKM fit from $K \rightarrow \pi \nu \bar{\nu}$, almost comparable to present global analysis, but not ultimate goal

*Buras et al. '05, '06

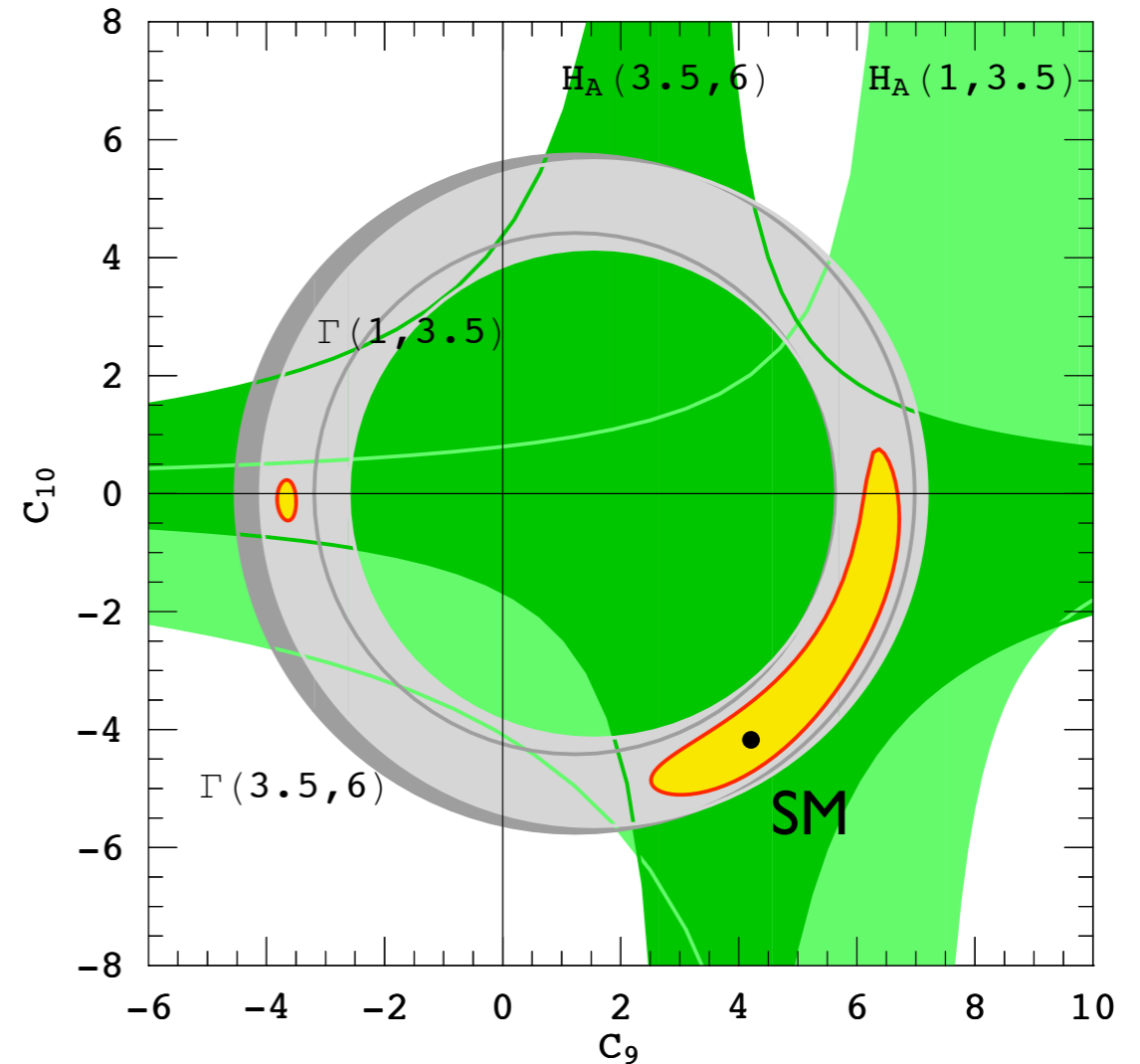
$\bar{B} \rightarrow X_s l^+ l^-$: learning effectively from 1 ab^{-1} data*

- angular decomposition:

$$\frac{d^2\Gamma}{dsdz} \sim \left\{ (1+z^2) \left[\left(C_9 + \frac{2}{s} C_7 \right)^2 + C_{10}^2 \right] + (1-z^2) \left[(C_9 + 2C_7)^2 + C_{10}^2 \right] - 4zsC_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right\}$$

$$\equiv \underbrace{H_T + H_L}_{\sim \Gamma} + \underbrace{H_A}_{\sim A_{\text{FB}}}$$

$$(s = q^2/m_b^2, z = \cos\theta, \theta : \angle b, l^+)$$



$$H_{T,L,A}(q_1^2, q_2^2) \equiv \int_{q_1^2}^{q_2^2} dq^2 H_{T,L,A}(q^2)$$

*Lee et al. '06

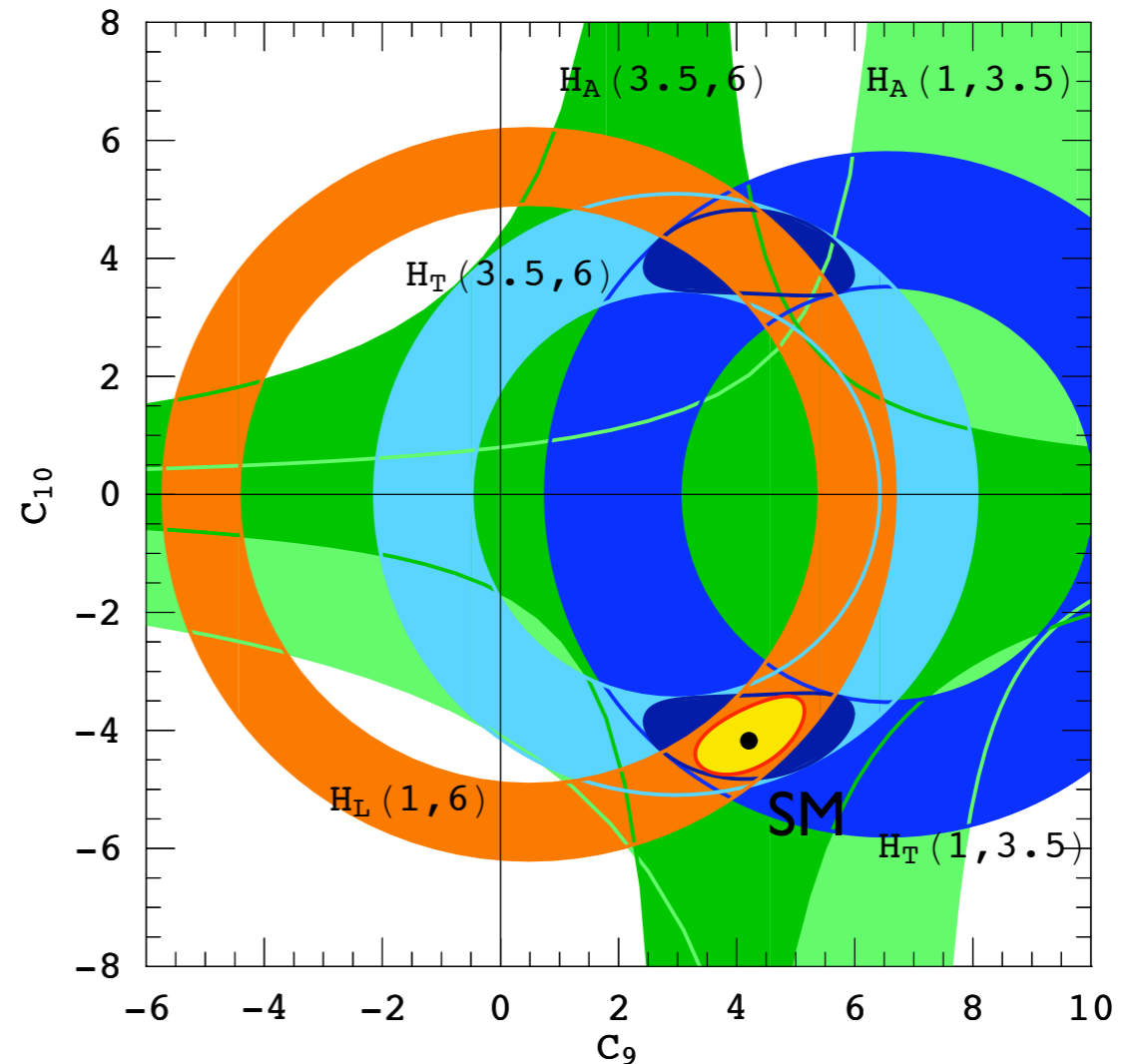
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$$\equiv \underbrace{H_T}_{\sim \Gamma} + \underbrace{H_L}_{\sim A_{\text{FB}}} + H_A$$

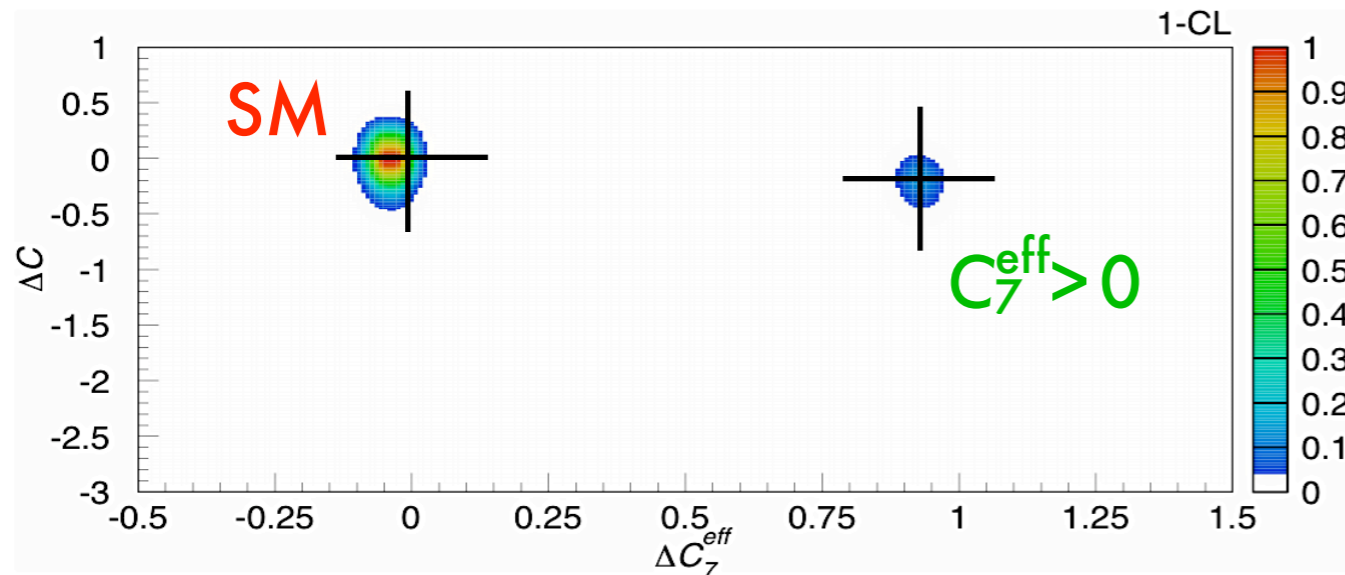
$$(s = q^2/m_b^2, z = \cos \theta, \theta : \angle b, l^+)$$



$$H_{T,L,A}(q_1^2, q_2^2) \equiv \int_{q_1^2}^{q_2^2} dq^2 H_{T,L,A}(q^2)$$

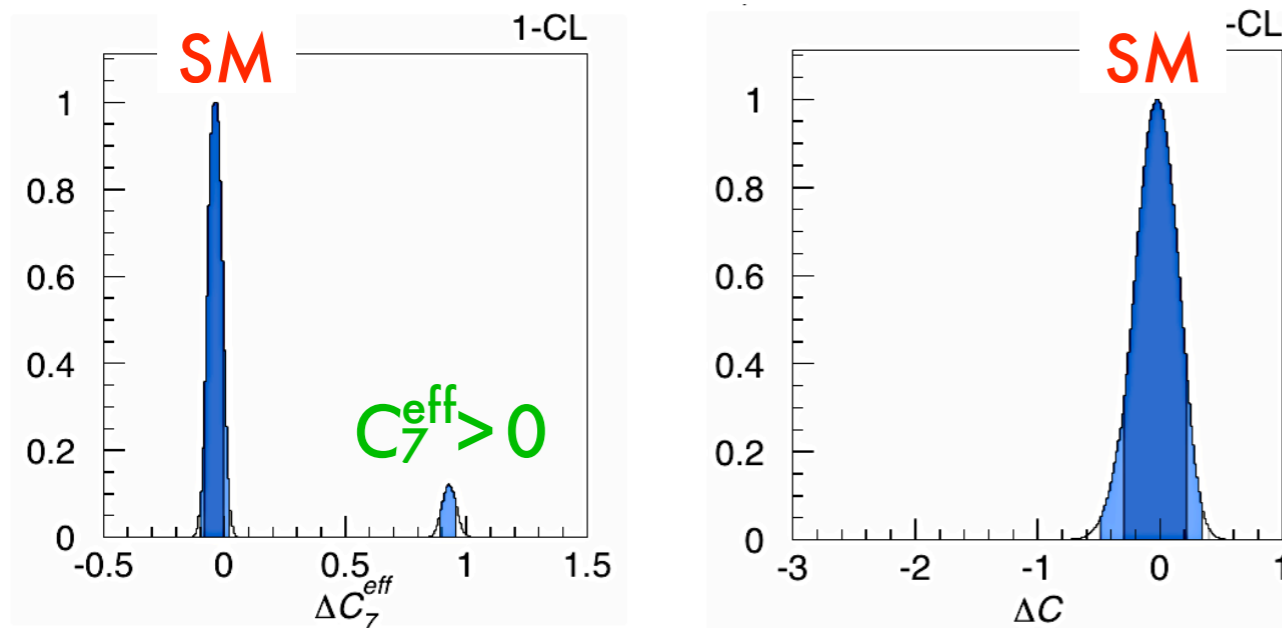
*Lee et al. '06

Bounds on ΔC & ΔC_7^{eff} from $\bar{B} \rightarrow X_s \gamma, l^+ l^-$ & $Z \rightarrow b \bar{b}$



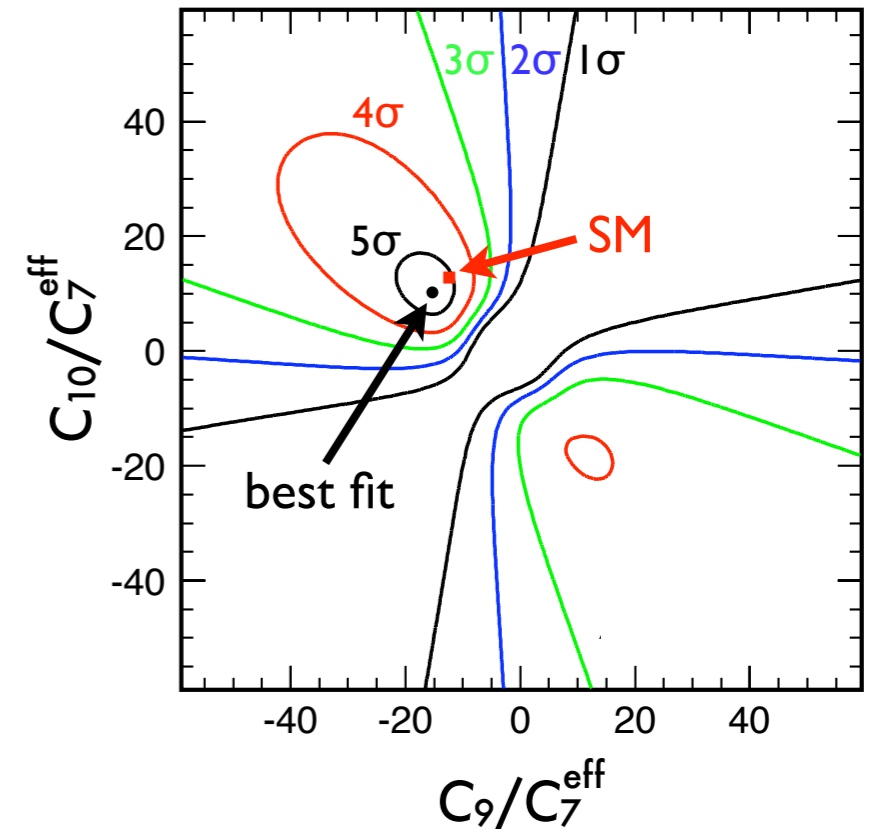
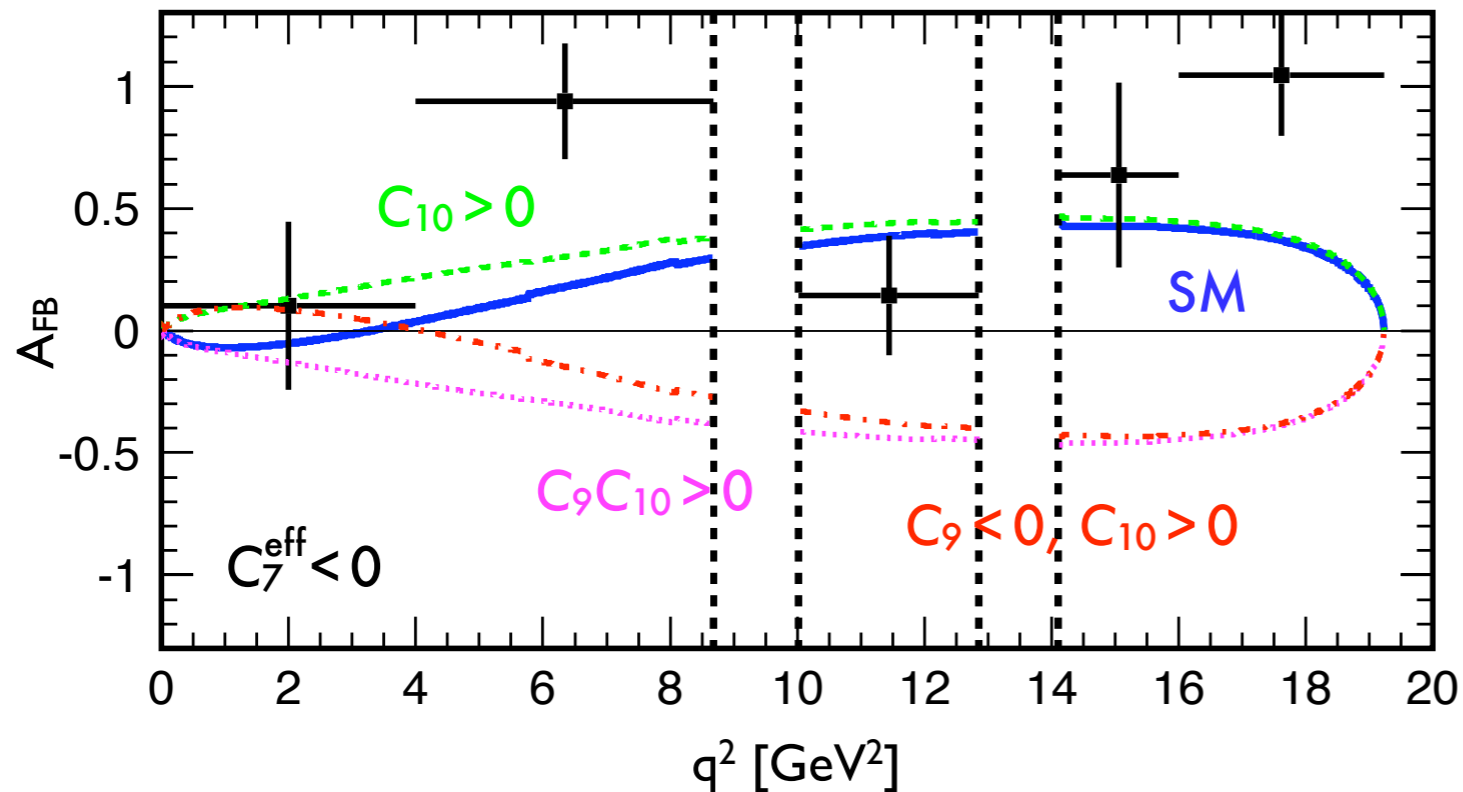
$$\Delta C = [-0.486, 0.366] \quad (95\% \text{ CL})$$

$$\Delta C_7^{\text{eff}} = [-0.104, 0.026] \cup [0.891, 0.968] \quad (95\% \text{ CL})$$



- large corrections to off-shell photon penguin still allow for “wrong” sign of ΔC_7^{eff}

Hunting Z-penguin with $\bar{B} \rightarrow K^* l^+ l^-$



- forward-backward asymmetry in $\bar{B} \rightarrow K^* l^+ l^-$ excludes $C_9 C_{10} > 0$ at 95% CL*

- hints towards exclusion of large destructive Z-penguin: $|\Delta C| \lesssim 1.5$

* Belle '05; BaBar '06