Status of Theory Calculations of the Muon Anomalous Magnetic Moment

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ROMA TRE UNIVERSITÀ DEGLI STUDI

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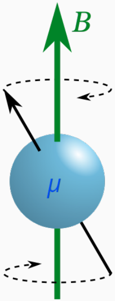
Introduction: the magnetic moment of a lepton



The magnetic moment $\boldsymbol{\mu}$ of a charged object parameterizes the torque that a static magnetic field exerts on it.

For a charged spin-$1/2$ particle:

$$\boldsymbol{\mu} = g\frac{e}{2m}\boldsymbol{S}$$

g is the well-known gyromagnetic factor.

In QFT the response of a charged lepton (say a muon μ) to a static and uniform e.m. field is encoded in ($k = p_1 - p_2$)

$$\langle \mu(p_2)|J^{\nu}_{\rm em}(0)|\mu(p_1)\rangle = -ie\bar{u}(p_1)\Gamma^{\nu}(p_1, p_2)u(p_2)$$

Lorentz invariance and e.m. current conservation constrain Γ^{ν}-structure:

$$\Gamma^{\nu}(p_1, p_2) = F_1(k^2)\gamma^{\nu} + \frac{i}{2m_{\mu}}F_2(k^2)\sigma^{\nu\rho}k_{\rho} + \text{P-violating terms}$$

The muon anomalous magnetic moment

Gyromagnetic factor g_μ related to form-factors $F_1(k^2)$ and $F_2(k^2)$ through

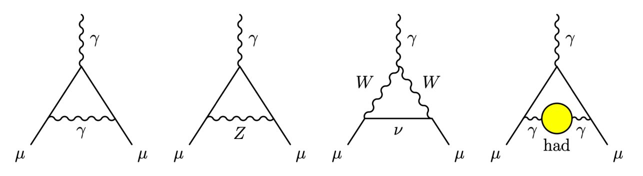
$$g_\mu = 2\left[F_1(0) + F_2(0)\right]$$

- Electric charge conservation $\implies F_1(0) = 1$.
- At tree level in the SM: $F_2(0) = 0 \implies g_\mu = g_\mu^{\mathrm{Dirac}} \equiv 2$.

The muon anomalous magnetic moment:

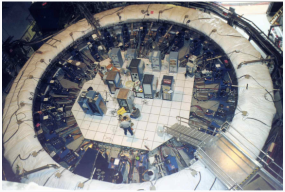
$$\boxed{a_\mu = \frac{g_\mu - 2}{2} = F_2(0)}$$

non-zero only at loop level. Contributions from all SM (and BSM) fields. E.g.



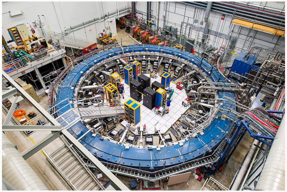
If very precisely measured can be a crucial probe of the completeness of the SM. Is it?

Latest update (August ’23) from FNAL experiment

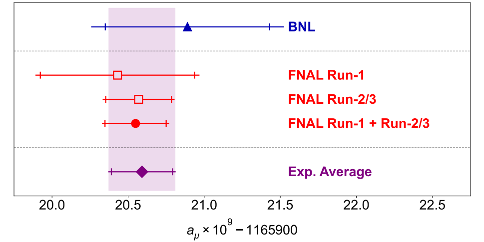


$g_\mu - 2$ @BNL (up to 2006) $\Longrightarrow$

transfer to Fermilab $\Longrightarrow$



$g_\mu - 2$ @Fermilab



$$\boxed{a_\mu^{\rm exp} = 116\,592\,059(22) \times 10^{-11} \quad [0.19\text{ppm}]}$$

Congratulations!!

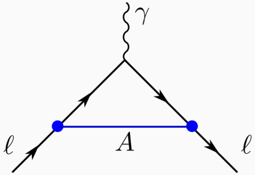
Results from Run-4/5/6 expected in 2025

Why did we pick the muon (and not e, τ) ?

Electron anomalous magnetic moment is measured with even higher precision (×1000):

$$\boxed{a_e^{\mathrm{exp}} = 1\,159\,652\,180.73(28) \times 10^{-12} \;\; [0.0002\,\mathrm{ppm}]}$$

However, NP contributions expected to be



$\ell = e, \mu, \tau$

$$\Longrightarrow \quad a_\ell^A \propto m_\ell^2 / m_A^2$$

$$m_\mu^2 / m_e^2 \simeq 43\,000$$

a_τ would have a much higher enhancement but not measured as accurately...

$$a_\tau = 0.0009^{+0.0032}_{-0.0031} \qquad \text{[CMS 2023, arXiv:2406.03975]}$$

The CMS's result ($\gamma\gamma \to \tau\tau$) dramatically improved the precision w.r.t. previous measurements.

Can we match, on the theory side, the experimental accuracy on a_μ?

The Muon $g-2$ Theory Initiative

The muon $g-2$ TI has been established in 2017 with the aim of matching the precision of the SM-theory prediction for a_μ with the experimental one.

https://muon-gm2-theory.illinois.edu

- Composed by experts in lattice QCD, dispersive approach, perturbative calculations, ...
- First white paper out in '20 [Physics Reports 887 (2020)]. Second out in a few months!!

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

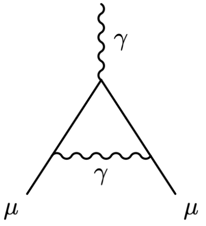
- Last TI meeting at KEK (Tsukuba, Japan).

The muon magnetic moment in the SM

a_μ can be decomposed into QED, weak and hadronic contributions

$$\boxed{a_\mu = \underbrace{a_\mu^{\text{QED}}}_{>99.99\%} + a_\mu^{\text{weak}} + \underbrace{a_\mu^{\text{had}}}_{\text{non-perturbative}}}$$

- The QED contribution to a_μ is completely dominant. LO (1-loop) contribution evaluated by J. Schwinger in 1948

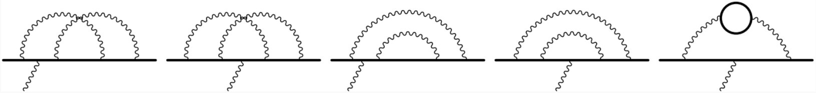


$$\Longrightarrow a_\mu^{\text{QED},1-\text{loop}} = \frac{\alpha}{2\pi}$$

- Since Schwinger's calculation many more QED-loops included...

The QED contribution a_μ^{QED}

Two-loops QED contributions to a_μ



To match experimental accuracy $\Delta a_\mu^{\exp} \simeq \mathcal{O}(10^{-10})$ several orders in the perturbative α expansion need to be considered

$$\boxed{a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + \sum_{n=2}^{\infty} C_\mu^n \left(\frac{\alpha}{\pi}\right)^n}$$

- Number of Feynman diagrams quickly rises with n: $1, 7, 72, 891, 12672, \ldots$
- Heroic effort to compute them up to five-loops [T. Aoyama et al. PRLs, 2012]

$C_\mu^6 \left(\frac{\alpha}{\pi}\right)^6 \simeq C_\mu^6 \times 10^{-16}$ requires unnaturally large $C_\mu^6 \simeq \mathcal{O}(10^6)$ to be relevant!!

$$\boxed{a_\mu^{\mathrm{QED}} = 116\,584\,718.931(104) \times 10^{-11} \checkmark}$$

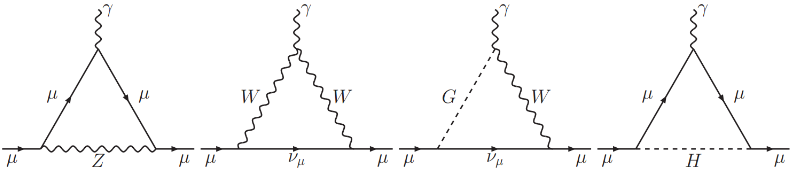
The weak contribution a_μ^{weak}

a_μ^{weak} defined as the sum of all loop diagrams containing at least a W, H, Z.

- Smallest of the three contributions due to Fermi-scale suppression:

$$\boxed{a_\mu^{\text{weak}} \propto \alpha_W^2 \frac{m_\mu^2}{M_W^2} \simeq \mathcal{O}(10^{-9})}$$

Sample of one-loop weak diagrams:



- At target precision of ~ 0.1 ppm two-loops calculation is sufficient [Czarnecki et al PRD (2006), Gnendiger et al PRD (2013)].

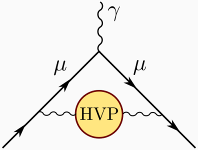
$$\boxed{a_\mu^{\text{weak}} = 153.6(1.0) \times 10^{-11} \;\checkmark}$$

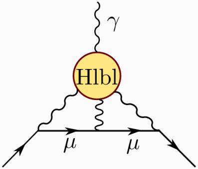
The hadronic contribution a_μ^{had}

Contributions to a_μ^{had} at target accuracy of $\mathcal{O}(10^{-10})$:

$$a_\mu^{\text{had}} = \underbrace{a_\mu^{\text{HVP-LO}}}_{\mathcal{O}(7\times 10^{-8})} + \underbrace{a_\mu^{\text{Hlbl}}}_{\mathcal{O}(10^{-9})} + \underbrace{a_\mu^{\text{HVP-NLO}}}_{\mathcal{O}(10^{-}9)} + \underbrace{a_\mu^{\text{HVP-NNLO}}}_{\mathcal{O}(10^{-}10)}$$

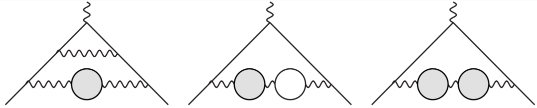
HVP-LO $\Longrightarrow$





$\Longleftarrow$ Hlbl

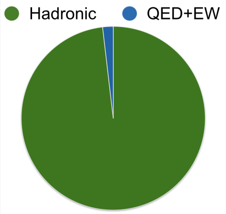
- NLO and NNLO HVP contributions relevant at target accuracy. At NLO:



- However, they can obtained from same non-perturbative input of $a_\mu^{\text{HVP-LO}}$. Hence we shall discuss only the latter.

How important are hadronic contributions?

The uncertainty in the theory prediction for a_μ dominated by the hadronic contribution, despite its smallness



Dominant source of uncertainty is $a_\mu^{\mathrm{HVP-LO}}$

- Hadronic contributions are fully non-perturbative.
- Two main approaches to evaluate them:

Dispersive approach:

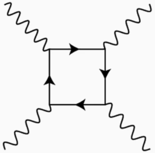
- Relates full $a_\mu^{\mathrm{HVP-LO}}$ to $e^+e^- \to$ hadrons cross-section via optical theorem.
- For Hlbl (only) low-lying intermediate-states contributions can expressed in terms of transition form-factors TFFs.

Lattice QCD:

- Only known first-principles SM method to evaluate both a_μ^{HVP} and a_μ^{Hlbl}.
- In the past the accuracy of the predictions were not good enough. The situation has recently changed.

The hadronic light-by-light contribution

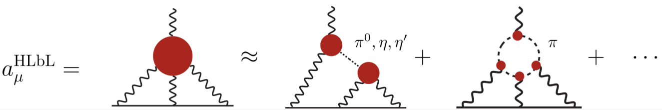
a_μ^{Hlbl} occurs at $\mathcal{O}(\alpha^3)$. Related to $2 \to 2$ (generally virtual) photons scattering



It involves the fourth-rank VP tensor:

$$T\langle 0|J^\mu J^\nu J^\rho J^\sigma|0\rangle = \Pi^{\mu\nu\rho\sigma}(k_1, \ldots, k_4)$$

- In the dispersive framework [Colangelo et al. JHEP09 (2015)] one isolates the dominant intermediate-states contributions:



- parameterized by transition form-factors **TFFs**. For dominant π^0-pole contr.

$$i\int d^4x e^{iqx} T\langle 0|J^\mu(x)J^\nu(0)|\pi^0(p)\rangle = \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta F_{\pi^0\gamma^*\gamma^*}(q^2, (q-p)^2)$$

TFFs from dispersion relations (using available exp. input) or recently from LQCD.

The hadronic light-by-light on the lattice

The cleanest, assumptions-independent, way of computing a_μ^{Hlbl} is given by Lattice QCD. The lattice QCD input is the 4-point correlation function of e.m. currents

$$\Pi^{\mu\nu\rho\sigma}(x,y,z,w) = T\langle 0|J^\mu(x)J^\nu(y)J^\rho(z)J^\nu(w)|0\rangle$$

- Long distance contribution very noisy. Noise rapidly increases reaching m_π^{phys}.
- Clever tricks employed to reduce computational cost. Lattice input can be compressed into

$$i\hat{\Pi}^{\rho,\mu\nu\lambda\sigma}(x,y) = \int dz\, z^\rho \langle 0|J^\mu(x)J^\nu(y)J^\sigma(z)J^\lambda(0)|0\rangle$$

$$a_\mu^{\mathrm{Hlbl}} = \frac{m_\mu e^6}{3}\int d^4y \int d^4x\, \underbrace{\mathcal{L}_{[\rho,\sigma],\mu\nu\lambda}(x,y)}_{\text{QED kernel}}\, \underbrace{i\,\hat{\Pi}^{\rho,\mu\nu\lambda\sigma}(x,y)}_{\text{QCD input}}$$

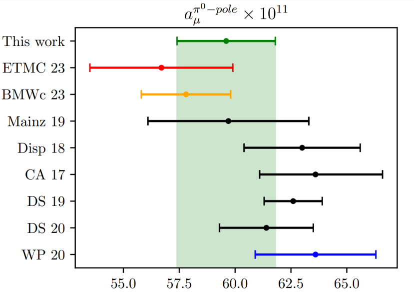
- So far three lattice Collaborations have fully computed a_μ^{Hlbl}:

RBC/UKQCD ('21, '23), MAINZ ('22) and BMW ('24)

Summary of current status for $a_\mu^{\rm Hlbl}$

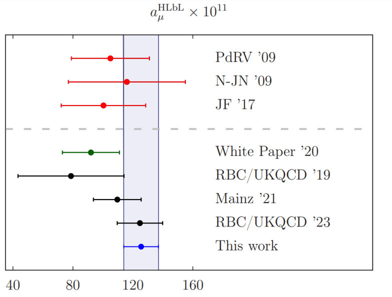
taken from RBC/UKQCD '24 ePrint:2411.06349 [1]

This work=RBC/UKQCD



taken from BMW '24 ePrint:2411.11719 [2]

This work=BMW

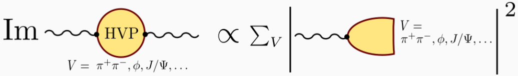


- LQCD calculations of $a_\mu^{\rm Hlbl}$ in line with the dispersive result from WP '20 (and with smaller uncertainties).
- LQCD calculations of $a_\mu^{{\rm Hlbl};\pi^0}$ in reasonable good agreement with dispersive ones.
- 10% accuracy goal for $a_\mu^{\rm Hlbl}$ achieved!.

The LO hadronic-vacuum-polarization (HVP) contribution

$a_\mu^{\mathrm{HVP-LO}}$ is the largest of the hadronic contributions.

- Until '20 LQCD calculations above percent level accuracy.
- However, $a_\mu^{\mathrm{HVP-LO}}$ is related to $\sigma(\gamma^* \to \text{hadrons})$ through optical theorem...

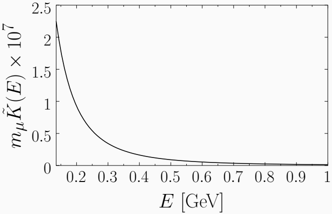


- In terms of the $e^+e^- \to$ hadron cross-section or actually the R-ratio:

$$R(E) = \frac{\sigma(e^+e^-(E) \to \text{hadrons})}{\sigma(e^+e^-(E) \to \mu^+\mu^-)}$$

- one has a very simple formula for $a_\mu^{\mathrm{HVP-LO}}$

$$a_\mu^{\mathrm{HVP-LO}} = \int_{m_\pi}^{\infty} dE\, R(E)\, \underbrace{\tilde{K}(E)}_{\text{analytic function}}$$



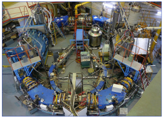
$a_\mu^{\mathrm{HVP-LO}}$ from the dispersive approach (I)

The central idea is to replace $R(E) \to R^{\exp}(E)$ and use previous formula.

$e^+e^- \to$ hadrons measured since '60 in various experiments

KLOE @ DAΦNE
FRASCATI

BABAR @ SLAC
STANFORD

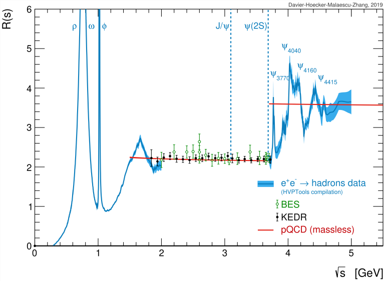


CMD3 @ VEPP-2000
NOVOSIBIRSK

Inclusive measurement of $R^{\exp}(E)$ obtained summing more than fourty exclusive channel measurements (comb. of various exp. , dominated by π's).

WP '20, pre-CMD3

	DHMZ19	KNT19
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)
Total $a_\mu^{\mathrm{HVP,\,LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\mathrm{DV+QCD}}$	692.8(2.4)



Two main groups involved in the analysis: DHMZ, KNT.

DHMZ = Davier-Hoecker-Malaescu-Zhang, KNT = Keshavarzi-Nomura-Teubner

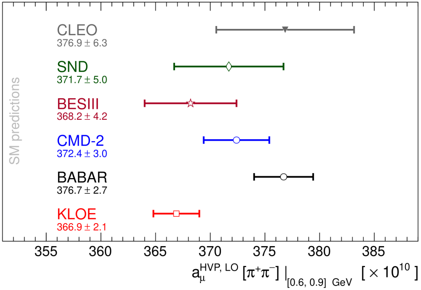
$a_\mu^{\mathrm{HVP-LO}}$ from the dispersive approach (II)

Combination of DHMZ and KNT results gives:

$$a_\mu^{\mathrm{HVP-LO}}[\mathrm{disp.}] = 6931(40) \times 10^{-11} \qquad [\mathrm{WP}\ \text{'}20]$$

Replacing the theoretical prediction with the experimental $R(E)$ is OK if:

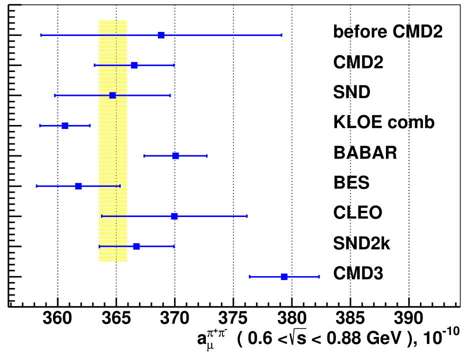
- All relevant decay channels identified.
- No underestimated uncertainty in any of the relevant channels (ISR & hadron/lepton VP insertion subtracted properly?).
- No NP contamination in the measurement (e.g. $e^+e^- \to A^*_{NP} \to$ hadrons).



The CMD-3 result

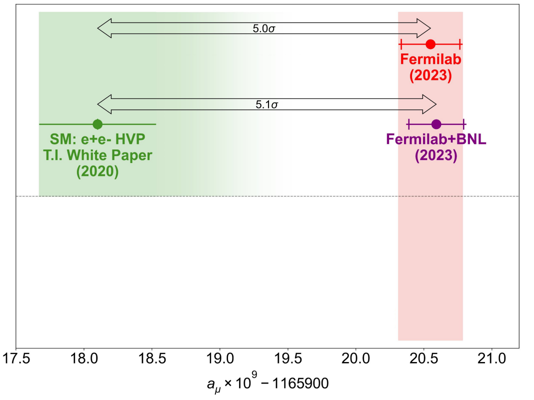
A new measurement of $e^+e^- \to \pi^+\pi^-$ with CMD detector at VEPP-2000 in 2023,

found significant deviations from previous measurements



- At the moment the situation of exp. $e^+e^- \to$ hadrons needs to be clarified.
- However since 2020 the situation changed since LQCD calculations have reached the level of precision ($< 1\%$) required.

The $g-2$ puzzle: let's start from scratch (WP'20)



- Using a_μ^{HVP} from dispersive analysis as in WP '20 a $> 5\sigma$ discrepancy present.
- In WP'20 precision of lattice result not good enough.
- Final value (quoted as "SM") was obtained from dispersive approach.

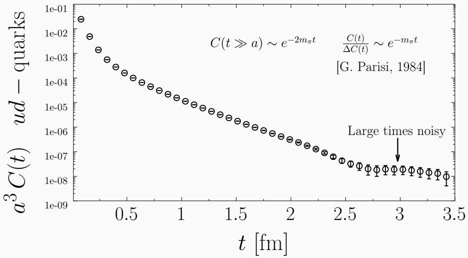
$a_\mu^{\mathrm{HVP-LO}}$ from lattice QCD

On the lattice, evaluating $a_\mu^{\mathrm{HVP-LO}}$ is much easier than a_μ^{Hlbl}.

The QCD input is the 2-point Euclidean correlation function of e.m. currents:

$$\boxed{C(t) = \frac{1}{3}\int d^3x\, \langle 0|J^i_{\mathrm{em}}(t,\boldsymbol{x})J^i_{\mathrm{em}}(0)|0\rangle \qquad J^i_{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma^i u - \frac{1}{3}\bar{d}\gamma^i d - \frac{1}{3}\bar{s}\gamma^i s + \frac{2}{3}\bar{c}\gamma^i c}$$

$$a_\mu^{\mathrm{HVP-LO}} = \int_0^\infty dt\ \underbrace{K(t)}_{\text{analytic kernel}}\ C(t) \qquad K(t) \overset{t\gg m_\mu^{-1}}{\to} t^2 \qquad [\text{Enhancement of } C(t) \text{ tail}]$$



Main difficulties for subpercent accuracy:

- S/N problem at large times.
- Large lattice volumes $V = L^3$ required to fit the light $\pi\pi$ states.
- Isospin-breaking effects $\alpha^3, \alpha^2(m_d - m_u)$ needs to be computed at target accuracy.

BMWc crosses the Rubicon [Nature 593 (2021)]

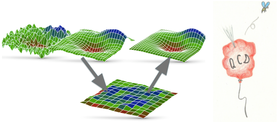
Leading hadronic contribution to the muon magnetic moment from lattice QCD

Sz. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato, K. K. Szabo, F. Stokes, B. C. Toth, Cs. Torok & L. Varnhorst

Nature **593**, 51–55 (2021) | Cite this article

- Order of magnitude improvement in stat. accuracy
- Large lattice volumes up to $L \simeq 11$ fm
- Seven lattice spacings to control UV cut-off effects.

Modern algorithms and new methods



Adaptative solvers & eigendeflation

Isospin symmetric
- connected light: 633.7(2.1)(4.2)
- connected strange: 53.393(89)(68)
- connected charm: 14.6(0)(1)
- disconnected: -13.36(1.18)(1.36)

QED isospin-breaking: sea
- connected: 0.37(21)(24)
- disconnected: -0.040(33)(21)

Etc.
- bottom; higher order; perturbative: 0.11(4)

QED isospin-breaking: valence
- connected: -1.23(40)(31)
- disconnected: -0.55(15)(10)

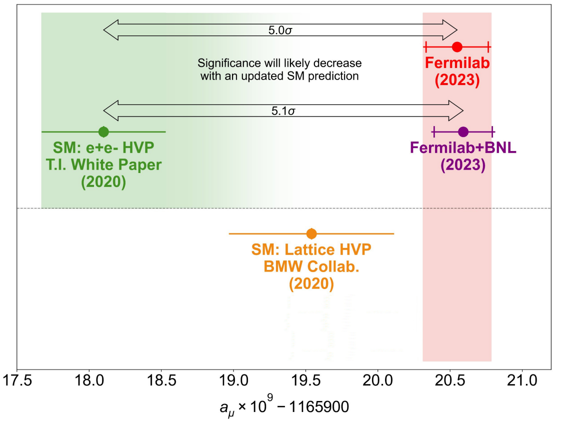
Strong isospin-breaking
- connected: 6.60(63)(53)
- disconnected: -4.67(54)(69)

QED isospin-breaking: mixed
- connected: -0.0093(86)(95)
- disconnected: 0.011(24)(14)

Finite-size effects
- isospin-symmetric: 18.7(2.5)
- isospin-breaking: 0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

The a_μ discrepancy after BMWc's result 2020-2021



- BMWc's result is 2.1σ larger then $a_\mu[\text{disp.}]$.
- ...and only 1.7σ smaller than FNAL+BNL results.
- To scrutinize e^+e^- data in '22-'24 many LQCD collaborations started to look at the so-called Euclidean-time window

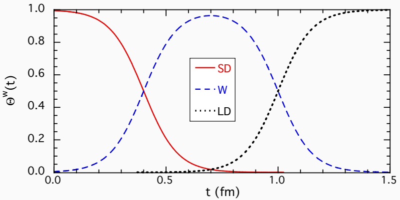
The Euclidean windows to test $e^+e^- \to$ hadrons

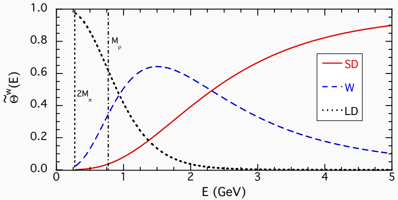
To perform stringent tests of $R(E)$ we are not bound to $a_\mu^{\mathrm{HVP-LO}}$

$$\underbrace{\int_0^\infty dt\ K(t)\ C(t)}_{\text{lattice, SM}} = a_\mu^{\mathrm{HVP-LO}} = \underbrace{\int_{M_\pi}^\infty dE\ \tilde{K}(E)\ R^{\exp}(E)}_{\text{dispersive, experimental}}$$

$$\Downarrow \qquad \Downarrow$$

$$\underbrace{\int_0^\infty dt\ K(t)\ C(t)\,\underline{\Theta^w(t)}}_{\text{lattice, SM}} = a_\mu^w = \underbrace{\int_{M_\pi}^\infty dE\ \tilde{K}(E)\ R^{\exp}(E)\,\underline{\tilde{\Theta}^w(E)}}_{\text{dispersive, experimental}}$$





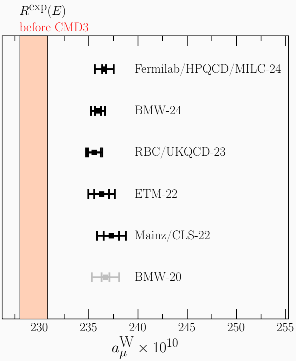
- $\Theta^{\mathrm{SD}} + \Theta^{\mathrm{W}} + \Theta^{\mathrm{LD}} = 1$. $w = \{\mathrm{SD}, \mathrm{W}, \mathrm{LD}\}$ probe $R(E)$ at different energies.
- $a^{\mathrm{SD/W}}$ very precise on the lattice $\implies$ may enhance differences with $R^{\exp}(E)$.

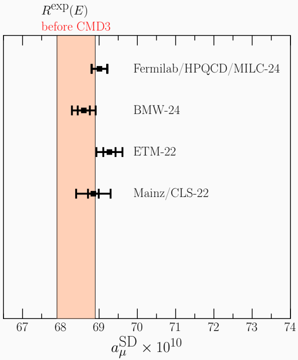
The short- and intermediate-distance windows

In 22-24 several LQCD results for a_μ^{W} and a_μ^{SD}. Many appeared before CMD3.

intermediate-distance $\implies$ $E \lesssim 1$ GeV $(\pi\pi, \pi\pi\pi)$

short-distance $\implies$ Large E $\gtrsim 1$GeV





- Many more lattice results for (dominant) ℓ-quark contribution. All in line ✓.
- A big achievement for the lattice community.
- Striking tension with $R^{\mathrm{exp}}(E)$-based results for a_μ^{W} which is dominated by $e^+e^- \to \rho \to \pi^+\pi^-$. High-energy part of R-ratio in line with experiments.

What about computing $R(E)$ directly on the lattice?

Can we compute $R(E)$ directly on the lattice?

$$\boxed{C(t) = \frac{1}{12\pi^2} \int_0^\infty dE\, e^{-Et}\, R(E)\, E^2}$$

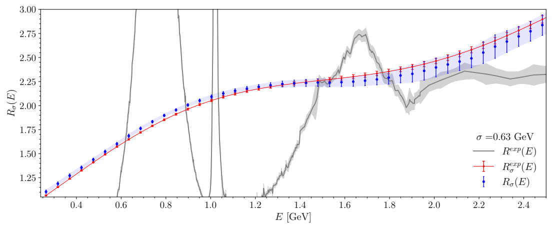
- Inverting the previous relation to obtain $R(E)$ from $C(t)$ (our lattice input) is an ill-posed problem if...
- ... $C(t)$ affected by statistical uncertainties and known only at a discrete and finite number of times (typical situation encountered in lattice calculation).
- But... this is not the end of the story.
- We have a new numerical technique, the Hansen-Lupo-Tantalo (HLT) method, which allows us to obtain on the lattice an energy-smeared version of $R(E)$.

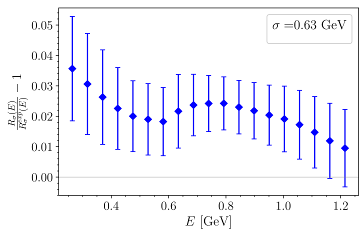
The energy-smeared $R-$ratio

In PRL 130 (2023) we (ETM) exploited the HLT method to evaluate on the lattice:

$$\boxed{R_\sigma(E) = \int_0^\infty d\omega\, R(\omega)\, \underbrace{N(E-\omega,\sigma)}_{\text{Gaussian}}}$$

$R_\sigma(E)$ is a "sort of" energy-binned version of $R(E)$ (with bin-size $\sim \sigma$).

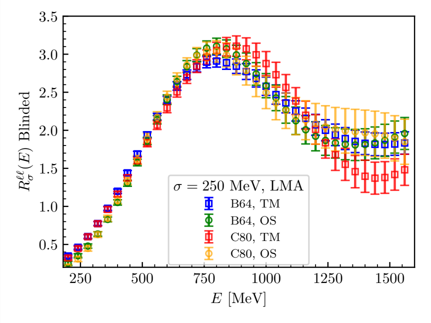


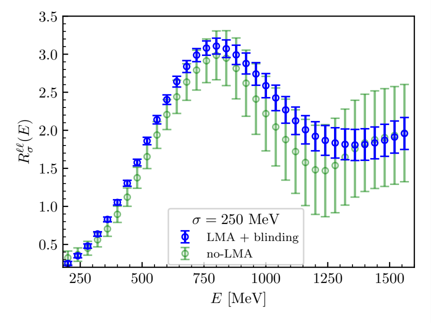


- In the low-energy region, for $\sigma \simeq 0.6$ GeV, we observe a $\approx 3\sigma$ (or $2.5-3\%$) deviation w.r.t. e^+e^- experimental results.
- Similar conclusions as from $a_\mu^{\rm W}$ $\Longrightarrow$ **higher SM value** w.r.t. $R^{\rm exp}(E)$ results around the ρ resonance.

Quick update by ETMC

We (ETM) started improving on the R-ratio using the new generation of LQCD vector-vector correlators (higher precision due to Low-Mode-Averaging techniques).





- New dataset still blinded. Huge reduction of error w.r.t. previous work.
- We are able to achieve good precision for smearing down to 250 MeV.
- Results so far obtained on two lattice spacing ensemble (B64 and C80).[1]

[1] I am grateful to F. Margari for providing me with the plots of the energy-smeared R-ratio derived from the new generation of data.

The long-distance window: 2024 results

In '24 many LQCD results appeared for the long-distance contributions (none of them published as of today).

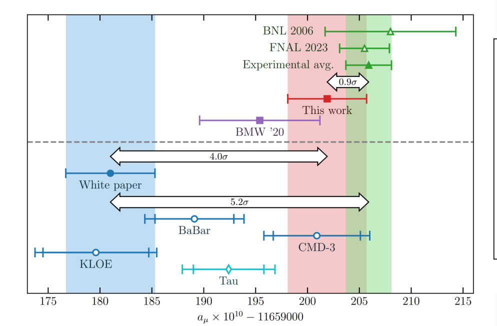
- BMW-24: The BMW Coll. reported an update of their previous result for $a_\mu^{\mathrm{HVP-LO}}$. New results obtained by combining LQCD data for the Euclidean VV-correlator and dispersive results. The latter are used in the large time region $t > 2.8$ fm [ePrint:2407.10913].
- RBC/UKQCD-24: The RBC/UKQCD Coll. reported a calculation of the light-connected contribution to a_μ^{LD} [ePrint:2410.20590].
- Mainz/CLS-24: The MAINZ/CLS Coll. reported an almost full calculation of $a_\mu^{\mathrm{HVP-LO}}$ (only some subleading IB diagrams missing) [ePrint:2411.07969].
- Fermilab/HPQCD/MILC-24: The Fermilab/HPQCD/MILC Coll. reported a calculation of the light-connected contribution to a_μ^{LD} [ePrint:2412.18491].

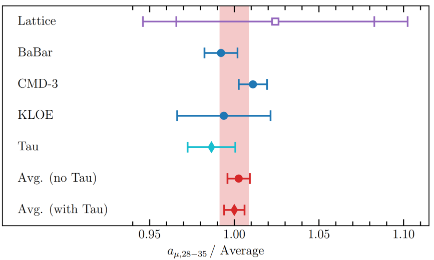
The light-connected contribution to a_μ^{LD} is the most challenging on the lattice:

- Affected by large finite-size effects (two pions in a box).
- Shows non-linear cut-off effects if the lattice discretization adopted suffers from significant distortion of the pion-spectrum.
- Large statistical uncertainty (exponentially decreasing S/N).

The BMW-24 result

[Plots taken from the BMW-24 paper ePrint:2407.10913]

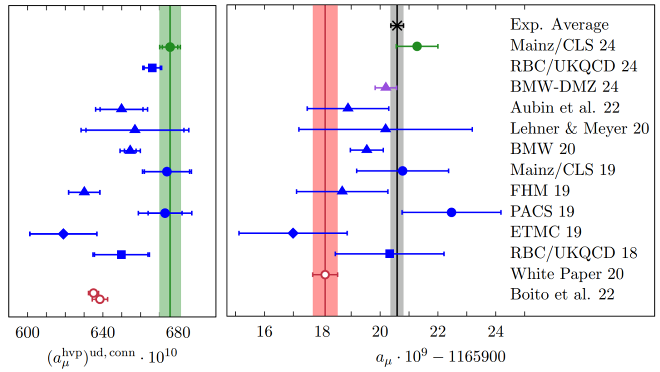




- Results shifted 1σ upward w.r.t. BMW-20.
- Reduced uncertainty partially/mainly due to the use of data-driven results for $t \gtrsim 2.8$ fm.
- Replacing the tail of the LQCD VV-correlator with the dispersive one motivated by agreement between experiments in the very low-energy region (incl. CMD3).
- Contribution of data-driven tail is $\simeq 28 \times 10^{-10}$, not a small effect!

The Mainz/CLS-24 result

[Plot taken from the Mainz/CLS paper ePrint:2411.07969]



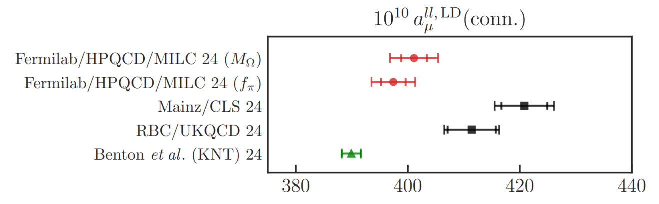
- Mainz/CLS LQCD result is $a_\mu^{\mathrm{LO-HVP}} = 724.9(5.0)_{\mathrm{stat}}(4.9)_{\mathrm{syst}} \times 10^{10}$
- Slightly larger than BMW-20 and (to lesser extent) BMW-24 which is based on both LQCD and data-driven methods.
- The $a_\mu^{\mathrm{LO-HVP}}$ by Mainz/CLS leads to a total $a_\mu = (g_\mu - 2)/2$, in line with the world-average of exp. results.

$a_\mu^{\mathrm{LD}}(\ell)$ from RBC/UKQCD and Fermilab/HPQCD/MILC

$$a_\mu^{\mathrm{LO-HVP}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

$$a_\mu^{\mathrm{LD}} = a_\mu^{\mathrm{LD}}(\ell) + a_\mu^{\mathrm{LD}}(s) + a_\mu^{\mathrm{LD}}(c) + a_\mu^{\mathrm{LD}}(\mathrm{disc.}) + \mathrm{IB-effects}$$

[Figure taken from Fermilab/HPQCD/MILC-24 , ePrint:2412.18491]

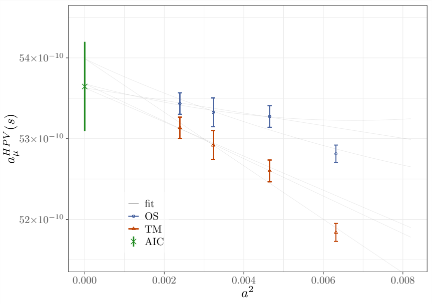


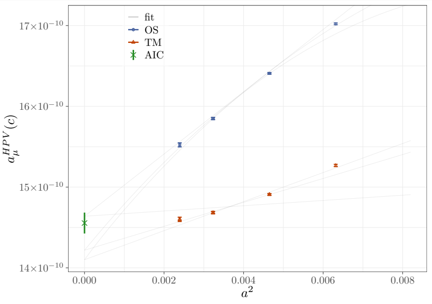
- Three lattice results available for $a_\mu^{\mathrm{LD}}(\ell)$.
- Separation between isoQCD ($m_u - m_d = \alpha_{\mathrm{em}} = 0$) and IB contributions is conventional. Indications that scheme ambiguities lead to larger diff. for a_μ^{LD}.
- Fermilab/HPQCD/MILC result lower than RBC/UKQCD and Mainz/CLS but...
- ...results shown obtained in different isoQCD-schemes. (one is not really comparing apple to apple).
- Important that all collaboration provide results in a given isoQCD-scheme.

Status of the ETM calculation

We (ETM) have recently produced results for $a_\mu^{\mathrm{LO-HVP}}(s)$ and $a_\mu^{\mathrm{LO-HVP}}(c)$

Taken from ePrint:2411.08852 (ETM)





$$a_\mu^{\mathrm{LO-HVP}}(s) = (53.57 \pm 0.63) \times 10^{-10} \, , \qquad a_\mu^{\mathrm{LO-HVP}}(c) = (14.56 \pm 0.13) \times 10^{-10}$$

We will publish our results for $a_\mu^{\mathrm{HVP-LO}}(\ell)$ in a few months.

Summary

Where are we?

$a_\mu^{\mathrm{HVP-LO}}$

$e^+e^- \to$ hadrons

- BMW gave an update of their ’20 paper, using an hybrid approach which combines LQCD data and dispersive results.
- Mainz/CLS produced a new and almost complete result for $a_\mu^{\mathrm{HVP-LO}}$ which is slightly higher than BMW-20 and leads to an a_μ compatible with Fermilab exp.
- Two additional collaborations produced results for $a_\mu^{\mathrm{LD}}(\ell)$.
- It is conceivable that the SM value of $a_\mu^{\mathrm{HVP-LO}}$ in the next WP update will be entirely based on lattice results.
- Warning: None of the new results reviewed has been published!

- Lattice QCD has signalled an inconsistency between previous $e^+e^- \to$ hadron measurements and the SM value.
- NP, unknown systematic in measurements?
- The new CMD-3 result can provide an explanation.
- The situation needs to be clarified.

Thank you for the attention