Nonanalytic correlation length in the Ising field theory

István Csépányi, Márton Kormos

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HUNGARY

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Background

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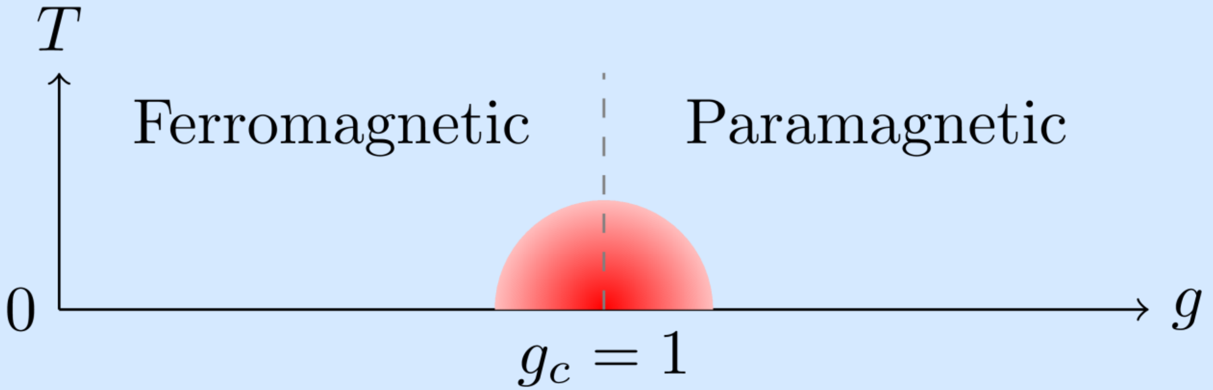

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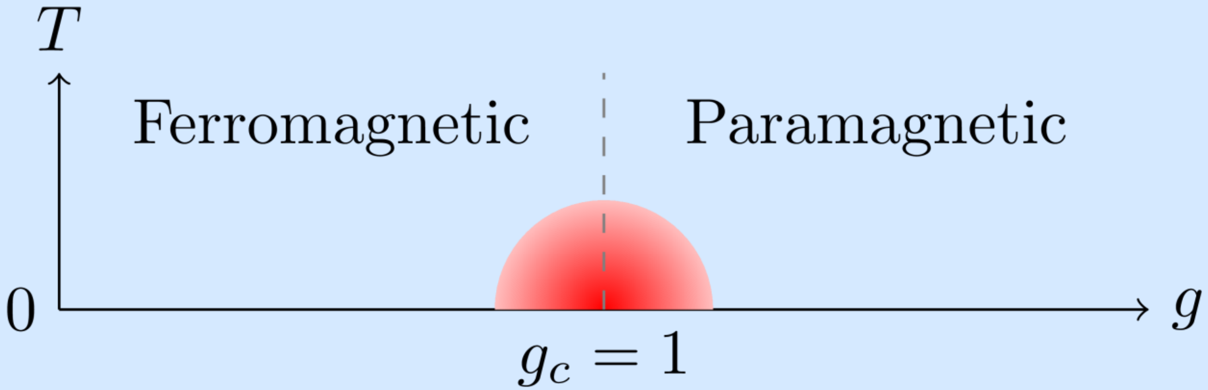

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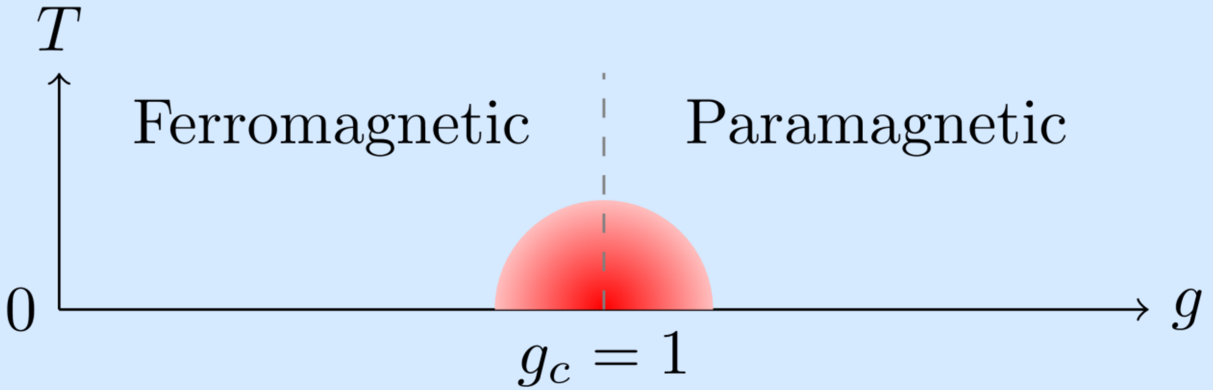

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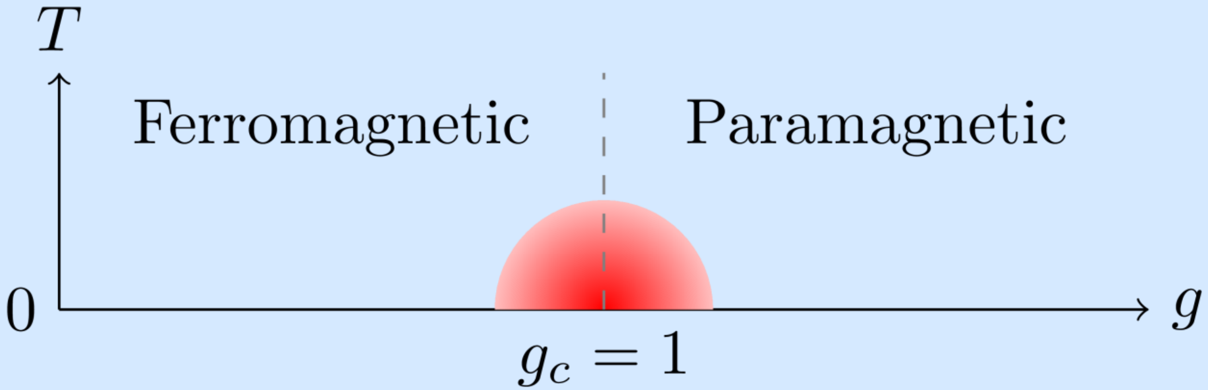

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Four parameters: x, $t = \zeta x$, β and PM/FM.

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Finite temperature form factor expansion

$$\left|\langle\theta_1,\ldots\theta_n|\hat{\sigma}|\theta'_1,\ldots\theta'_m\rangle\right| = m^{1/8}\,\frac{\prod\limits_{1\le i<j\le n}\tanh\left(\frac{\theta_i-\theta_j}{2}\right)\prod\limits_{1\le k<l\le m}\tanh\left(\frac{\theta'_k-\theta'_l}{2}\right)}{\prod\limits_{i=1}^{n}\prod\limits_{k=1}^{m}\tanh\left(\frac{\theta_i-\theta'_k}{2}\right)}$$

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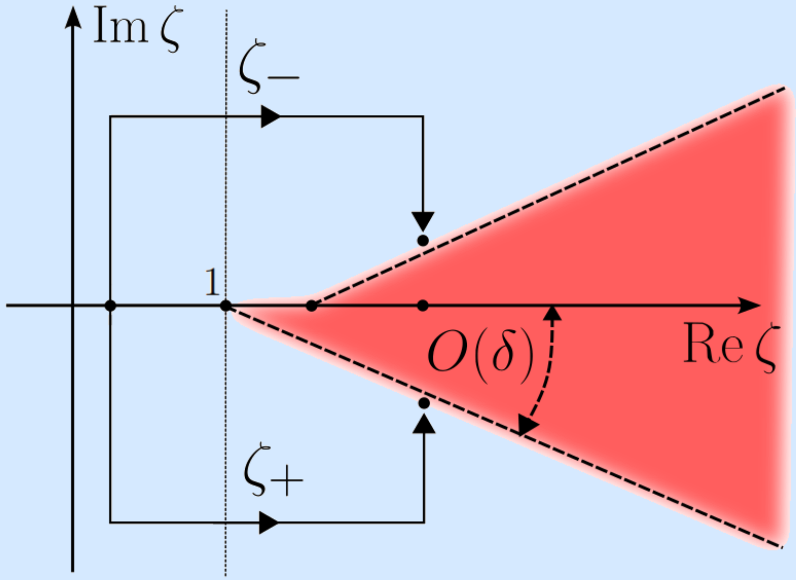

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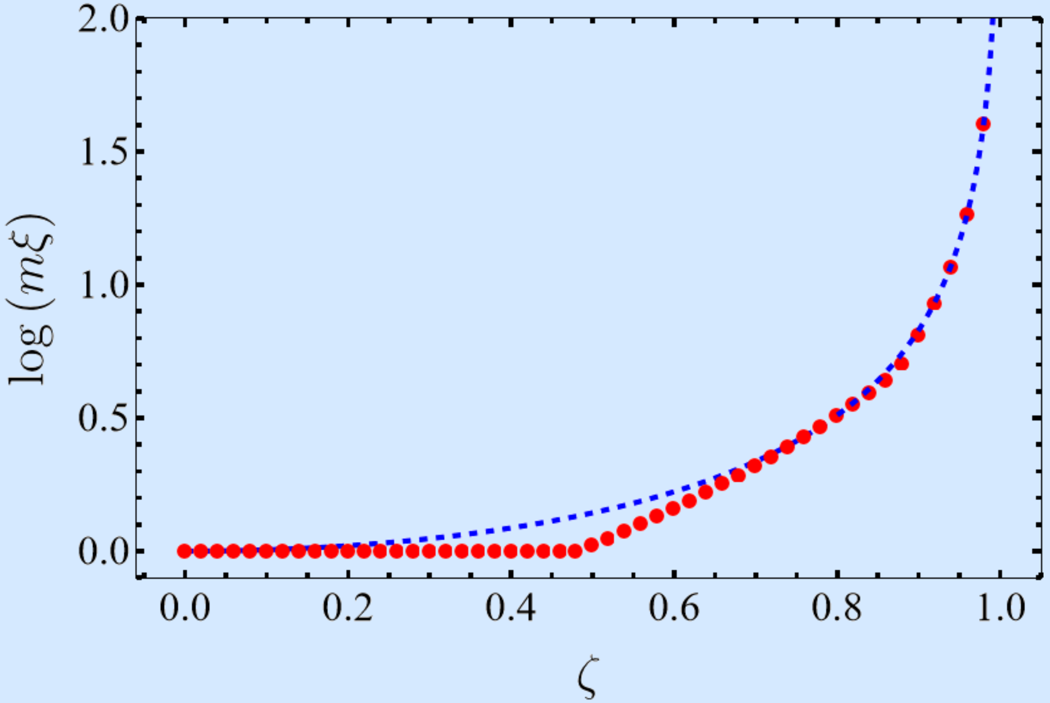

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