Theoretical progress on $|V_{us}|$ on lattice

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Kaon07, May 21, 2007

1. introduction

determinations of $|V_{us}|$

decays	$|V_{us}|$ *(PDG2006,CKM2006)*	lattice can provide...
K_{l3} decays	0.2257(21)	$f_+(0)$
hyperon decays	0.2250(27)	form factors
$K_{\mu 2}$, $\pi_{\mu 2}$ decays	0.2245(+12/-31)	f_K/f_π
hadronic τ decays	0.2002(46)	m_s

this talk: an overview of current status of lattice studies

- recent unquenched calculations of kaon matrix elements
- range of $m_{ud,\text{sea}}$, consistency among {lattice data, experiment, ChPT}

1. introduction

outline

- f_K / f_π
  - results from Kogut-Susskind (KS) quarks
  - studies w/ Wilson-type/chiral fermions
- K_{l3} form factor
  - recent unquenched studies
- hyperon / τ decays
  - hyperon decay form factors
  - strange quark mass
- summary

2. f_K/f_π

2.1 $|V_{us}|$ from $K_{\mu 2}$ and $\pi_{\mu 2}$ decays

$|V_{us}|$ from $K_{\mu 2}$ and $\pi_{\mu 2}$ decays *(Marciano, 2004)*

$$\frac{\Gamma(K \to \mu \bar{\nu}_\mu)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{M_K \left(1 - m_\mu^2/M_K^2\right)^2}{M_\pi \left(1 - m_\mu^2/M_\pi^2\right)^2} \left\{1 + \frac{\alpha}{\pi}\left(C_K - C_\pi\right)\right\}$$

- $\Gamma_{K_{\mu 2}}/\Gamma_{\pi_{\mu 2}}$: experiments $\Rightarrow \Delta|V_{us}| \lesssim 0.2$ %
- $|V_{ud}|$: super-allowed nuclear β decays $\Rightarrow \Delta|V_{us}| < 0.1$ %
- $C_K - C_\pi$: radiative corrections $\Rightarrow \Delta|V_{us}| \lesssim 0.2$ %
- f_K/f_π from lattice
  - from simplest matrix element
  - doesn't need non-perturbarive (NP) Z_A, a, ...
  - need to simulate small m_{ud} $\to$ reliable chiral extrap.
  - w/ f_K/f_π from KS fermions *(MILC,2004)*

    $f_K/f_\pi = 1.201(8)(15) \;\to\; |V_{us}| = 0.2236(30)$ *(Marciano, 2004)*

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2.2 updates on KS results: simulations

MILC's simulations

- improved gauge + improved KS: computationally cheap
- $N_f = 2+1$
- $m_{ud,\text{sim}} \gtrsim m_{s,\text{phys}}/10$
- $L \simeq 2.4-3.4$ fm
- $a^{-1} \simeq 1.6, 2.2$ GeV (2004)
- latest update *(Lattice 2006)*
  simulations extended to
  - $(m_{ud}, a^{-1}) = (0.1\, m_s, 2.2\,\text{GeV})$
  - $(m_{ud}, a^{-1}) = (0.4\, m_s, 3.3\,\text{GeV})$
  - at $a^{-1} = 1.3$ GeV

taste symmetry breaking

- formulation of 4 flavors(=tastes) by construction
- SU(4) symmetric in $a=0$ limit

forth-root trick

to simulate quark w/ $N_f < 4$

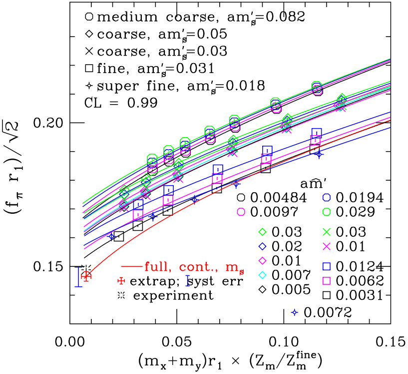
$$Z \;=\; \int [dU]\, (\text{det}[D_{\text{KS}}])^{1/4}\, \exp[-S_g]$$

(D_{KS} = Dirac operator; U = link variable)

- non-local at $a \neq 0$
  (Jansen et al.,2004, Bernard et al.,2006)
- correct continuum limit (?)

2.2 updates on KS results: data analysis

analysis method



($r_1 \sim$0.33fm $\Leftarrow$ static potential)

- w/ partially quenched data ($m_{\rm val} \neq m_{\rm sea}$)

- $M_{\rm PS}$, $F_{\rm PS}$: FSE corrected
- chiral/continuum extrapolation using rooted Staggered ChPT
  (Lee-Sharpe,1999;Aubin-Bernard,2003)
  - effects of taste symmetry breaking
  - effects of rooting tricks
  - LO, NLO (analytic,logs)
  - NNLO, a part of NNNLO (analytic)
- curvature as $m_{ud} \to 0$: chiral log
- $L_4 = 0.1(2)(3)$, $L_5 = 2.0(3)(2)$

  $\Leftrightarrow$ $L_4 = 0.0(8)$, $L_5 = 2.3(1)$
  (@ $\mu = M_\eta$, in units of 10^{-3})
  (Amoros *et al.*,2001)

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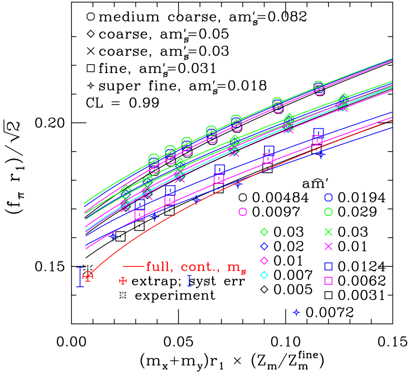

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2.2 update on KS results: results

result for decay constant

- $f_\pi = 128.6(0.4)(3.0)$ MeV,
  $f_K = 155.3(0.4)(3.1)$ MeV
- $f_K/f_\pi = 1.208(2)(+7/-14)$

  $\Leftrightarrow$ *(MILC,2004)*

  $f_\pi = 129.5(0.9)(3.5)$ MeV,

  $f_K = 156.6(1.0)(3.6)$ MeV

  $f_K/f_\pi = 1.210(4)(13)$

- largest systematic error

  $\Leftarrow$ chiral/continuum extrap.

  $\Leftarrow$ difficult to improve drastically

- effects of taste symmetry breaking, rooting trick: taken into account carefully ...
  - good consistency with $f_{\pi,\text{ex}} = 130.7(4)$ MeV
  - independent calculation with different discretization

2.3 studies w/ Wilson-type/chiral fermions: recent simulations

recent algorithmic improvements $\Rightarrow$ enables to explore much smaller $m_{\rm sea}$

Hasenbusch trick + multiple time scale *(Hasenbusch, 2001, Sexton-Weingarten, 1992)*;

domain decomposition HMC *(Lüscher, 2003)* rational HMC *(Clark-Kennedy,2003)*

w/ (improved) Wilson fermions

- Wilson : Del Debbio *et al.*
- clover: QCDSF, PACS-CS
- cheap $\Rightarrow$ large, fine lattices
- explicit chiral symmetry breaking

w/ twisted mass Wilson fermions

$$D = D_W + i\mu\gamma_5\tau_3$$

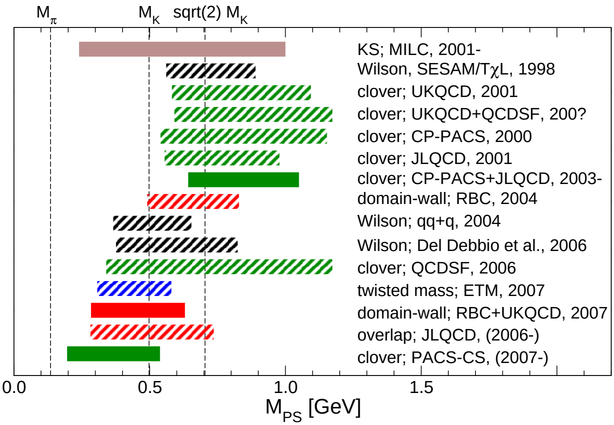
- large scale simulations by ETM
- $O(a)$ improvement (maximal twist)
- cheap
- renormalization is simple
- parity, isospin (?)

w/ chiral fermions

- RBC+UKQCD: domain-wall
  - 5D formulation : $L_5 \to$ chiral symmetry breaking
  - good chiral symmetry ($m_{\rm res} \lesssim 3$ MeV)
- JLQCD : overlap quarks
  - 4D formulation :
  - exact chiral symmetry
  - more expensive than domain-wall
  - explore fixed topological sectors

2.3 studies w/ Wilson-type/chiral fermions

sea quark masses simulated by recent simulations

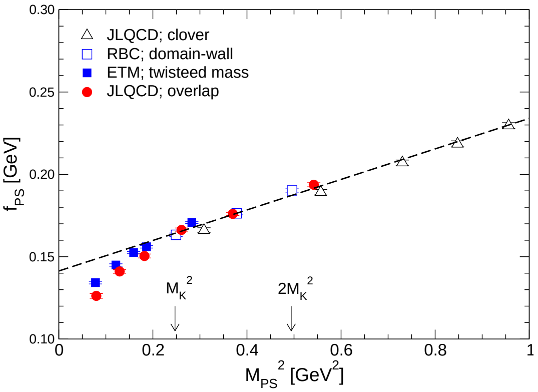


- previous large-scale simulations : limited to $m_{\text{sea}} \gtrsim m_s/2$
- recent simulations : exploring $m_{\text{sea}} \lesssim m_s/2$

2.3 studies w/ Wilson-type/chiral fermions

$f_{\rm PS}$ vs $M_{\rm PS}^2$

$N_f = 2$ data, NP Z_A, a from r_0
$m_{\rm sea} \gtrsim m_s/2$: JLQCD(clover), RBC(domain-wall)
$m_{\rm sea} \lesssim m_s/2$: ETM(twisted mass), JLQCD(overlap)



- curvature at $m_{ud,\rm sim} \lesssim m_{s,\rm phys}$
  $\Rightarrow$ chiral log ?

- ETM: simultaneous NLO ChPT fit to $M_{\rm PS}^2$, $F_{\rm PS}$
  (FSEs corrected, stat. err. only)
  $F = 121.3(7)$, $\bar{l}_4 = 4.52(6)$
  $\Leftrightarrow$ $F = 122.5(3.1)$ *(MILC, 2006)*
  $F = 121.6(1.0)$ *(JLQCD, 2006, prelim.)*
  $\bar{l}_4 = 4.4(2)$ *(Colangelo et al., 2001)*
  $\Downarrow$
  exploring quark masses small enough to make contact with NLO ChPT!

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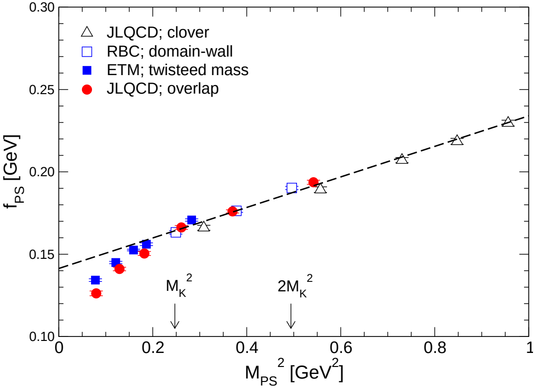

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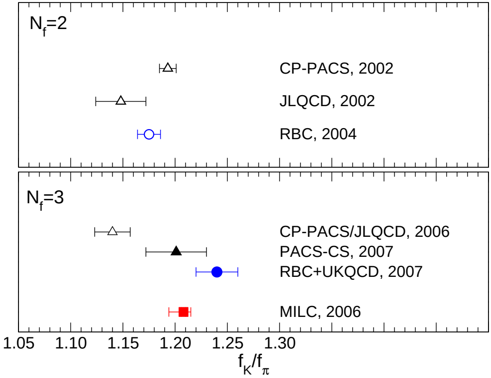
2.3 summary on f_K/f_π

comparison of lattice data



SESAM/TχL, 1999 $\sim$ 1.12 *QCDSF, 2006* $\sim$ 1.19 – 1.21

- simulations at heavy $m_{\rm sea}$
  $\Rightarrow$ overestimate f_π
  $\Rightarrow$ underestimate f_K/f_π
- PACS-CS : very preliminary
- RBC+UKQCD : result on $16^3\times32$
  runs on $24^3\times64$: on-going
- JLQCD (overlap) ?
- MILC (2001-)
  full budget of uncertainties
  (error due to $a\neq0$, chiral fit, ...)

- best estimate : updated MILC (KS): $f_K/f_\pi = 1.208(2)(+7/-14)$
- estimates from Wilson-type/chiral fermions will be improved in the near future

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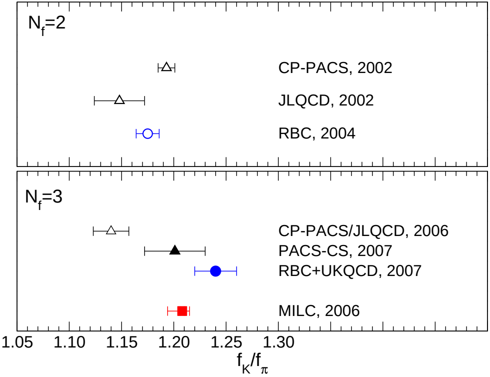

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3. K_{l3} form factors

3.1 $|V_{us}|$ from K_{l3} decays

$$\Gamma(K \to \pi l \bar{\nu}_l) = \frac{G_F^2}{192\pi^3} M_K^5\, C^2\, I\, |V_{us}|^2\, |f_+(0)|^2\, S_{\rm EW}\, \left(1 + \Delta_{\rm EM} + \Delta_{SU(2)}\right)$$

- $\Gamma_{K_l 3}$: experiments $\Rightarrow \Delta|V_{us}| \sim 0.1-0.5\%$ *
- I : phase space int. ($\Leftarrow$ form factor shape) $\Rightarrow \Delta|V_{us}| \lesssim 0.2\%$ *
- $\Delta_{SU(2)}$: isospin breaking effects $\Rightarrow \Delta|V_{us}| \sim 0.2\%$ *
- $\Delta_{\rm EM}$: long-distance EM corrections: $\Rightarrow \Delta|V_{us}| \sim 0.1-0.4\%$ *

  ("*" = from FlaviaNet summary *(Moulson,CKM2006)*)
- $|f_+(0)|$: vector form factor at $q^2 = 0$

$$f_+(0) = 1 + f_2 + f_4 + O(p^6)$$

- ChPT: $f_2 = -0.023$ *(Gasser-Leutwyler,1985)* w/o LECs *(Ademollo-Gatto theorem)*
- quark model: $f_4 = -0.016(8)$ *(Leutwyler-Roos,1984)*
- $O(p^6)$ ChPT (+ large N_c,...) : $f_4 = -0.003(11) - +0.007(12)$
  (Bijnens-Talavera,2003; Jamin-Oller-Pich,2004; Cirigliano-Neufeld-Pichl,2004; Cirigliano et al.,2005)
- lattice : $f_4 + O(p^6)$

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3.2 lattice studies

first study in quenched QCD:

Bećirević et al., 2005

- demonstrate that calculation with 1% accruracy is feasible
  1. calculate $f_0(q^2_{\rm max})$ accurately
  2. study q^2 of dependence and interpolate to q^2
  3. carry out chiral extrapolation of $f_+(0) = f_0(0)$
- $f_+(0) = 0.960(5)(6)$

unquenched calculations:

follow strategy proposed in the quenched calculation

	N_f	action	a[fm]	L[fm]	$M_{\rm PS}$[MeV]
JLQCD (2005)	2	clover	0.09	1.8	$\gtrsim$ 550
RBC (2006)	2	domain-wall	0.12	1.9	$\gtrsim$ 490
MILC (2005)	3	KS (d=clover)	0.12	2.5	$\gtrsim$ 500
RBC+UKQCD (2007)	3	domain-wall	0.12	1.9, 2.9	$\gtrsim$ 300

- details of RBC+UKQCD calculation $\longrightarrow$ talk by Jüttner
- this talk: compare calculation method and resuls of 4 studies

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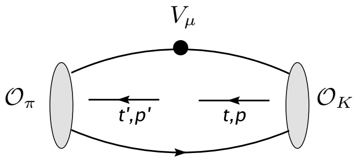
3.3 calculation of $f_0(q^2_{max})$

- 3 point functions

$$C_\mu^{K\pi}(t,t';\mathbf{p};\mathbf{p}') \propto \langle \pi(p')|V_\mu|K(p)\rangle$$

  - overlap of state: $|\mathcal{O}^\dagger_{K(\pi)}\rangle = Z_{\mathcal{O}}\ |K(\pi)\rangle$
  - renorm. factor: $V_\mu = Z_V\ V_{\mathrm{lat},\mu}$
  - exp. factor: $e^{-E_K(\mathbf{p})\,t}$, $e^{-E_\pi(\mathbf{p}')\,t'}$



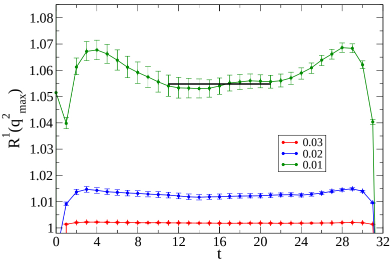
- double ratio *(Hashimoto et al.,1999)*

$$R(t,t') = \frac{C_4^{K\pi}(t,t';0,0)\,C_4^{\pi K}(t,t';0,0)}{C_4^{KK}(t,t';0,0)\,C_4^{\pi\pi}(t,t';0,0)} \rightarrow \frac{(M_K+M_\pi)^2}{4M_K M_\pi}|f_0(q^2_{max})|^2$$

  - cancel unnecessary factors
  - f_0: very accurate data at q^2_{max}

  f_+: need $C_k(|\mathbf{p}|,|\mathbf{p}'|\neq 0)$

R vs t; $N_f=3$ *(RBC+UKQCD)*



- toward larger $|m_{ud}-m_s|$: less accurate $f_0(q^2_{max})$

  RBC($N_f=2$): O(0.01%) error @ $m_{ud}=(4/5)\,m_s$ $\rightarrow$ O(0.1%) @ $m_s/2$

  $\rightarrow$ even larger @ $< m_s/2$???

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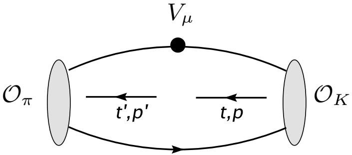

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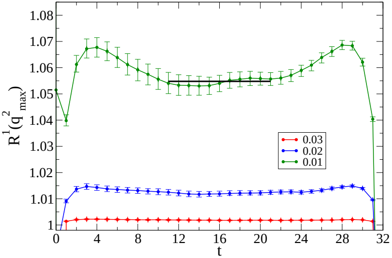
R vs t; $N_f=3$ *(RBC+UKQCD)*



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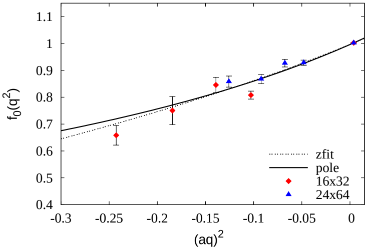
3.4 q^2 interpolation

- calculate f_0 at $q^2 \neq q^2_{\mathrm{max}}$
  - another ratios using $C_\mu(t, t'; \mathbf{p}, \mathbf{p}')$ w/ $\mu \neq 4$, and/or $|\mathbf{p}|, |\mathbf{p}'| \neq 0$
  - much less accurate
- interpolate to $q^2 = 0$
  - pole, linear, quadratic
  - z-param: *(Bourrely, 1981; Boyd et al., 2005)*

    analyticity, unitarity : $t = q^2 \to z$

$$f_0(t) = \frac{1}{\phi(t, t_0, Q^2)} \sum_{k=0}^{\infty} a_k(t_0, Q^2)\, z(t, t_0)^k$$

$f_0(q^2)$ vs q^2; $N_f = 3$ *(RBC+UKQCD)*



fit form	JLQCD	RBC	MILC	RBC+UKQCD
pole: $f_0(q^2) = f_0(0)/(1 - \lambda_0\, q^2)$	–	●	×	●
linear: $f_0(q^2) = f_0(0)\,(1 + \lambda_0\, q^2)$	–	○	–	○
quad.: $f_0(q^2) = f_0(0)\,(1 + \lambda_0'\, q^2 + \lambda_0''\, q^4)$	●	○	–	○
z-param	–	–	–	○

● = adopted; ○ = tested; × = correct $f_0(q^2_{\mathrm{max}})$ to $f_0(0)$

- MILC : correct $f_0(q^2_{\mathrm{max}})$ to $f_0(0)$ using λ_0 from experiment
- JLQCD : employ general quad. form → larger error
- RBC(+UKQCD) : test several forms, check consistency and estimate sys. error

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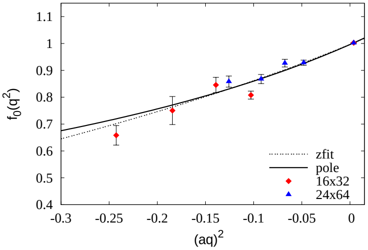
- interpolate to $q^2=0$
  - pole, linear, quadratic
  - z-param: *(Bourrely, 1981; Boyd et al., 2005)* analyticity, unitarity : $t=q^2 \to z$

$$f_0(t) = \frac{1}{\phi(t,t_0,Q^2)} \sum_{k=0}^{\infty} a_k(t_0,Q^2)\, z(t,t_0)^k$$

$f_0(q^2)$ vs q^2; $N_f=3$ *(RBC+UKQCD)*



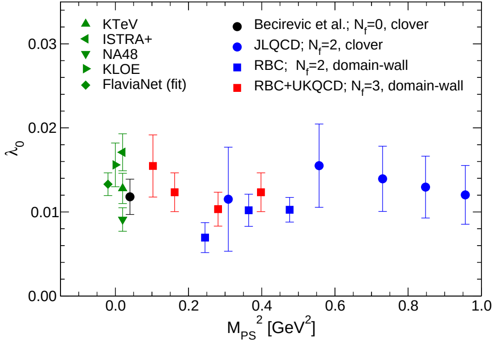
fit form	JLQCD	RBC	MILC	RBC+UKQCD
pole: $f_0(q^2) = f_0(0)/(1-\lambda_0\, q^2)$	–	●	×	●
linear: $f_0(q^2) = f_0(0)\,(1+\lambda_0\, q^2)$	–	○	–	○
quad.: $f_0(q^2) = f_0(0)\,(1+\lambda_0'\, q^2 + \lambda_0''\, q^4)$	●	○	–	○
z-param	–	–	–	○

●= adopted; ○= tested; × = correct $f_0(q^2_{\rm max})$ to $f_0(0)$

- MILC : correct $f_0(q^2_{\rm max})$ to $f_0(0)$ using λ_0 from experiment
- JLQCD : employ general quad. form → larger error
- RBC(+UKQCD) : test several forms, check consistency and estimate sys. error

3.4 q^2 interpolation

comparison of form factor shape



- $m_{s,\text{sim}} \simeq m_{s,\text{phys}}$
- RBC (+UKQCD): from pole fit
- JLQCD:
  quad. fit $\rightarrow$ large error

- reasonably consistent w/ experiment
- rigorous comparison $\Leftarrow$ $m_{s,\text{sim}} = m_{s,\text{phys}}$, scale a, chiral fit

3.5 chiral extrapolation

- $f_+(0) = f_0(0)$
- $f_+(0) = 1 + f_2 + \Delta f$
- availability of f_2 for lattice setup:
  - (P)QChPT ($m_{\rm sea} \neq m_{\rm val}$): $N_f = 0, 2, 3$ *(Bećirević-Martinelli-Villadoro et al., 2006)*
  - finite V : $N_f = 0, 3$ *(Bećirević et al., 2005)*
  - PQChPT f_2 at finite V ?
- chiral extrap. of $f_+(0)$ = extrap. of Δf
  - NNLO analytic term : single term at $q^2 = 0$
    $\propto (M_K^2 - M_\pi^2)^2$ w/ LEC of $O(p^6)$ $\mathcal{L}_\chi$
  - NNLO loops : very complicated...
    many terms w/ LECs, or estimated numerically
  - lattice studies: use a ratio motivated by Ademollo-Gatto theorem
    $R_{\Delta f} = \Delta f/(M_K^2 - M_\pi^2)^2$

3.5 chiral extrapolation

chiral fit of $R_{\Delta f}$

- unquenched simulations: limited # of simulated quark masses
  $\Rightarrow$ use simple polynomial fits for $R_{\Delta f}$

- constant : $R_{\Delta f} = c_0$ $\Leftarrow$ Δf is dominated by NNLO analytic term
- linear : $R_{\Delta f} = c_0 + c_1 (M_K^2 + M_\pi^2)$
- quadratic : $''linear'' + c_2 (M_K^2 + M_\pi^2)^2 + \cdots$

  $\Leftarrow$ hoping these are good description of NNLO and higher

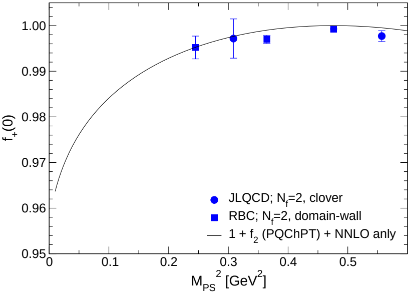
fit	JLQCD	RBC	MILC	RBC+UKQCD
const. fit	–	○	–	○
linear fit	●	●	●	●
quad. fit	–	×	–	–

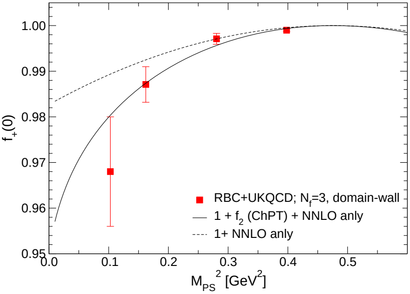
●= adopted; ○= tested; × = ill-determined coefficients

- describes unquenched data reasonably well
- need accurate data at small m_{ud} to be confident with the simple chiral extrap.

3.5 chiral extrapolation

m_{ud} dependence of $f_+(0)$





- JLQCD ($N_f = 2$), RBC ($N_f = 2$): $m_{\rm sea} > m_s/2$
- RBC+UKQCD ($N_f = 3$)

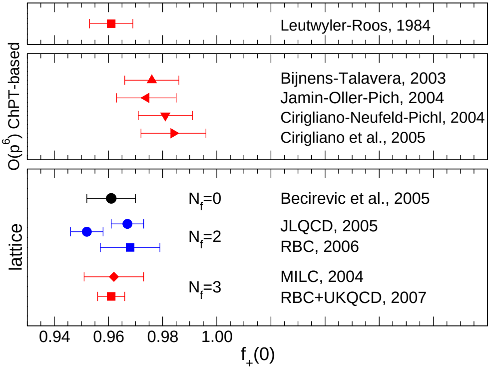
  rapid change toward $m_{ud} = 0$ $\rightarrow$ consistent w/ ChPT log ?

  larger error as $m_{ud} \rightarrow 0$

  - larger error of $f_0(q^2_{\rm max})$; larger $q^2_{\rm max}$ $\rightarrow$ error of q^2 interp.
  - twisted boundary condition, low-mode averaging, all-to-all propagator
- m_q dependence of NNLO logs ?

3.6 comparison of $f_+(0)$

theoretical estimates of $f_+(0)$



- good consistency between lattice and LR
- systematically smaller than ChPT results ?

systematic errors:

- FSE : RBC+UKQCD $\Rightarrow$ consistency between $L=2$ and 3 fm
- $a \neq 0$: consistency among various discretization
- chiral extrap.: need deeper understanding of chiral behavior

4.1 hyperon decays

in quenched QCD

	action	a[fm]	L[fm]	$M_{\rm PS}$[MeV]	$f_1(0)$
Guadagnoli et al. (2006)	clover	0.08	1.8	$\geq$ 700	$-0.948(29)$
RBC (2006)	domain-wall	0.15	2.4	$\geq$ 550	$-0.953(24)$

- through ratios of correlators (cf. studies of K_{l3} form factors)
- heavy $m_{\rm sea}$: $f_1(0)+1$ = dominated by contrib. of local terms of chiral expansion ?

chiral logs from HBChPT

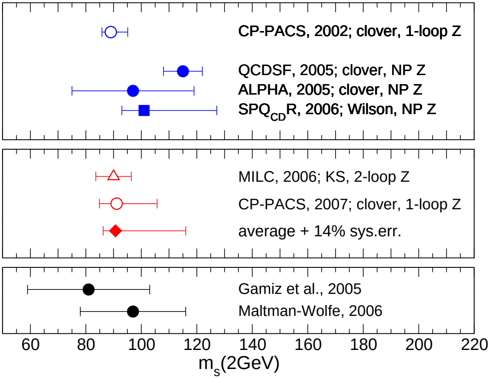
- *Villadoro, 2005; Guadgnoli et al., 2006*
  - studies $O(p^2)$, $O(p^3)$, $O(1/M_{\rm oct})$ contributions for various decay modes
    $\Rightarrow$ poor convergency
  - $|m_{\rm dec} - m_{\rm oct}| \sim \Lambda_{\rm QCD}$ at heavy m_q
    $\Rightarrow$ decuplet contributions : huge

unquenched calculation

- *Lin-Orginos, 2007*
  - mixed action (KS + domain-wall), $a=0.13$ fm, $L=2.6$ fm, $M_{\rm PS} \gtrsim 350$ MeV
  - $f_1(0) = -0.90(7)$ (VERY preliminary)

4.2 strange quark mass

recent estimate of $m_s^{\overline{\rm MS}}(\mu=2\text{GeV})$



- $N_f=2$
  - *QCDSF, ALPHA* (clover), *SPQCDR* (Wilson)
  - non-perturbative renorm.

    $\Rightarrow$ $\sim$ 14 % larger *(SPQCDR)*
- $N_f=3$
  - *MILC* (KS), *CP-PACS* (clover)
  - $\simeq$ 90 MeV
  - perturbative renorm.

    $\Rightarrow$ $91(+25/-5)$ MeV

- reasonably consistent w/ m_s from τ decays

5. summary

f_K/f_π

- MILC (KS): $f_K/f_\pi = 1.208(+7/-14)$ $\Rightarrow |V_{us}| = 0.2226(+26/-15)$
- well matured : difficult to improve drastically

K_{l3} form factor

- RBC+UKQCD (domain-wall): $f_+(0) = 0.9609(51)$ $\Rightarrow |V_{us}| = 0.2255(13)$
- chiral extrapolation

... picked up a single calculation $\Leftarrow$ $N_f = 3$, range of $m_{ud,\mathrm{sim}}$

other studies ?

- $m_{ud,\mathrm{sim}} \lesssim m_{s,\mathrm{phys}}/2$
- algorithmic improvements
  - $\Rightarrow$ simulations by *Del Debbio et al*, PACS-CS, ETM, JLQCD, ...
  - $\Rightarrow$ independent calculations with Wilson-type / chiral fermions in the near future