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BH scattering, inspirals and ringdowns

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Motivations

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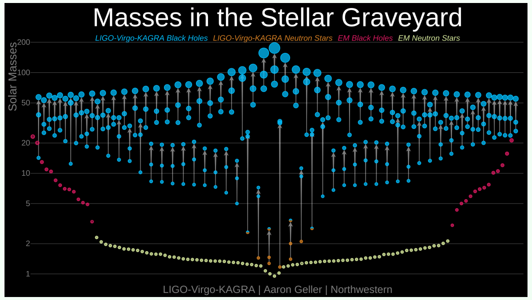
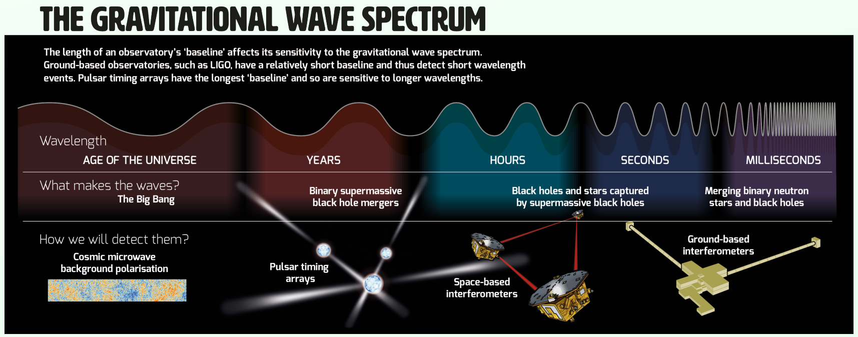
- GW Astronomy Era



- Ground based detectors: LIGO, VIRGO, KAGRA, ET, CE



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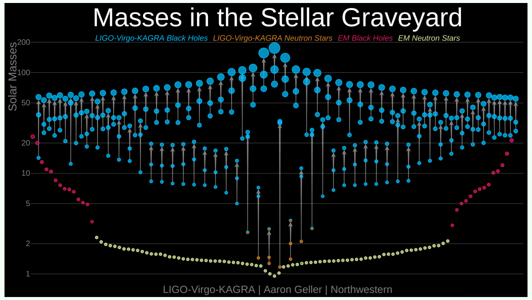

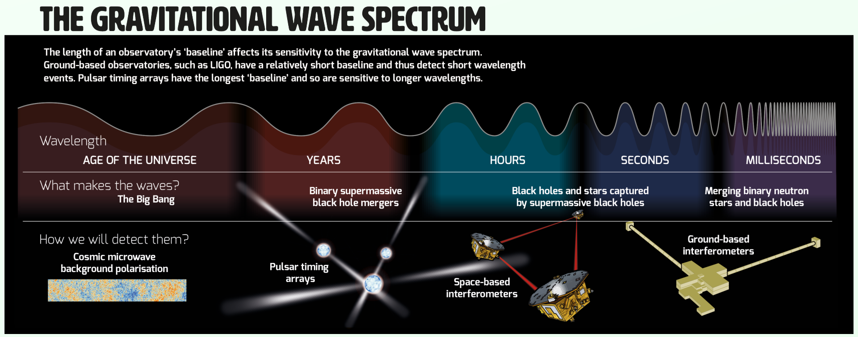

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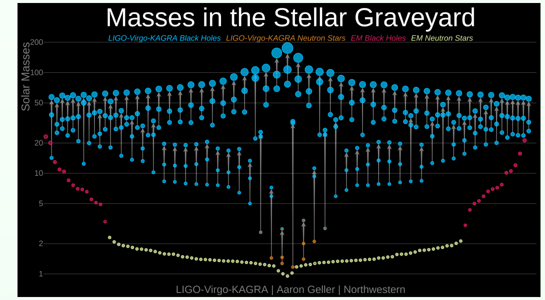

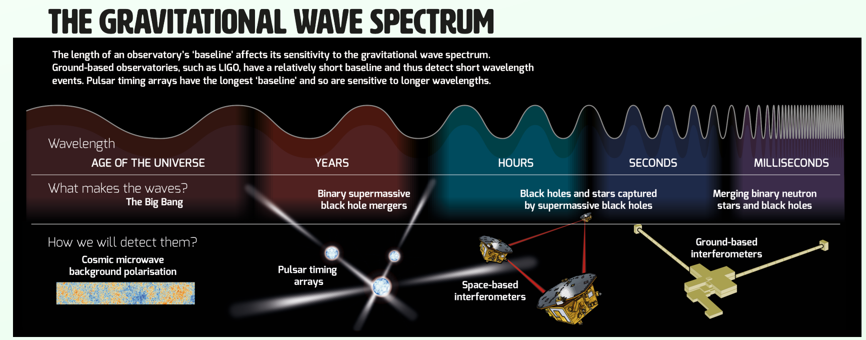

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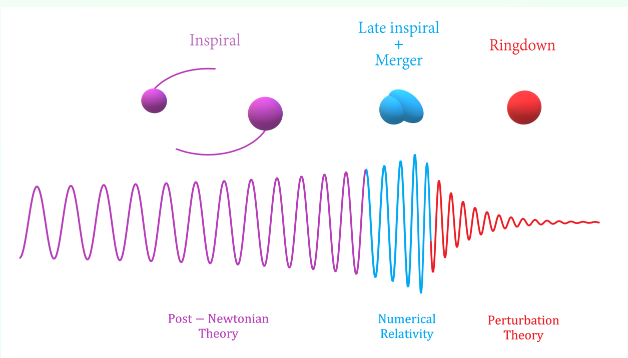
- **What we have seen ?**



Binary system collision: Inspiral, Merging and Ringdown

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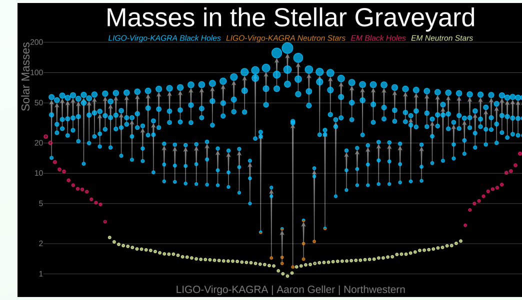

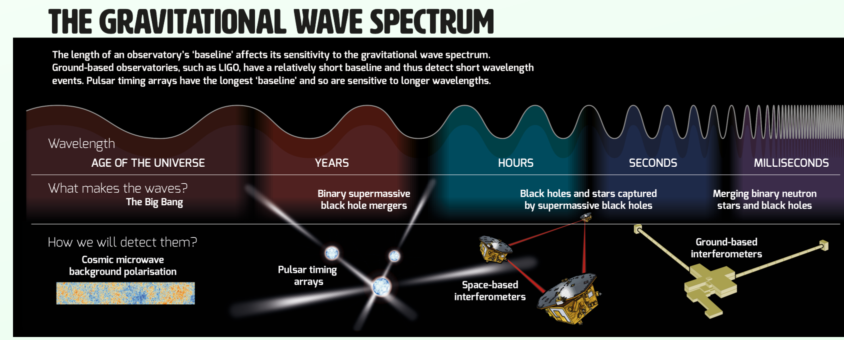

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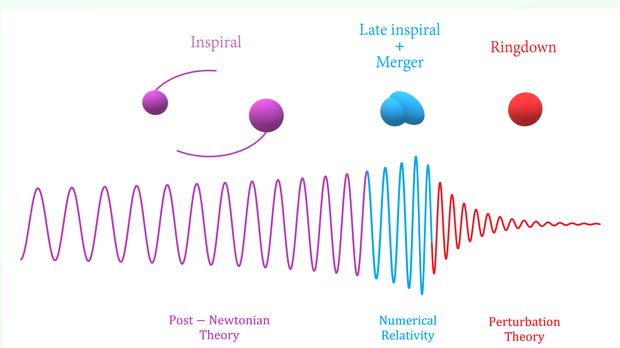

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⟹ All observations consistent with GR !

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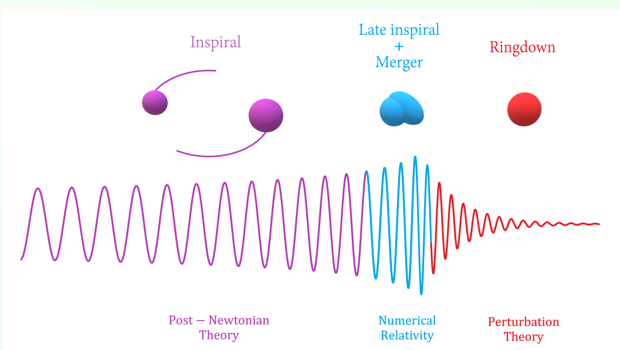

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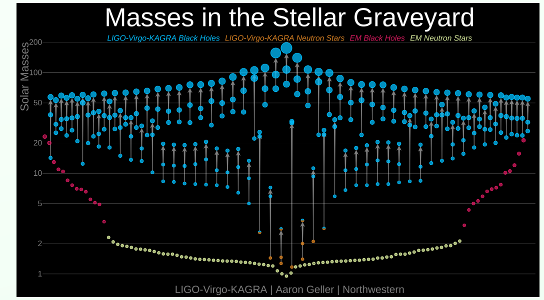

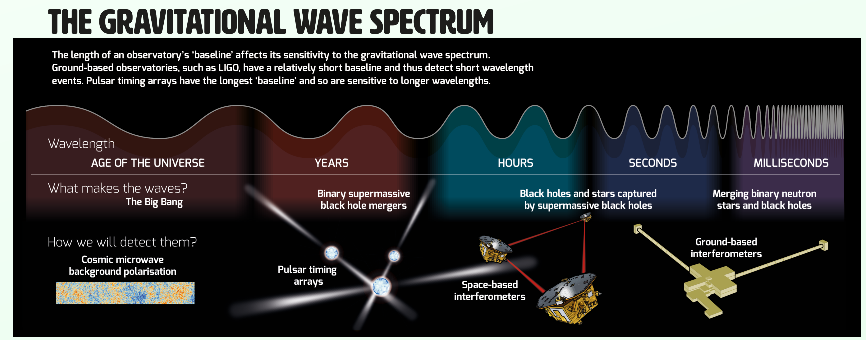

All observations consistent with GR !

Neutron stars and BH's (2020 R. Penrose Nobel prize)

"for the discovery that BH formation is a robust prediction of GR"

BHs vs ECO's

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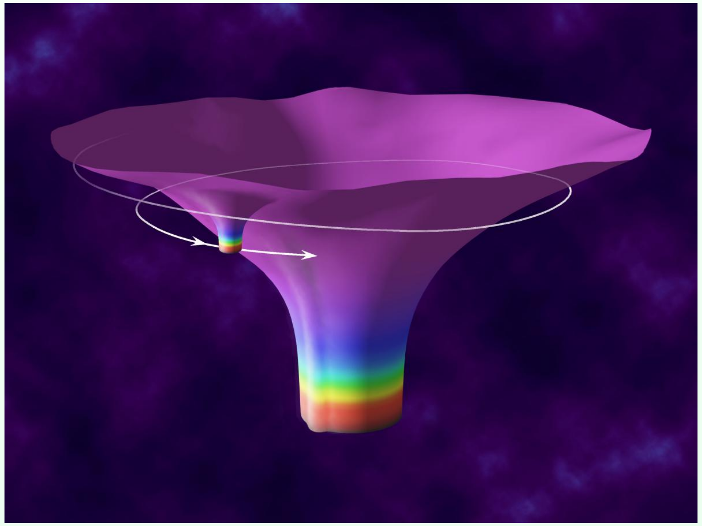
- Black holes ?



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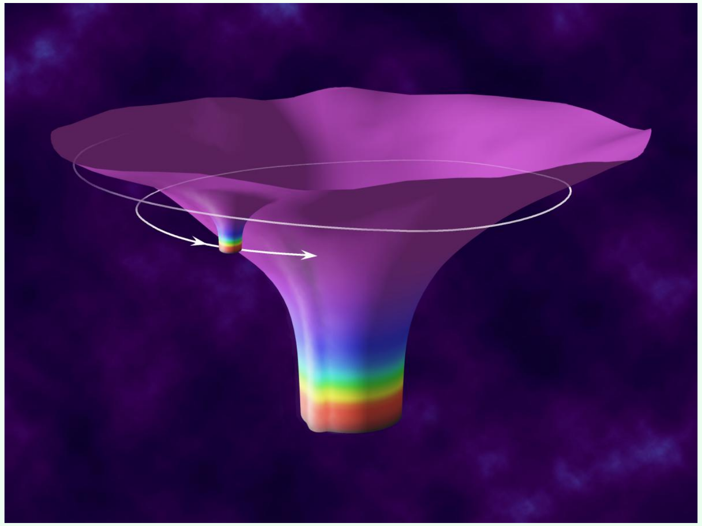

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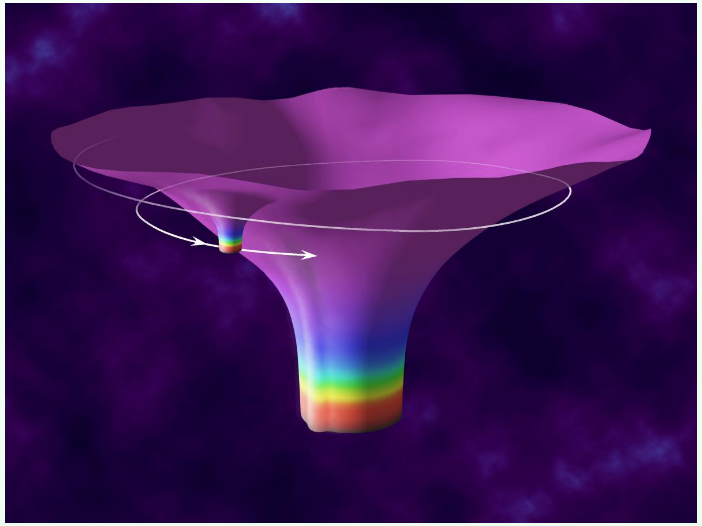

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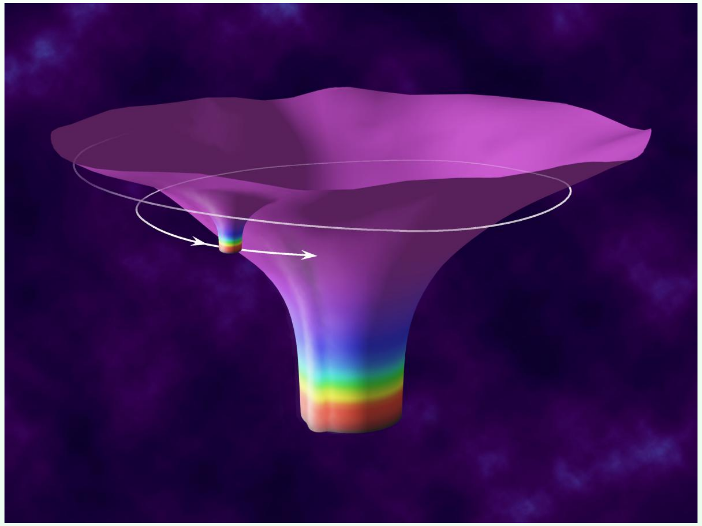

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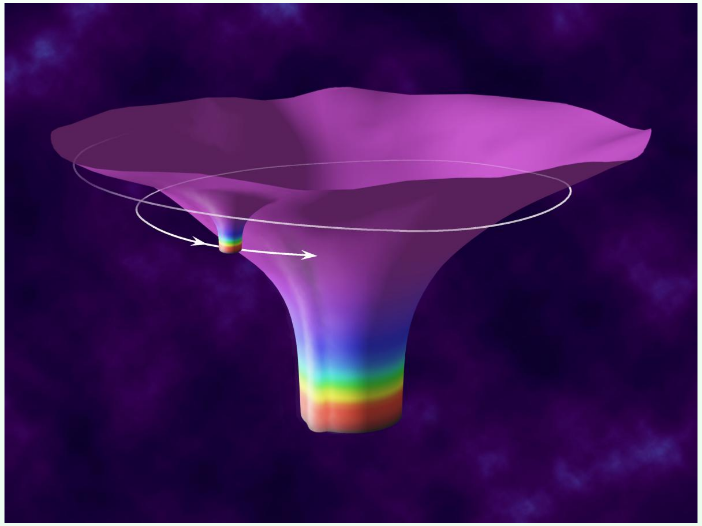

Inspiral Ringdown

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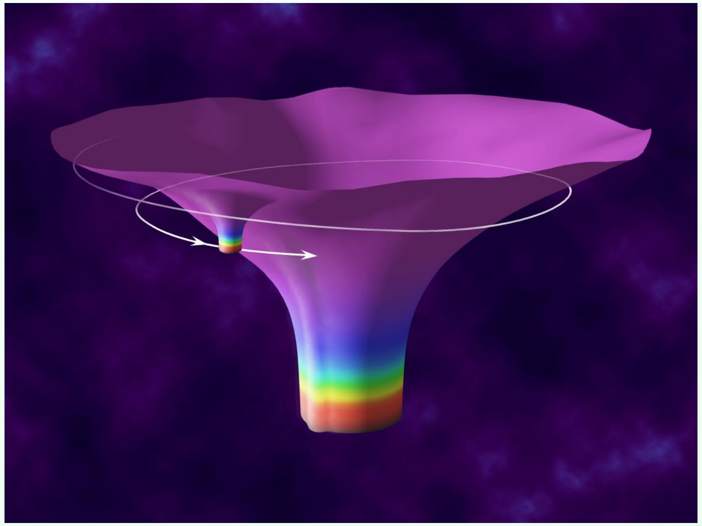

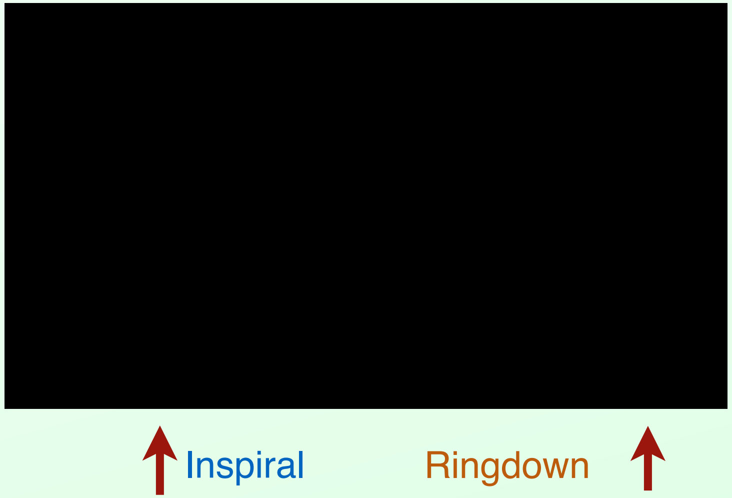

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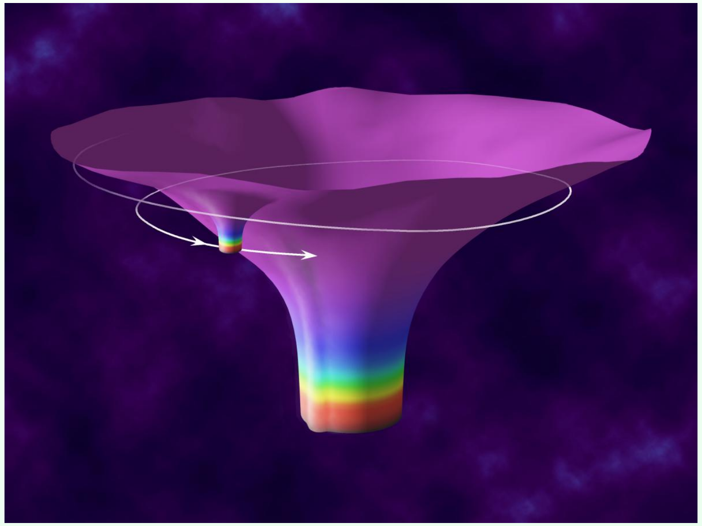

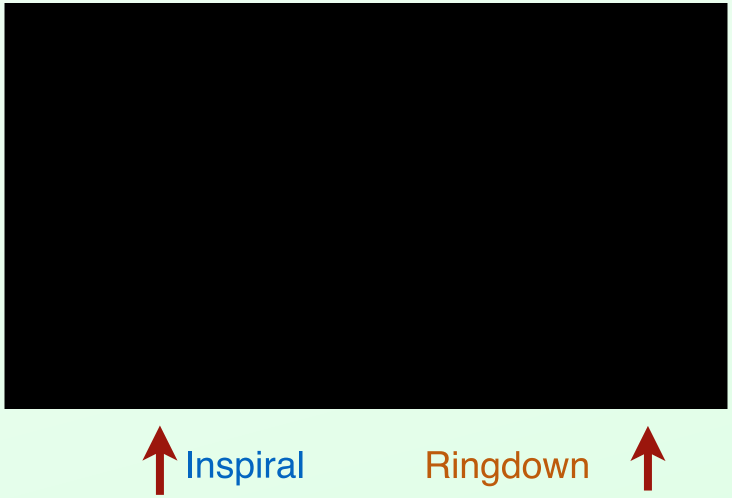

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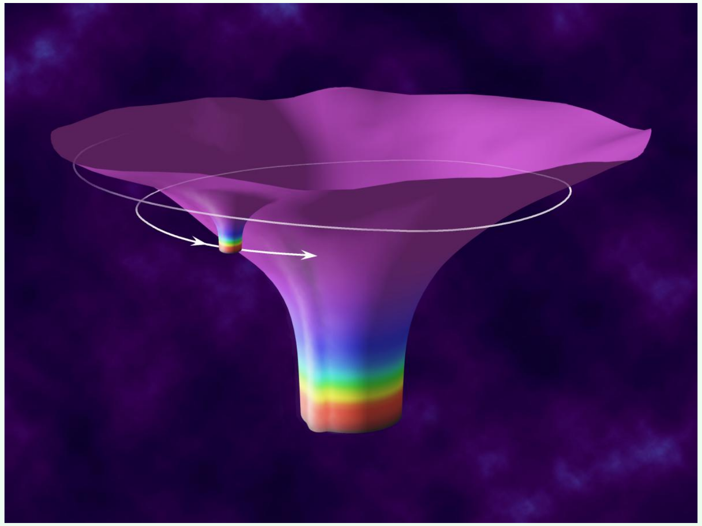

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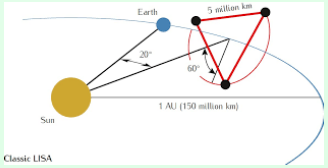
  - EMRI's: Extreme mass ratio inspiral $\frac{\mu}{M} << 1$ $10^4 - 10^6$ orbits

Inspiral Ringdown

LISA



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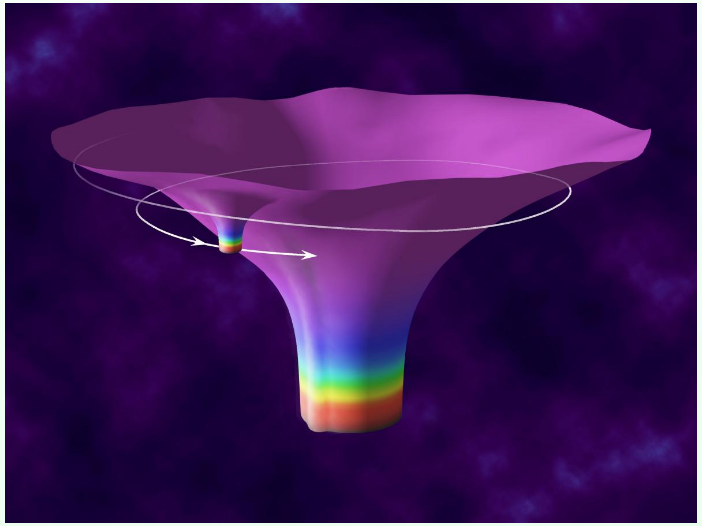
Alternatives: Multipole expansion, amplitudes



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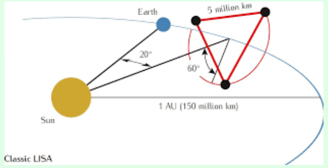

Outline

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- The Physical problem

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  - Black hole perturbation

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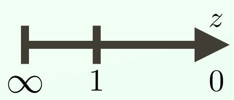

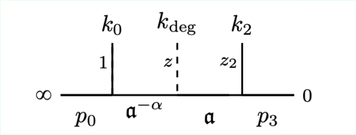

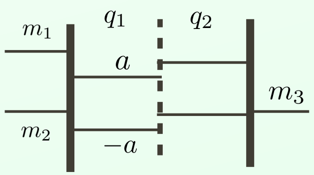

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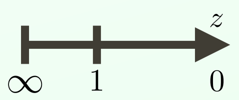

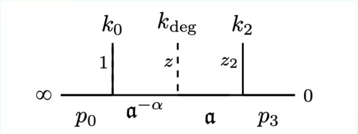

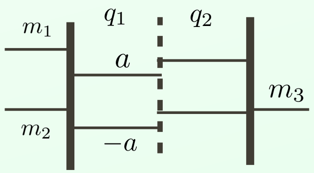

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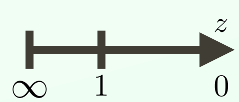

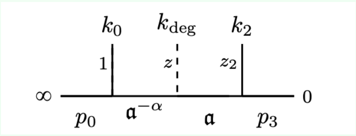

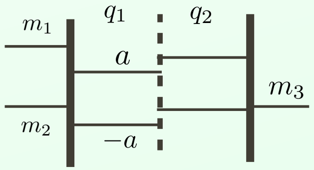

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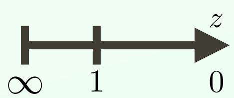

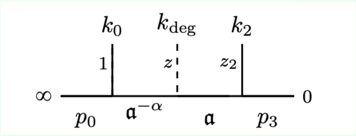

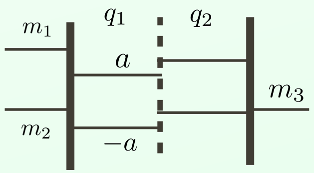

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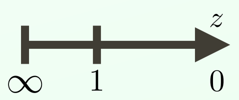

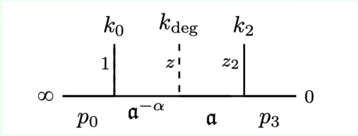

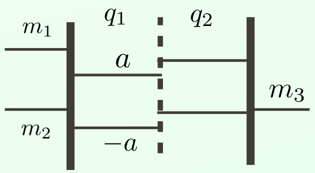

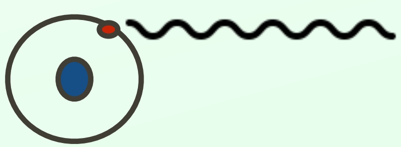

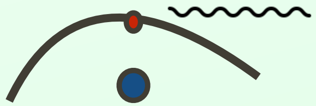

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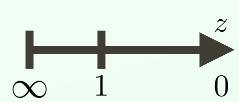

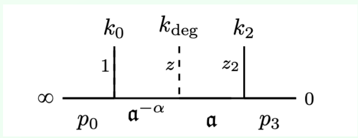

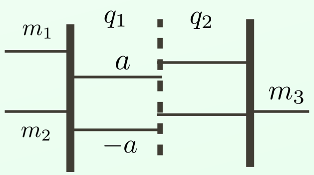

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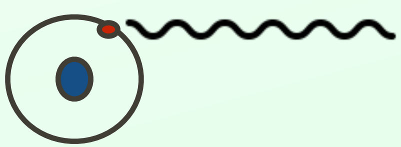

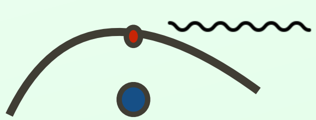

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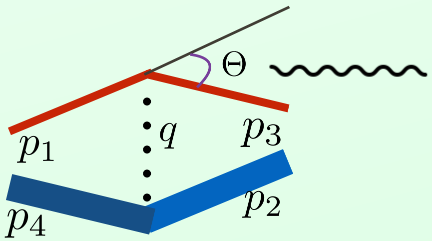
  - Soft limit of gravitational scattering   Memory effects and universal log towers



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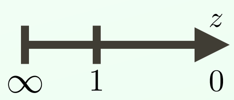

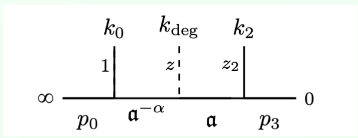

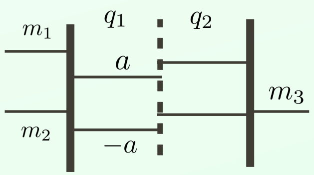

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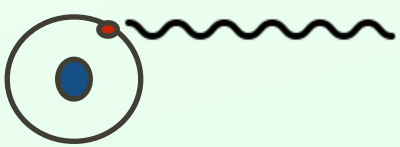

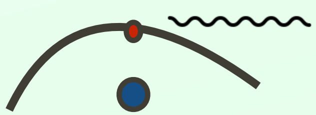

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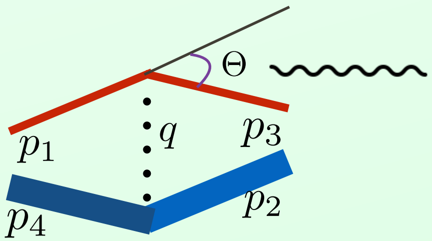

- **Conclusions:**

Black hole perturbation theory

Black hole perturbation theory

- Linearized Einstein Equations

$$G_{MN} = 8\pi T_{MN} \qquad g_{MN} = g^0_{MN} + h_{MN} \qquad r^4 C_{n\bar{m}n\bar{m}} = \int d\omega \sum_{\ell,m} e^{\mathrm{i}m\phi - \mathrm{i}\omega t} R_{\ell m}(r)\, {}_s S_{\ell m}(\chi)$$

Schwarzschild metric

Weyl tensor

$$\chi = \cos\theta \qquad f(r) = 1 - \frac{2M}{r}$$

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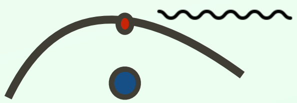
⇒ Teukolski equation

$$r^4 f(r)^2 \frac{d}{dr}\left[\frac{R'_{\ell m}(r)}{r^2 f(r)}\right] + \left(\frac{\omega^2 r^2 + 4\mathrm{i}(r-M)\omega}{f(r)} - 8\mathrm{i}\omega r - (\ell+2)(\ell-1)\right) R_{\ell m}(r) = T_{\ell m}(r)$$



radial distance

wave frequency

mass

angular momentum

Stress energy tensor

Black hole perturbation theory

- **Linearized Einstein Equations**

$$G_{MN} = 8\pi T_{MN} \qquad g_{MN} = g^0_{MN} + h_{MN} \qquad r^4 C_{n\bar{m}n\bar{m}} = \int d\omega \sum_{\ell,m} e^{\mathrm{i}m\phi - \mathrm{i}\omega t} R_{\ell m}(r)\, {}_s S_{\ell m}(\chi)$$

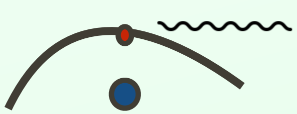
Schwarzschild metric — Weyl tensor

$$\chi = \cos\theta \qquad f(r) = 1 - \frac{2M}{r}$$

⇨ Teukolski equation

$$r^4 f(r)^2 \frac{d}{dr}\left[\frac{R'_{\ell m}(r)}{r^2 f(r)}\right] + \left(\frac{\omega^2 r^2 + 4\mathrm{i}(r-M)\omega}{f(r)} - 8\mathrm{i}\omega r - (\ell+2)(\ell-1)\right) R_{\ell m}(r) = T_{\ell m}(r)$$

radial distance — wave frequency — mass — angular momentum — Stress energy tensor



- **Singularities**
  - Two regular singularities at: $r = 0, 2M$
  - One irregular singularity at: $r = \infty$

} Confluent Heun Equation !

Black hole perturbation theory

- ## Linearized Einstein Equations

$$G_{MN} = 8\pi T_{MN} \qquad g_{MN} = g^0_{MN} + h_{MN} \qquad r^4 C_{n\bar{m}n\bar{m}} = \int d\omega \sum_{\ell,m} e^{im\phi - i\omega t} R_{\ell m}(r)\, {}_sS_{\ell m}(\chi)$$

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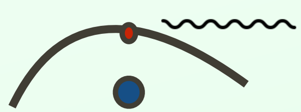
radial distance

wave frequency

mass

angular momentum

Stress energy tensor



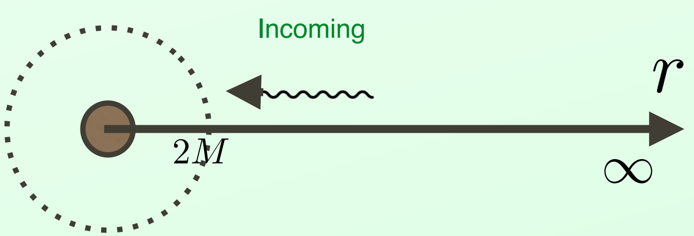
- ## Singularities
  - Two regular singularities at: $r = 0, 2M$
  - One irregular singularity at: $r = \infty$

} Confluent Heun Equation !

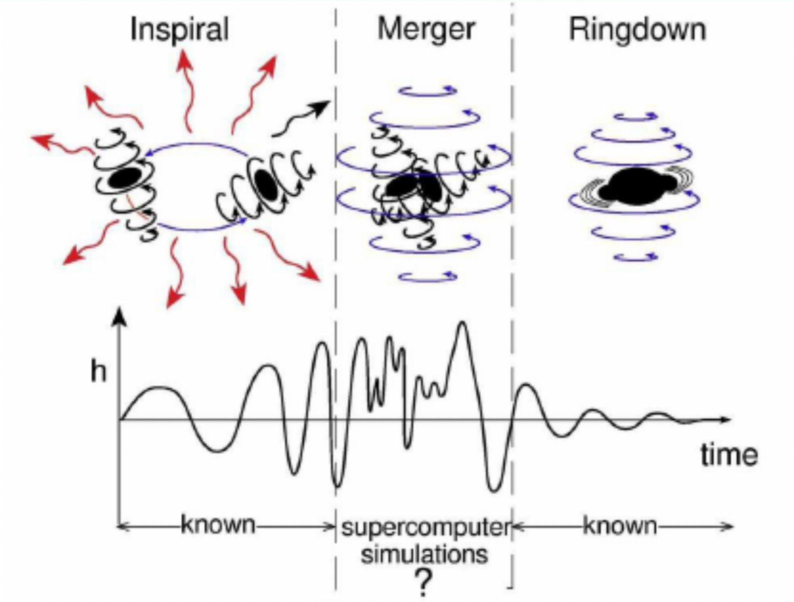
- ## Boundary conditions

$$R_{\ell m}(r) \underset{r\to 2M}{\approx} D_- \left(1 - \frac{2M}{r}\right)^{-1-2i\omega M} \cancel{+ D_+ \left(1 - \frac{2M}{r}\right)^{2i\omega M}}$$

Incoming wave

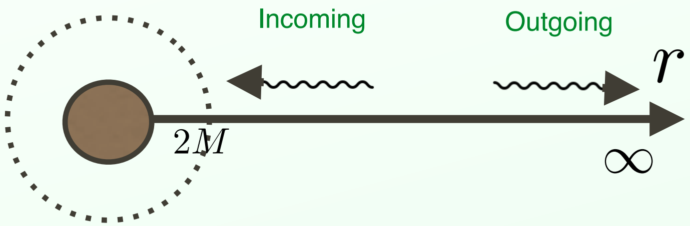


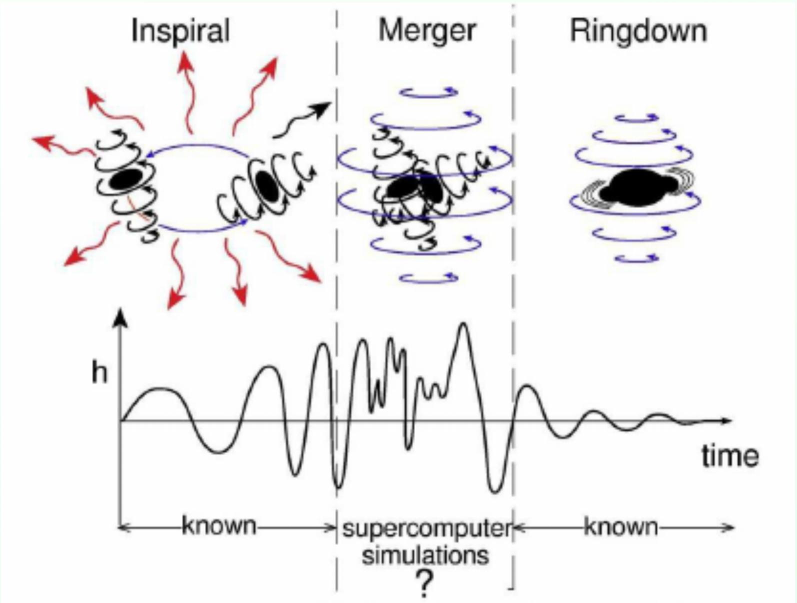
Ringdown & Inspiral



Ringdown & Inspiral

- ## Ringdown
  - Dominated by: QuasiNormalModes's





Ringdown & Inspiral

- **Ringdown**
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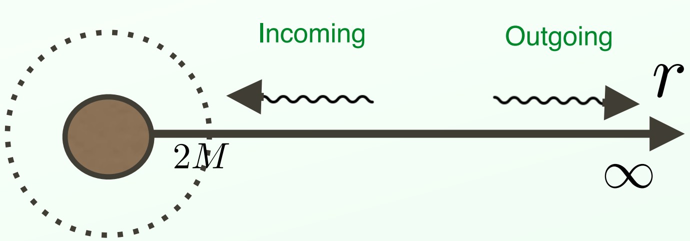
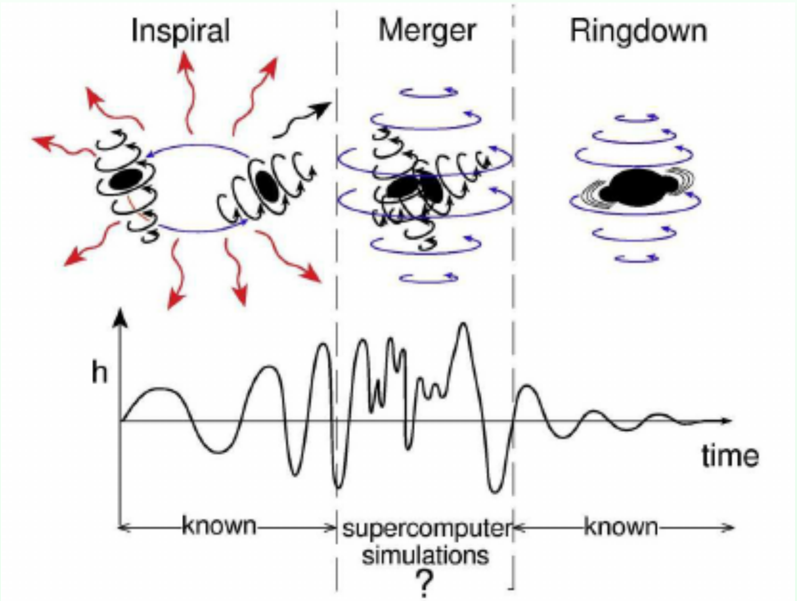


$\Rightarrow$ Connection formulas



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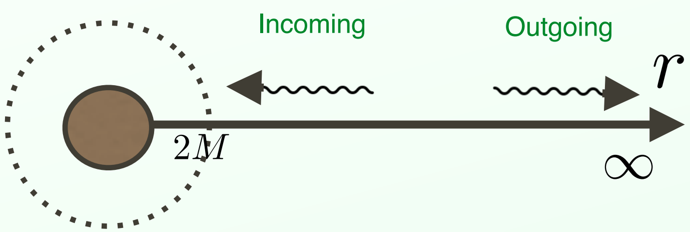

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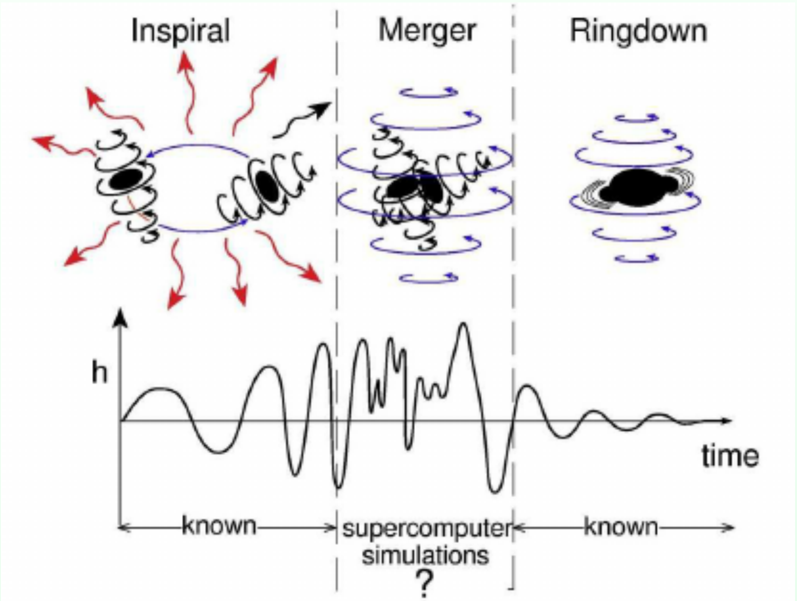
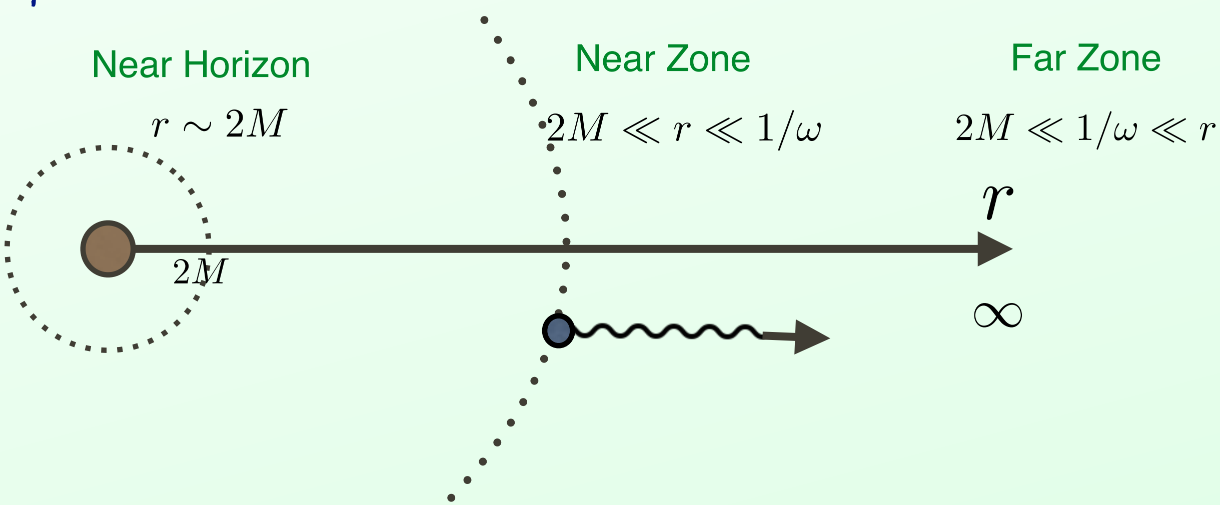


- ## Inspiral



Ringdown & Inspiral

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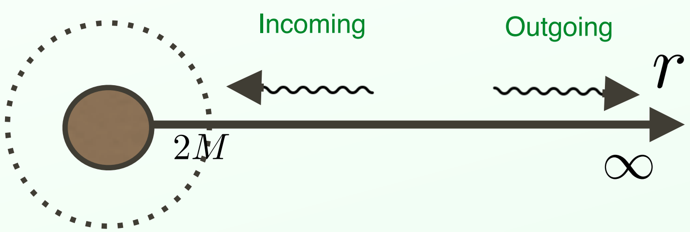

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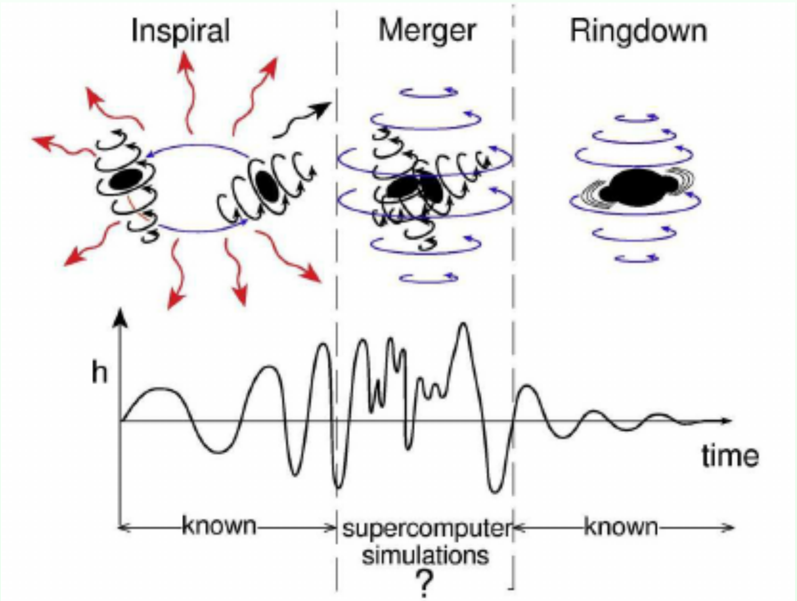
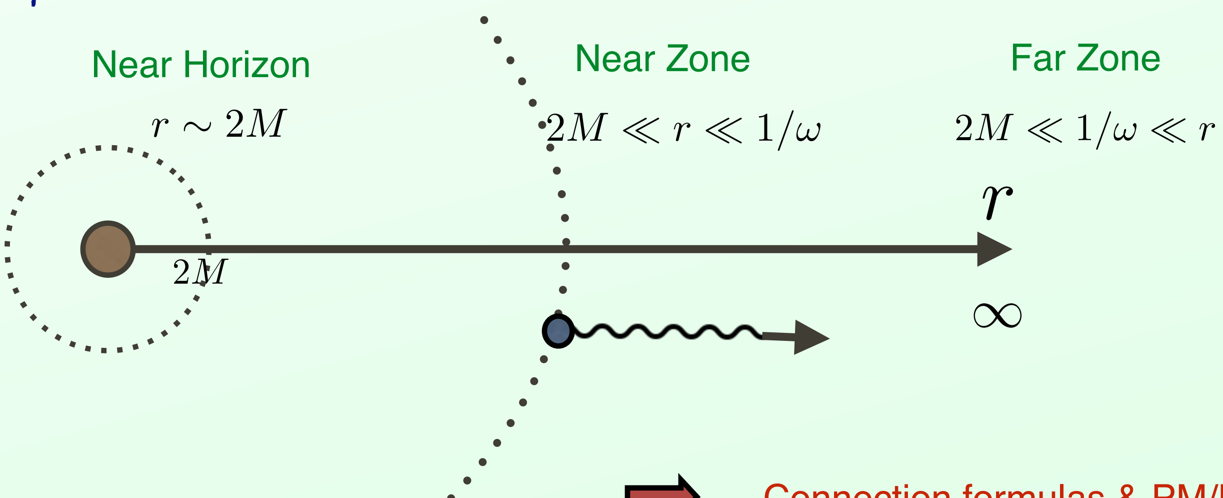


- **Inspiral**



$\Rightarrow$ Connection formulas & PM/PN expansions ?

Heun gravity dictionary

Heun gravity dictionary

- **Homogenous Equation** Schrodinger like form

$$\Psi''(z) + Q(z)\Psi(z) = 0 \qquad R(r) = \frac{(1-z)^{\frac{1}{2}}}{z^2}\Psi(z) \qquad z = \frac{2M}{r}$$

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$$Q_{\text{gauge}} = -\frac{x^2}{4z^4} + \frac{x m_3}{z^3} + \frac{1-(m_1-m_2)^2}{4(z-1)z} + \frac{1-(m_1+m_2)^2}{4(z-1)^2 z} + \frac{u-\frac{1}{4}+\frac{x}{2}(m_1+m_2-1)}{(z-1)z^2}$$

with

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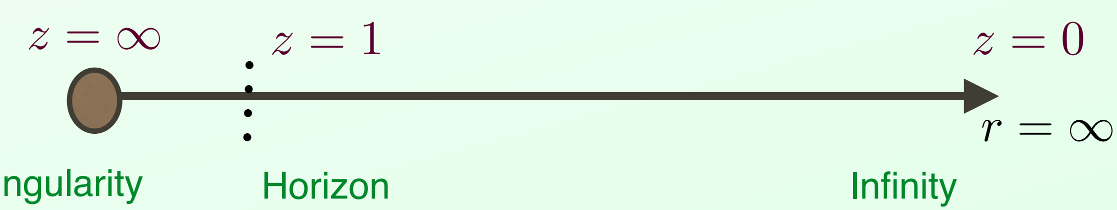

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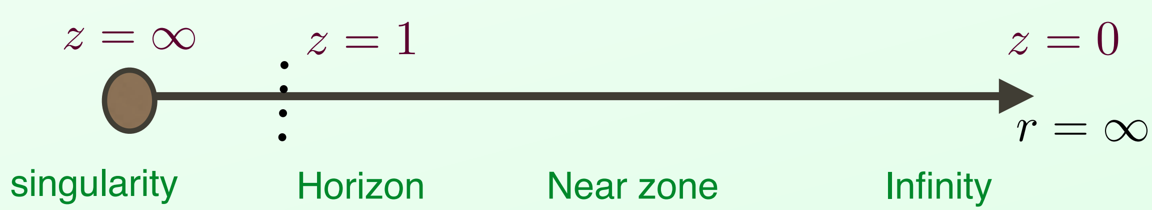

$$\sum_{\alpha'=\pm} F_{\alpha\alpha'}\Psi^H_{\alpha'}(z) = \Psi_\alpha(z) = \sum_{\alpha'=\pm} B_{\alpha\alpha'}\Psi^\infty_{\alpha'}(z)$$

Heun gravity dictionary

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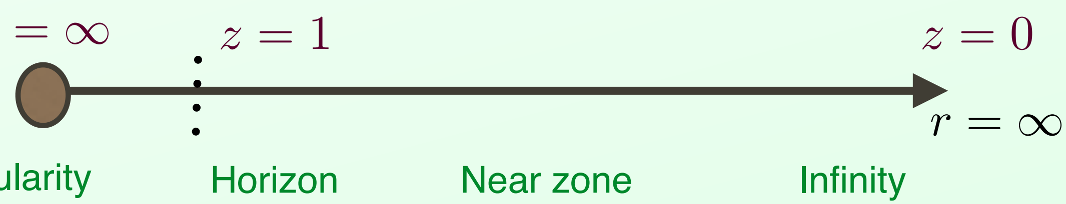

$$\sum_{\alpha'=\pm} F_{\alpha\alpha'}\Psi^{H}_{\alpha'}(z) = \Psi_\alpha(z) = \sum_{\alpha'=\pm} B_{\alpha\alpha'}\Psi^{\infty}_{\alpha'}(z)$$

Connection coefficients !

Gauge/CFT/Gravity

Aminov, Grassi, Hatsuda 2006.06111
Bianchi, Consoli, Grillo, Morales 2105.04245
Bonelli, Iossa, Lichtig, Tanzini 2105.04483

Gauge/CFT/Gravity

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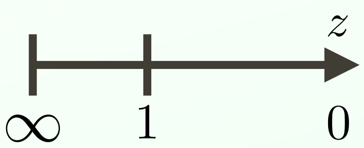
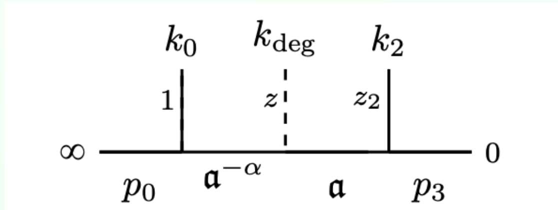
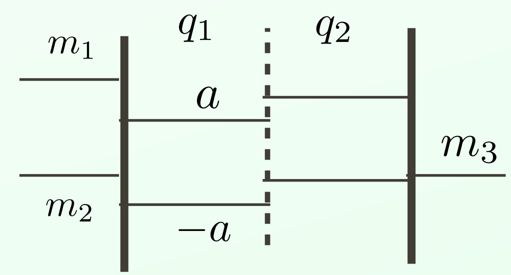


Null

$$\langle p_0 \left| V_{p_1}(1) V_{k_0}(z) \mathcal{L} V_{k_{\rm deg}}(z) V_{k_2}(z_2) \right| p_3 \rangle = 0$$



$$P_L(x)y^2 + P(x)y + P_R(x) = 0$$



GRAVITY

2D CFT

AGT duality

GAUGE

Gauge/CFT/Gravity

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Bianchi, Consoli, Grillo, Morales 2105.04245
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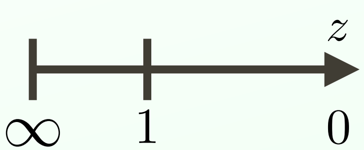

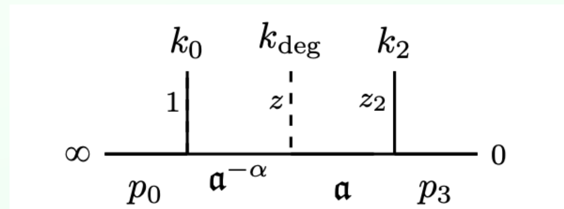

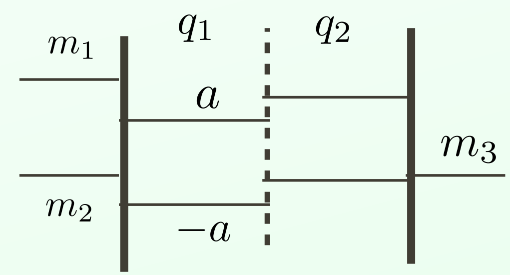

GRAVITY

2D CFT

GAUGE

AGT duality

Wave function

Degenerate Correlator

Partition function

Gauge/CFT/Gravity

Aminov, Grassi, Hatsuda 2006.06111
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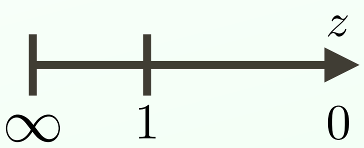

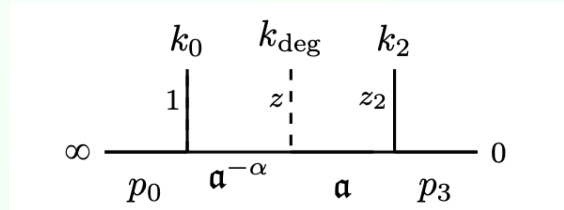

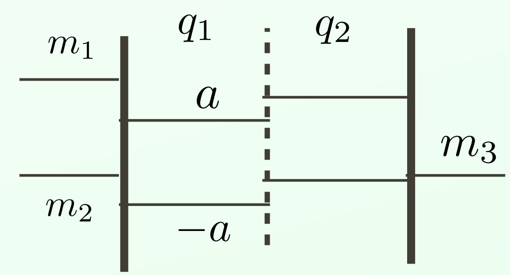

GRAVITY ⟷ 2D CFT ⟷ GAUGE

AGT duality

GRAVITY	2D CFT	GAUGE
Wave function	Degenerate Correlator	Partition function
Wave equation	Ward Identity	Quantum SW curve

Gauge/CFT/Gravity

Aminov, Grassi, Hatsuda 2006.06111
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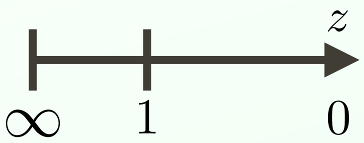

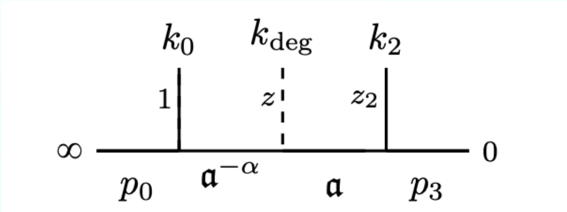

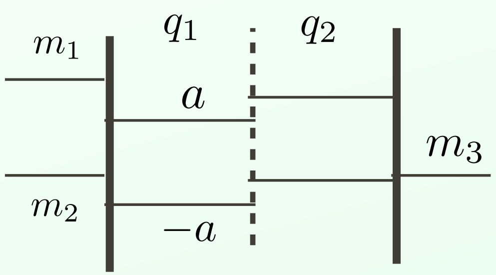

GRAVITY ⟷ 2D CFT ⟷ GAUGE

AGT duality

GRAVITY	2D CFT	GAUGE
Wave function	Degenerate Correlator	Partition function
Wave equation	Ward Identity	Quantum SW curve
Singularities & horizons	Insertion points	Gauge couplings

Gauge/CFT/Gravity

Aminov, Grassi, Hatsuda 2006.06111
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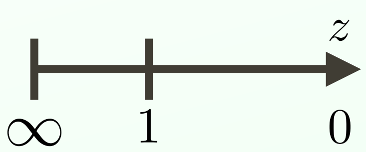

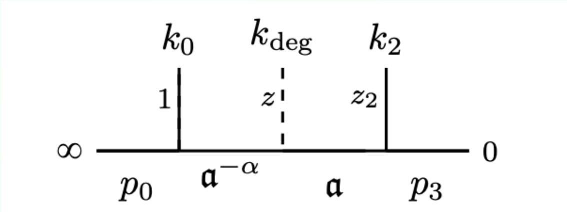

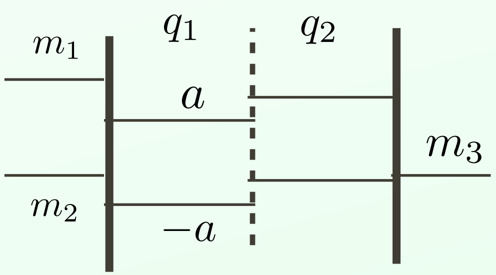

GRAVITY	2D CFT	GAUGE
	AGT duality	
Wave function	Degenerate Correlator	Partition function
Wave equation	Ward Identity	Quantum SW curve
Singularities & horizons	Insertion points	Gauge couplings
Connection formulas	Crossing symmetry	S-dualities

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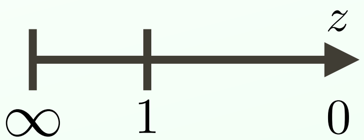

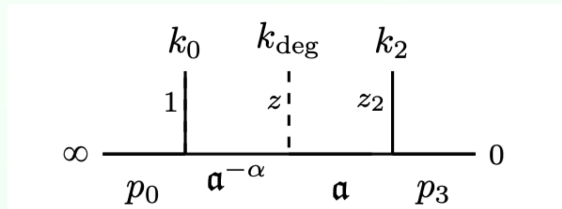

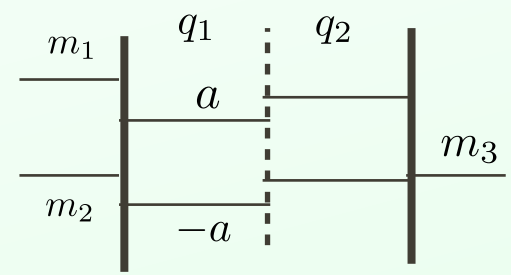

GRAVITY	2D CFT	GAUGE
Wave function	Degenerate Correlator	Partition function
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Connection formulas	Crossing symmetry	S-dualities
BH and wave charges	Momenta	Masses

AGT duality

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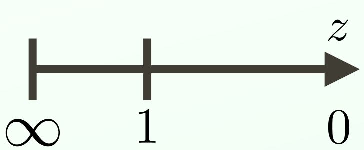

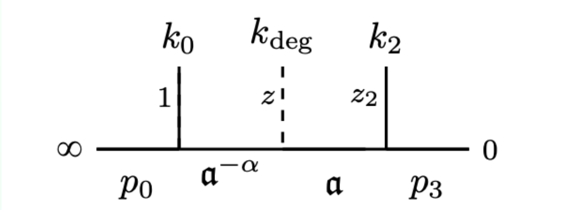

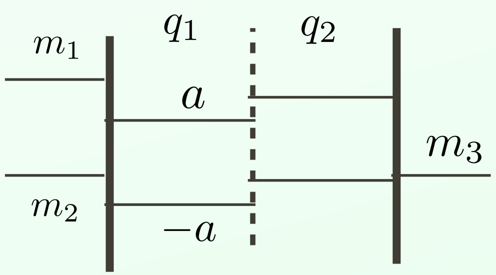

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BH and wave charges	Momenta	Masses

AGT duality

$$\Rightarrow \quad u = -x\partial_x \mathcal{F}(a, x)$$

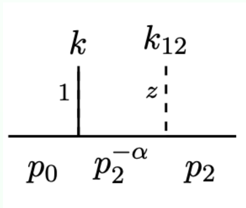
Gauge prepotential

!

CFT Brainding and Fusion

CFT Braiding and Fusion

- Hypergeometric



$$= (1-z)^{\frac{1}{2}+k} z^{\frac{1}{2}+\alpha p_2} {}_2F_1\left(\frac{1}{2}+k+p_0+\alpha p_2, \frac{1}{2}+k-p_0+\alpha p_2, 1+2\alpha p_2; z\right)$$

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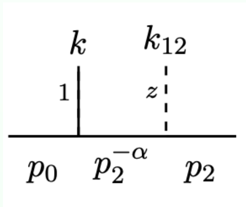

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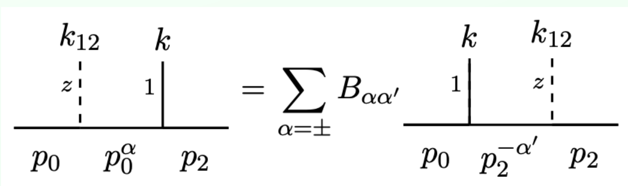
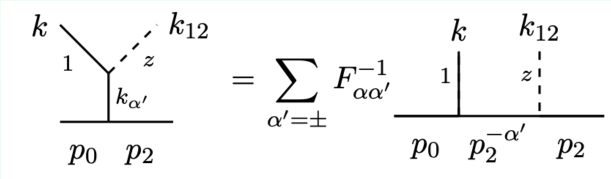
- Brainding fusion relations

Crossing symmetry



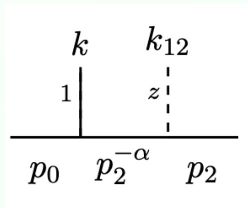
$$B_{\alpha\alpha'} = \frac{e^{i\pi(\alpha p_0 - \alpha' p_2)}\Gamma\left(1-2\alpha p_0\right)\Gamma\left(-2\alpha' p_2\right)}{\Gamma\left(\frac{1}{2}-\alpha p_0-\alpha' p_2+k\right)\Gamma\left(\frac{1}{2}-\alpha p_0-\alpha' p_2-k\right)}$$



$$F^{-1}_{\alpha\alpha'} = \frac{\Gamma(1-2\alpha k)\Gamma\left(-2\alpha' p_2\right)}{\Gamma\left(\frac{1}{2}-\alpha k+p_0-\alpha' p_2\right)\Gamma\left(\frac{1}{2}-\alpha k-p_0-\alpha' p_2\right)}$$

CFT Braiding and Fusion

- **Hypergeometric**



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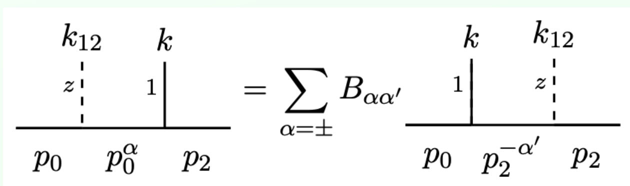
Crossing symmetry



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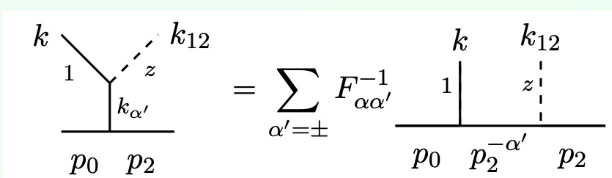

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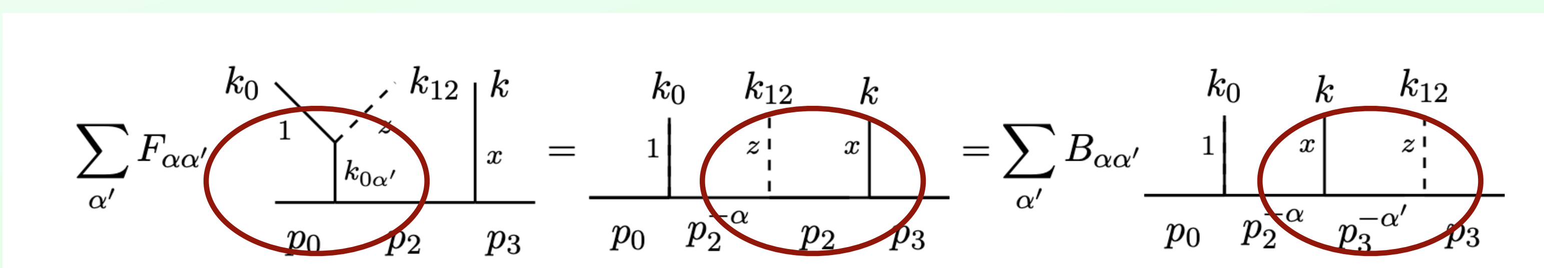

CFT Braiding and Fusion

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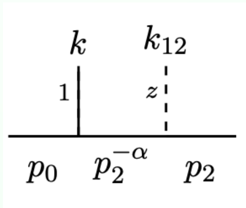

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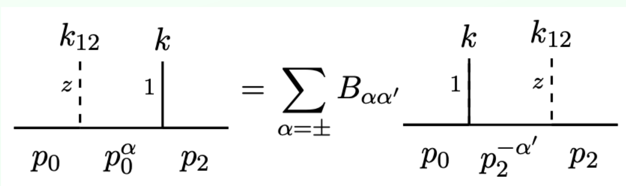

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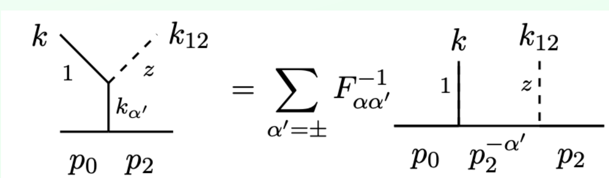

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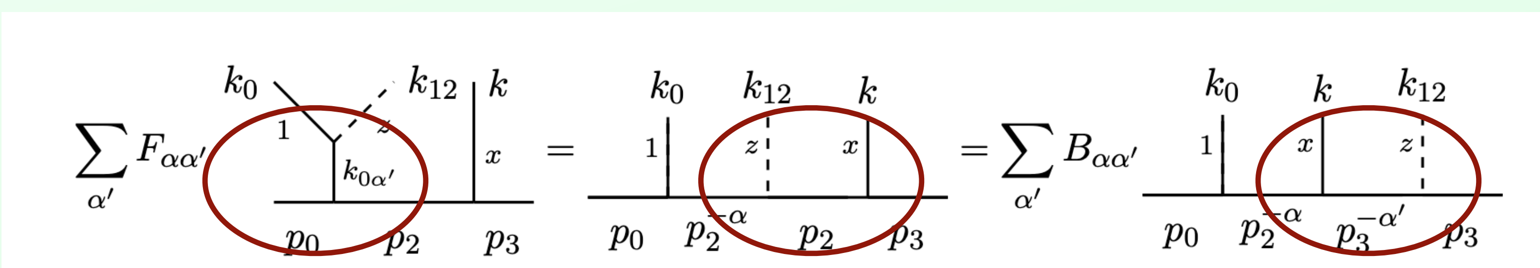
$p_2 = a$

SW period !



PM expansion

PM expansion

- PM expansion $\qquad x = 4\mathrm{i}\omega M \qquad y = 2\mathrm{i}\omega r$

$$R_\alpha(y) = e^{-\frac{y}{2}} \left(1 - \frac{x}{y}\right)^{2-\frac{x}{2}} y^{-1-\frac{x}{2}} \left[P_0(y) H^0_\alpha(y) + \widehat{P}_0(y) H^{0\prime}_\alpha(y)\right]$$

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$$H^0_\alpha(y) = y^{\frac{1}{2}-\alpha a - m_3} \, {}_1F_1\left(\frac{1}{2} - \alpha a - m_3; 1 - 2\alpha a; y\right)$$

and $P_0, \widehat{P}_0$ Finite polynomials of y and x/y. !

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Boundary conditions

- **Fundamental solutions:** $$R_{\mathrm{in,up}}(z) = \sum_\alpha c^{\mathrm{in,up}}_\alpha R_\alpha(z)$$

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BHs vs ECOs ?

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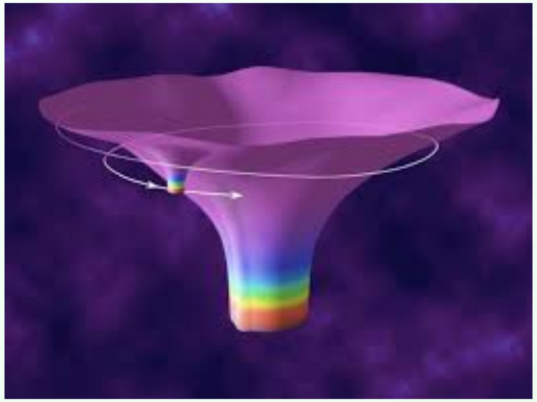
$$R_\alpha(y) = y^{\frac{3}{2}-\alpha a} \left[1 + \frac{2y}{1-2\alpha a} + \frac{(17-2\alpha a)y^2}{16(2\alpha a - 1)(\alpha a - 1)} + \frac{(2\alpha a - 3)x}{4y} + \dots\right]$$

SW period

with

$$a(\ell) = \ell + \frac{1}{2} + \frac{\left(15\ell^4 + 30\ell^3 + 28\ell^2 + 13\ell + 24\right) x^2}{8\ell(\ell+1)(2\ell+1)\left(4\ell^2 + 4\ell - 3\right)} + \dots$$

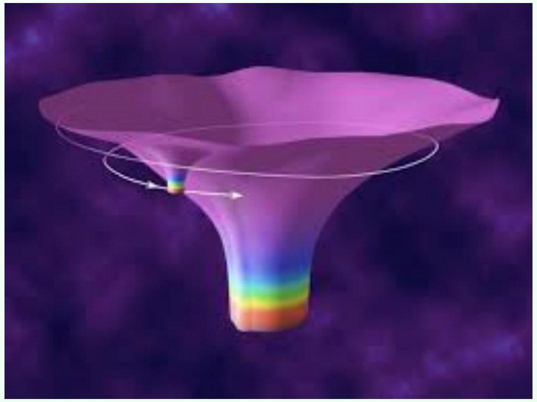
Inspiral motion



Inspiral motion

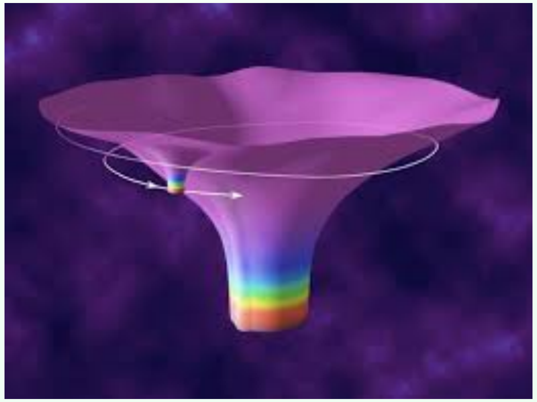
- **Stress energy:**

$$\mathcal{T}^{\mu\nu} = \int d\tau \frac{u^\mu u^\nu}{r^2 \sin\theta} \delta^{(4)} \left[\mathbf{x} - \mathbf{x}_0(\tau)\right] \qquad u^\mu = \partial_\tau x^\mu$$



$$\partial_\tau t = \frac{E}{f(r)} \quad , \quad \partial_\tau r = \sqrt{E^2 - f(r)\left(\mu^2 + \frac{J^2}{r^2}\right)} \quad , \quad \partial_\tau \theta = 0 \quad , \quad \partial_\tau \phi = \frac{J}{r^2}$$

Inspiral motion



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- **Wave form:**

$$h = h_+ - \mathrm{i} h_\times = \frac{2}{r} \int d\omega \, e^{\mathrm{i}\omega(r^* - t)} \sum_{\ell,m} h_{\ell m} Y^{\ell m}_{-2}(\theta,\phi)$$

$$h_{\ell m} = \frac{Z_{\ell m}}{(\mathrm{i}\omega)^2} = \frac{1}{(\mathrm{i}\omega)^2} \int_{r_+}^{\infty} \mathfrak{R}_{\mathrm{in}}(r') \frac{T_{\ell m}(r')}{\Delta(r')^2} dr'$$

homogenous incoming wave at horizon

$$\mathfrak{R}_{\mathrm{in}} = (-2\mathrm{i}\omega r)^{\widehat{\ell}+2} (-2\mathrm{i}\omega M)^{\mathrm{i}\omega M} \frac{\Gamma(\widehat{\ell} - 1 - 2\mathrm{i}\omega M)}{\Gamma(2\widehat{\ell} + 2)} \left(1 + \frac{2i\omega r}{1+\ell} - \frac{2M\left(1 + \frac{\ell}{2}\right)}{r} - \frac{(9+\ell)\omega^2 r^2}{6 + 10\ell + 4\ell^2} + \dots\right)$$

Inspiral motion

- Stress energy:

$$T^{\mu\nu} = \int d\tau \frac{u^\mu u^\nu}{r^2 \sin\theta} \delta^{(4)} [\mathbf{x} - \mathbf{x}_0(\tau)] \qquad u^\mu = \partial_\tau x^\mu$$

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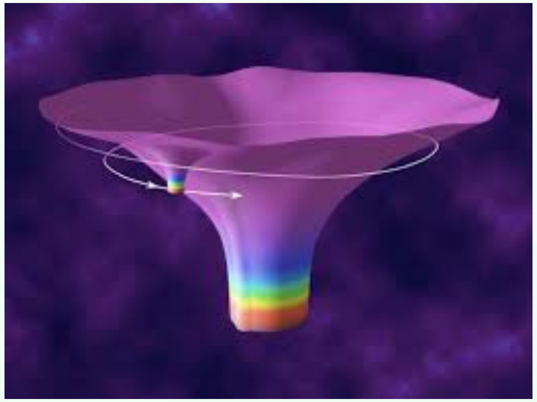

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Tail of tails

Double expansion with rational coefficients !

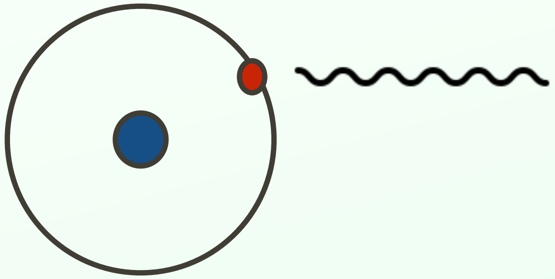
⇨ All log's and transcendental numbers are exponentiated !

Blanchet 1310.1528
Thorne, Damour, Will, Blanchet, Poisson
Mino,Nakamura, Sasaki, Shibata, Tagoshi, Tanaka

PN expansion of gravitational waves

PN expansion of gravitational waves

- Circular orbits:



$$r_0 = \frac{M}{v^2} \qquad \omega = \frac{m v^3}{M}$$

PN expansion of gravitational waves

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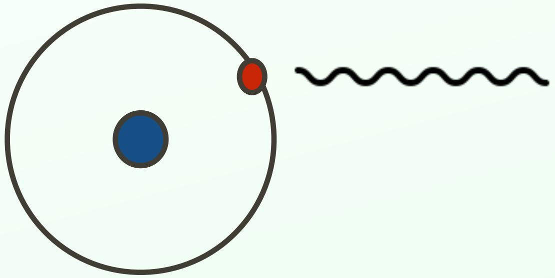

$$r_0 = \frac{M}{v^2} \qquad \omega = \frac{mv^3}{M}$$

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Mino,Nakamura, Sasaki, Shibata, Tagoshi, Tanaka

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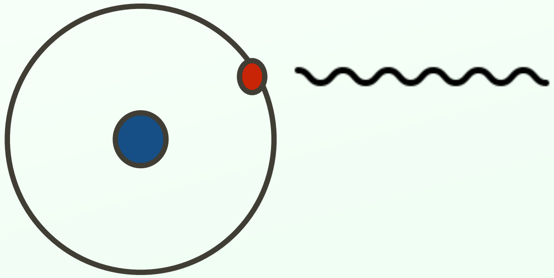

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Mino, Nakamura, Sasaki, Shibata, Tagoshi, Tanaka

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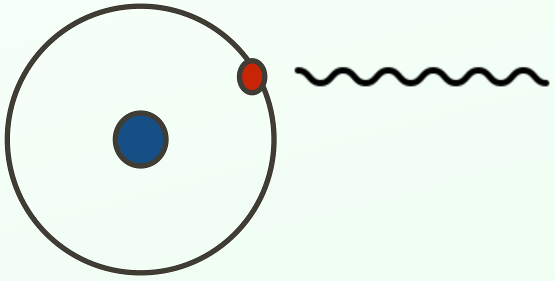

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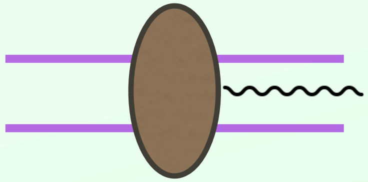
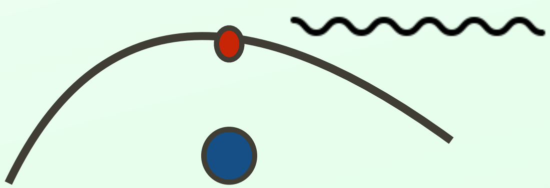
Mino, Nakamura, Sasaki, Shibata, Tagoshi, Tanaka

- Hyperbolic encounters:





Multipolar Post Minkowskian — Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng 2402.06604

Amplitudes — Georgoudis, Heisenberg, Russo 2402.0636

PN expansion of gravitational waves

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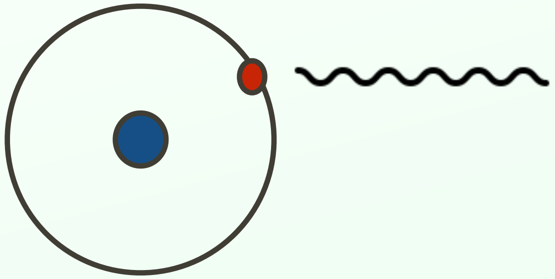

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Mino, Nakamura, Sasaki, Shibata, Tagoshi, Tanaka

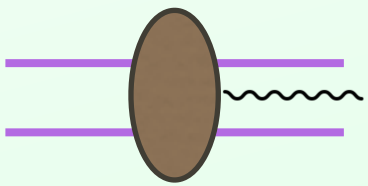
- **Hyperbolic encounters:**

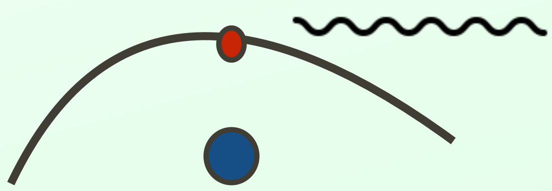
eccentricity

$$r_H(v) = M a_r (e_r \cosh(v) - 1)$$
$$t(v) = M a_r^{3/2} K_t (e_t \sinh(v) - v) + \ldots$$

Velocity

$$\phi(v) = 2K_\phi \mathrm{Arctan}\left(\sqrt{\frac{e_\phi + 1}{e_\phi - 1}} \tanh\left(\frac{v}{2}\right)\right) + \ldots$$





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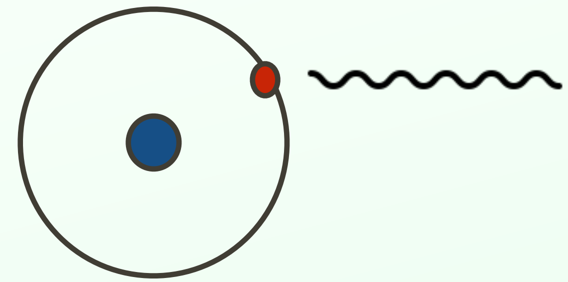

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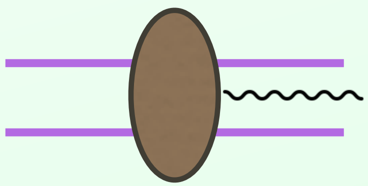
- **Hyperbolic encounters:**

eccentricity

$$r_H(v) = M a_r (e_r \cosh(v) - 1)$$
$$t(v) = M a_r^{3/2} K_t (e_t \sinh(v) - v) + \dots$$

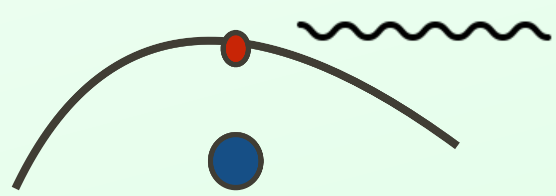
Velocity

$$\phi(v) = 2K_\phi \mathrm{Arctan}\left(\sqrt{\frac{e_\phi + 1}{e_\phi - 1}} \tanh\left(\frac{v}{2}\right)\right) + \dots$$



Multipolar Post Minkowskian

Amplitudes



Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng 2402.06604

Georgoudis, Heisenberg, Russo 2402.0636

$$h_{20} = \sqrt{\frac{2\pi a_r}{15}}\,\mu M \left(2K_0(u) - \frac{40K_0(u) + 45uK_1(u)}{7a_r}\right) + \dots \qquad u = \omega M e_r a_r^{3/2}$$

Soft limit

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- **Soft theorem:** Weinberg 65

$$h^{\mu\nu} \underset{\omega\to 0}{\approx} \sum_i \frac{p_i^\mu p_i^\nu}{p_i \cdot k}$$

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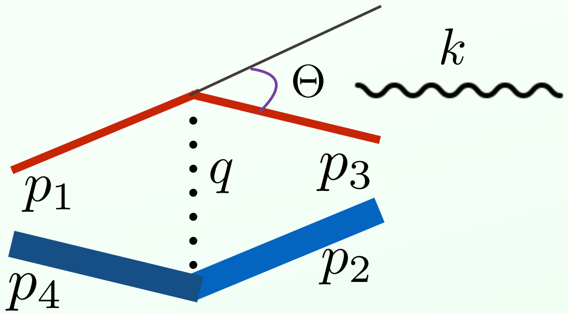

$$k = \omega(1, \vec{n})$$

$$p_1 + p_2 = (E_T, \vec{0})$$

$$E_T^2 = M^2 + \mu^2 + 2M\mu\sqrt{1+p_\infty^2}$$

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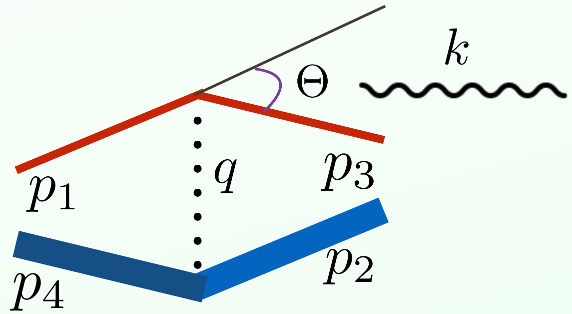

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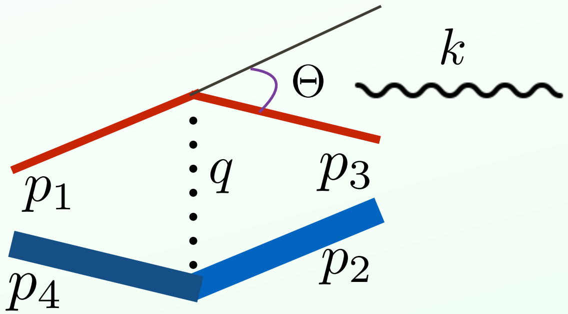

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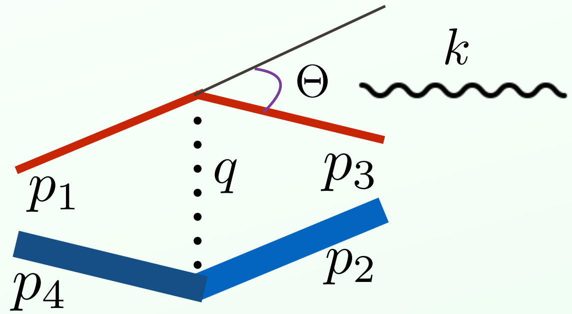

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$$h_{\mathrm{soft}} \underset{\omega\to 0}{\approx} -\mathrm{i}\omega^{2\mathrm{i}\omega M}\left[\frac{(p_{\mathrm{in}} \cdot e)^2}{p_{\mathrm{in}} \cdot k}\omega^{-\mathrm{i}\,\omega M C} + \frac{(p_{\mathrm{out}} \cdot e)^2}{p_{\mathrm{out}} \cdot k}\omega^{\mathrm{i}\,\omega M C}\right] ,$$

Resume whole series of universal log towers: !

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- **ECO vs BHs:** Differences show up at order $\left(\frac{GM}{r}\right)^5$! LISA ?

Thank you !