Marciana 2025 Workshop, Marciana Marina, Isola d'Elba, Italy, 26.06.2025

Precision calculations of A=2-4 radii

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in collaboration with

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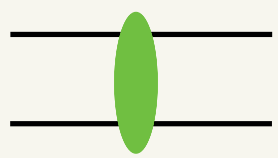
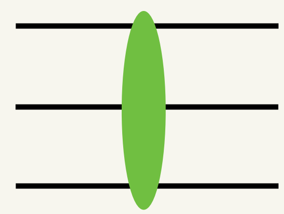
Nuclear structure using chiral effective field theory

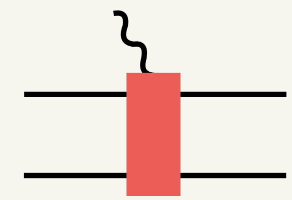
Low-energy **chiral effective field theory** of the standard model



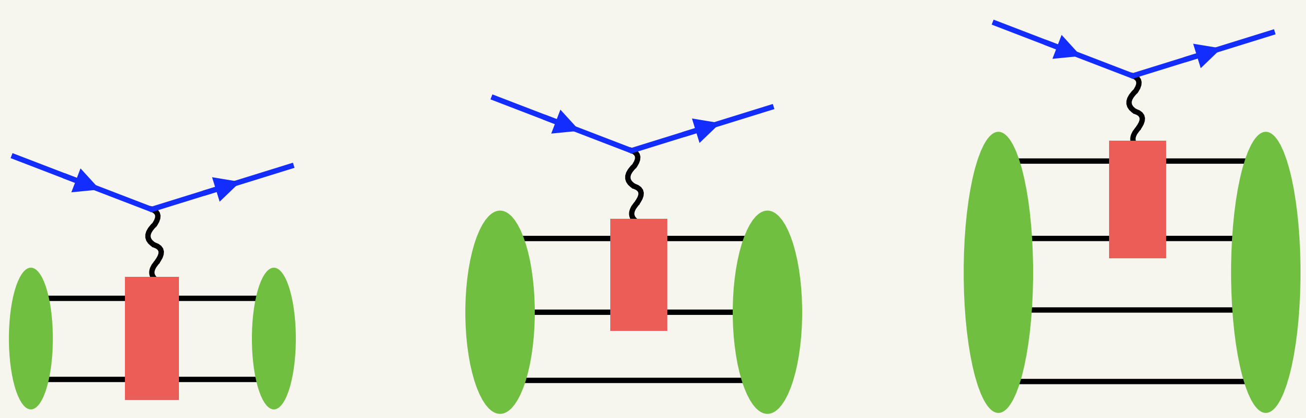
Chiral two-nucleon forces



Chiral three-nucleon forces

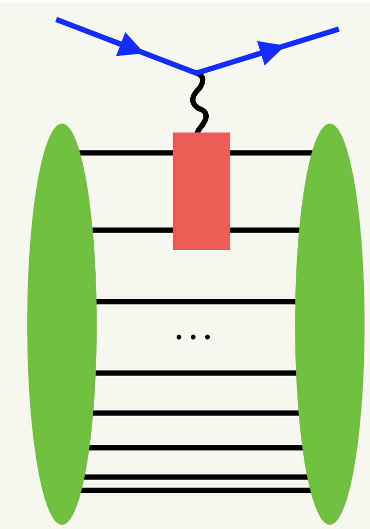


Chiral electromagnetic currents



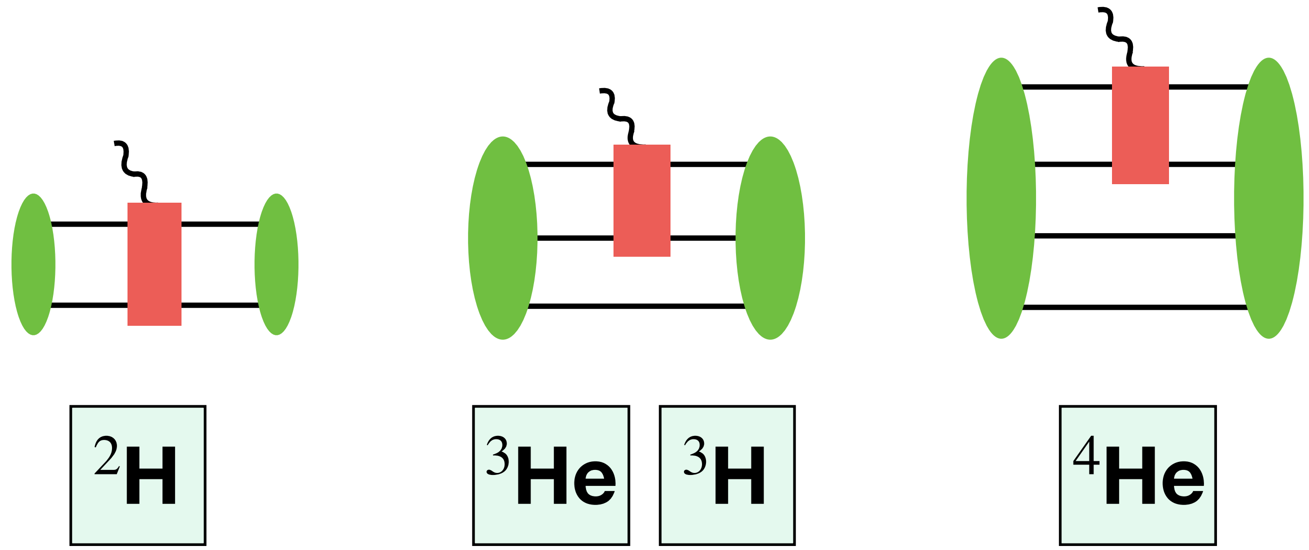
^{2}**H** ^{3}**He** ^{3}**H** ^{4}**He**

charge form factors and **radii** of light nuclei



structure of medium-mass nuclei

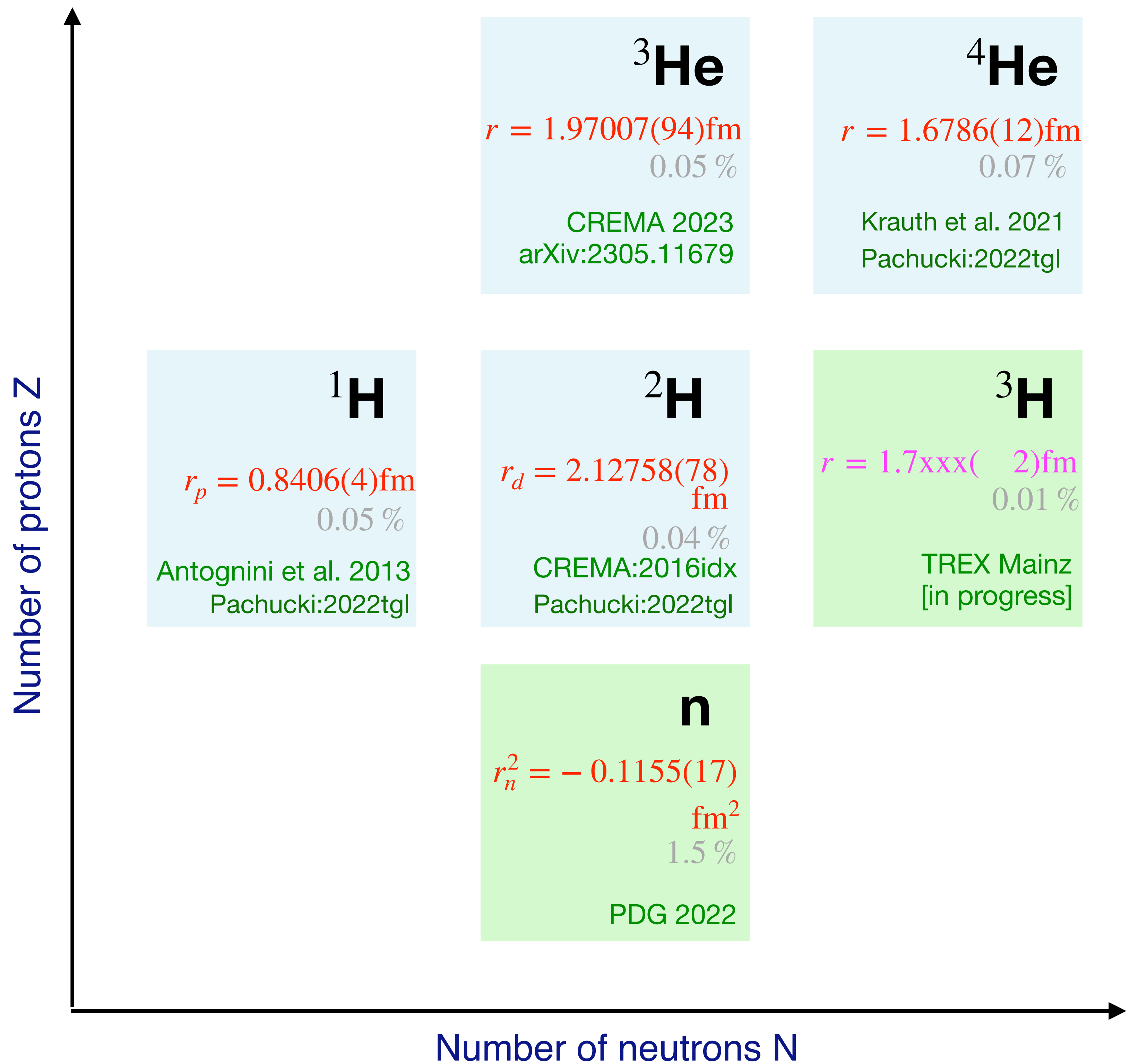
Precise calculations of charge radii of super-light nuclei



Motivation:

- Precision **tests of nuclear chiral effective field theory (Chiral EFT)**
- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**
- A new way to **extract the neutron and the proton charge radii** from few-nucleon data

Charge radii of A ≤ 4 nuclei — experimental data



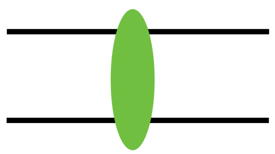
Pohl:2013yb,
CREMA:2016idx,
Pohl:2016glp,
Schmidt:2018kjc,
Krauth:2021foz
...

Data from isotope shift

$(r_d^2 - r_p^2) = 3.82070(31) fm^2$
0.01 %

Pachucki et al. 2018
Jentschura et al. 2011

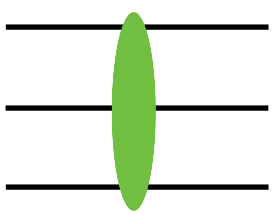
Chiral effective field theory - **precise, accurate** and **consistent**



N^4LO^+

New **high-precision chiral NN forces** (N^4LO^+) Reinert et al. PRL 126, 092501 (2021)

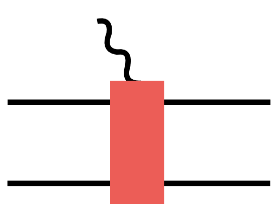
- Nearly perfect description of pp and pn scattering data up to pion production threshold



N^2LO

Chiral 3N forces (general N^2LO; selected terms at N^4LO) Epelbaum:2019kcf

- LECs cD and cE (N^2LO) are fitted to RIKEN Nd DCS data and ^{3}He binding energy
- **Consistent** regularisation of N^3LO is finished, PWD in progress (Talk by Hermann Krebs)

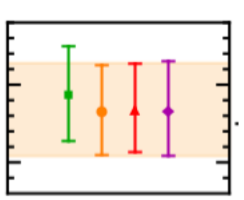


N^2LO
Isoscalar: N^4LO

2N Chiral electromagnetic currents (general N^2LO; isoscalar N^4LO)

- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)

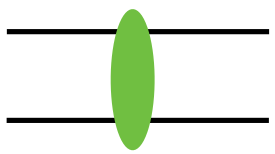


Reliable methods to quantify truncation uncertainty of the EFT expansion

Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017), Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

See also talk by Jason Bub

Chiral effective field theory - **precise, accurate** and **consistent**



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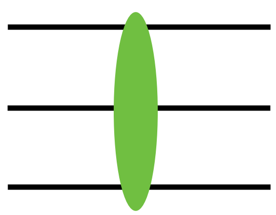

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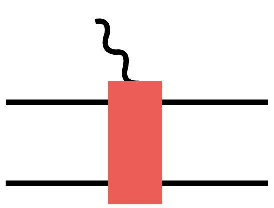

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- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
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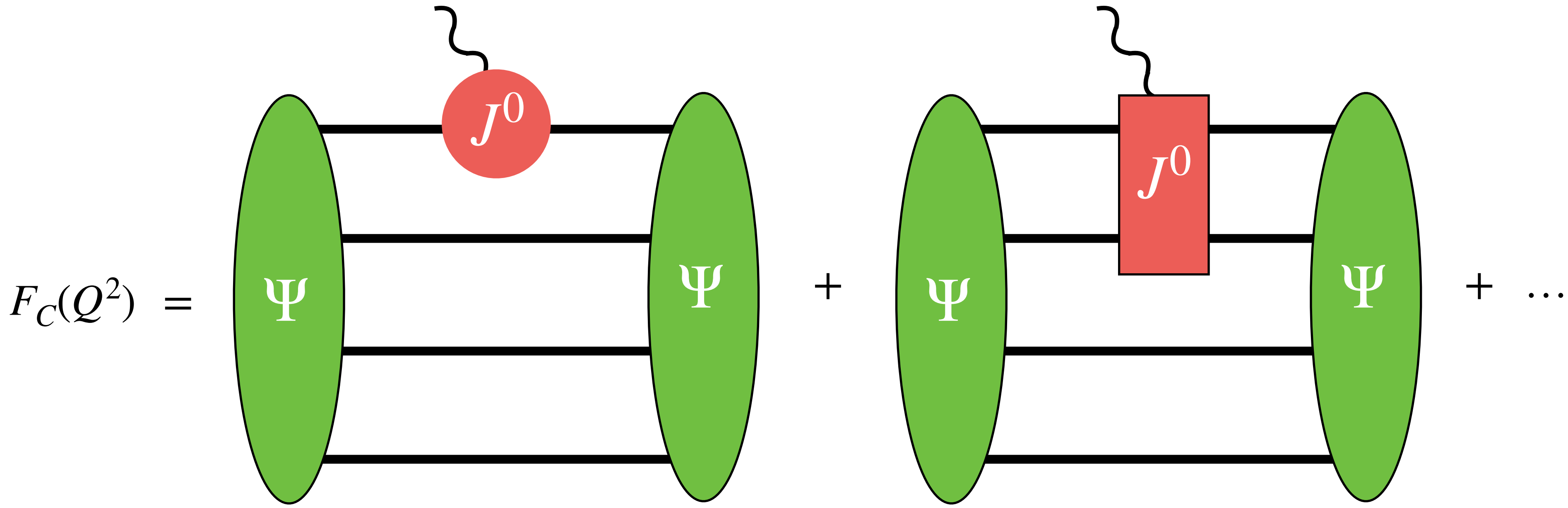
Goals of this study:

- consistent χEFT calculation of isoscalar structure radii of A = 2, 3, 4 nuclei
- aim at N^4LO level of accuracy even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, numerical and other)

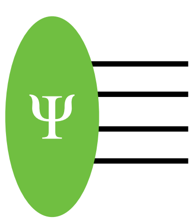
Chiral EFT calculation of the nuclear charge radius

Charge radius r_C is related to the charge form factor $F_C(Q^2)$

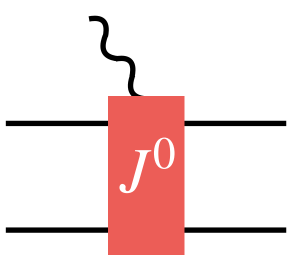
$$r_C^2 = (-6)\frac{\partial}{\partial Q^2} F_C(Q^2)\Big|_{Q=0}$$



The matrix element is a convolution of nuclear wave function and charge density operator



Nuclear wave function based on high-precision chiral EFT interactions



Charge density operator - consistent with chiral nuclear forces

Structure radius

Nuclear **charge radius** can be decomposed into structure, proton and neutron radii

$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{N}{Z} r_n^2$$

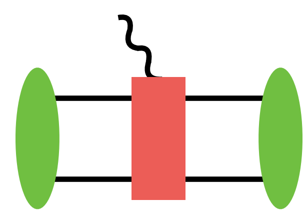
Nuclear structure radius

- depends on distribution of matter (proton and neutrons) inside the nuclei
- depends on many-body electromagnetic currents
- **can be accurately calculated using chiral nuclear forces and EM currents**

Structure radius = charge radius if protons and neutrons have point-like charge distributions

Deuteron structure radius

& extraction of the neutron charge radius

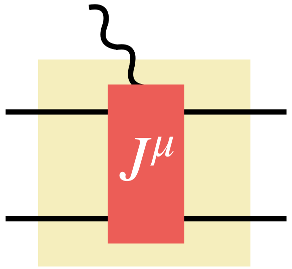


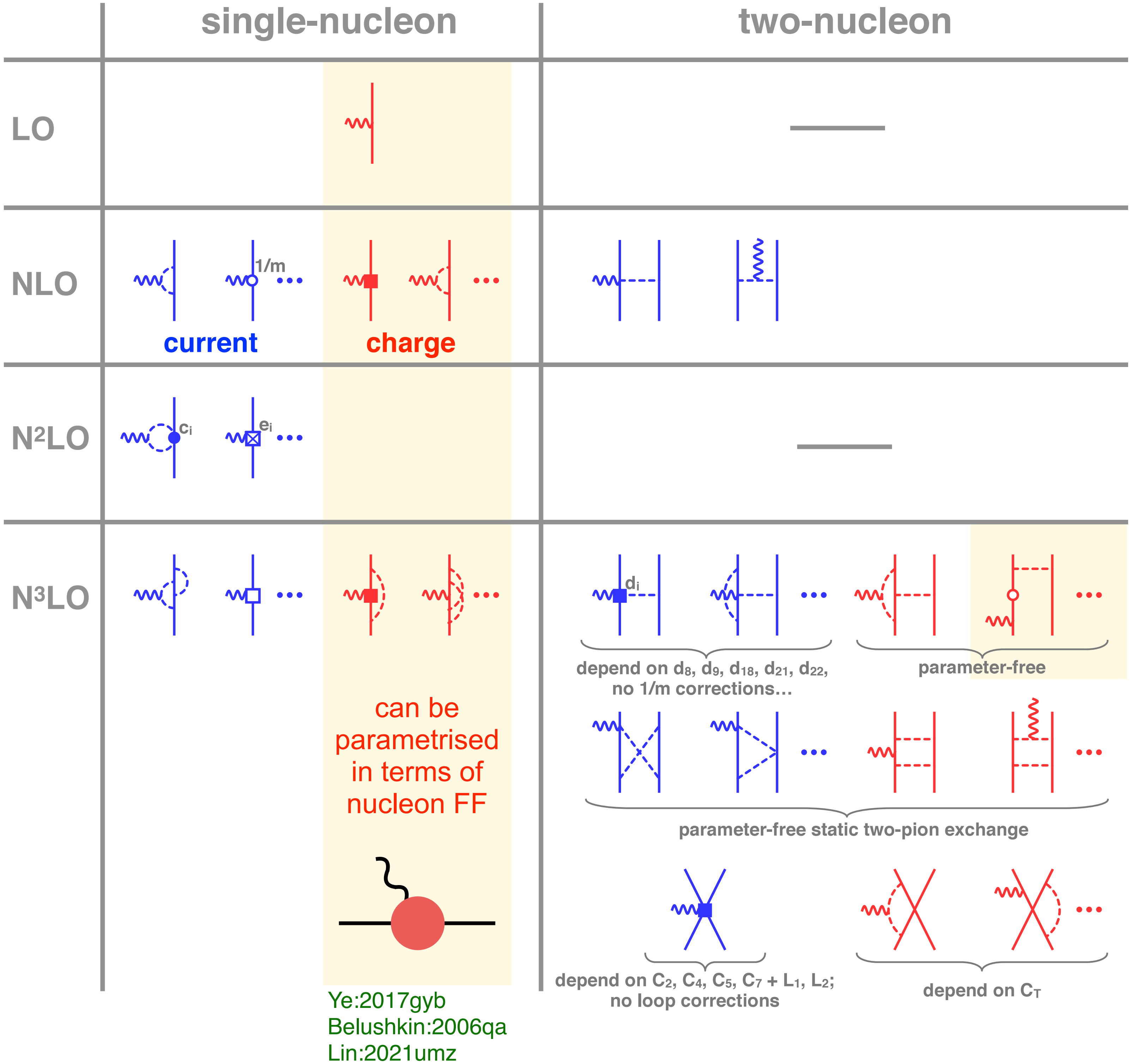
^{2}**H**

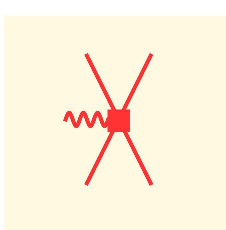
PRL124 082501 (2020)
Phys.Rev.C 103, 024313 (2021)

Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka
Review: H. Krebs, EPJA 56 (2020) 240



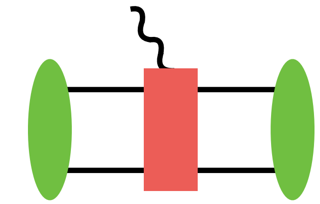




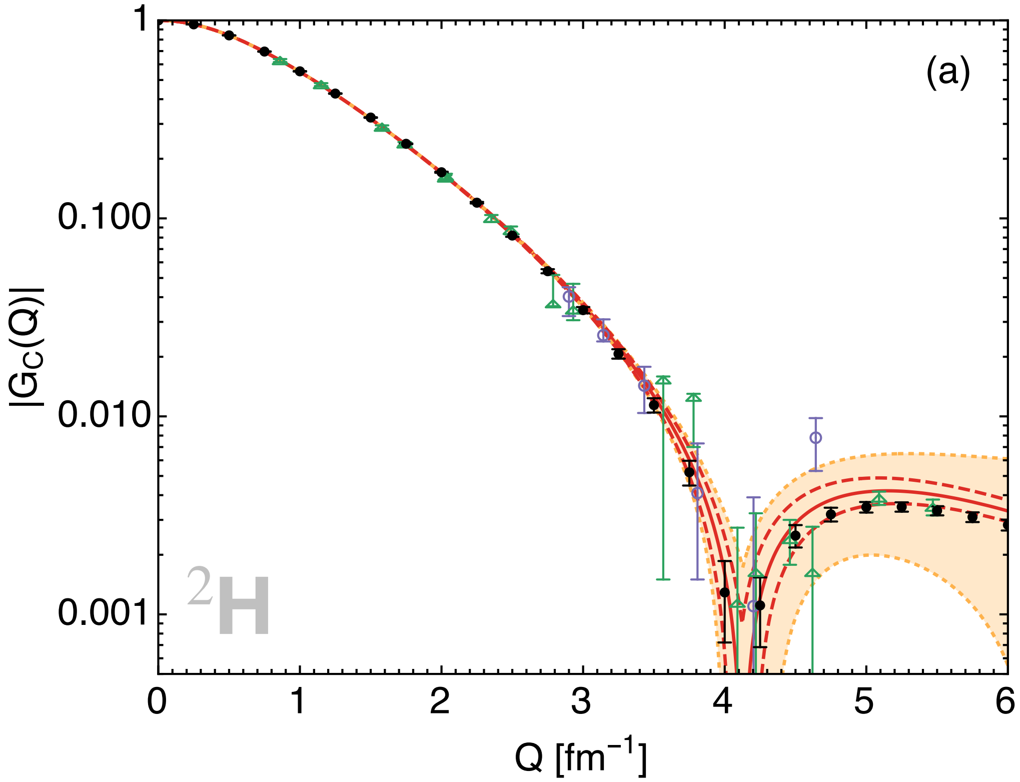
depend on **3 parameters (LECs)**
$^3S_1$-$^3S_1$ - can be fitted to deuteron FF data
$^3S_1$-$^3D_1$ - this one too
$^1S_0$-$^1S_0$ - can be fitted to ^{4}He FF data

Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

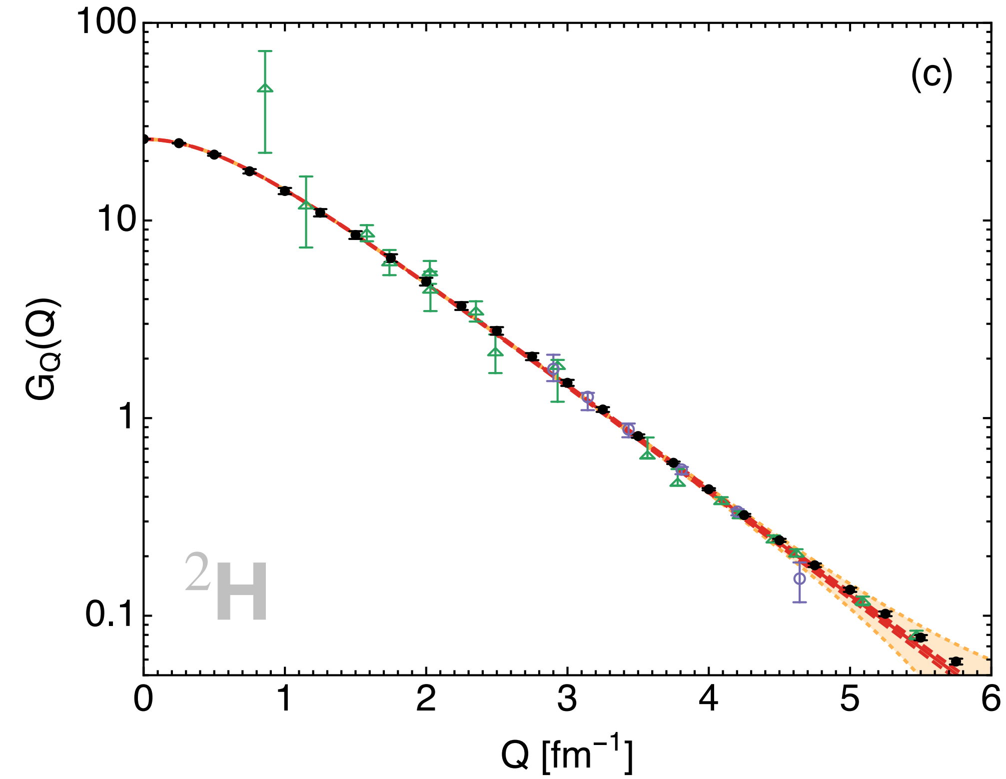
Deuteron charge and quadrupole form factors



Deuteron charge form factor



Deuteron quadrupole form factor



our result + N⁴LO truncation uncertainty

statistical uncertainty

Experimental data

Parameterisation by I.Sick (not used in the fit)

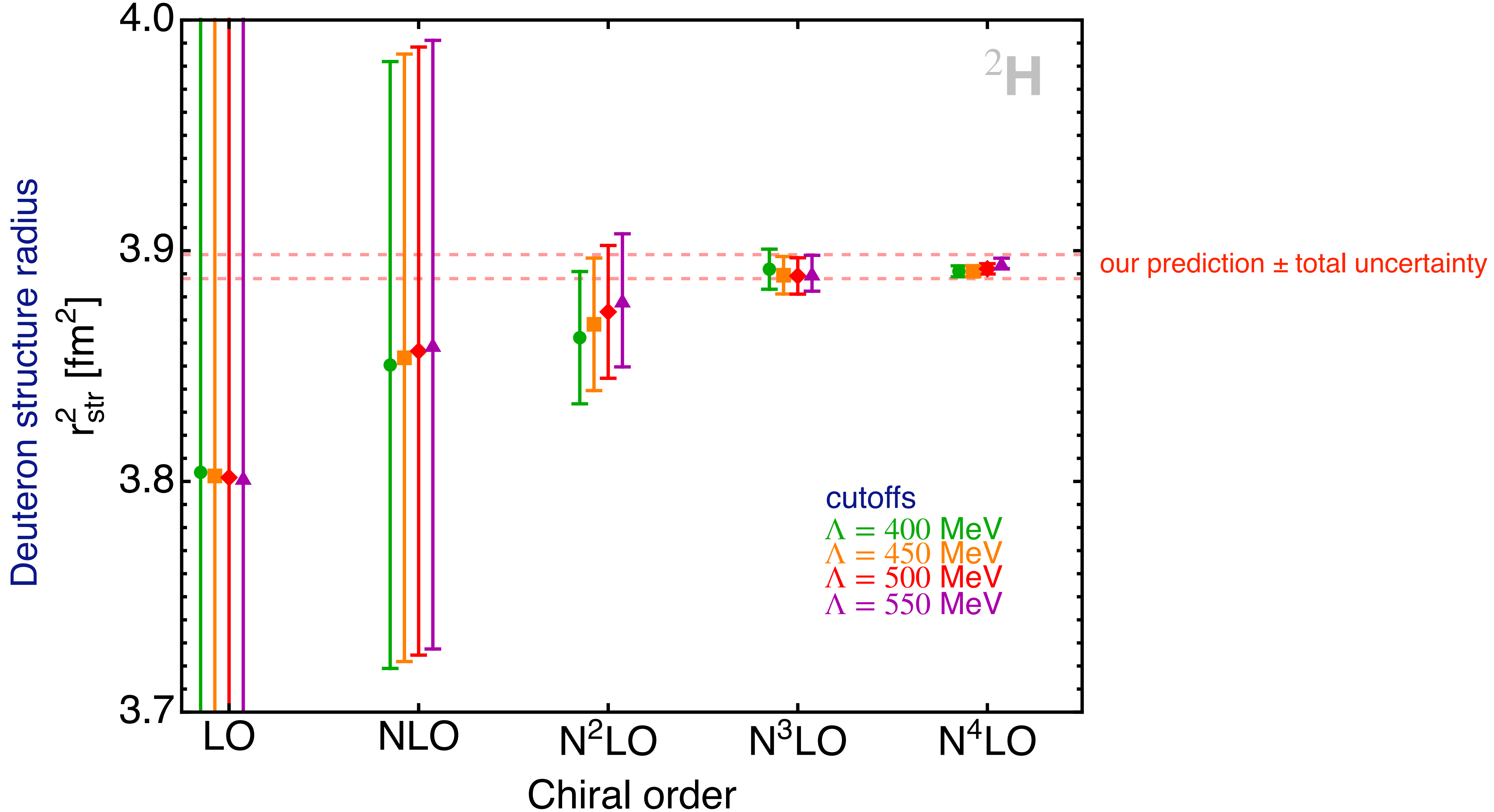
Extraction of deuteron structure radius and quadrupole moment

$$r_{str} = 1.9729^{+0.0015}_{-0.0012}\, fm \qquad Q_d = 0.2854^{+0.0038}_{-0.0017}\, fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Truncation uncertainty of ^{2}H structure radius

Using Bayesian model to estimate truncation uncertainty at each order Epelbaum et al. EPJA 56, 92 (2020)

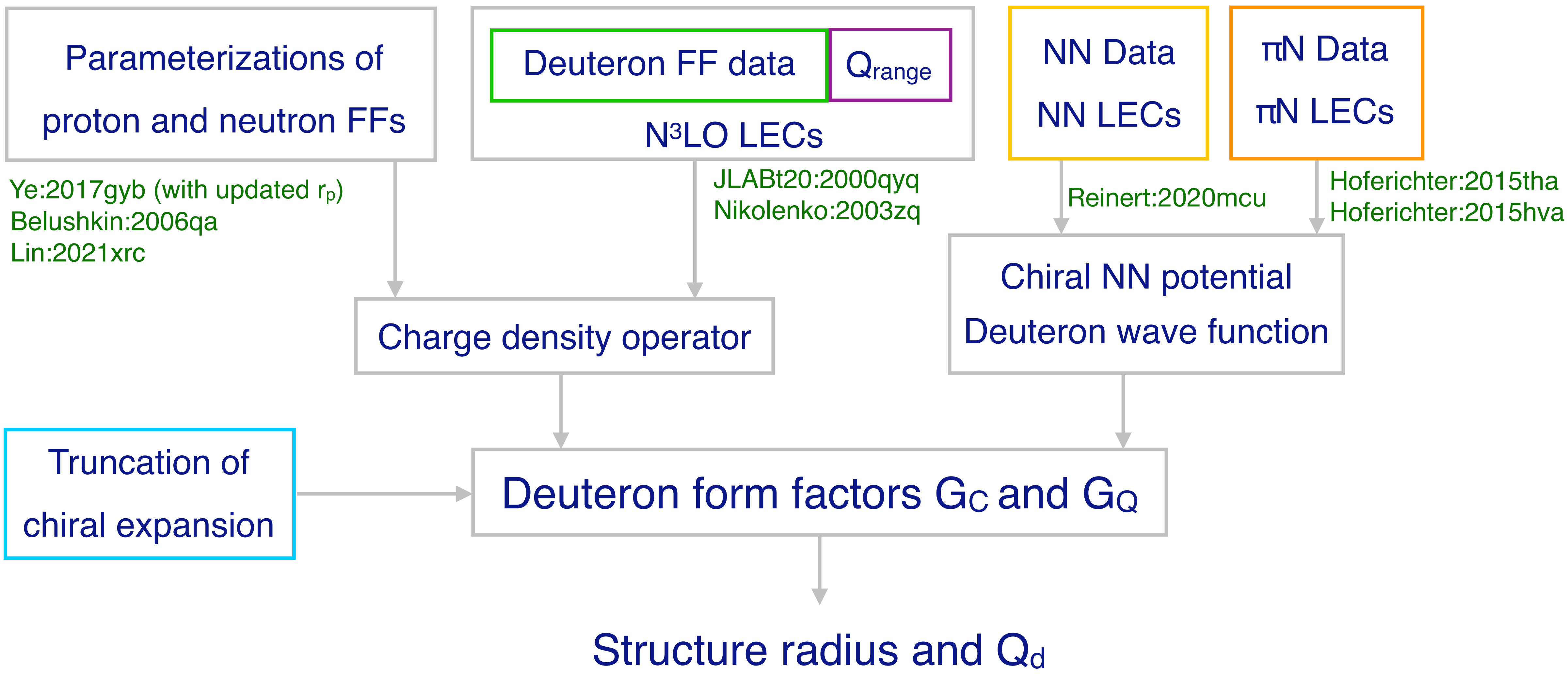


Chiral EFT expansion converges very well

Cutoff dependence is smaller than the truncation uncertainty

Uncertainty analysis of deuteron structure radius

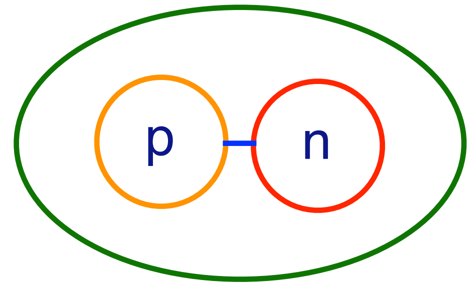
We propagate uncertainties from multiple sources



	Central	Truncation	$\rho^{\text{reg}}_{\text{Cont}}$	πN LECs RSA	2N LECs and f_i^2	Q range	Total
r^2_{str} (fm^2)	3.8925	± 0.0030	± 0.0024	± 0.0003	± 0.0025	$+0.0035$ -0.0005	$+0.0058$ -0.0046
Q_d (fm^2)	0.2854	± 0.0005	± 0.0007	± 0.0003	± 0.0016	$+0.0035$ -0.0005	$+0.0038$ -0.0017

Neutron charge radius from high-accuracy χEFT calculation of deuteron structure radius

^{2}H



$$r_d^2 = r_{str}^2(^2\mathrm{H}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Deuteron-proton radius difference

$$(r_d^2 - r_p^2) = 3.82070(31) fm^2$$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)
Jentschura et al. PRA 83 (2011)

Our accurate **χEFT calculation**

of the **deuteron structure** radius

New method of determination

of the neutron charge radius

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2(^2\mathrm{H})$$

$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

Neutron charge radius in PDG 2022

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm^2)	*DOCUMENT ID*		*COMMENT*
−0.1155±0.0017 OUR AVERAGE			
−0.115 ±0.002 ±0.003	KOPECKY	97	*n e* scattering (Pb)
−0.124 ±0.003 ±0.005	KOPECKY	97	*n e* scattering (Bi)
−0.114 ±0.003	KOESTER	95	*n e* scattering (Pb, Bi)
−0.115 ±0.003	[1] KROHN	73	*n e* scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •			
−0.1101±0.0089	[2] HEACOCK	21	*n* interferometry
$-0.106\,^{+0.007}_{-0.005}$	[3] FILIN	20	chiral EFT analysis
$-0.117\,^{+0.007}_{-0.011}$	BELUSHKIN	07	Dispersion analysis
−0.113 ±0.003 ±0.004	KOPECKY	95	*n e* scattering (Pb)
−0.134 ±0.009	ALEKSANDR...	86	*n e* scattering (Bi)
−0.114 ±0.003	KOESTER	86	*n e* scattering (Pb, Bi)
−0.118 ±0.002	KOESTER	76	*n e* scattering (Pb)
−0.120 ±0.002	KOESTER	76	*n e* scattering (Bi)
−0.116 ±0.003	KROHN	66	*n e* scattering (Ne, Ar, Kr, Xe)

[1] KROHN 73 measured -0.112 ± 0.003 fm^2. This value is as corrected by KOESTER 76.

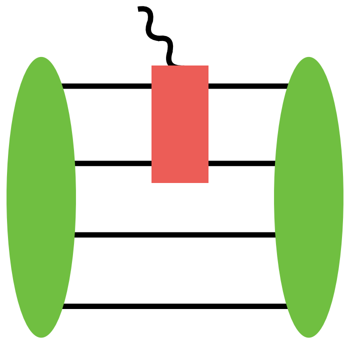
[2] HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

[3] FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

Extracted rn value shows 2σ deviation from the PDG (2022) weighted average $r_n^2 = -0.1155(17) fm^2$

^{4}He charge radius

Precision test of the chiral EFT for ^{4}He



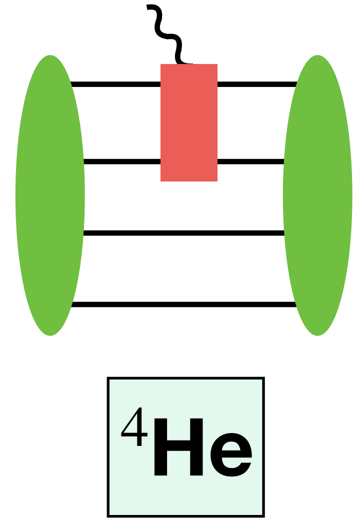
^{4}**He**

Preliminary results

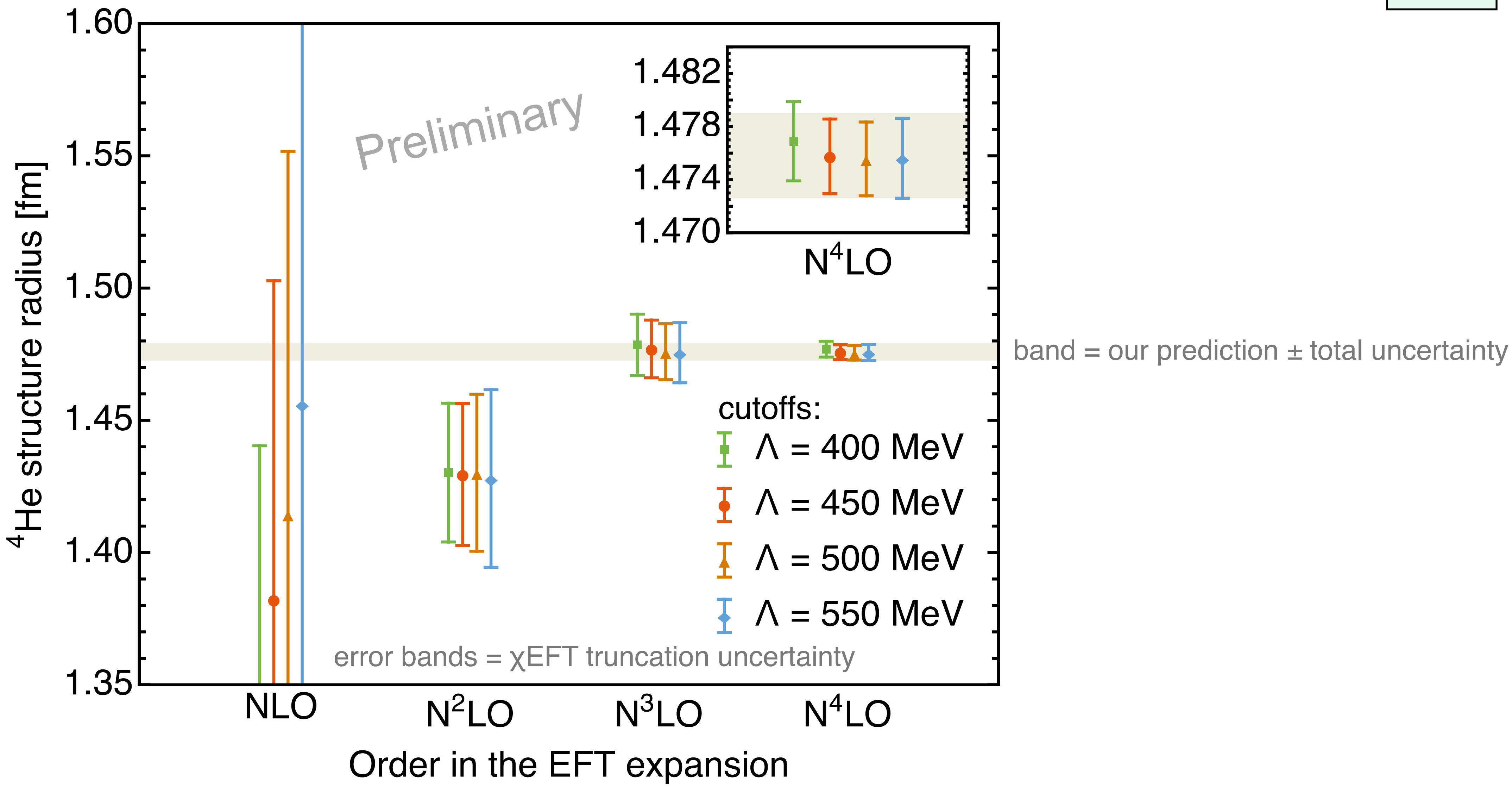
Prediction of ^{4}He structure radius

Our preliminary prediction for ^{4}He structure radius:

$$r_{str}(^4\mathbf{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}}\ fm$$ (Preliminary)



Estimation of truncation error:

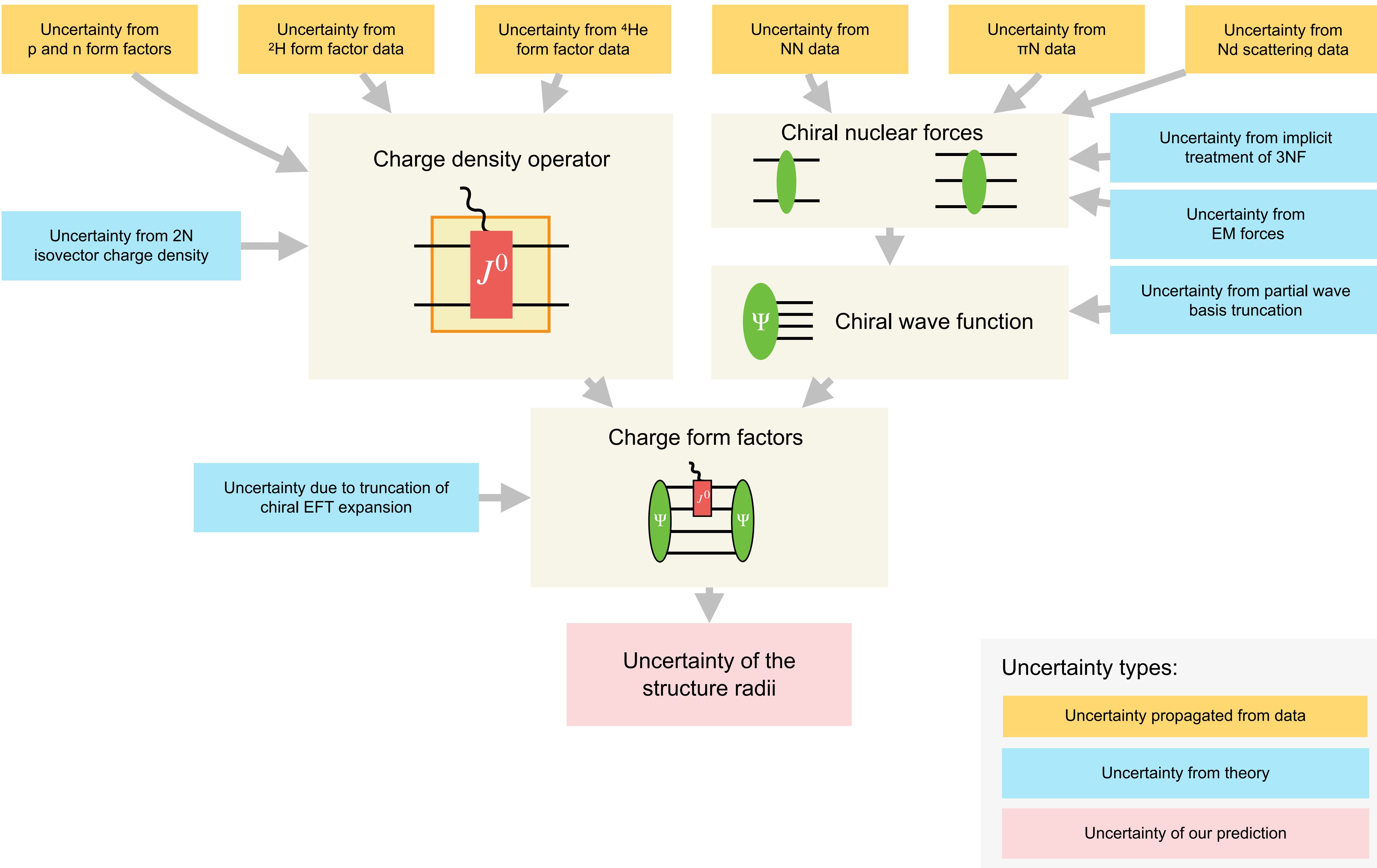


Cutoff dependence is smaller than the truncation uncertainty

Chiral EFT expansion converges well

Extensive uncertainty analysis

Propagation of uncertainties from data and theory



Prediction for ^{4}He charge radius

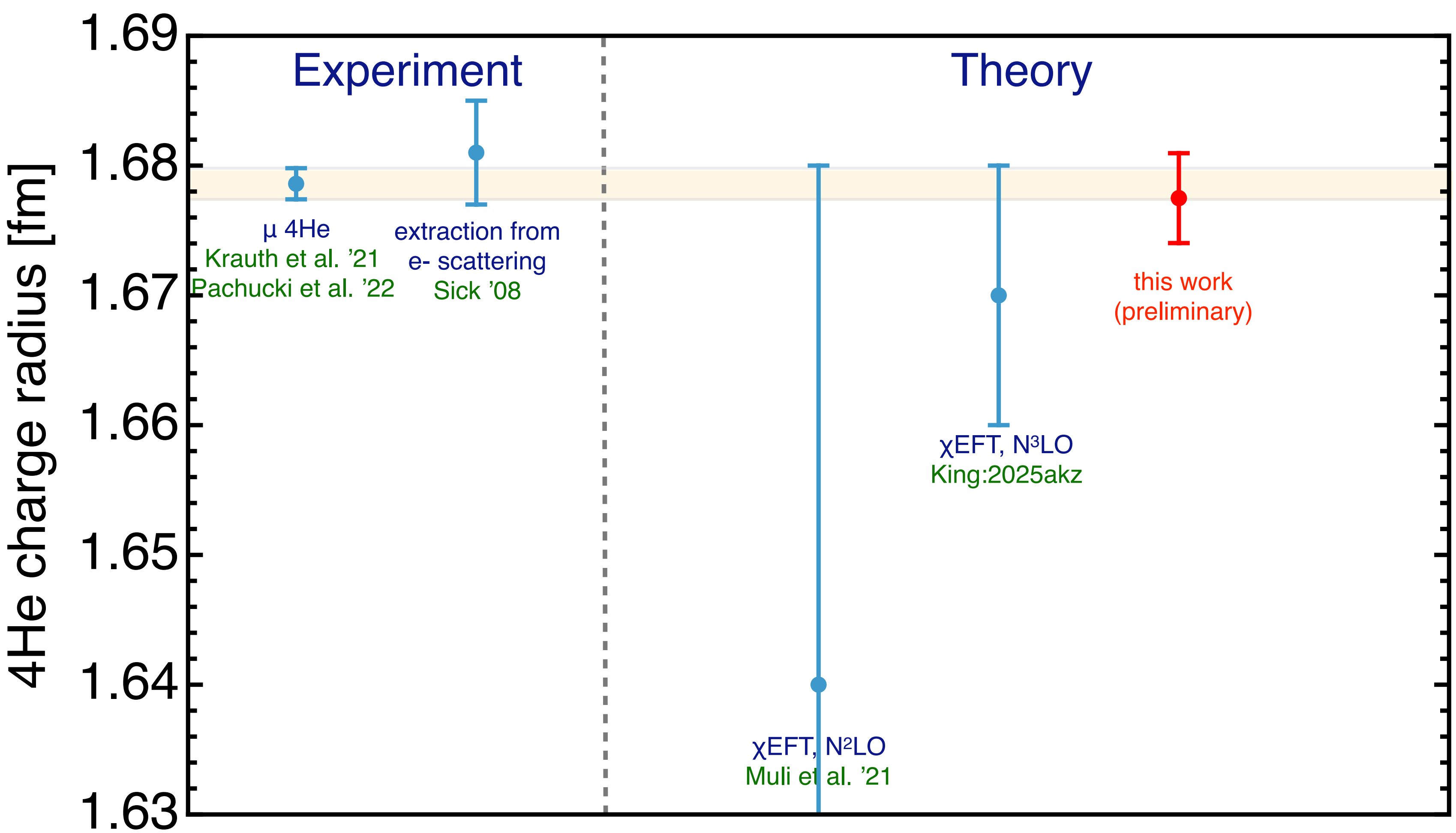
$$r_{str}(^4\mathbf{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}}\ fm$$ (Preliminary)

Our prediction for ^{4}He **charge** radius

$$r_C(^4\mathbf{He}) = (1.6775 \pm 0.0035)\, fm$$

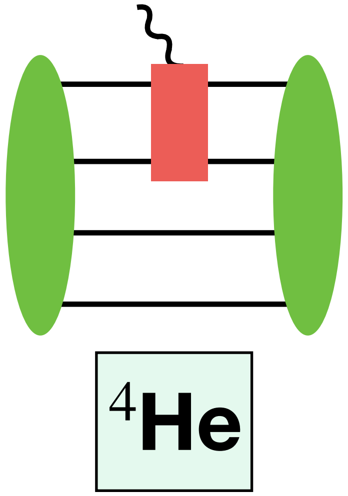
preliminary, using CODATA 2018 r_p and own determination of r_n

$$r_C(^4\mathbf{He}) = r_{str}^2(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$



Our prediction for ^{4}He charge radius is fully consistent with the muonic-atom spectroscopy

Isoscalar nucleon charge radius from data on ^{4}He



Our prediction for ^{4}He **structure** radius

Experimental ^{4}He charge radius
Krauth et al., Nature 589 (2021) 7843, 527-531
+ theory update from Pachucki et al. (2022)

$$r_C(^4\mathbf{He}) = r^2_{str}(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Determination

of the isoscalar nucleon charge radius

$$(r_n^2 + r_p^2) = (0.607 \pm 0.010)\, fm^2$$

preliminary

Proton charge radius from isoscalar nucleon radius

Our determination of the

isoscalar nucleon charge radius from ^{4}He

$(r_n^2 + r_p^2) = (0.607 \pm 0.010)\, fm^2$ preliminary

Our determination of the

neutron charge radius from ^{2}H

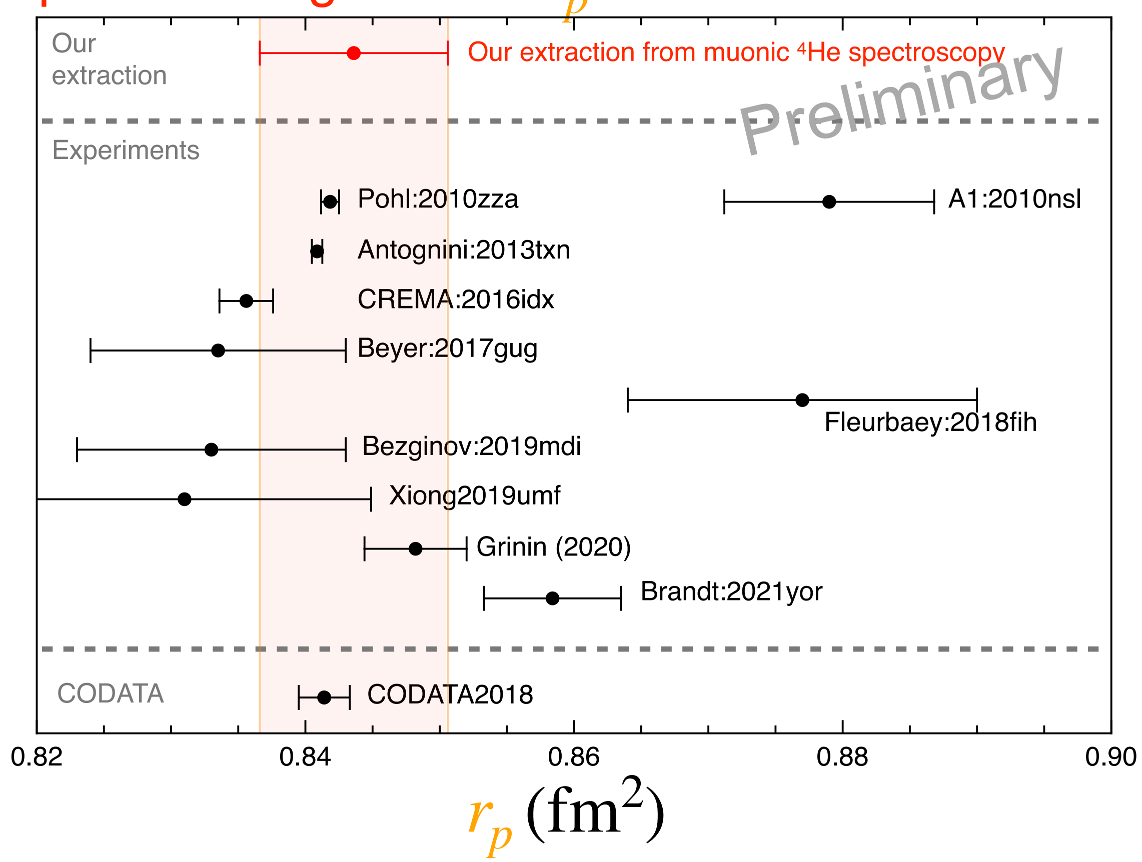
$r_n^2 = -0.105^{+0.005}_{-0.006}\, fm^2$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius: $r_p = (0.844 \pm 0.007)\, fm$

preliminary

proton charge radius r_p



Our extraction supports the „small“ proton radius

Prediction for isoscalar 3N charge radius

With all LECs being fixed, we can predict the isoscalar 3N charge radius: $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Our result is 10x more precise than current experimental data:

the ^3H charge radius from e^- scattering experiments: $r_C^{3H} = (1.7550 \pm 0.0860)\, fm$ (5 %) Amroun et al. '94 (world average)

the ^3He charge radius from muonic ^3He: $r_C^{3He} = (1.9701 \pm 0.0009)\, fm$ (0.05 %) CREMA 2023 arXiv:2305.11679

Exp. **3N isoscalar charge radius**: (using muonic ^3He and old ^3H) $r_{C,exp.}^{isoscalar3N} = (1.9010 \pm 0.0260)\, fm$ (1.4 %)

T-REX experiment in Mainz [Pohl et al.] aims at measuring r_C^{3H} within ±0.0002 fm (400x more precise)

The isoscalar 3N radius will be then known within ±0.0009 fm

⇒ **precision tests of nuclear chiral EFT!**

Summary

Precise and accurate calculation of A = 2, 3, 4 isoscalar charge radii in chiral effective field theory

Extensive uncertainty analysis

Sub-percent accuracy for charge radii

Charge radii of neutron and proton from light nuclei:

- ^{2}H r_{str} combined with isotope-shift data => extracted the neutron charge radius (2σ tension with PDG)
- ^{4}He r_{str} combined with spectroscopic data => extracted isoscalar nucleon and proton charge radii preliminary

^{4}He calculation: preliminary

- calculated ^{4}He charge radius (0.2% accuracy) agrees with the new μ^{4}He measurement

^{3}H-^{3}He: preliminary

- predicted the isoscalar 3N charge radius r_C (0.1% accuracy)
- our r_C is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX (^{3}H) exp. in Mainz will allow for a precision test of nuclear chiral EFT

Outlook

In progress:

- **Consistent** inclusion of **isovector 2N currents at N^3LO** and **N^4LO**
- **Consistent** inclusion of **N^3LO** and **N^4LO three-nucleon forces**
- Analysis of **magnetic form factors** (PhD thesis of Daniel Möller)
- Application to processes with two photons (**polarizabilities**, …)
- Calculation of isoscalar N4LO **charge radii for nuclei with A>4** (work in progress by LENPIC collaboration)