Reduced fidelities for free fermions out of equilibrium

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Reduced fidelity

- Fidelity between two density matrices:

$$\mathcal{F}(\rho,\sigma) = \frac{\mathrm{Tr}(\rho\sigma)}{\sqrt{\mathrm{Tr}(\rho^2)\,\mathrm{Tr}(\sigma^2)}}$$

- $0 \leqslant \mathcal{F} \leqslant 1$, equals 1 iff $\rho = \sigma$
- $\mathcal{F}(\rho,\sigma) = |\langle\psi_\rho|\psi_\sigma\rangle|^2$ for $\rho = |\psi_\rho\rangle\langle\psi_\rho|$ and $\sigma = |\psi_\sigma\rangle\langle\psi_\sigma|$
- **Reduced fidelity out of equilibrium:** fidelity between reduced density matrices on a subsystem A, evaluated at different times after a quench
- Quench: $|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}Ht}|\Psi_0\rangle$ with

$$H(h,\gamma) = -\sum_{j=1}^{L}\left(\frac{1+\gamma}{4}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{4}\sigma_j^y\sigma_{j+1}^y + \frac{h}{2}\sigma_j^z\right)$$

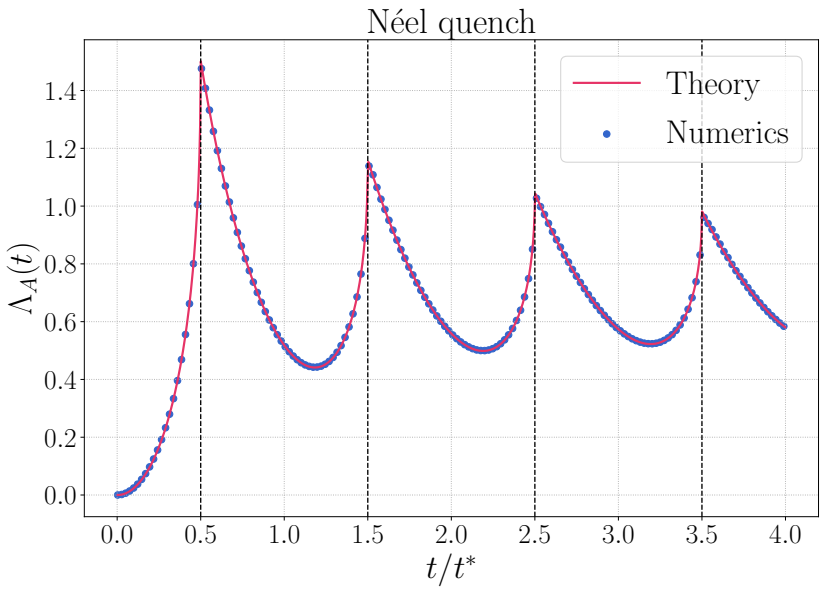
and Gaussian initial states

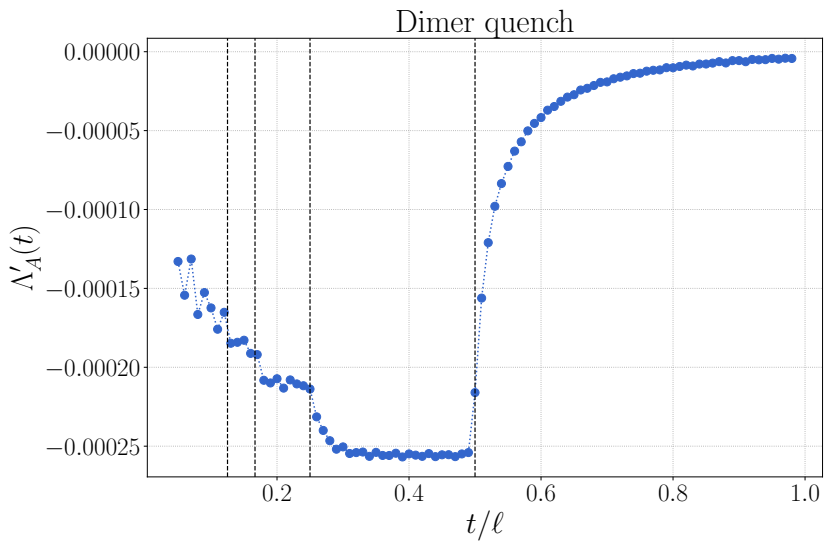
Reduced Loschmidt echo

- Defined as fidelity between $\rho_A(0)$ and $\rho_A(t)$:

$$\mathcal{F}_A(t) = \frac{\mathrm{Tr}(\rho_A(0)\rho_A(t))}{\sqrt{\mathrm{Tr}(\rho_A(0)^2)\,\mathrm{Tr}(\rho_A(t)^2)}}, \quad \Lambda_A(t) = -\frac{1}{\ell}\log(\mathcal{F}_A(t))$$

- Exhibits singularities at critical times $t = (m + 1/2)t^*$
- Related to **dynamical quantum phase transitions**
- Nested lightcones structure in the hydrodynamic regime





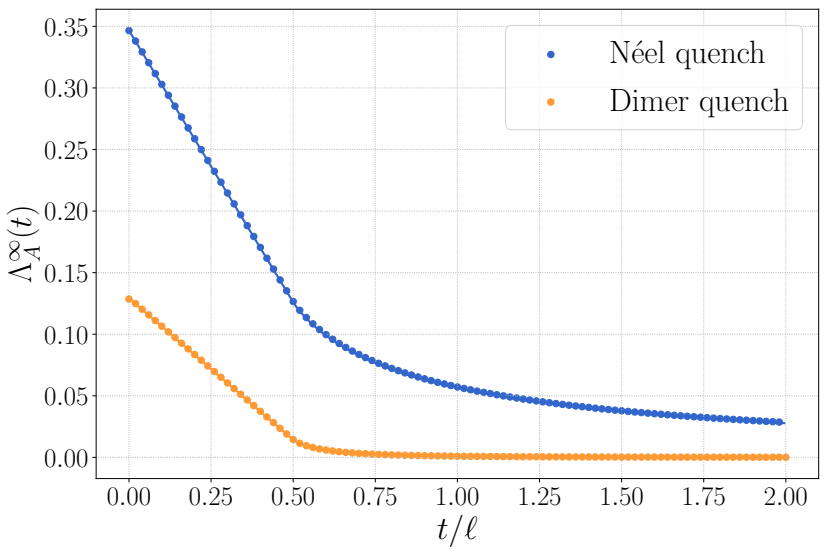
Final-state fidelity

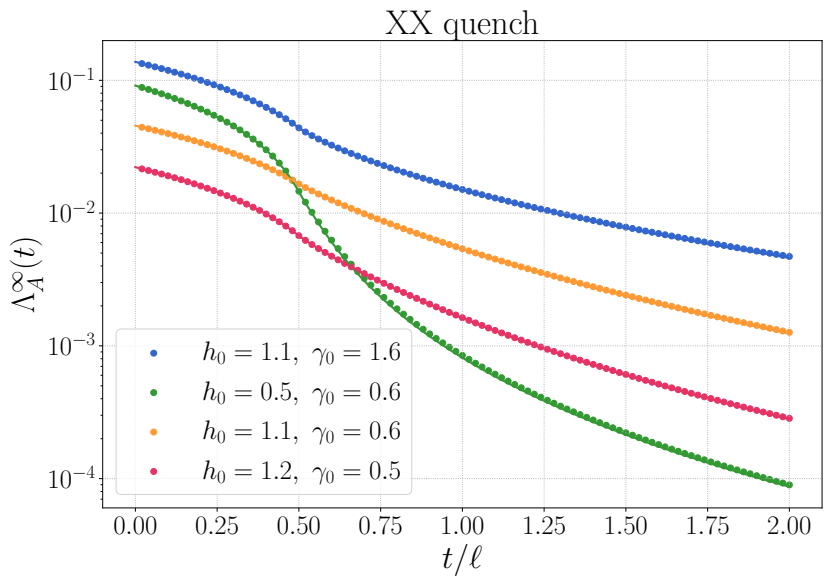
- Compare $\rho_A(t)$ with its infinite-time limit ρ_A^∞
- Analytic expressions and quasiparticle picture interpretation

$$\Lambda_A^\infty(t) = \int_{-\pi}^{\pi} \frac{\mathrm{d}k}{2\pi} \left(1 - \min(2|v_k|t/\ell, 1)\right) \Lambda_k$$

$$\Lambda_k = \frac{1}{2} \log\left(\frac{1 + (2n_k - 1)^2}{2}\right) - \log\left(\frac{\sqrt{1 + 3(2n_k - 1)^2}}{2}\right)$$

- Tool to study thermalization and **quantum Mpemba effects**





RAQIS in SEPT 2026