Large-scale fluctuations in the sine-Gordon field theory

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HUN-REN-BME-BCE Quantum Technology Research Group,
Budapest University of Technology and Economics

joint work with
Botond Nagy, Gábor Takács

Giuseppe Del Vecchio Del Vecchio, Alvise Bastianello, Benjamin Doyon

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SciPost Phys. **16**, 145 (2024)
Phys. Rev. Lett. **131**, 263401 (2023)

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BME PHYSICS

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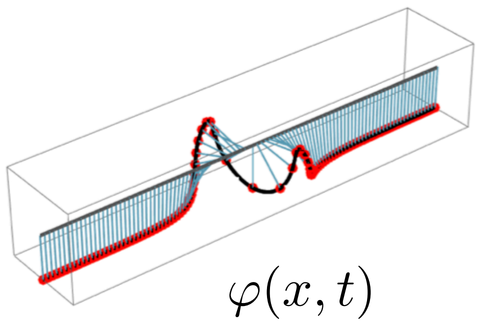

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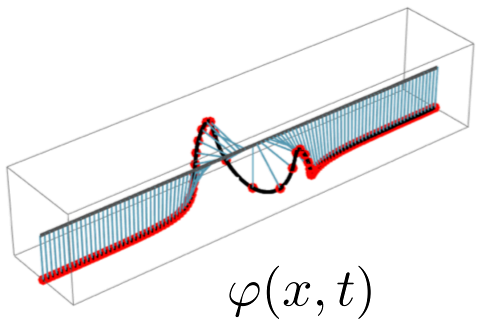

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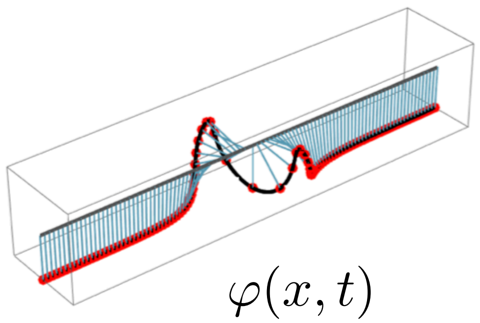
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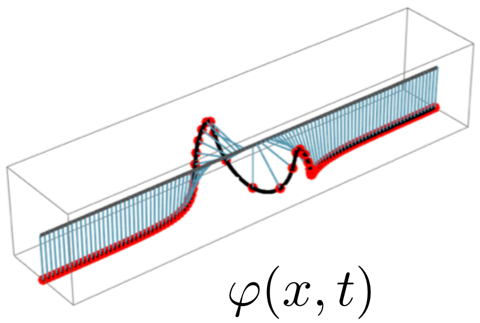

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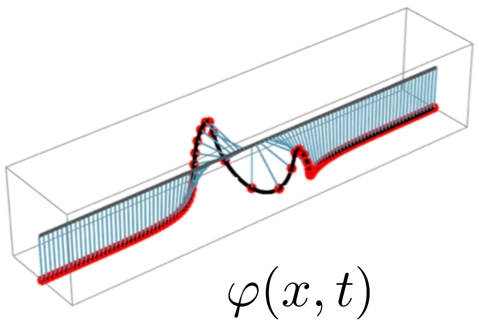

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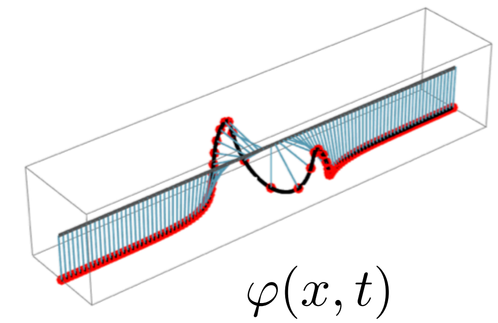

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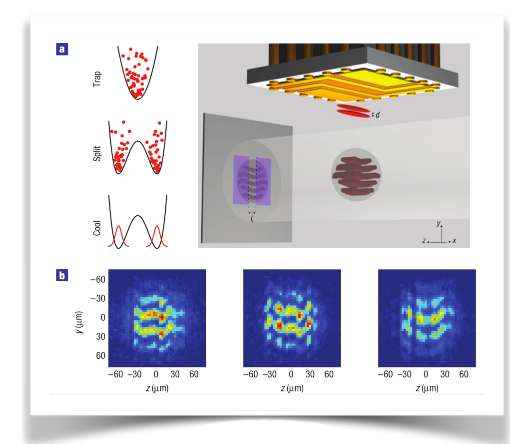

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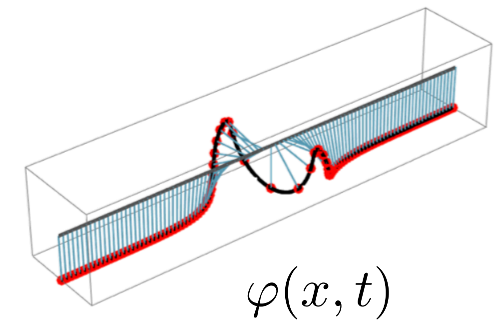

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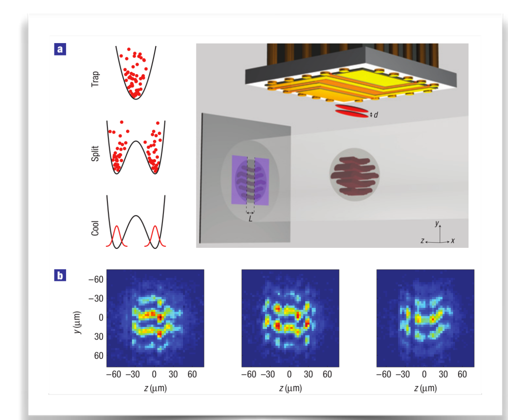

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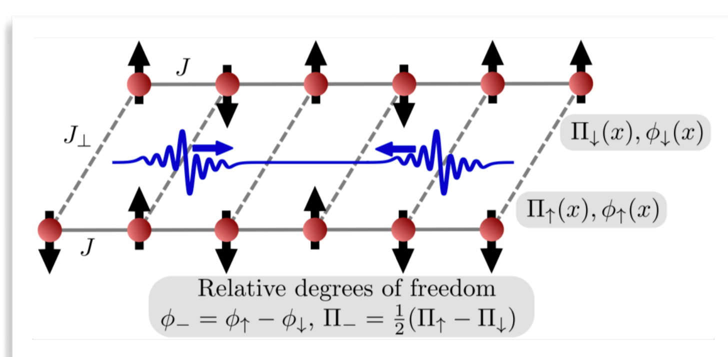

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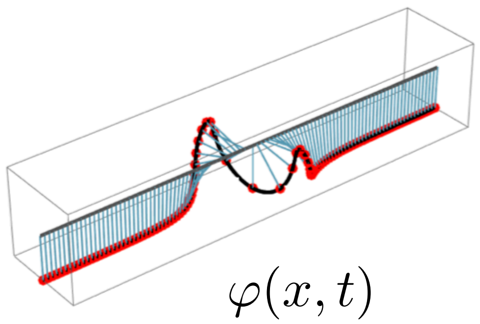

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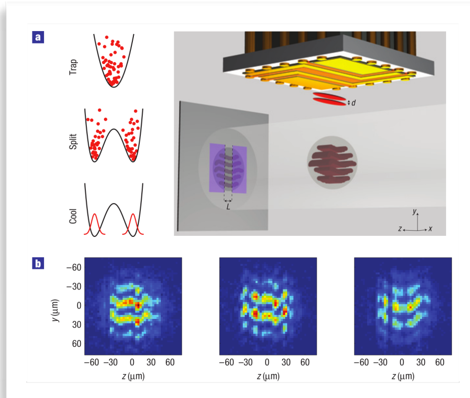

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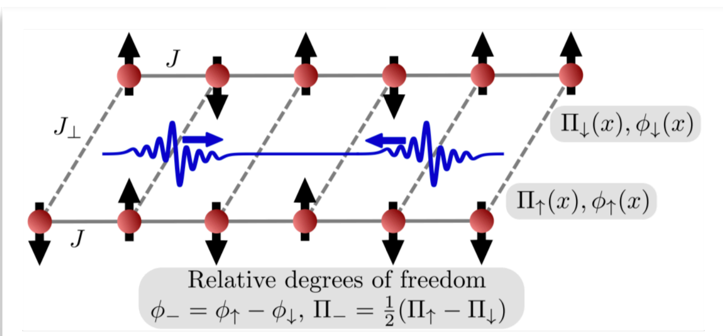

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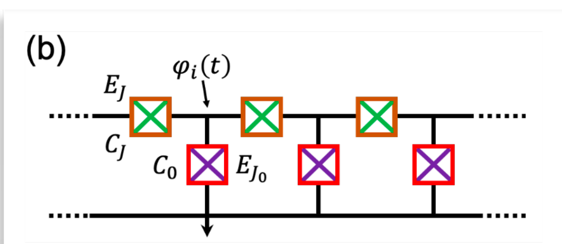
- quantum circuits of Josephson junctions



A. Roy et al., Nucl. Phys. B **968**, 115445 (2021)

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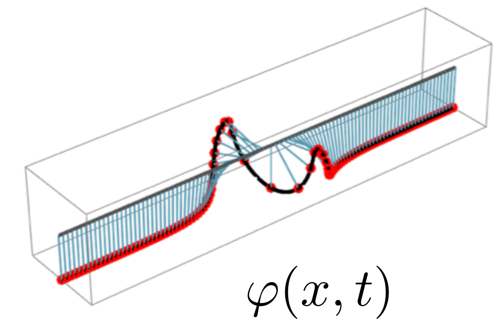

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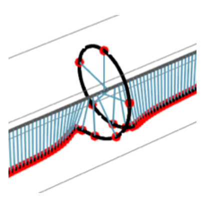

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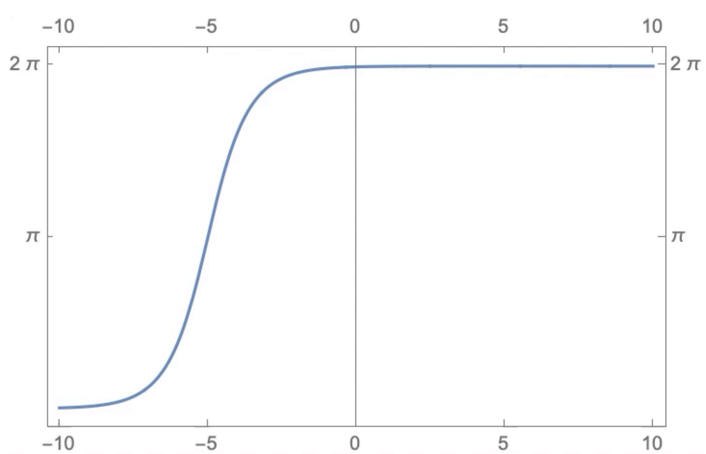

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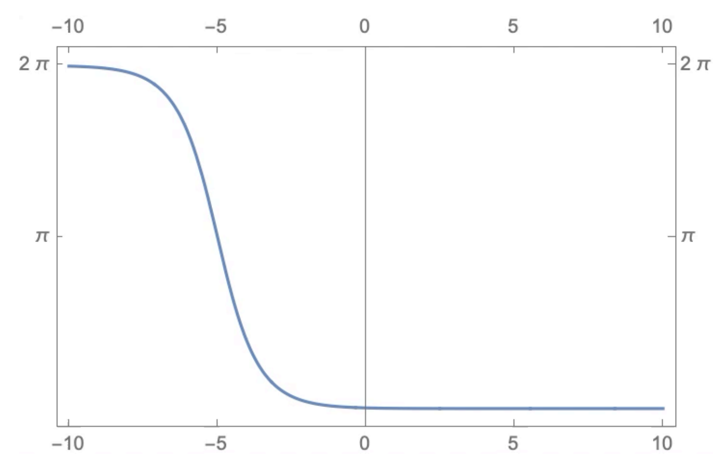

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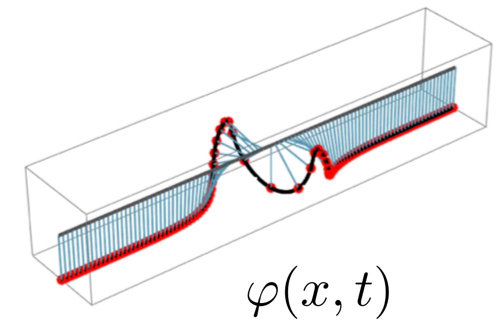

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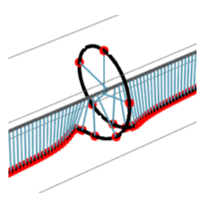

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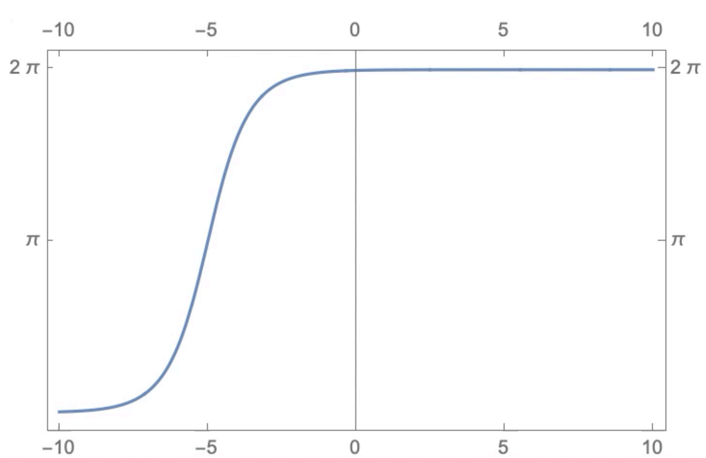

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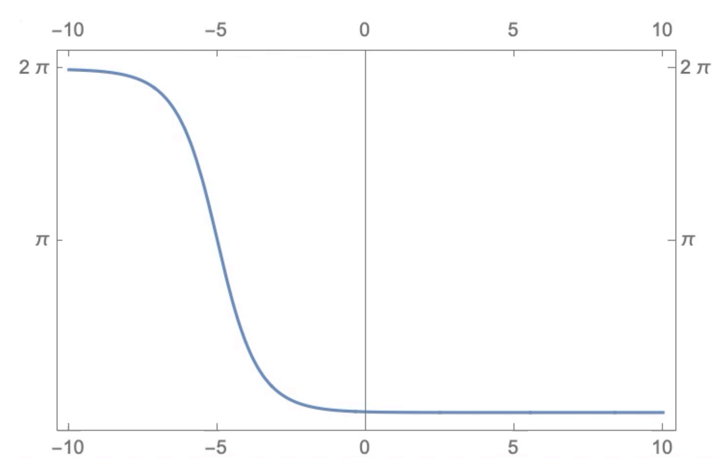

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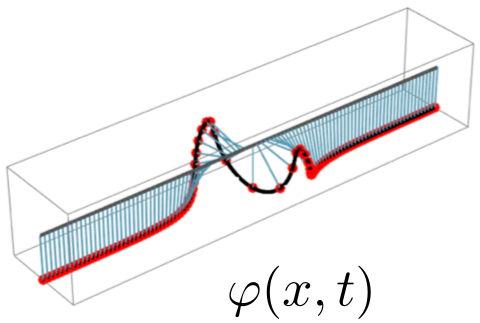
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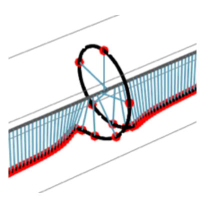

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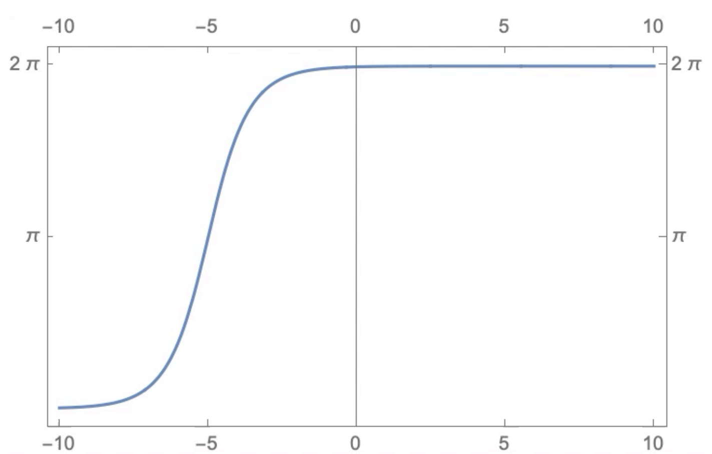

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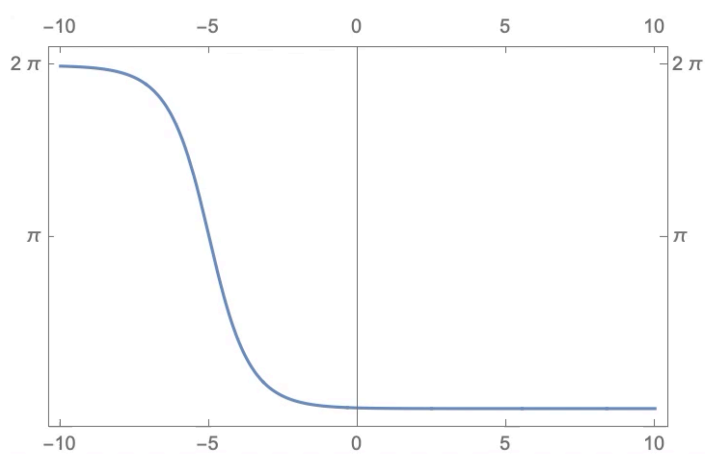

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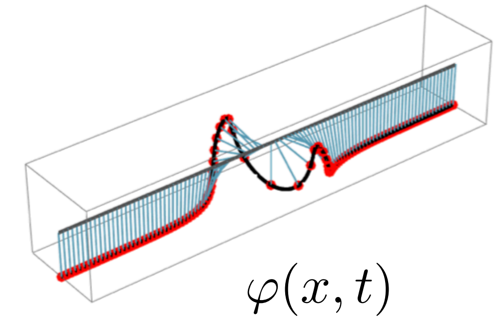

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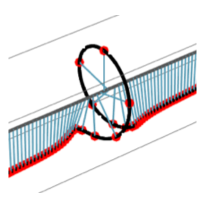

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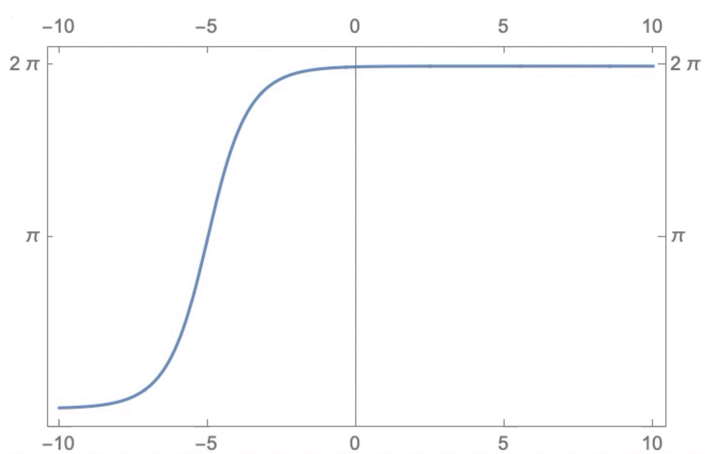

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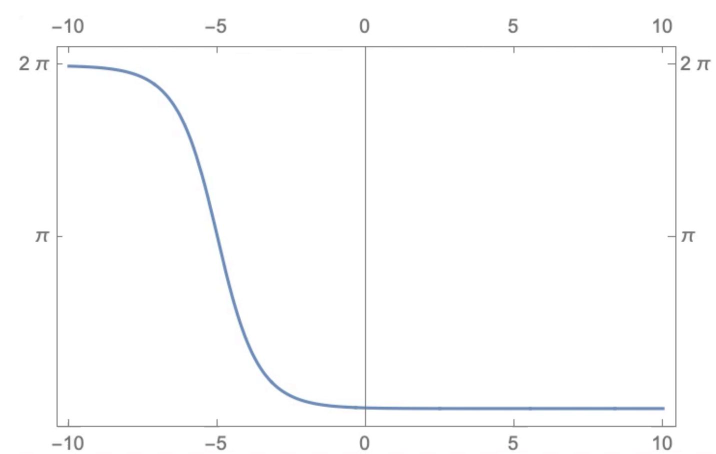

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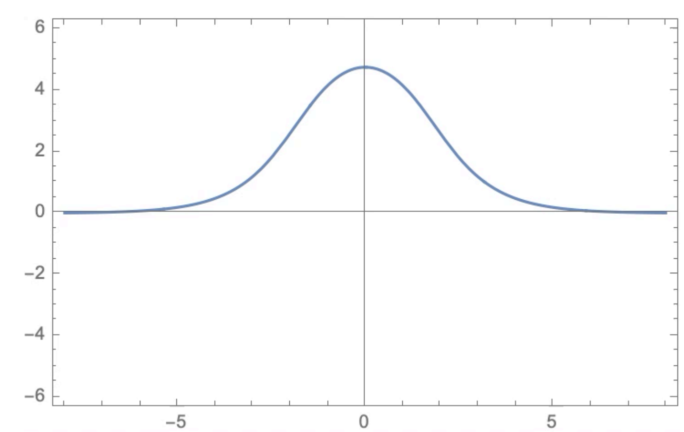
$Q = 0$



breather

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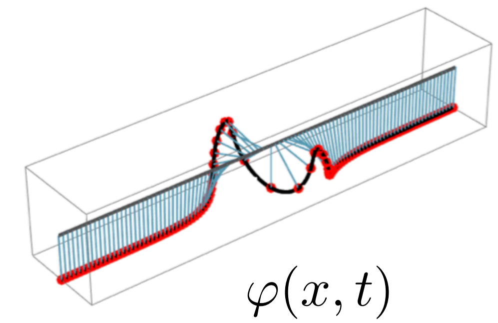

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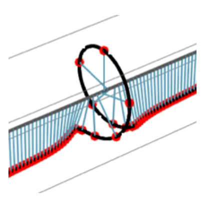

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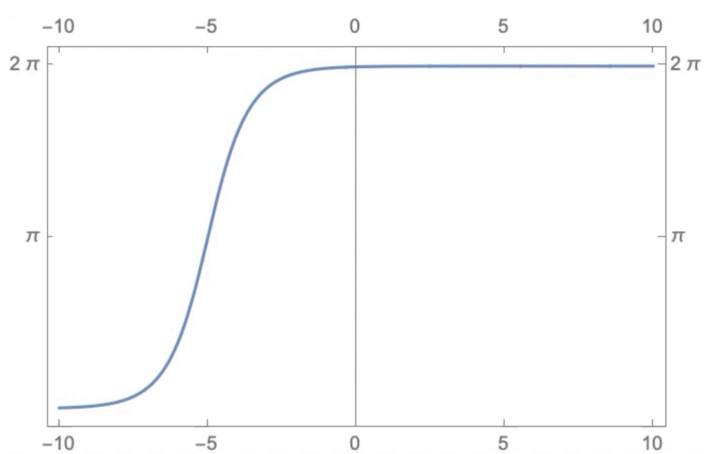

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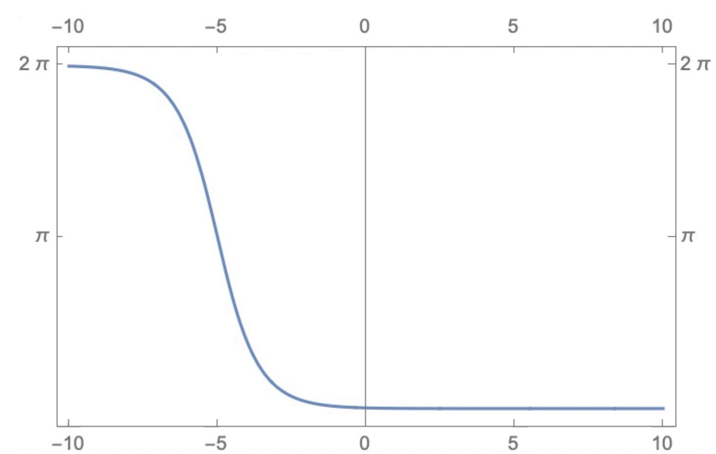

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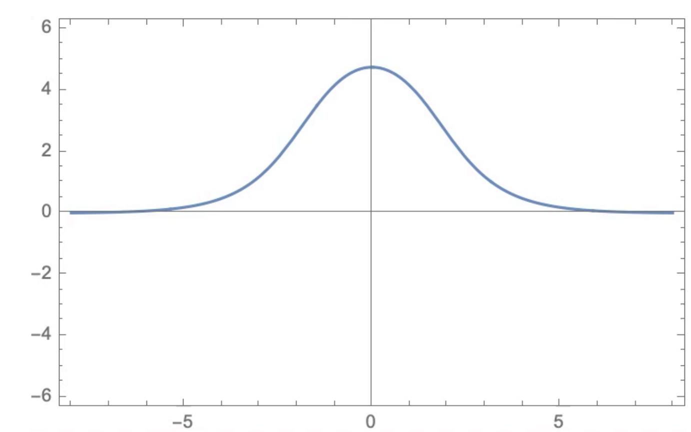

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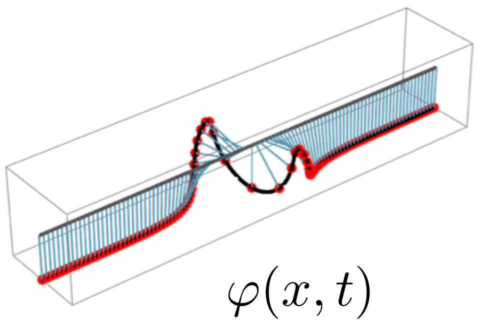

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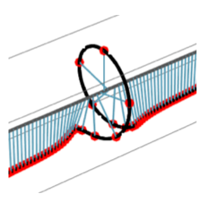

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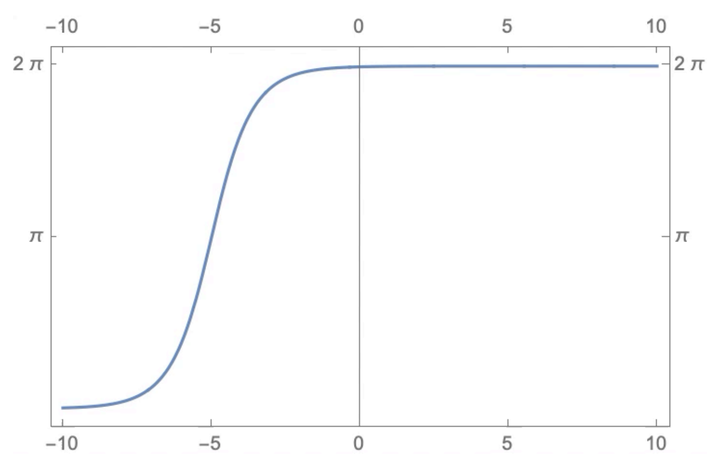

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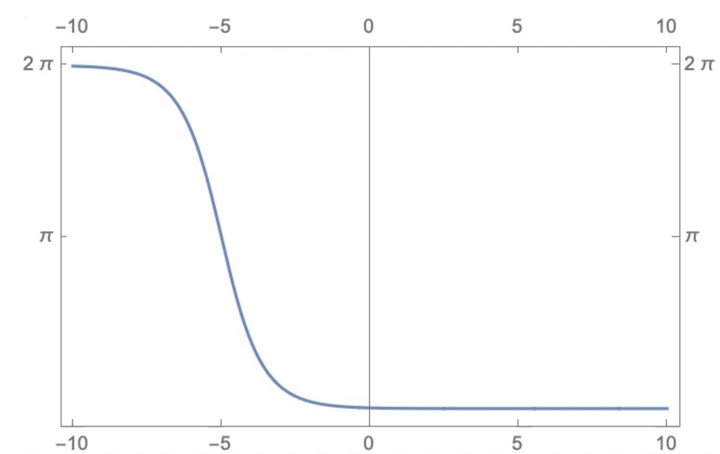

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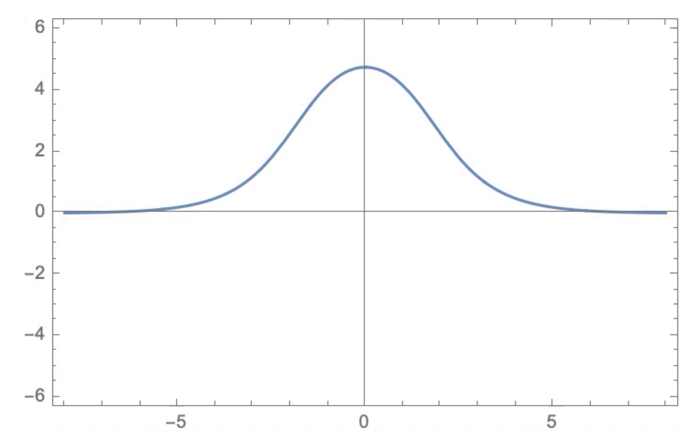

breather

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topological charge

$$Q = \int \mathrm{d}x\, q(x) = \int \mathrm{d}x \frac{\beta}{2\pi}\partial_x \varphi(x) = \frac{\beta}{2\pi}[\varphi(x=+\infty) - \varphi(x=-\infty)]$$

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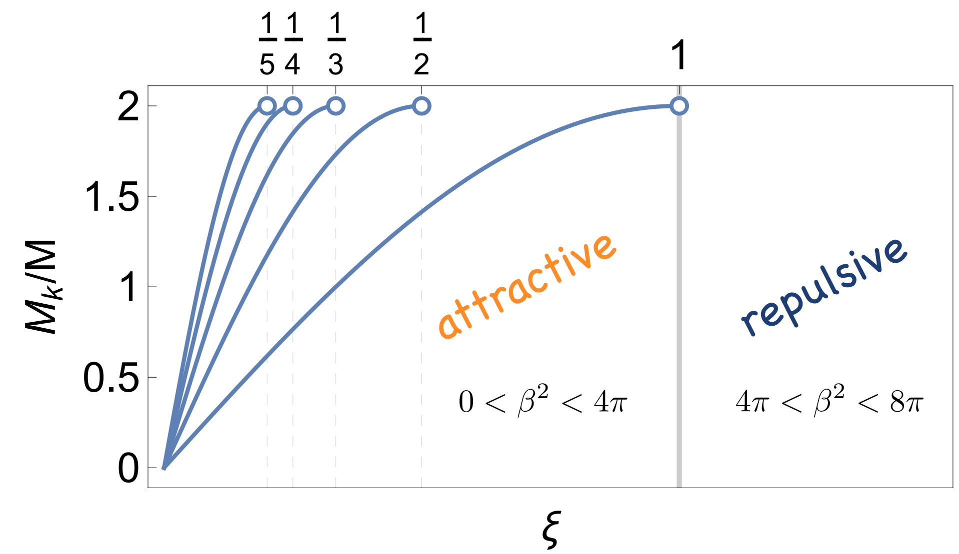

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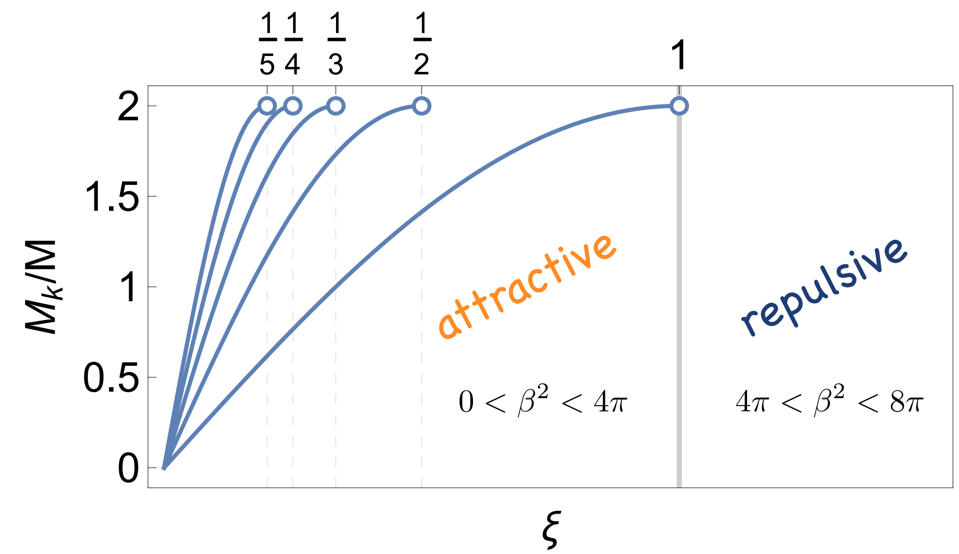

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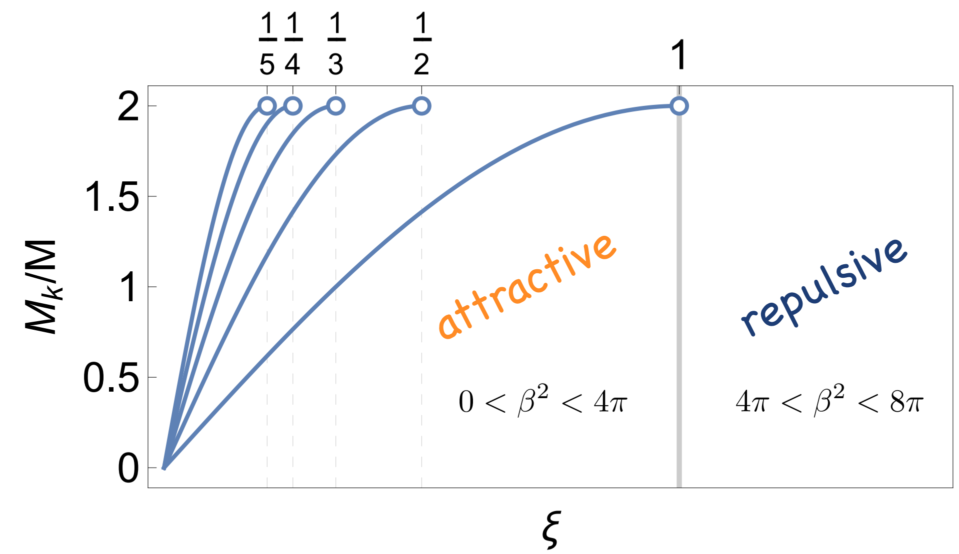

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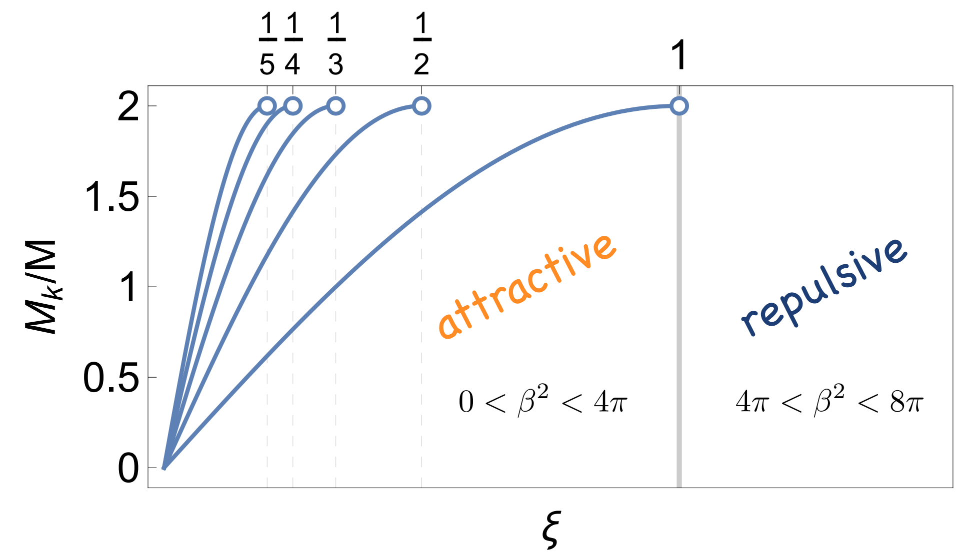

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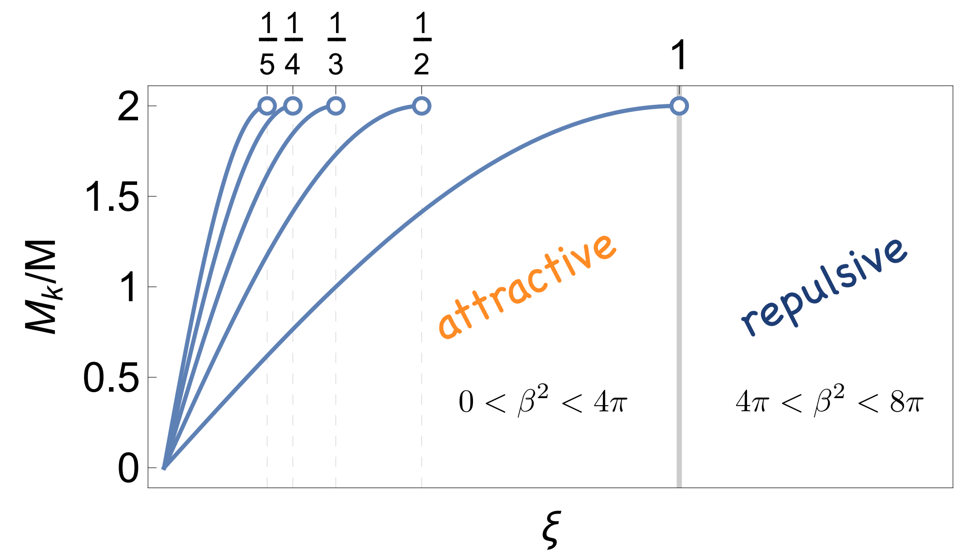

$$M_k = 2M \sin\left(\frac{k\pi}{2}\xi\right)$$

Purely elastic, factorised scattering

exact 2-body S-matrices are known

$$
\begin{array}{cc}
 & \begin{array}{cccc} |++\rangle & |+-\rangle & |-+\rangle & |--\rangle \end{array} \\
S = & \left(\begin{array}{c|cc|c}
S_0 & 0 & 0 & 0 \\ \hline
0 & S_R & S_T & 0 \\
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$$

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Zamolodchikov^2 '79

$$|S_{\mathrm{T}}(\theta)|^2 + |S_{\mathrm{R}}(\theta)|^2 = 1$$

Quantum sine-Gordon

Quantum sG is also integrable

Kinks and breathers become massive particles

Renormalised coupling: $\xi = \dfrac{\beta^2}{8\pi - \beta^2}$

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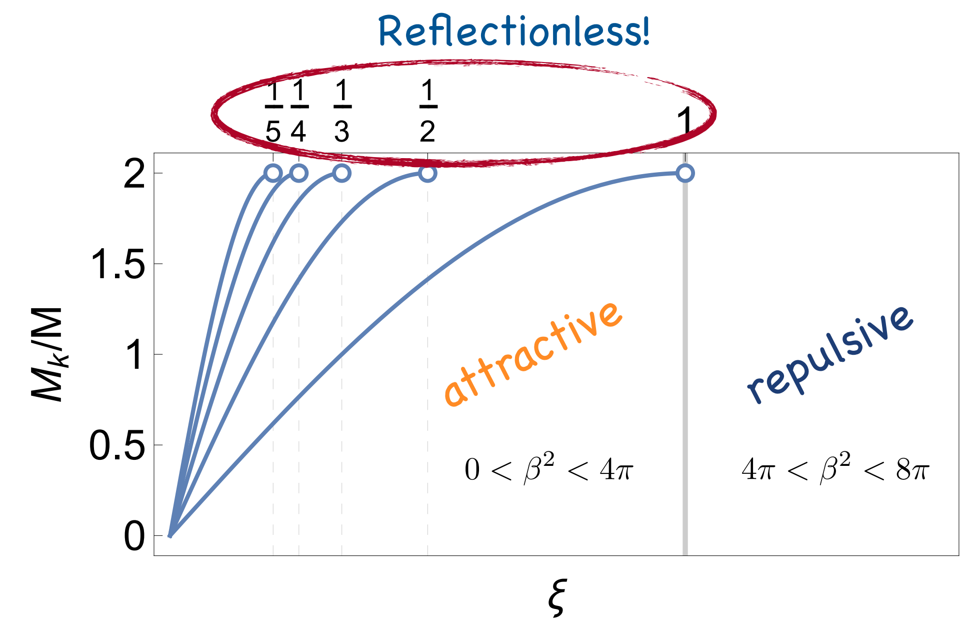

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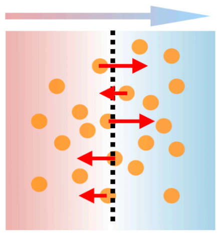

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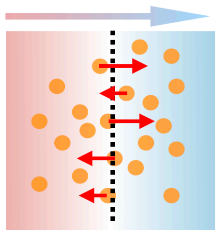
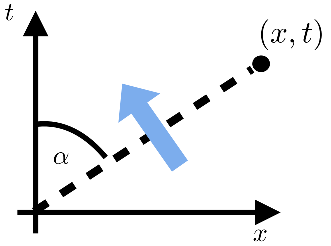


Generalisation: $$P\left[X = \int_{(0,0)}^{(x,t)} [dt'\, j(x',t') - dx'\, q(x',t')]\right] = ?$$ path independent



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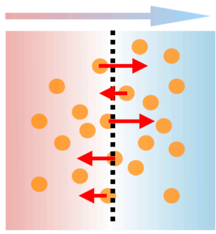

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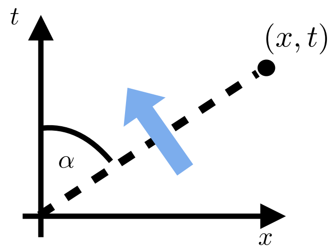

Moment generating function (full counting statistics):

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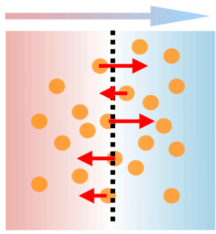

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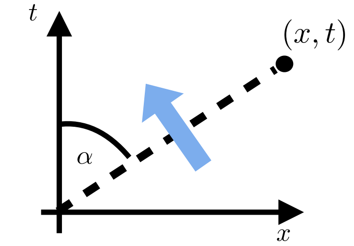

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Topological charge in sine-Gordon: fluctuations of the field difference!

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M. Borsi et al., PRX '20
B. Pozsgay, PRL '20

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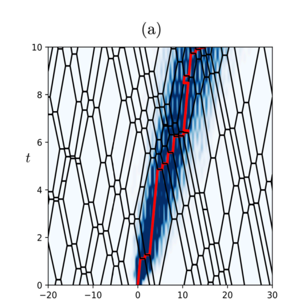

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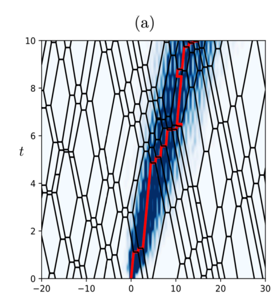

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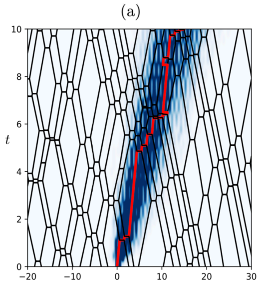

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→ GHD

Ballistic Fluctuation Theory

J. Myers et al., ’20; Doyon, Myers '20

For large x, t

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J. Myers et al., '20; Doyon, Myers '20

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$$C(\lambda) = \left\langle e^{\lambda \int_{(0,0)}^{(x,t)} [dt'\, j(x',t') - dx'\, q(x',t')]} \right\rangle \asymp e^{\ell F_\alpha(\lambda)}$$

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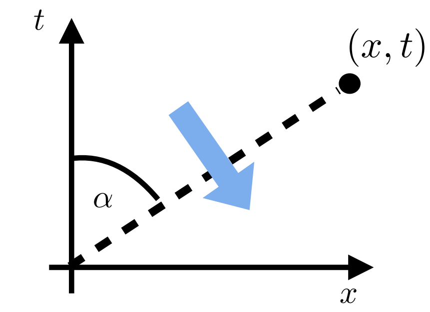

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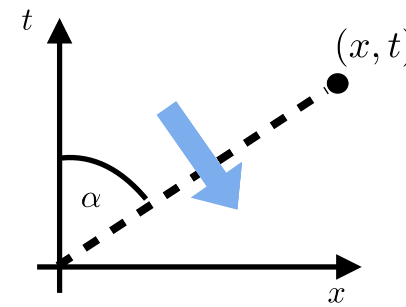

All cumulants scale extensively

Scaled cumulants:
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$F_\alpha(\lambda)$: Full Counting Statistics (FCS), scaled cumulant generating function

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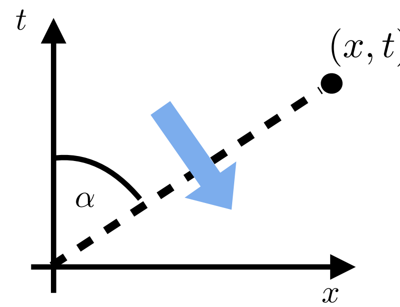

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Flow in the space of GGE-s: $$\partial_{\bar\lambda}\epsilon_{\bar\lambda}(\theta) = \mathrm{sgn}\left(\sin\alpha - \cos\alpha\, v^{\rm eff}_{\bar\lambda}(\theta)\right) q^{\rm dr}_{\bar\lambda}(\theta)$$

$$F_\alpha(\lambda) = \int_0^\lambda \mathrm{d}\bar\lambda \,(\cos\alpha\, \mathrm{j}_{\bar\lambda} - \sin\alpha\, \mathrm{q}_{\bar\lambda})$$

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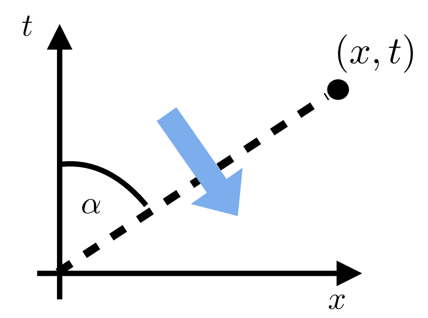
J. Myers et al., '20; Doyon, Myers '20

For large x, t

$$C(\lambda) = \left\langle e^{\lambda \int_{(0,0)}^{(x,t)} [dt'\, j(x',t') - dx'\, q(x',t')]} \right\rangle \asymp e^{\ell F_\alpha(\lambda)}$$

$$\ell = \sqrt{x^2 + t^2} \qquad \tan\alpha = \frac{x}{t}$$



All cumulants scale extensively

Scaled cumulants: $$c_n(\alpha) = \frac{1}{\ell}\langle X^n \rangle_c = \left.\frac{\mathrm{d}^n F_\alpha(\lambda)}{\mathrm{d}\lambda^n}\right|_{\lambda=0}$$

$F_\alpha(\lambda)$: Full Counting Statistics (FCS), scaled cumulant generating function

Flow in the space of GGE-s: $$\partial_{\bar\lambda}\epsilon_{\bar\lambda}(\theta) = \mathrm{sgn}\left(\sin\alpha - \cos\alpha\, v^{\mathrm{eff}}_{\bar\lambda}(\theta)\right) q^{\mathrm{dr}}_{\bar\lambda}(\theta)$$

$$F_\alpha(\lambda) = \int_0^\lambda \mathrm{d}\bar\lambda \left(\cos\alpha\, \mathrm{j}_{\bar\lambda} - \sin\alpha\, \mathrm{q}_{\bar\lambda}\right)$$

Explicit expressions for cumulants

$$c_2(\alpha) = \sum_a \int \mathrm{d}\theta\ \rho^{\mathrm{r}}_a(\theta)(1-\vartheta_a(\theta))\left|\cos\alpha\ v^{\mathrm{eff}}_a - \sin\alpha\right| q^{\mathrm{dr}}_a(\theta)^2$$

$$c_3(\alpha) = \sum_a \int \mathrm{d}\theta\ \rho^{\mathrm{r}}_a(1-\vartheta_a)\left|\cos\alpha\ v^{\mathrm{eff}}_a - \sin\alpha\right| q^{\mathrm{dr}}_a \left[3\left(s(1-\vartheta_a)(q^{\mathrm{dr}}_a)^2\right)^{\mathrm{dr}} + \eta_a s(\vartheta_a - 2)(q^{\mathrm{dr}}_a)^2\right]$$

Semiclassical limit

G. Del Vecchio[2], MK, A. Bastianello, B. Doyon, PRL '23

$\beta \to 0$ weak coupling limit

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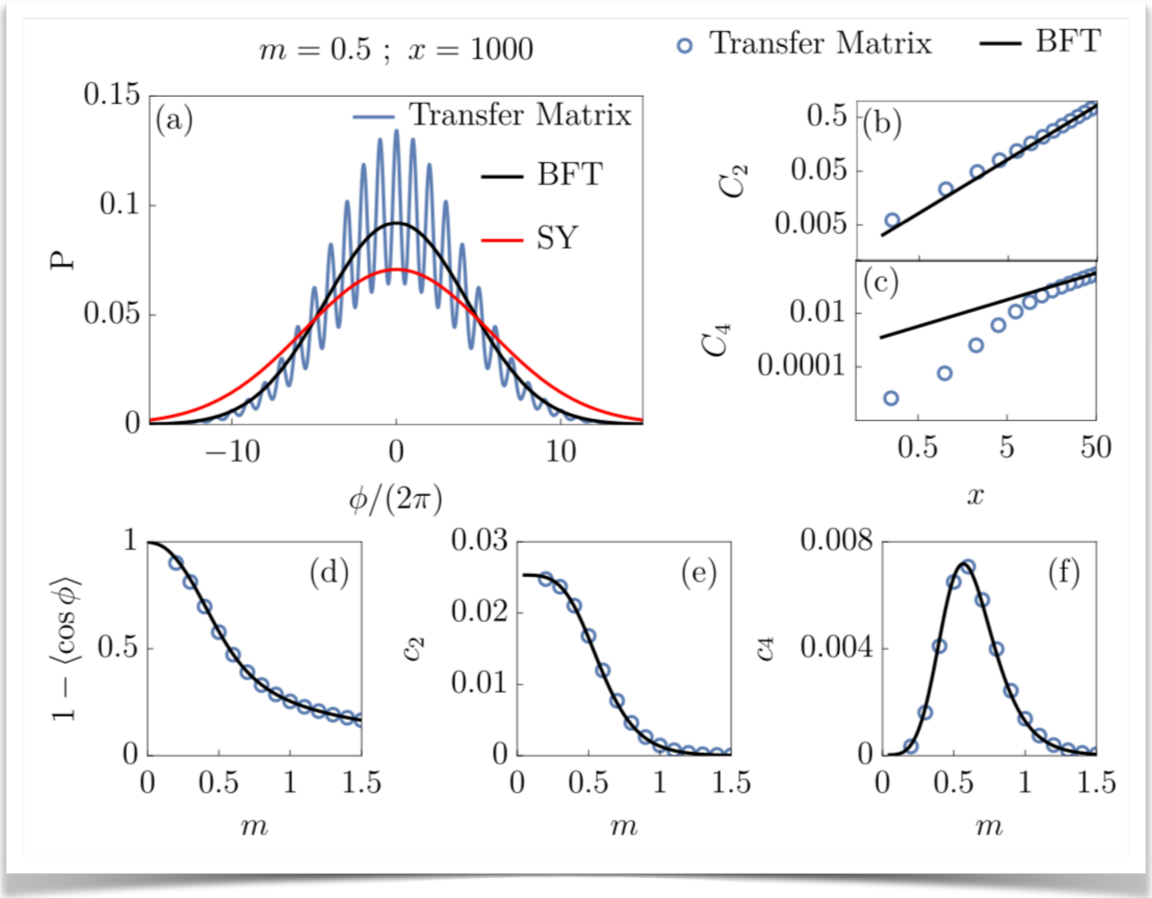
Equal time ($\alpha = \pi/2$)



thermal equilibrium (canonical ensemble)

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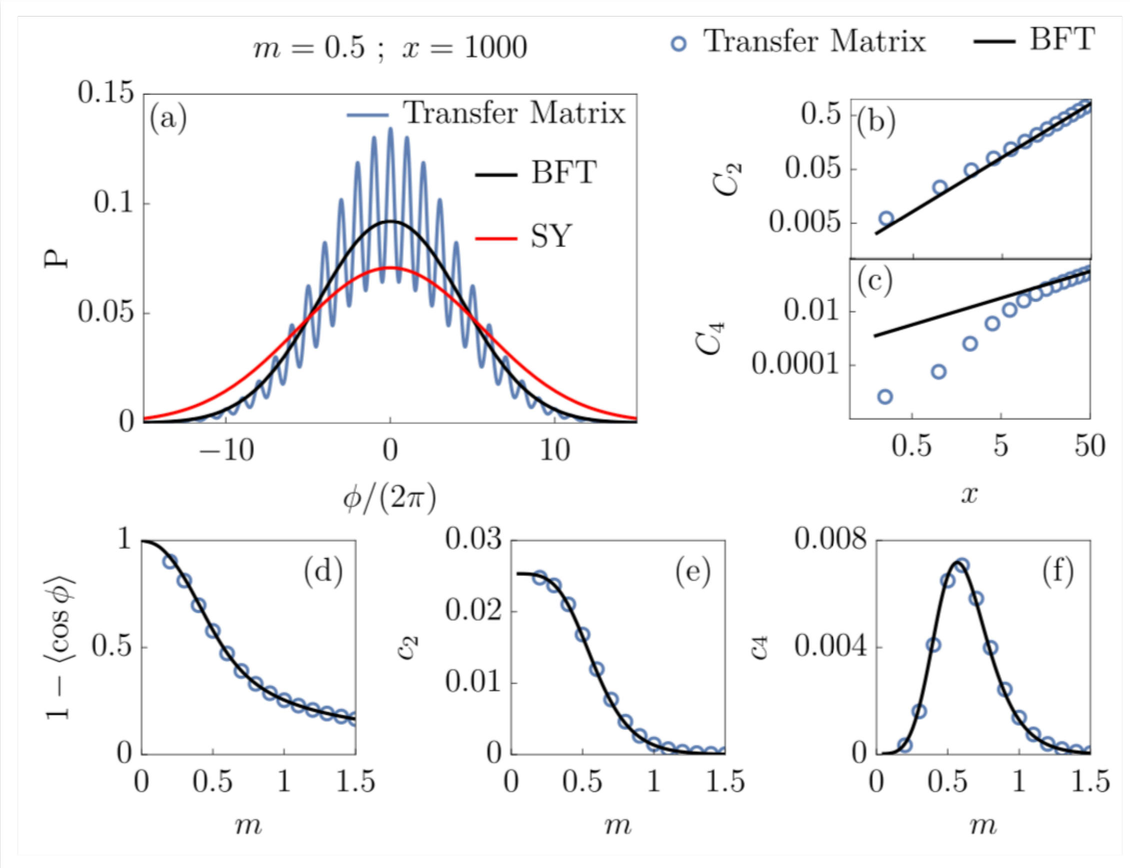
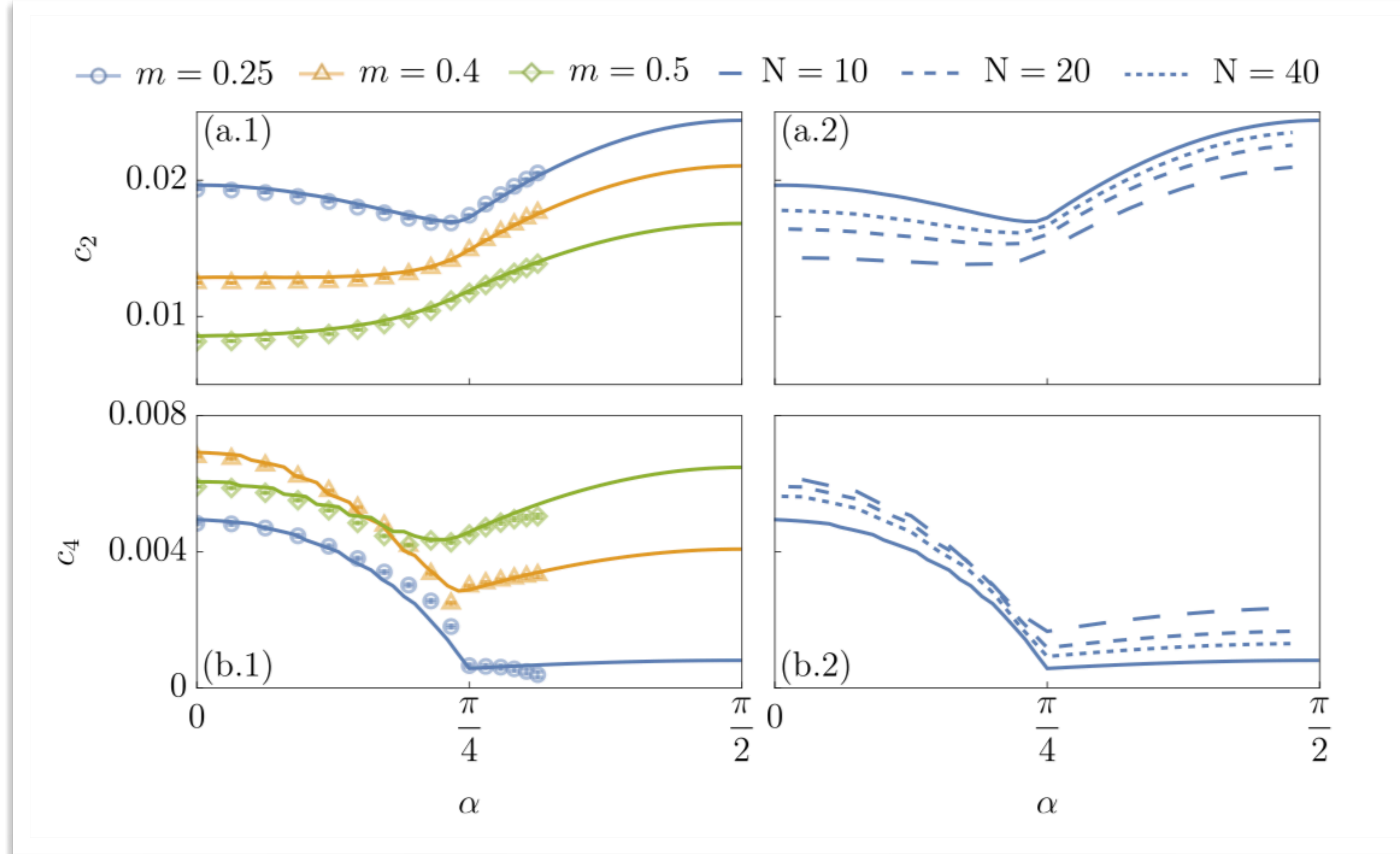


Space-time direction (α) dependence



thermal equilibrium (canonical ensemble)

2nd cumulants

Generic coupling

$$c_2(\alpha) = \sum_a \int \mathrm{d}\theta \; \rho_a^{\mathrm{r}}(\theta)(1-\vartheta_a(\theta)) \left|\cos\alpha \; v_a^{\mathrm{eff}} - \sin\alpha\right| q_a^{\mathrm{dr}}(\theta)^2$$

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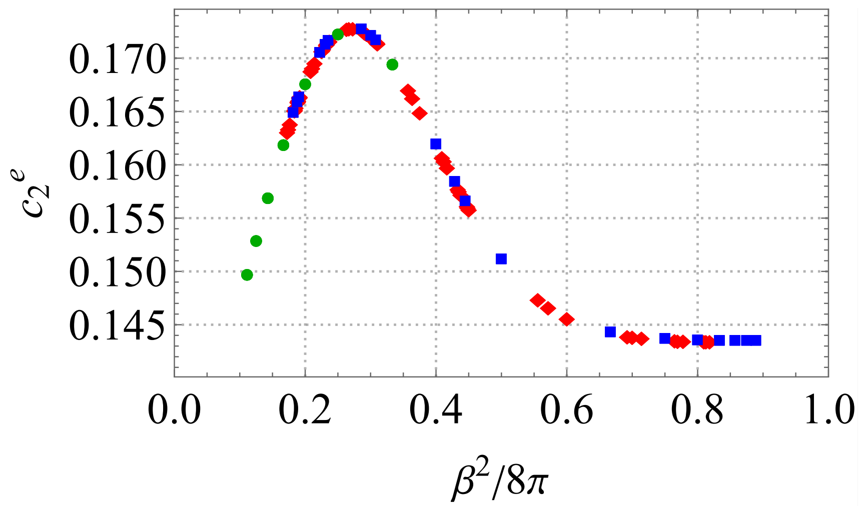
Energy

$\alpha = \pi/2$



$T = 0.5M$

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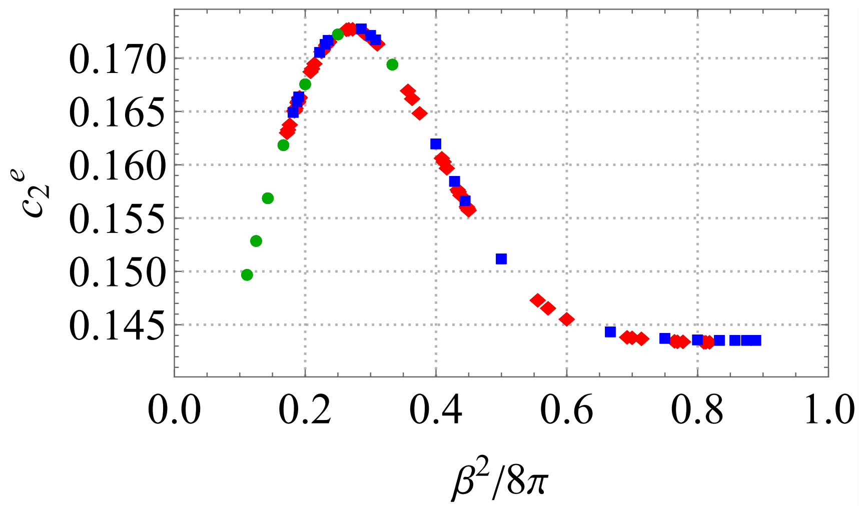

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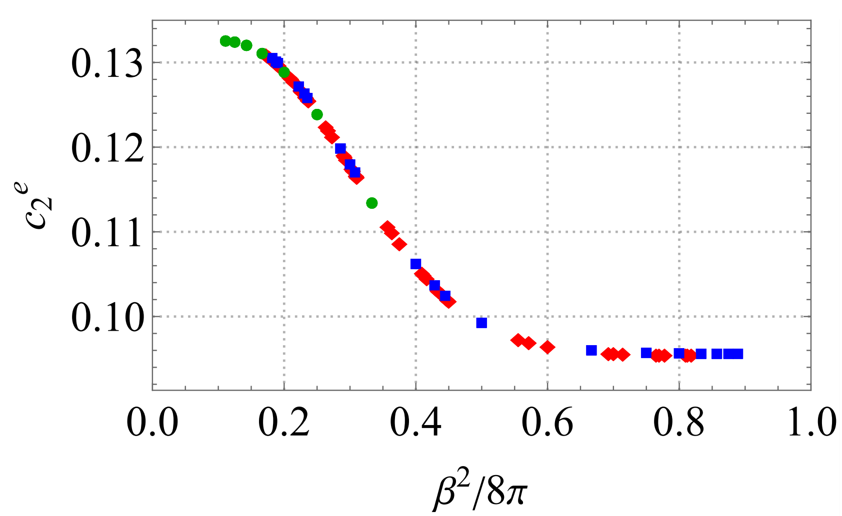

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Energy

Topological charge

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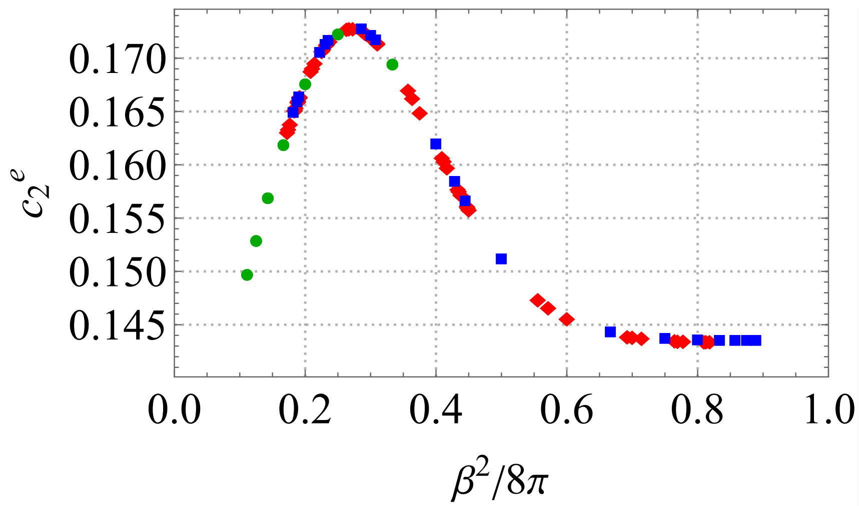

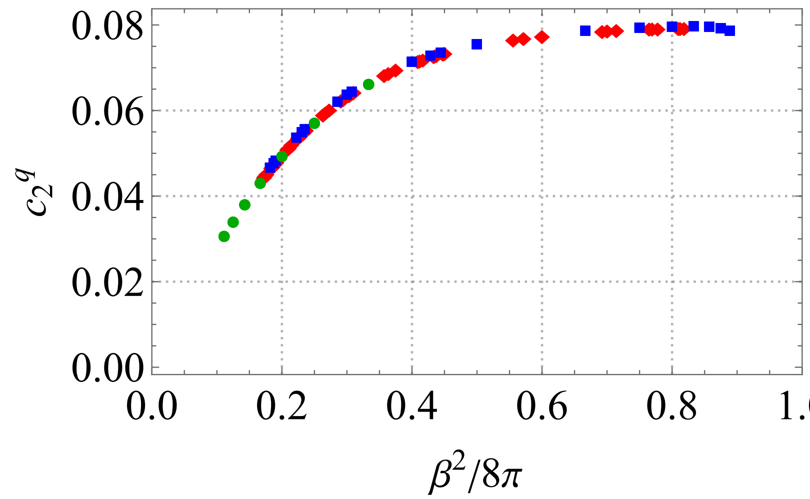

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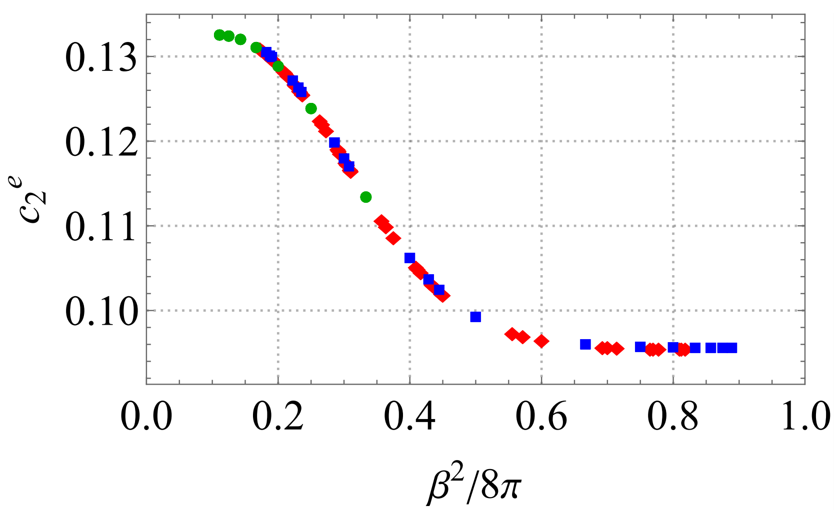

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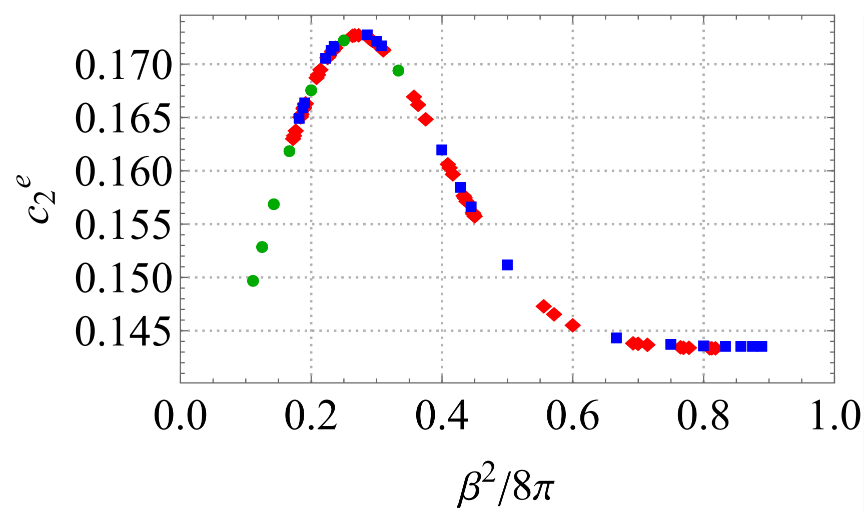

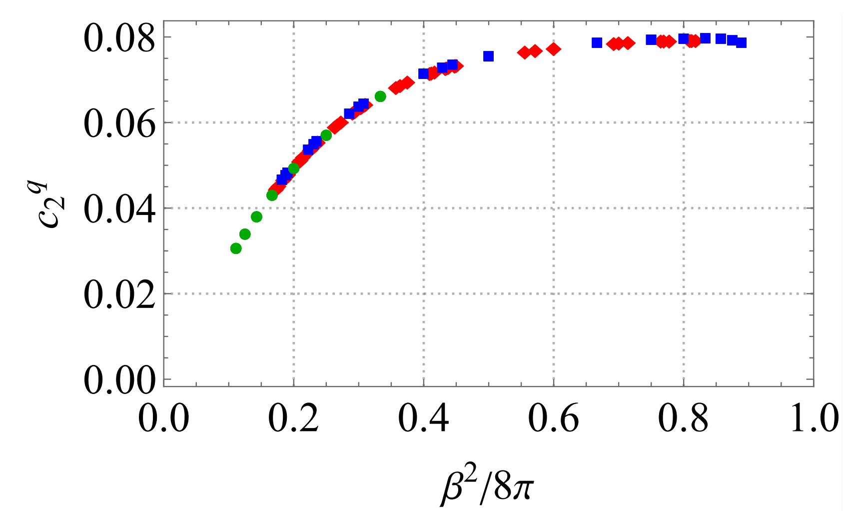

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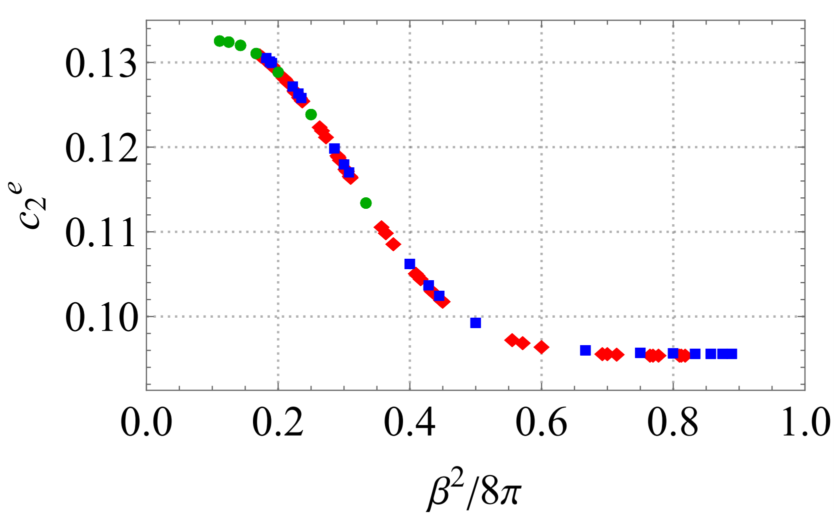

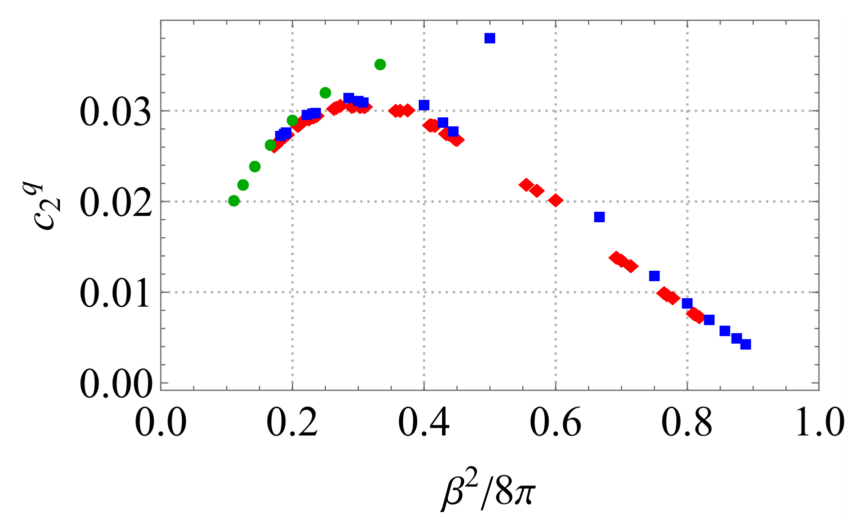

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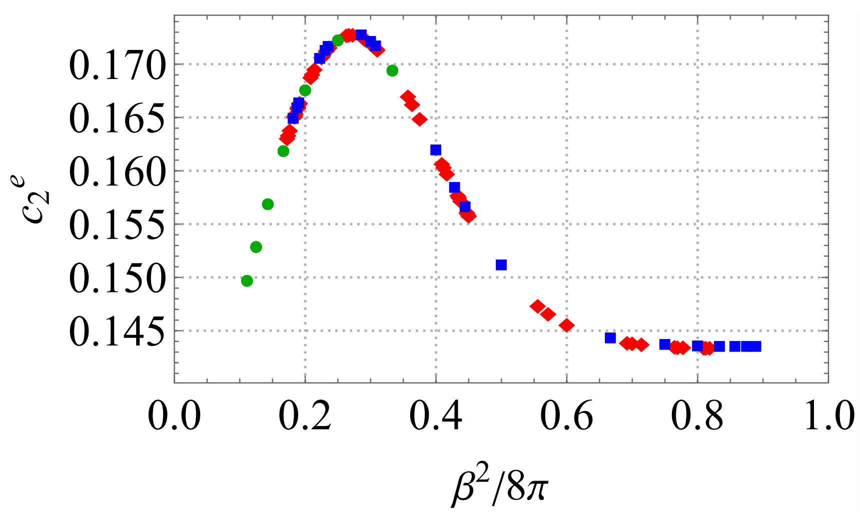

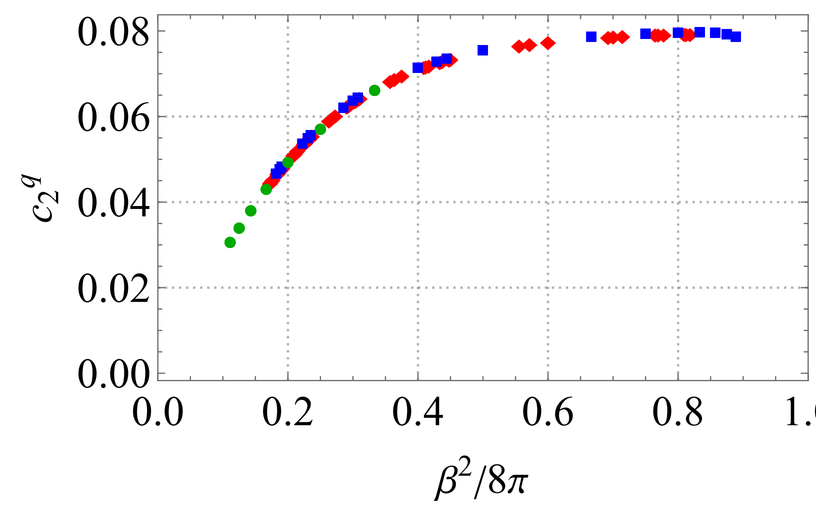

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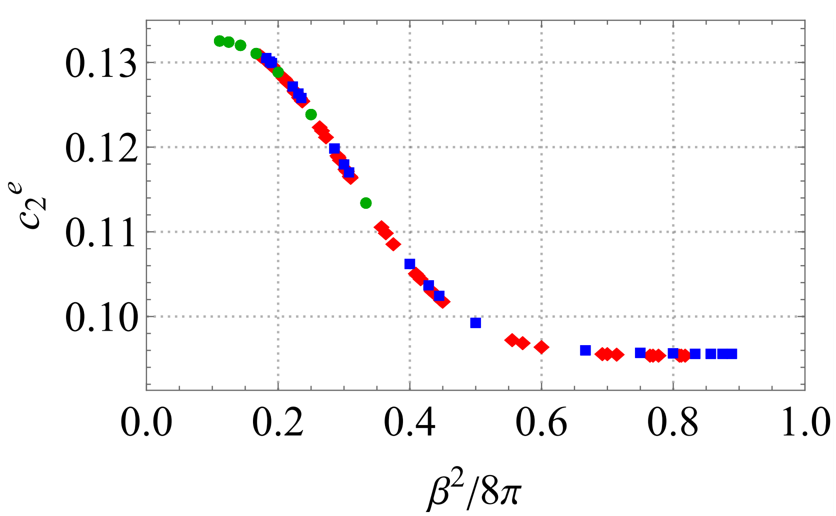

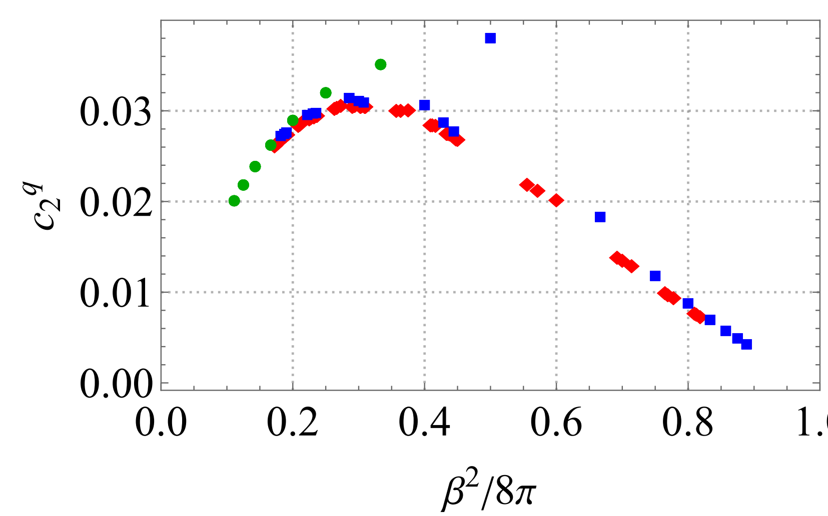

Fractal (popcorn)!

Magnons

Scattering of solitons and antisolitons is non-diagonal!

$$
\begin{array}{cc}
 & \begin{array}{cccc} |++\rangle & |+-\rangle & |-+\rangle & |--\rangle \end{array} \\
S = & \left(\begin{array}{c|cc|c} S_0 & 0 & 0 & 0 \\ \hline 0 & S_R & S_T & 0 \\ 0 & S_T & S_R & 0 \\ \hline 0 & 0 & 0 & S_0 \end{array}\right) \begin{array}{c} |++\rangle \\ |+-\rangle \\ |-+\rangle \\ |--\rangle \end{array}
\end{array}
$$

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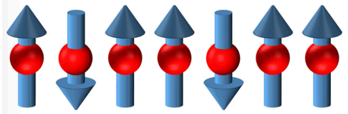
Magnons: "waves of charge flips" over the all-soliton state

$$Q = N_s - 2N_m$$



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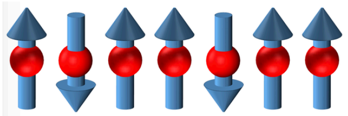

auxiliary particles having zero (bare) momentum and energy but nonzero charge

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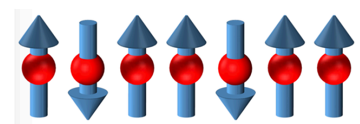

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Magnonic part of (nested) Bethe Ansatz coincides with that of the gapless XXZ spin chain

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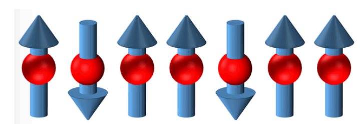

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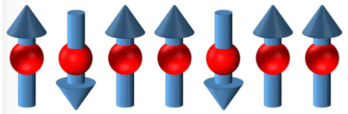

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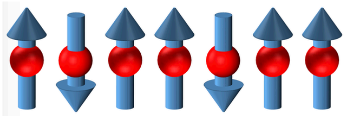

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Takahashi's book

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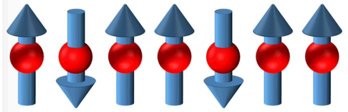

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Takahashi's book

1 kink, n_B breathers, n_m magnon species.

$q = +1$ $\qquad 0 \qquad$ $-2\ell_a$

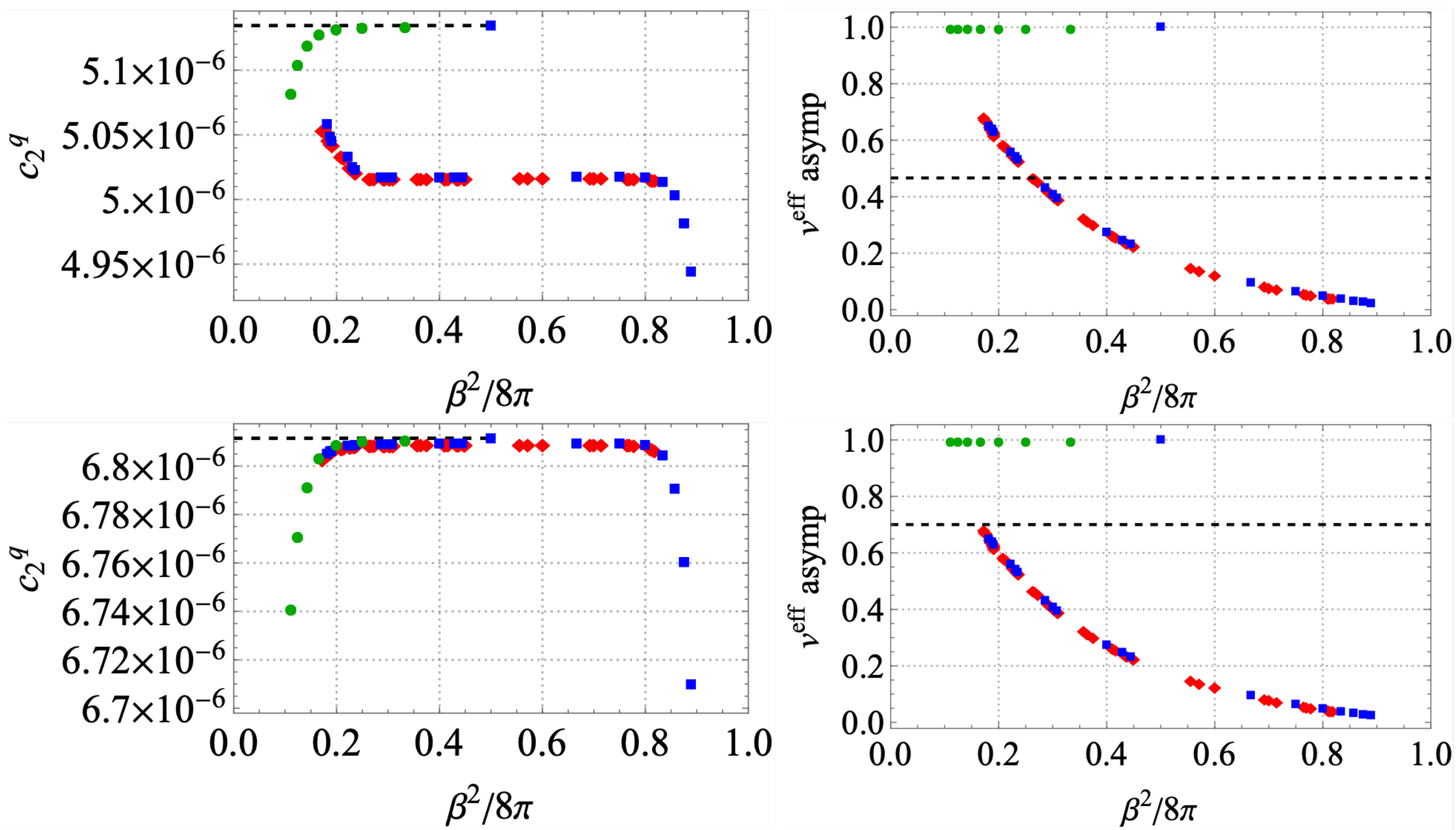
Fractal behaviour

Disappears outside the light cone of the magnons $\tan\alpha = x/t > |v_j^{\mathrm{eff}}(\theta)|$

$T = 0.1M$

$\alpha = 5\pi/36$

$\alpha = 7\pi/36$



High temperature limit $T \gg M$

Free energy: recover the CFT (=free massless boson) value (Rogers dilogarithm):

$$f = -\frac{\pi}{6R^2} - \frac{\mu^2}{2}\left(\frac{\beta}{2\pi}\right)^2 \qquad R = 1/T$$

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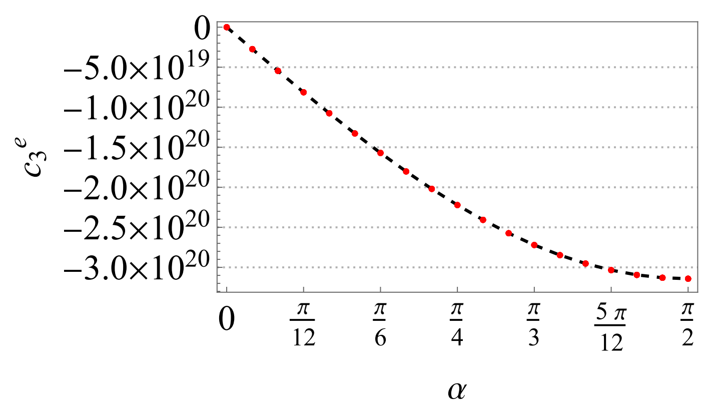
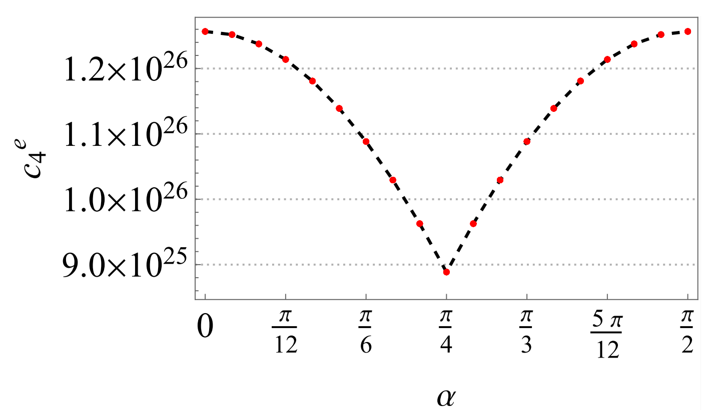
Energy





$T = 10^5 M \qquad \xi = 3$

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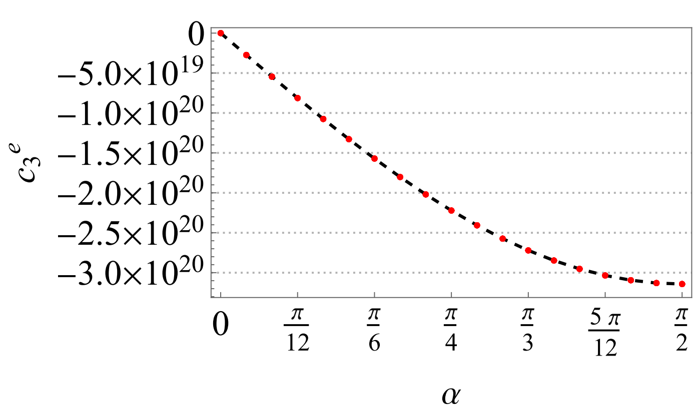
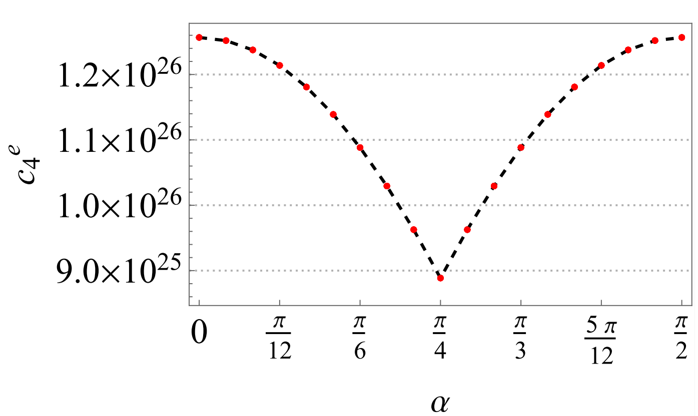
Energy





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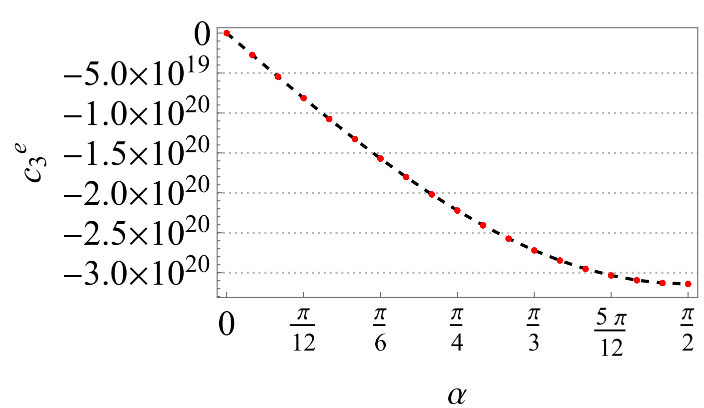
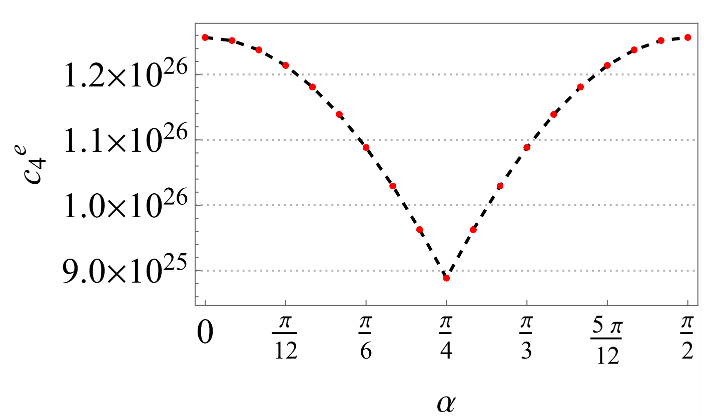
Energy





$T = 10^5 M \qquad \xi = 3$

High temperature limit $T \gg M$

Free energy: recover the CFT (=free massless boson) value (Rogers dilogarithm):

$$f = -\frac{\pi}{6R^2} - \frac{\mu^2}{2}\left(\frac{\beta}{2\pi}\right)^2 \qquad R = 1/T$$

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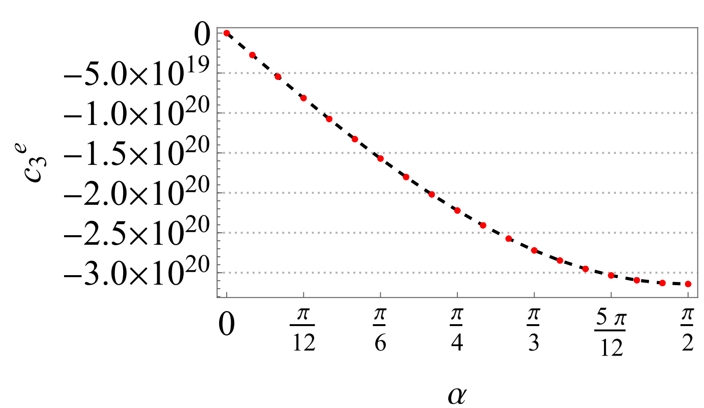
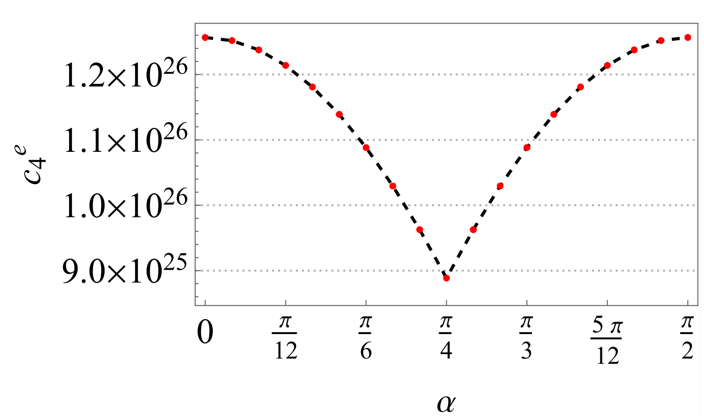
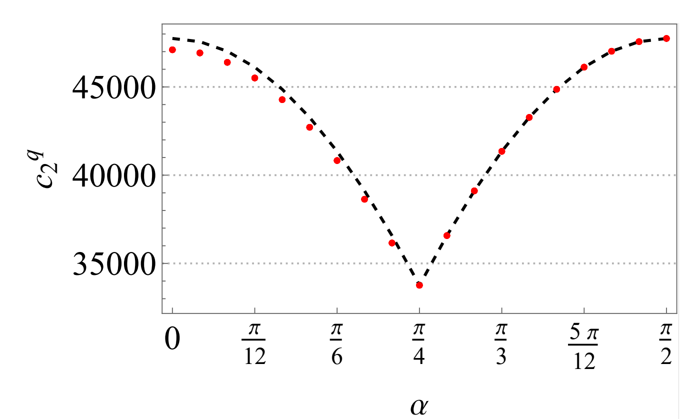




Topological charge



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agrees with Bernard-Doyon '14

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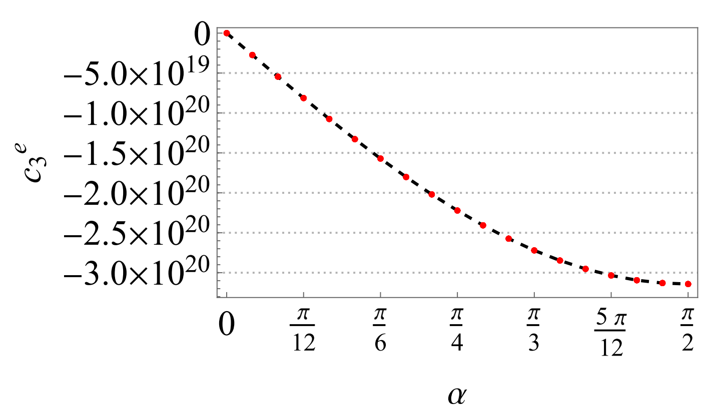

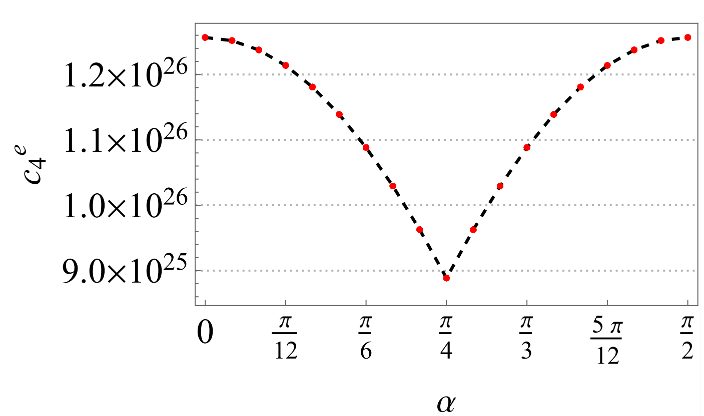

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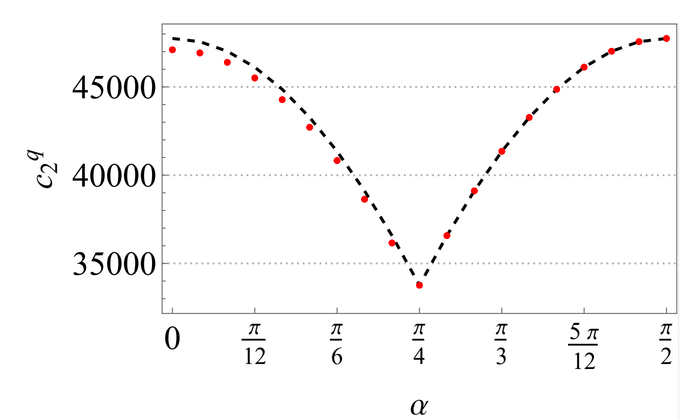

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agrees with Bernard-Doyon '14

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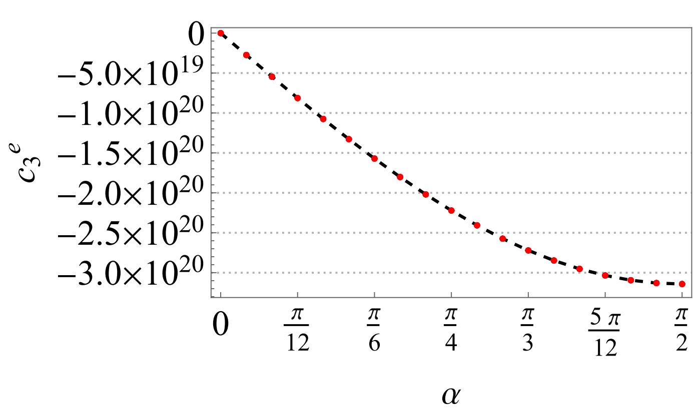
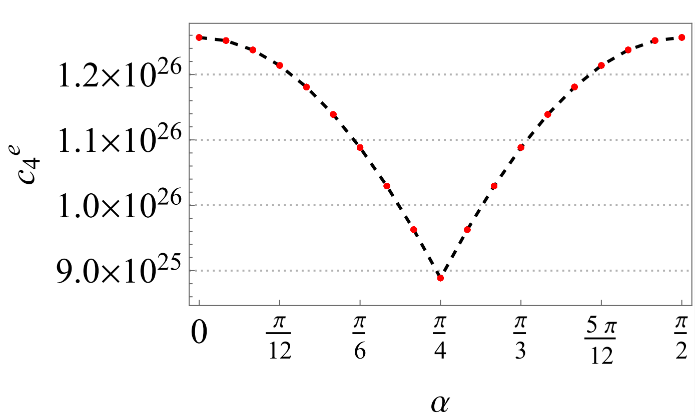
Energy





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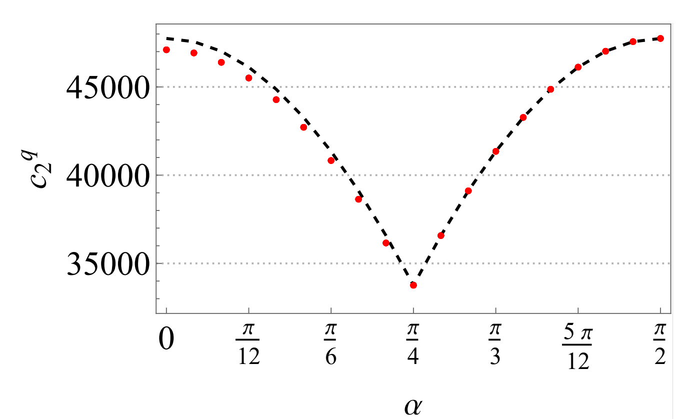

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Low temperature limit $T \ll M$

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Reflectionless points

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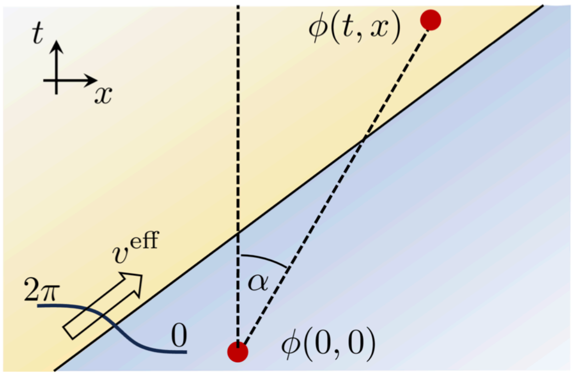
$$F_\alpha(\lambda) = 4\sinh^2(\lambda/2)\int \frac{d\theta}{2\pi} M\cosh(\theta) e^{-M\cosh\theta/T} |\tanh\theta\cos\alpha - \sin\alpha|$$

Low temperature limit $T \ll M$

Semiclassical picture



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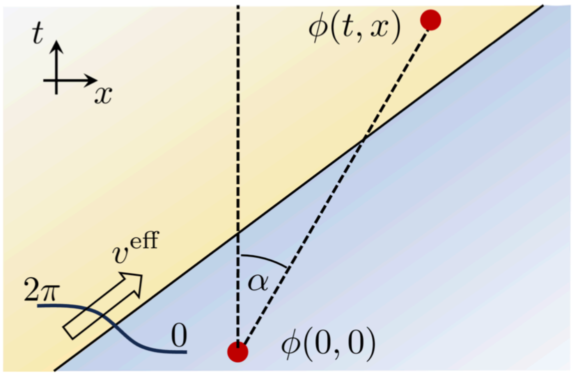

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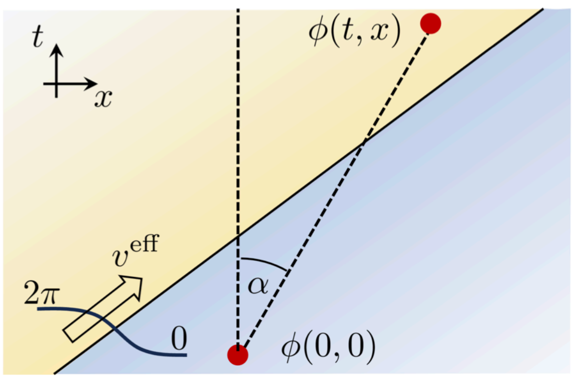

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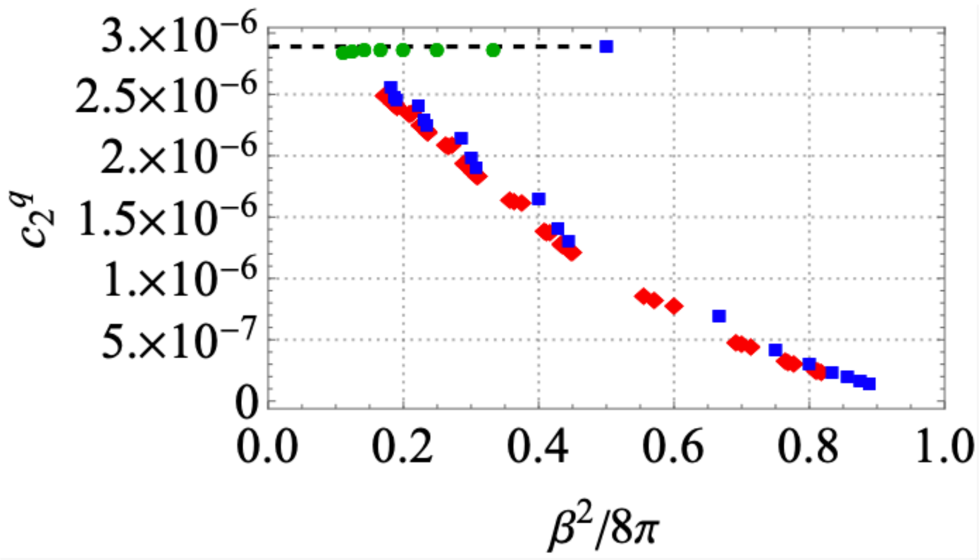
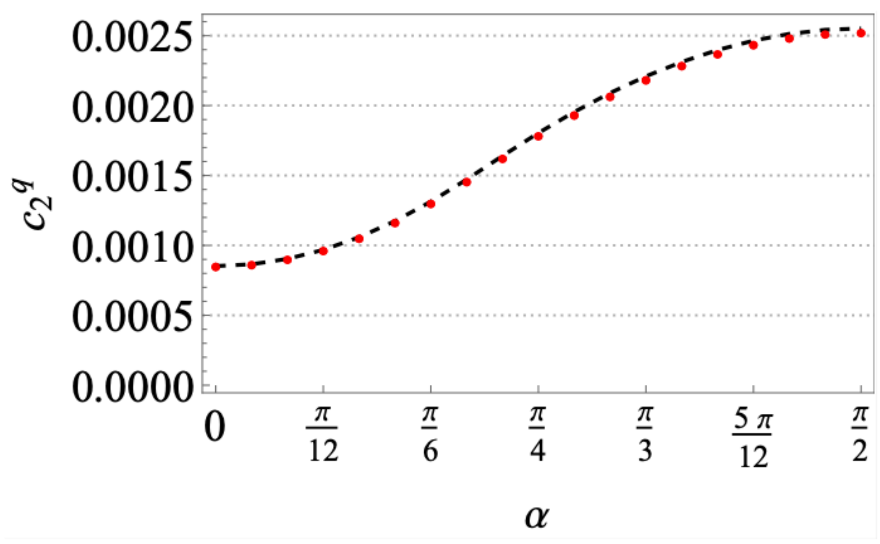


$T = 0.2M \qquad \xi = 1/3$



Low temperature limit $T \ll M$

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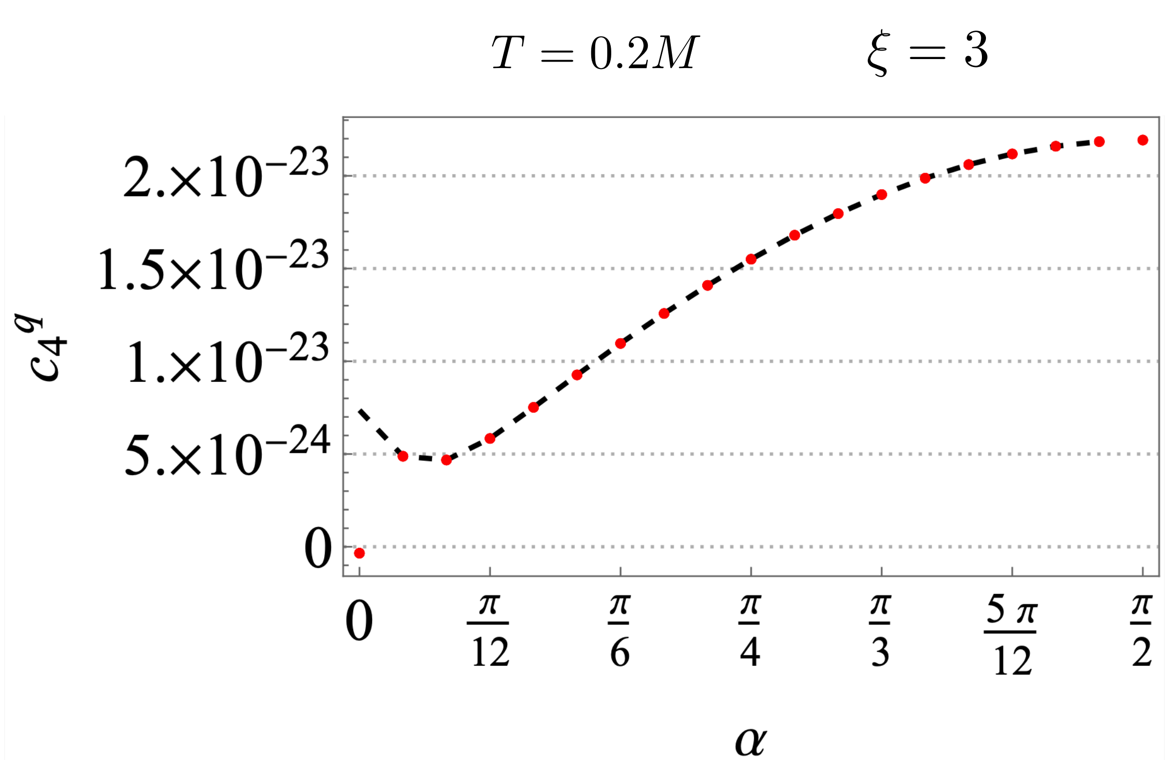

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Thank you for your attention!