Confinement and false vacuum decay on the Potts quantum spin chain

O. Pomponio, A. Krasznai and G. Takács, SciPost Phys. 18 (2025) 082

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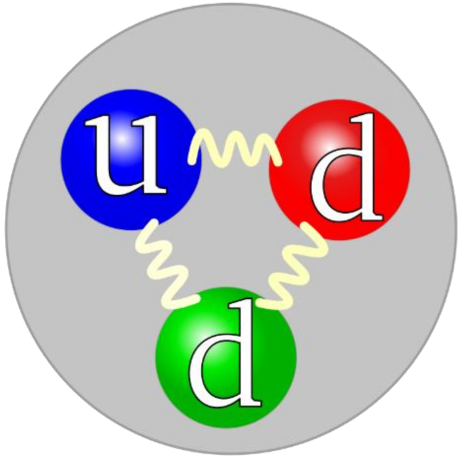
Motivation

- Strong interactions: central concept: confinement
- Underlying mechanism: linear potential → can be realised in condensed matter systems



Motivation

- Simplest example: FM mixed-field Ising spin chain: $$H = -\sum_j \left(\hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + g\hat{\sigma}_j^x + h\hat{\sigma}_j^z\right)$$
  - Small h , FM phase: $0 < g < 1$
  - Lifted $\mathbb{Z}_2$ symmetry
  - Advantage: experimentally realisable [W. L. Tan et al., Nat. Phys. 17 (2021) 742–747]
  - Disadvantage: the colours are restricted to 2 → only mesons, no baryons

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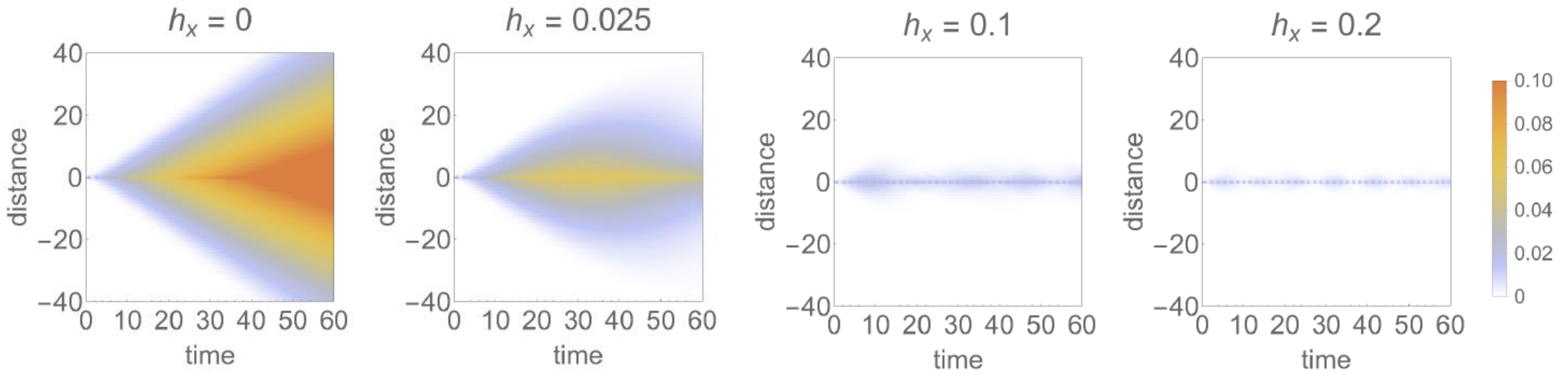
  - Disadvantage: the colours are restricted to 2 → only mesons, no baryons
- FM mixed-field 3-state Potts model: $$H = -\sum_i \sum_{\alpha=1}^{3} \left(P_i^\alpha P_{i+1}^\alpha + h_\alpha P_i^\alpha\right) - g\sum_i \tilde{P}_i$$
  - Small h_α , FM phase: $0 < g < 1$
  - Lifted $\mathbb{S}_3$ symmetry
  - Advantage: 3 colours → mesons and baryons!
  - Disadvantage: no experimental realisations yet

Motivation

Confinement: [B. McCoy, T. Wu, Phys. Rev. D 18, 1259 (1978)]

- Dynamics is also seriously altered by confinement!
  - Observed in the Ising before: confinement leads to the localisation of quasiparticles (and correlations) after global quenches [M. Kormos, et al., Nat. Phys. 13, 246–249 (2017)]
  - In the Potts: much more options for global quenches



Outline of the talk

- Pure transverse Potts model
- Excitation spectrum of the mixed-field model
- Non-equilibrium dynamics (global quenches)

Pure transverse Potts-model

Pure transverse Potts

- Hamiltonian: $$H = -\sum_i \sum_{\alpha=1}^{3} P_i^\alpha P_{i+1}^\alpha - g \sum_i \tilde{P}_i$$ $$\mathcal{H} = \bigotimes_i (\mathbb{C}^3)_i$$

- Order parameters: $$P^\alpha = |\alpha\rangle\langle\alpha| - \frac{1}{3}\mathbb{I}_{3\times 3}$$ $$\sum_{\alpha=1}^{3} P^\alpha = 0$$

$$|1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad |2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad |3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

- “Transverse field”: $$\tilde{P} = \frac{1}{3}(|1\rangle + |2\rangle + |3\rangle)(\langle 1| + \langle 2| + \langle 3|)$$

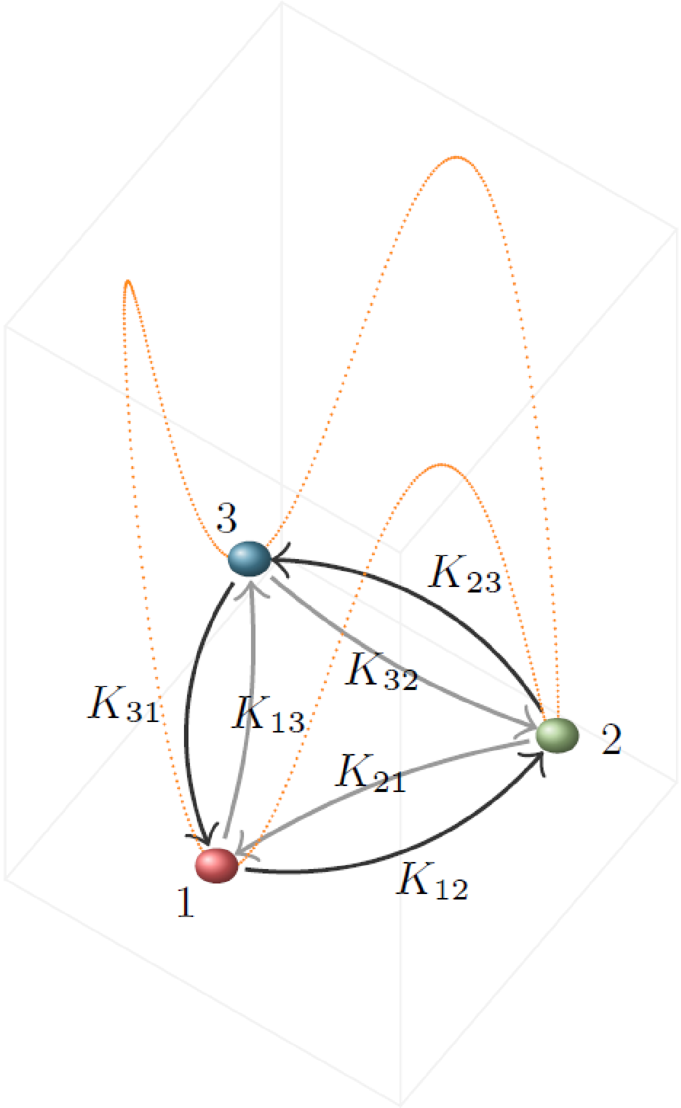
- No analytical solution, perturbation theory

[Á. Rapp et al., Phys. Rev. B 74 (2006) 014433]

Pure transverse Potts

- FM phase: $0 < g < 1$
- $\mathbb{S}_3$ symmetry, spontaneously broken in FM phase
- 3 ground states $|R\rangle, |G\rangle, |B\rangle$, for small g approximately

$$\bigotimes_i |1\rangle_i, \bigotimes_i |2\rangle_i, \bigotimes_i |3\rangle_i$$



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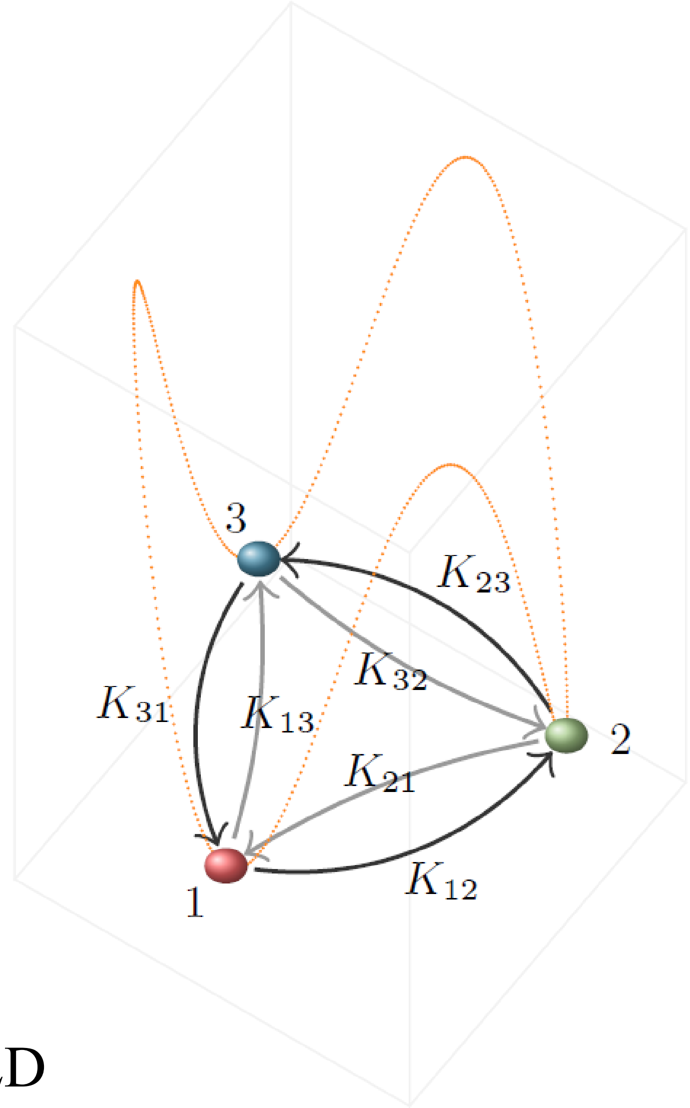
$$\bigotimes_i |1\rangle_i, \bigotimes_i |2\rangle_i, \bigotimes_i |3\rangle_i$$

- Elementary excitations: kinks/antikinks $K_{\mu\nu}$
- Dispersion relation of a kink $\epsilon(k)$: ED data to be fitted with $\epsilon(k) = \sqrt{A + B \cos k}$
- Scattering $\rightarrow$ Phase shifts can be numerically calculated from ED



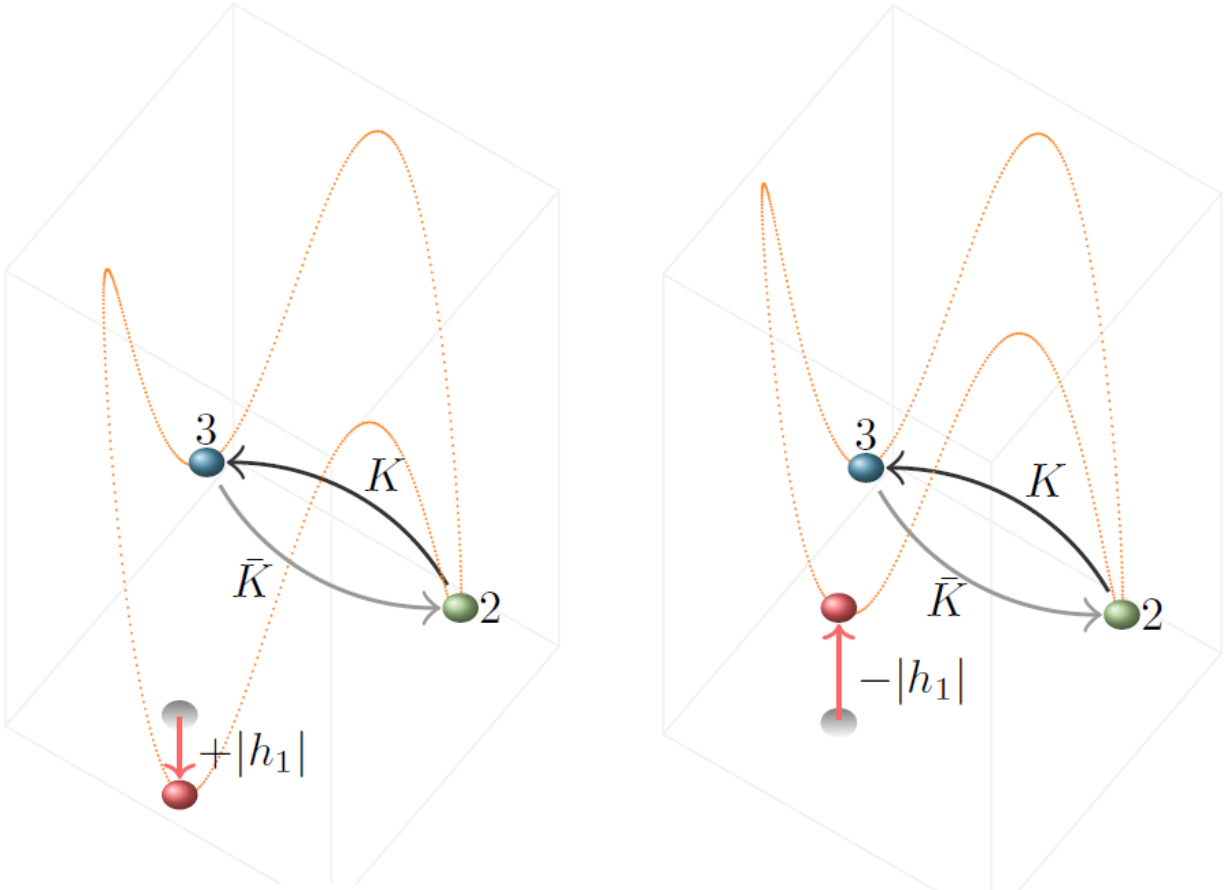
Excitation spectrum of the mixed-field Potts-model

Turning on one longitudinal field

- Hamiltonian: $$H = -\sum_i \sum_{\alpha=1}^{3} \left(P_i^\alpha P_{i+1}^\alpha + h_\alpha P_i^\alpha\right) - g \sum_i \tilde{P}_i$$

- Only one of $h_\alpha \neq 0$ and $0 < g < 1$

- Remaining $\mathbb{Z}_2$ symmetry



Turning on one longitudinal field

- Hamiltonian: $$H = -\sum_i \sum_{\alpha=1}^{3} \left(P_i^\alpha P_{i+1}^\alpha + h_\alpha P_i^\alpha\right) - g \sum_i \tilde{P}_i$$
- Only one of $h_\alpha \neq 0$ and $0 < g < 1$
- Remaining $\mathbb{Z}_2$ symmetry
- h_α induces a linear potential between kinks/antikinks: $V(d) = \pm \chi d$ $\rightarrow$ bound states
- Fit function: $\chi = |h_\mu| \left(1 - g^2\right)^\alpha$ $\rightarrow$ from numerical data: $\alpha \approx 0.102$

Semiclassical quantisation

- Neglecting short-range interactions:
- Effective 2-kink Hamiltonian: $\mathcal{H} = \underbrace{\epsilon(k_1) + \epsilon(k_2)}_{\text{kinetic part}} + \underbrace{\chi|x_2 - x_1|}_{\text{potential part}}$

Semiclassical quantisation

- Neglecting short-range interactions:
- Effective 2-kink Hamiltonian: $\mathcal{H} = \epsilon(k_1) + \epsilon(k_2) + \chi|x_2 - x_1|$

$$X = \frac{x_1 + x_2}{2}\,, \qquad x = x_2 - x_1$$

$$K = k_1 + k_2\,, \qquad k = \frac{k_2 - k_1}{2}$$

$$\mathcal{H} = \omega(k;K) + \chi|x|$$

$$\omega(k;K) = \epsilon(K/2 + k) + \epsilon(K/2 - k)$$

- Bohr-Sommerfeld quantisation $\oint dx\, k(x) = 2\pi(\nu + 1/2)$

Semiclassical quantisation

- $\chi > 0$: mesons

➢ Collisional:

1,2,3…..

0 (even) or 1 (odd) spatial symmetry

$$2\,E_n^{(\kappa)}(K)\,k_a - \int_{-k_a}^{k_a} dk\,\omega(k;K) = 2\pi\chi\left(n - \frac{1}{2} + \frac{(-1)^\kappa}{4}\right)$$

$$E_n(K) = \omega(k_a;K)$$

Semiclassical quantisation

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$$E_n(K) = \omega(k_a;K)$$

➢ Collisionless (localised due to Bloch oscillation):

$$E_n = n\chi + \frac{1}{\pi}\int_{-\pi}^{\pi} dk\, \epsilon(k)$$

Semiclassical quantisation

- $\chi > 0$: mesons

1,2,3…..

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➢ Collisional:

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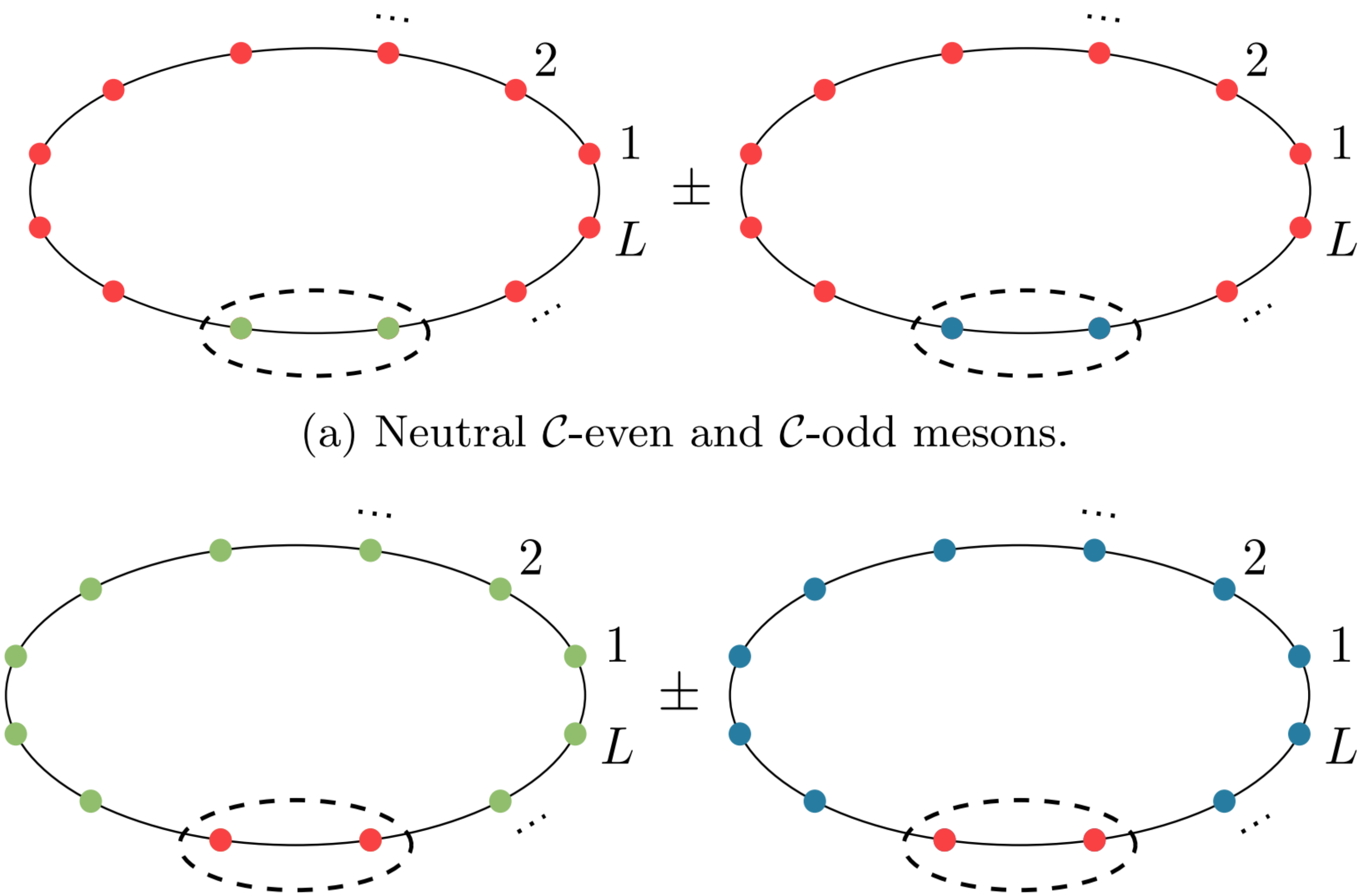
$$E_n(K) = \omega(k_a;K)$$

➢ Collisionless: $$E_n = n\chi + \frac{1}{\pi}\int_{-\pi}^{\pi} dk\,\epsilon(k)$$

- $\chi < 0$: bubbles $\rightarrow$ quantisation similarly, collisionless particles are the same

Mesons and bubbles with PBC

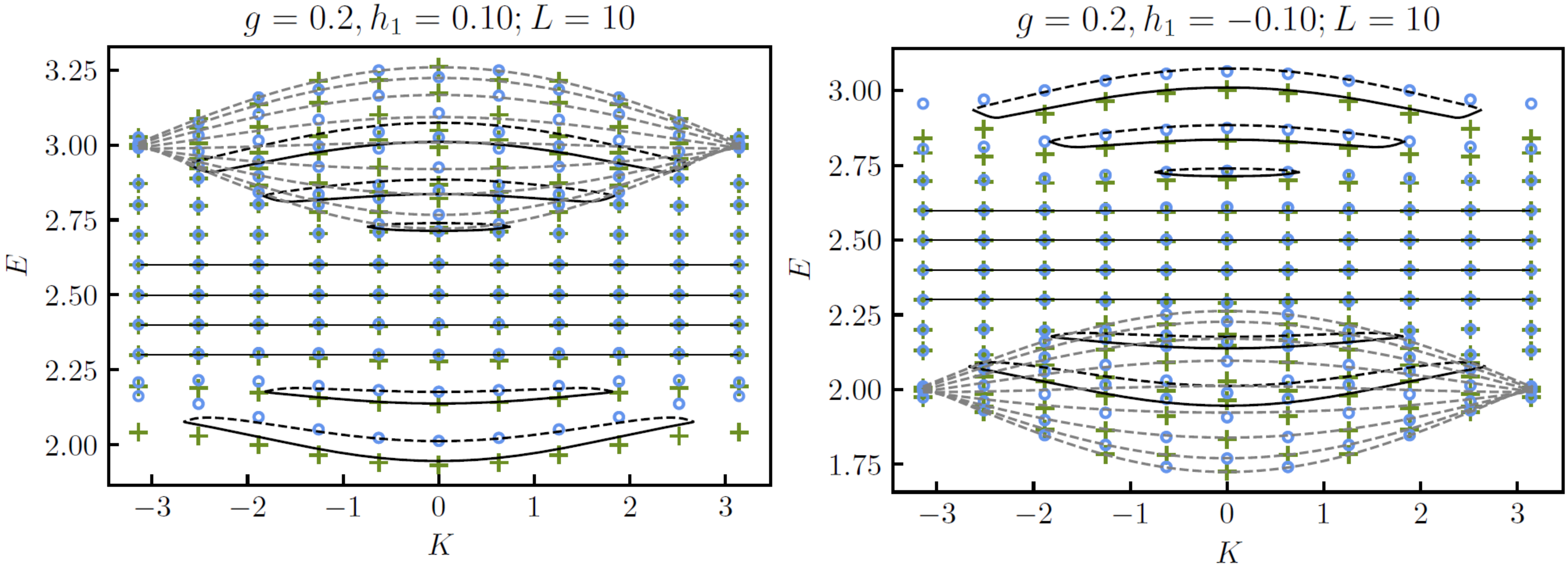
$h_1 > 0$



(a) Neutral $\mathcal{C}$-even and $\mathcal{C}$-odd mesons.

(b) Neutral $\mathcal{C}$-even and $\mathcal{C}$-odd bubbles.

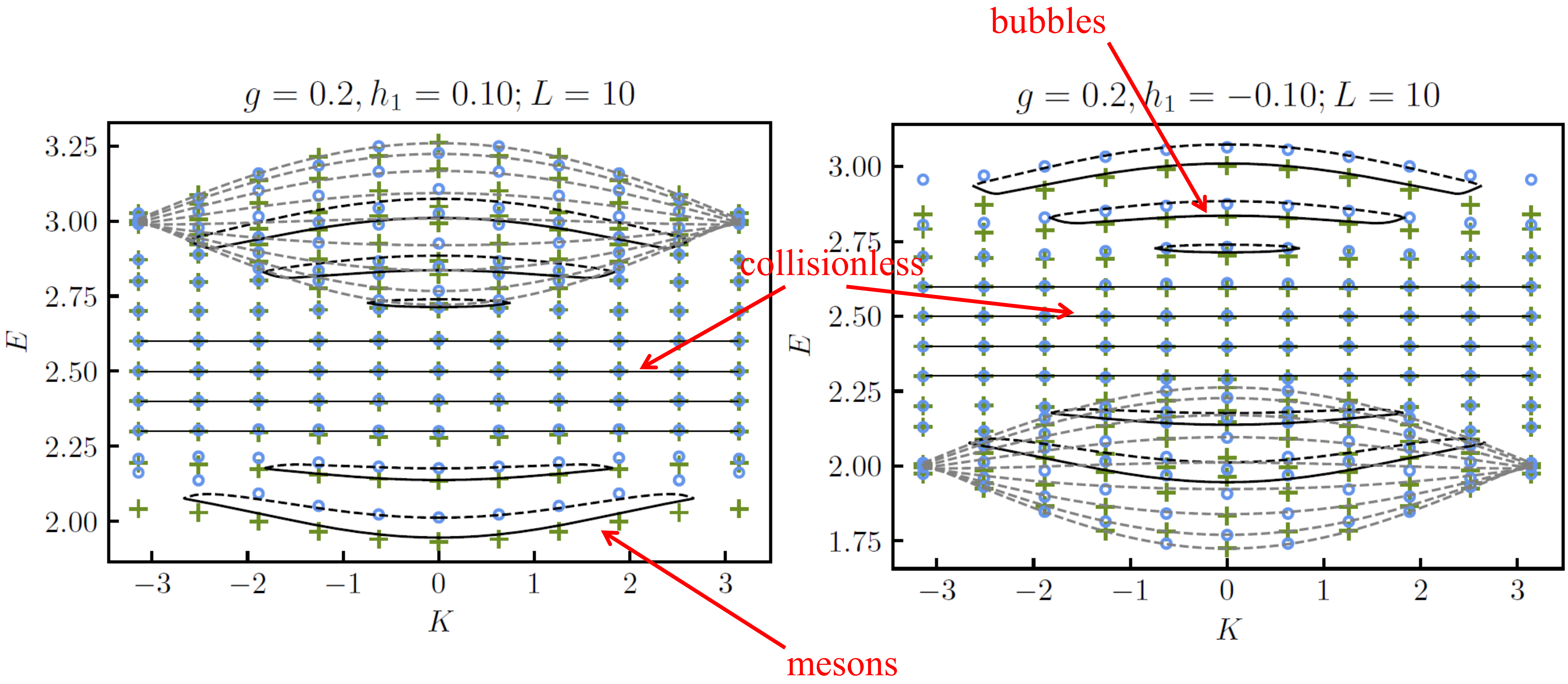
Exact Diagonalisation vs Semiclassical spectra



TV: $|R\rangle$ FV: $|G\rangle$, $|B\rangle$

TV: $|G\rangle$, $|B\rangle$ FV: $|R\rangle$

ED vs Semiclassical spectra



TV: $|R\rangle$ FV: $|G\rangle$, $|B\rangle$

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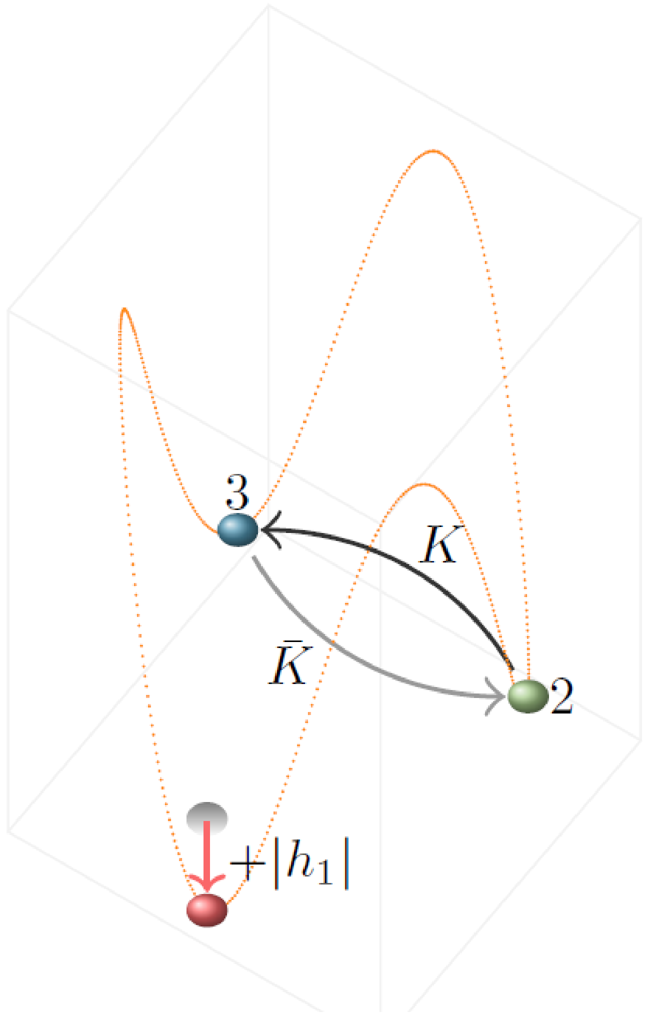
Free 2-kink continuum and resonances

- Residual $\mathbb{Z}_2$ symmetry $\rightarrow$ free kinks
- In infinite volume: free-kink continuum



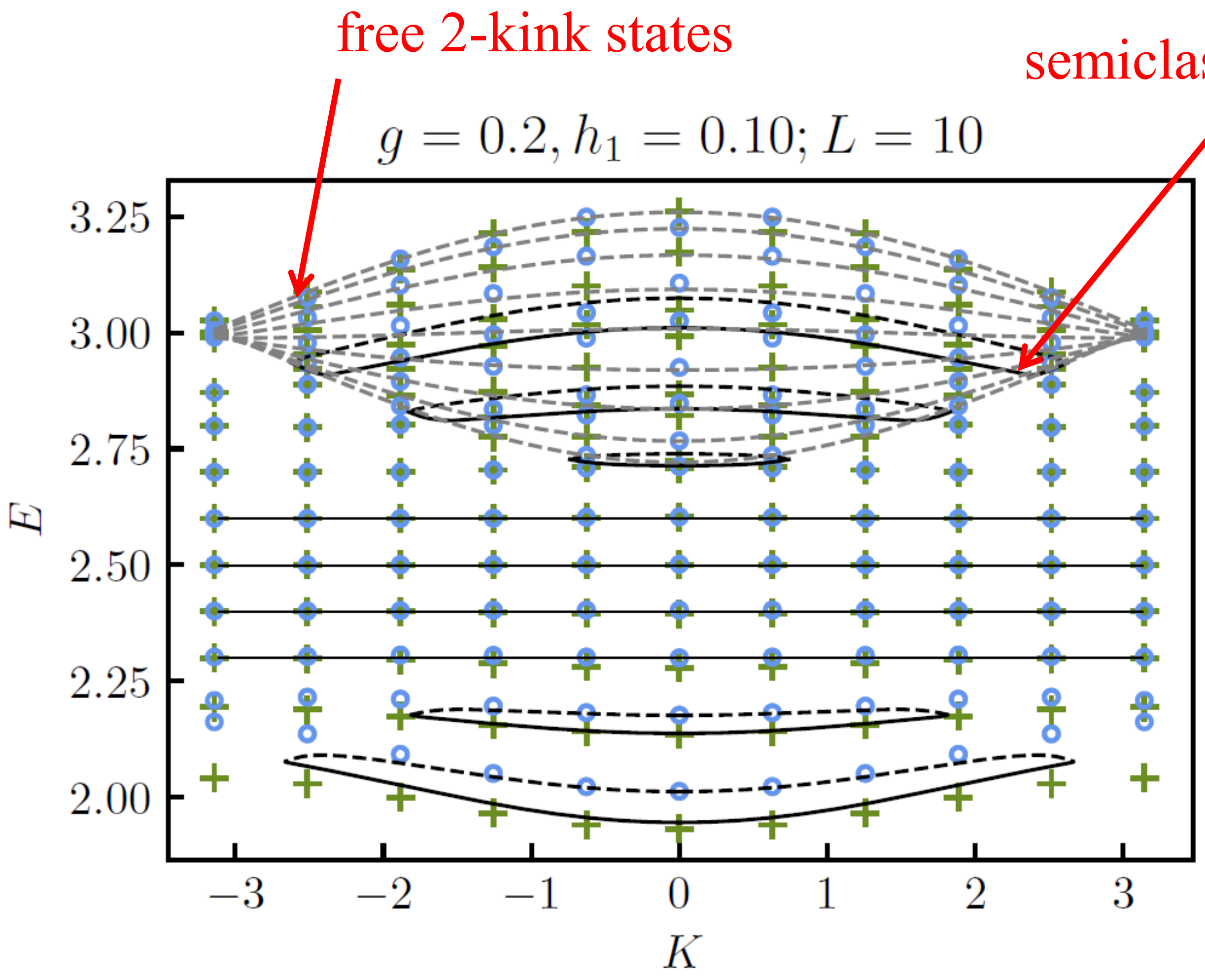
Free 2-kink continuum and resonances

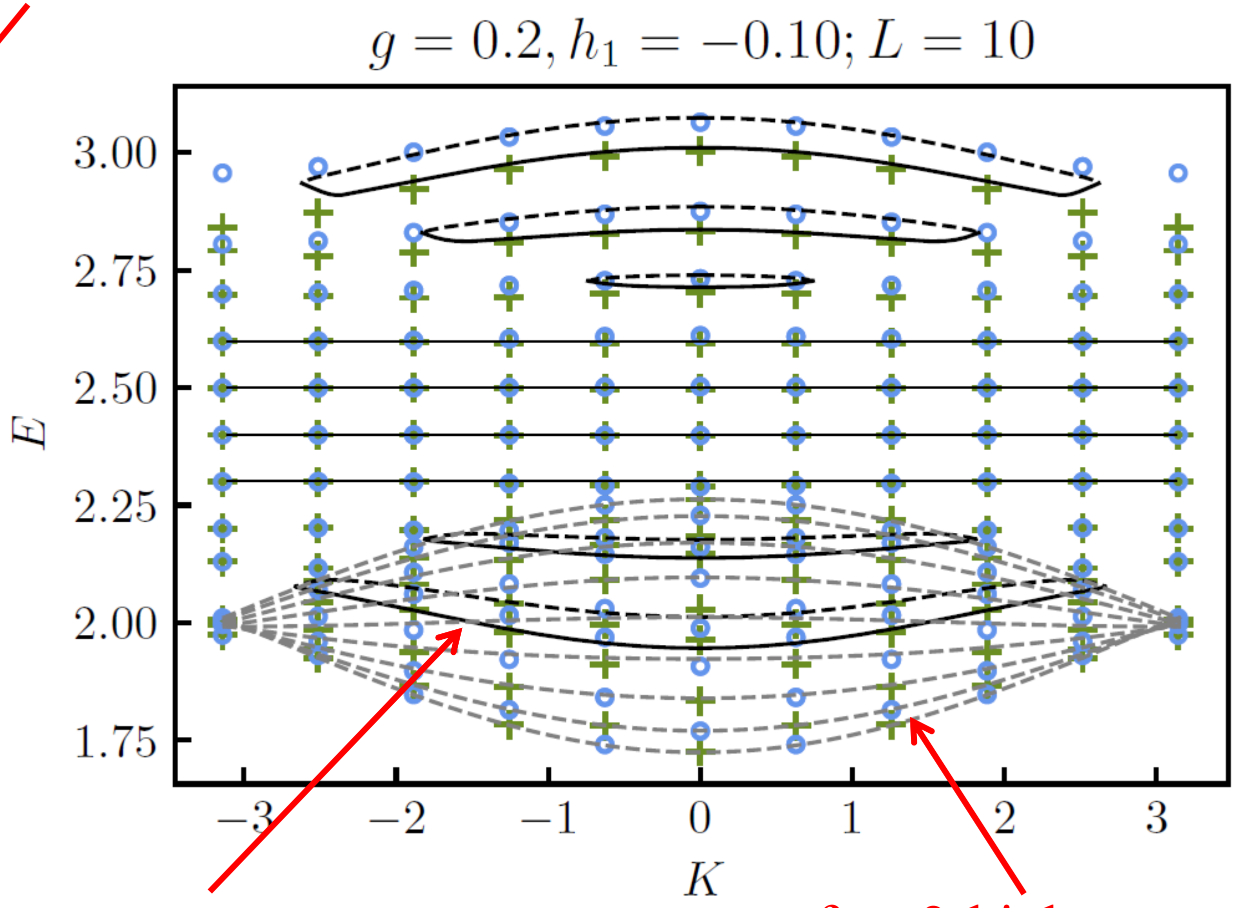
- Residual $\mathbb{Z}_2$ symmetry $\rightarrow$ free kinks
- In infinite volume: free-kink continuum
- Free-kink continuum hybridizes with the bubbles/mesons
- They form a continuous spectrum with resonances
- Resonances cannot be treated by ED or semiclassics but perturbatively yes! $\rightarrow$

  A. Krasznai, S. Rutkevich and G. Takács, "*Confinement in the three-state Potts quantum spin chain in extreme ferromagnetic limit*", arXiv:2508.20821

- No classification of the ED states is possible in the free 2-kink regime

ED vs Semiclassical spectra





TV: $|R\rangle$ FV: $|G\rangle$, $|B\rangle$

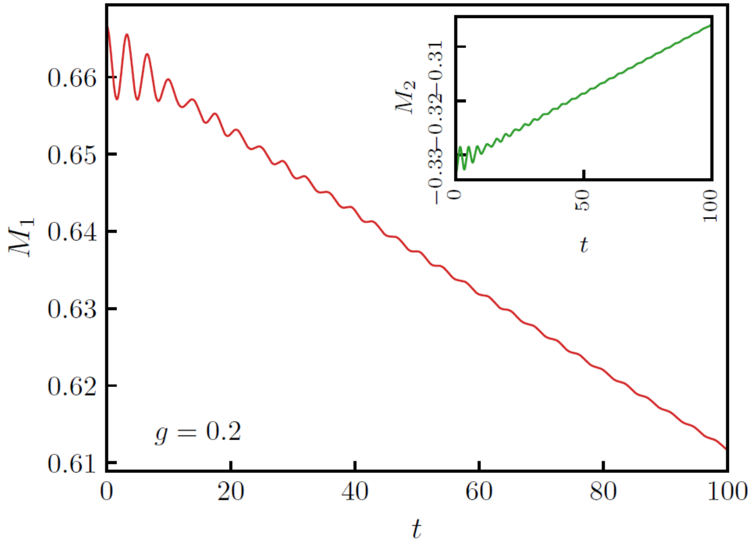
TV: $|G\rangle$, $|B\rangle$ FV: $|R\rangle$

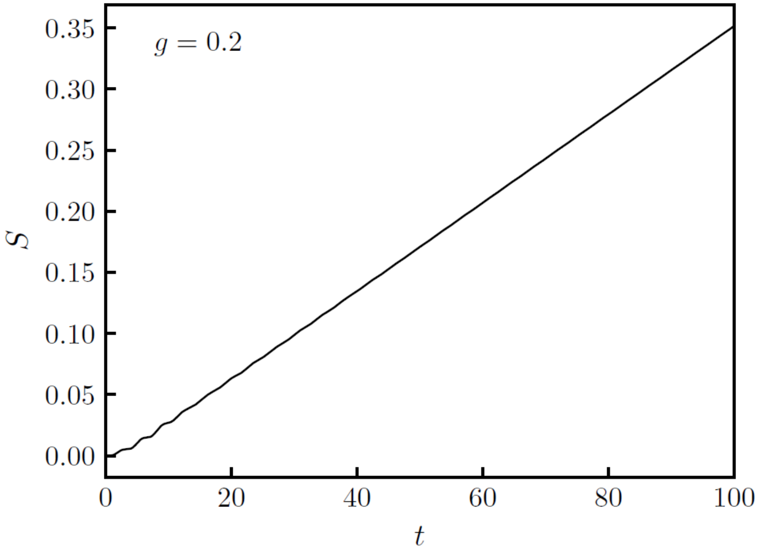
Non-equilibrium dynamics (global quenches)

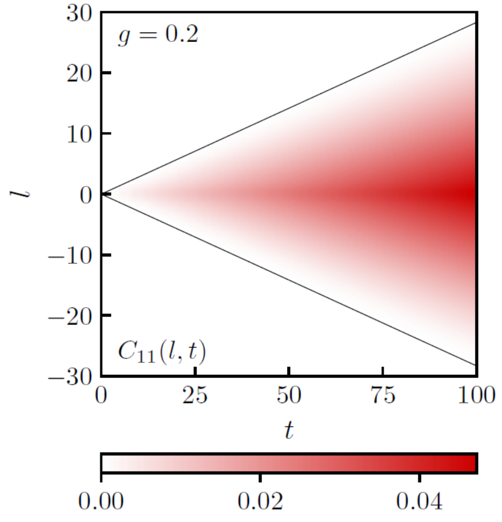
Global quenches of the Potts model

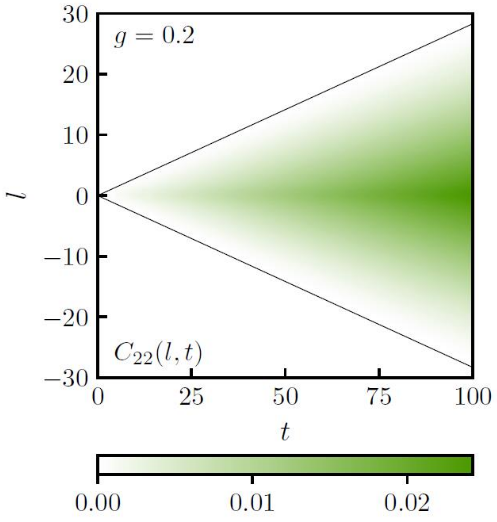
- Quench: initial state: $|\psi_0\rangle = \bigotimes_{i=1}^{L} |1\rangle_i \;\rightarrow\; |\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$
- Global quench: translational invariance, infinite volume
- iTEBD method to get numerical data
- Magnetisations: $M_\mu(t) = \langle\psi(t)|\, P^\mu \,|\psi(t)\rangle$
- Correlators: $C_{\mu\nu}(l,t) = \langle\psi(t)|\, P_l^\mu P_0^\nu \,|\psi(t)\rangle - M_\mu(t) M_\nu(t)$
- FM phase: $0 < g < 1$

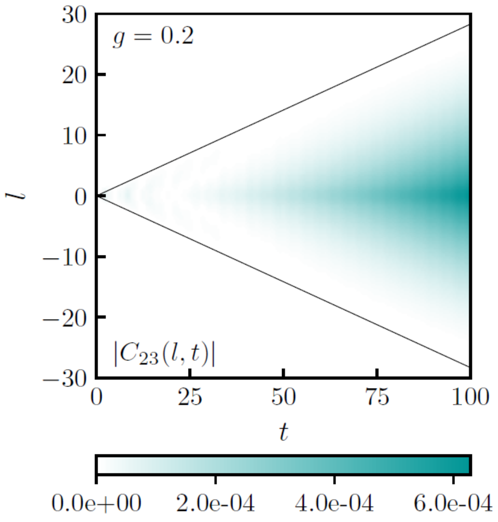
Global quenches of the pure transverse Potts



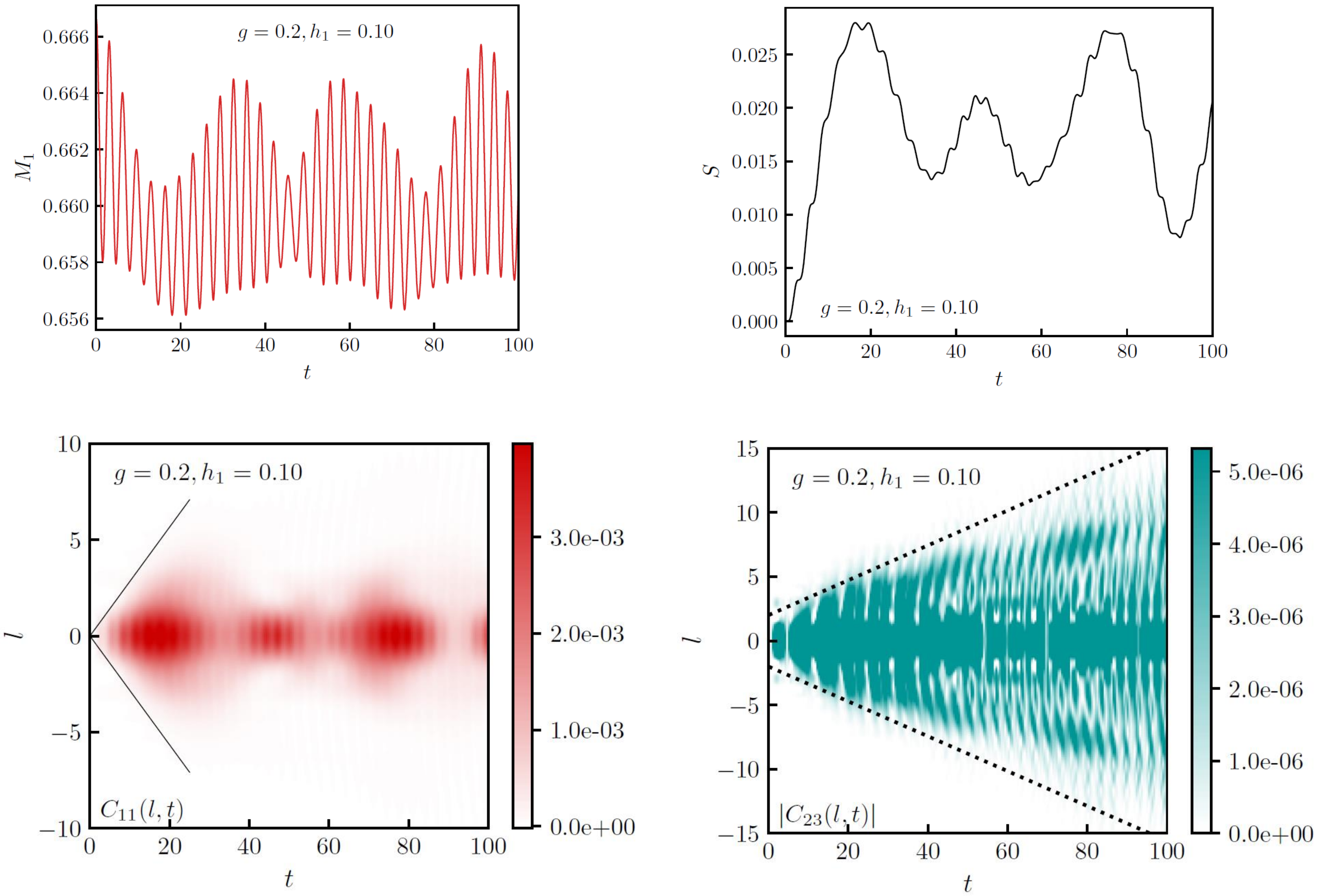




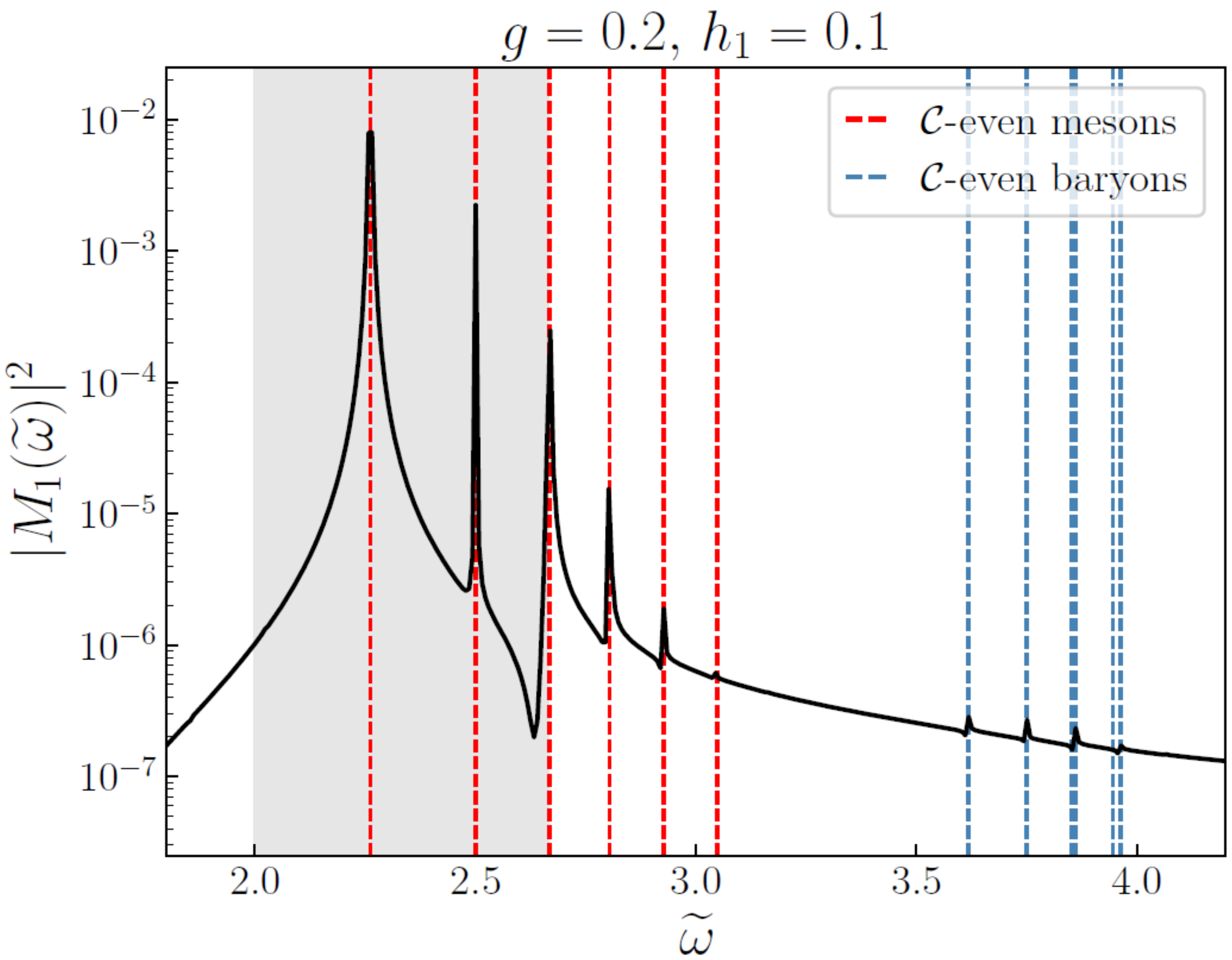


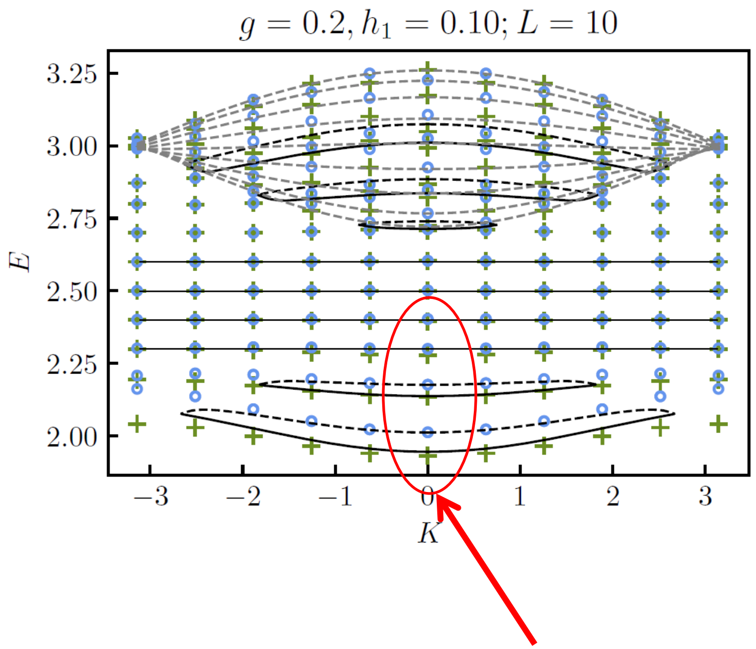


Global quenches of the Potts: confining $h_1 > 0$



Global quenches of the Potts: confining $h_1 > 0$

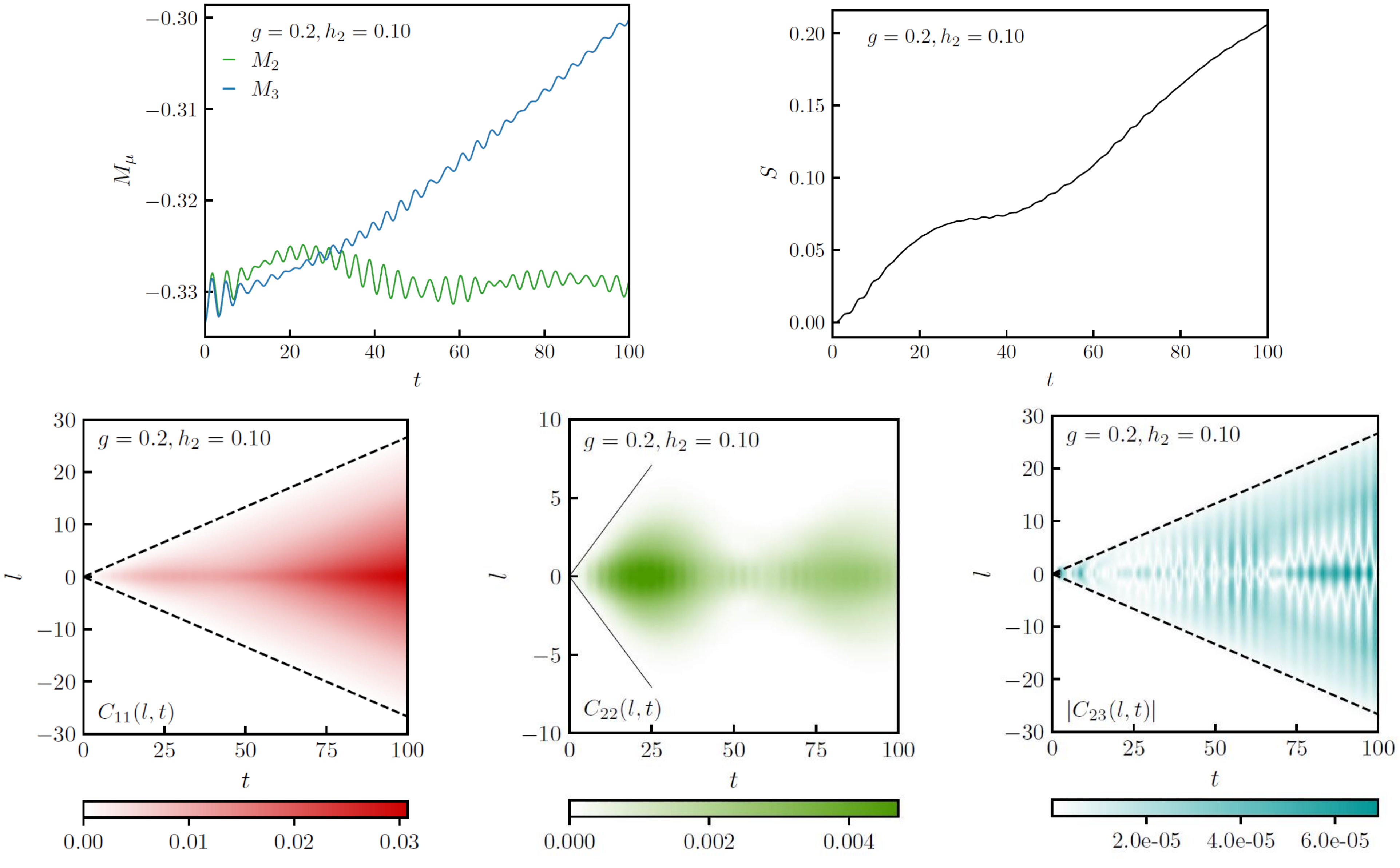




- Translation invariance → K=0
- Init state: close to TV
- Symmetry of the quench

Mesons and baryons!!

Global quenches of the Potts: positive oblique $h_2 > 0$



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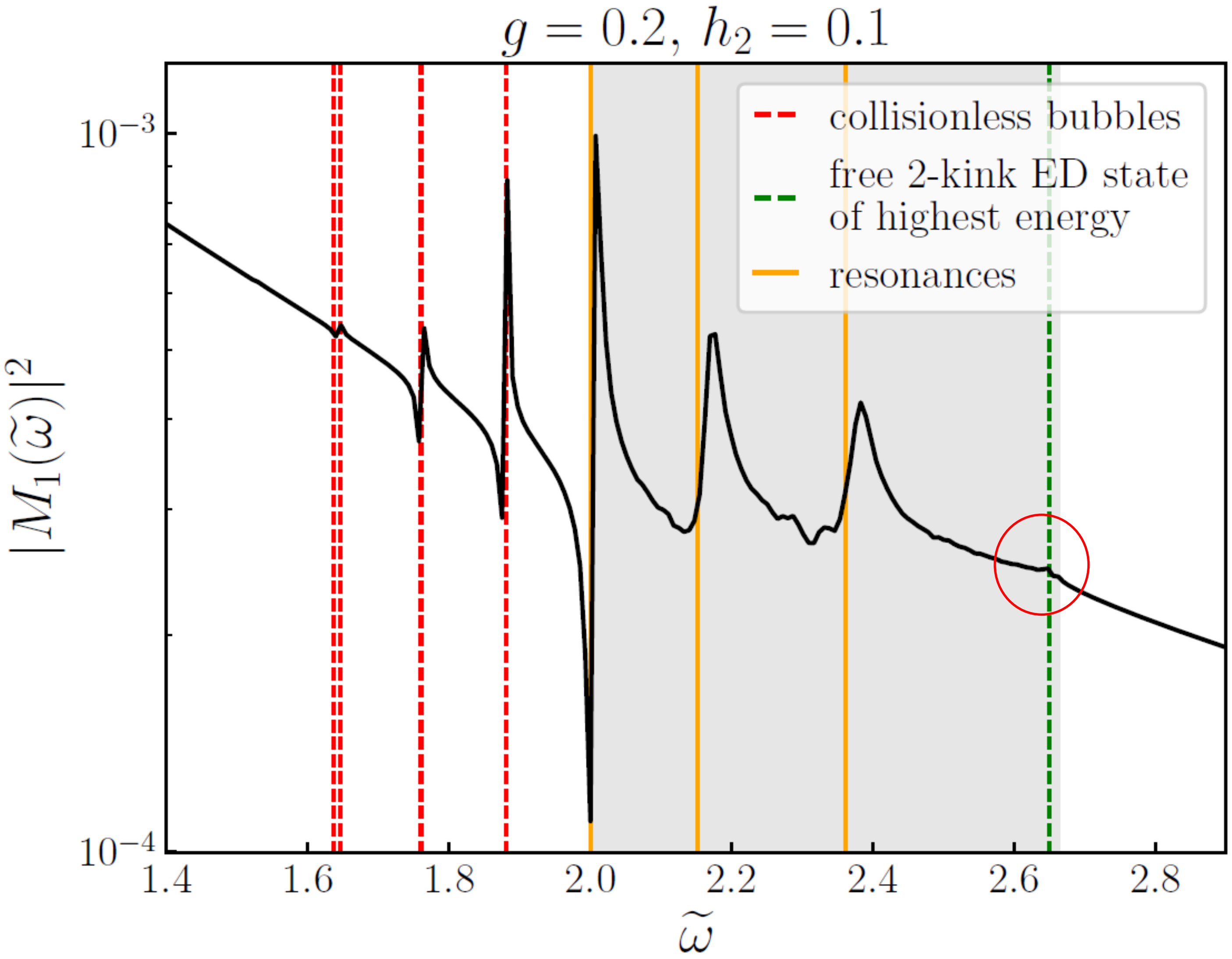

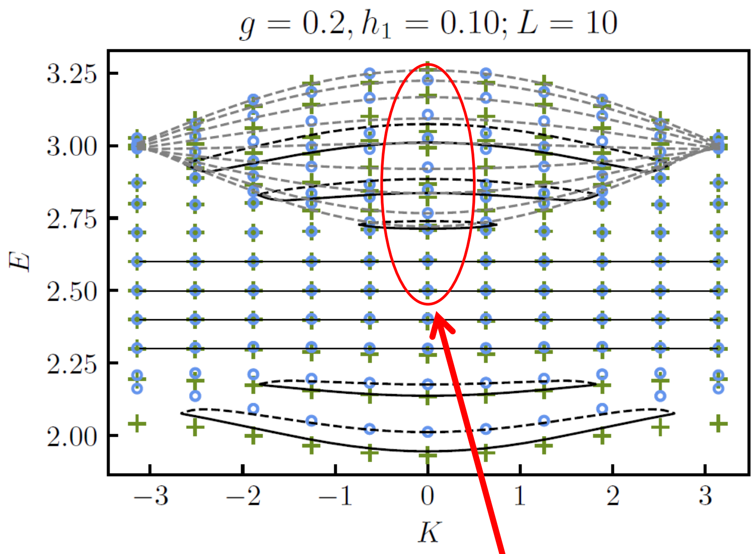

- Translation invariance → K=0
- Init state: close to FV
- No symmetry of the quench

Bubbles and free kinks!

↓

light-cone structure in correlators!

Perspetives

- Direct observation of the baryons?
- Full quantum mechanics of the mesons, bubbles, baryons
- Experimental realisation of the Potts (so far only for critical)

[R. Samajdar, et al., Phys. Rev. A 98 (2018) 023614]

[H. Bernien, et al., Nature 551 (2017) 579–584]

Summary

- Pure transverse Potts-model

- Construction of the low-lying energy spectrum of the mixed-field Potts model with semiclassics and ED

- Interpretation of the dynamics using the quasiparticle spectrum

    - Pure transverse: magnetization relaxes, correlators: light-cone structure

    - Parallel quenches: quasiparticle localisation, magnetisation: persistent oscillations, entropy saturates, correlators: light-cones get suppressed

    - Oblique quenches: partial localisation, correlators: some light-cones remain, entropy grows

Thank you for your attention!

Anna Krasznai

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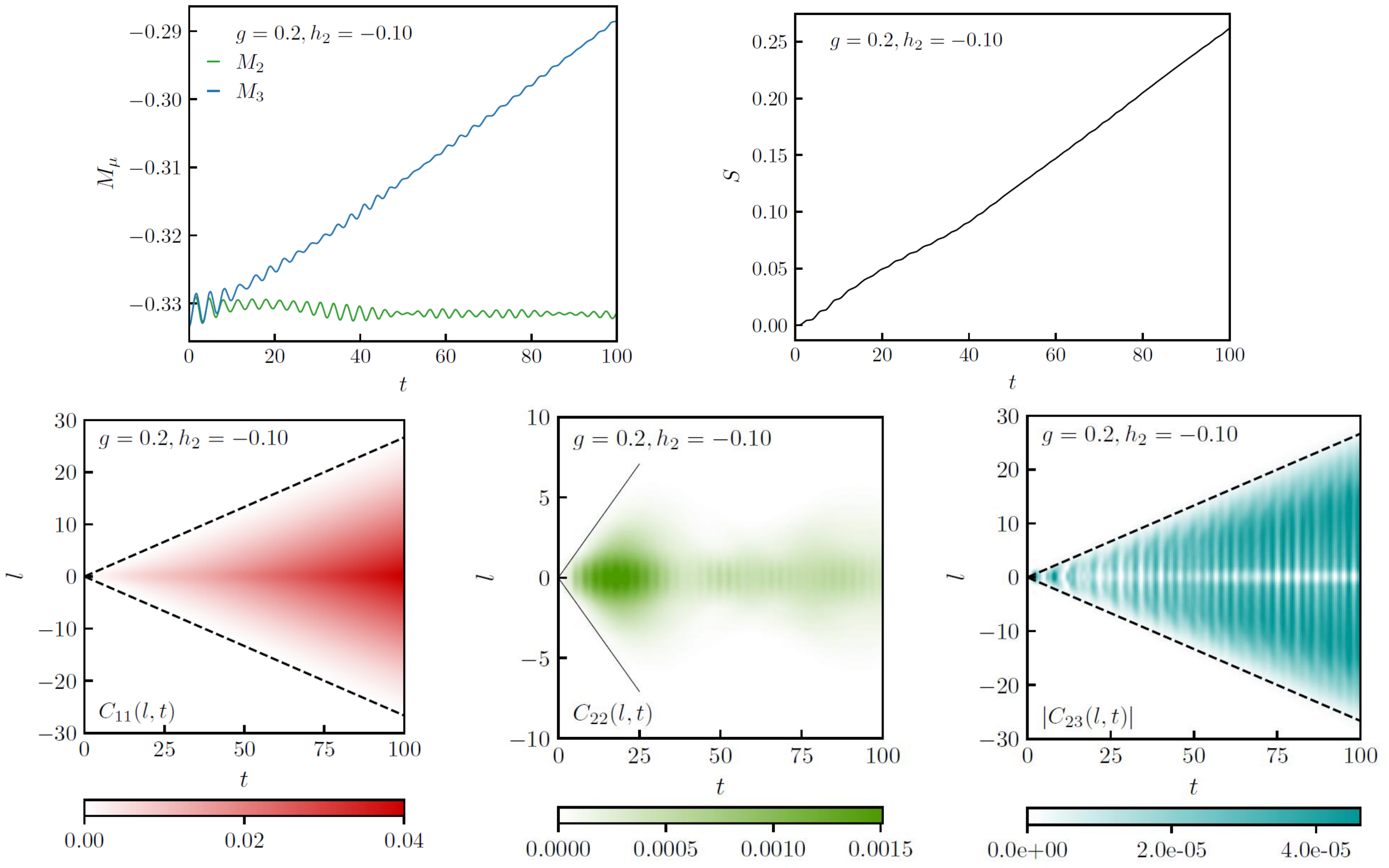
Global quenches of the Potts: negative oblique $h_2 < 0$



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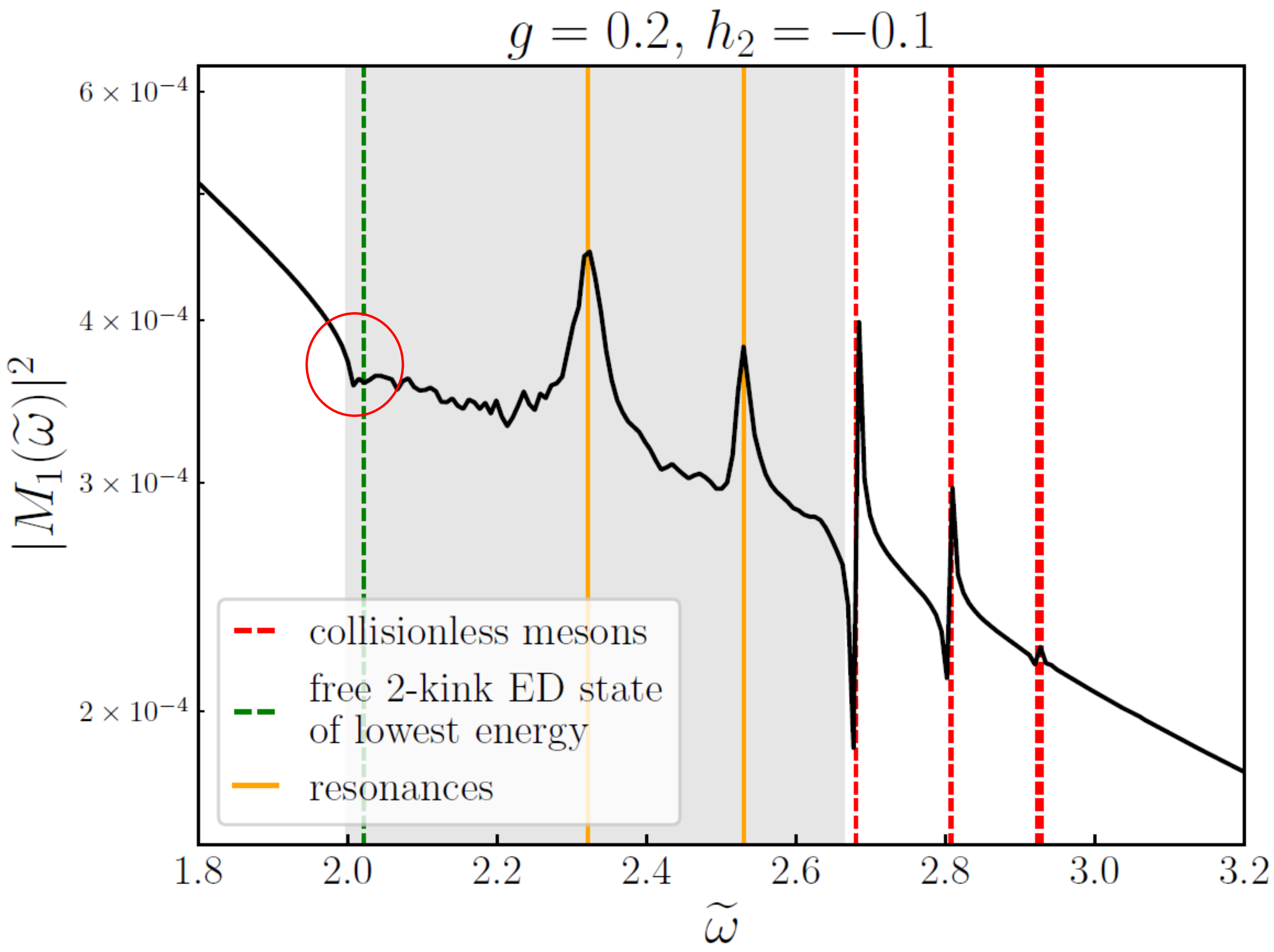
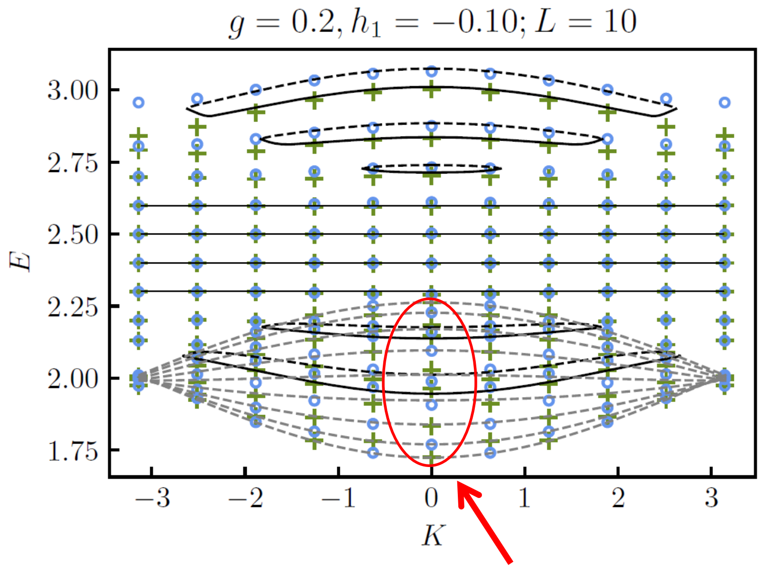




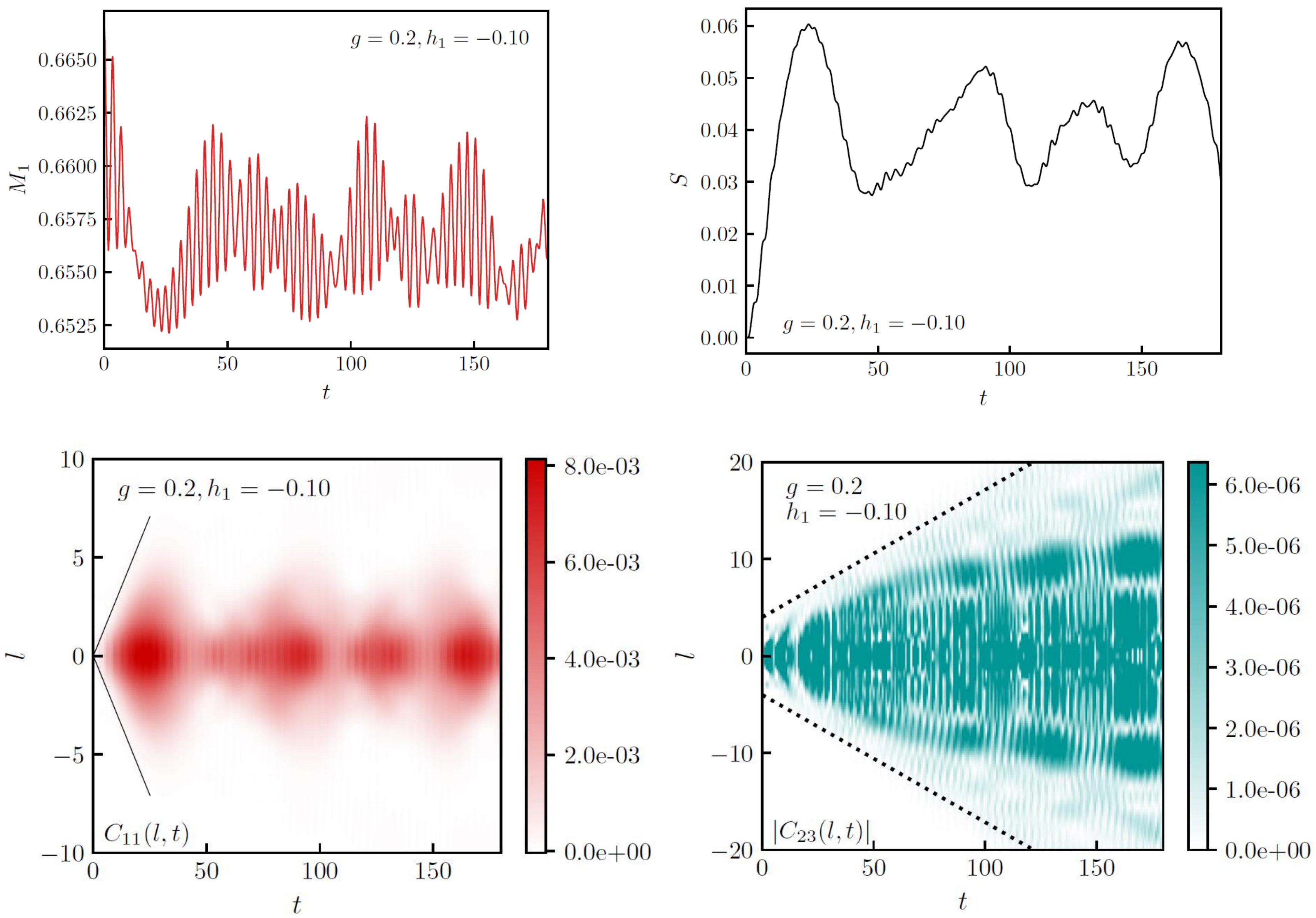
- Translation invariance → K=0
- Init state: close to TV
- No symmetry of the quench

Mesons and free kinks!

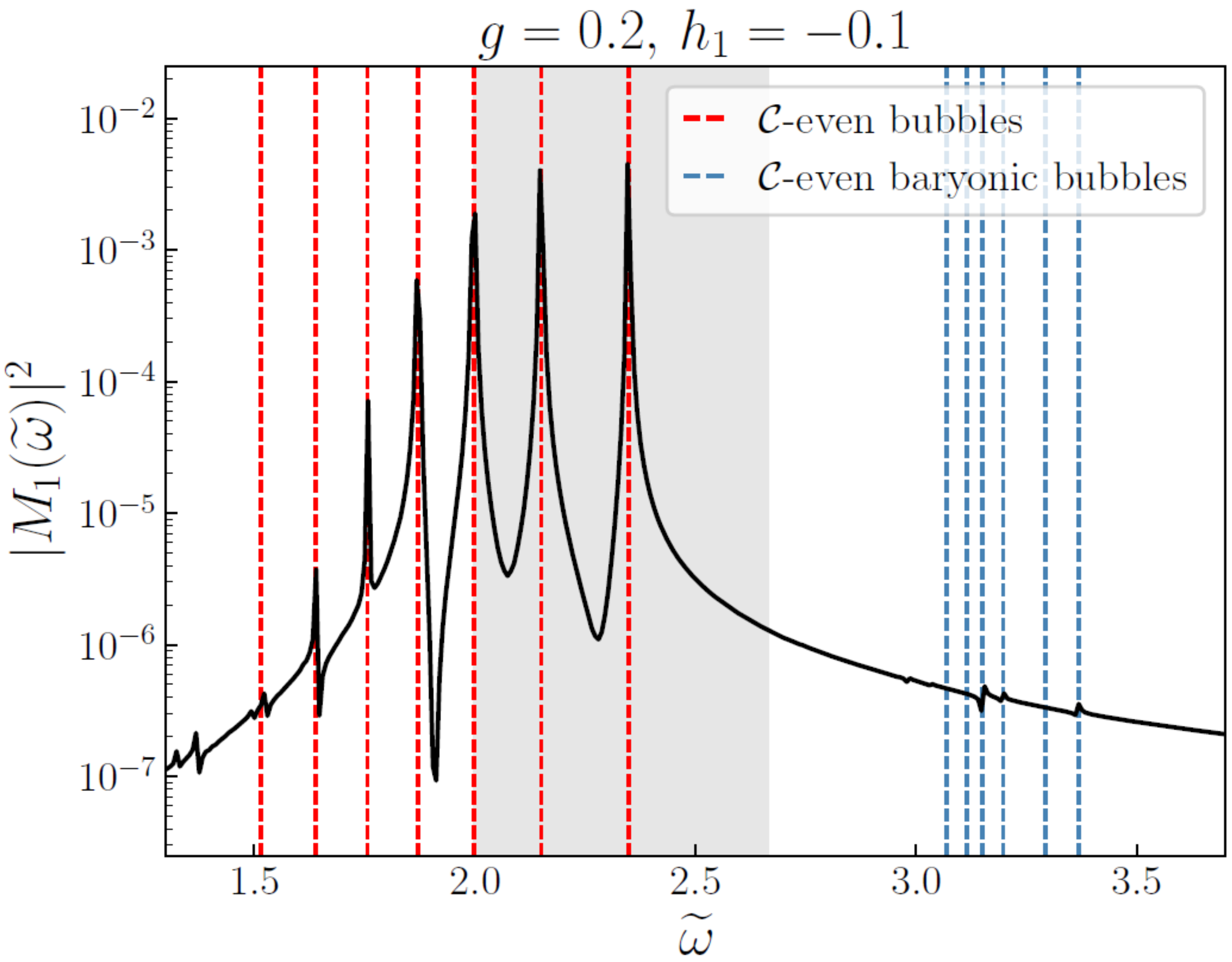
↓

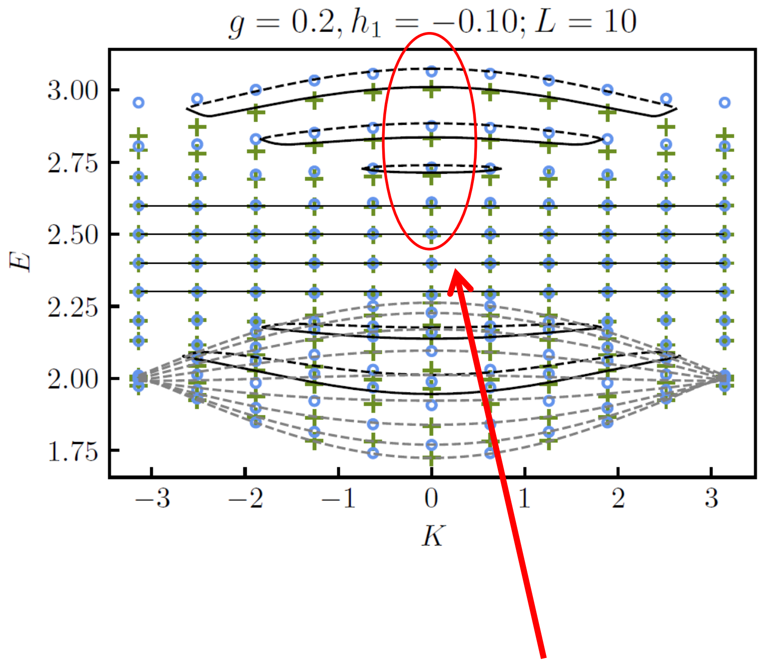
light-cone structure in correlators!

Global quenches of the Potts: anticonfining $h_1 < 0$



Global quenches of the Potts: anticonfining $h_1 < 0$





- Translation invariance → K=0
- Init state: close to FV
- Symmetry of the quench

Bubbles and baryonic bubbles!!

Scattering of kinks

- Scattering matrix: $$S(k_1,k_2) = \begin{pmatrix} s_3(k_1,k_2) & 0 & 0 & 0 \\ 0 & s_1(k_1,k_2) & s_2(k_1,k_2) & 0 \\ 0 & s_2(k_1,k_2) & s_1(k_1,k_2) & 0 \\ 0 & 0 & 0 & s_3(k_1,k_2) \end{pmatrix}$$

[Á. Rapp et al., New J. Phys. 15 (2013) 013058]

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- Kink-kink, antikink-antikink scattering:

$$K_{\alpha\gamma}(k_1)K_{\gamma\beta}(k_2) = s_3(k_1,k_2)K_{\alpha\gamma}(k_2)K_{\gamma\beta}(k_1), \qquad \alpha \neq \beta.$$

- Kink-antikink scattering:

$$K_{\alpha\gamma}(k_1)K_{\gamma\alpha}(k_2) = s_1(k_1,k_2)K_{\alpha\gamma}(k_2)K_{\gamma\alpha}(k_1) + s_2(k_1,k_2)K_{\alpha\beta}(k_2)K_{\beta\alpha}(k_1), \quad \beta \neq \gamma.$$

[Á. Rapp et al., New J. Phys. 15 (2013) 013058]

Scattering of kinks

- Symmetric and antisymmetric combinations:

$$s_t(k_1,k_2) = e^{i\delta_t(k_1,k_2)} = s_1(k_1,k_2) + s_2(k_1,k_2)$$
$$s_s(k_1,k_2) = e^{i\delta_s(k_1,k_2)} = s_1(k_1,k_2) - s_2(k_1,k_2)$$

- Phase shifts can be numerically calculated from ED

Semiclassical quantisation

- 2-kink effective Hamiltonian:

$$\mathcal{H} = \omega(k;K) + \chi|x|$$

$$\omega(k;K) = \epsilon(K/2+k) + \epsilon(K/2-k)$$

- Canonical EOM:

$$\dot{X}(t) = \frac{\partial\omega(k;K)}{\partial K}, \qquad \dot{K}(t) = 0$$

$$\dot{x}(t) = \frac{\partial\omega(k;K)}{\partial k}, \qquad \dot{k}(t) = -\chi\text{sign}(x(t))$$

total momentum is constant

Semiclassical quantisation

- Canonical EOM:

$$\dot{X}(t) = \frac{\partial\omega(k;K)}{\partial K}, \qquad \dot{K}(t) = 0$$

$$\dot{x}(t) = \frac{\partial\omega(k;K)}{\partial k}, \qquad \dot{k}(t) = -\chi \mathrm{sign}(x(t))$$

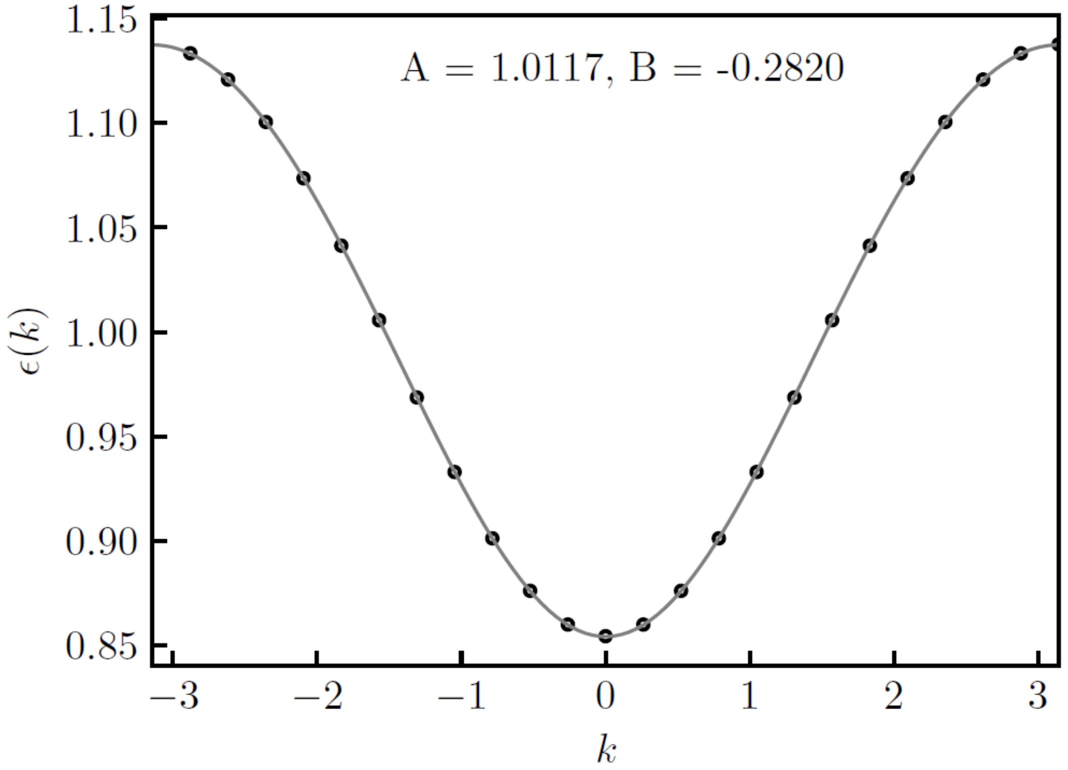
- Periodic motion irrespectively of the sign of χ due to Bloch-oscillations

- Bohr-Sommerfeld quantisation $\oint dx\, k(x) = 2\pi(\nu + 1/2)$

Measuring the kink dispersion relation

- Pure transverse + twisted boundary condition: $P^{\mu}_{L+1} = P^{\mu\pm1}_{L+1}$
- Fit function: $\epsilon(k) = \sqrt{A + B\cos k}$

	$g = 0.2$	$g = 0.3$	$g = 0.4$	$g = 0.5$	$g = 0.6$
A	1.0117	1.0291	1.0565	1.0955	1.1479
B	-0.2820	-0.4321	-0.5863	-0.7434	-0.9023



Calculation of the string-tension

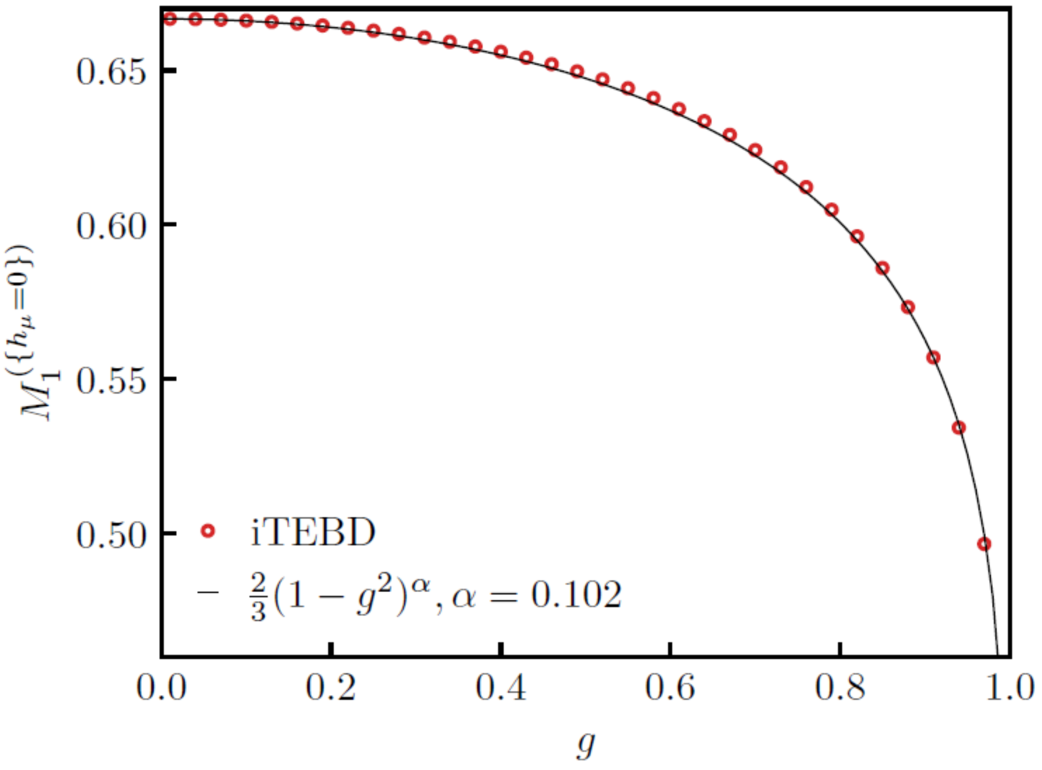
- The spontaneous magnetization should be calculated with ED

$$V(d) = \pm\chi d$$

$$\chi = |h_\mu|(M_\mu - M_{\nu\neq\mu})$$

$$\chi = |h_\mu|\,(1-g^2)^\alpha$$

$$\alpha \approx 0.102$$



Calculation of the phase shift

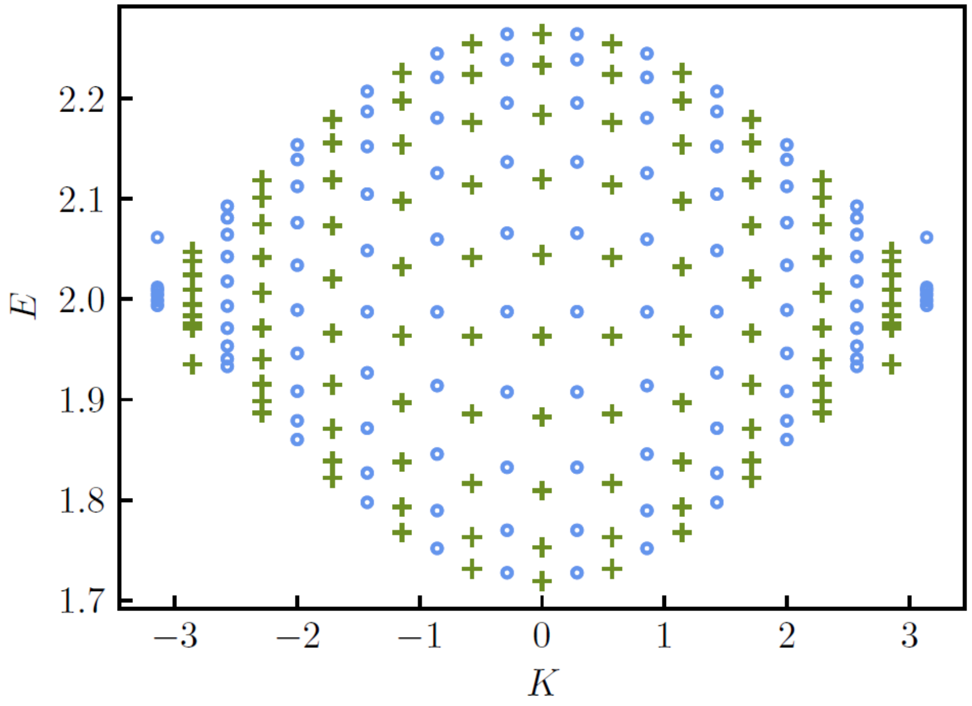
- Kink-antikink scattering
- ED with partially twisted sector $\tilde{T}_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$K^{(\kappa)} = k_1 + k_2 \qquad \tilde{T}_L^2 = \mathbb{I}$$

$$K^{(\kappa)} = \frac{2\pi}{\tilde{L}}\left(n_1^{(\kappa)} + n_2^{(\kappa)}\right), \qquad \tilde{L} = 2L$$

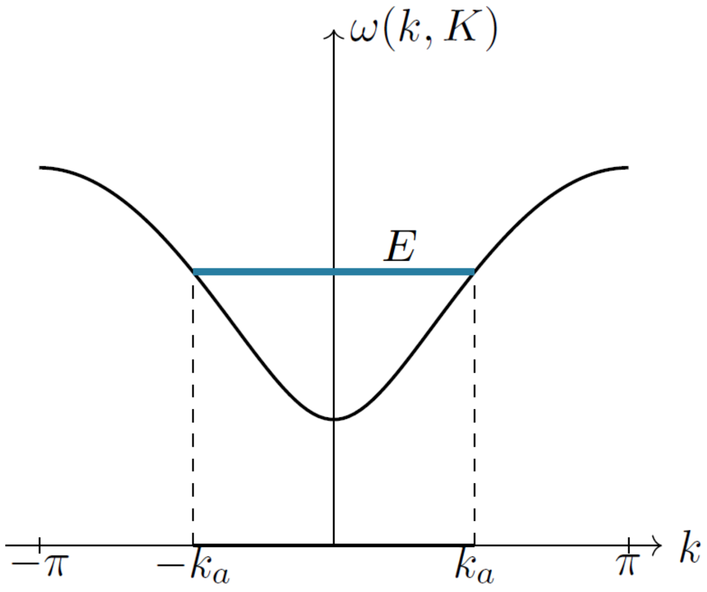
$$k_1\tilde{L} + \delta^{(\kappa)}(k_1, k_2) = 2\pi n_1^{(\kappa)}$$
$$k_2\tilde{L} + \delta^{(\kappa)}(k_2, k_1) = 2\pi n_2^{(\kappa)},$$

$$E = \epsilon(K/2 + k) + \epsilon(K/2 - k)$$

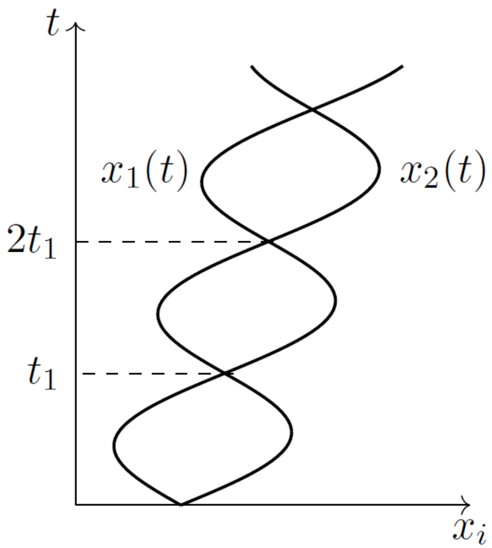


Semiclassical trajectories

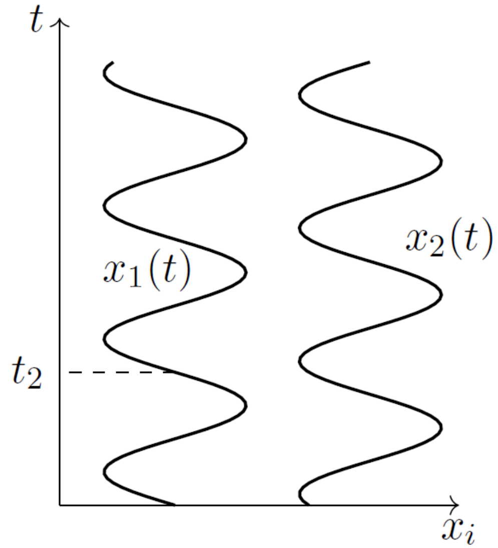
Mesons



Collisional:

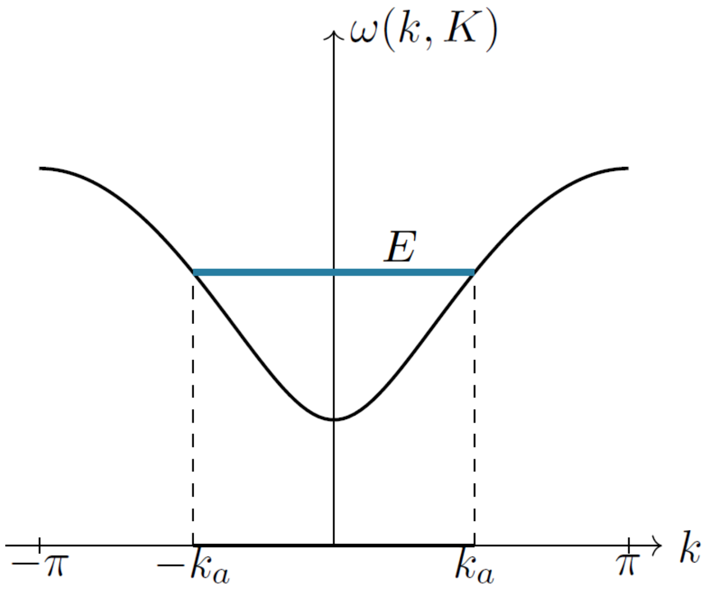


Collisionless:

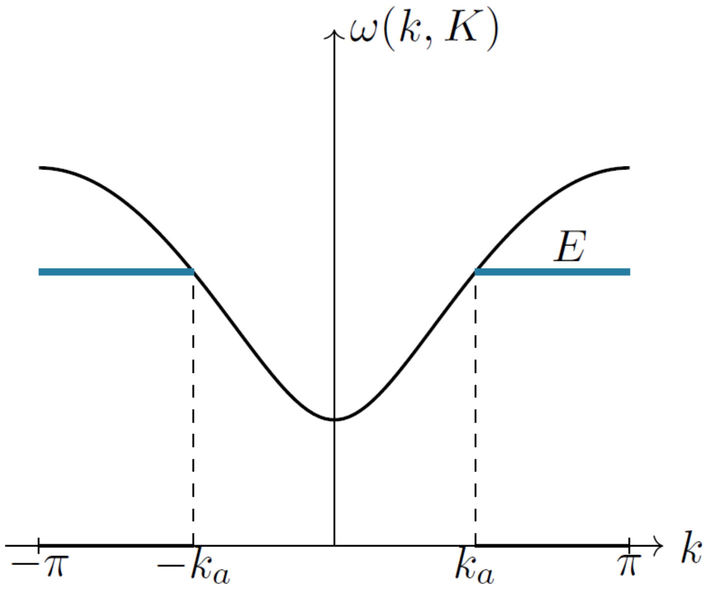


Semiclassical trajectories

Mesons



Bubbles



ED vs Semiclassical spectra

- Exact diagonalisation with PBC
- Operator to select the 2-kink states:

$$\mathcal{O}_{\mathrm{POTTS}} = \sum_i \left(P_i^1 P_{i+1}^2 + P_i^2 P_{i+1}^1 + P_i^2 P_{i+1}^3 + P_i^3 P_{i+1}^2 + P_i^3 P_{i+1}^1 + P_i^1 P_{i+1}^3 + \frac{2}{3} \right)$$