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Quantum and Classical Dynamics with Random Permutation Circuits

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Motivation: quantum many-body dynamics and thermalization

not simple in eigenbasis of H
short-range correlated

Quantum quench: $|\Psi_0\rangle \rightarrow |\Psi_t\rangle = e^{-it\hat{H}} |\Psi_0\rangle$

local

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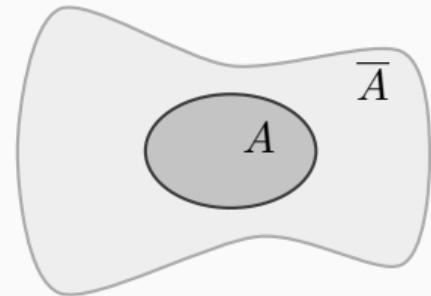
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$$\lim_{t \rightarrow \infty} \lim_{|\bar{A}| \rightarrow \infty} \langle \Psi_t | \mathcal{O}_A | \Psi_t \rangle = \text{tr}[\rho_{\text{th}} \mathcal{O}_A]$$



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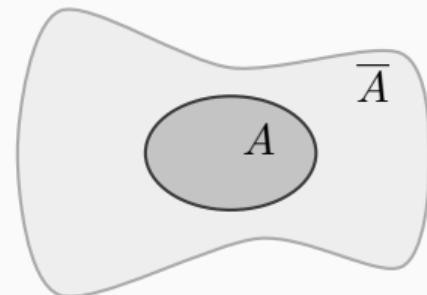
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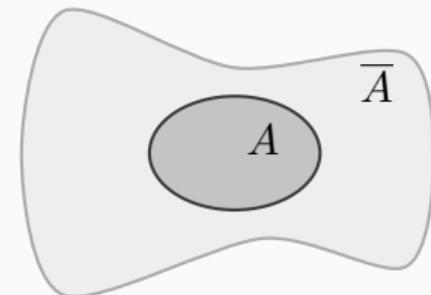
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Q: What happens under classical deterministic dynamics?

⇐ \exists a basis : $|\underline{s}\rangle \rightarrow |\underline{s}'\rangle$

Classical deterministic dynamics

$|\Psi_t\rangle =$ local permutation evolution

$|\uparrow\rangle$ $|\uparrow\rangle$ $|\downarrow\rangle$ $|\downarrow\rangle$ $|\uparrow\rangle$ $|\uparrow\rangle$ $|\uparrow\rangle$ $|\downarrow\rangle$

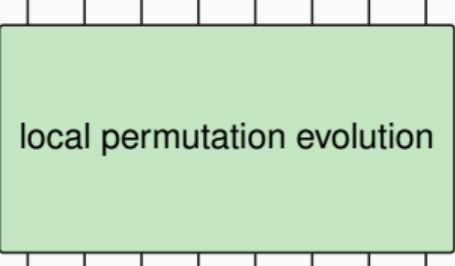
Classical deterministic dynamics

$$|\Psi_t\rangle = \boxed{\text{local permutation evolution}} = \text{product state}$$

The diagram illustrates the concept of local permutation evolution. A central green rectangular box contains the text "local permutation evolution". Above and below this box are two horizontal lines with vertical tick marks at regular intervals. These lines are connected by horizontal lines to a row of 10 quantum states below, represented by symbols like $|\uparrow\rangle$ and $|\downarrow\rangle$.

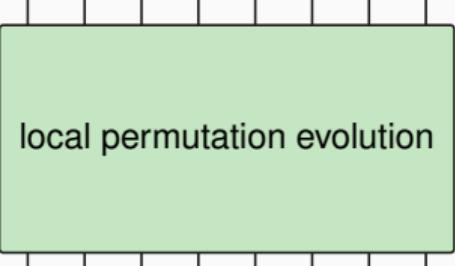
$$|\uparrow\rangle |\uparrow\rangle |\downarrow\rangle |\downarrow\rangle |\uparrow\rangle |\uparrow\rangle |\uparrow\rangle |\uparrow\rangle |\downarrow\rangle$$

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$|\Psi_t\rangle =$  local permutation evolution = product state \implies cannot thermalise

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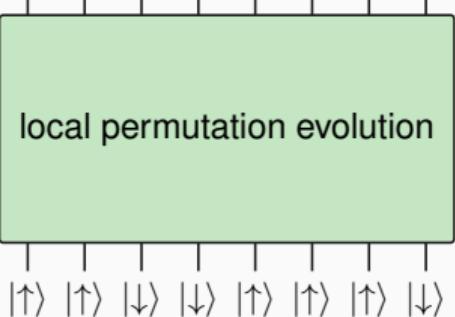
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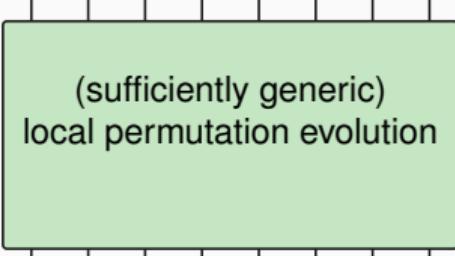
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This statement is basis dependent!

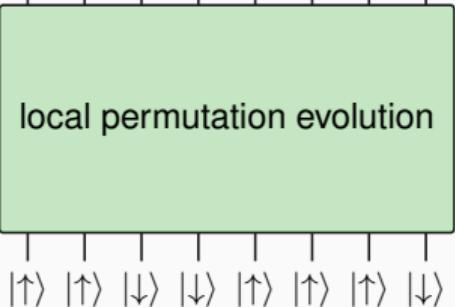
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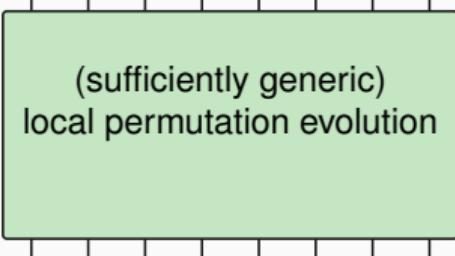
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$|\Psi_t\rangle =$  (sufficiently generic)
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Is there a simple way to distinguish it from generic quantum dynamics?

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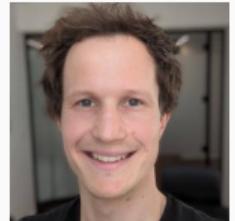
B. Bertini, KK, P. Kos, D. Malz, Phys. Rev. X 15, 011015 (2025)



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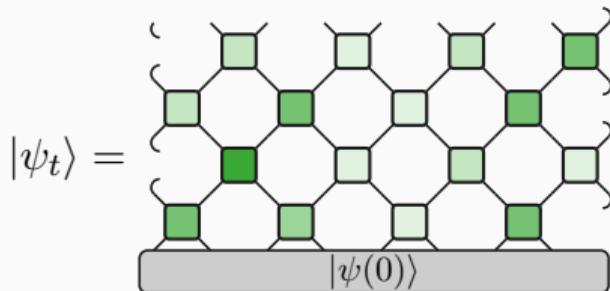
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In analogy with random-unitary circuits:

chain of L qudits with locally applied randomly chosen permutations



$$\square \in S(d^2)$$

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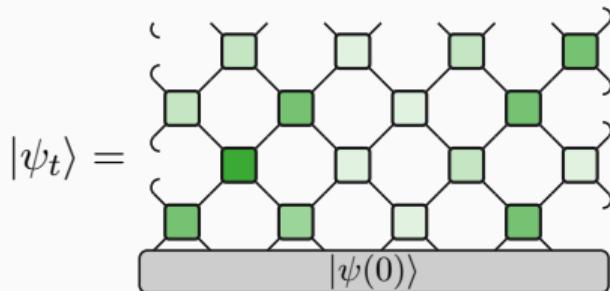
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$$\text{Diagram of a single hexagonal node with three outgoing edges, labeled } \in S(d^2).$$

How do the averaged quantities (e.g. OTOCs and purity) behave?

Technical tool: Averages over the permutation group I

Basic building block:

$$\mathcal{Q}^{(m)} := \frac{1}{d^2!} \sum_{U \in S(d^2)} \underbrace{U \otimes U \otimes U \otimes \cdots \otimes U}_m = \boxed{\text{Diagram}}$$

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Detour: Uniform averages over $d \times d$ unitary matrices

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- Symmetry under reshuffling of replicas: $|\sigma\rangle = \sum_{s_1, \dots, s_n=0}^{d-1} |s_1 s_{\sigma(1)} s_2 s_{\sigma(2)} \dots s_n s_{\sigma(n)}\rangle \quad \forall \sigma \in S(n)$

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Example: $2n = m = 4$

$B_4 = 15$ partition states (and $2! = 2$ permutation states)

$$\pi_1 = \{\{0, 1, 2, 3\}\}$$

$$\pi_2 = \{\{0, 1, 2\}, \{3\}\}$$

$$\pi_3 = \{\{0, 1, 3\}, \{2\}\}$$

$$\pi_4 = \{\{0, 2, 3\}, \{1\}\}$$

$$\pi_5 = \{\{1, 2, 3\}, \{0\}\}$$

$$\pi_6 = \{\{0, 1\}, \{2, 3\}\}$$

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$$\pi_{13} = \{\{1, 2\}, \{0\}, \{3\}\}$$

$$\pi_{15} = \{\{0\}, \{1\}, \{2\}, \{3\}\}$$

$$\pi_2 = \{\{0, 1, 2\}, \{3\}\}$$

$$\pi_4 = \{\{0, 2, 3\}, \{1\}\}$$

$$\pi_6 = \{\{0, 1\}, \{2, 3\}\}$$

$$\pi_8 = \{\{0, 2\}, \{1, 3\}\}$$

$$\pi_{10} = \{\{0, 2\}, \{1\}, \{3\}\}$$

$$\pi_{12} = \{\{1, 3\}, \{0\}, \{2\}\}$$

$$\pi_{14} = \{\{2, 3\}, \{0\}, \{1\}\}$$

$$|\textcircled{O}\rangle = \frac{1}{d} \sum_{a,b=0}^{d-1} |aabb\rangle$$

$$|\square\rangle = \frac{1}{d} \sum_{a,b=0}^{d-1} |abba\rangle$$

$$|\times\rangle = \frac{1}{\sqrt{d}} \sum_{a=0}^{d-1} |aaaa\rangle$$

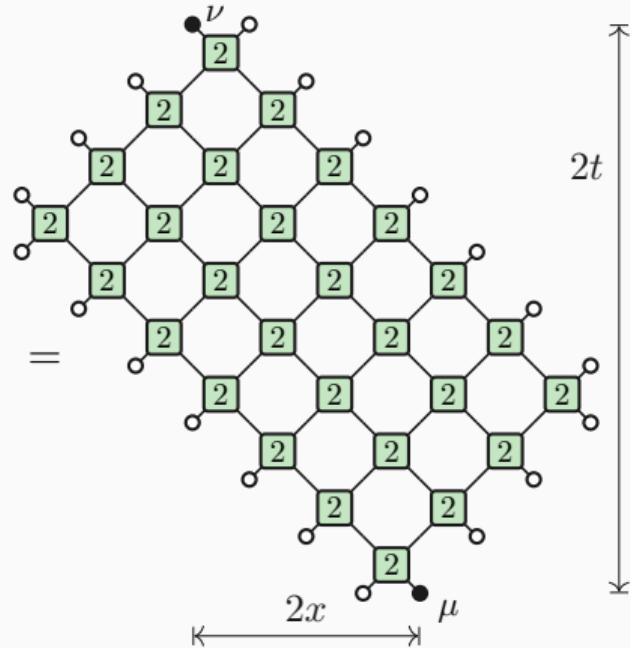
$$|-\rangle = \frac{1}{d^2} \sum_{a,b,c,e=0}^{d-1} |abce\rangle$$

Averaged infinite-temperature two-point correlation function I

$$C_{\mu\nu}(x, t) = \frac{1}{\text{tr } \mathbb{1}} \langle \text{tr}[\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)] \rangle_Q$$

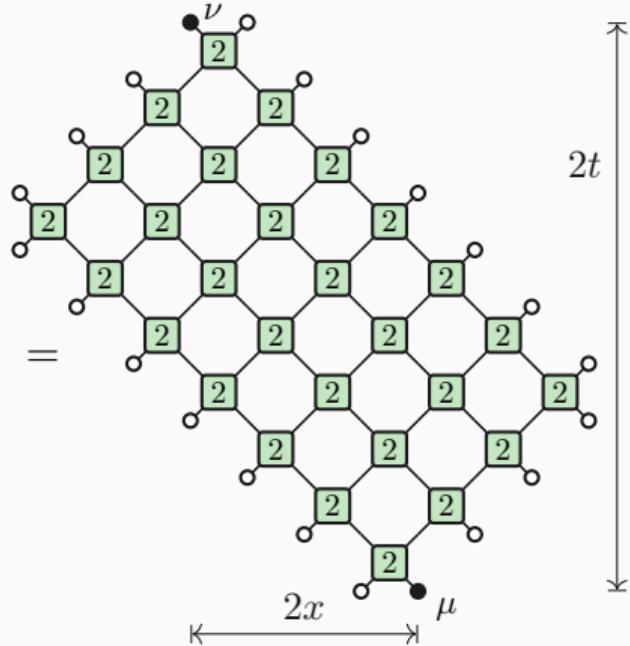
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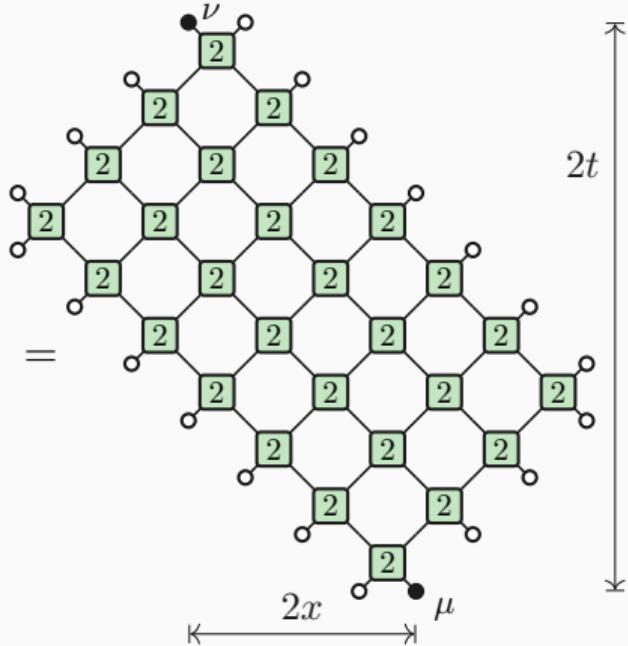
$$\boxed{\begin{array}{l} \text{---} \\ |2\rangle \end{array}} = \mathcal{Q}^{(2)}$$

$$\boxed{|O\rangle = \frac{1}{\sqrt{d}} \sum_{s=0}^{d-1} |ss\rangle \propto |\pi_1\rangle}$$

$$\boxed{|\bullet_\mu\rangle = \mathcal{O}_\mu \otimes \mathbb{1} |O\rangle}$$

Averaged infinite-temperature two-point correlation function I

$$C_{\mu\nu}(x, t) = \frac{1}{\text{tr } \mathbb{1}} \langle \text{tr}[\mathcal{O}_\mu(x, t) \mathcal{O}_\nu(0, 0)] \rangle_Q$$



$$\boxed{\begin{array}{c} 2 \\ \square \end{array}} = \mathcal{Q}^{(2)}$$

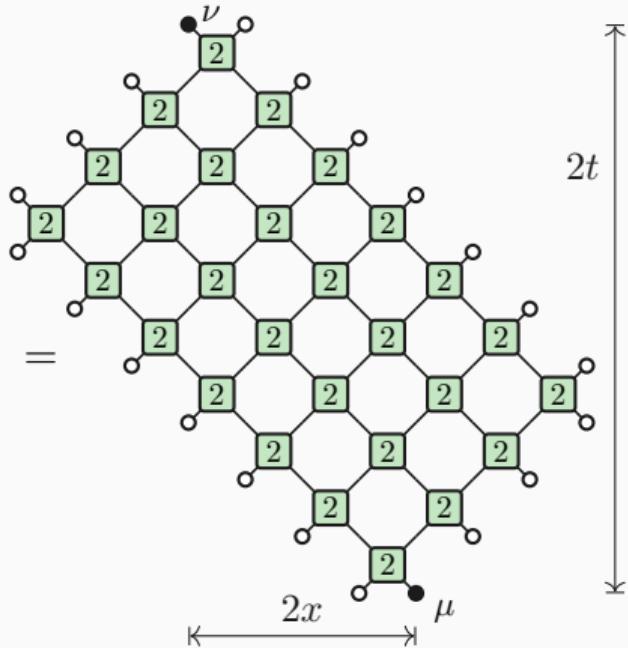
$$\boxed{|\mathcal{O}\rangle = \frac{1}{\sqrt{d}} \sum_{s=0}^{d-1} |ss\rangle \propto |\pi_1\rangle}$$

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$B_2 = 2 \implies C_{\mu\nu}(x, t)$ nontrivial

Averaged infinite-temperature two-point correlation function I

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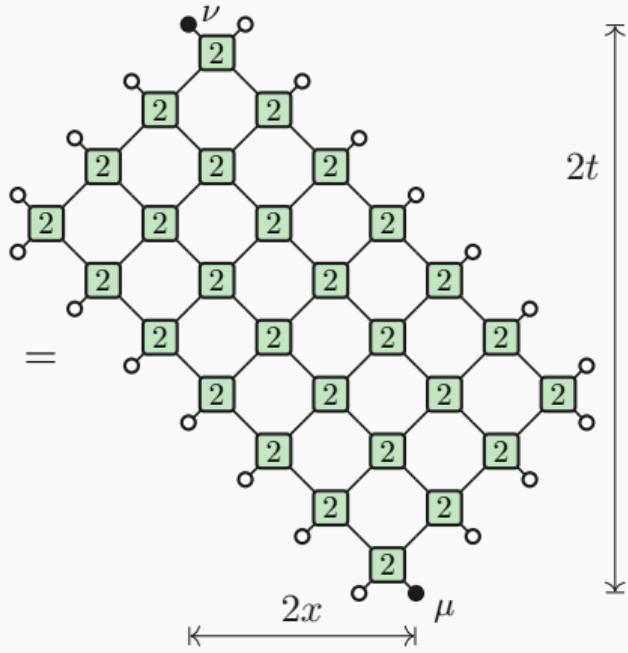
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$$\pi_1 = \{\{0, 1\}\} \quad \pi_2 = \{\{0\}, \{1\}\}$$

$$\boxed{|- \rangle = \frac{1}{d} \sum_{s_1, s_2=0}^{d-1} |s_1 s_2\rangle \propto |\pi_2\rangle}$$

Averaged infinite-temperature two-point correlation function I

$$C_{\mu\nu}(x, t) = \frac{1}{\text{tr } \mathbb{1}} \langle \text{tr}[\mathcal{O}_\mu(x, t) \mathcal{O}_\nu(0, 0)] \rangle_Q$$



$$\boxed{2} = \mathcal{Q}^{(2)}$$

$$|\mathcal{O}\rangle = \frac{1}{\sqrt{d}} \sum_{s=0}^{d-1} |ss\rangle \propto |\pi_1\rangle$$

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$$\pi_1 = \{\{0, 1\}\} \quad \pi_2 = \{\{0\}, \{1\}\}$$

$$|-\rangle = \frac{1}{d} \sum_{s_1, s_2=0}^{d-1} |s_1 s_2\rangle \propto |\pi_2\rangle$$

First step: project $|\bullet_{\mu,\nu}\rangle$ to $\{ |\mathcal{O}\rangle, |-\rangle \}$

Averaged infinite-temperature two-point correlation function II

$$C_{\mu\nu}(x, t) = \frac{o_\mu o_\nu}{(1 - 1/d)^2} \left[C_{--}(x, t) - \frac{1}{d} \right]$$

Averaged infinite-temperature two-point correlation function II

$$C_{\mu\nu}(x, t) = \frac{o_\mu o_\nu}{(1 - 1/d)^2} \left[C--(x, t) - \frac{1}{d} \right]$$

$$C--(x, t) = \begin{array}{c} \text{A hexagonal lattice with nodes labeled '2' in green squares. The nodes are arranged in a grid pattern, with the top node having an arrow pointing upwards. The nodes are connected by edges forming a hexagonal mesh. The labels '2' are placed at various nodes, such as the top node, some internal nodes, and nodes along the edges. The overall structure is a 5x5 grid of hexagons.} \end{array}$$

$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

Averaged infinite-temperature two-point correlation function II

$$C_{\mu\nu}(x, t) = \frac{o_\mu o_\nu}{(1 - 1/d)^2} \left[C--(x, t) - \frac{1}{d} \right]$$

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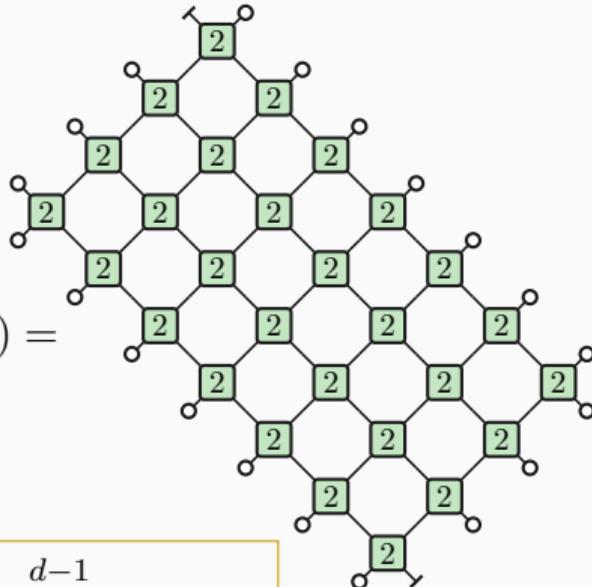
$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

$$\begin{array}{ccc} \text{Diagram 1: } & \text{Diagram 2: } & \text{Diagram 3: } \\ \text{Diagram 4: } & \text{Diagram 5: } & \text{Diagram 6: } \\ \text{Diagram 7: } & \text{Diagram 8: } & \text{Diagram 9: } \end{array}$$

$$\begin{aligned} \text{Diagram 1: } &= \text{Diagram 2: } \\ \text{Diagram 4: } &= \text{Diagram 5: } = \frac{\sqrt{d}}{1+d} (\text{Diagram 2: } + \text{Diagram 3: }) \end{aligned}$$

Averaged infinite-temperature two-point correlation function II

$$C_{\mu\nu}(x,t) = \frac{o_\mu o_\nu}{(1 - 1/d)^2} \left[C_{--}(x,t) - \frac{1}{d} \right]$$



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$$\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} = \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} = \frac{\sqrt{d}}{1+d} \left(\begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} + \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} \right)$$

Analogous to $2n = 4$ for random unitaries:

$$\left. \frac{C_{\mu\nu}(x,t)}{o_\mu o_\nu} \right|_{d \rightarrow d^2} = \left\langle \left(\frac{\text{tr}[O_\mu(x,t)O_\nu(0,0)]}{\text{tr} \mathbb{1}} \right)^2 \right\rangle_U$$

Averaged infinite-temperature two-point correlation function II

$$C_{\mu\nu}(x, t) = \frac{o_\mu o_\nu}{(1 - 1/d)^2} \left[C_{--}(x, t) - \frac{1}{d} \right]$$

$$C_{--}(x, t) = \begin{array}{c} \text{A hexagonal lattice with sites labeled } 2 \\ \text{at each vertex. The lattice has } x \text{ columns and } t \text{ rows.} \end{array}$$

$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

$$\begin{aligned} \text{Diagram 1: } & \text{A square loop with a central site labeled } 2 \text{ and four external sites.} \\ & \text{Value: } 1 \\ \text{Diagram 2: } & \text{A square loop with a central site labeled } 2 \text{ and four external sites.} \\ & \text{Value: } 1 \\ \text{Diagram 3: } & \text{A square loop with a central site labeled } 2 \text{ and four external sites.} \\ & \text{Value: } \frac{\sqrt{d}}{1+d} \left(1 + 1 \right) \end{aligned}$$

Analogous to $2n = 4$ for random unitaries:

$$\left. \frac{C_{\mu\nu}(x, t)}{o_\mu o_\nu} \right|_{d \rightarrow d^2} = \left\langle \left(\frac{\text{tr}[O_\mu(x, t) O_\nu(0, 0)]}{\text{tr } \mathbb{1}} \right)^2 \right\rangle_U$$

Asymptotic scaling:

$$C_{\mu\nu}(x, t) \simeq \frac{o_\mu o_\nu c(d)}{t^2} \left(\frac{4/d}{(1 + 1/d)^2} \right)^{2t} e^{-2x^2/t}$$

Out-of-time-ordered correlation functions I

$$O_{\mu\nu}(x, t) = \frac{1}{2} \frac{\left\langle \text{tr} \left[|[\mathcal{O}_\mu(x, t), \mathcal{O}_\nu(0, 0)]|^2 \right] \right\rangle_Q}{\text{tr} \mathbb{1}}$$

Out-of-time-ordered correlation functions I

$$O_{\mu\nu}(x, t) = \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu^2(x, t)\mathcal{O}_\nu^2(0, 0)] \rangle_Q}{\text{tr} \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(1)}(x, t) \quad = 1 + \text{subleading terms}} - \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)] \rangle_Q}{\text{tr} \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(2)}(x, t)}$$

$$\begin{aligned}\text{tr}[\mathcal{O}_\mu] &= 0 \\ \text{tr}[\mathcal{O}_\mu^2] &= d\end{aligned}$$

Out-of-time-ordered correlation functions I

$$O_{\mu\nu}(x, t) = \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu^2(x, t)\mathcal{O}_\nu^2(0, 0)] \rangle_Q}{\text{tr} \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(1)}(x, t) \text{ = 1+subleading terms}} - \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)] \rangle_Q}{\text{tr} \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(2)}(x, t)}$$

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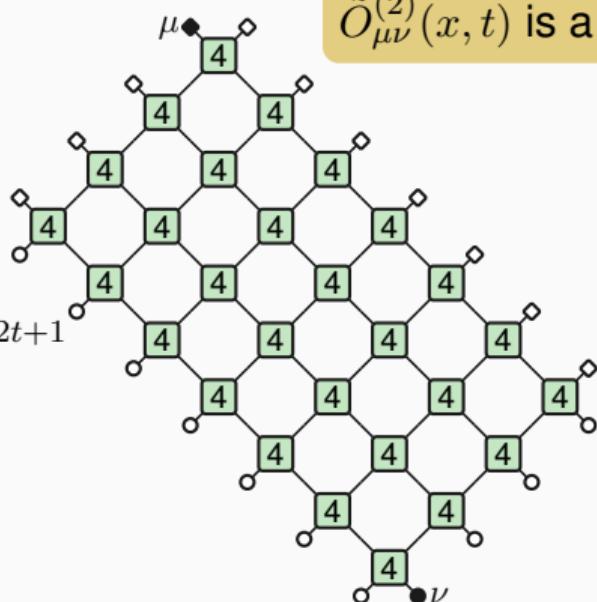
$\tilde{O}_{\mu\nu}^{(2)}(x, t)$ is a $m = 4$ quantity

Out-of-time-ordered correlation functions I

$$O_{\mu\nu}(x, t) = \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu^2(x, t)\mathcal{O}_\nu^2(0, 0)] \rangle_Q}{\text{tr } \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(1)}(x, t) = 1 + \text{subleading terms}} - \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)] \rangle_Q}{\text{tr } \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(2)}(x, t)}$$

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$\tilde{O}_{\mu\nu}^{(2)}(x, t)$ is a $m = 4$ quantity



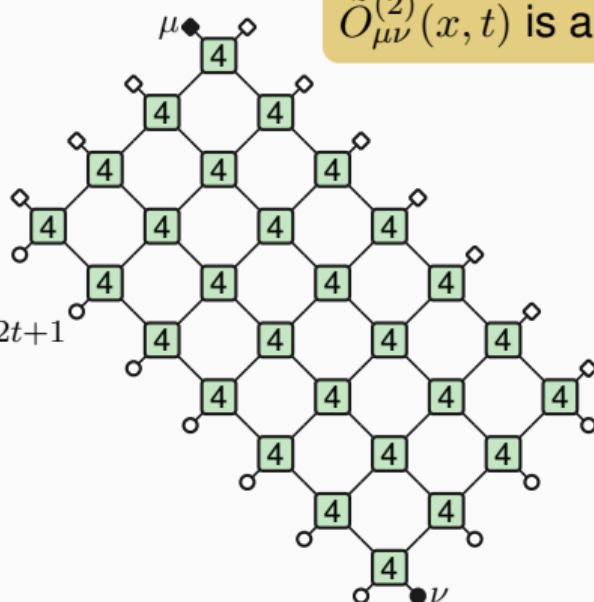
$$\tilde{O}_{\mu\nu}^{(2)}(x, t) = d^{2t+1}$$

Out-of-time-ordered correlation functions I

$$O_{\mu\nu}(x, t) = \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu^2(x, t)\mathcal{O}_\nu^2(0, 0)] \rangle_Q}{\text{tr } \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(1)}(x, t) = 1 + \text{subleading terms}} - \underbrace{\frac{\langle \text{tr}[\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)\mathcal{O}_\mu(x, t)\mathcal{O}_\nu(0, 0)] \rangle_Q}{\text{tr } \mathbb{1}}}_{\tilde{O}_{\mu\nu}^{(2)}(x, t)}$$

$\text{tr}[\mathcal{O}_\mu] = 0$
 $\text{tr}[\mathcal{O}_\mu^2] = d$

$\tilde{O}_{\mu\nu}^{(2)}(x, t)$ is a $m = 4$ quantity



$$\tilde{O}_{\mu\nu}^{(2)}(x, t) = d^{2t+1}$$

$B_4 = 15$ states!

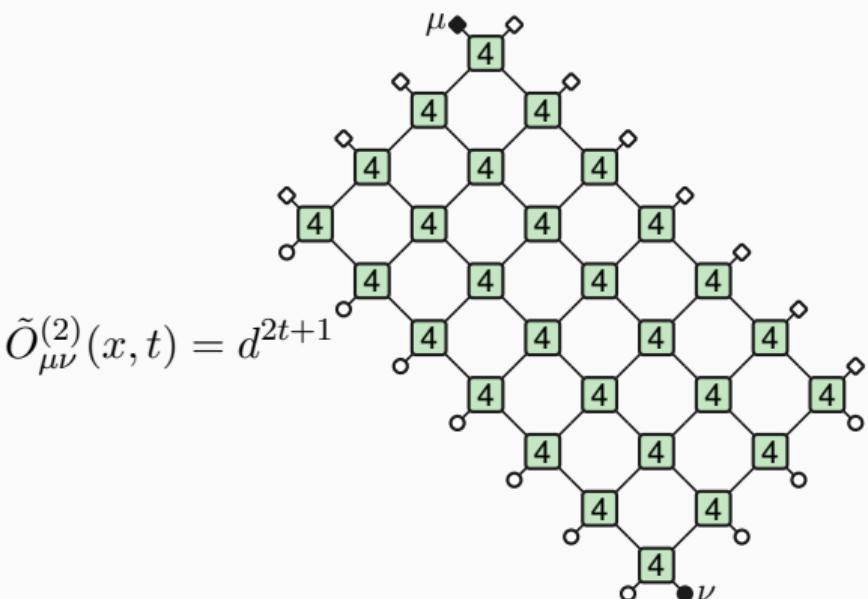
$$|\textcircled{O}\rangle = \frac{1}{d} \sum_{a,b=0}^{d-1} |aabb\rangle \quad |\square\rangle = \frac{1}{d} \sum_{a,b=0}^{d-1} |abba\rangle$$

$$|\times\rangle = \frac{1}{\sqrt{d}} \sum_{a=0}^{d-1} |aaaa\rangle \quad |-\rangle = \frac{1}{d^2} \sum_{a,b,c,e=0}^{d-1} |abce\rangle$$

$$|\bullet_\nu\rangle = (\mathcal{O}_\nu \otimes \mathbb{1})^{\otimes 2} |\textcircled{O}\rangle \quad |\blacksquare_\mu\rangle = (\mathcal{O}_\mu \otimes \mathbb{1})^{\otimes 2} |\square\rangle$$

Out-of-time-ordered correlation functions II

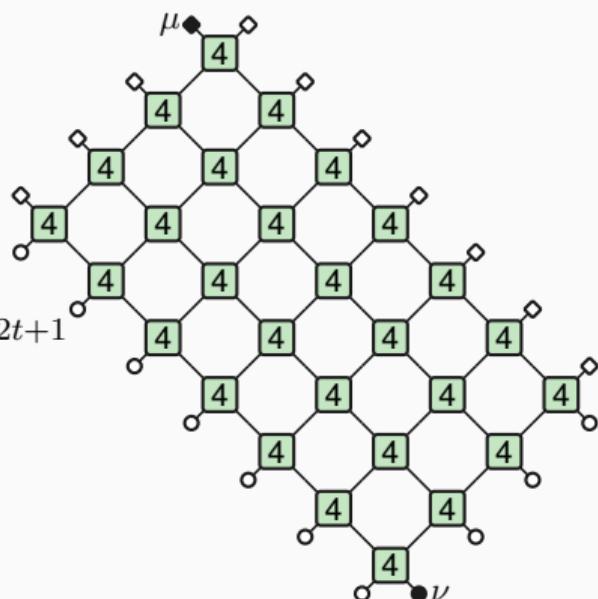
Problem: generic $|\blacksquare_\mu\rangle$, $|\bullet_\nu\rangle$ are projected to all 15 states $\Rightarrow \tilde{O}^{(2)}(x, t)$ hard to evaluate



Out-of-time-ordered correlation functions II

Problem: generic $|\blacksquare_\mu\rangle$, $|\bullet_\nu\rangle$ are projected to all 15 states $\Rightarrow \tilde{O}^{(2)}(x, t)$ hard to evaluate

Additional structure for $\mathcal{O}_\nu \rightarrow \mathcal{O}_d$ diagonal

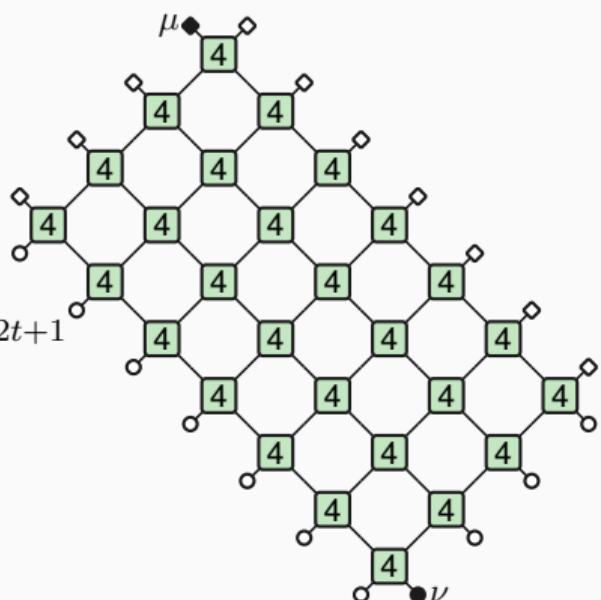


Out-of-time-ordered correlation functions II

Problem: generic $|\blacksquare_\mu\rangle$, $|\bullet_\nu\rangle$ are projected to all 15 states $\Rightarrow \tilde{O}^{(2)}(x, t)$ hard to evaluate

Additional structure for $\mathcal{O}_\nu \rightarrow \mathcal{O}_d$ diagonal

$$|\bullet_d\rangle \mapsto \frac{\sqrt{d}}{d-1} |\times\rangle - \frac{1}{d-1} |\circ\rangle$$



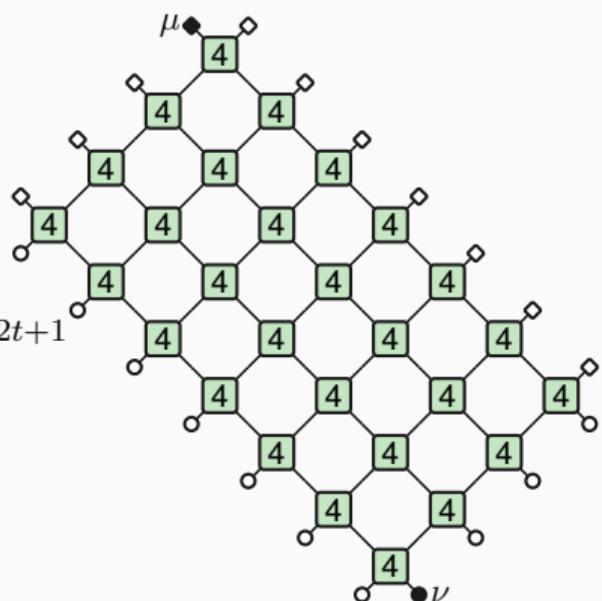
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Action of $\mathcal{Q}^{(4)}$ on these states closes:



$$\begin{aligned} \text{Diagram 1: } & \text{Top hexagon } 4 \text{ with open edges} = \text{Two vertical lines} \\ \text{Diagram 2: } & \text{Top hexagon } 4 \text{ with '+' signs at top and bottom} = \text{Two crossed lines} \\ \text{Diagram 3: } & \text{Bottom hexagon } 4 \text{ with '+' signs at top and bottom} = \frac{\sqrt{d}}{1+d} \left(\text{Two vertical lines} + \text{Two crossed lines} \right) \end{aligned}$$

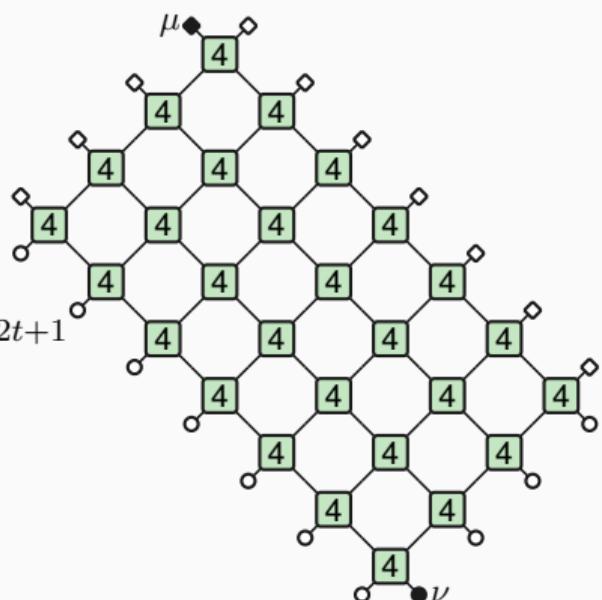
Out-of-time-ordered correlation functions II

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Action of $\mathcal{Q}^{(4)}$ on these states closes:



$$\tilde{O}_{\mu\nu}^{(2)}(x, t) = d^{2t+1}$$

$$\begin{aligned} \text{Diagram 1: } & \text{A hexagon with a green square containing '4' and two open circles on its left side. It equals a pair of vertical lines: } \textcircled{1} \text{ } \textcircled{1} \\ \text{Diagram 2: } & \text{A hexagon with a green square containing '4' and two '+' signs on its top and right sides. It equals a pair of crossed lines: } \text{X} \text{ } \text{X} \\ \text{Diagram 3: } & \text{A hexagon with a green square containing '4' and one open circle on its left and one '+' sign on its right. It equals a sum: } \frac{\sqrt{d}}{1+d} \left(\textcircled{1} \text{ } \textcircled{1} + \text{X} \text{ } \text{X} \right) \end{aligned}$$

We can obtain the closed-form expression

Out-of-time-ordered correlation functions III

$$C_{\mu\nu}(x, t) = \frac{d^2 o_\mu o_\nu}{d - 1} f(x, t) \quad O_{\mu d}(x, t) = \frac{1 - o_{\mu, 2}}{d - 1} [f(x, t)]_{d \mapsto \frac{1}{d}}$$

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$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

$$o_{\mu,2} = \frac{1}{d} \sum_{i=0}^{d-1} \langle i | \mathcal{O}_\mu | i \rangle^2$$

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$$O^{(\text{RU})}(x, t) = O_{\mu d}(x, t)|_{o_{\mu,2} \mapsto 0, d \mapsto d^2}$$

$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

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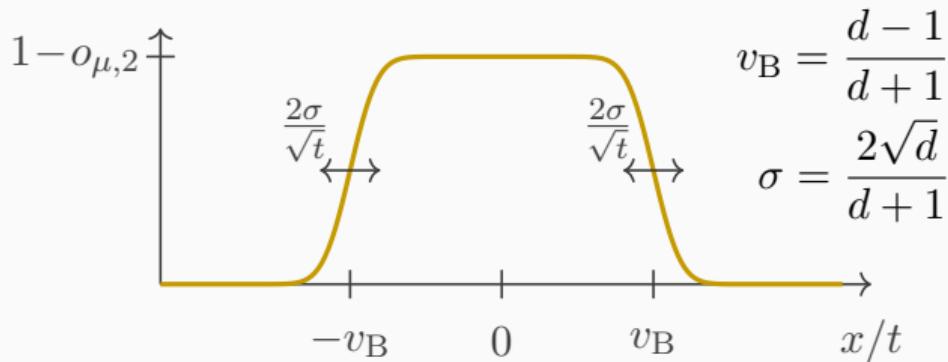
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Asymptotic scaling:



Out-of-time-ordered correlation functions III

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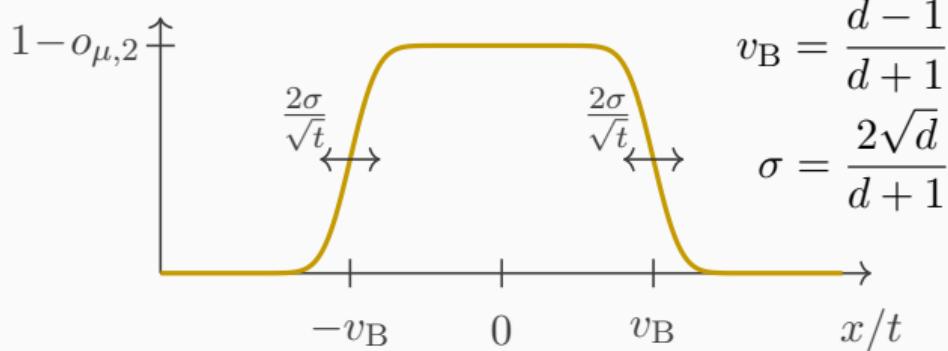
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Asymptotic scaling:

Non-generic value inside the light-cone!



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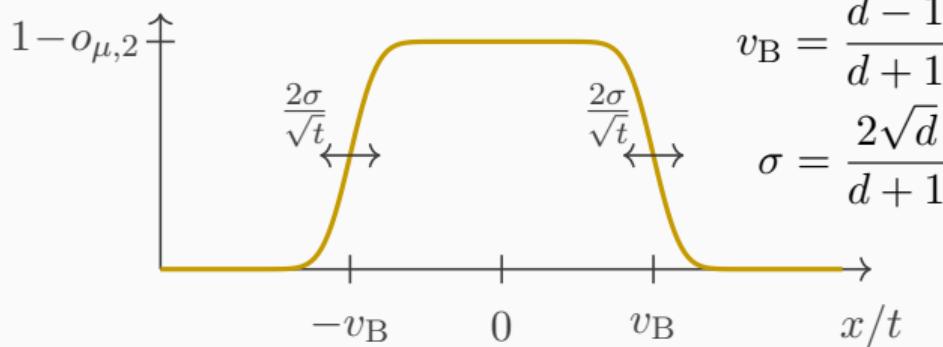
$$o_\mu = \frac{1}{d^{\frac{3}{2}}} \sum_{i,j=0}^{d-1} \langle i | \mathcal{O}_\mu | j \rangle$$

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$$o_{\mu, 2} = \frac{1}{d} \sum_{i=0}^{d-1} \langle i | \mathcal{O}_\mu | i \rangle^2$$

Asymptotic scaling:

Non-generic value inside the light-cone!



$$v_B = \frac{d-1}{d+1}$$

$$\sigma = \frac{2\sqrt{d}}{d+1}$$

Dynamics preserves diagonal operators

$$[\mathcal{O}_{d,1}(0, 0), \mathcal{O}_{d,2}(x, t)] = 0$$

Out-of-time-ordered correlation functions III

$$C_{\mu\nu}(x, t) = \frac{d^2 o_\mu o_\nu}{d-1} f(x, t)$$

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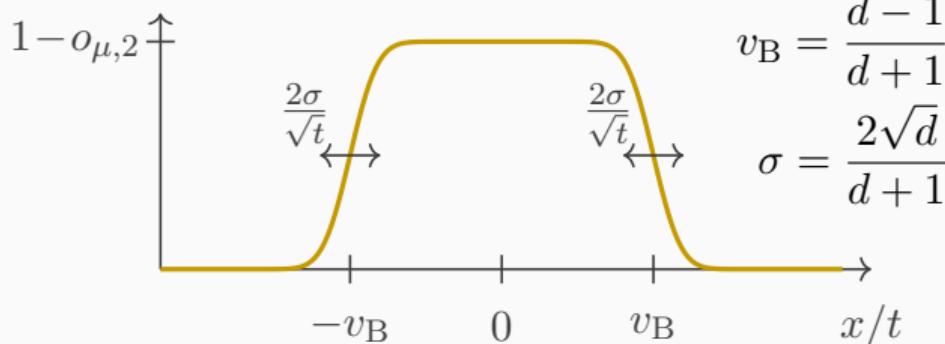
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$$O^{(\text{RU})}(x, t) = O_{\mu d}(x, t)|_{o_{\mu,2} \mapsto 0, d \mapsto d^2}$$

$$o_{\mu,2} = \frac{1}{d} \sum_{i=0}^{d-1} \langle i | \mathcal{O}_\mu | i \rangle^2$$

Asymptotic scaling:

Non-generic value inside the light-cone!



$$v_B = \frac{d-1}{d+1}$$

$$\sigma = \frac{2\sqrt{d}}{d+1}$$

Dynamics preserves diagonal operators

$$[\mathcal{O}_{d,1}(0, 0), \mathcal{O}_{d,2}(x, t)] = 0$$

Expectation: $O_{\mu\nu}(0, t) \rightarrow 1 - o_{\mu,2} o_{\nu,2}$

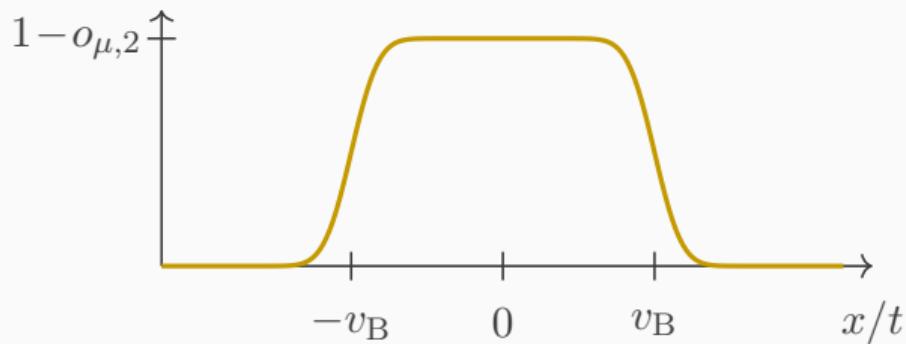
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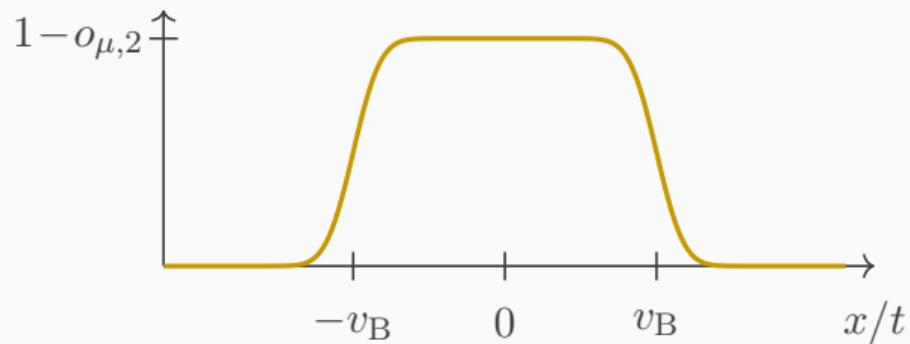
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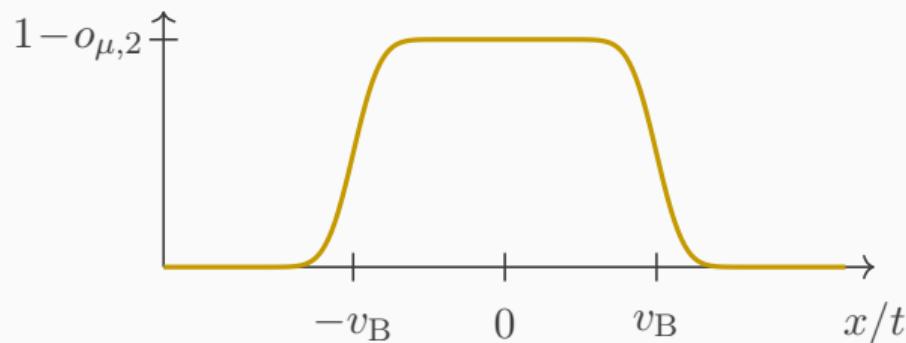
What about entanglement? \leftarrow cf. Michele's talk

D. Szász-Schagrin, M. Mazzoni, B. Bertini, KK, L. Piroli, 2025, arXiv:2505.06158

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Higher replica quantities: local operator entanglement

B. Bertini, KK, P. Kos, D. Malz, 2025, arXiv:2508.10890