Bayes-ic Interactions -
Bayesian analysis of nucleon-nucleon scattering data in pionless effective field theory

Jason Bub

In collaboration with: Maria Piarulli, Dick Furnstahl, Saori Pastore, and Daniel Phillips

Marciana 2025 - Lepton Interactions with Nucleons and Nuclei
26 June 2025

WashU

Outline

- Motivation/Bayesian EFT Model Calibration
- BUQEYE Formalism
- Interaction Choice
- Results

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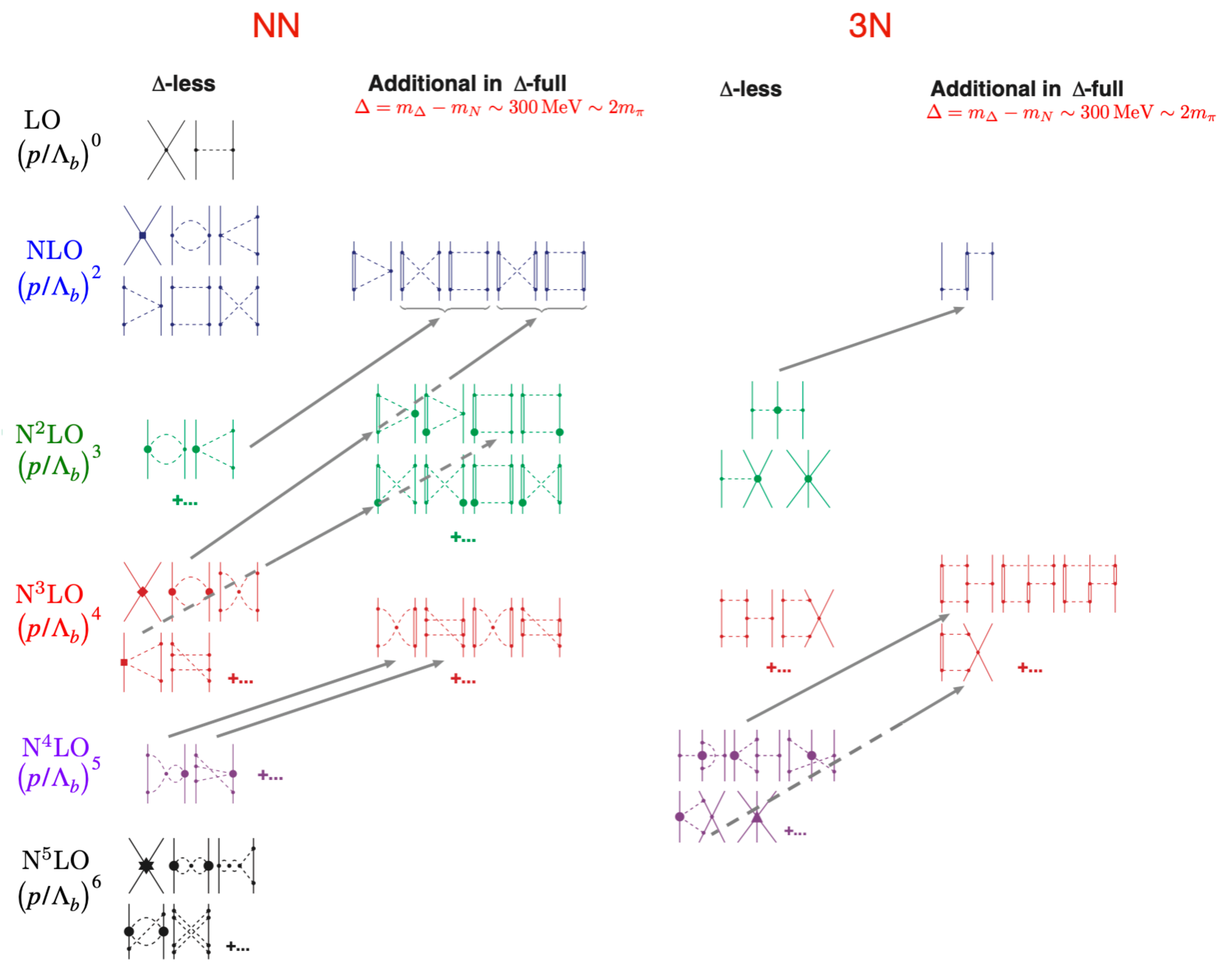

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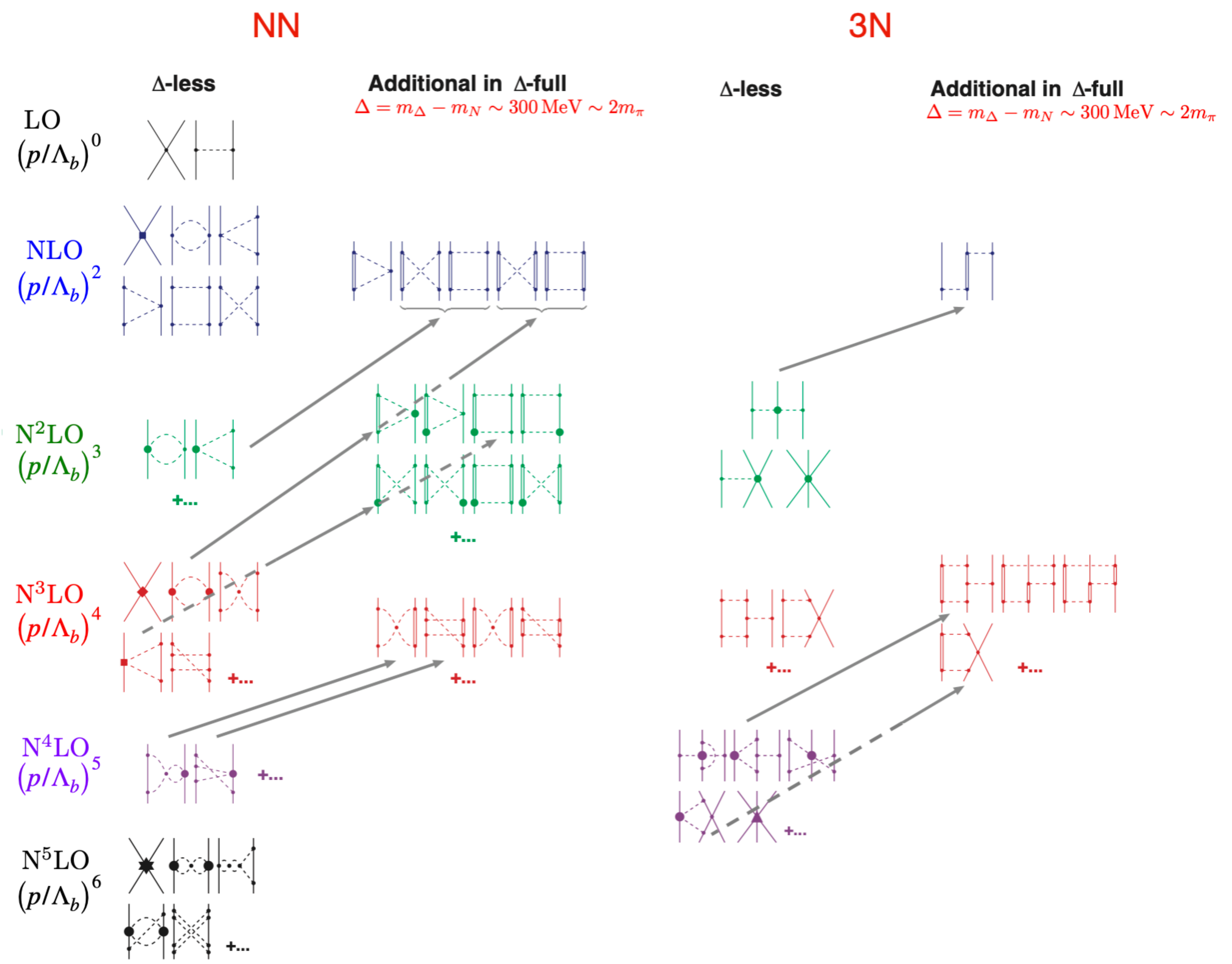

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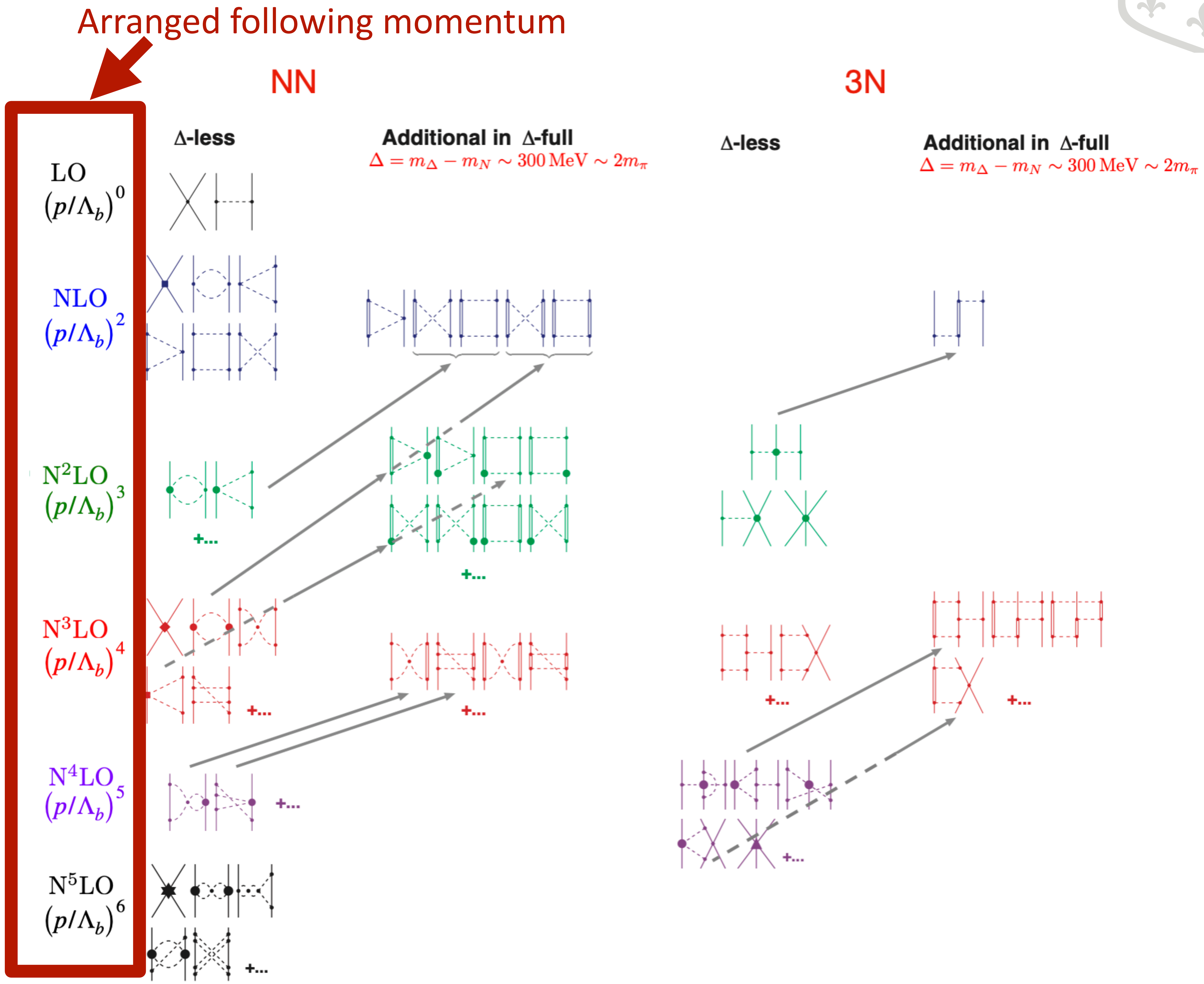

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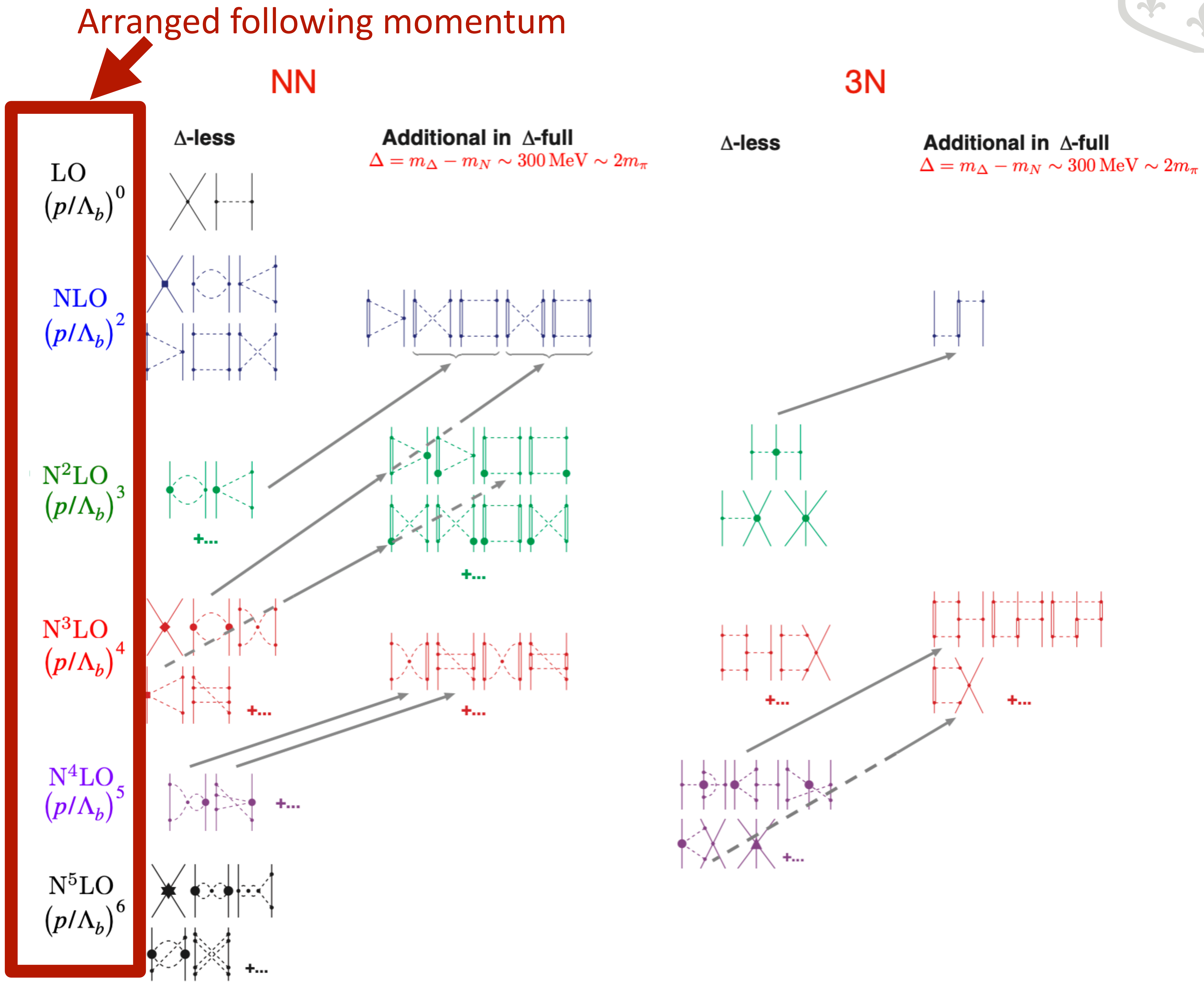

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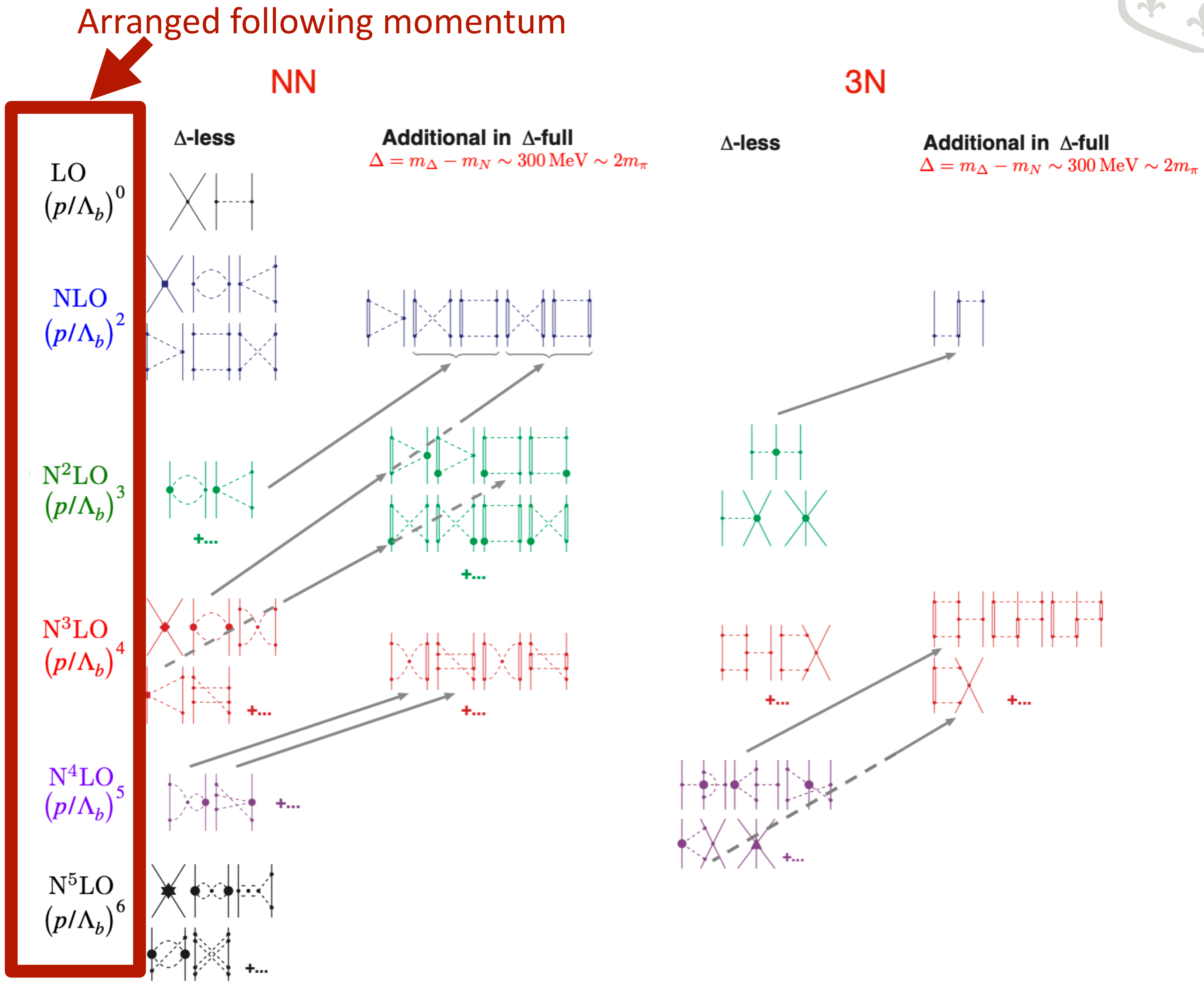

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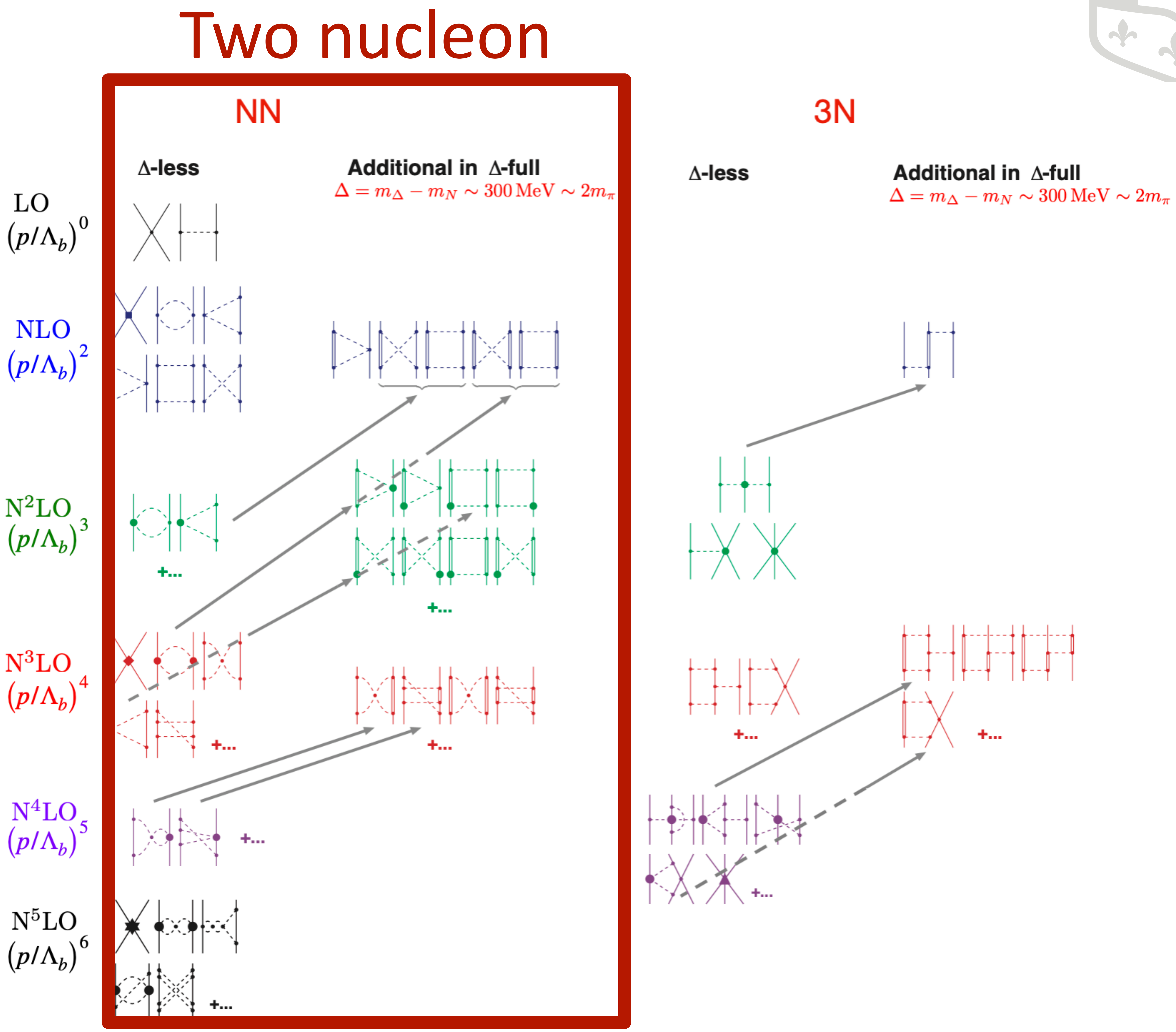

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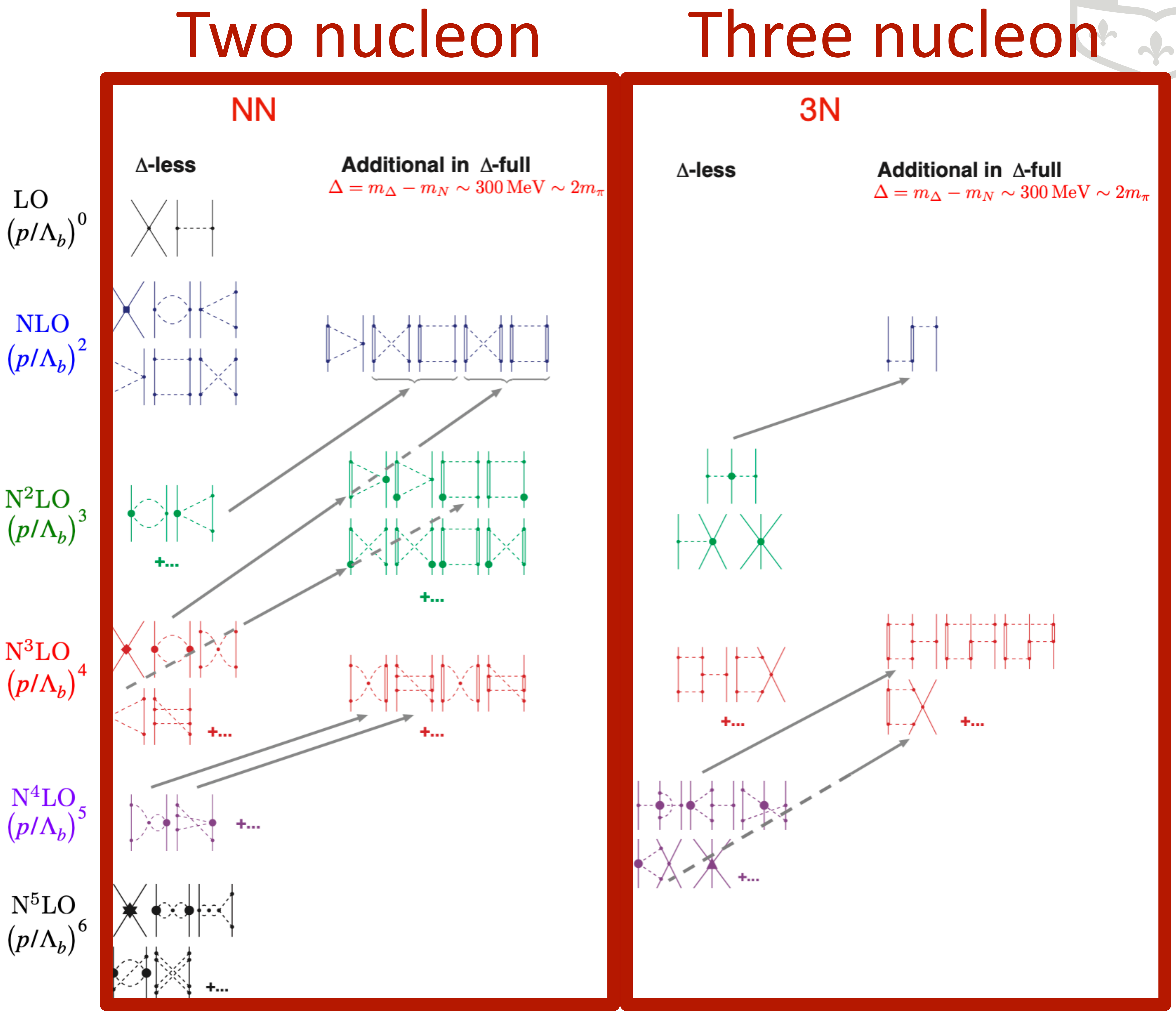

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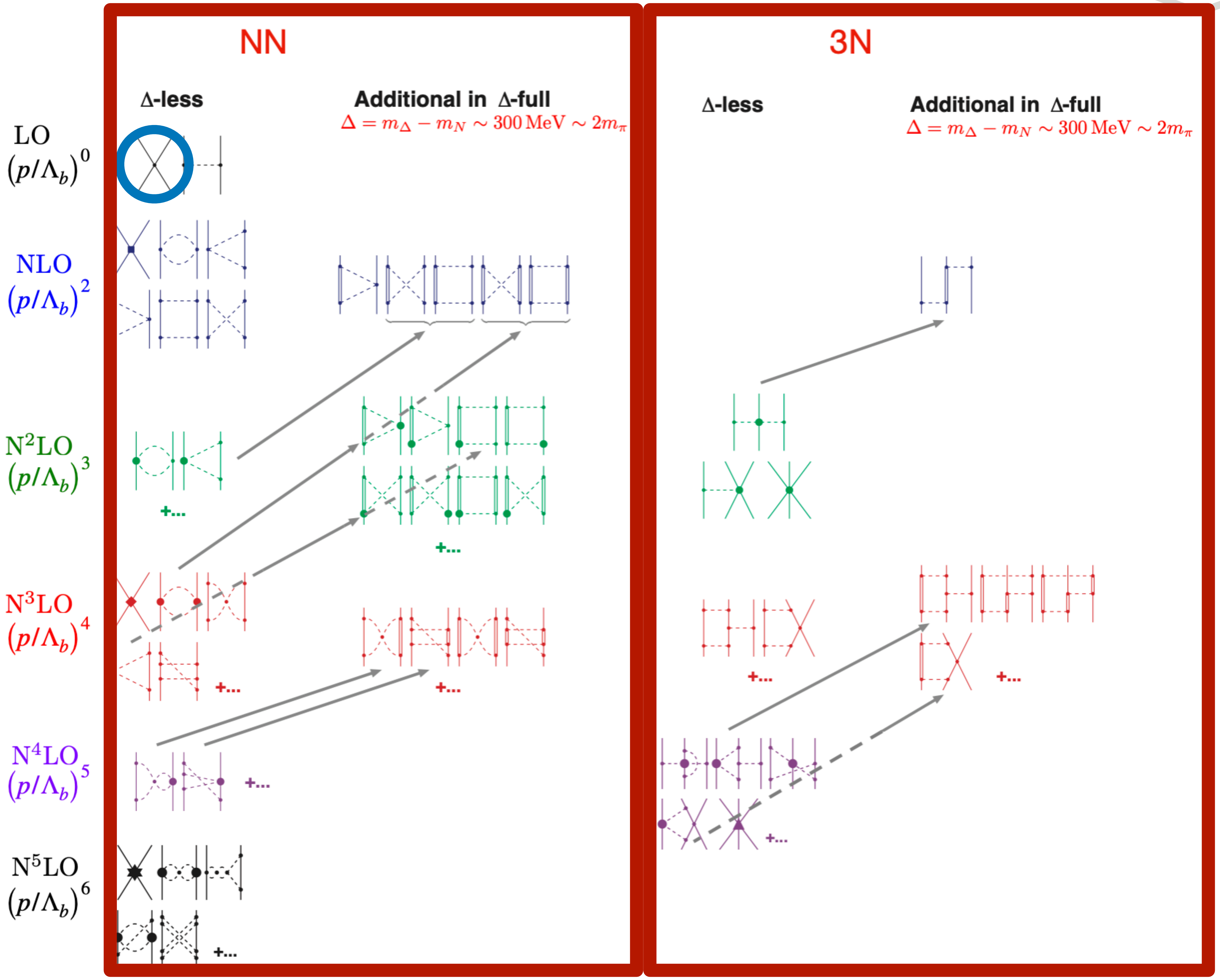

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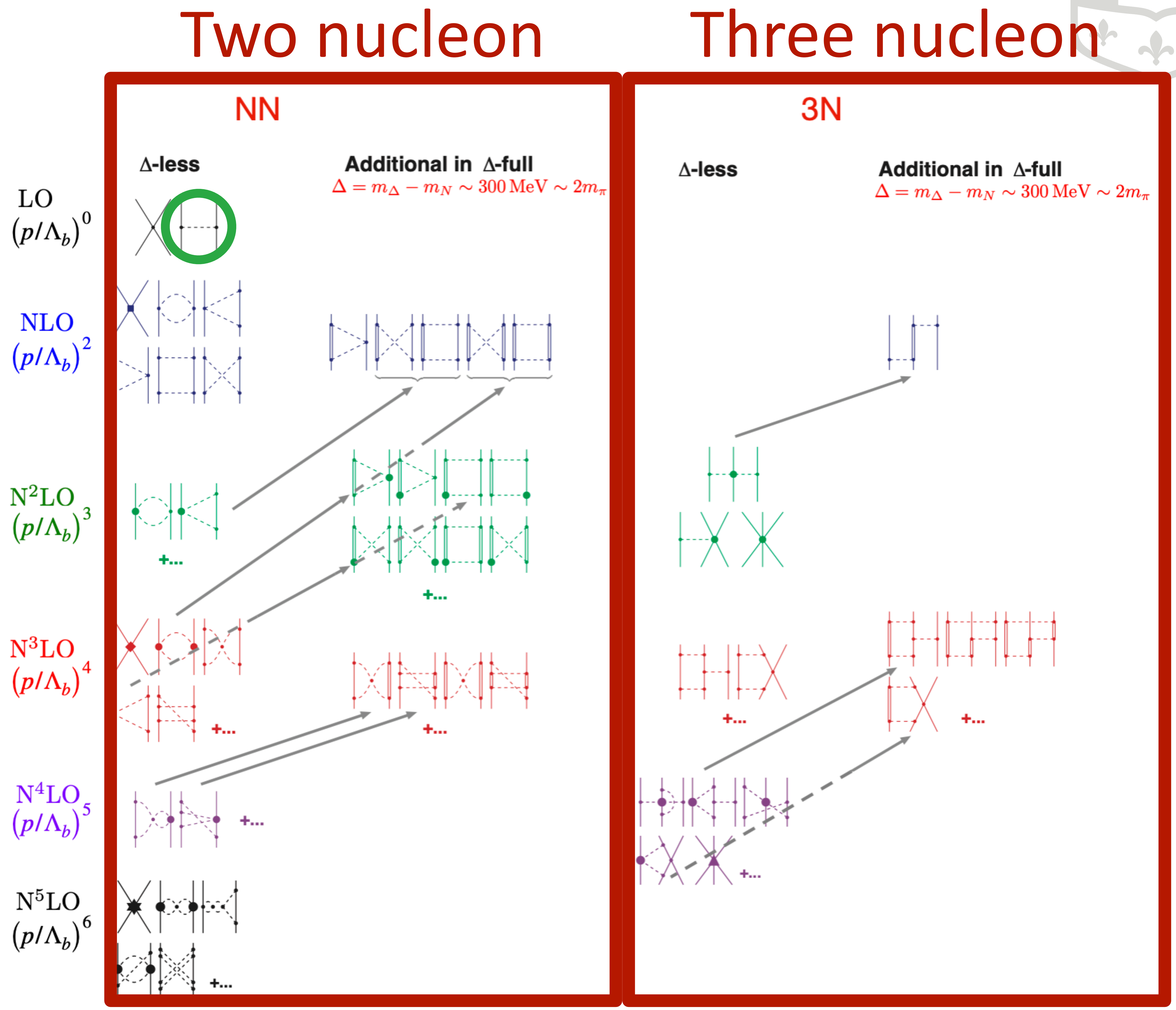

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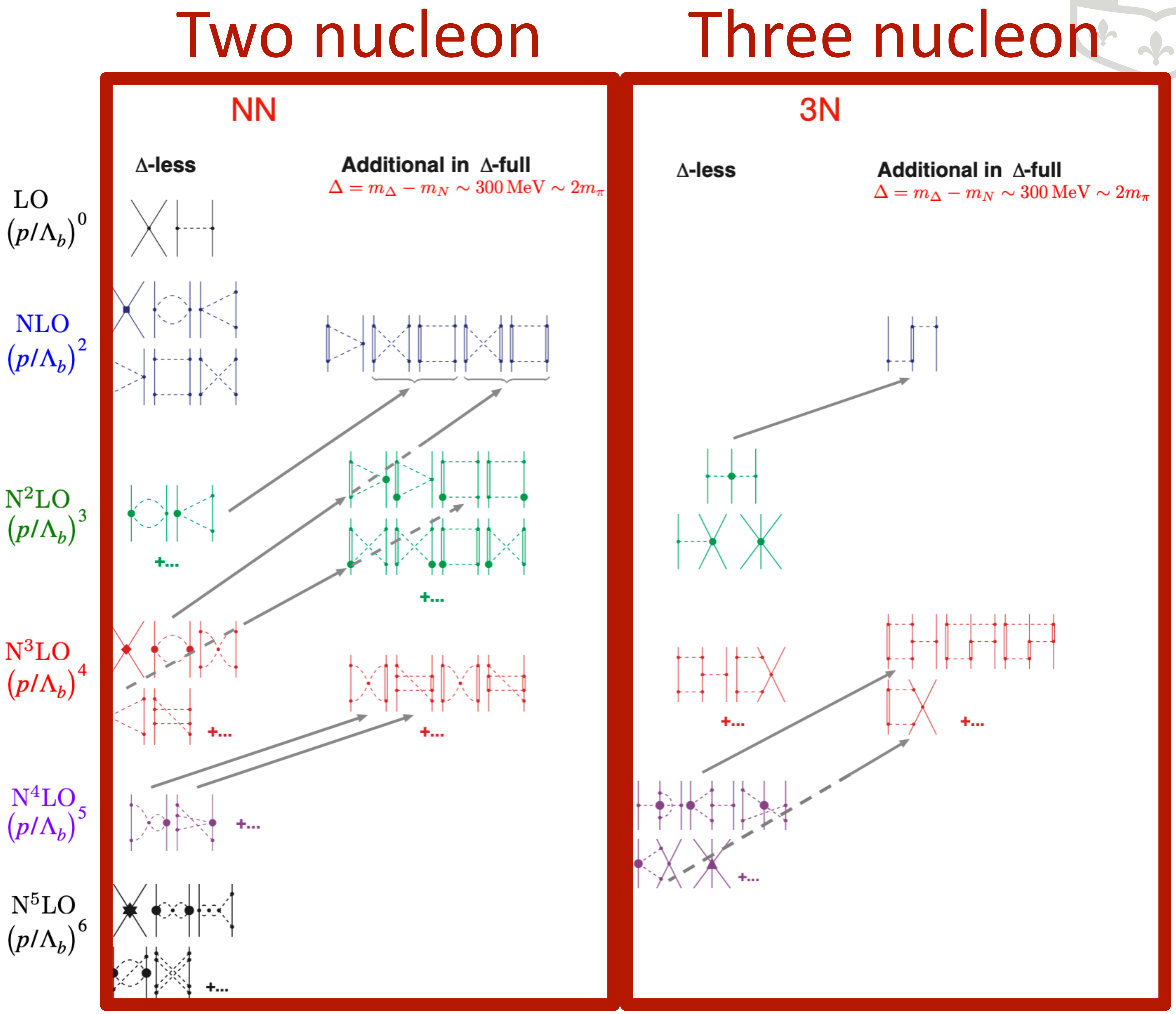
A specific organization is known as a power counting.



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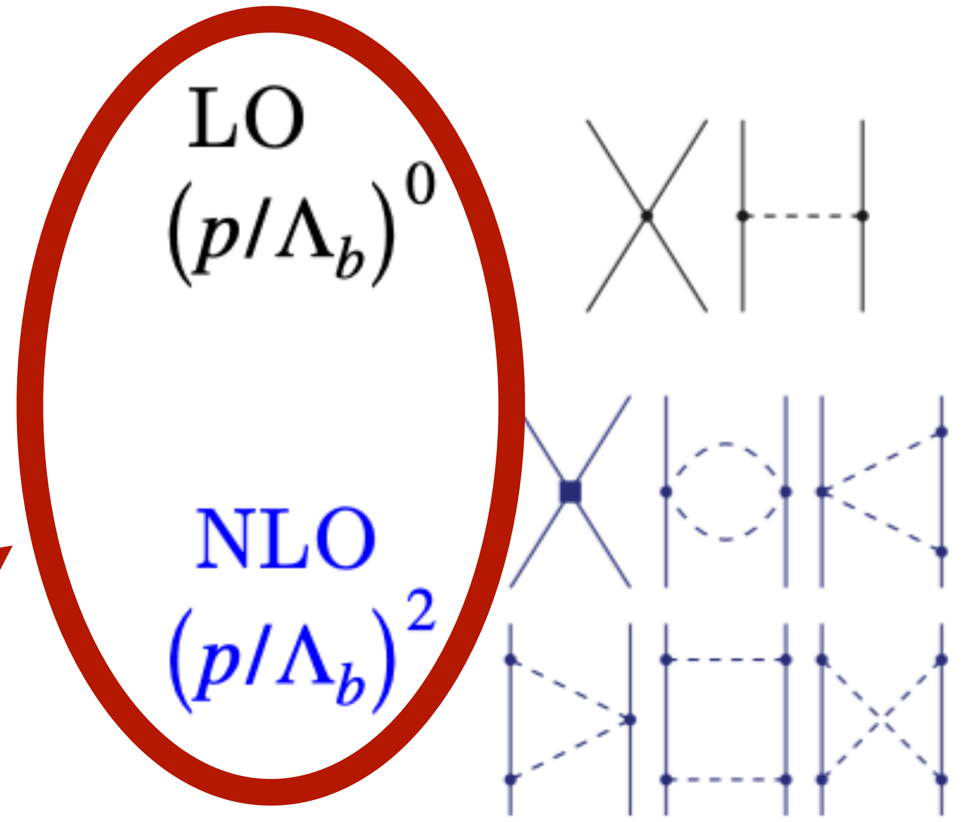

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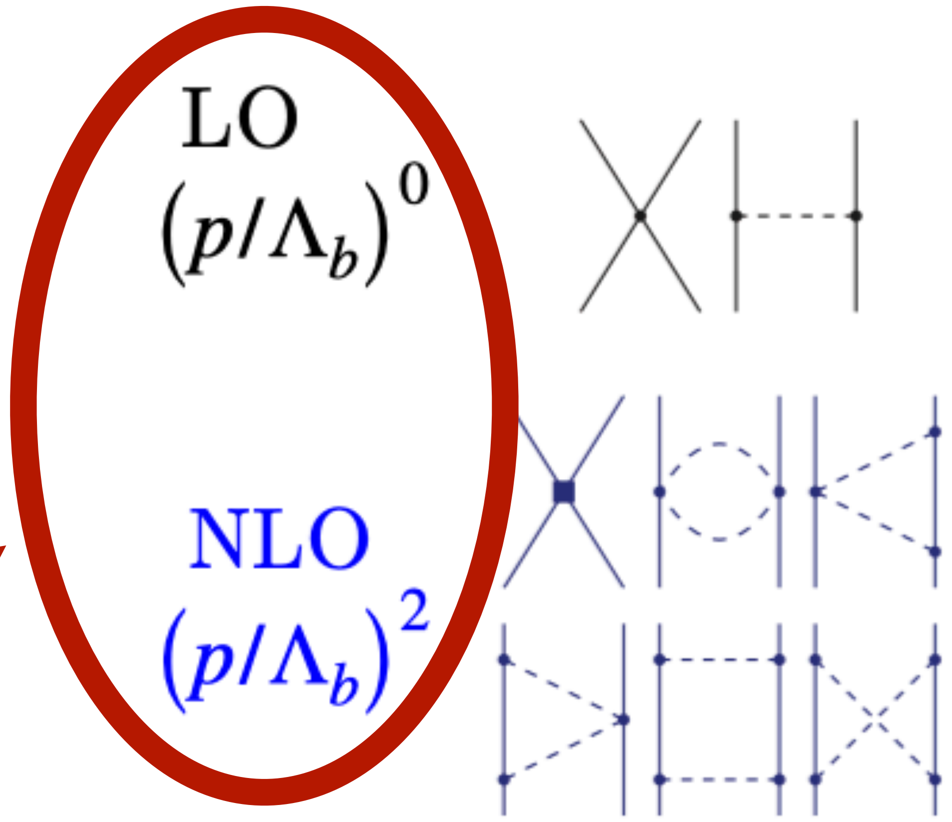

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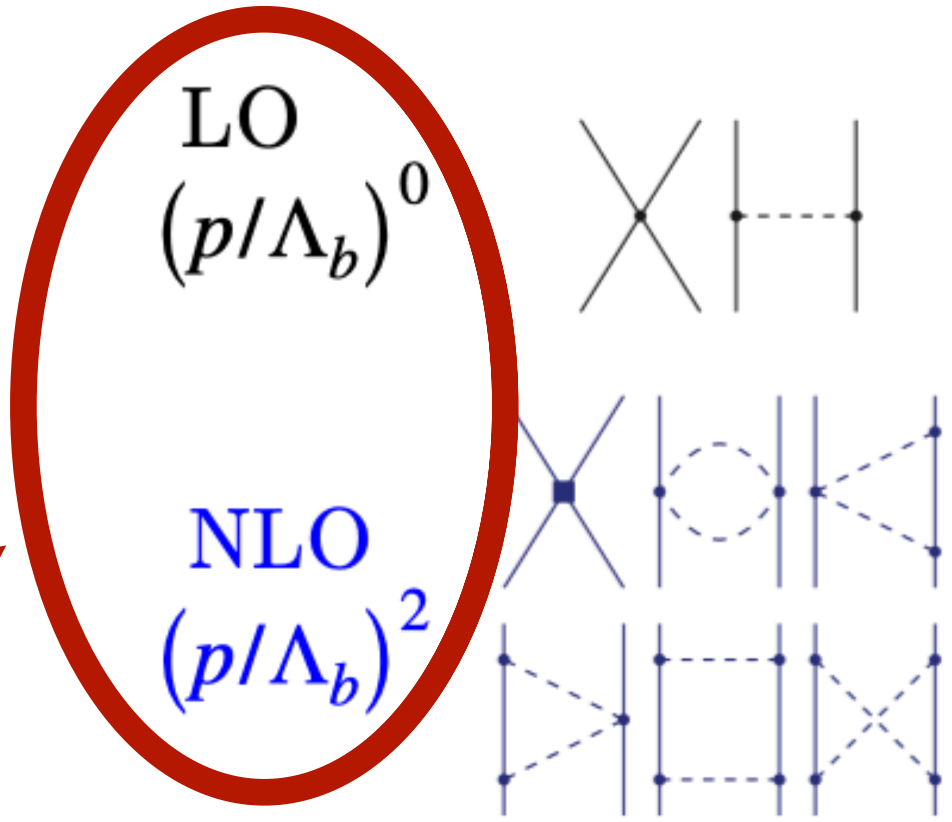

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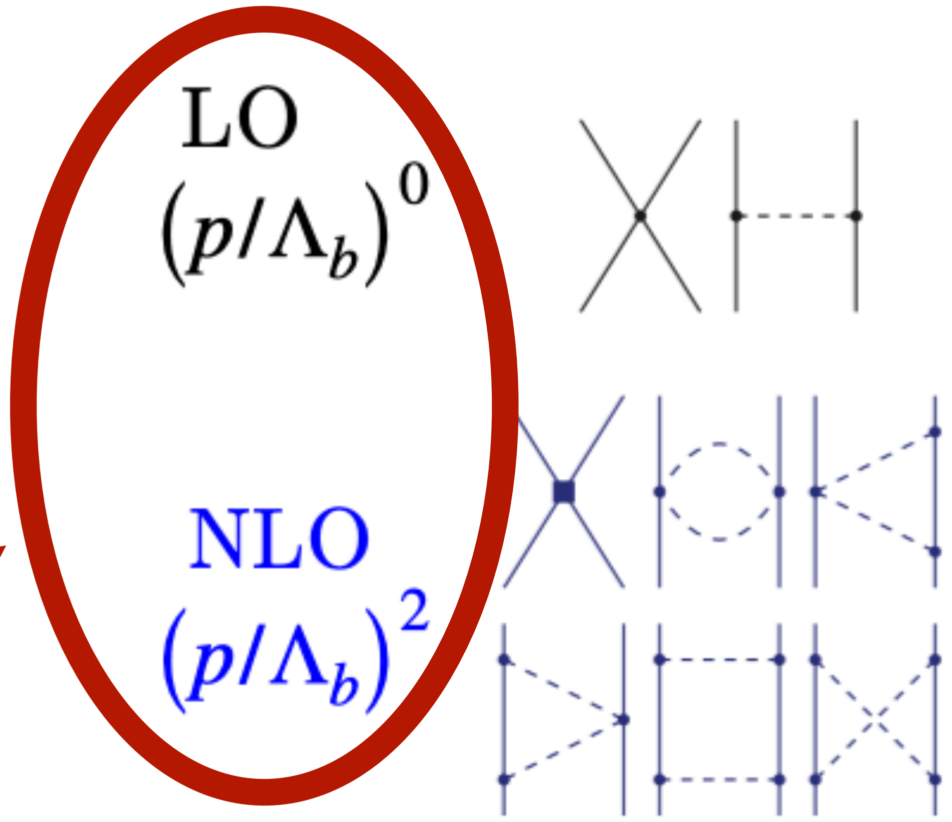

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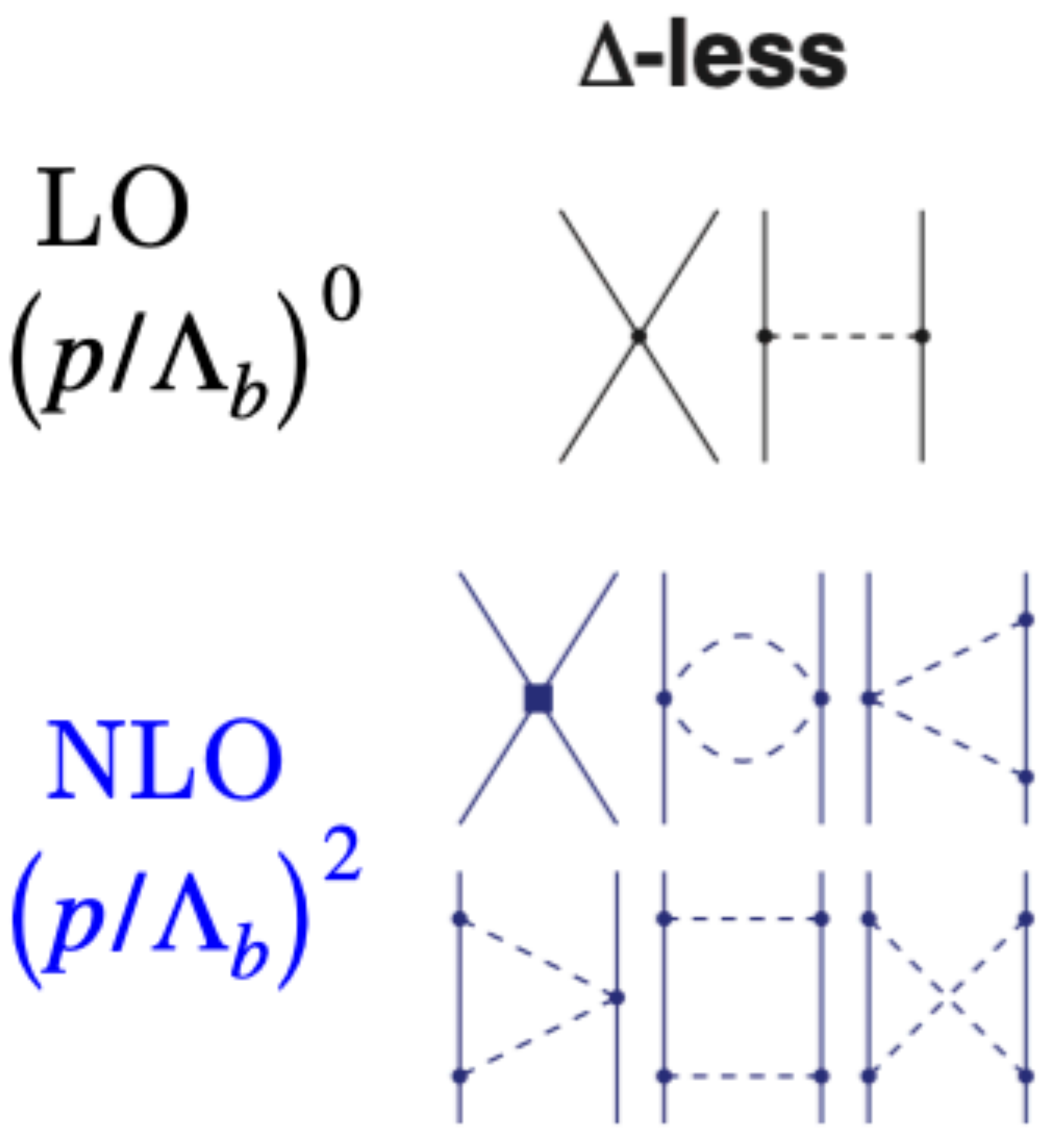

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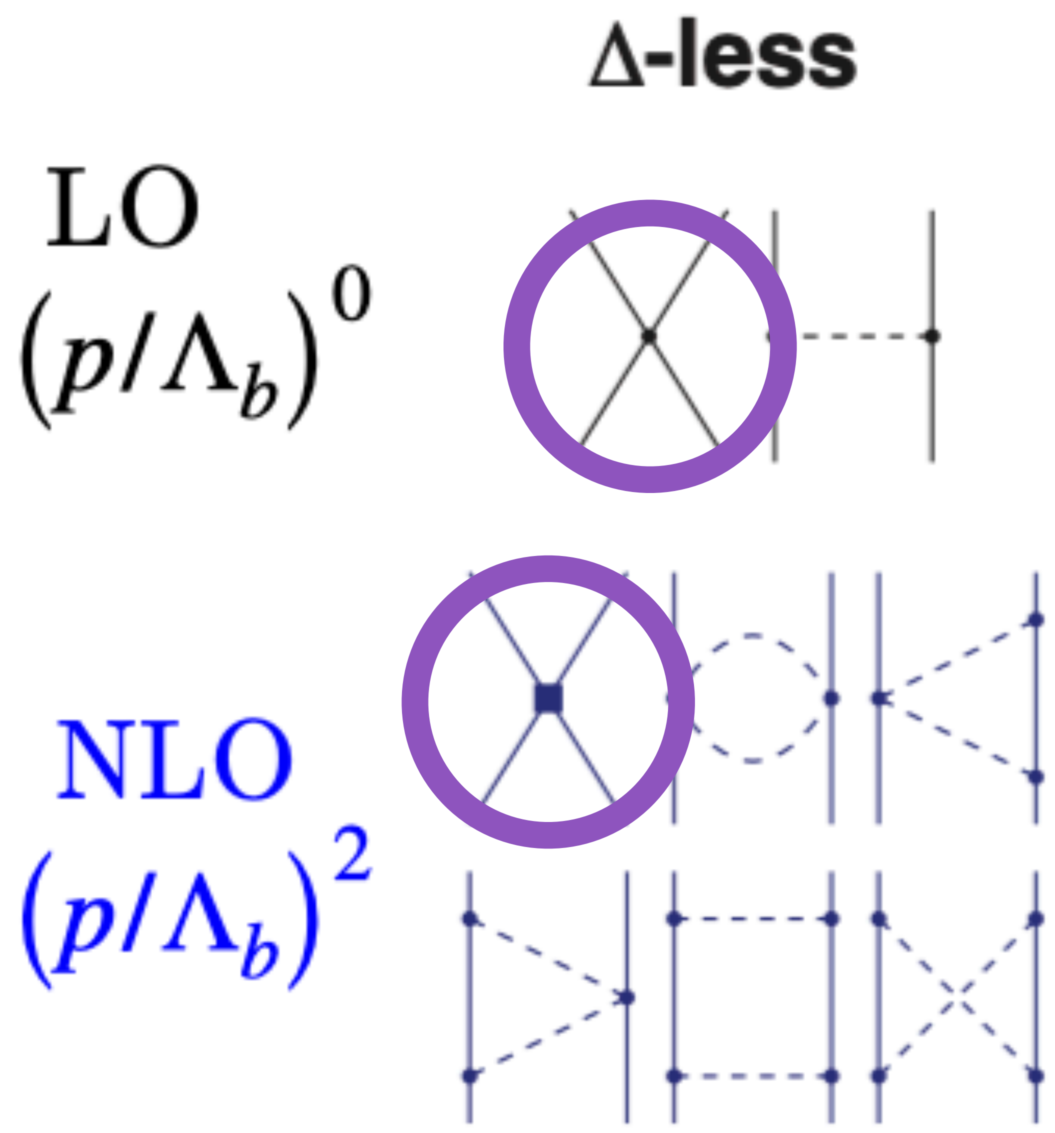

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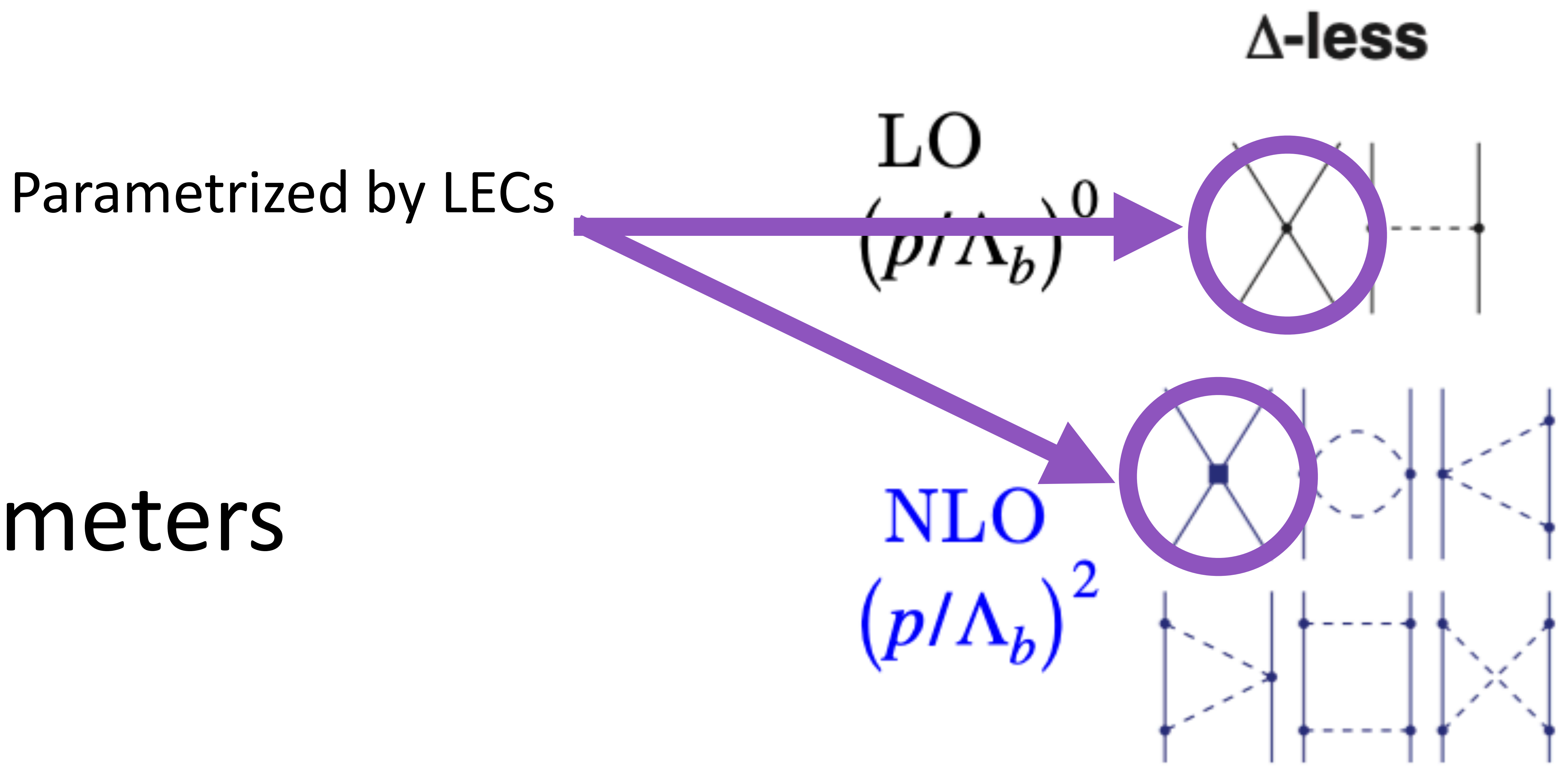

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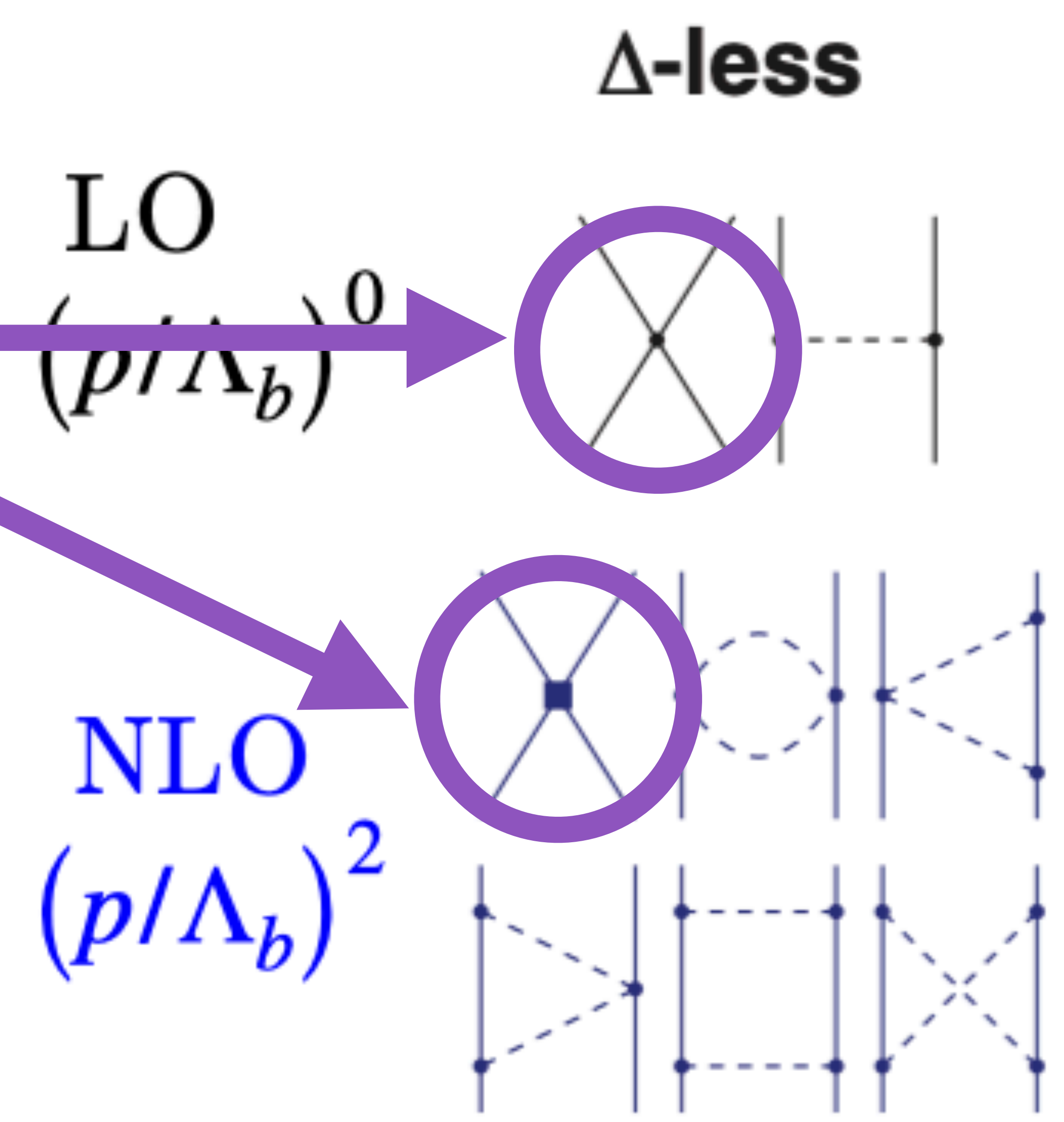

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$$\Sigma_{ij}^{\text{th}} = \mathbb{E}\left[\delta_{\text{th},i}^{(k)} \delta_{\text{th},j}^{(k)}\right] - \mathbb{E}\left[\delta_{\text{th},i}^{(k)}\right] \mathbb{E}\left[\delta_{\text{th},j}^{(k)}\right] = \frac{\left(y_{\text{ref},i} \bar{c} Q_i^{k+1}\right)\left(y_{\text{ref},j} \bar{c} Q_j^{k+1}\right)}{1 - Q_i Q_j}$$

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Priority to understand scale of physical processes

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and "scale" of the χ^{-2} distribution

$$\tau^2 = \frac{1}{\nu}\left(\nu_0 \tau_0 + \sum_{n,i} c_{n,i}^2\right)$$

Outline

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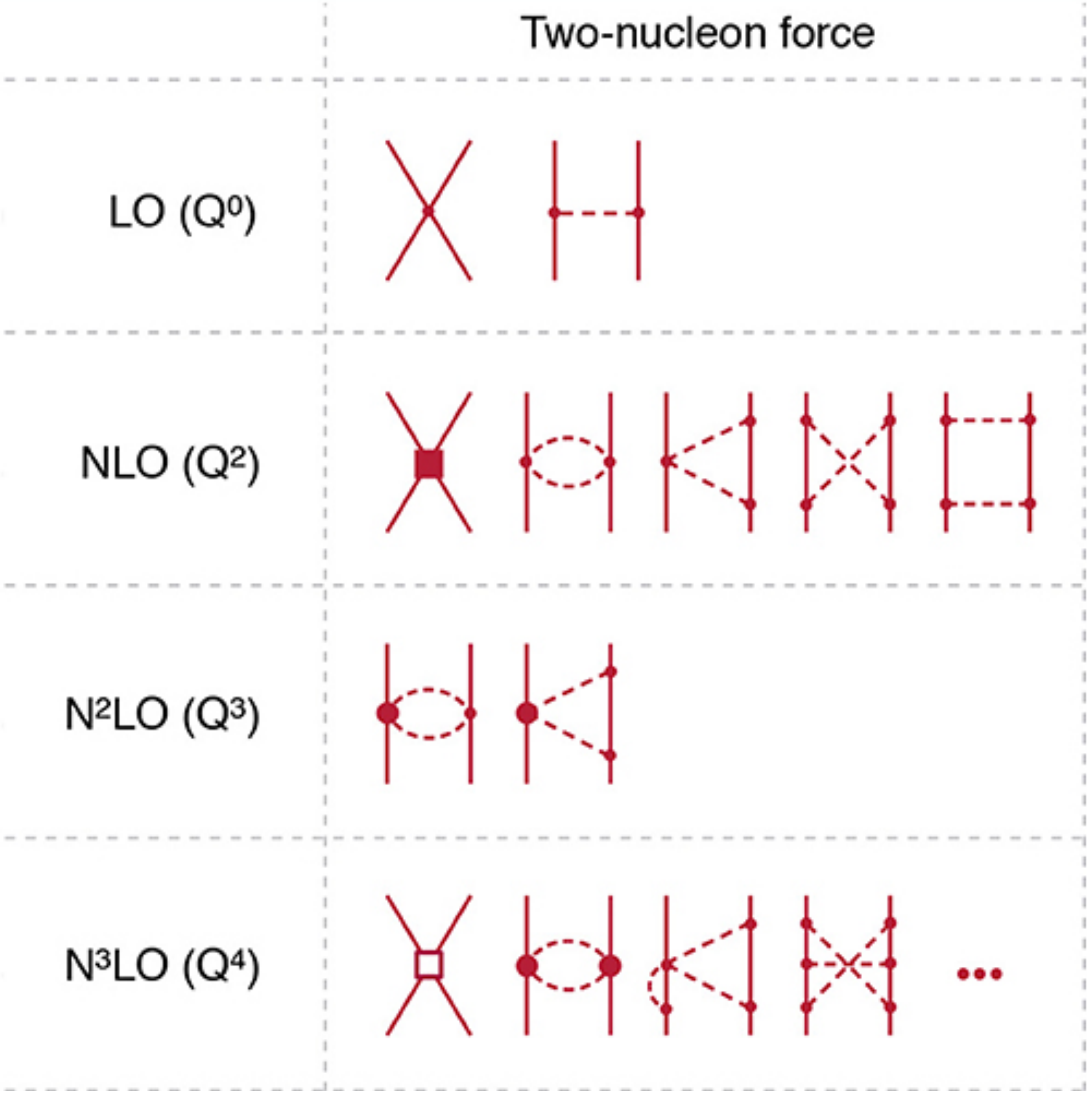

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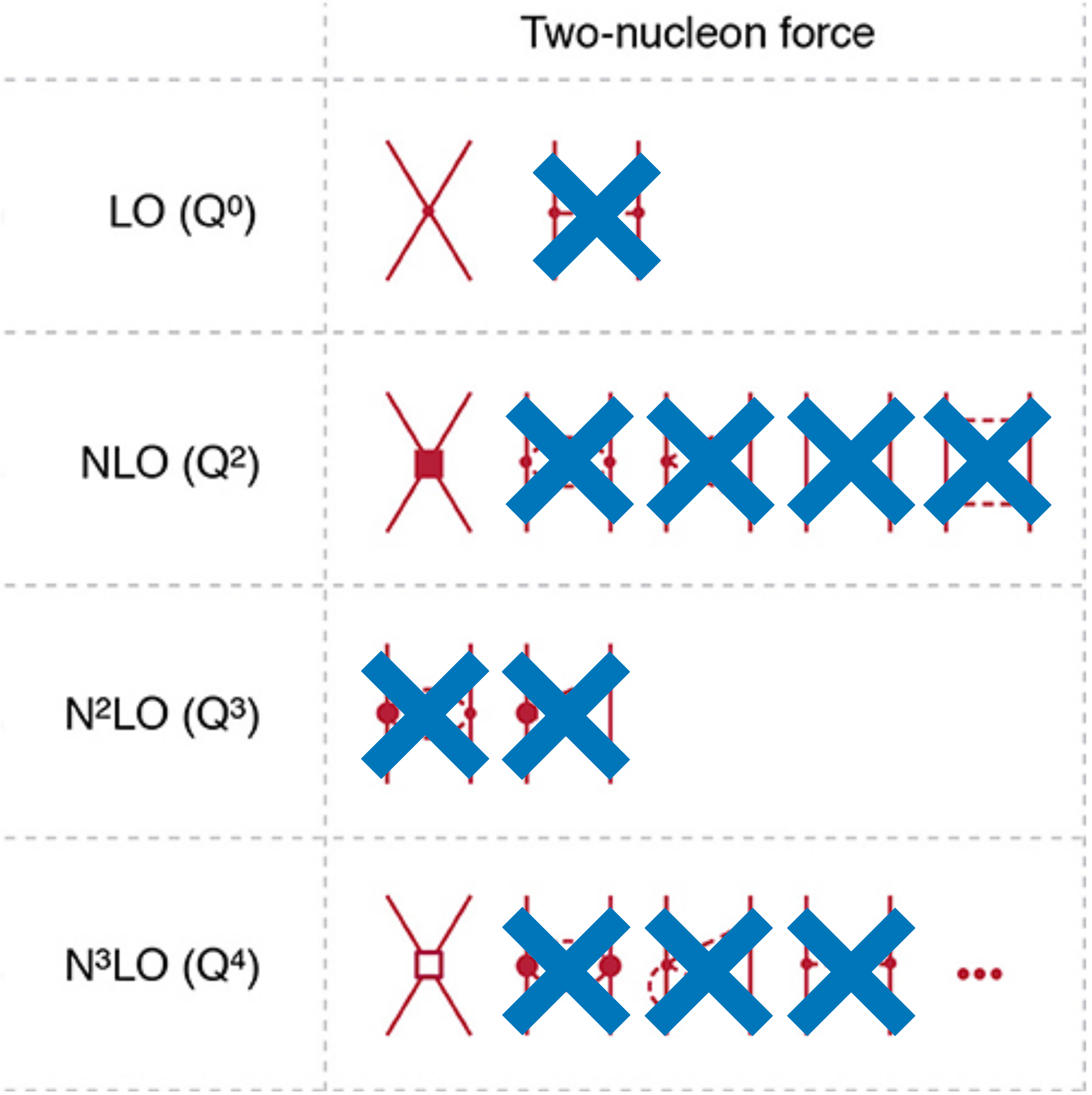

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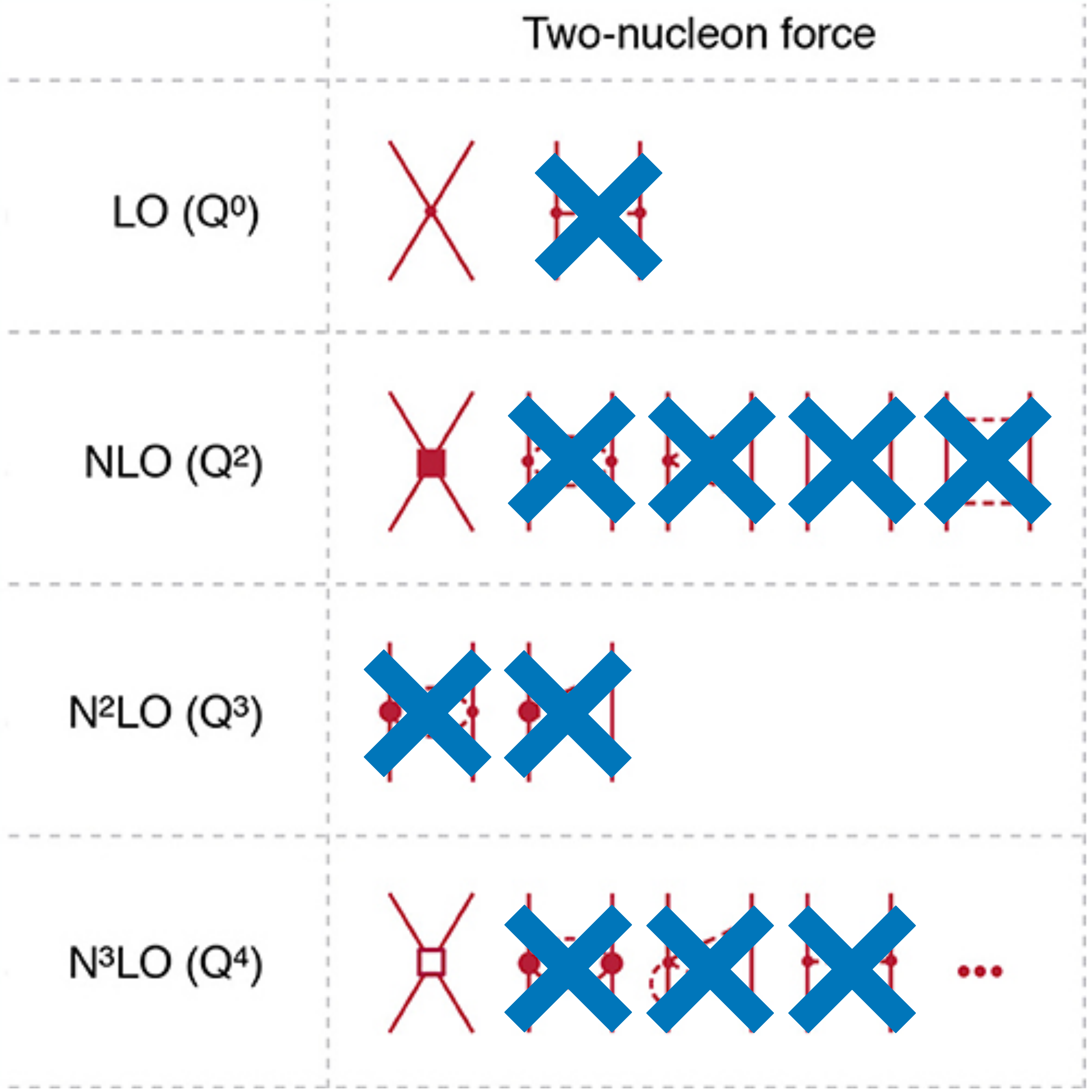

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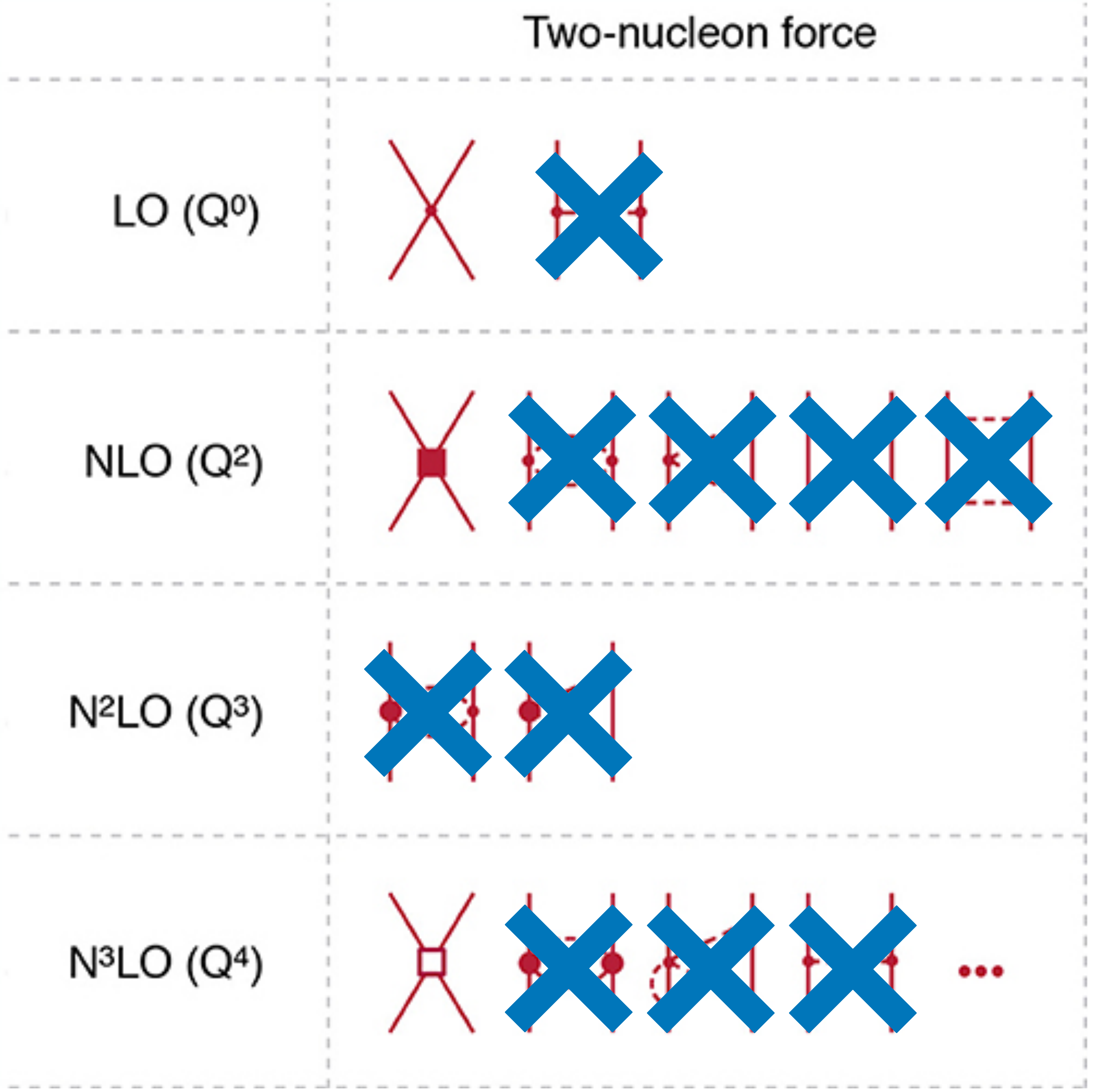

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We choose $R_s \in [1.5, 2.0, 2.5]$ fm which are ~$\frac{400}{R_s}$ MeV in momentum space.

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Our choice of data is the pp and np Granada database (251 differential cross sections, 133 total cross sections, 4 polarized cross sections) up to 5 MeV + deuteron binding energy + nn scattering length.

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We then use Markov Chain Monte Carlo (MCMC) to sample the posteriors at LO $\left(Q^0\right)$, NLO $\left(Q^2\right)$, and N3LO $\left(Q^4\right)$, allowing for the order-by-order convergence analysis for LO→NLO and NLO→N3LO to estimate $\bar{c}$ and Λ_b.

Prior Choices

- $\mathrm{pr}(\vec{\mathrm{a}}\,|\,\mathrm{I}) \sim \mathcal{N}\left(\vec{\mathrm{a}}^{\mathrm{MAP}}_{\mathrm{p.s.}}, \overrightarrow{10^2}\right)$
- $\mathrm{pr}(\Lambda_{\mathrm{b}}\,|\,\mathrm{I}) \sim \mathcal{N}\left(500\ \text{MeV}, 1000^2\ \text{MeV}^2\right)$
- $\mathrm{pr}(\bar{\mathrm{c}}^2\,|\,\mathrm{I}) \sim \chi^{-2}(\nu_0 = 1.5, \tau_0^2 = 1.5^2)$
- $r(x_i, x_j; \vec{l}) = e^{|p_i - p_j|/2l_p} e^{|\theta_i - \theta_j|/2l_\theta} \delta_{\mathrm{type}_i, \mathrm{type}_j}, \quad l_p = 0.3\ \text{MeV}, l_\theta = 20^\circ$
- $p_{\mathrm{soft}} = \begin{cases} p_d \sim 45\ \text{MeV}/c, & \text{for } np \text{ scattering} \\ 1/{}^1a_{\mathrm{pp}} \sim 25\ \text{MeV}, & \text{for } pp \text{ scattering} . \end{cases}$

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Ex: NLO Posterior

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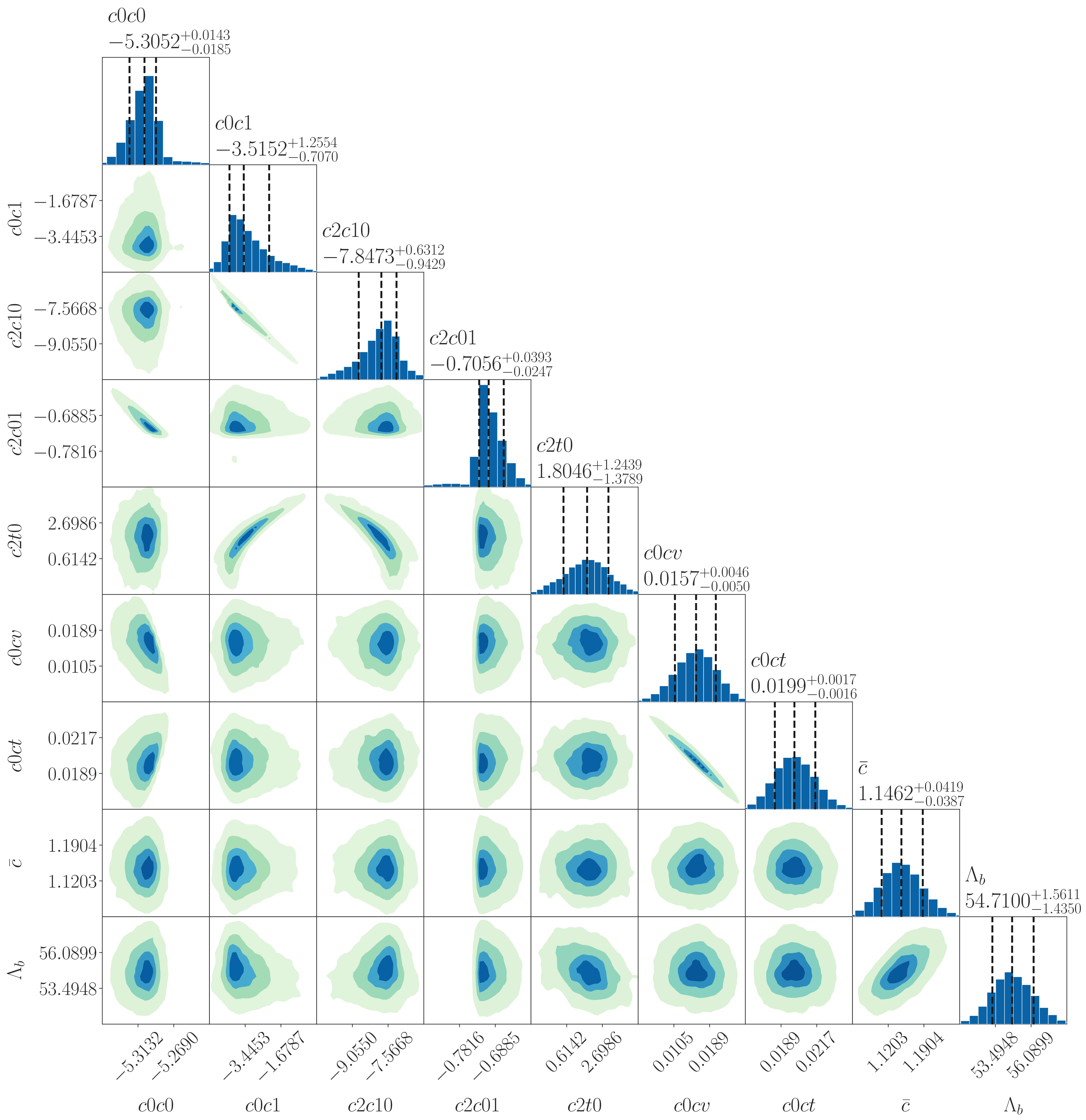

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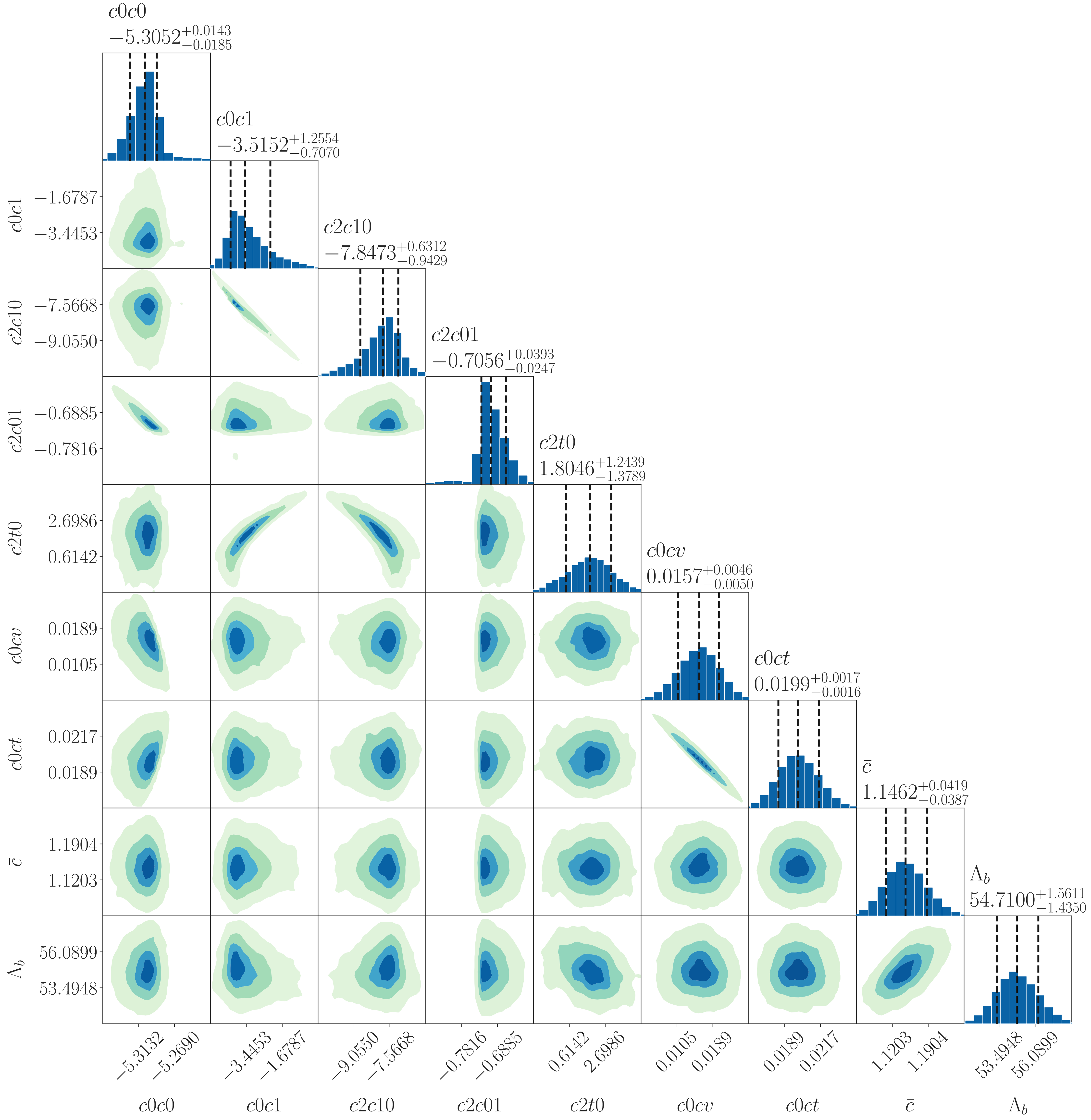

More figures

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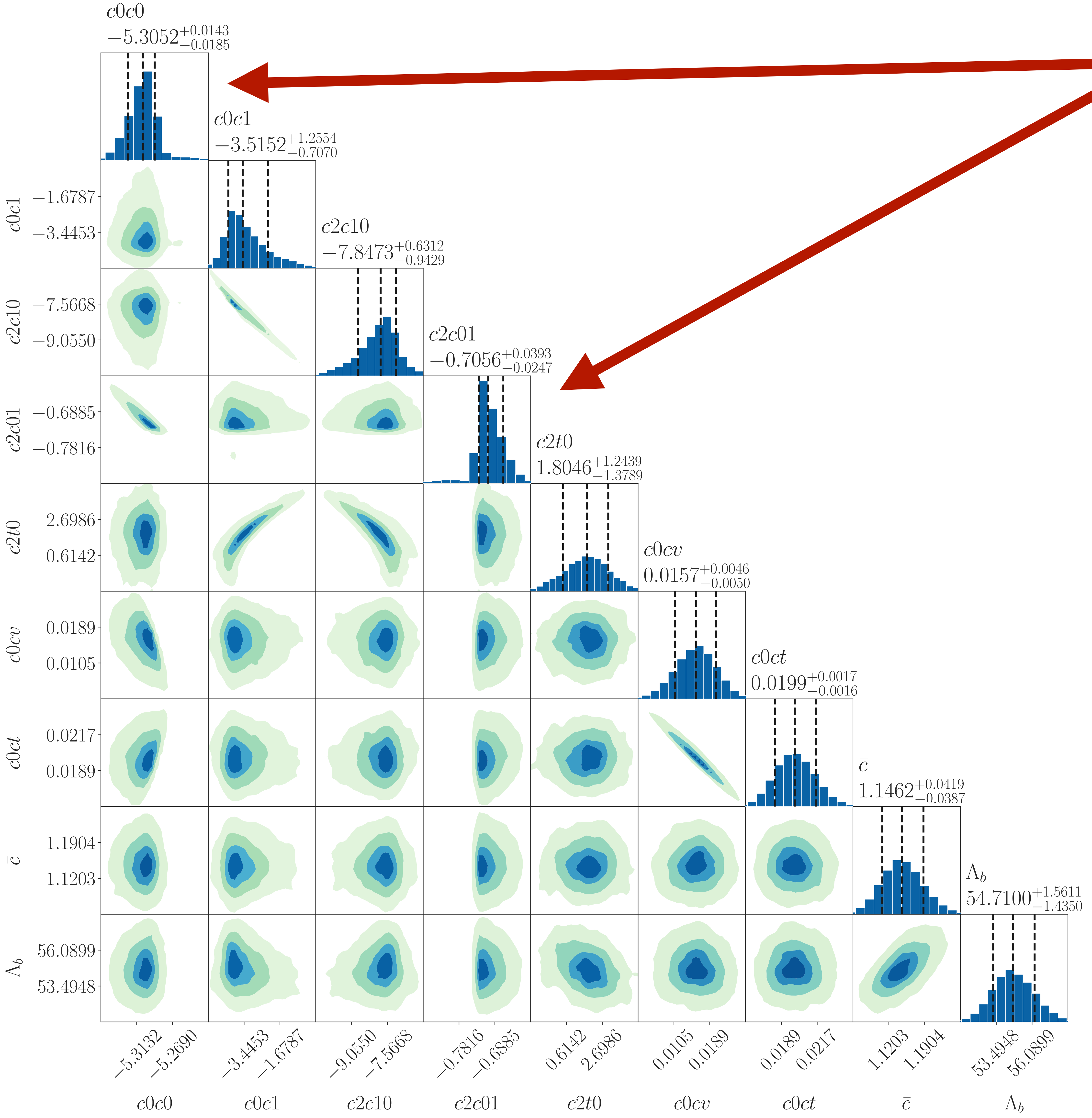

$(S, T) = (0,1)$

More figures

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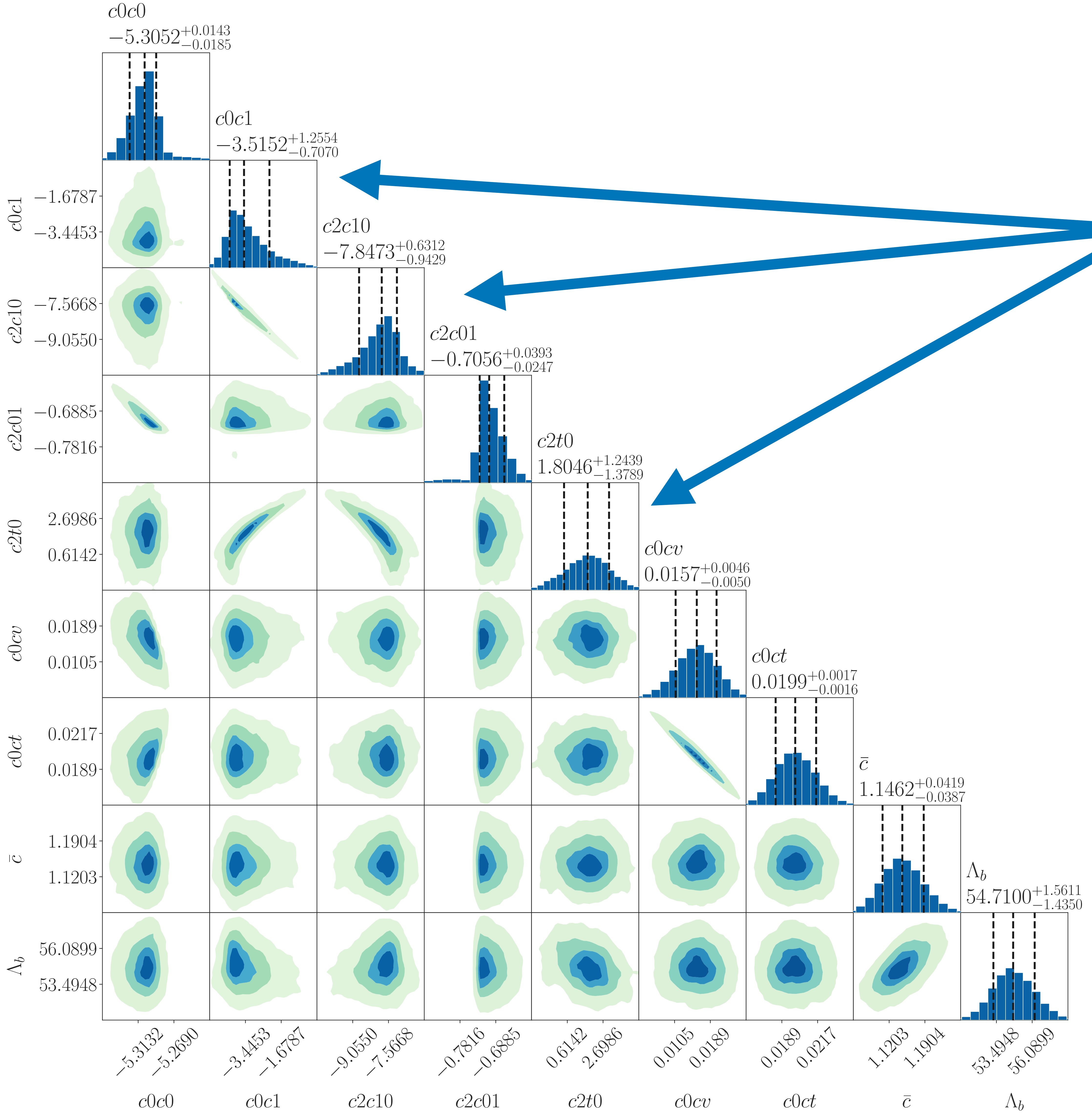

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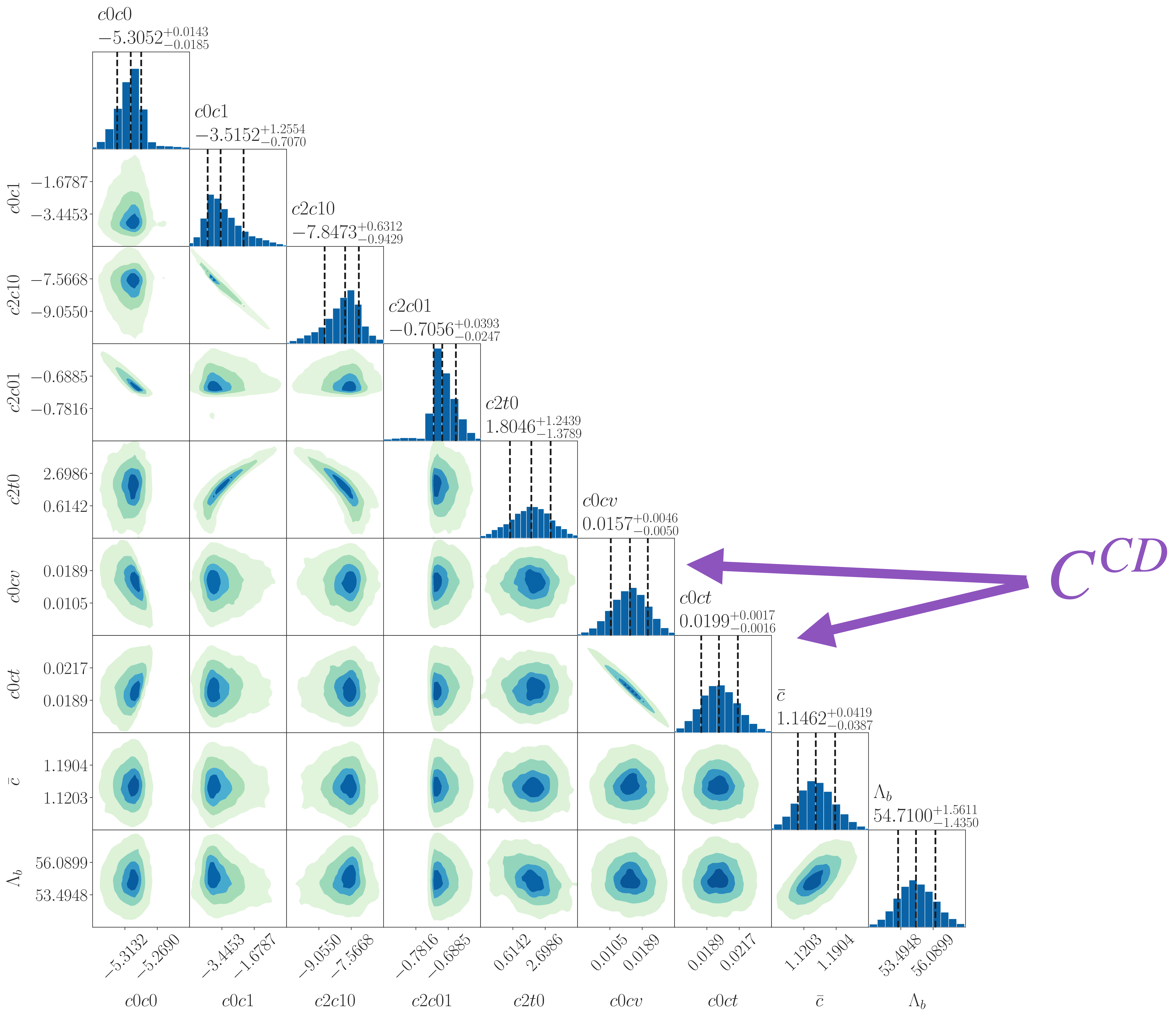
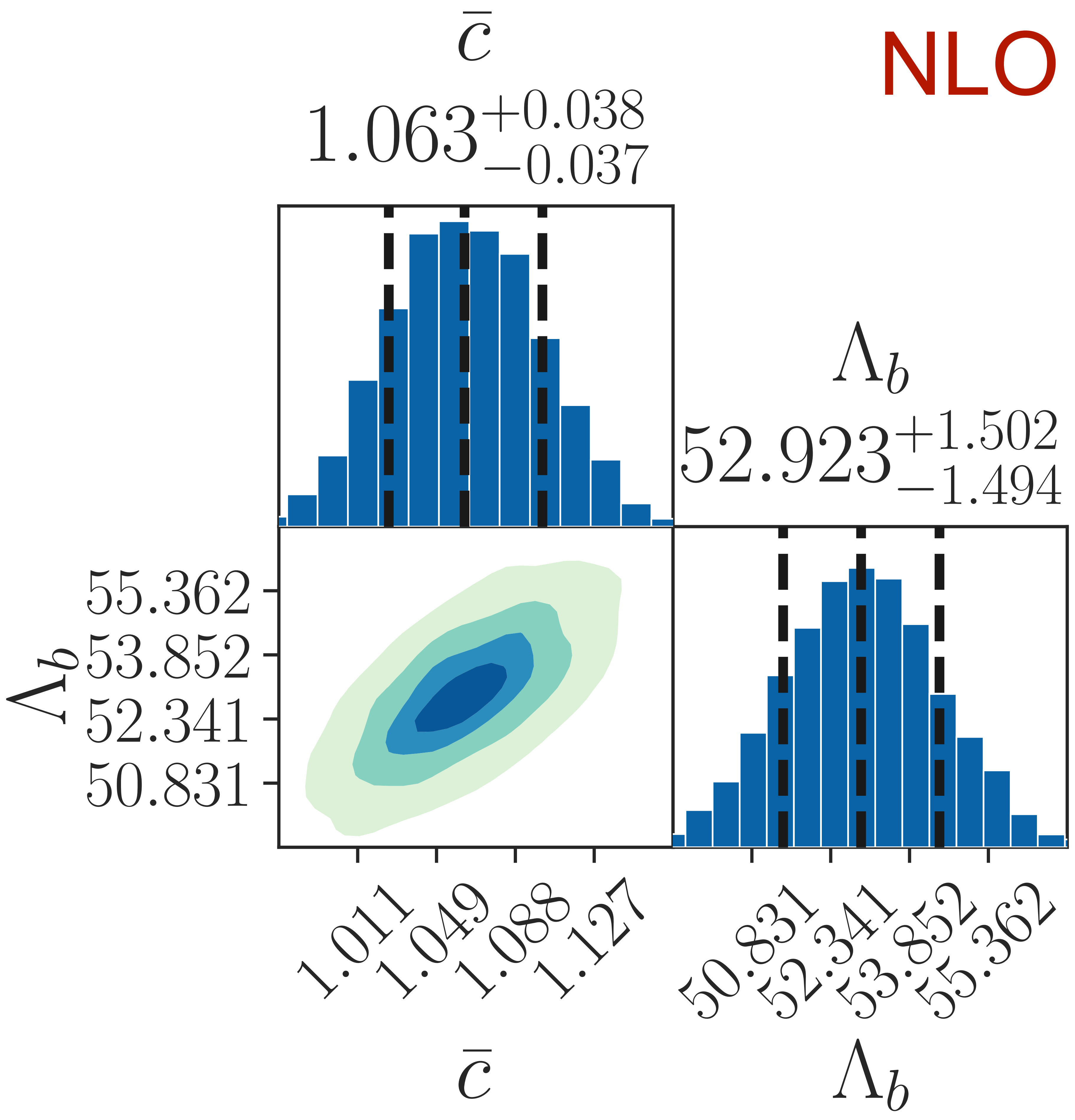
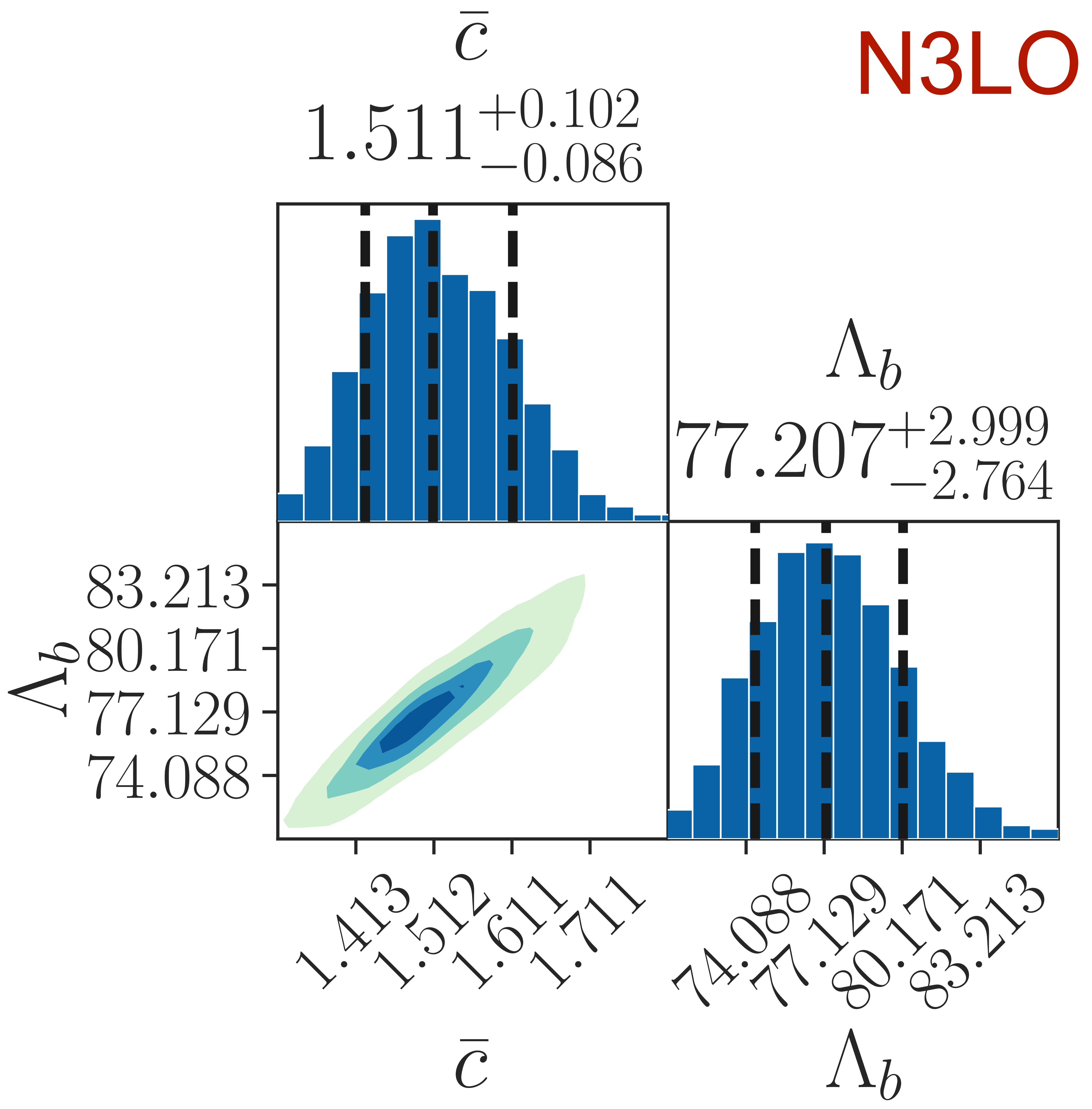


More figures

2.5 fm $\bar{c}$ and Λ_b Posteriors





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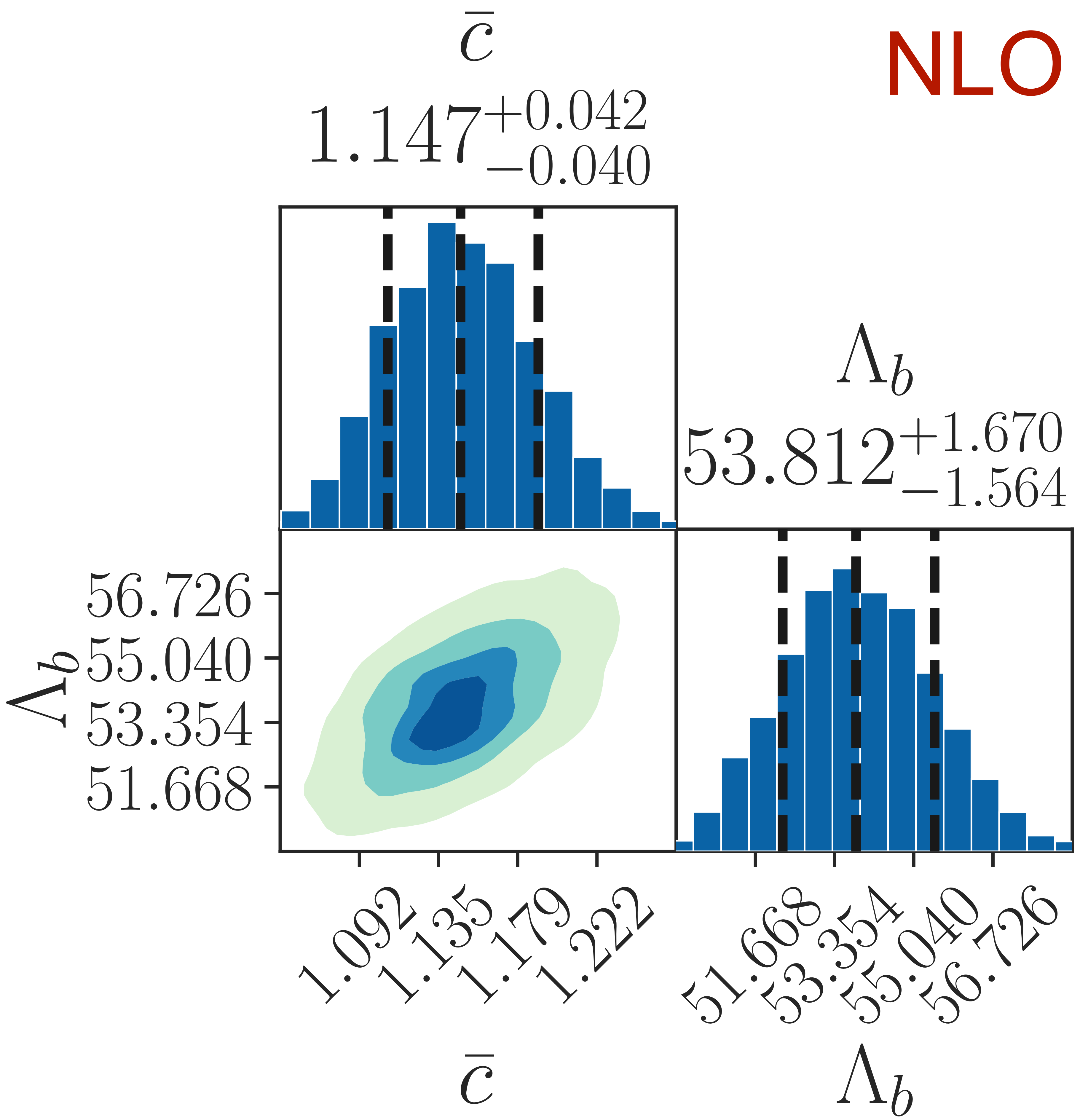

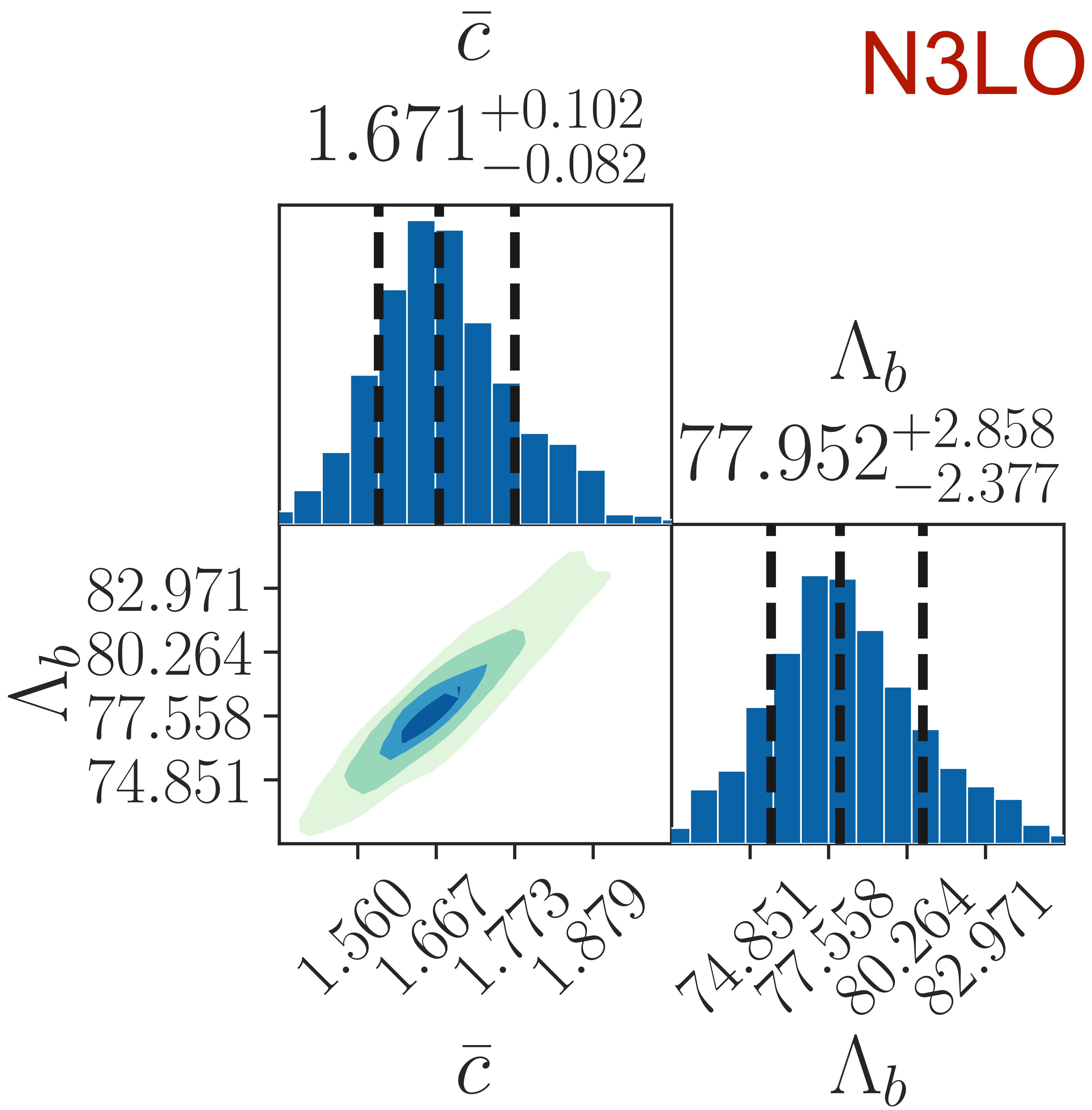

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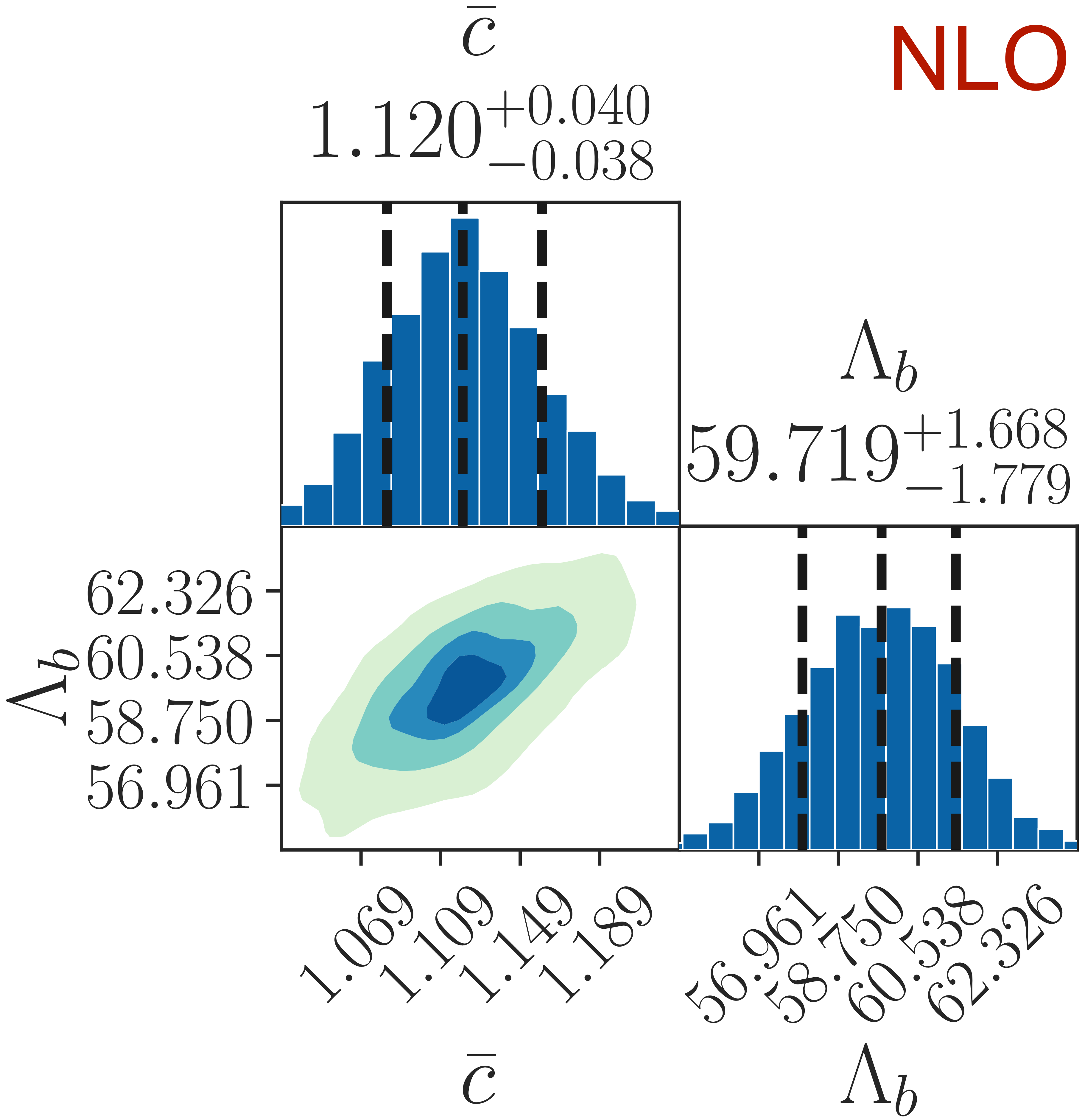
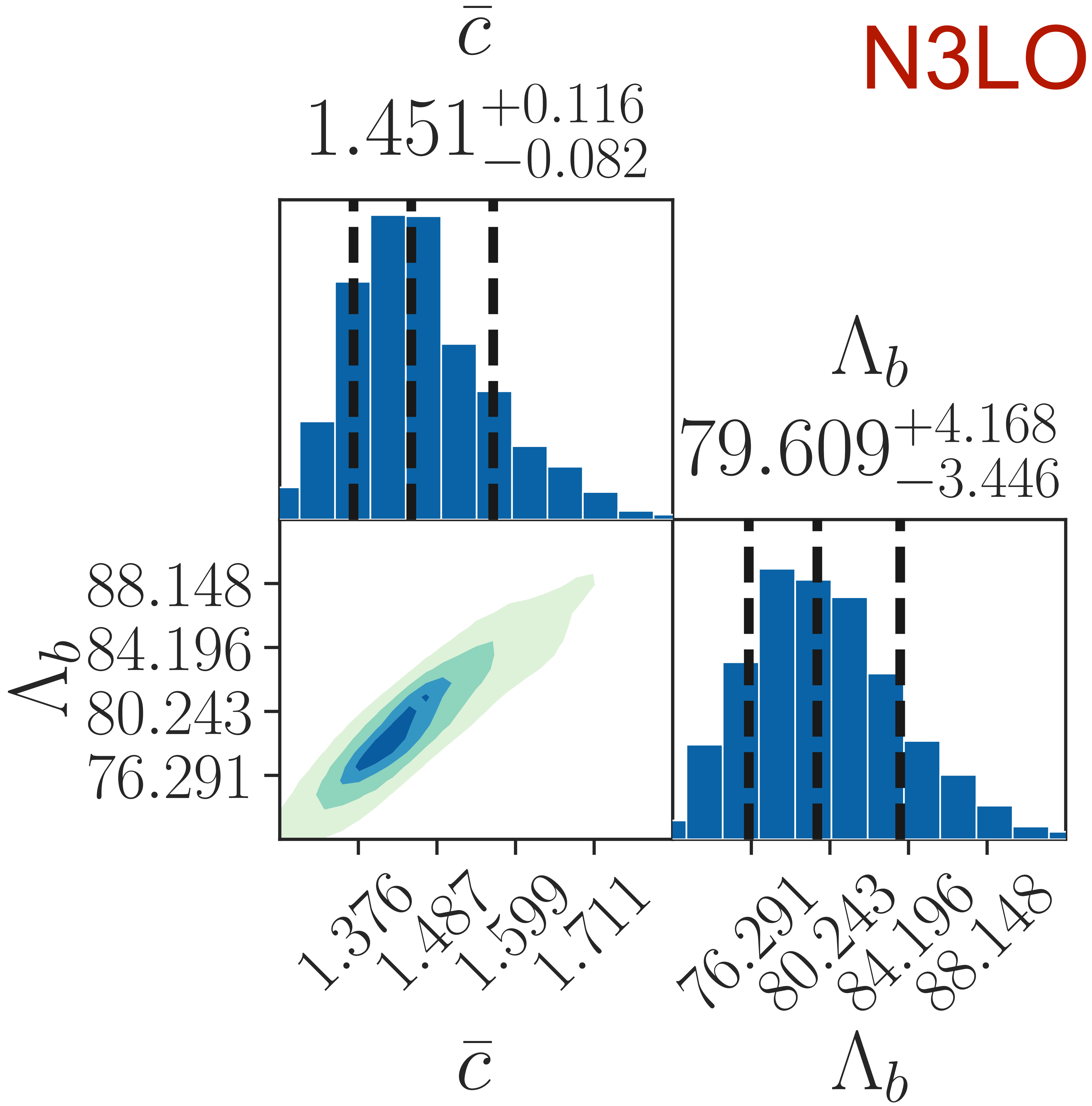
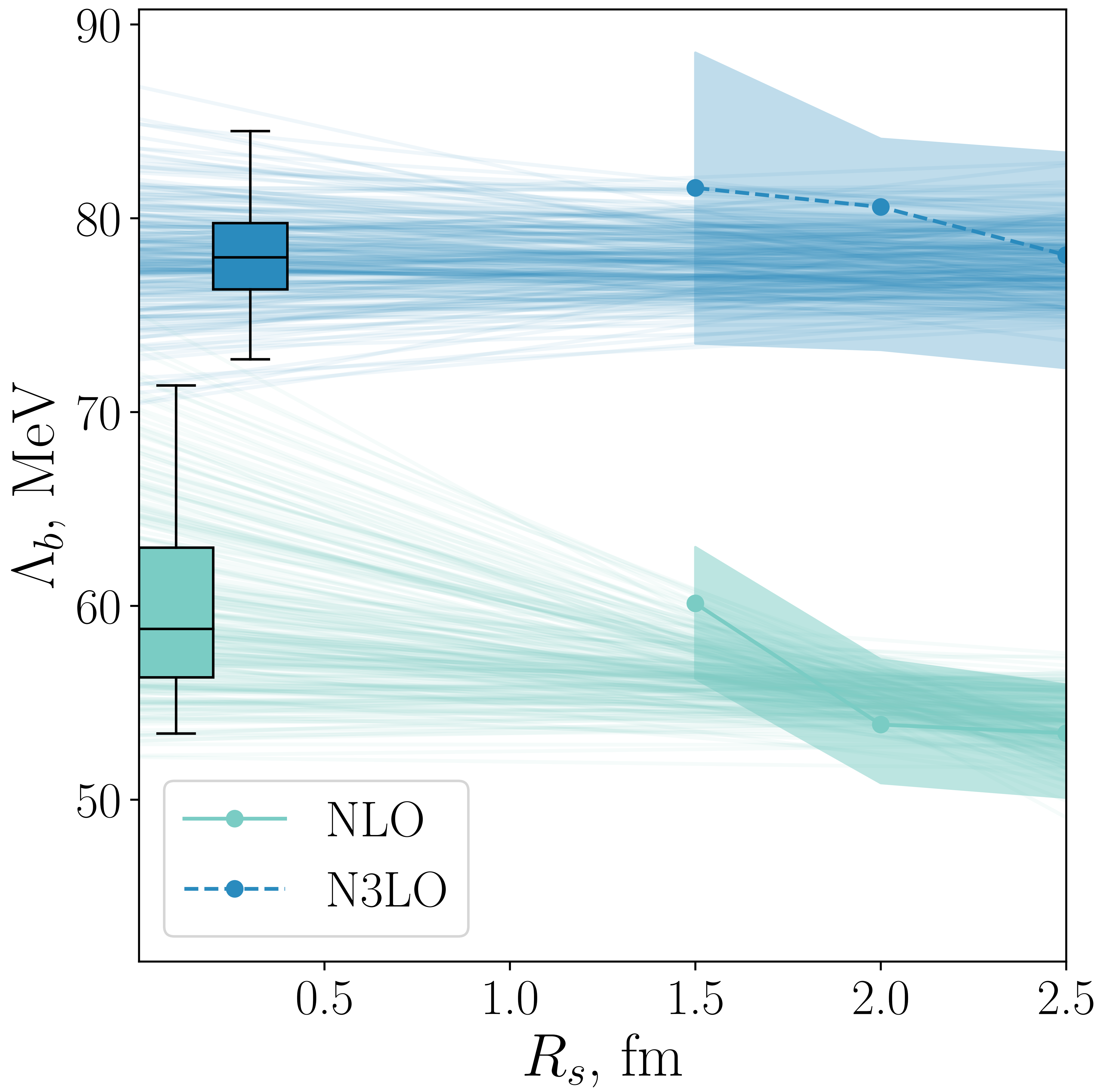




Order Dependence of Λ_b



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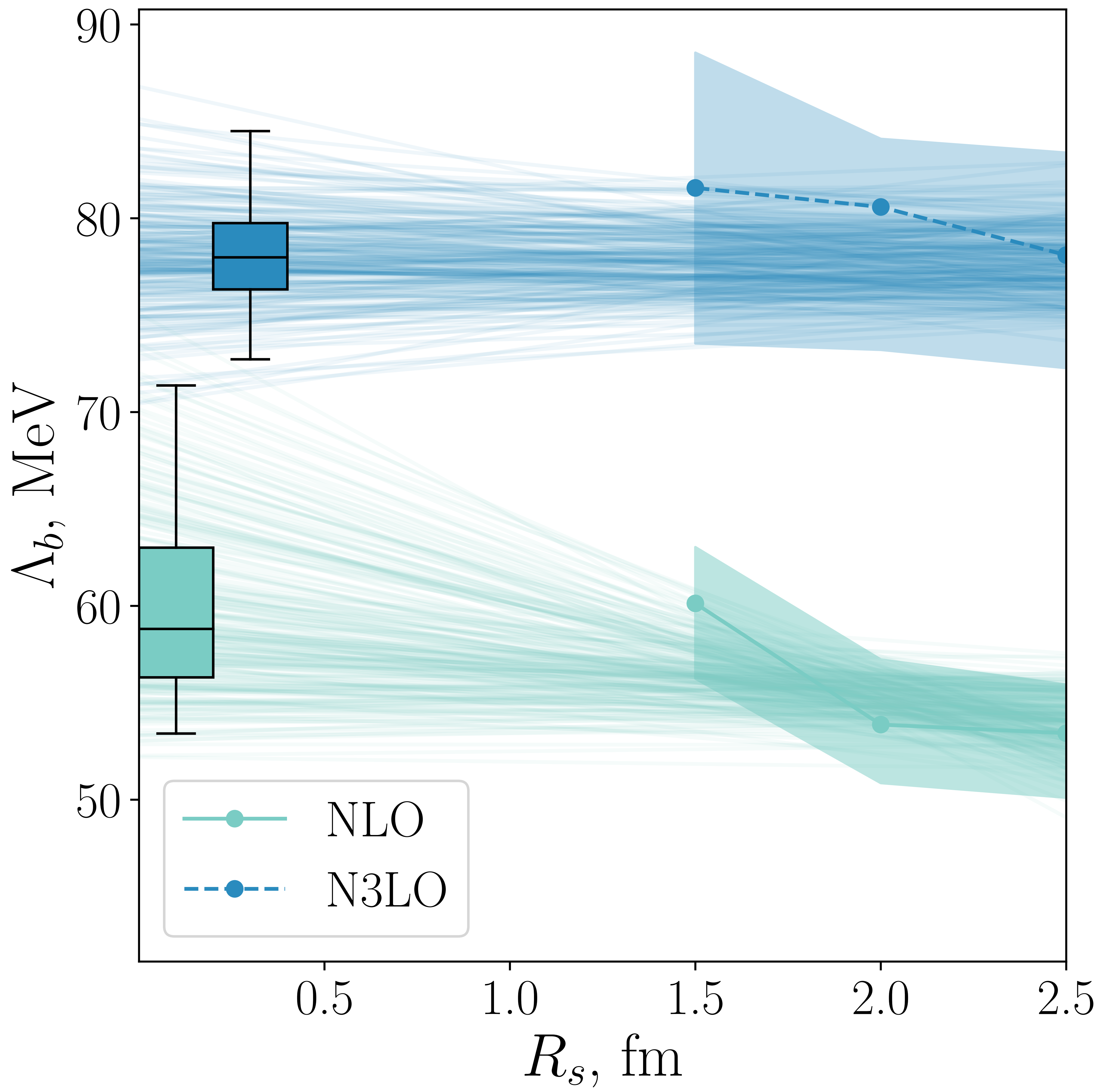

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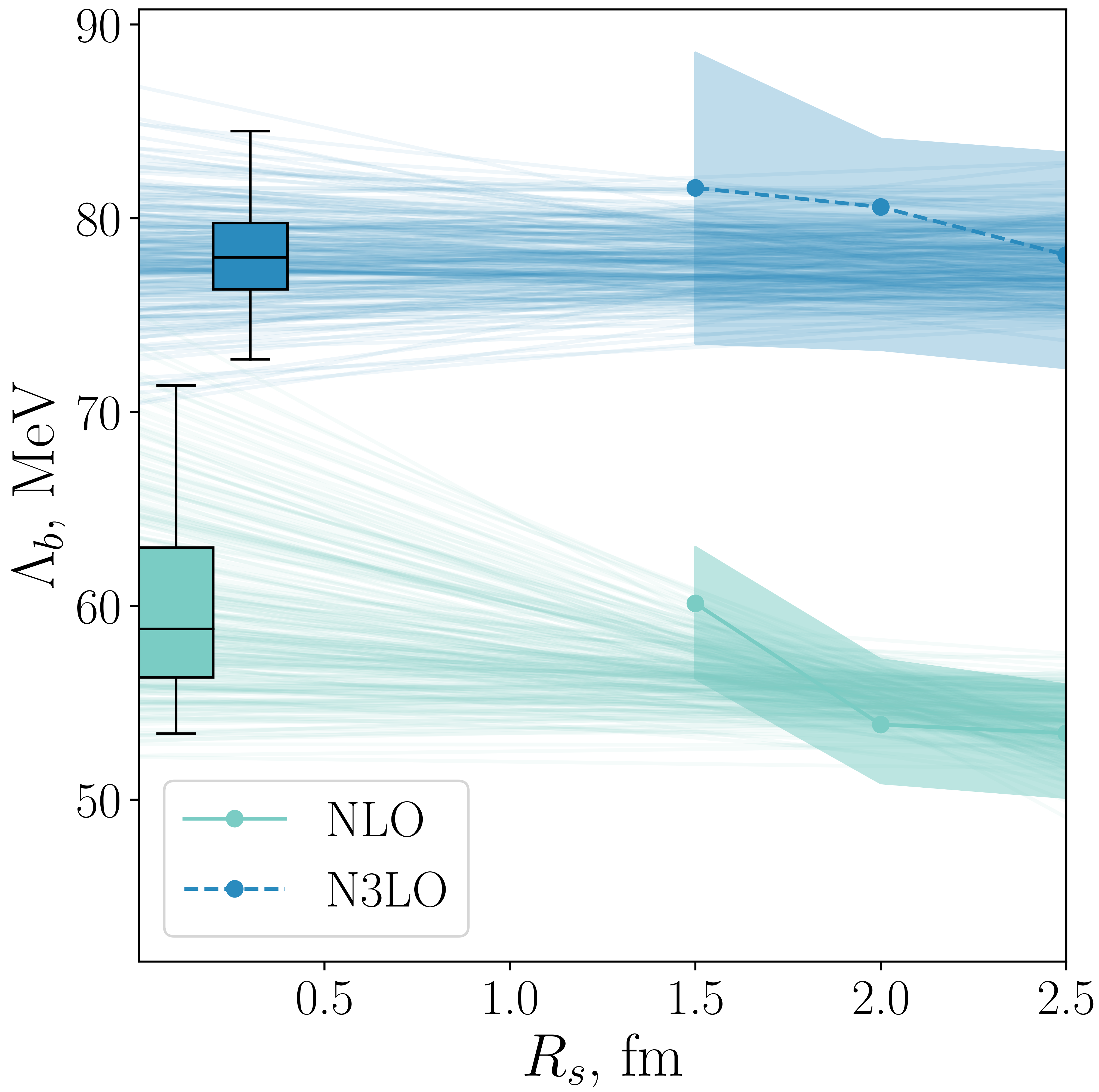

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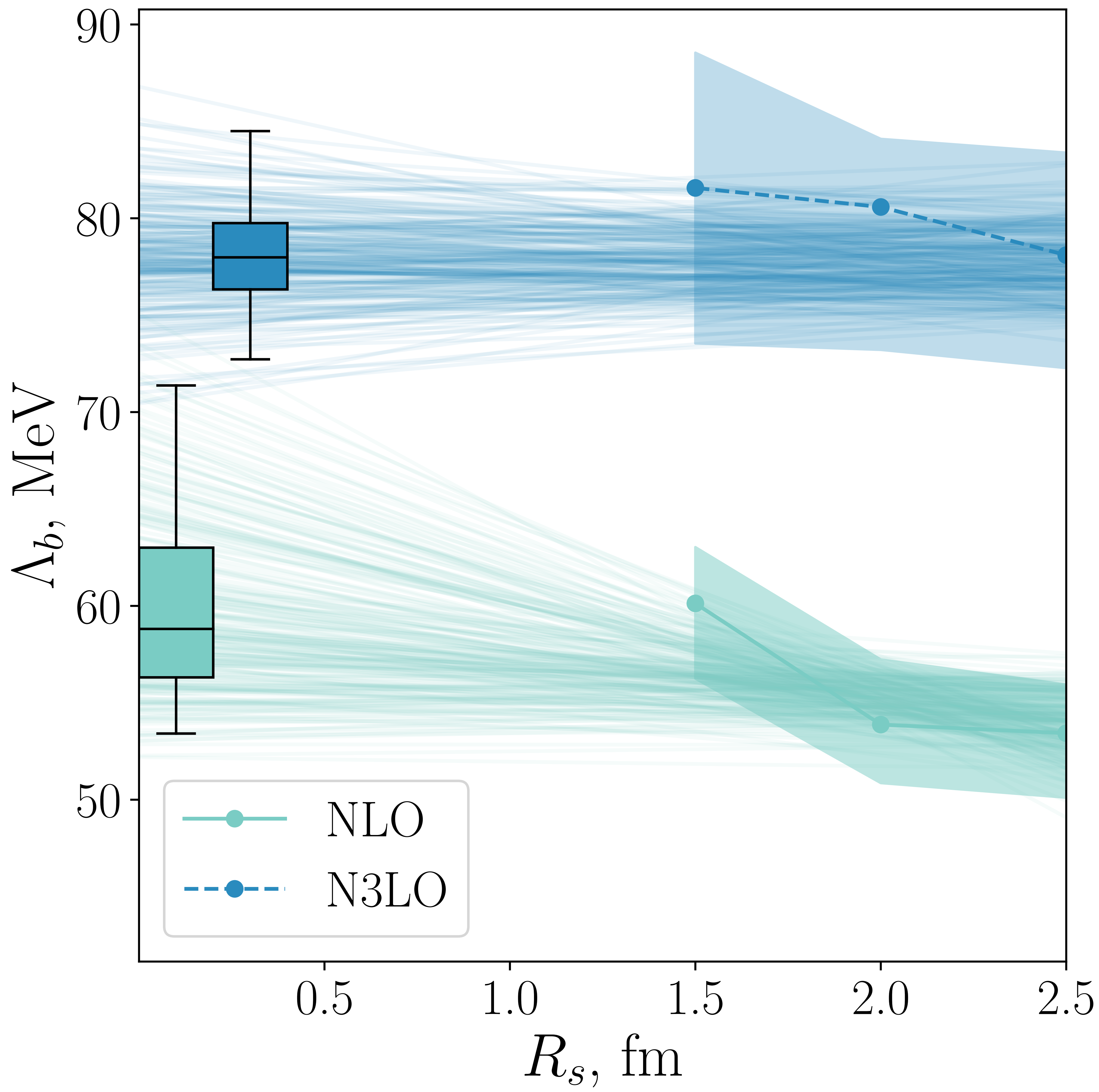

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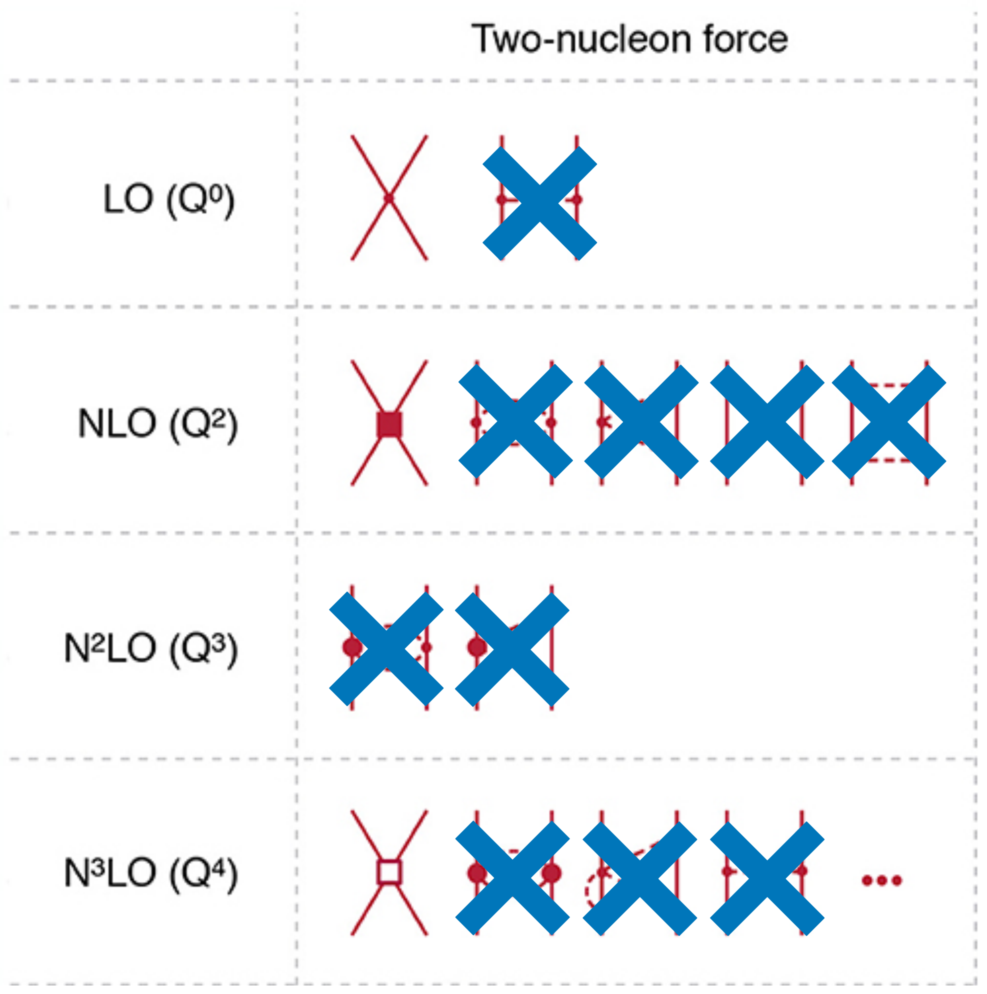

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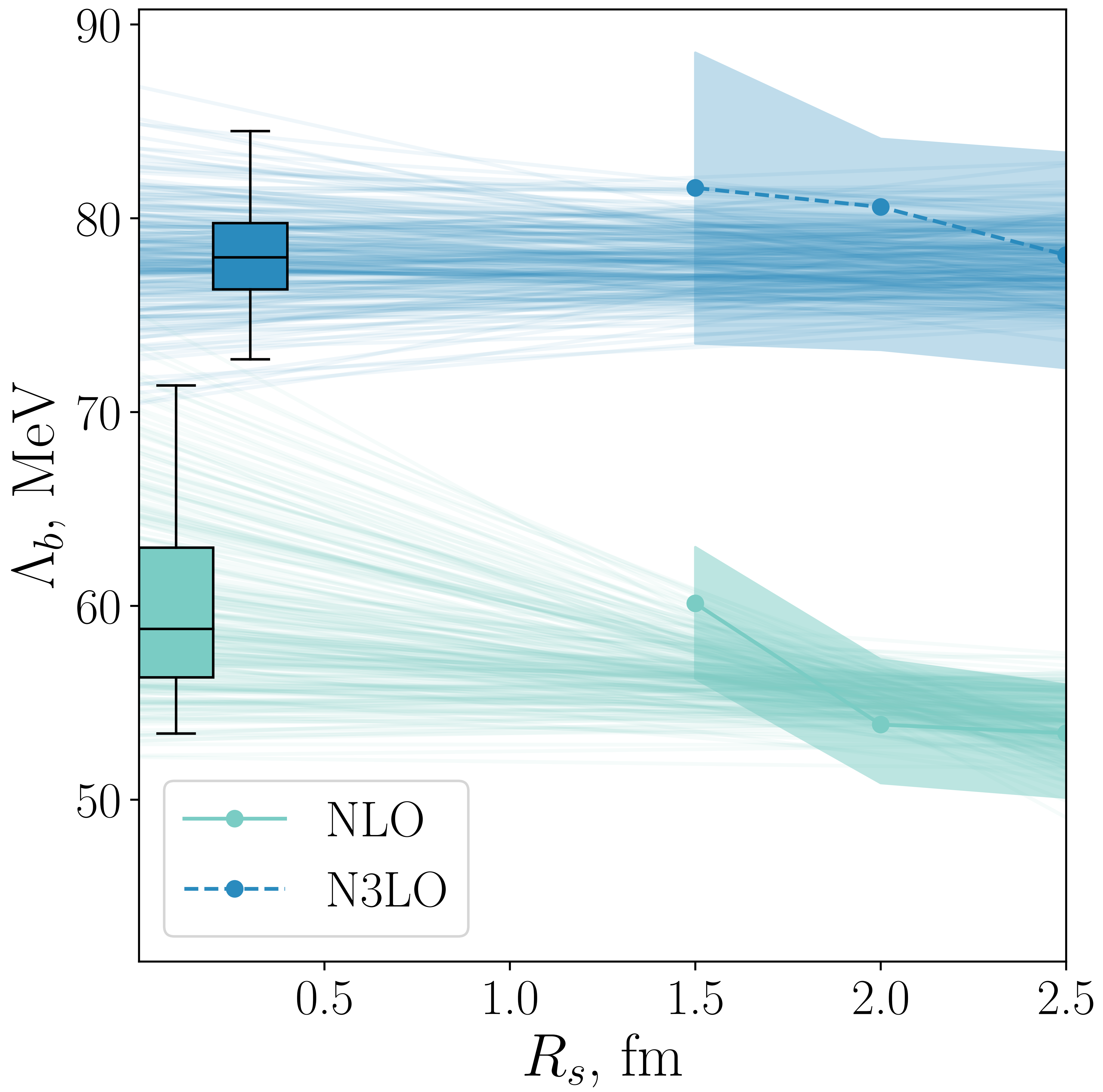

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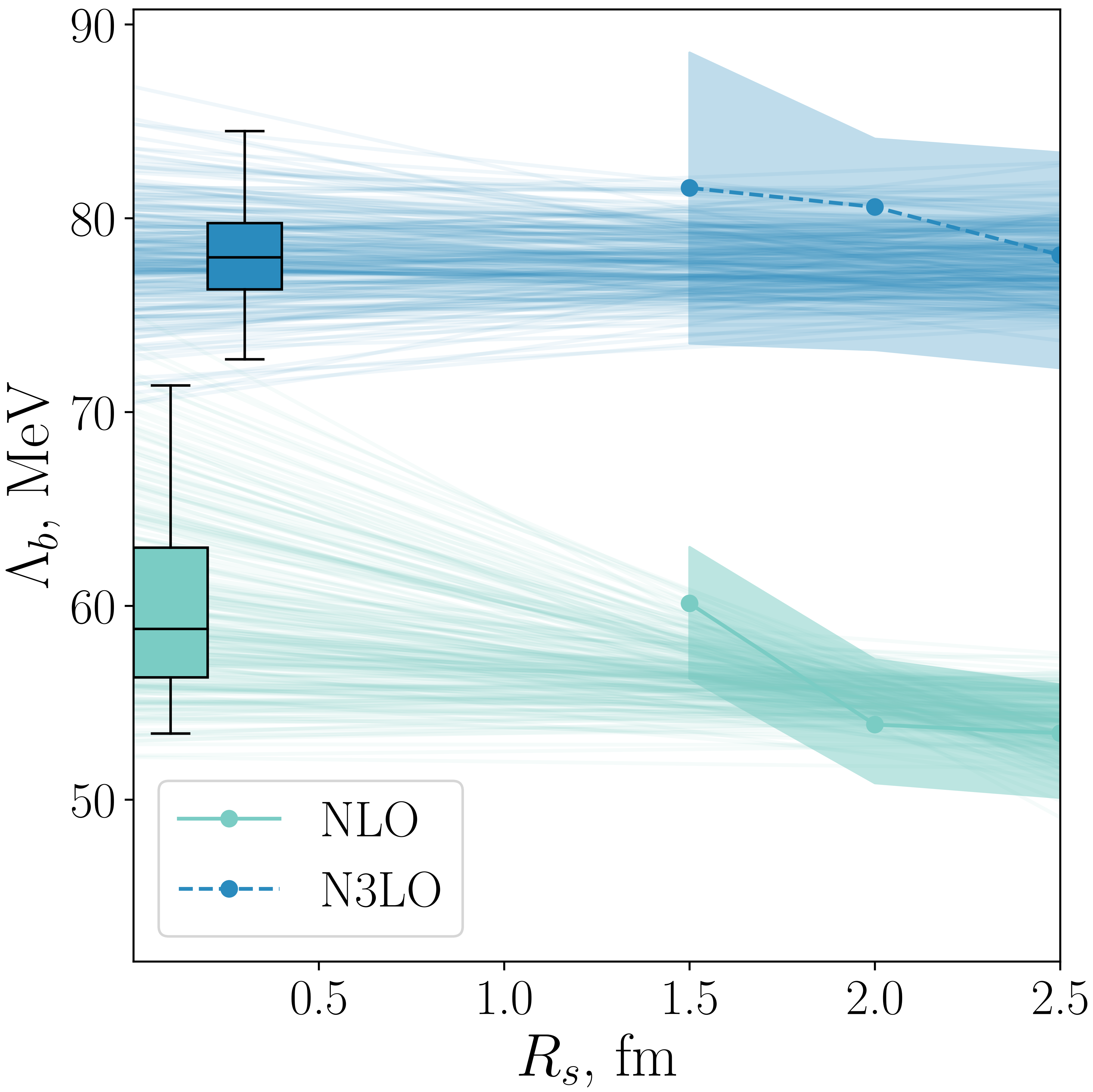

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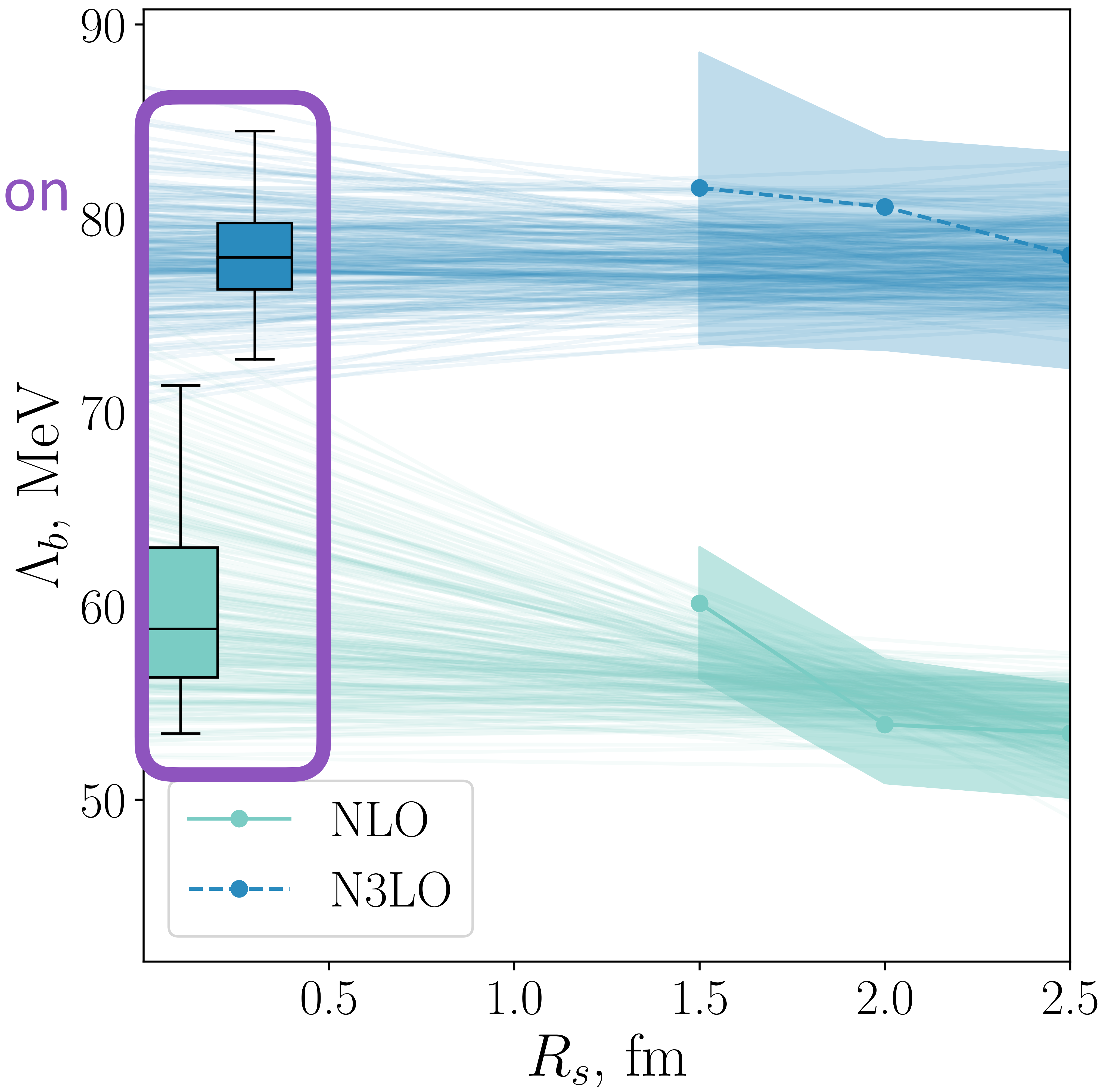

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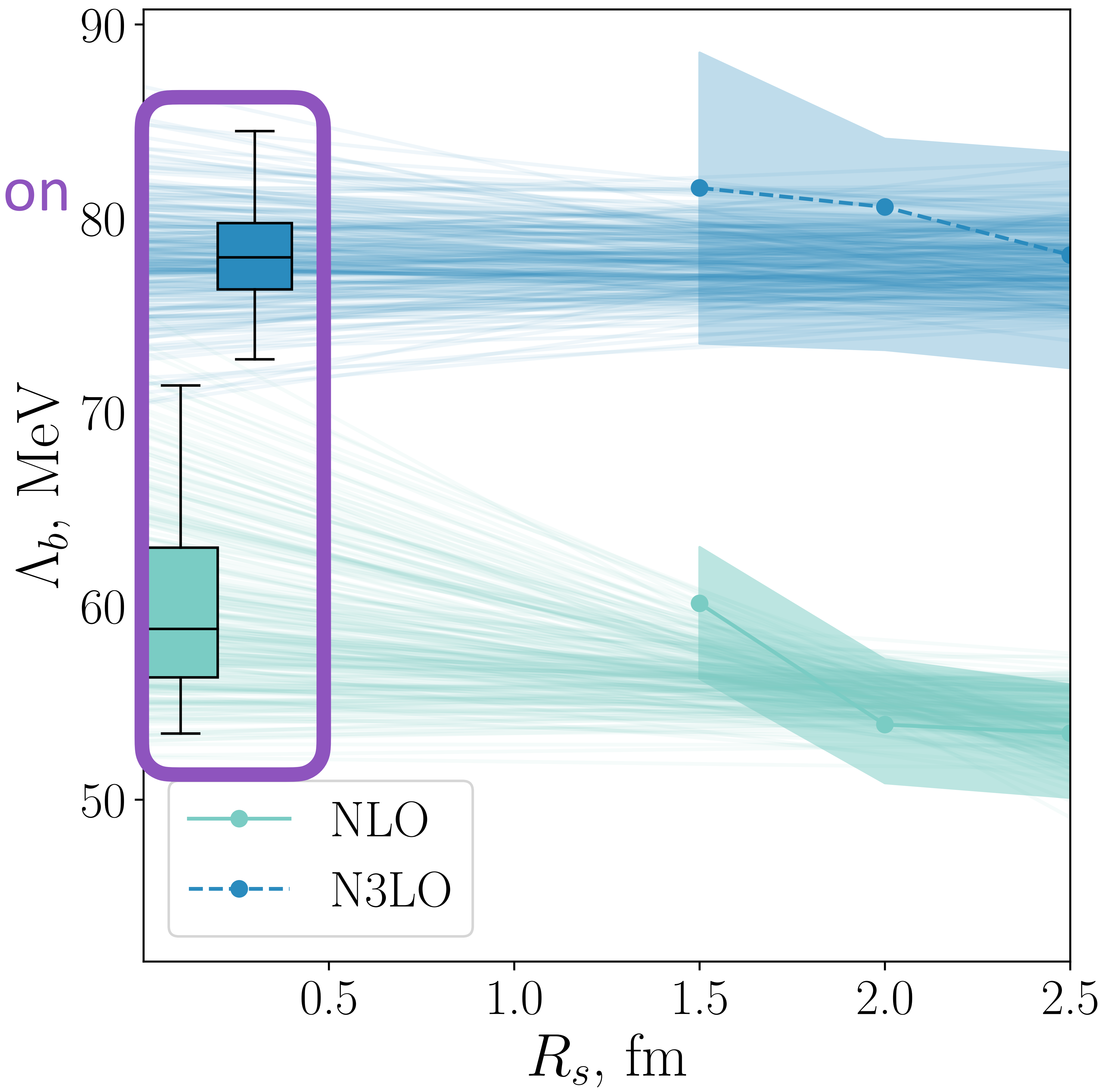

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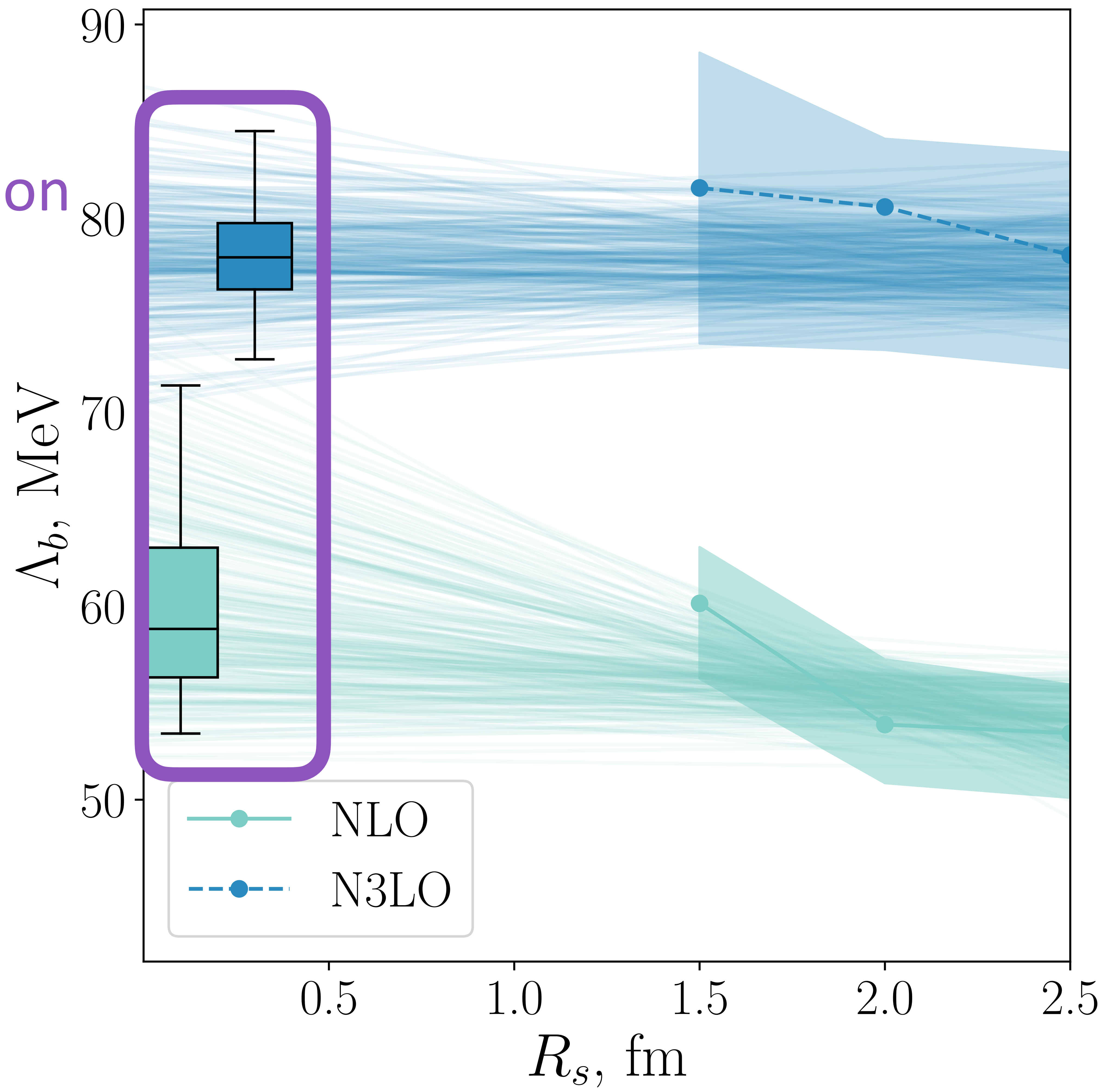

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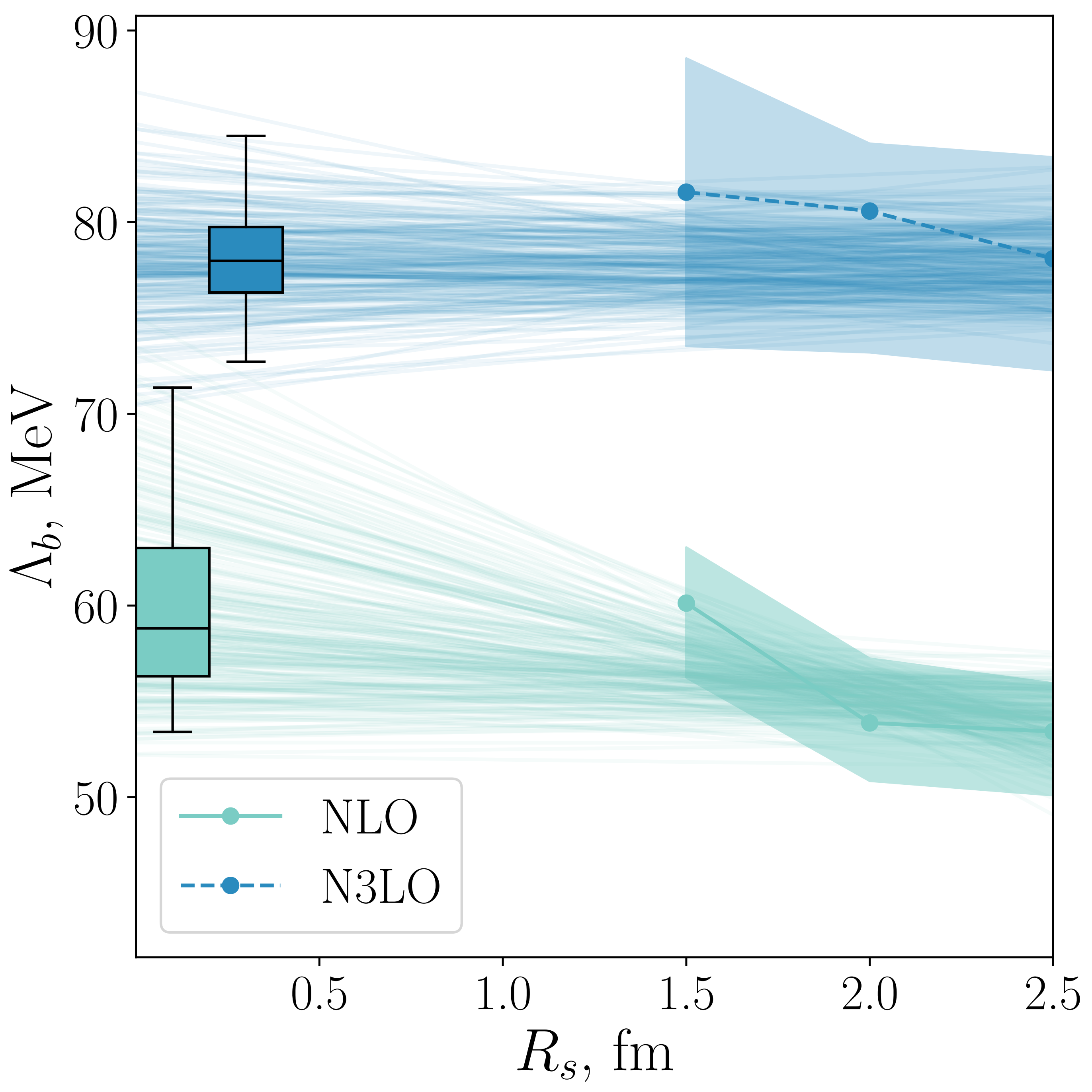

JB et al. Phys. Rev. C **111**, 034005 (2025)

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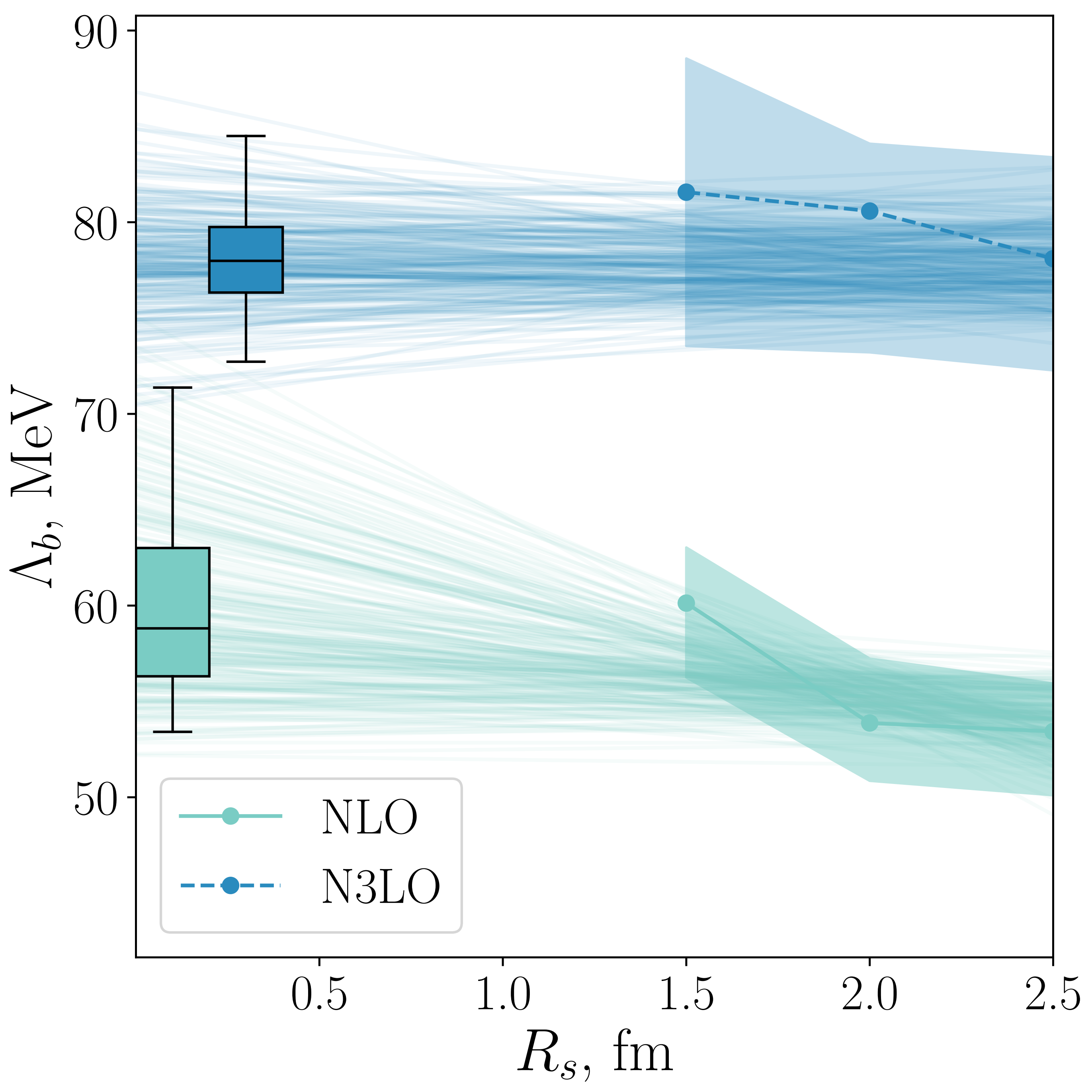

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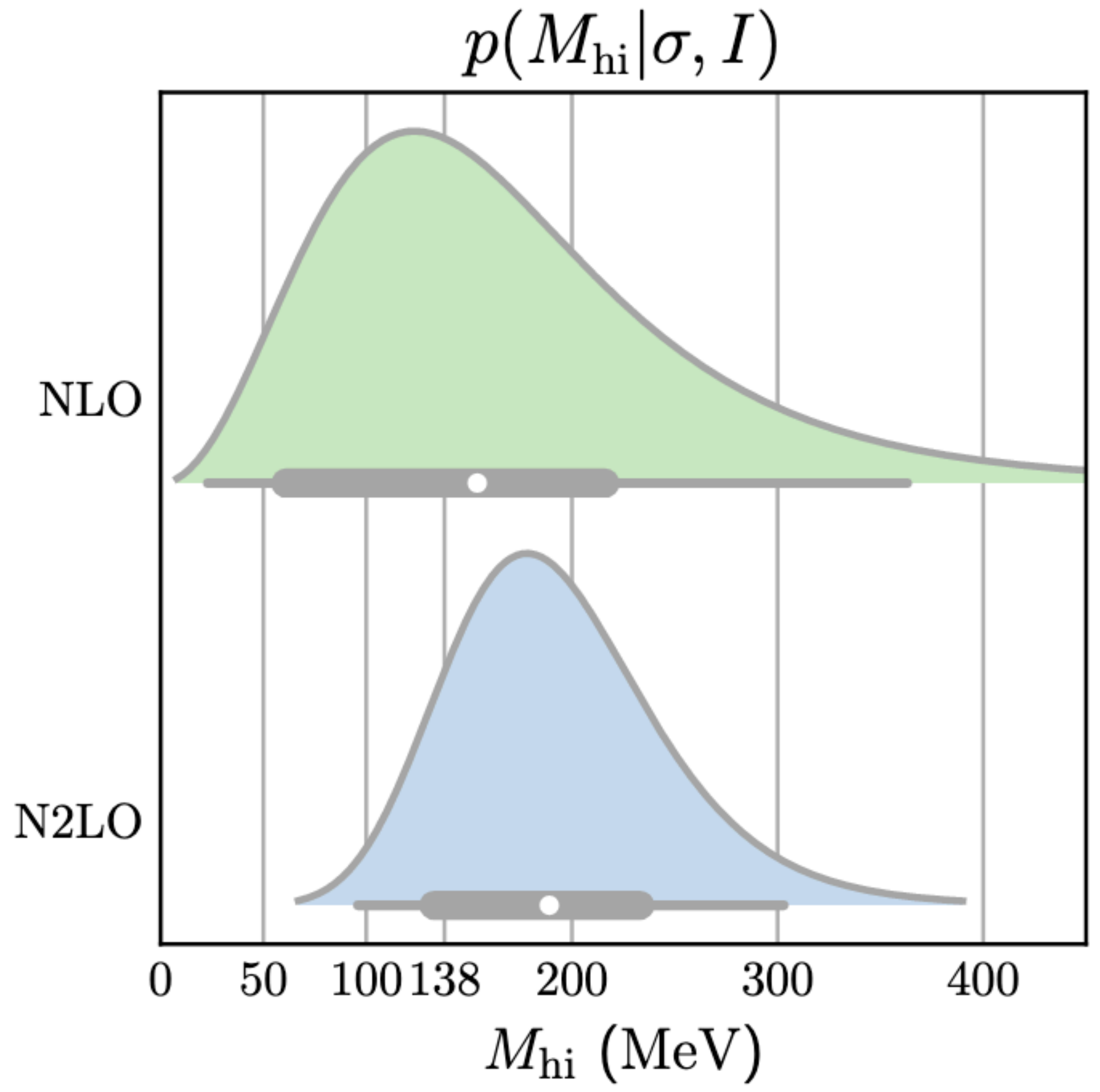
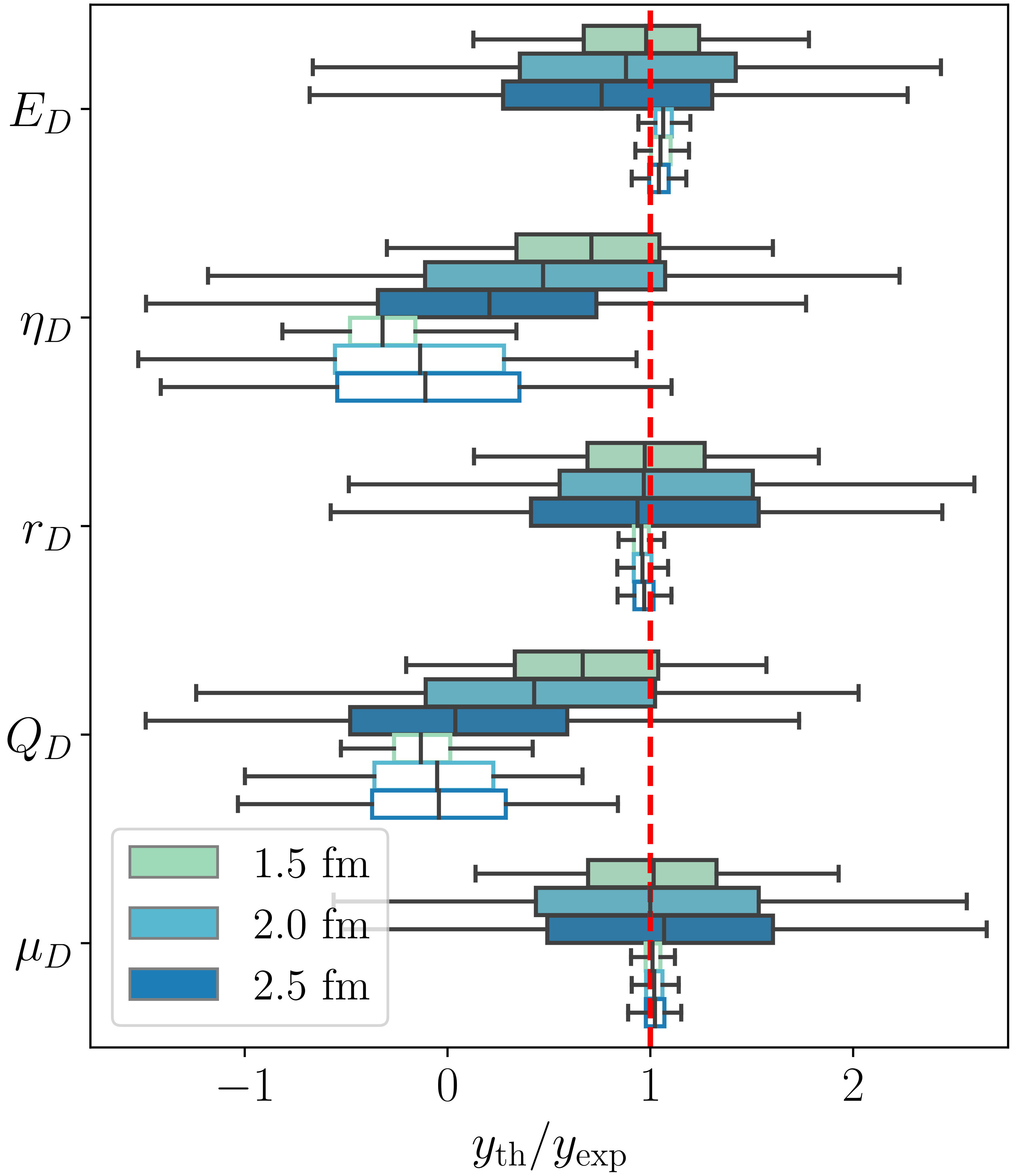


Ekström and Platter, *Phys.Lett.B* **860** 139207 (2025)

Propagation of Errors for Deuteron

Filled: NLO

Empty: N3LO



$$\mathrm{pr}(\mathbf{y}_{\rm th}\,|\,\mathbf{y},\mathbf{x},I) = \int d\mathbf{a}\, d\bar{c}^2\, d\Lambda_b\, \mathcal{N}\left(\mathbf{y}_{\rm th}(\mathbf{a},\mathbf{x}), \Sigma_{\rm th}\left(\bar{c}^2,\Lambda_b\right)\right) \times$$
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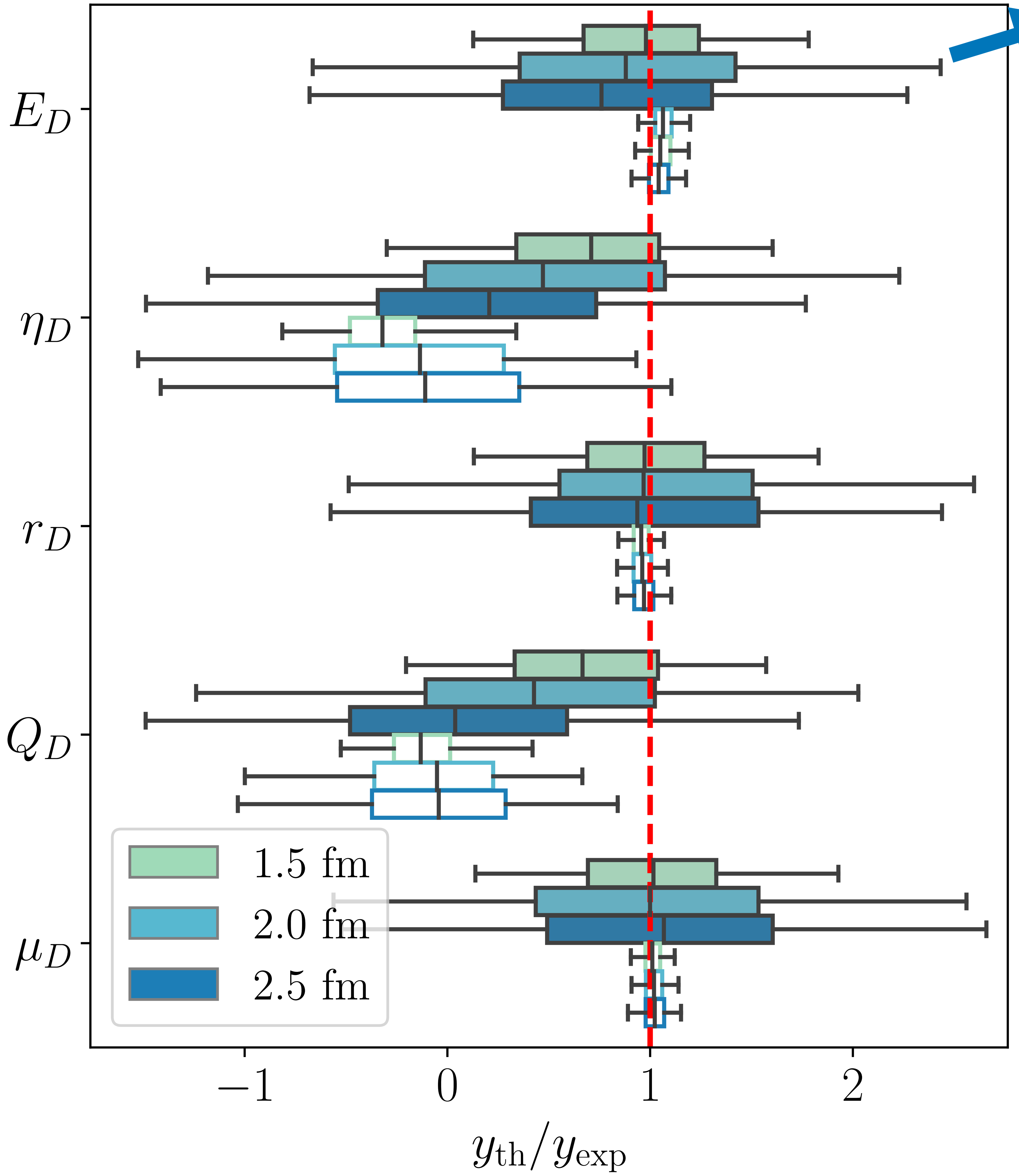
Propagation of Errors for Deuteron

Filled: NLO

Empty: N3LO



$95\,\%$ Confidence interval $\rightarrow \sim 150\,\%$ uncertainty

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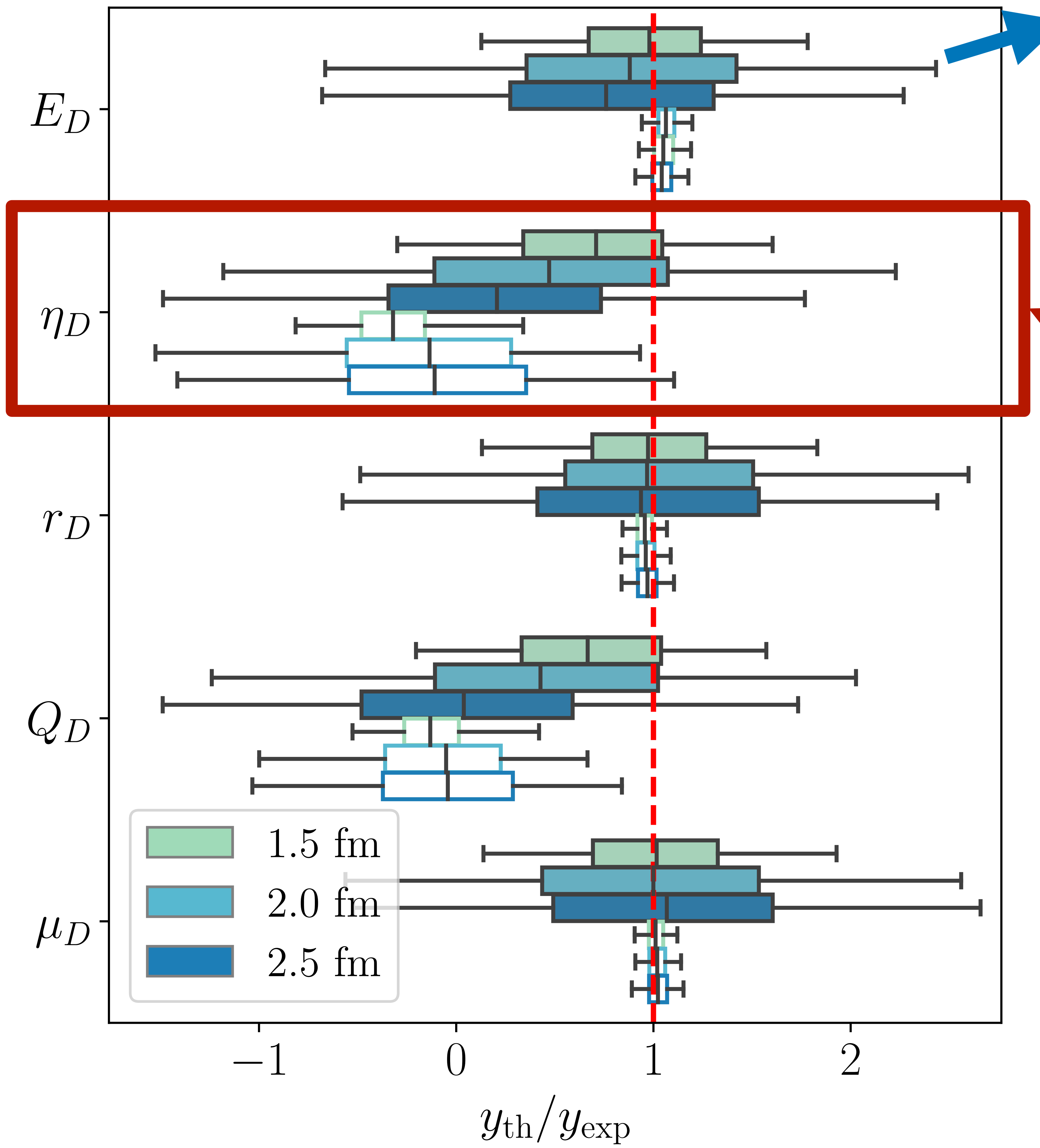

95 % Confidence interval → ~ 150 % uncertainty

$$\text{pr}(\mathbf{y}_{\text{th}} \mid \mathbf{y}, \mathbf{x}, I) = \int d\mathbf{a}\, d\bar{c}^2\, d\Lambda_b\, \mathcal{N}\left(\mathbf{y}_{\text{th}}(\mathbf{a}, \mathbf{x}), \Sigma_{\text{th}}\left(\bar{c}^2, \Lambda_b\right)\right) \times \text{pr}(\mathbf{a} \mid \mathbf{y}, I)\text{pr}(\bar{c}^2 \mid \Lambda_b, \mathbf{a}, I)\text{pr}(\Lambda_b \mid \mathbf{a}, I),$$

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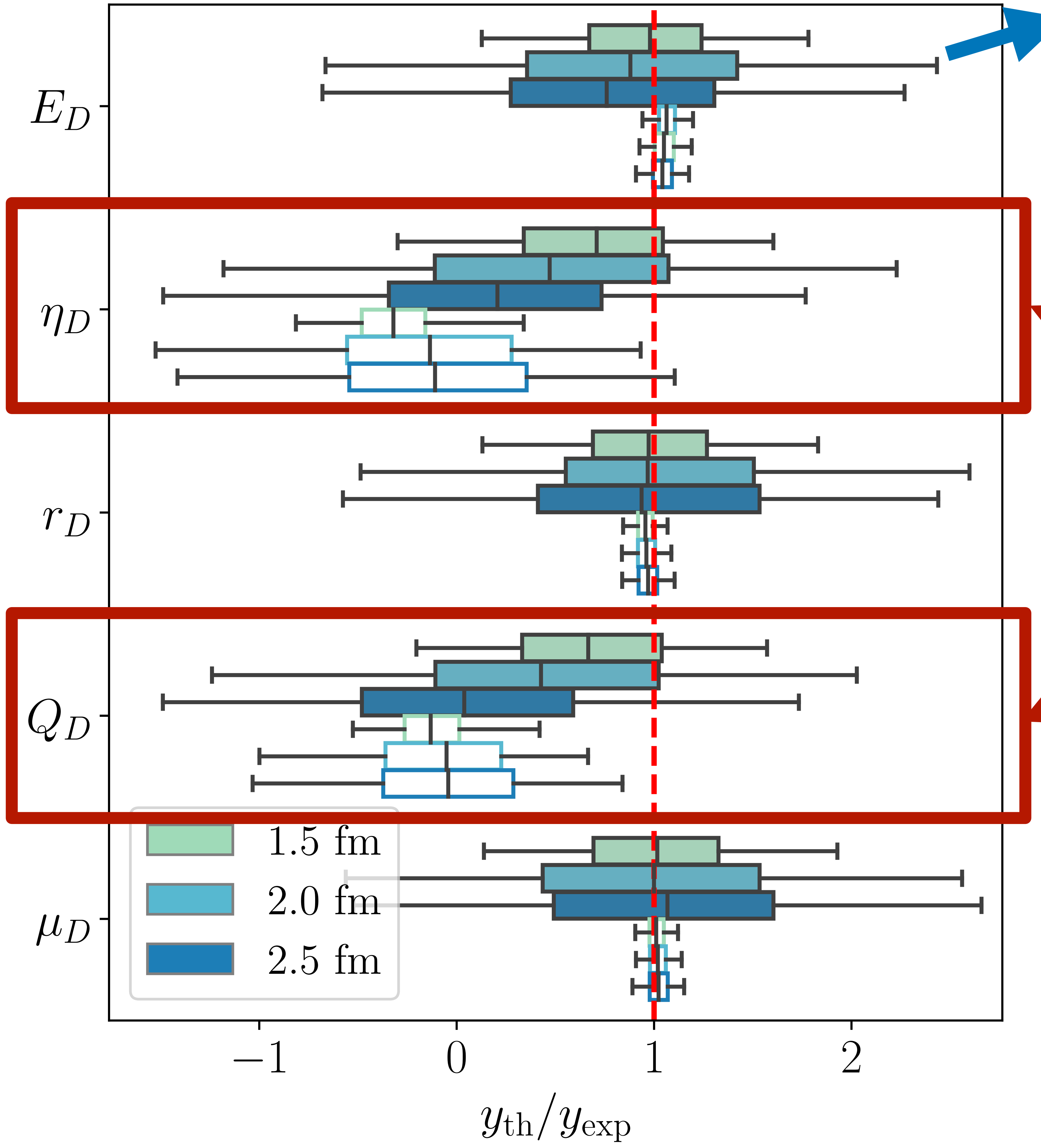

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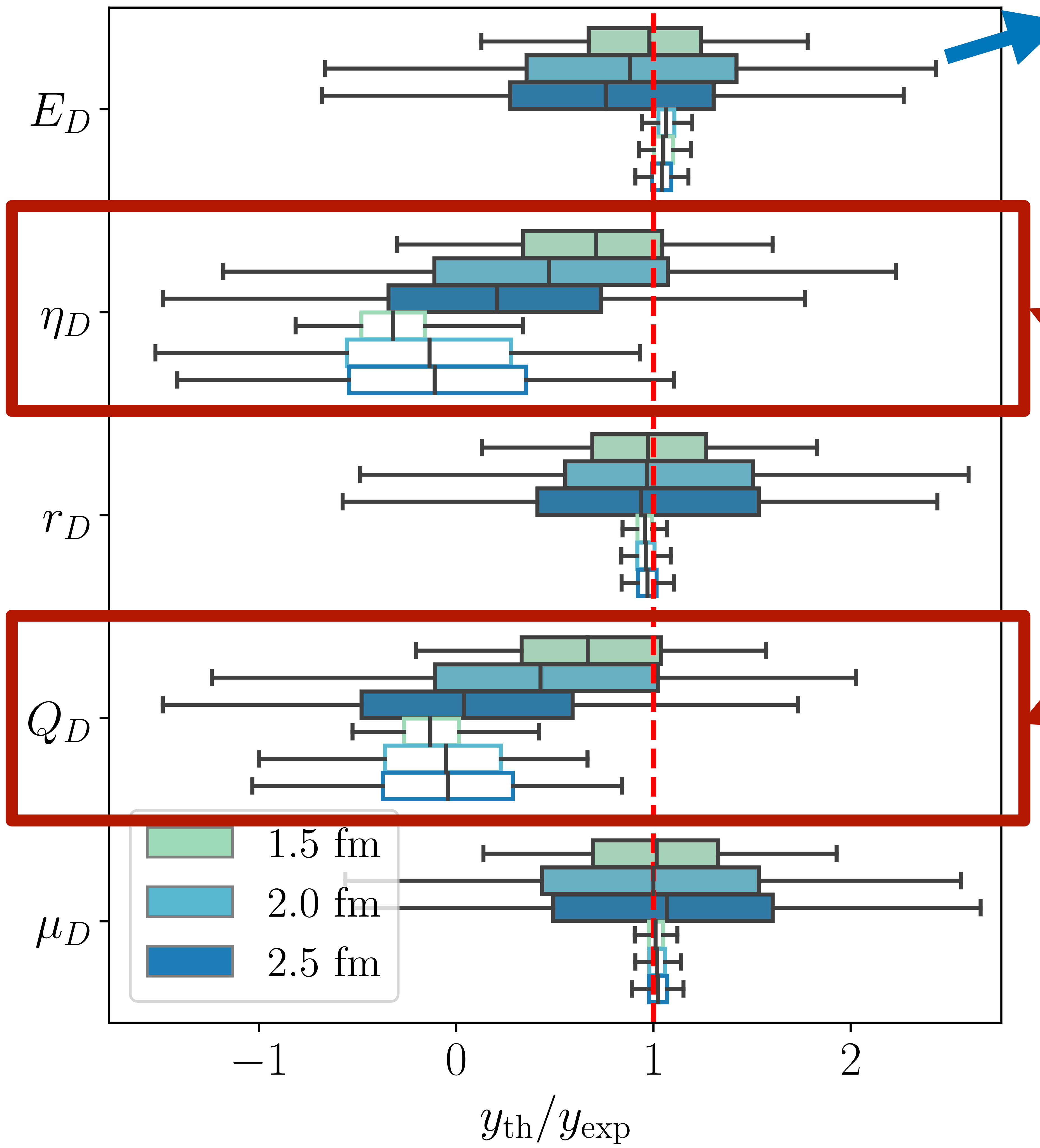

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→SHOULD BE CONSISTENT WITH 0

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  - Easy for chiral interactions: $p \approx m_\pi$
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- three-nucleon interactions are important, and we do not have uncertainty quantification for these

Improving Power-Counting

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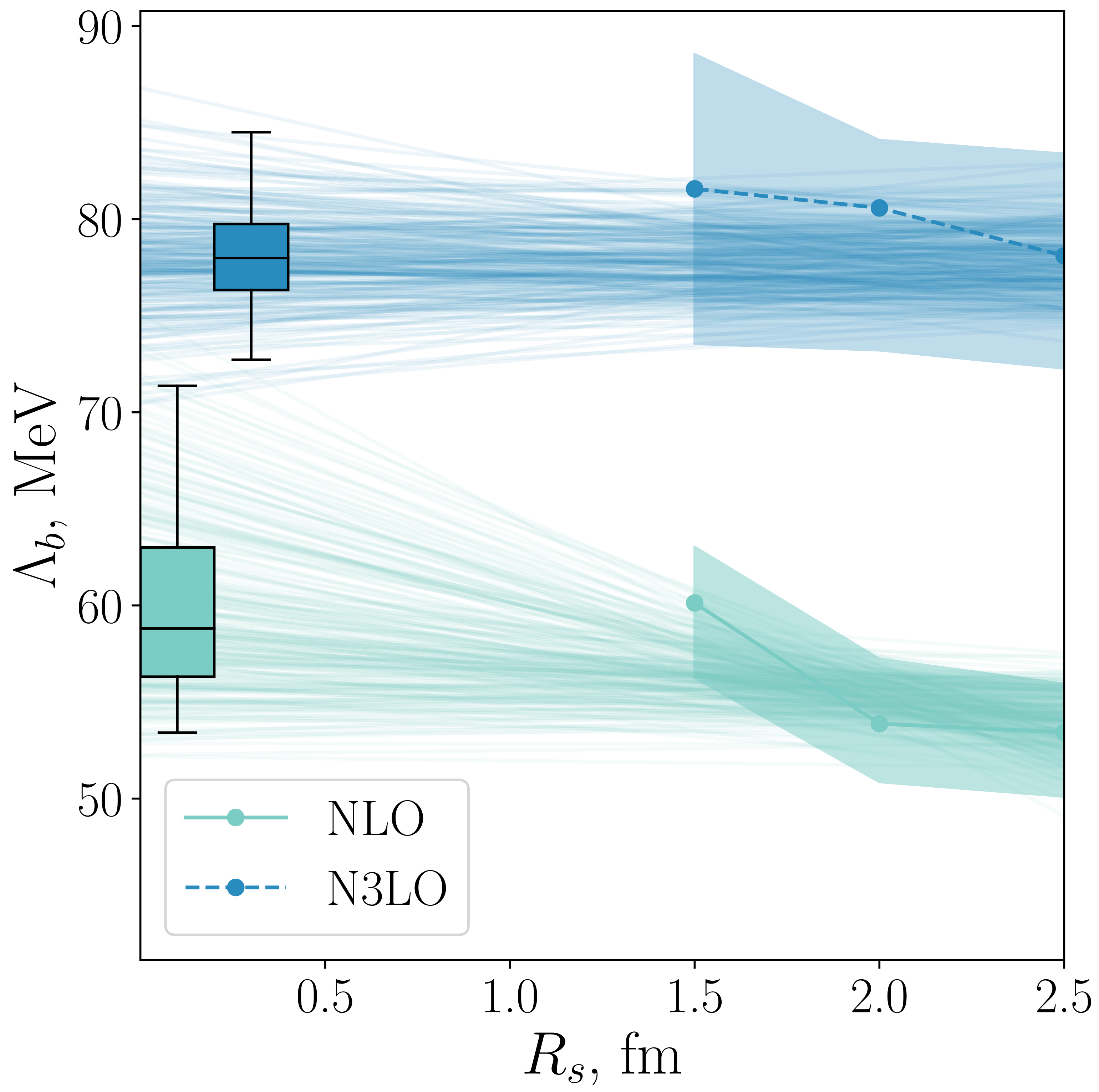

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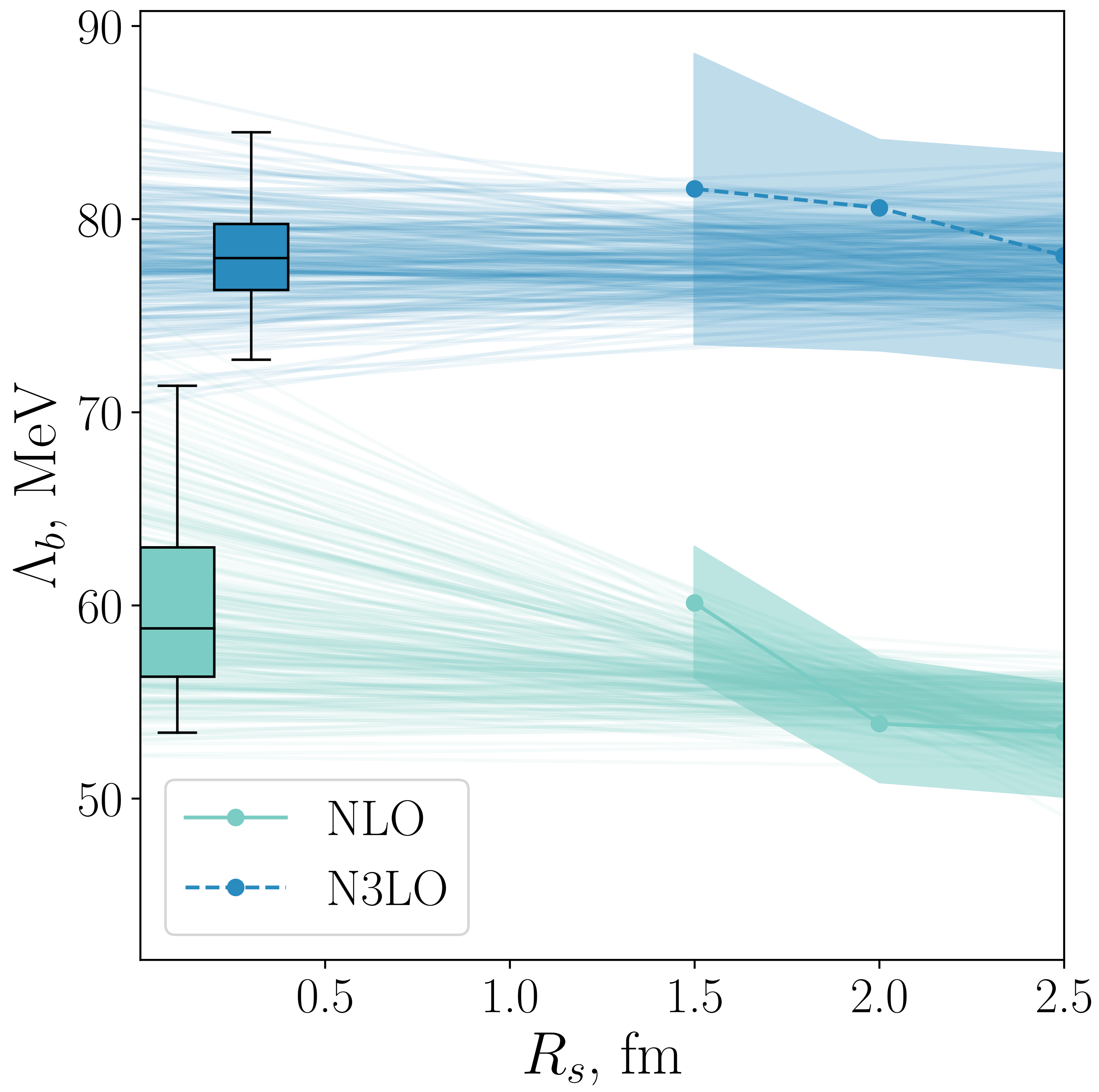

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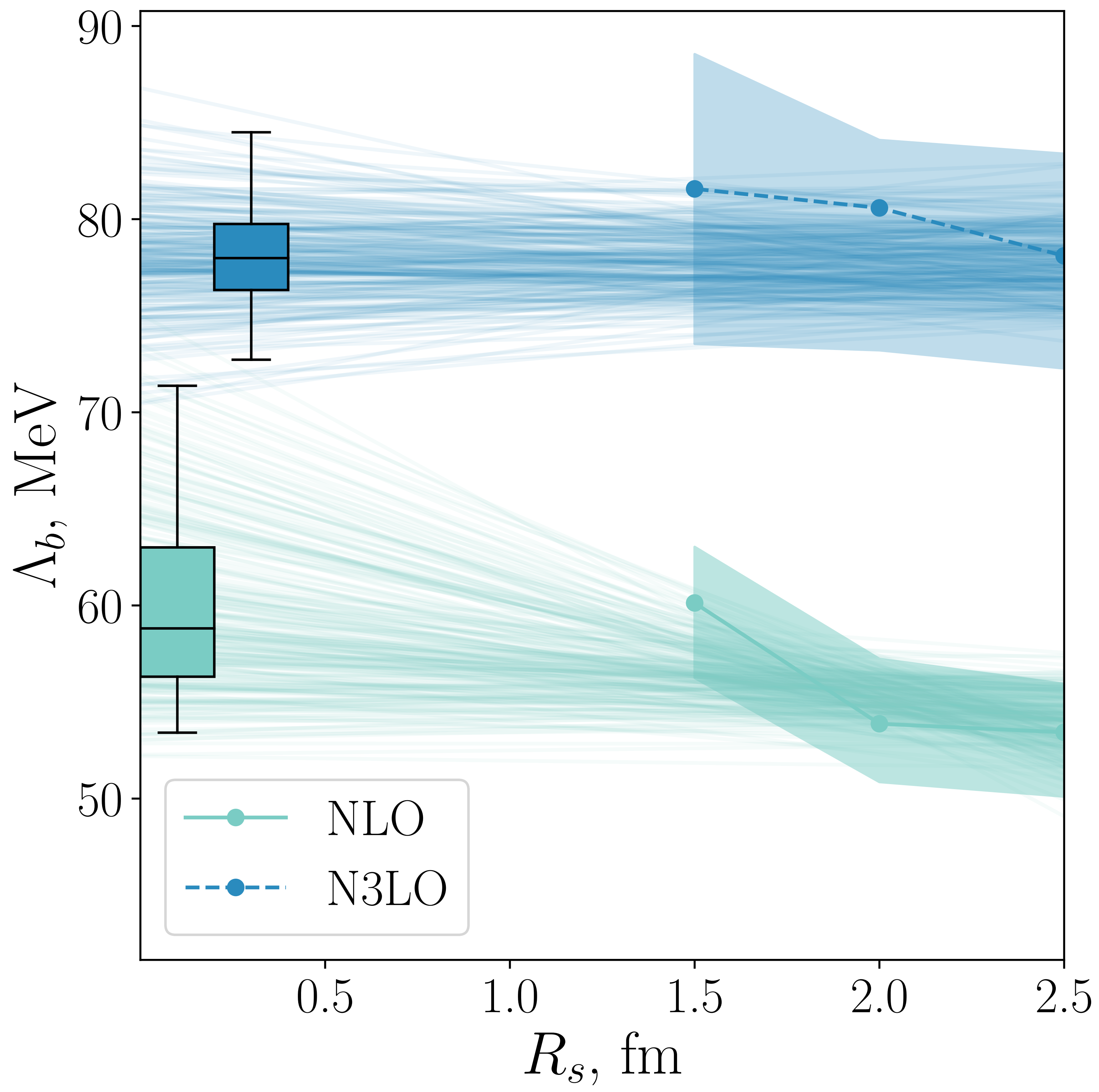

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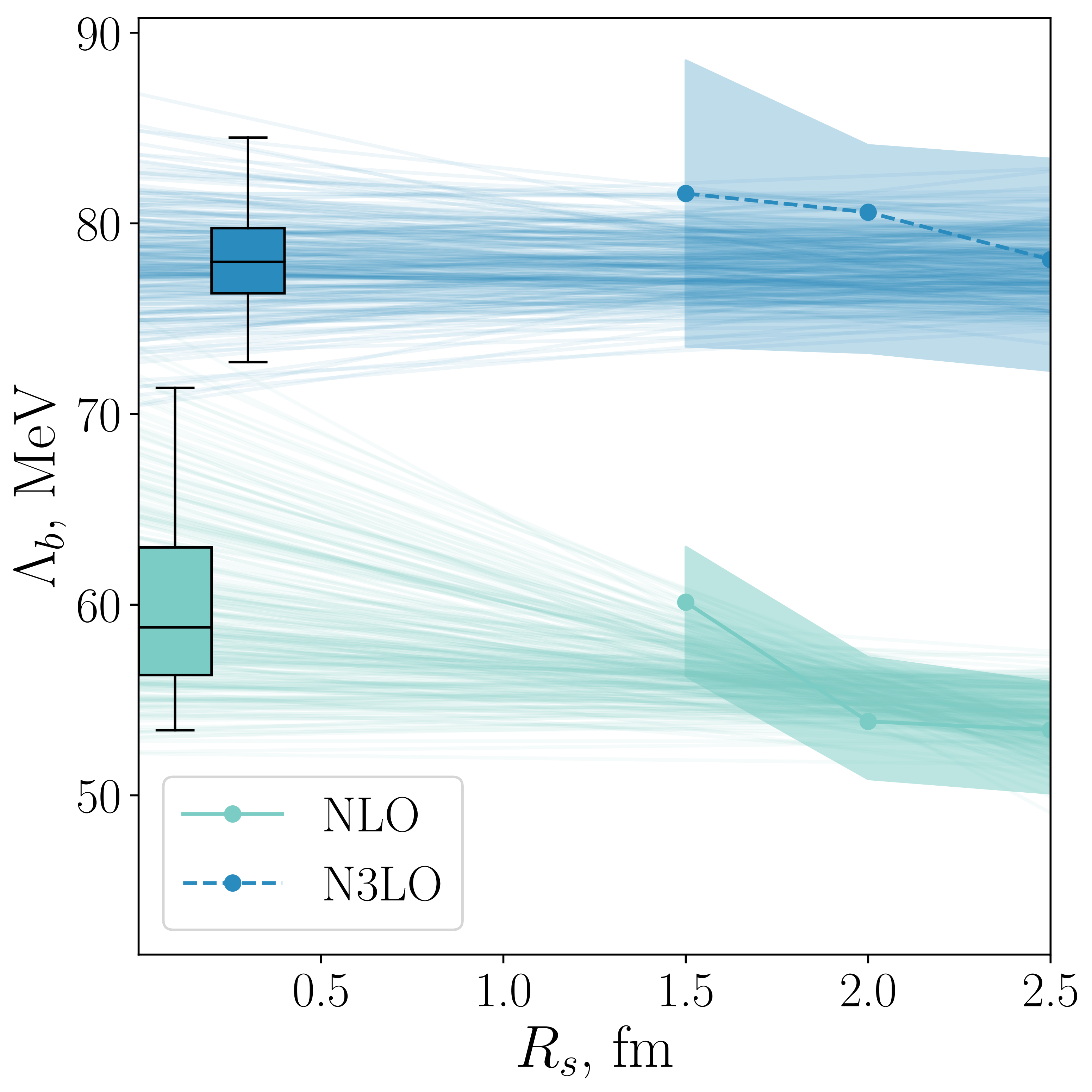

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- Re-counting the index of interactions

Summary

- EFT interactions provide a systematic framework for microscopic nuclear interactions.
  - Enable rigorous uncertainty quantification.
- Bayesian analysis is a useful tool in EFT contexts.
  - Uncertainty quantification
  - Extract information of physics from data
    - E.g., inconsistencies in EFT power counting

"Bayesian analysis is like having a discussion with data."

What can the data tell us about the physics we want to explore?

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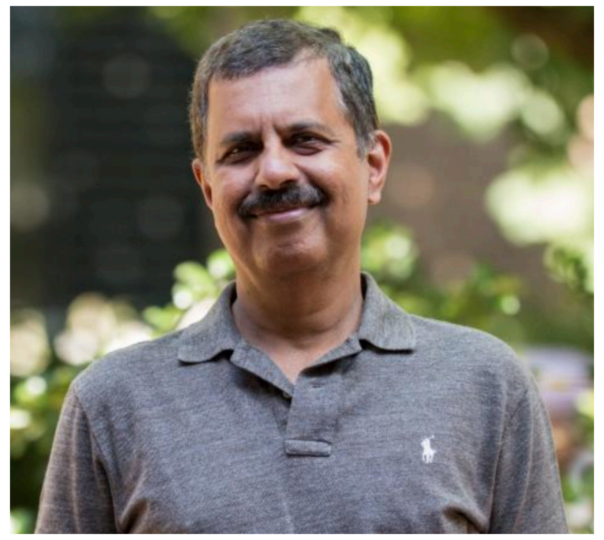
JJ Silva (GS)



Sai Iyer
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Dick Furnstahl
OSU

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Bayesian Analysis of Nuclear Dynamics

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NTNP

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