European Strategy for Particle Physics

PARALLEL 1 / **ELECTROWEAK PHYSICS**

# Theory Uncertainty Scenarios


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23-27 JUNE 2025 Lido di Venezia

# Outline



- Introduction
- What are theory uncertainties?
- How to estimate theory uncertainties?
- Theory for e+e- colliders (FCC-ee and LCF)
- Theory for hadron colliders (HL-LHC and FCC-hh)
- Theory for muon colliders
- Conclusions

# What are "theory uncertainties"?
# and why are they important?



- **"Theory uncertainties"** is an umbrella term that cover many different uncertainties connected to the **theoretical/modelling framework** used in a given analysis, and in its interpretation
- Due to their nature, the estimation of theory uncertainties is subject to a **high degree of arbitrariness**. Thus, ideally should be subdominant. For future precision measurements, this will require significant work.
- **Purpose of this presentation:** Review, update and extension of previous theory error estimates (mostly for e+e- colliders)
- Consider what is known today and **scenarios for future improvements**
- No attempt to forecast which scenario is likely to be achieved, or to determine a hard boundary of what improvements are possible, just estimate which improvements are needed to reach a certain precision.

# Examples of theory/modelling uncertainties



Examples of theory uncertainties are:

- **Missing higher-order uncertainties** (MHOUs) in perturbative calculations.
- **PDF uncertainties:** depends on the exp. uncertainties and th. uncertainties for the observables used in the PDF fits; and modelling uncertainties (parameterization, heavy-quark evolution) of the PDFs themselves.
- **Non-perturbative** (NP) effects (e.g. hadronization, running couplings, …); usually implemented via physics inspired modelling, and requires input from data**.**
- **Parametric uncertainties:** determination of physical quantities used the in the calculations (e.g. the value of a mass or coupling) requires theory input to extract from data

# How to estimate a "theory uncertainty"?



**MHOUs:**

- Count prefactors
- Extrapolation of perturbative series
- Renormalization scale dependence
- Renormalization scheme dependence

**NP modeling:**

- Variation of model parameters
- Comparison of models
- Lattice computations with error evaluation
- etc.

e.g.

- $g^2/(4\pi^2)*N_f$ for ew. loop order
- $\alpha_S/\pi*C$ for strong loop order

($N_f$ = # of fermion species)

(C = Casimir factor)

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Assumption that the perturbation series follows a geometric series

e.g.

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2),$$

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- etc.

Missing orders for MSbar quantities ~ difference between different values of the renormalization scale.

e.g. for Z-pole observables typically $\mu = M_Z; M_Z/2; 2\ M_Z$

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- Lattice computations with error evaluation
- etc.

Comparison between different renormalization schemes
(e.g. OS vs MSbar)

# How to estimate a "theory uncertainty"?



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**NP modeling:**

- Variation of model parameters
- Comparison of models
- Lattice computations with error evaluation
- etc.

Theory errors in general do not have a clear statistical interpretation

**For fits within ESPPU '26:**
Use Gaussian pdfs with width given by theory errors

# Theory for e+e− colliders



- Focus on **electroweak precision observables (EWPOs)** and **Higgs precision observables (HPOs)**
- EWPOs/HPOs are **pseudo-observables (POs)** — theory input is needed to extract them from data (backgrounds, QED/QCD radiation, Monte-Carlo modelling, etc.)
- Results for POs are compared to SM (or BSM) predictions to **probe new physics**
- **Both of the above** are sources of theory uncertainties
  ➔ attempt to provide estimates for as many sources as possible, *but not comprehensive*
- **Three scenarios:**
  a) Current status
  b) “Conservative” future theory developments
  c) “Aggressive” future theory developments

# Theory for e+e− colliders



**Experimental precision goals of e+e- colliders:**
(from ESPPU submissions; final inputs used in fits may differ; see also presentations by C. Grefe, M. Selvaggi)

| | LCF-250 | FCC-ee |
|---|---|---|
| H ➔ bb [%] | 0.41 | 0.21 |
| H ➔ cc [%] | 2.5 | 1.6 |
| H ➔ ττ [%] | 1.0 | 0.6 |
| H ➔ WW [%] | 1.4 | 0.8 |
| H ➔ ZZ [%] | 5.5 | 2.5 |
| H ➔ γZ [%] | | 12 |
| H ➔ γγ [%] | 10 | 3.6 |
| ee ➔ ZH [%] | 0.62 | 0.31 |

| | LCF | FCC-ee |
|---|---|---|
| $\sigma_{had}$ [pb] | | 0.8 |
| $R_\ell$ [$10^{-3}$] | 6 | 0.07 |
| $R_c$ [$10^{-5}$] | 5.9 | 0.05 |
| $R_b$ [$10^{-5}$] | 1.4 | 0.04 |
| $\Gamma_Z$ [MeV] | 0.5 | 0.013 |
| $A_\ell$ [$10^{-5}$] | 3 | 1.4 |
| $A_b$ [$10^{-5}$] | 48 | 10 |
| $m_W$ [MeV] | 1.8 | 0.31 |
| B(W➔eν) [%] | | 0.015 |

# Higgs decays



- Uncertainties dominated by MHOU
- Also some parametric uncertainties from strong coupling, W/Z/t/b/c masses

See also: 1404.0319, 1906.05379, 2206.08326

$\alpha$ = ew loop, $\alpha_t$ = ew loop with top Yukawa, $\alpha_S$= QCD loop

| | current | | future (conservative) | | future (aggressive) | |
|---|---|---|---|---|---|---|
| | **th. err. estimate** | **available orders** | **th. err. estimate** | **additional orders** | **th. err. estimate** | **additional orders** |
| H ➔ bb / cc [%] | < 0.4 | $\alpha_S{}^4 + \alpha + \alpha_t{}^2 + \alpha_t\alpha_S$ | 0.2 | $\alpha^2 + \alpha\alpha_S$ | 0.1 | $\alpha_S{}^5$ |
| H ➔ ττ / μμ [%] | < 0.3 | $\alpha + \alpha_t{}^2 + \alpha_t\alpha_S$ | < 0.1 | $\alpha^2 + \alpha\alpha_S$ | | |
| H ➔ WW / ZZ [%] | 0.5 | $\alpha_S + \alpha + \alpha_t{}^2 + \alpha_t\alpha_S$ | 0.3 | $\alpha_S{}^2$ | | |
| H ➔ gg [%] | 2.3 | $\alpha_S{}^3 + \alpha$ | 1.0 | $\alpha_S{}^4$ | 0.5 | $\alpha_S{}^5 + \alpha\alpha_S$ |
| H ➔ γγ [%] | < 1.0 | $\alpha_S{}^2 + \alpha$ | | | | |
| H ➔ Zγ [%] | 1.5 | $\alpha_S + \alpha$ | | | | |

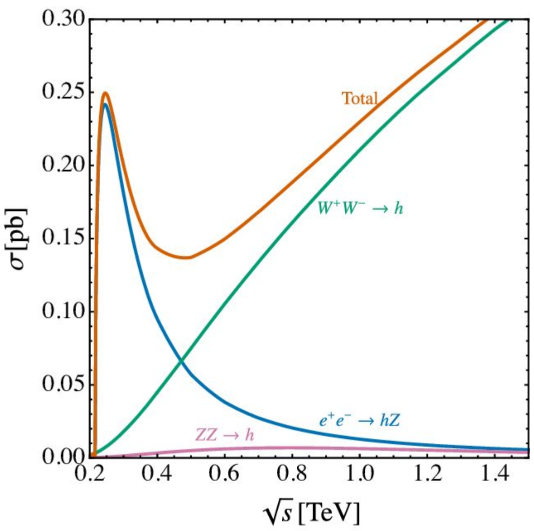
# Higgs production



- H+Z dominant @ 240/250 GeV (for current theory prediction see 2305.16547)
- WW fusion more important at higher energies (NLO known for hadron colliders, 0710.4749)
- At very high energies, corrections dominated by $\log^2(E/m_W)$ contributions (apply naive scaling)

$\alpha$ = ew loop, $\alpha_f$ = ew fermion loop, $\alpha_S$= QCD loop

| | collider energy | current | | future (conservative) | |
|---|---|---|---|---|---|
| | | **th. err. estimate** | **available orders** | **th. err. estimate** | **additional orders** |
| HZ [%] | 240 GeV | 0.3 | $\alpha + \alpha\alpha_S + \alpha\alpha_f$ | < 0.1 | $\alpha^2$ |
| WW fusion [%] | < 500 GeV | 1 | $\alpha$ | 0.1 | $\alpha^2 + \alpha\alpha_S$ |
| | 1 TeV | 2 | | 0.2 | |
| | 3 TeV | 4 | | 0.4 | |



# Z-pole (extraction of POs)



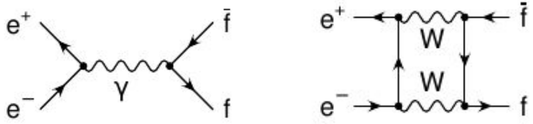
- Subtraction of $\gamma$-exchange, $\gamma$–$Z$ interference, box contributions:

$$\sigma_{\mathrm{hard}} = \sigma_{\mathrm{Z}} + \underbrace{\sigma_{\gamma} + \sigma_{\gamma\mathrm{Z}} + \sigma_{\mathrm{box}}}$$

- $Z$-pole contribution:

computed in SM

$$\sigma_{\mathrm{Z}} = \frac{R}{(s - \overline{M}_{\mathrm{Z}}^2)^2 + \overline{M}_{\mathrm{Z}}^2 \overline{\Gamma}_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$



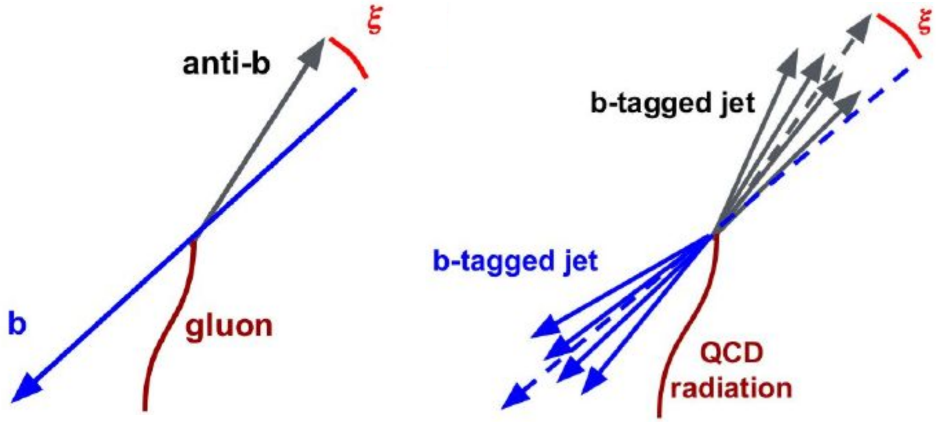

| | current (NLO) | future (NNLO) |
|---|---|---|
| $\sigma_{had}$ [pb] | 4 | 0.5 |
| $R_\ell$ [$10^{-3}$] | 2.1 | 0.27 |
| $R_c$ [$10^{-6}$] | 2 | 0.25 |
| $R_b$ [$10^{-6}$] | 1.4 | 0.17 |
| $\Gamma_Z$ [MeV] | 0.1 | 0.013 |
| $\sin^2\theta_{eff,lept}$ (from $A_{FB}^{\ell}$) | $20 \times 10^{-4}$ | $2.4 \times 10^{-4}$ |
| $\sin^2\theta_{eff,lept}$ (from $A_{FB}^{b}$) | $1.2 \times 10^{-4}$ | $0.15 \times 10^{-4}$ |

- Method: evaluate SM predictions with and without NLO corrections
- Estimate unknown higher orders by applying prefactors

# Z-pole (extraction of POs)



- Need Monte-Carlo (MC)  simulations to account for acceptances, cut, particle ID, etc.
- Uncertainties of QED MC estimated according to 1903.09895
- Future scenario includes full $O(\alpha^2)$, log-enhanced $O(\alpha^3)$, and fermion-pair production
- QCD MC uncertainty driven by gluon splitting, NP hadronization (see 2010.08604, 2209.08078)
- Conservative (aggressive) future scenario assumes fixed-order NNLO (N3LO) QCD corrections, and factor 5 (50) NP improvement




J. Alcaraz, 20 Jan2022, FCC-ee EW, EF04

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| | **current** | **future (conservative)** | **future (aggressive)** |
|---|---|---|---|
| $\sigma_{had}$ [pb] | 25 | 1.5 | |
| $R_\ell$ [$10^{-3}$] | 12 | 0.3 | |
| $R_c$ [$10^{-5}$] | 170 | 17 | 3.5 |
| $R_b$ [$10^{-5}$] | 40 | 4 | 0.8 |
| $\Gamma_Z$ [MeV] | 0.2 | 0.03 | |
| $\sin^2\theta_{eff,lept}$ (from $A_{FB}^{\ell}$) | $2.8 \times 10^{-4}$ | $0.05 \times 10^{-4}$ | |
| $A_b / A_c$ [$10^{-4}$] | 10 | 2 | 0.3 |

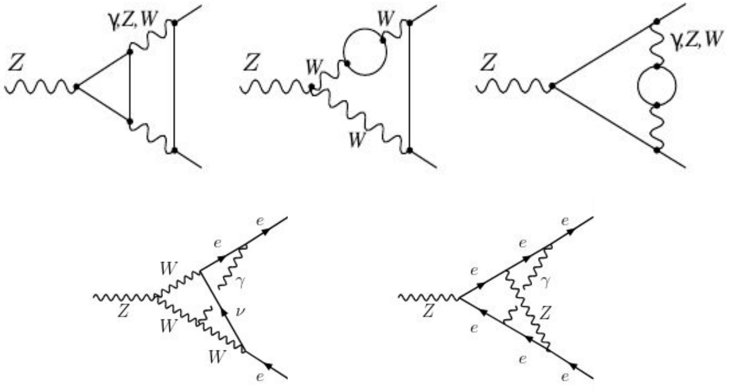
# Z–pole (SM predictions for POs)



For past work see: 1809.01830, 1906.05379

- Comparison of POs extracted from data with SM predictions used to test SM and probe BSM physics
- EW SM corrections are relatively large
  ➔ need multi-loop contributions

- *Current:* NNLO + $m_t$-enhanced N3LO*
- *"Conservative":* N3LO with $N_f \geq 2$ or $N(\alpha_S) \geq 1$ + $m_t$-enhanced N4LO
- *"Aggressive":* N4LO with $N_f \geq 2$ or $N(\alpha_S) \geq 2$ + partial $m_t$-enhanced N5LO




* also $O(\alpha_t \alpha_S^3)$

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- *“Aggressive”*: N4LO with $N_f \geq 2$ or $N(\alpha_S) \geq 2$ + partial $m_t$-enhanced N5LO

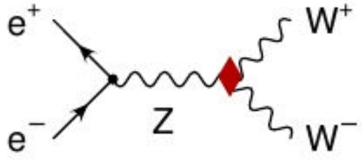
| | **current** | **future (conservative)** | **future (aggressive)** |
| --- | --- | --- | --- |
| $\sigma_{had}$ [pb] | 6 | 1.6 | 0.3 |
| $R_\ell$ [$10^{-3}$] | 6 | 1.2 | 0.2 |
| $R_c$ [$10^{-5}$] | 5 | 1 | 0.2 |
| $R_b$ [$10^{-5}$] | 10 | 2 | 0.35 |
| $\Gamma_Z$ [MeV] | 0.4 | 0.08 | 0.016 |
| $\sin^2\theta_{eff,lept}$ [$10^{-5}$] | 4.5 | 0.7 | 0.06 |
| $m_W$ [MeV] | 4 | 1 | 0.1 |

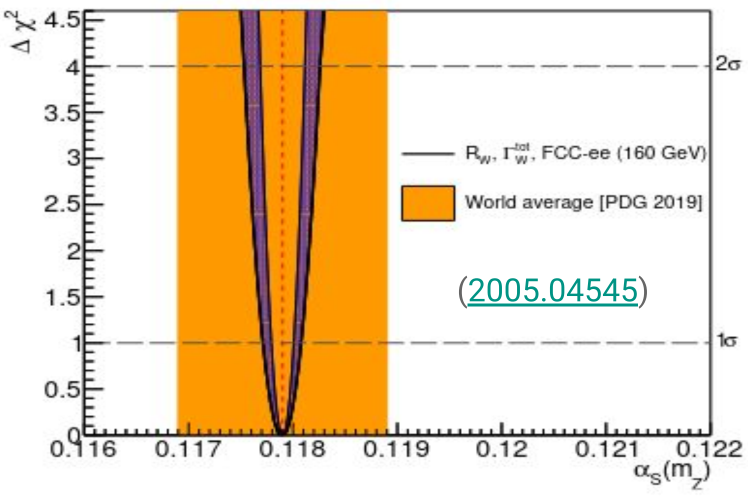
* also $O(\alpha_t \alpha_S^3)$

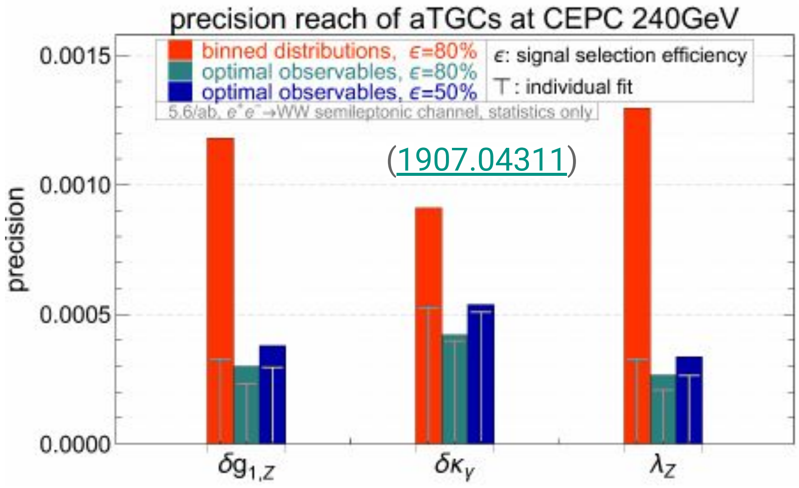
# W production and decay



- ee➔WW useful to constrain anomalous gauge-boson couplings (aGC) (current SM predictions: hep-ph/0502063)
- W decays useful, e.g., to constrain $\alpha_S$ (current SM predictions: 2005.04545)



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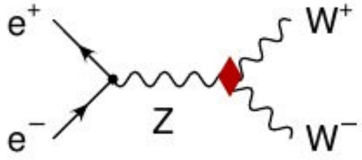
- W decays useful, e.g., to constrain $\alpha_S$ (current SM predictions: 2005.04545)
- Assume $\sqrt{s}$ = 240 GeV




$\alpha$ = ew loop, $\alpha_S$= QCD loop

| | current | | future (conservative) | |
|---|---|---|---|---|
| | **th. err. estimate** | **available orders** | **th. err. estimate** | **additional orders** |
| σ(ee➔WW) [%] | 0.4 | $\alpha$ | 0.07 | $\alpha^2 + \alpha\alpha_S$ |
| Γ(W➔lept.) [%] | 0.1 | $\alpha$ | 0.013 | $\alpha^2 + \alpha\alpha_S$ |
| Γ(W➔had.) [%] | 0.1 | $\alpha + \alpha\alpha_S + \alpha_S^4$ | 0.015 | $\alpha^2 + \alpha\alpha_S^2$ |
| had./lept. BR [%] | 0.015 | $\alpha + \alpha\alpha_S + \alpha_S^4$ | <0.01 | $\alpha^2 + \alpha\alpha_S^2$ |

# SM parameters



- SM predictions require precise inputs for $m_t$, $m_W$, $\alpha_S(m_Z)$, $\Delta\alpha = 1 - \alpha(0)/\alpha(m_Z)$ [also $m_{b,c}$]
- Can be extracted from e+e- data, but requiring additional theory input

| | current | future (conservative) | future (aggressive) | Comments |
|---|---|---|---|---|
| $m_t$ [MeV] | 50 | 30 | | from tt threshold; th. error based on 1711.10429, 1906.05379 |
| $m_W$ [MeV] | 3 | 0.7 | | from WW threshold; th. error for EW/QCD corrections from 1906.05379, for QED MC based on 1903.09895 |
| $\alpha_S(m_Z)$ [$10^{-3}$] | 1 | 0.3 | 0.1 | mostly based on lattice improvements (see QCD group) |
| $\Delta\alpha$ [$10^{-5}$] | <10 | <5 | 1–3 | “conservative” from e+e- data (based on doi:10.23731/CYRM-2020-003.9), “aggressive” from direct Z-pole measurement (1512.05544, 2501.05508) |

# Open issues for e+e− colliders



**No available theory uncertainty estimates for:**

- EWPOs from radiative return ee ➔ γZ
- Luminosity determination from ee ➔ ee, ee ➔ γγ (see e.g. 2506.15390)
- MC generator effects for W/Higgs/top production and decay
- Unique challenges in cross-over region between threshold and continuum for WW and ttbar production, e.g. $\sqrt{s}$ ~ 360–365 GeV for ee ➔ tt (see 1712.02220, 2209.14259)

# Theory for hadron colliders



**HL-LHC** (2504.00672)

- In general terms future theory uncertainties are assumed to be **half the present ones**.
  ➔ Requires significant theory improvements

- **W-mass:** 5 MeV (ATLAS, Snowmass) – **important caveat**: obtained assuming 1 $fb^{-1}$ of low pile-up runs, and as sources of errors only stat+HL-LHC improved PDFs.

  Developments on theory side likely required to push the theory uncertainties (well) below 5 MeV. More studies needed for a more robust error estimation.

  Possible improvement with LHeC PDFs down to 3 MeV*.

- **Top-quark mass:** currently most precise LHC measurements with $\delta_{exp}m_t$ ~ 0.3 GeV suffer from ambiguity in definition of top "pole" mass of $\delta_{MC}m_t$ ~ 0.5 GeV (1807.06617, 2309.00547)
  ➔ Improvement of MC generators and/or analysis techniques needed

* Not considered in fits

# Theory for hadron colliders



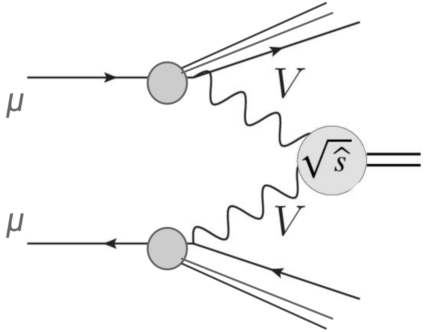
**FCC-hh:**

- Single-Higgs processes: specific production ratios among final state, reconstructed within similar fiducial regions for the parent Higgs, are chosen so that the theoretical uncertainties are reduced.
- Determination of Higgs couplings is obtained using FCC-ee absolute measurements.
- Higgs / gauge boson pair production: dedicated analyses are necessary.
- Z/W/H boson "radiation" will be ubiquitous (see e.g. 2203.11129 and refs. therein)
    - High-multiplicity matrix elements or electroweak parton showers?

# Theory for muon colliders



- Dedicated studies on theoretical uncertainties at the muon collider are in their early stages.
- The expectation is that theory uncertainties will not be a limiting factor for physics, with a few exceptions, e.g. $m_W$ and $m_t$ from threshold scans (EDMS doc)
- Higgs production from WW fusion at 10 TeV ~ 8% (current) - 0.8% (with NNLO corrections), based on scaling the estimated th. error @ 240 GeV by $\log^2(E/m_W)$
- High-energy, log-enhanced, EW corrections are a major source of theory uncertainty. Resummation of leading Sudakov double logarithms necessary.
- Different approaches to the problem:
  - EW PDFs: Currently known only at leading logarithm order. Extension to NLL possible but complex and requiring foundational work.
  - QED PDFs matched with fixed order NLO to produce partial NNLO EW corrections (arXiv:2506.10733) ~ %-level of accuracy possibly achievable. More studies needed.



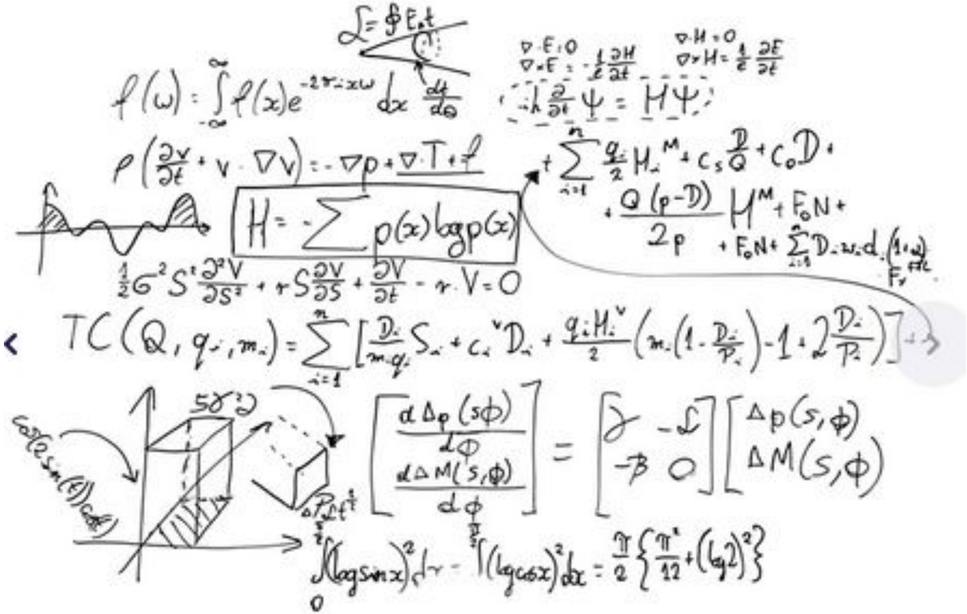
# Conclusions and Outlook



- Numbers shown are preliminary; updates will follow after Venice.
- Theory work is crucial: many areas require 1–2 perturbative orders improvement; key building blocks include multi-loop corrections, resummations, MC generators, and lattice inputs.
- For Higgs and WW continuum, extensions of existing methods likely suffice to make theory errors subdominant.
- For Z-pole, WW/tt thresholds, more fundamental advances in techniques and tools are needed.
- Theory vs. experimental uncertainties are not always cleanly separable.
- These are not predictions: projections are uncertain, and theory progress is hard to anticipate.
- Precise BSM calculations (for specific models or EFTs) may be needed for the accurate interpretation of potential discrepancies or limits.
- Finally, although significant effort will be required from the theory community, there is currently no hard limitation that prevents theory uncertainties from becoming small enough to fully exploit the precision of future colliders.

# Questions?

**What part of**



**did you not understand?**