Future prospects from Lattice QCD

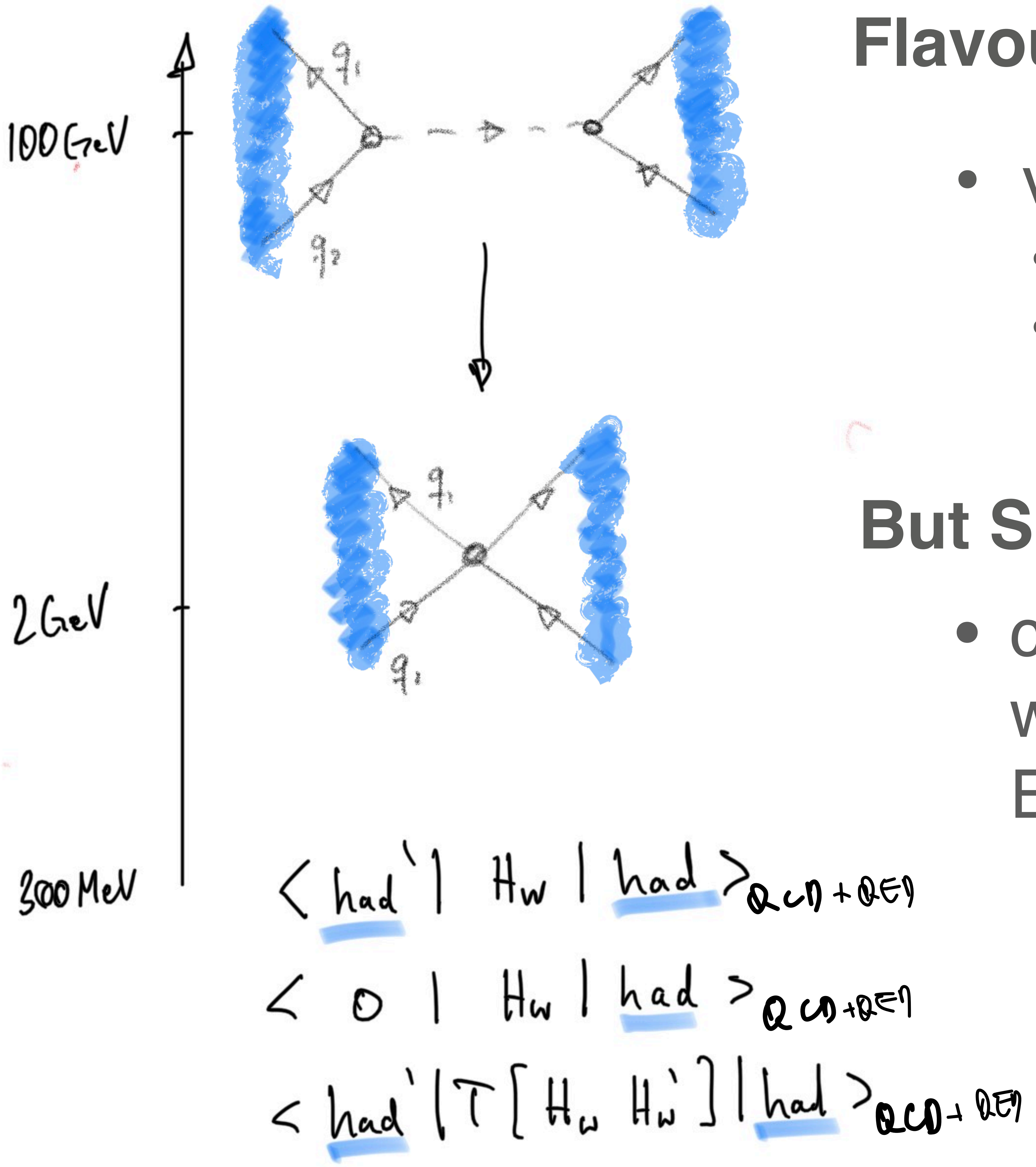
[for flavour]

23.06.2025

Andreas Jüttner

CERN
University of Southampton

The role of lattice QCD in flavour physics



Flavour involves EW, EM and QCD physics

- vast range of energy scales
  - EW gauge bosons → $O(100)\mathrm{GeV}$
  - photons, quarks, gluons [mesons, baryons] → few to hundreds of MeV where **QCD nonperturbative**

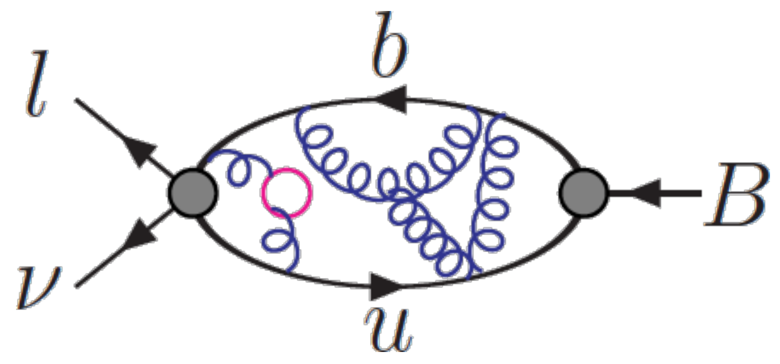
But SM helps us a bit:

- current and future precision requirements allow to replace weak gauge bosons by point-interaction described by an Effective Hamiltonian H_W

Require model-independent tool to predict nonperturbative QCD(+QED) contributions in the Standard Model

An example — leptonic decay

e.g tree level leptonic *B* decay:



$(\langle 0 | H_w | \text{had} \rangle)$

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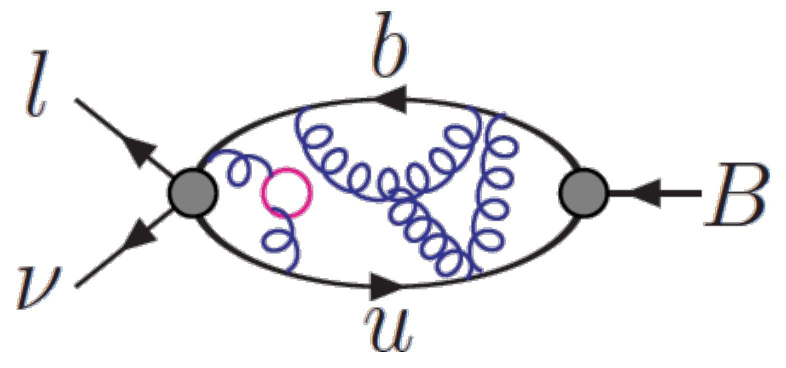

$$\left(\langle 0 \,|\, H_w \,|\, \text{had} \rangle\right)$$

$$\Gamma_{\text{exp.}}(B \to l\nu_l) \overset{???}{=} V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

$$\underbrace{\phantom{\Gamma_{\text{exp.}}(B \to l\nu_l)}}_{\text{experiment}} \approx \underbrace{|V_{\text{ub}}|^2}_{\text{output}} \underbrace{\frac{m_B}{8\pi} G_F^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2}_{\text{theory prediction/kinematics}}$$

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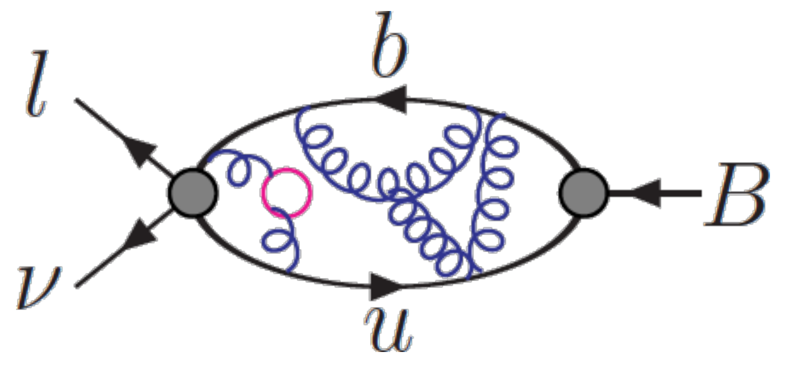

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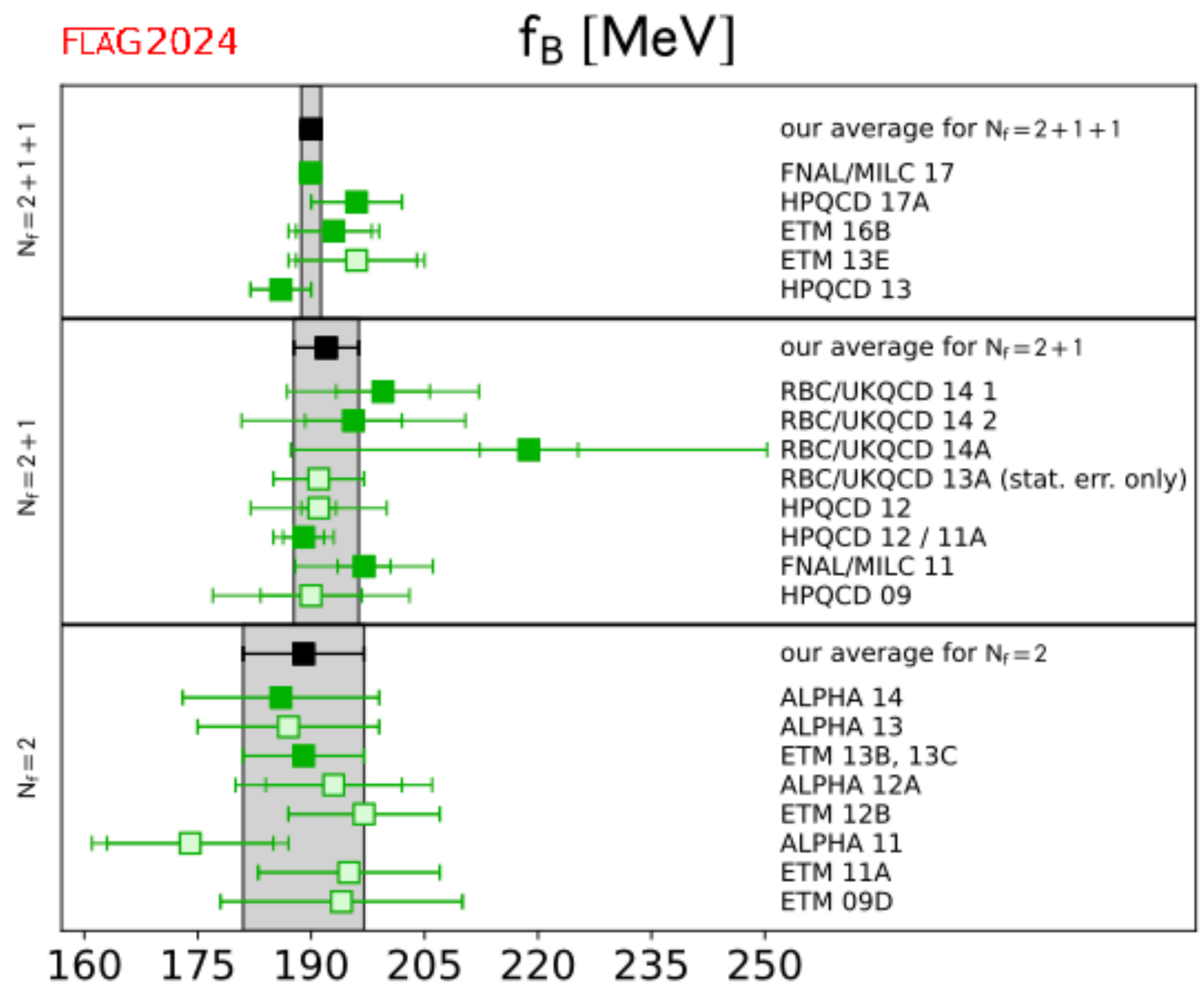
experiment — output — theory prediction/kinematics



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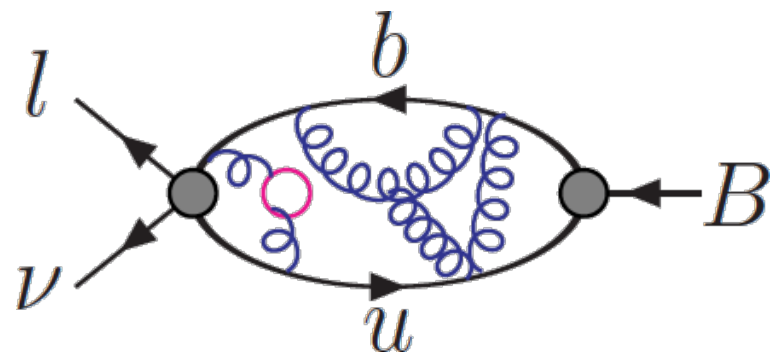

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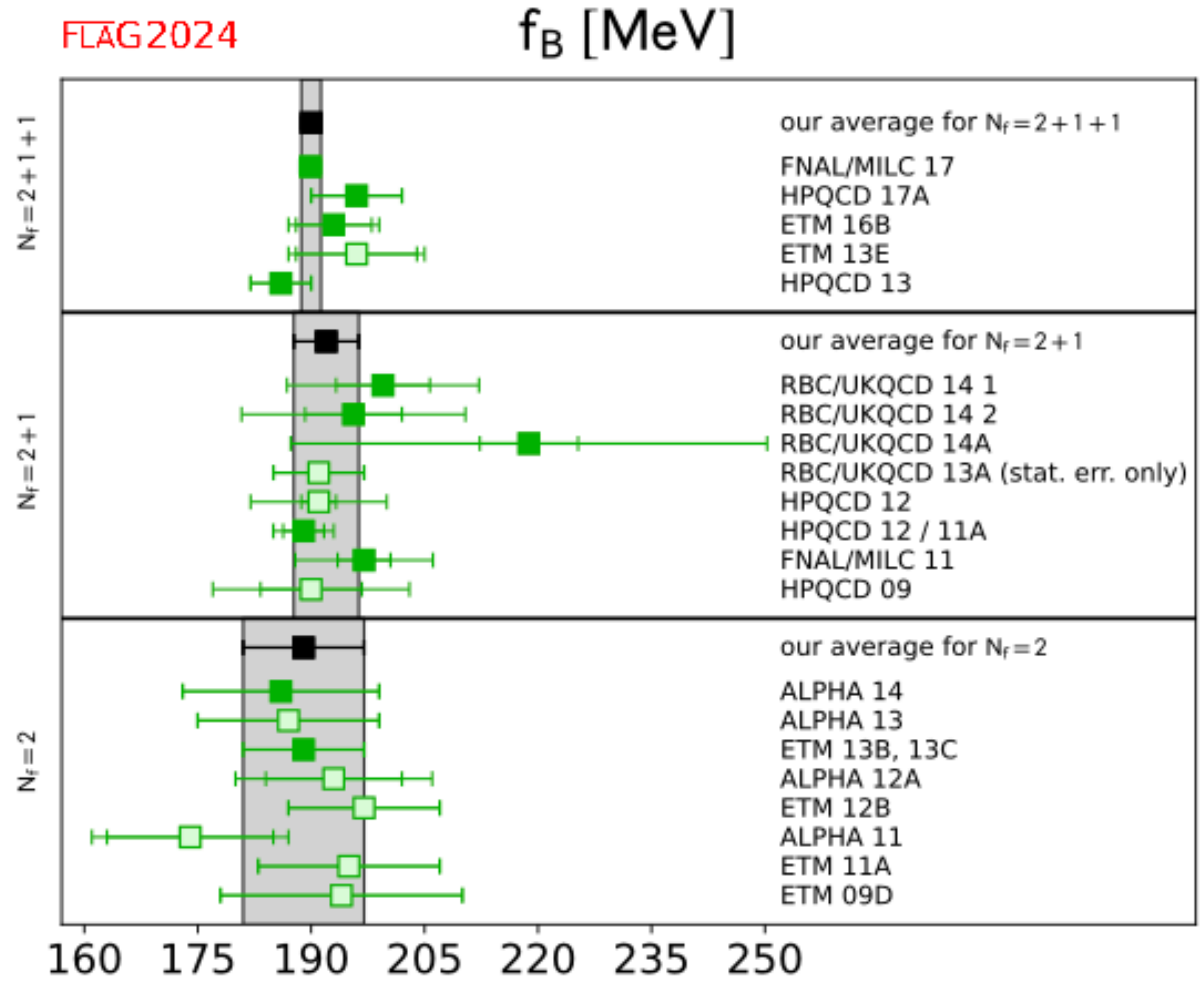

SM tests:

- A) *over-constraining of SM params:* experiment (lhs) + theory prediction (rhs) → CKM
- B) *SM prediction* tested by experiment (e.g. rare decays such as $d\Gamma(B \to K^*\ell\ell)/dq^2$)

same principle for many other processes (rare and tree decays, mixing, …)

Lattice QCD

Regulate quantum field theory:

- **Finite volume** of length L as **IR regulator**
- **Discrete spacetime** (lattice spacing a) as **UV regulator**

$$\frac{1}{L} \ll \Lambda_{\text{physics}} \ll \frac{1}{a}$$

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Compute hadronic matrix elements as expectation values of Euclidean lattice path integral:

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-S_{\text{lat}}[U,\psi,\bar{\psi}]}$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f$$

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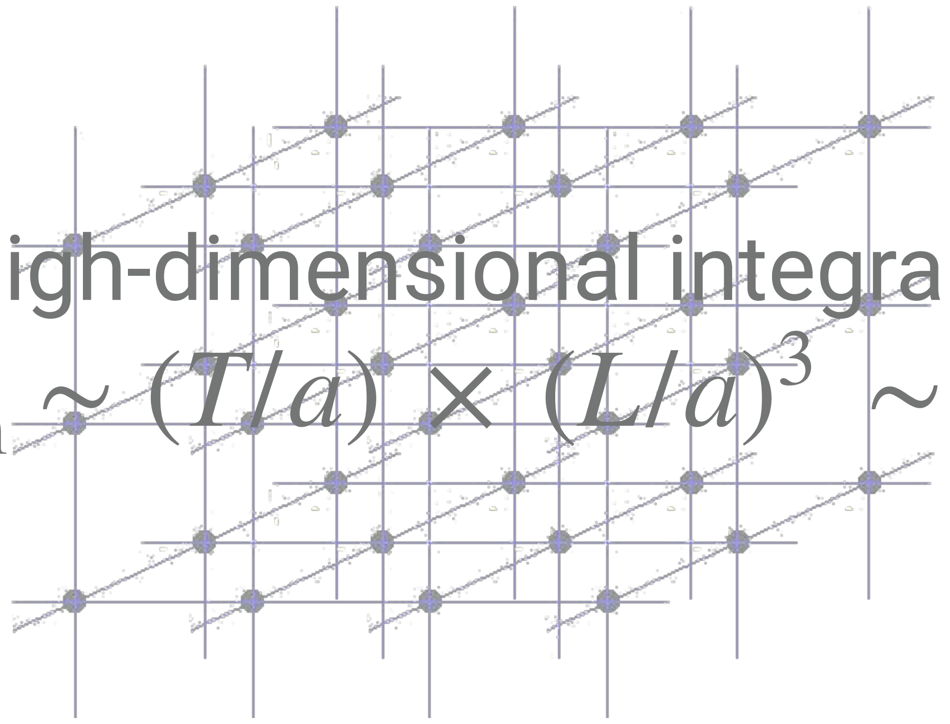

high-dimensional integral:

$$N_{\text{sim}} \sim (T/a) \times (L/a)^3 \sim 10^9$$

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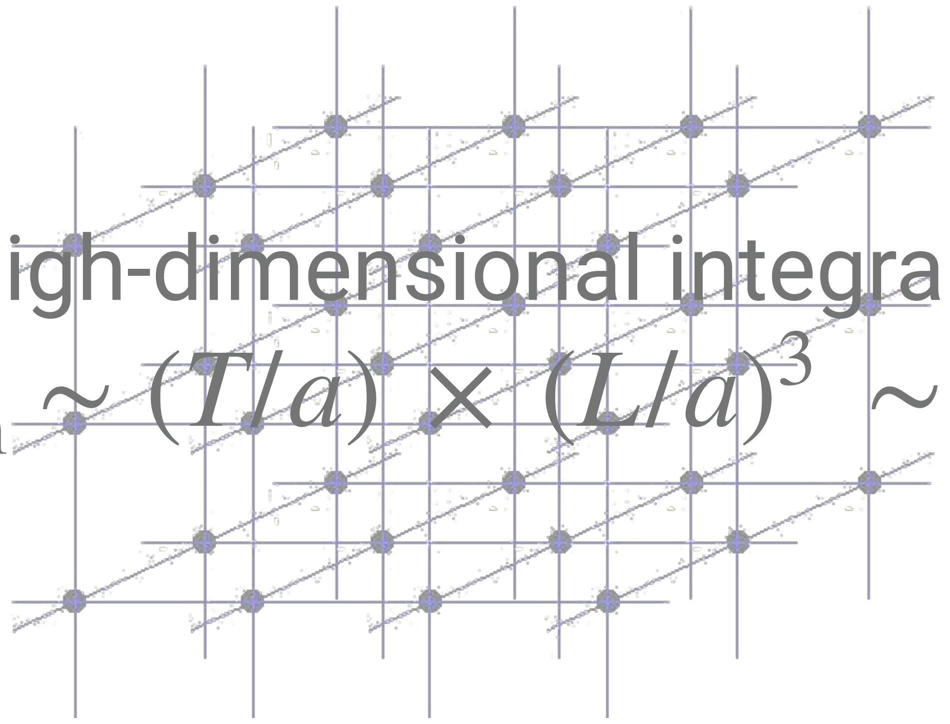

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Fix free QCD parameters:

- gauge coupling $g \rightarrow \alpha_s = g^2/4\pi$
- quark masses $m_f = u,d,s,c,b,t$

One measured input per QCD parameter from PDG:

e.g. $M_\pi, f_\pi, M_K, M_{D_s}, M_\Omega$

Continuum/infinite-volume limits need to be taken.
Lattice QCD predictive *from first principles*,
i.e. no model assumptions, incorporating u, d, s, c vacuum polarisation effects

What's state-of-the-art of lattice for pheno?

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
m_{ud}[MeV]	4.1.1	3.427(51)	[7–9]	3.387(39)	[10–16]		
m_s[MeV]	4.1.1	93.46(58)	[7–9, 17, 18]	92.4(1.0)	[11–15, 19]		
m_s/m_{ud}	4.1.2	27.227(81)	[7, 8, 20, 21]	27.42(12)	[12–14, 19, 22]		
			[9, 23]	2.27(9)			
			[9, 23]	4.67(9)			
			[23, 25]	0.485(19)			
$\overline{m}_c$(3 GeV)[GeV]	4.2.2	0.989(10)	[7–9, 18, 26, 27]	0.991(6)	[15, 28–32]		
m_c/m_s	4.2.3	11.766(30)	[7–9, 18]	11.82(16)	[29, 33]		
			[9, 34–37]	4.171(20)			
			[38, 39]	0.9677(27)			
			[20, 42–45]	1.1916(34)			
$f_{\pi^\pm}$[MeV]	5.6			130.2(8)	[10, 12, 46, 47]		
$f_{K^\pm}$[MeV]	5.6	155.7(3)	[21, 42, 43, 45]	155.7(7)	[10, 12, 46, 47]		
				$1.50(4)(14) \times 10^{-8}$			
				$-8.34(1.03) \times 10^{-13}$			
			[52]	0.7533(91)			
			[52]	0.488(15)			
B_3	6.4	0.79(6)	[52]	0.757(27)	[55, 56, 58]	0.78(4)(2)	[57]
B_4	6.4	0.78(2)(4)	[52]	0.903(14)	[55, 56, 58]	0.76(2)(2)	[57]
B_5	6.4	0.49(3)(3)	[52]	0.691(14)	[55, 56, 58]	0.58(2)(2)	[57]

quark masses

input to theory

kaon decay

NA62/KOTO(-II)

kaon mixing

$\Delta M_K, \epsilon_K, \epsilon'/\epsilon$

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
f_D[MeV]	7.1	212.0(7)	[20, 43]	210.4(1.5)	[28, 59–61]		
f_{D_s}[MeV]	7.1	249.9(5)	[20, 43]	247.7(1.2)	[28, 29, 60–62]		
				1.174(7)	[28, 59–61]		
				0.666(29)	[64]		
				0.747(19)	[66]		
f_B[MeV]	8.1	190.0(1.3)	[20, 36, 67, 68]	192.0(4.3)			
				228.4(3.7)			
				1.201(16)			
				225(9)			
$f_{B_s}\sqrt{\hat{B}_{B_s}}$[MeV]	8.2	256.1(5.7)	[77]	274(8)			
			[77]	1.30(10)			
			[77]	1.35(6)			
ξ			[77]	1.206(17)			
B_{B_s}/B_{B_d}	8.2	1.008(25)	[77]	1.032(38)	[71, 79]	1.007(21)	[73]

Quantity	Sec.	$N_f = 2+1$ and $N_f = 2+1+1$	Refs.		
$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$[MeV]	9.10	338(10)	[15, 18, 80–87]		

$D_{(s)}$-meson decay

$B_{(s)}$-meson decay

$B_{(s)}$-mixing

STCF
LHCb
BESIII
BelleII
terra Z

strong-coupling constant

input to theory

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.
g_A^{u-d}	10.3.1	1.263(10)	[88–91]	1.265(20)	[92–96]
g_S^{u-d}	10.3.1	1.085(114)	[90]	1.083(69)	[93–97]
g_T^{u-d}	10.3.1	0.981(21)	[90, 98]	0.993(15)	[93–96]
g_A^u	10.4.1	0.777(25)(30)	[99]		
g_T^u	10.4.1	0.784(28)(10)	[100]		
g_T^d	10.4.1	−0.204(11)(10)	[100]		
g_T^s	10.4.1	−0.0027(16)	[100]		
$\sigma_{\pi N}$[MeV]	10.4.4	60.9(6.5)	[26, 101]	42.2(2.4)	[102–106]
σ_s[MeV]	10.4.4	41.0(8.8)	[107]	44.9(6.4)	[102–108]
$\langle x \rangle_{u-d}$	10.5.1	0.158(32)	[98, 109]	0.153(13)	[96, 110, 111]
$\langle x \rangle_{\Delta u - \Delta d}$	10.5.1	0.213(27)	[109]	0.200(13)	[96, 110]
$\langle x \rangle_{\delta u - \delta d}$	10.5.1	0.195(25)	[98, 109]	0.206(17)	[96, 110]

nucleon matrix elements

nucleon structure
nEDM

Decay	form factor	fit	Sec.	Fig.	Tab. $N_f = 2+1+1$	Refs.		
$D \to K\ell\nu$	f_+, f_0	lat	7.2	18	30	[63, 65, 123]		
$D \to K\ell\nu$		lat+exp	7.5	19	32	[63, 65, 123]		
B				25				
B_s				26				
B								
B				27				
$B \to D\ell\nu$	f_+, f_0	lat	8.4.1	28				
B_s					46	[134]		
B				29	47	[135]		
B_s				30		[135]		
B				31				41]
$B \to D\ell\nu$	f_+, f_0	lat+exp	8.9	32				43]
$B \to D^*\ell\nu$	g, f, F_1, F_2	lat+exp	8.9	33, 34	53	[135, 144–148]		48]

$D_{(s)}$ SL decays

$B_{(s,c)}$ SL decays

STCF
LHCb
BESIII
BelleII
terra Z

FLAG
Flavour Lattice Averaging Group
http://flag.unibe.ch
FLAG 24 arXiv:2411.04268

FLAG tries to answer:
"What is currently the best lattice value for a particular quantity?"
in a way that is readily accessible to those who are not expert in lattice methods.

Precision quantities discussed in FLAG24

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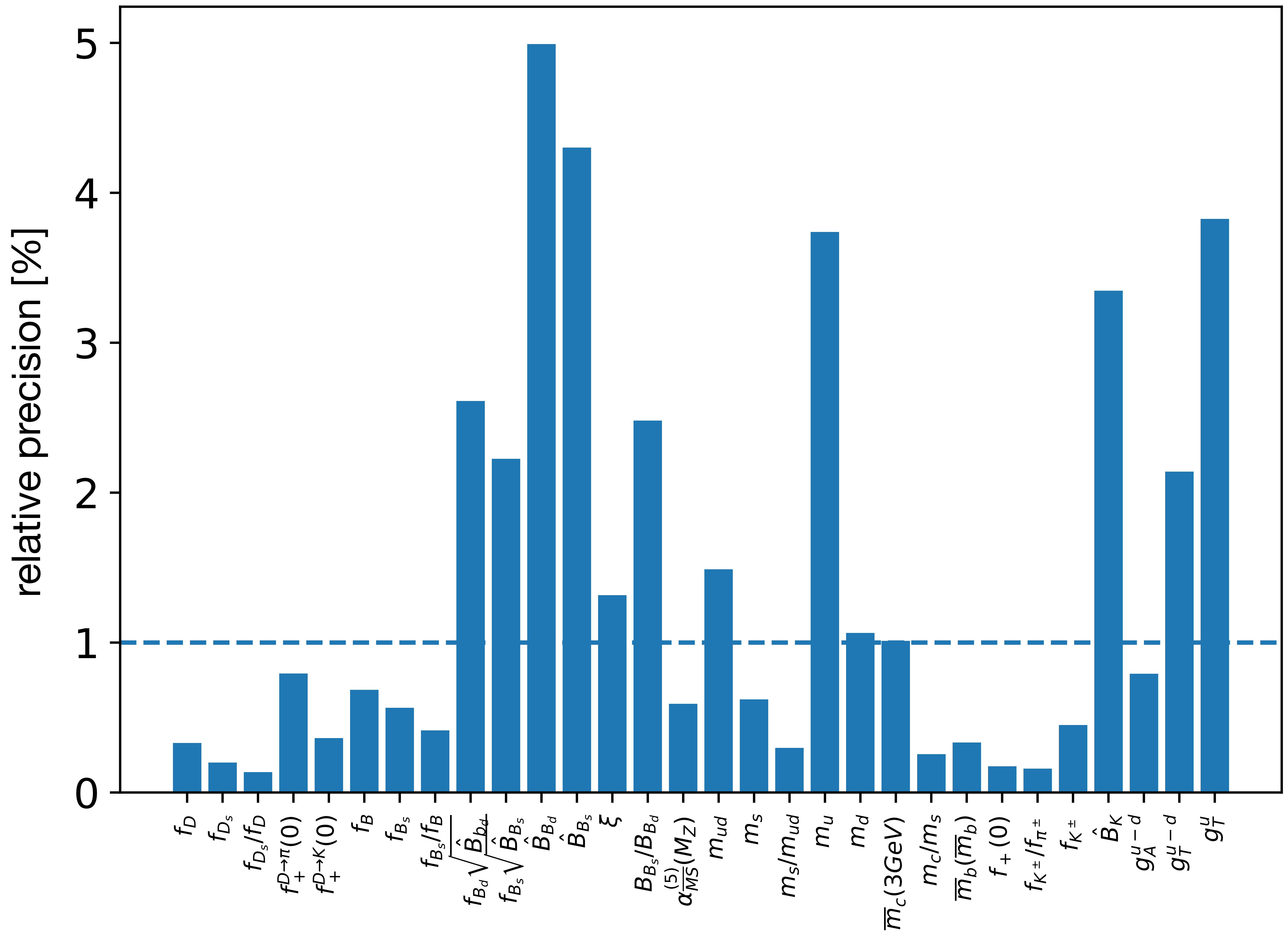


Exploiting future experimental facilities in full sets a high bar for future (5, 10, 20 years) lattice simulations in terms of

- software/computing/algorithm
- precision
- set of calculated observables

Relevance for European Strategy

Flavour and nonperturbative physics in the European Strategy submissions

pions:

- $\pi \to \ell\nu$
- $\pi^+ \to \pi^0 e^+ \nu$
- $|V_{ud}|$

[115] PIONEER

charm:

- $D - \bar{D}$ mixing
- D (rare) decays
  $(D \to \pi\ell\ell, D \to hh\ell\ell, \ldots)$
- D hadronic decays
- $|V_{cd}|, |V_{cs}|$

[81] LHCb Upgrade II
[188] LEP3
[196] FCC
[205] Belle-II/Super KEK B
[223] ATLAS, Belle-II, CMS, LHCb
[231] STCF

bottom:

- $B_{u,d,s,c}$ (rare) L/SL decays
  $(\bar{B}_s^0 \to K\ell\nu, B \to D^{**}\ell\bar{\nu},$
  $B_{(s)}^0 \to \mu^+\mu^-, B^0 \to K^{*0}\mu^+\mu^-, \ldots)$
- Λ_b SL/radiative decays
  $(\Lambda_b \to p\ell\nu, \Lambda_b^0 \to \Lambda\gamma)$
- $B_{(s)} - \bar{B}_{(s)}$ mixing
- hadronic decays
- inclusive/exclusive
- $|V_{ub}|, |V_{cb}|$

[81] LHCb Upgrade II
[188] LEP3
[196] FCC
[205] Belle-II/Super KEK B
[223] ATLAS, Belle-II, CMS, LHCb

kaons:

- $K - \bar{K}$ mixing
- K (rare) L/SL decays
  $(K \to \pi\ell\ell(\nu\nu), K \to \ell\ell)$
- rare hyperon decay
- $K_L - K_S$ interference,
- $|V_{us}|$

[55] Kaon community
[81] LHCb Upgrade II
[155] KOTO(-II)

nEDM:

$\sum_i L_i(\mu)\langle n(p')|\mathcal{O}_i^{\overline{\mathrm{MS}}}|\gamma^*(k)n(p)\rangle$

[158] EDM community

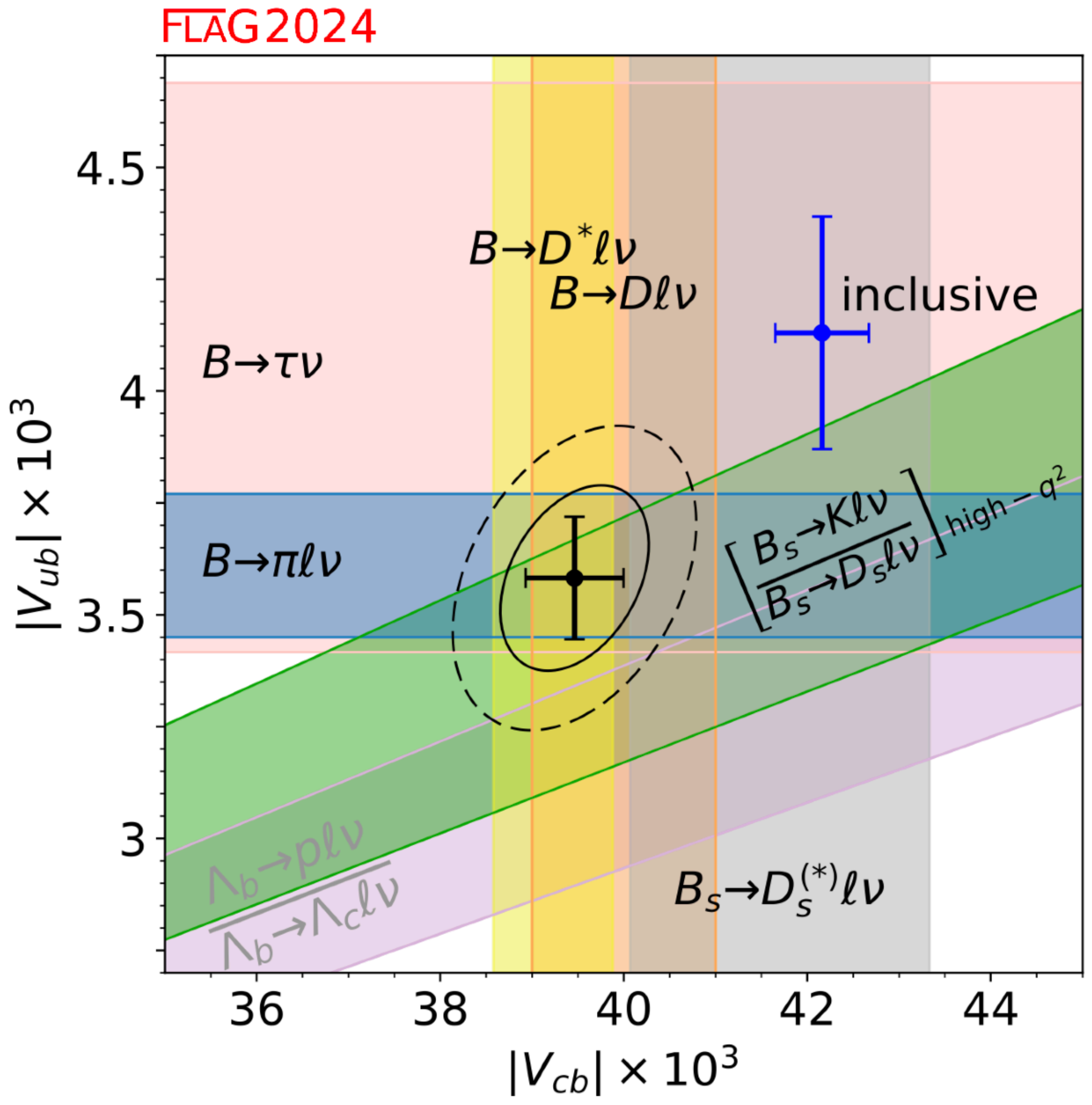
L: leptonic
SL: semileptonic

Example: CKM ME determination

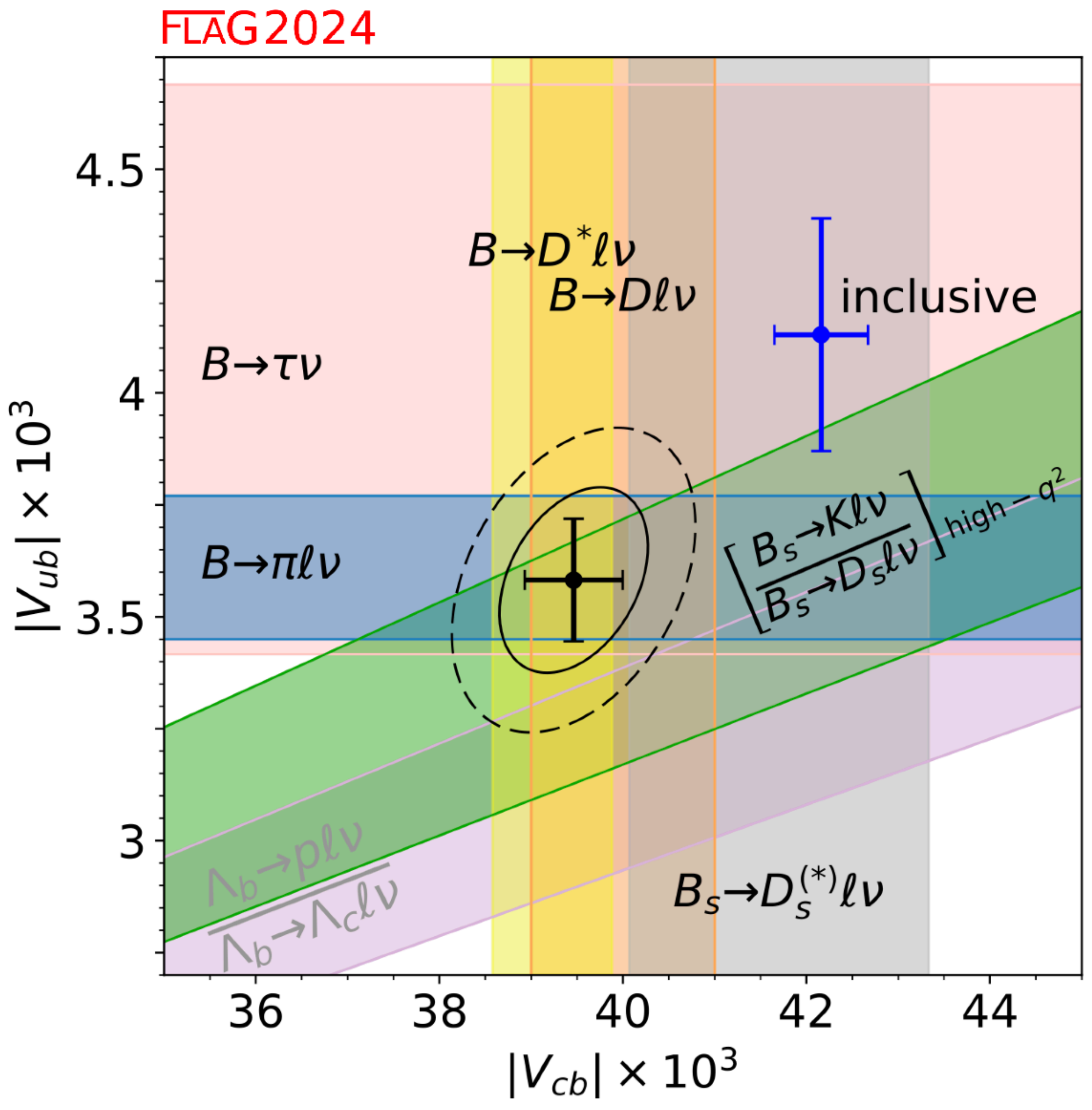
- lattice computes hadronic matrix element (decay constant, form factor, mixing-matrix elements)
- combined analysis of state-of-the-art lattice results & experiment allow for $O(\%)$-precision determination of CKM ME
- precision over-shadowed by tension between inclusive and exclusive determinations — needs to be understood and resolved (e.g. $|V_{ub}|, |V_{cb}|$) lattice now working on this (next slide)
- future experimental projections
  - PIONEER $\delta|V_{ud}| \sim 0.05\,\%$ [115]
  - STCF $\delta\{|V_{us}|, |V_{cd}|, |V_{cs}|\} \sim O(10^{-3})$ [231]
  - ATLAS/Belle-II/CMS/LHCb $\delta|V_{ub}| \sim 1.2\,\%, \delta|V_{cb}| \sim 1\,\%$ [223]

Lattice input for CKM with 1%-goal will be possible on experimental time scales.
Going beyond requires theoretical and algorithmic developments, which is happening now.

Exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ based on lattice simulations



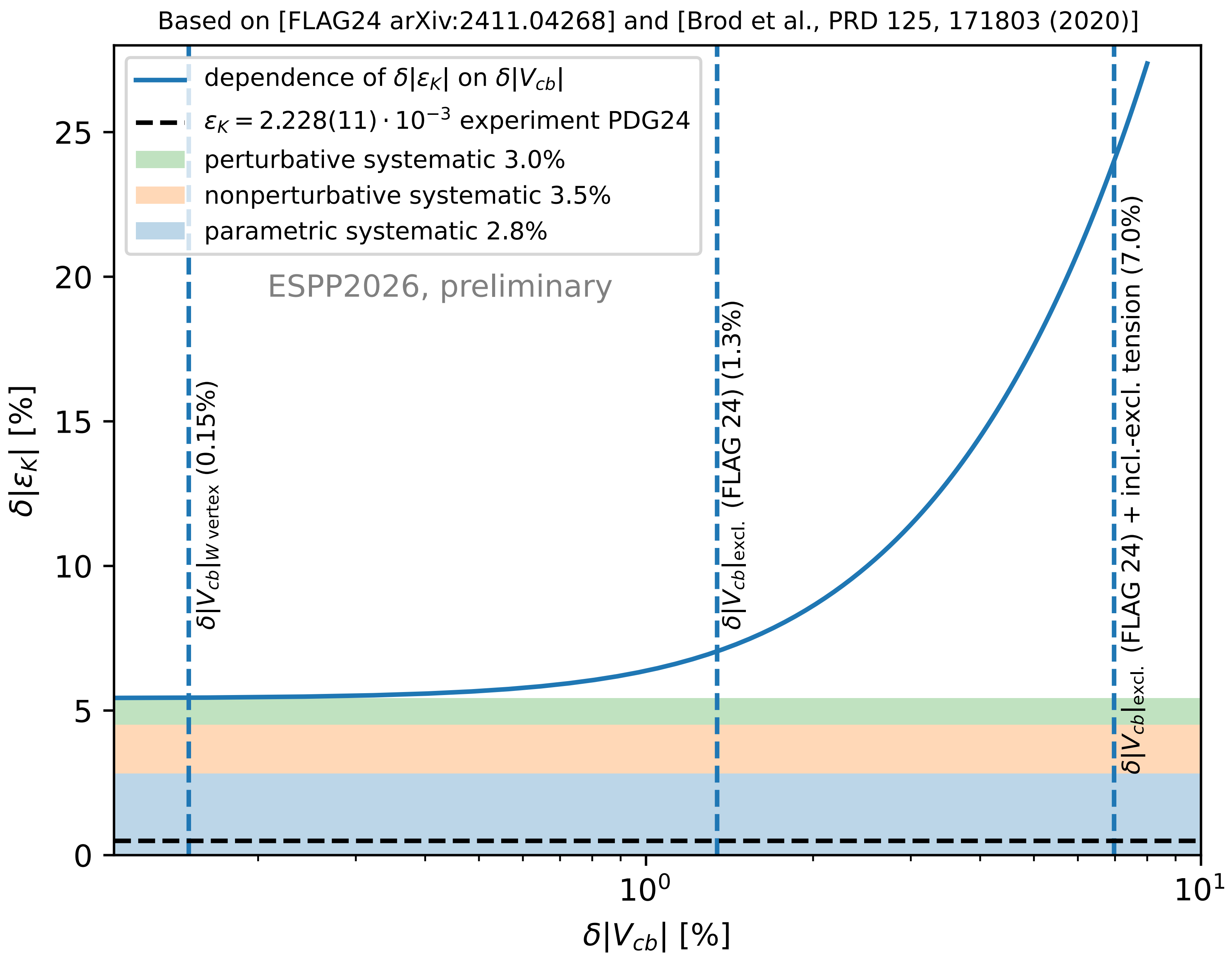
$|V_{cb}|$ tension

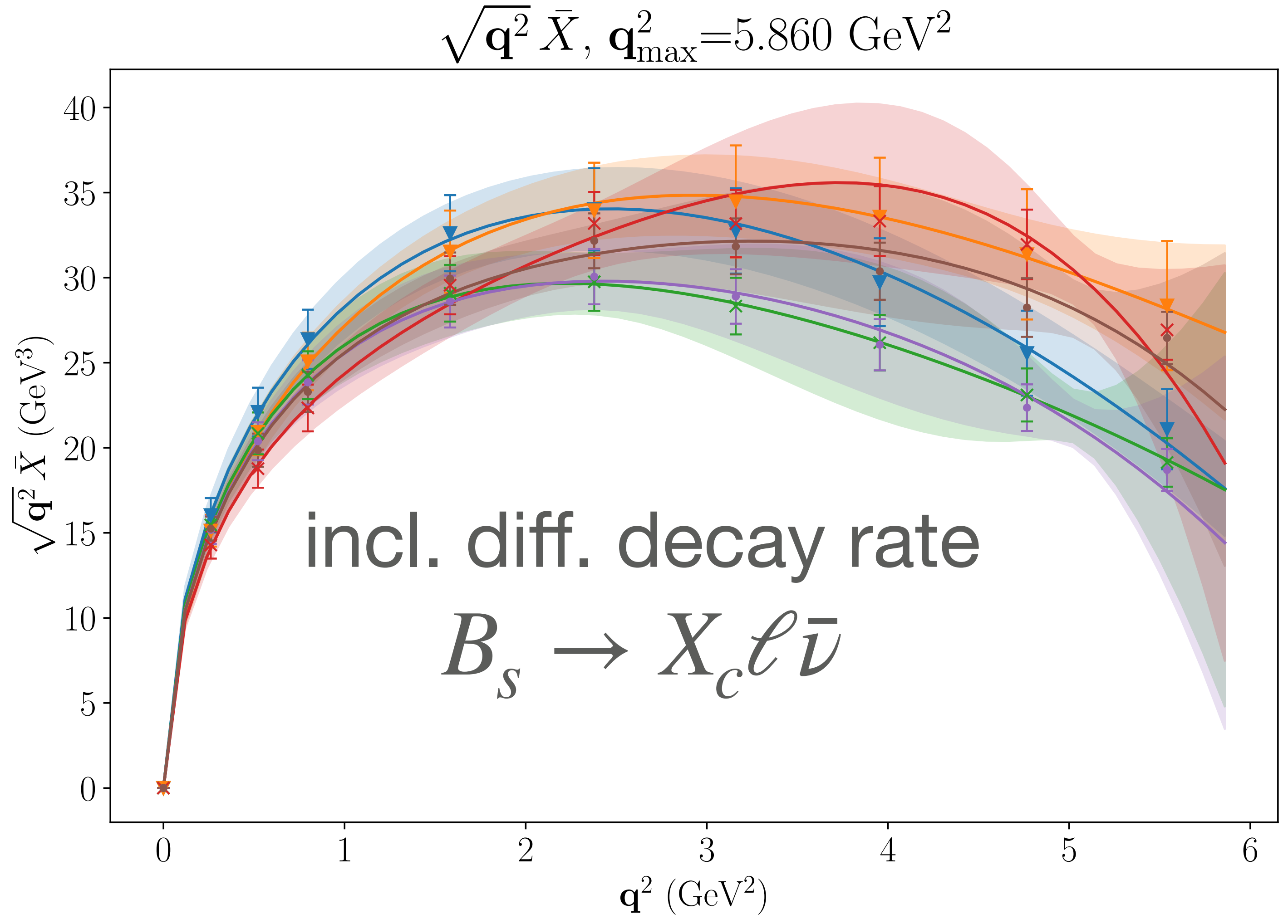


Inclusive decay $B \to X_{u,c}\ell\bar{\nu}_\ell$:

- tension with exclusive persisting and real bottle-neck for pheno
- existing determinations OPE based
- new ideas allow for lattice computations:
  - first result for $B_s \to X_c\ell\bar{\nu}$ inclusive diff. decay rate [Barone et al. JHEP 07 (2023) 145]
  - for $D_s \to X\ell\bar{\nu}$ first CKM result $|V_{cs}| = 0.951(35)$ [De Santis et al. arXiv:2504.06064]

Hansen et al. (2017) PRD 96 094513 (2017)
Hashimoto PTEP 53-56 (2017)
Bailas et al. PTEP 43-50 (2020)
Gambino and Hashimoto PRL 125 32001 (2020)
Gambino et al. JHEP 07 (2022) 083
Barone et al. JHEP 07 (2023) 145
Barone et al. arXiv:2504.03358
De Santis et al. arXiv:2504.06063
De Santis et al. arXiv:2504.06064





Progress in lattice QCD could be decisive in resolving the $|V_{cb}|$ and $|V_{ub}|$ puzzles, strengthening crucial SM tests.

Going below %-level precision

- **Strong isospin breaking (sIB)**

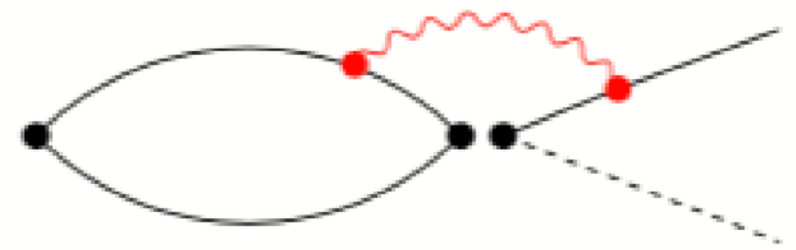
Numbers from FLAG 24

$$m_u = 2.14(8)\text{MeV} \qquad m_d = 4.70(5)\text{MeV} \qquad \frac{m_u - m_d}{\Lambda_{\text{QCD}}} \sim O(1\%)$$

$\overline{\text{MS}}(2\text{GeV})$

- **QED**

$$\alpha_{\text{QED}} \approx \frac{1}{137} \sim O(1\%)$$



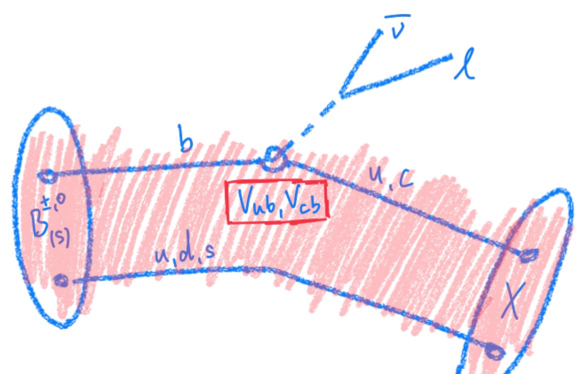
  - Combined QCD+QED/siB treatment needed
  - Already known how to include QED/sIB for hadron spectra, $(g-2)_\mu$ and some leptonic decays
  - Further theory/algorithmic development is required for extending to more complicated quantities (SL decay, scattering, …), but there are no known show-stoppers

    [Duncan et al. PRL 76 1996, Hayakawa, Uno Prog.Th.Ph 120 2008, Endres et al. PRL 117 2016, Lucini et al. JHEP 02 2016, …], [BMWc Science 347 2015, Lubicz et al. PRD 95 2017, Davoudi, Savage, PRD 90 2014, Endres et al. PRL 117 2016, Lucini et al. JHEP 02 2016, Davoudi et al. PRD 99 2019, J. Bijnens et al. PRD 100, 014508 (2019), Hansen PRL 123 (2019) 172001], [RM123+SOTON, PRD120 (2018), PRDD100 (2019)], [Carrasco et al. PRD 91 2016]

- **continuum and infinite-volume limits, signal-to-noise (from MC integration)**
  - algorithmic research (classical MC as well as new directions AI/ML)
  - ties into scaling and performance on novel compute architecture

    [Finkrenrath, arXiv:2402.11704]

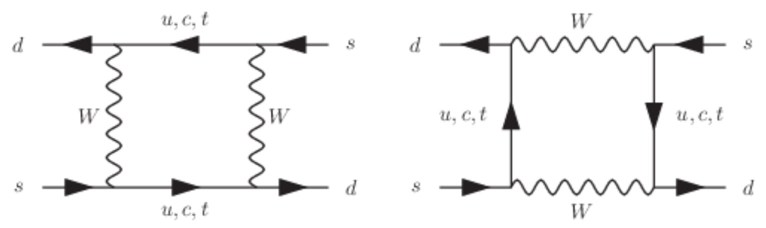
There are dedicated efforts under way to tackle these issues (e.g. within NGT@CERN)



Inclusive semileptonic $B_{(s)}$ and $D_{(s)}$ decay

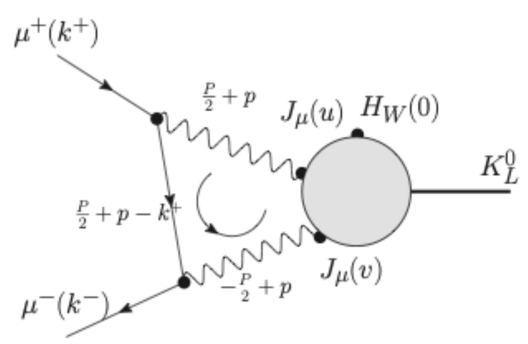
[Barone et al. JHEP (2023)], [Barone et al. arXiv:2504.03358], [De Santis et al. arXiv:2504.06063], [De Santis et al. arXiv:2504.06064]

illustration arXiv:2404.02297



$\epsilon_K, \epsilon'/\epsilon, \Delta M_K$

[Bai et al. PRD 2024]

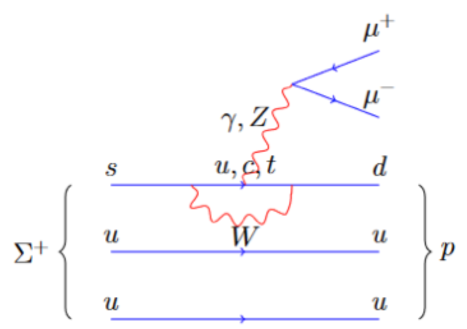


long-distance effects in rare leptonic decays

e.g. $K \to \ell\ell$

[Chao et al. PRD 2024]

illustration LHCb

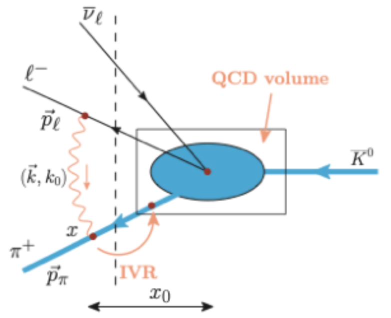


rare hyperon decay

[Erben et al. arXiv:2504.07727]

Examples for new and exploratory directions in lattice QCD for flavour

	γ	quarks	μ	e	τ	ν
$3_q \to 2_q$	b → s γ	$B_s \leftrightarrow \bar{B}_s$	b → s μμ b → c μν	R_{bs} (e/μ) R_{bc} (e/μ)	b → s ττ b → c τν	b → s νν
$3_q \to 1_q$	b → d γ	$B_d \leftrightarrow \bar{B}_d$	b → d μμ b → u μν	R_{bd} (e/μ) R_{bu} (e/μ)	b → d ττ b → u τν	b → d νν
$2_q \to 2_q, 1_q$	c → u γ	$D \leftrightarrow \bar{D}$	c → u μμ c → s μν c → d μν	R_{cu} (e/μ) R_{cs} (e/μ) R_{cd} (e/μ)		c → u νν



QCD+QED for semileptonic decays

[Christ et al. arXiv:2402.08915]

long-distance contributions to rare kaon decay

[Boyle et al. PRD 2023]

long-distance contributions to charm mixing

[Di Carlo, Erben, Hansen arXiv:2504.16189]

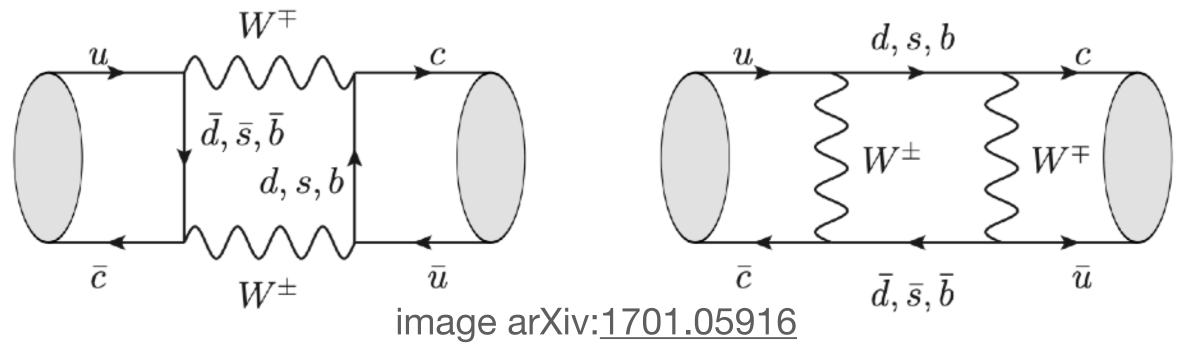
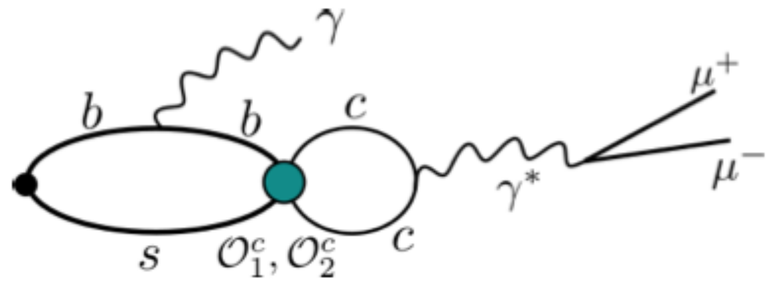


image arXiv:1701.05916

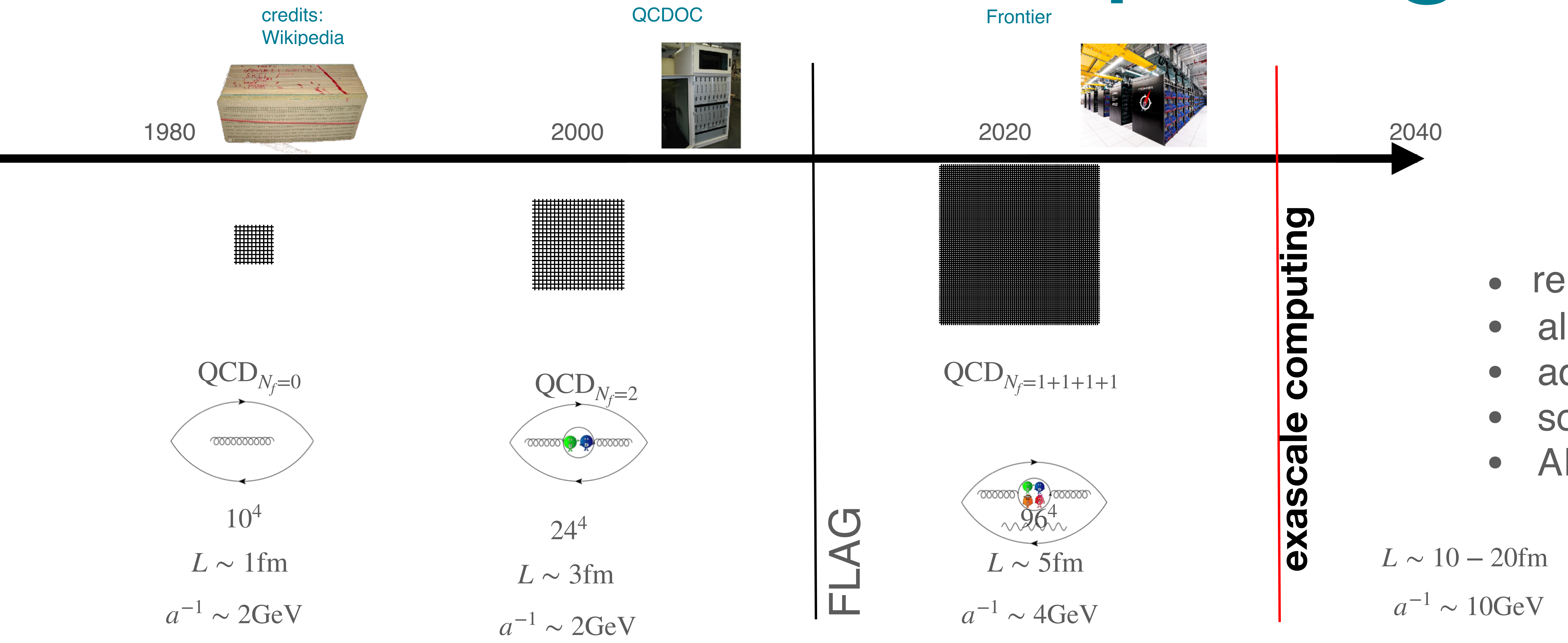


short/long-distance contributions to radiative decays

e.g. $B_s \to \mu\mu\gamma, D_s \to \ell\nu_\ell\gamma$

[Frezzotti et al. PRD 2024], [Frezzotti et al. PRD 2024]

Computing



- removing the regulators ($a \to 0, L \to \infty$)
- algorithm development
- adaption of new hardware paradigms
- software-development and data curation
- AI/ML

Lattice community submission [29], [168]:

- Access to Tier-0/1 computing resources
- Access to large-scale data storage (processing and data curation)
- Support for training and career-perspectives for research-software engineers (RSE)

Summary

- The physics in many of the flavour submissions rely on first-principles predictions of hadronic matrix elements → lattice QCD(+QED)

- Lattice QCD(+QED) will be playing an increasingly prominent role in flavour physics
  - in the HL-LHC/Belle-II/BESIII era and beyond
  - at the precision frontier (e.g. leptonic, semileptonic tree decays)
  - in terms of the computation of a increasing set of long-distance contributions and rare processes

- Current and future experimental landscape provides excellent incentives for lattice QCD to further develop. The lattice community has demonstrated to be creative and able to deliver, in particular if clear expectations and objectives exist (see $(g-2)_\mu$)

- In order to deliver, the lattice community will require support on ([29] and [168]):
  - access to state-of-the-art high-performance computing resources
  - increasingly also storage and data curation
  - support for hardware-specific software development