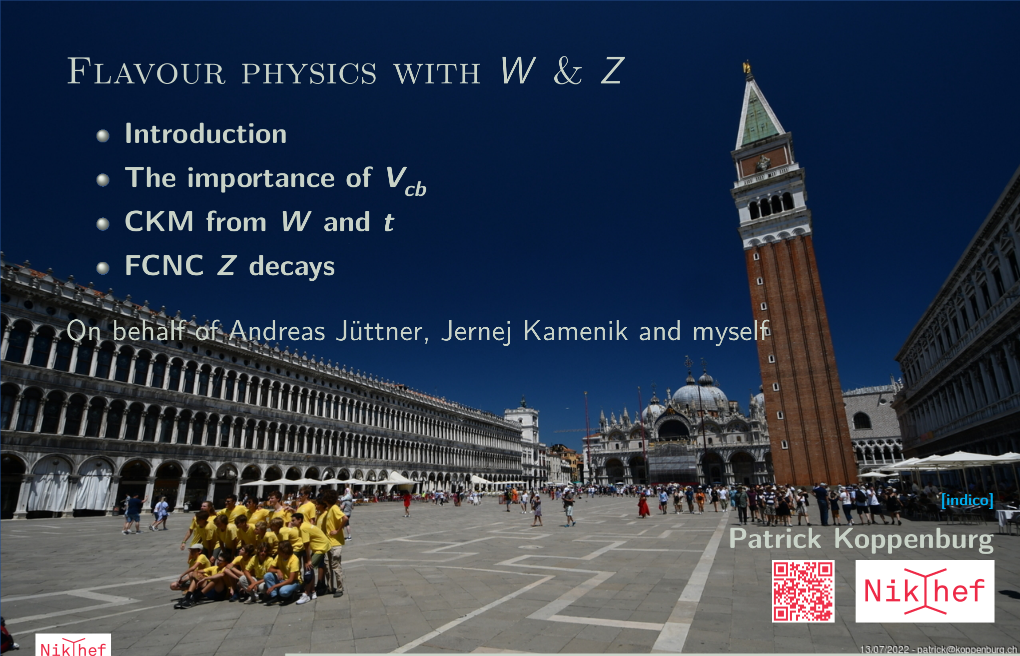
FLAVOUR PHYSICS WITH W & Z

- Introduction
- The importance of V_{cb}
- CKM from W and t
- FCNC Z decays

On behalf of Andreas Jüttner, Jernej Kamenik and myself



[indico]

Patrick Koppenburg

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Why?

22/06/2025 - patrick@koppenburg.ch

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

THE CKM MATRIX

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Wolfenstein parametrisation in $\lambda = \sin\theta_{\mathrm{C}} \approx 0.23$ at order $\mathcal{O}(\lambda^3)$:

$$\begin{pmatrix} 1 \quad -\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 \quad -\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3[1-(\rho - i\eta)] & -A\lambda^2 & 1 \end{pmatrix}$$

The Higgs and the *W* disagree on what a quark is!

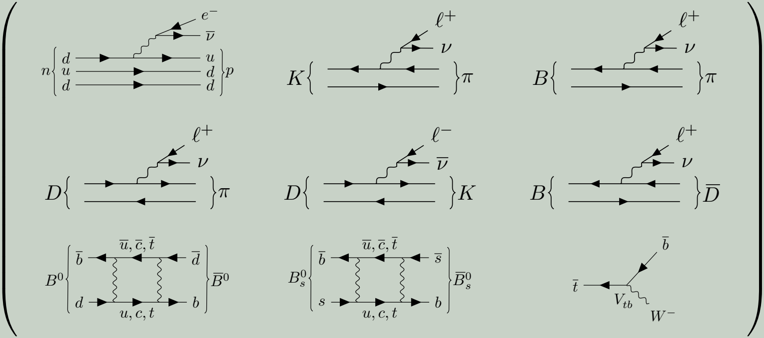
[Wolfenstein, PRL 51 (1983) 1945]

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

THE CKM MATRIX

...and how it is actually measured

$$V_{\mathrm{CKM}} =$$



The top-quark row is measured somewhat differently

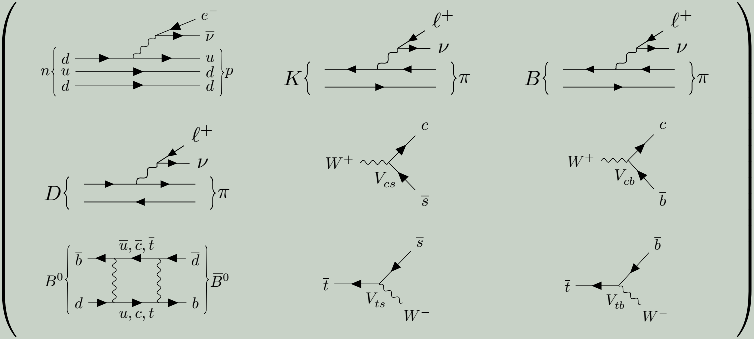
[B]

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

THE CKM MATRIX

. . . and how it could be measured

$$V_{\text{CKM}} =$$



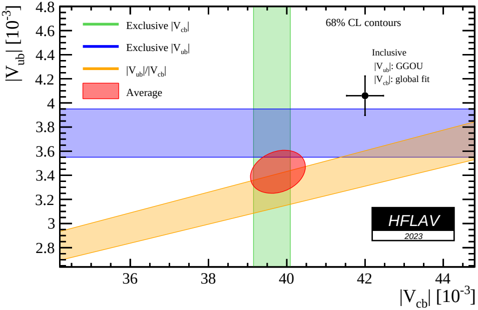
A $WW/t\overline{t}$ factory could measure the bottom right corner directly

[B]

[HFlav, arXiv:2411.18639]

$|V_{ub}|$ AND $|V_{cb}|$

HFLAV



Discrepancy between exclusive and inclusive V_{ub} (18%) and V_{cb} (5.5%) determinations. Both more than 3σ.

$$|V_{ub}|_{\text{excl}} = (\ \ 3.43 \pm 0.12) \times 10^{-3} \quad (3\%)$$

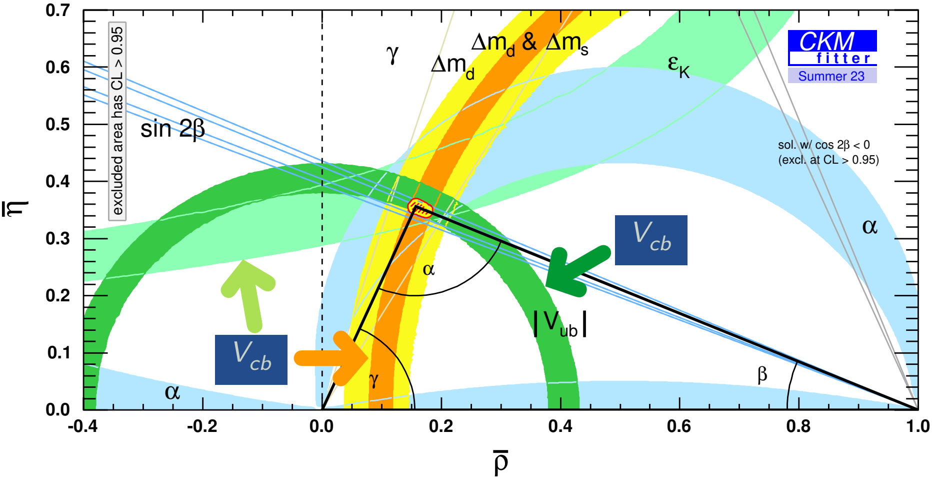
$$|V_{cb}|_{\text{excl}} = (39.77 \pm 0.46) \times 10^{-3} \quad (1\%)$$

$$|V_{ub}|_{\text{incl}} = (\ \ 4.06 \pm 0.16) \times 10^{-3} \quad (3\%)$$

$$|V_{cb}|_{\text{incl}} = (41.97 \pm 0.48) \times 10^{-3} \quad (1\%)$$

[CKMfitter 07/23]

UNITARITY TRIANGLE



[Brod, Gorbahn, Stamou, PRL 125 (2020) 171803, arXiv:1911.06822]

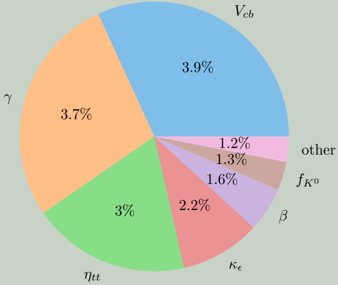
ϵ_K FROM CKM UNITARITY

With [Buras *et al.*, NPB 574 (2000) 291]

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times \Big(|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} \mathcal{S}(x_t) - \eta_{ut} \mathcal{S}(x_c, x_t) \Big)$$

one gets the numerical values

$$\begin{aligned} |\epsilon_K| = {} & (2.161 \pm 0.153_{\text{param.}} \pm 0.064_{\eta_{tt}} \\ & \pm 0.008_{\eta_{ut}} \pm 0.027_{\widehat{B}_K} \\ & \pm 0.052_{\xi_s} \pm 0.046_{\kappa_\epsilon}) \times 10^{-3}\,, \\ = {} & (2.161 \pm 0.153_{\text{param.}} \\ & \pm 0.076_{\text{non-pert.}} \\ & \pm 0.065_{\text{pert.}}) \times 10^{-3}\,, \\ = {} & 2.16(18) \times 10^{-3}\,. \end{aligned}$$



Error budget on ϵ_K [Buras, Stangl, EPJC 85 (2025) 519]

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Uncertainties [Brod, Gorbahn, Stamou, PRL 125 (2020) 171803] :

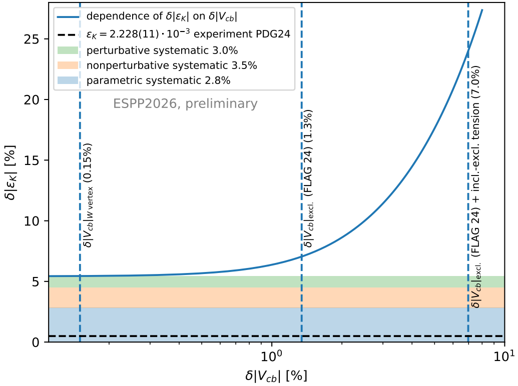
β: 0.3°

$\xi_s = \dfrac{F_{B_s^0}\sqrt{\widehat{B}_s}}{F_{B_s^0}\sqrt{\widehat{B}_d}}$: 1.4% (in $\bar{\eta}, \bar{\rho}$)

κ_ϵ : 2%

$\widehat{B}_K$: 4.2%

Based on [FLAG24 arXiv:2411.04268] and [Brod et al., PRD 125, 171803 (2020)]



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Uncertainty assumptions for beyond 2040:

- β: 0.3° (now) ➜ 0.08°
- $\xi_s = \dfrac{F_{B_s^0}\sqrt{\widehat{B}_s}}{F_{B_s^0}\sqrt{\widehat{B}_d}}$: 1.4% ➜ 0.5%
- κ_ϵ : 0.7% (present Lattice, but Brod uses 2%)
- $\widehat{B}_K$: 4.2% ➜ 1.3%

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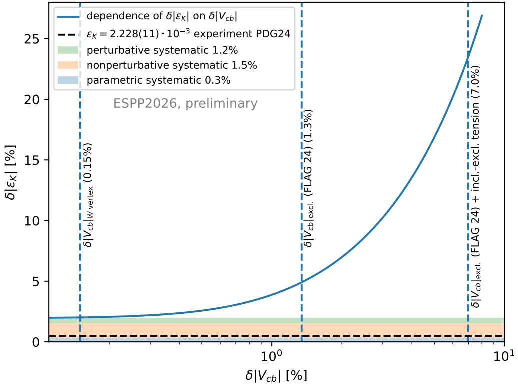

[B]

CKM from W

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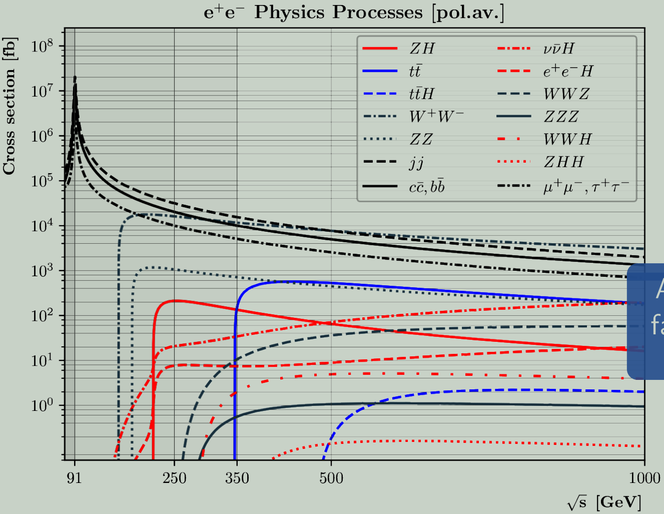
[J.R. Reuter]

e^+e^- CROSS-SECTIONS



Any Higgs factory can do WW

[Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390]

CKM FROM WW

Abstract [#141]

Assuming $\mathcal{O}(10^8)$ WW pairs, which comes as a by-product of large $ZH(H)$ samples, or from dedicated WW runs, one measures

$$\mathcal{B}_{ij} = \frac{|V_{ij}|^2}{\sum_{l=u,c;\ m=d,s,b} |V_{lm}|^2} \mathcal{B}_{\text{had}}$$

V_{ub} and V_{cb} are feasible. The others have too high backgrounds from mis-ID.

Present precision on $|V_{cs}|$: 0.6%; on $|V_{cb}|$: 1% with 5% discrepancy.

Number of correctly tagged jets W (before other efficiencies) based on

[Lian *et al.*, arXiv:2310.03440]

V_{ij}	BF	yield
V_{ud}	3.18×10^{-1}	3.2×10^{6}
V_{us}	1.70×10^{-2}	3.4×10^{5}
V_{ub}	4.50×10^{-6}	1.2×10^{2}
V_{cd}	1.70×10^{-2}	3.5×10^{5}
V_{cs}	3.17×10^{-1}	1.3×10^{7}
V_{cb}	5.90×10^{-4}	3.3×10^{4}

[Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390]

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The precision is systematically limited by the knowledge of the jet flavour-tagging efficiency, which is calibrated from Z events.

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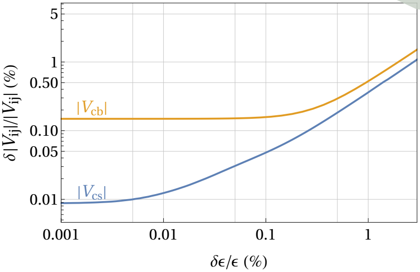
[B]

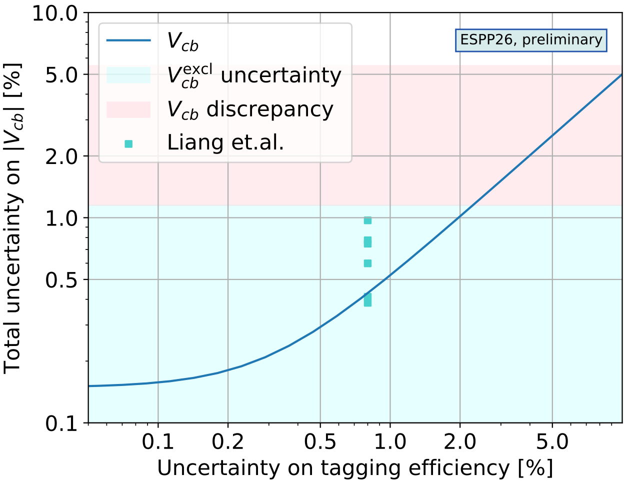


Plot from [Marzocca, Szewc, Tammaro, JHEP 11 (2024) 17], fast simulation

Typical jet-tagging efficiencies at e^+e^- colliders

	b	s	c	u	d	g
ϵ^b_β	0.8	0.0001	0.003	0.0005	0.0005	0.007
ϵ^c_β	0.02	0.008	0.8	0.01	0.01	0.01
ϵ^s_β	0.01	0.9	0.1	0.3	0.3	0.2

$W \rightarrow q\overline{q}$ AND $Z \rightarrow q\overline{q}$



V_{cb} precision compared to present uncertainty and discrepancy between exclusive and inclusive.

Study with full simulation gets $[\pm 0.36 \pm 0.20]\%$ for $20\,\text{ab}^{-1}$. [Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071]

My version of Fig. 2 in [Marzocca, Szewc, Tammaro, JHEP 11 (2024)

[B]

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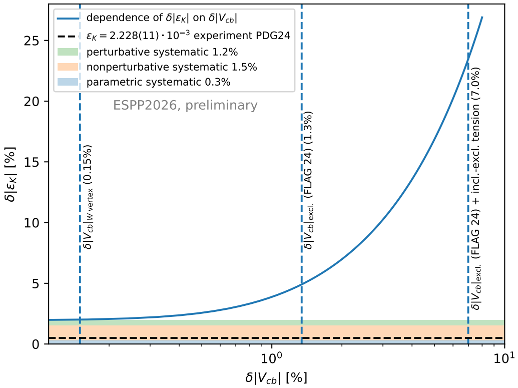

Top

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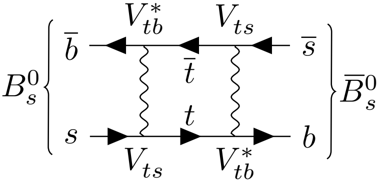
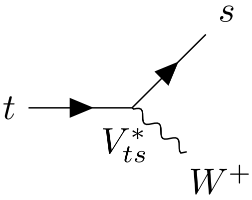
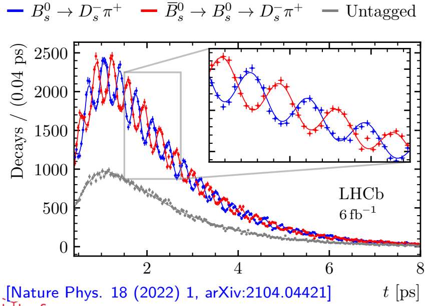
[Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390]

V_{ts} FROM TOP QUARK DECAYS

Abstract [#141]







[Nature Phys. 18 (2022) 1, arXiv:2104.04421]

Usually V_{ts} from B_s^0 mixing ➜ 2% precision, but assume no new physics

Study was done assuming 2M $t\bar{t}$ pairs. Observation of $t \to sW$ is possible with 15% precision on BF

➜ Precision on V_{ts}: 7.5%

FCNC Z decays

[Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390]

FCNC $Z \to q_i\overline{q}_j$

Abstract [#141]

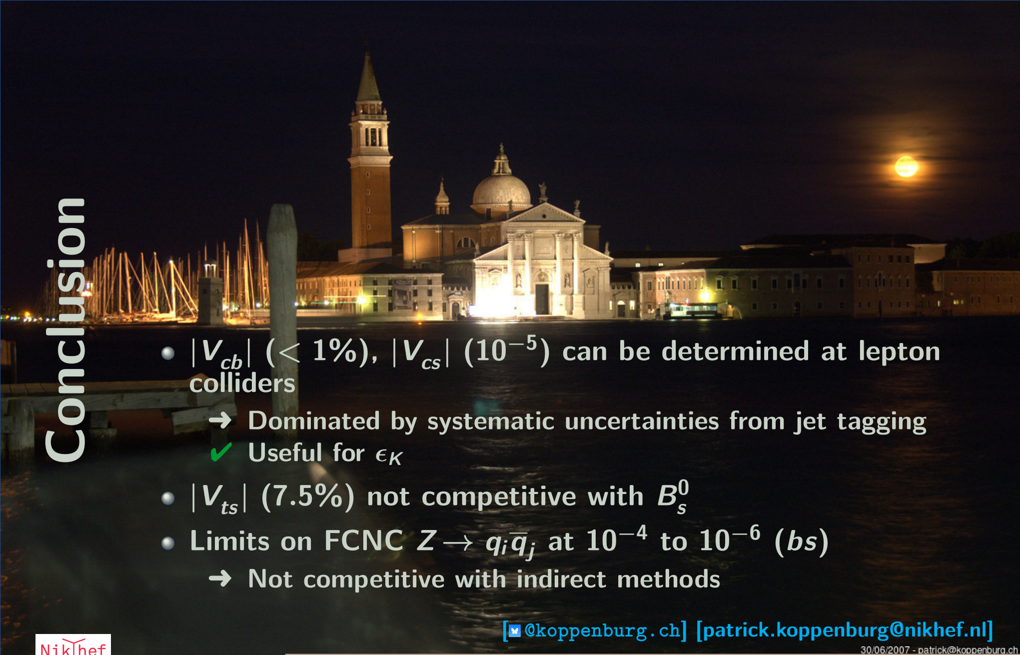
FCNC forbidden at tree level, but occur in loops as we know from penguin b decays. SM predictions are [Kamenik *et al.*, PRD 109 (2024) L011301]

$$\mathcal{B}(Z \to b\overline{s}) + \text{c.c.} = (4.2 \pm 0.7) \times 10^{-8}$$
$$\mathcal{B}(Z \to b\overline{d}) + \text{c.c.} = (1.8 \pm 0.3) \times 10^{-9}$$
$$\mathcal{B}(Z \to c\overline{u}) + \text{c.c.} = (1.4 \pm 0.2) \times 10^{-18}$$

Selections follow those of FCNC Higgs decays and backgrounds are dominated by single mistags. Assuming 0.1% systematic uncertainties, 6 tera Z can get limits of

$$\mathcal{B}(Z \to b\overline{s}) + \text{c.c.} < 7.3 \times 10^{-6}$$
$$\mathcal{B}(Z \to b\overline{d}) + \text{c.c.} < 2.4 \times 10^{-4}$$
$$\mathcal{B}(Z \to c\overline{u}) + \text{c.c.} < 4.1 \times 10^{-4}$$

"Indirect probes set BSM bounds that are much stronger." [Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390] [#141].


30/06/2007 - patrick@koppenburg.ch

Conclusion

- $|V_{cb}|$ ($< 1\%$), $|V_{cs}|$ (10^{-5}) can be determined at lepton colliders
  - → Dominated by systematic uncertainties from jet tagging
  - ✓ Useful for ϵ_K
- $|V_{ts}|$ (7.5%) not competitive with B^0_s
- Limits on FCNC $Z \to q_i \overline{q}_j$ at 10^{-4} to 10^{-6} (bs)
  - → Not competitive with indirect methods

[🦋 @koppenburg.ch] [patrick.koppenburg@nikhef.nl]

Backup

W STATISTICS

Project	W pairs	single W	Total	Comment
FCC	2.4×10^8		4.8×10^8	[FCC vol.1]
CEPC	2.1×10^8		2.2×10^8	[CEPC]
LCF 250	1.1×10^7	3.0×10^6	2.5×10^7	J.List
LCF 550	7.2×10^7	4.1×10^7	1.8×10^8	J.List
CLIC				
LHeC		8×10^4	8×10^4	[LHeC]
MuCol			10^8	[#207]

Add a comment on which matrix elements can be done

[B]

Z STATISTICS

Project	Z pole	additional Z	Total	Comment
FCC	6×10^{12}	1.3×10^{7}	6×10^{12}	[FCC vol.1] [1]
CEPC	4.1×10^{12}	2.2×10^{7}	4.1×10^{12}	[CEPC] [2]
LCF 250	6×10^{9}	?	6×10^{9}	
CLIC				
LHeC			$n \times 10^{5}$	[LHeC]
MuCol				

[1]: Extrapolating $2.2 \times 10^6 + 3.7 \times 10^5$ ZH events to $1.1 \times 10^7 + 1.8 \times 10^6$ ZZ

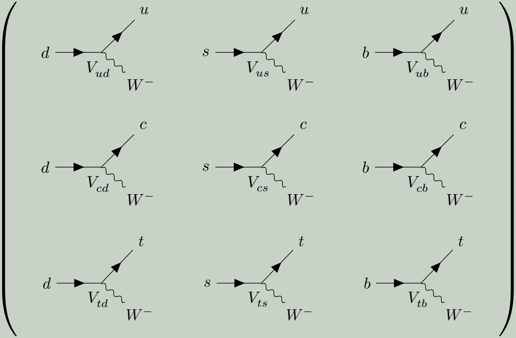
[2]: Extrapolating ratio of integrated luminosities wrt FCC

[B]

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

THE CKM MATRIX

$V_{\mathrm{CKM}} =$



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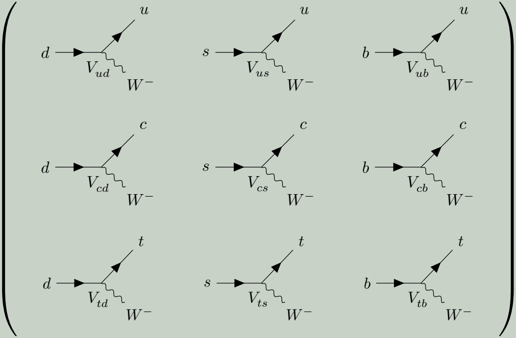

$$= \simeq \begin{pmatrix} 1 & 0.23 & 10^{-4} \\ -0.23 & 1 & 0.04 \\ 10^{-3} & -0.04 & 1 \end{pmatrix}$$

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

THE CKM MATRIX

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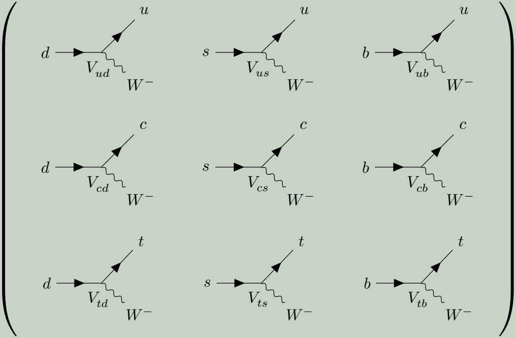


This matrix is

UNITARY: as much gets in as gets out

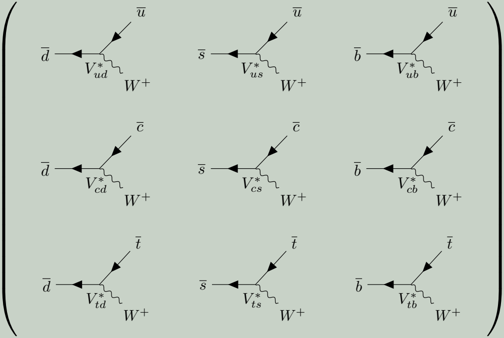
COMPLEX: it's quantum mechanics

➜ A SINGLE PHASE cannot be rotated away. It is the source of all known experimental *CP* violation effects. (PMNS also has it.)

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

The CKM matrix

$\overline{V}_{\text{CKM}} =$



This matrix is

Unitary: as much gets in as gets out

Complex: it's quantum mechanics

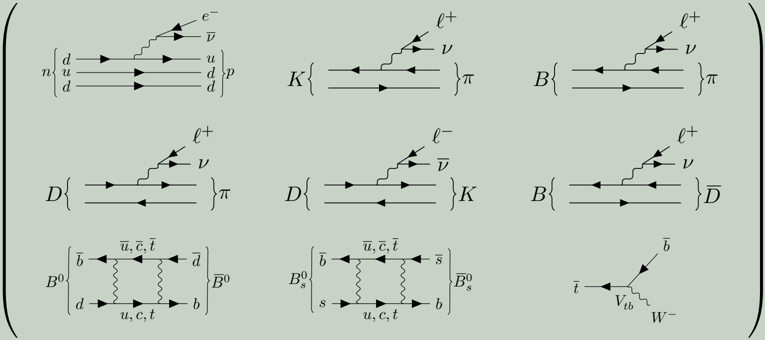
➜ A single phase cannot be rotated away. It is the source of all known experimental *CP* violation effects. (PMNS also has it.)

[Kobayashi, Maskawa, Progr. Theo. Phys. 49 (1973) 652]

The CKM matrix

…and how it is actually measured

$V_{\text{CKM}} =$



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THE CKM MATRIX

…and how it could be measured

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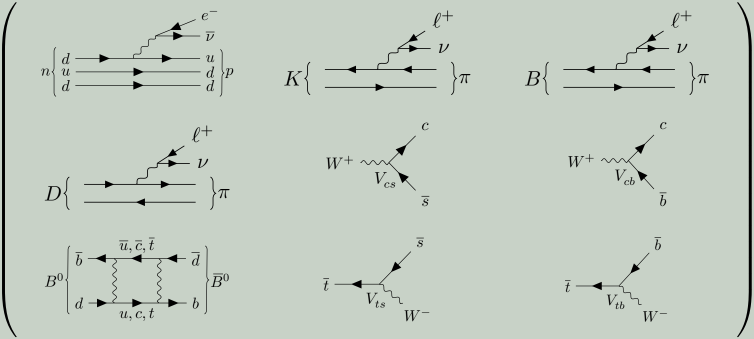
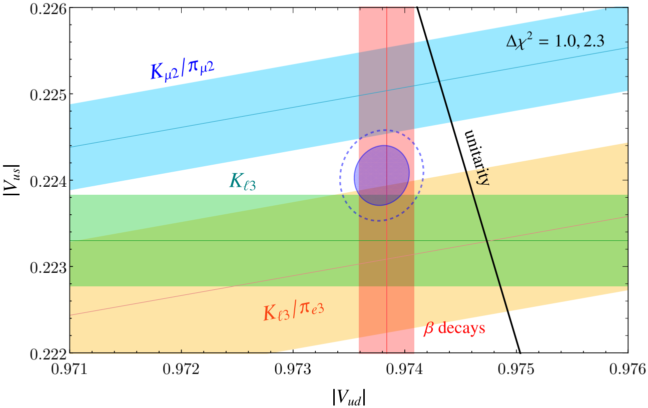


A $WW/t\overline{t}$ factory could measure the bottom right corner directly

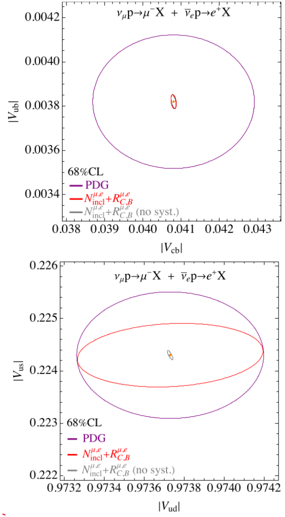
CABIBBO ANOMALY AND EW FITS



Some tension in kaon physics. May indicate some new right-handed current.

[ESPPU input, arXiv:2504.21417]

CKM MATRIX ELEMENTS



A 10 TeV muon collider would produce 10^8 W bosons, similar to FCC or LC, giving access to $|V_{cb}|$ and $|V_{cs}|$ [Marzocca, Szewc, Tammaro, JHEP 11 (2024) 017], however without a Z run for calibration.

The $\overline{\nu}_e$ and ν_μ from the straight sections can interact with a 1 ton target as $\nu_\mu p \to \mu^- X$ and alike. The number of DIS events $N^{\mu,e}_{c,b}$ and the ratios

$$R^{\mu,e}_{c,b} = \frac{N^{\mu,e}_{c,b}}{N^{\mu,e}_{\text{incl.}}}$$

get $|V_{cb}|$ and $|V_{ub}|$ to 0.1% and 0.5% precision. Similarly, for the Cabibbo anomaly, $|V_{ud}|$ and $|V_{us}|$ to 3×10^{-4} and 0.2% precision.

CKM FROM WW

W^-	$\overline{u}d$	$\overline{u}s$	$\overline{u}b$	$\overline{c}d$	$\overline{c}s$	$\overline{c}b$
BR	31.8%	1.7%	4.5×10^{-6}	1.7%	31.7%	5.9×10^{-4}
N_{ev}	32×10^6	1.7×10^6	450	1.7×10^6	32×10^6	59×10^3
$\delta^{stat}_{V_{ij}}$	0.018%	0.077%	4.7%	0.077%	0.018%	0.41%

Precision on CKM matrix elements assuming 10^8 W bosons

LHEC W PHYSICS

Process	$E_e = 50\,\text{GeV},\ E_p = 7\,\text{TeV}$ $p_T^e > 10\,\text{GeV}$	$E_e = 60\,\text{GeV},\ E_p = 7\,\text{TeV}$ $p_T^e > 10\,\text{GeV}$	$E_e = 60\,\text{GeV},\ E_p = 7\,\text{TeV}$ $p_T^e > 5\,\text{GeV}$
e^-W^+j	1.00 pb	1.18 pb	1.60 pb
e^-W^-j	0.930 pb	1.11 pb	1.41 pb
$\nu_e^-W^-j$	0.796 pb	0.956 pb	0.956 pb
ν_e^-Zj	0.412 pb	0.502 pb	0.502 pb
e^-Zj	0.177 pb	0.204 pb	0.242 pb

SM cross-sections

With at most 4 pb, expect 80k W in 20 fb^{-1}.

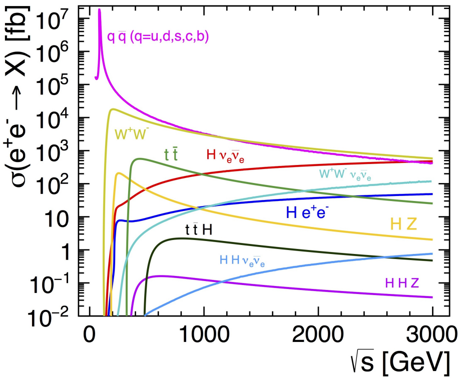
Likely not competitive with $e^+e^- \to WW$ for V_{cb} and alike

[Robson, Leonidopoulos, de Blas, Koppenburg, List, Maltoni *et al.*, arXiv:2506.15390]

WW PHYSICS

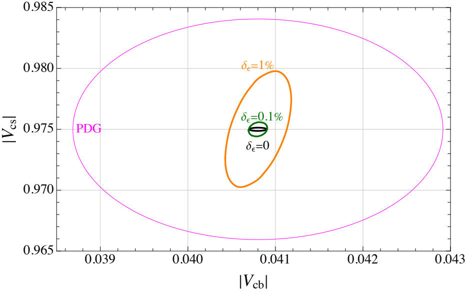
With WW pairs one can measure

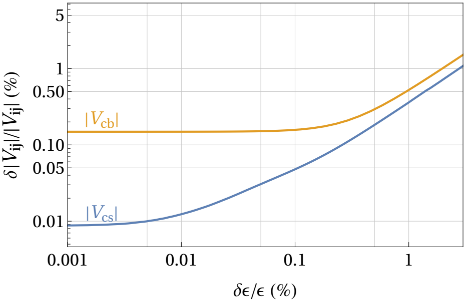
- THE W mass ($0.5\,\mathrm{MeV}/c^2$) with a threshold scan
- $W\to\ell\nu$ BFs to 10^{-4}
- CKM MATRIX ELEMENTS notably V_{cb}
- WW DIFFERENTIAL MEASUREMENT useful for SMEFT
- FRAGMENTATION FUNCTIONS relevant for H and top physics



[Marzocca, Szewc, Tammaro, JHEP 11 (2024) 017, arXiv:2405.08880]

CKM MATRIX ELEMENTS AT A WW MACHINE





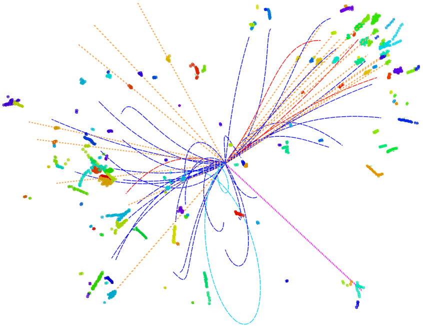
The precision on V_{cb} and V_{cs} depends on how well the jet tagging efficiency is known.

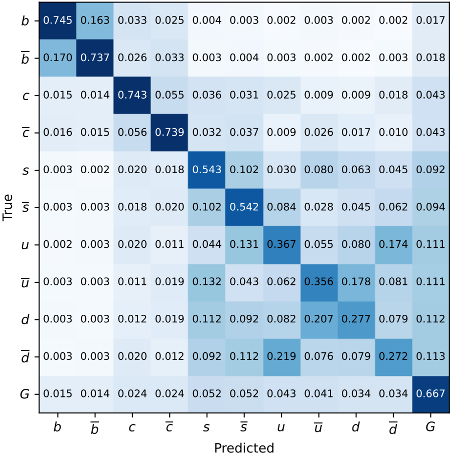
These numbers are quoted in FCC's [#196]

[Lian *et al.*, PRL 132 (2024) 221802, arXiv:2310.03440]

JET ORIGIN AT AN e^+e^- COLLIDER

Categorisation of jets in 10 (anti)quarks and gluon with all state-of-art techniques. Applied to $\nu,\overline{\nu}\ H$, $H \rightarrow jj$ events



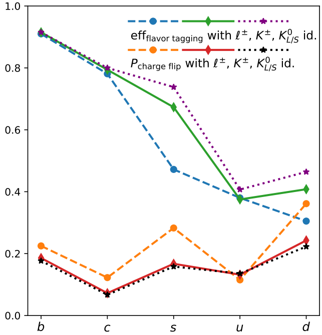


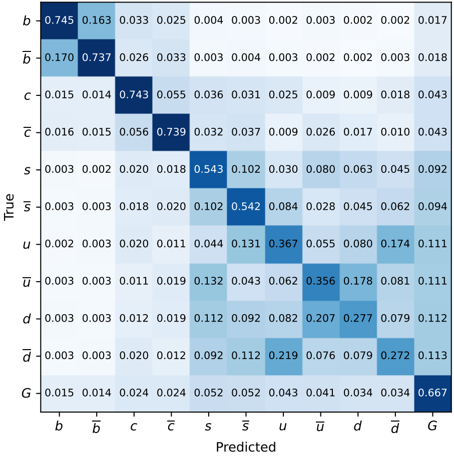
	b	$\overline{b}$	c	$\overline{c}$	s	$\overline{s}$	u	$\overline{u}$	d	$\overline{d}$	G
b	0.745	0.163	0.033	0.025	0.004	0.003	0.002	0.003	0.002	0.002	0.017
$\overline{b}$	0.170	0.737	0.026	0.033	0.003	0.004	0.003	0.002	0.002	0.003	0.018
c	0.015	0.014	0.743	0.055	0.036	0.031	0.025	0.009	0.009	0.018	0.043
$\overline{c}$	0.016	0.015	0.056	0.739	0.032	0.037	0.009	0.026	0.017	0.010	0.043
s	0.003	0.002	0.020	0.018	0.543	0.102	0.030	0.080	0.063	0.045	0.092
$\overline{s}$	0.003	0.003	0.018	0.020	0.102	0.542	0.084	0.028	0.045	0.062	0.094
u	0.002	0.003	0.020	0.011	0.044	0.131	0.367	0.055	0.080	0.174	0.111
$\overline{u}$	0.003	0.003	0.011	0.019	0.132	0.043	0.062	0.356	0.178	0.081	0.111
d	0.003	0.003	0.012	0.019	0.112	0.092	0.082	0.207	0.277	0.079	0.112
$\overline{d}$	0.003	0.003	0.020	0.012	0.092	0.112	0.219	0.076	0.079	0.272	0.113
G	0.015	0.014	0.024	0.024	0.052	0.052	0.043	0.041	0.034	0.034	0.667

[Lian *et al.*, PRL 132 (2024) 221802, arXiv:2310.03440]

JET ORIGIN AT AN e^+e^- COLLIDER

Categorisation of jets in 10 (anti)quarks and gluon with all state-of-art techniques. Applied to $\nu,\overline{\nu}\ H$, $H\to jj$ events





	b	$\overline{b}$	c	$\overline{c}$	s	$\overline{s}$	u	$\overline{u}$	d	$\overline{d}$	G
b	0.745	0.163	0.033	0.025	0.004	0.003	0.002	0.003	0.002	0.002	0.017
$\overline{b}$	0.170	0.737	0.026	0.033	0.003	0.004	0.003	0.002	0.002	0.003	0.018
c	0.015	0.014	0.743	0.055	0.036	0.031	0.025	0.009	0.009	0.018	0.043
$\overline{c}$	0.016	0.015	0.056	0.739	0.032	0.037	0.009	0.026	0.017	0.010	0.043
s	0.003	0.002	0.020	0.018	0.543	0.102	0.030	0.080	0.063	0.045	0.092
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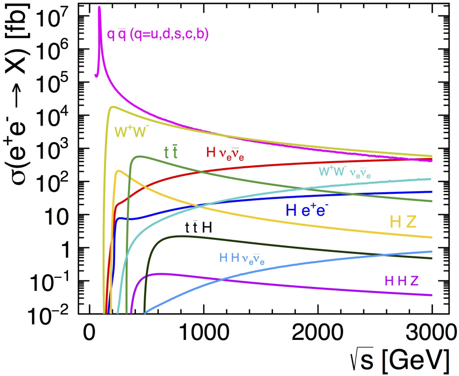
$\overline{d}$	0.003	0.003	0.020	0.012	0.092	0.112	0.219	0.076	0.079	0.272	0.113
G	0.015	0.014	0.024	0.024	0.052	0.052	0.043	0.041	0.034	0.034	0.667

[Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071, arXiv:2406.01675]

V_{cb} IN WW EVENTS (FULL SIMULATION)

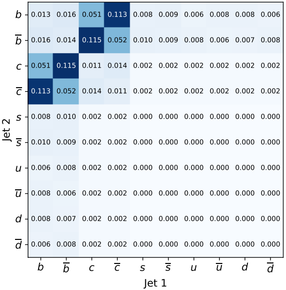
Most comprehensive study of V_{cb} from W, but predates submissions

- Running at the threshold is not ideal. Peak cross-section is around 200 GeV.
  - ➜ Assuming 10^8 W bosons at threshold ("WW")
- or FCC scenario: $20\,\mathrm{ab}^{-1}$
- Also consider polarised beams at 250 GeV
  - ➜ Cross-section increases by a factor $(1+0.8)(1+0.3)=2.34$ with (0.8,0.3) polarisation



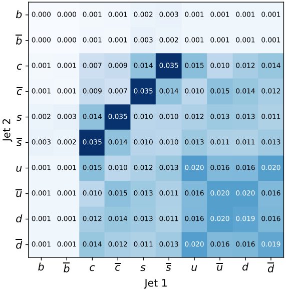
[Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071, arXiv:2406.01675]

V_{cb} IN WW EVENTS (FULL SIMULATION)



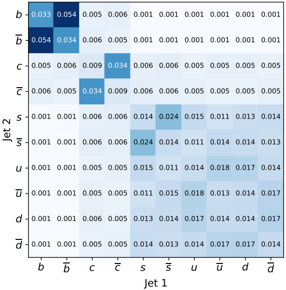
	b	$\bar{b}$	c	$\bar{c}$	s	$\bar{s}$	u	$\bar{u}$	d	$\bar{d}$
b	0.013	0.016	0.051	0.113	0.008	0.009	0.006	0.008	0.008	0.006
$\bar{b}$	0.016	0.014	0.115	0.052	0.010	0.009	0.008	0.006	0.007	0.008
c	0.051	0.115	0.011	0.014	0.002	0.002	0.002	0.002	0.002	0.002
$\bar{c}$	0.113	0.052	0.014	0.011	0.002	0.002	0.002	0.002	0.002	0.002
s	0.008	0.010	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{s}$	0.010	0.009	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
u	0.006	0.008	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{u}$	0.008	0.006	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
d	0.008	0.007	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{d}$	0.006	0.008	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000

$WW \to \mu\nu c\bar{b}$



	b	$\bar{b}$	c	$\bar{c}$	s	$\bar{s}$	u	$\bar{u}$	d	$\bar{d}$
b	0.000	0.000	0.001	0.001	0.002	0.003	0.001	0.001	0.001	0.001
$\bar{b}$	0.000	0.000	0.001	0.001	0.003	0.002	0.001	0.001	0.001	0.001
c	0.001	0.001	0.007	0.009	0.014	0.035	0.015	0.010	0.012	0.014
$\bar{c}$	0.001	0.001	0.009	0.007	0.035	0.014	0.010	0.015	0.014	0.012
s	0.002	0.003	0.014	0.035	0.010	0.010	0.012	0.013	0.013	0.011
$\bar{s}$	0.003	0.002	0.035	0.014	0.010	0.010	0.013	0.011	0.011	0.013
u	0.001	0.001	0.015	0.010	0.012	0.013	0.020	0.016	0.016	0.020
$\bar{u}$	0.001	0.001	0.010	0.015	0.013	0.011	0.016	0.020	0.020	0.016
d	0.001	0.001	0.012	0.014	0.013	0.011	0.016	0.020	0.019	0.016
$\bar{d}$	0.001	0.001	0.014	0.012	0.011	0.013	0.020	0.016	0.016	0.019

Other WW



	b	$\bar{b}$	c	$\bar{c}$	s	$\bar{s}$	u	$\bar{u}$	d	$\bar{d}$
b	0.033	0.054	0.005	0.006	0.001	0.001	0.001	0.001	0.001	0.001
$\bar{b}$	0.054	0.034	0.006	0.005	0.001	0.001	0.001	0.001	0.001	0.001
c	0.005	0.006	0.009	0.034	0.006	0.006	0.005	0.005	0.005	0.005
$\bar{c}$	0.006	0.005	0.034	0.009	0.006	0.006	0.005	0.005	0.005	0.005
s	0.001	0.001	0.006	0.006	0.014	0.024	0.015	0.011	0.013	0.014
$\bar{s}$	0.001	0.001	0.006	0.006	0.024	0.014	0.011	0.014	0.014	0.013
u	0.001	0.001	0.005	0.005	0.015	0.011	0.014	0.018	0.017	0.014
$\bar{u}$	0.001	0.001	0.005	0.005	0.011	0.015	0.018	0.013	0.014	0.017
d	0.001	0.001	0.006	0.005	0.013	0.014	0.017	0.014	0.014	0.017
$\bar{d}$	0.001	0.001	0.005	0.005	0.014	0.013	0.014	0.017	0.017	0.014

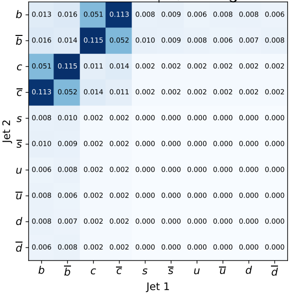
Non-W backgrounds

[Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071, arXiv:2406.01675]

V_{cb} IN WW EVENTS (FULL SIMULATION)

Systematic \ Mode $W^+W^- \to$	$\mu\nu cb$	$e\nu cb$	Combined	Syst.1	Syst.2	Comment
Unpolarized, Baseline (5 ab^{-1})	0.91%	1.2 %	0.72%	1.5%	0.20%	(2×CEPC)
Unpolarized, Extended (20 ab^{-1})	0.45%	0.60%	0.36%	1.5%	0.20%	(FCC)
WW Threshold (5×10^7 WW)	1.2 %	1.6 %	0.95%	1.5%	0.20%	
Unpolarized, Baseline + WW	0.72%	0.96%	0.58%	1.1%	0.15%	
Unpolarized, Extended + WW	0.42%	0.56%	0.34%	1.1%	0.18%	
Polarized, Baseline (0.5 ab^{-1})	1.9 %	2.5 %	1.5 %	1.5%	0.20%	(ILC stage 1)
Polarized, Extended (2 ab^{-1})	0.94%	1.3 %	0.75%	1.5%	0.20%	(ILC, $\frac{2}{3}$×LCF)

The extrapolated deviation of b and c tagging efficiency from MC simulation is about 0.8%. The systematic uncertainties depend on whether the FT uncertainty comes from comparing different MC (2%, syst.1), or if one can improve it to 0.25% (syst.2)



	b	$\bar{b}$	c	$\bar{c}$	s	$\bar{s}$	u	$\bar{u}$	d	$\bar{d}$
b	0.013	0.016	0.051	0.113	0.008	0.009	0.006	0.008	0.008	0.006
$\bar{b}$	0.016	0.014	0.115	0.052	0.010	0.009	0.008	0.006	0.007	0.008
c	0.051	0.115	0.011	0.014	0.002	0.002	0.002	0.002	0.002	0.002
$\bar{c}$	0.113	0.052	0.014	0.011	0.002	0.002	0.002	0.002	0.002	0.002
s	0.008	0.010	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{s}$	0.010	0.009	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
u	0.006	0.008	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{u}$	0.008	0.006	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
d	0.008	0.007	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000
$\bar{d}$	0.006	0.008	0.002	0.002	0.000	0.000	0.000	0.000	0.000	0.000

[Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071, arXiv:2406.01675]

V_{cb} IN WW EVENTS (FULL SIMULATION)

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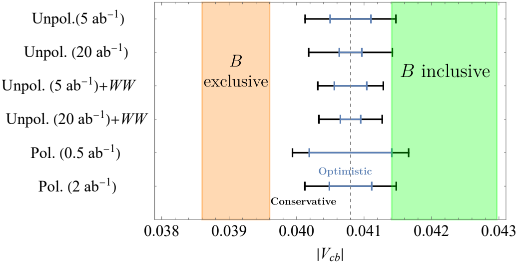
Polarized, Baseline (0.5 ab^{-1})	1.9 %	2.5 %	1.5 %	1.5%	0.20%	(ILC stage 1)
Polarized, Extended (2 ab^{-1})	0.94%	1.3 %	0.75%	1.5%	0.20%	(ILC, $\frac{2}{3}$×LCF)

The extrapolated deviation of b and c tagging efficiency from MC simulation is about 0.8%. The systematic uncertainties depend on whether the FT uncertainty comes from comparing different MC (2%, syst.1), or if one can improve it to 0.25% (syst.2)



[Liang, Li, Zhu, Shen, Ruan, JHEP 12 (2024) 071, arXiv:2406.01675]

V_{cb} IN WW EVENTS (FULL SIMULATION)

Topology	events
$\mu\nu cb$	40.3 k
$\mu\nu cd/s$	24.2 M
$\mu\nu qq_{\text{other}}$	24.2 M
$\mu_\tau\nu cb$	7.7 k
$\mu_\tau\nu cd/s$	4.2 M
$\mu_\tau\nu qq_{\text{other}}$	4.2 M
WW_{other}	194 M
$4f_{\text{other}}$	133 M
Higgs	4.0 M
$2f$	1.8 G

Stats for 20 ab^{-1}