THE HYPERON-NUCLEON INTERACTION IN LOW-ENERGY EFFECTIVE FIELD THEORY

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In collaboration with

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Aim of the project

Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

1. Introduction
2. Potential model derivation

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3. Cross section calculation
4. Fitting procedure

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Develop a **local** potential model for the ΛN interaction, in a **contact EFT** approach

1. Introduction
2. Potential model derivation
3. Cross section calculation
4. Fitting procedure
5. Preliminary results
6. Outlook

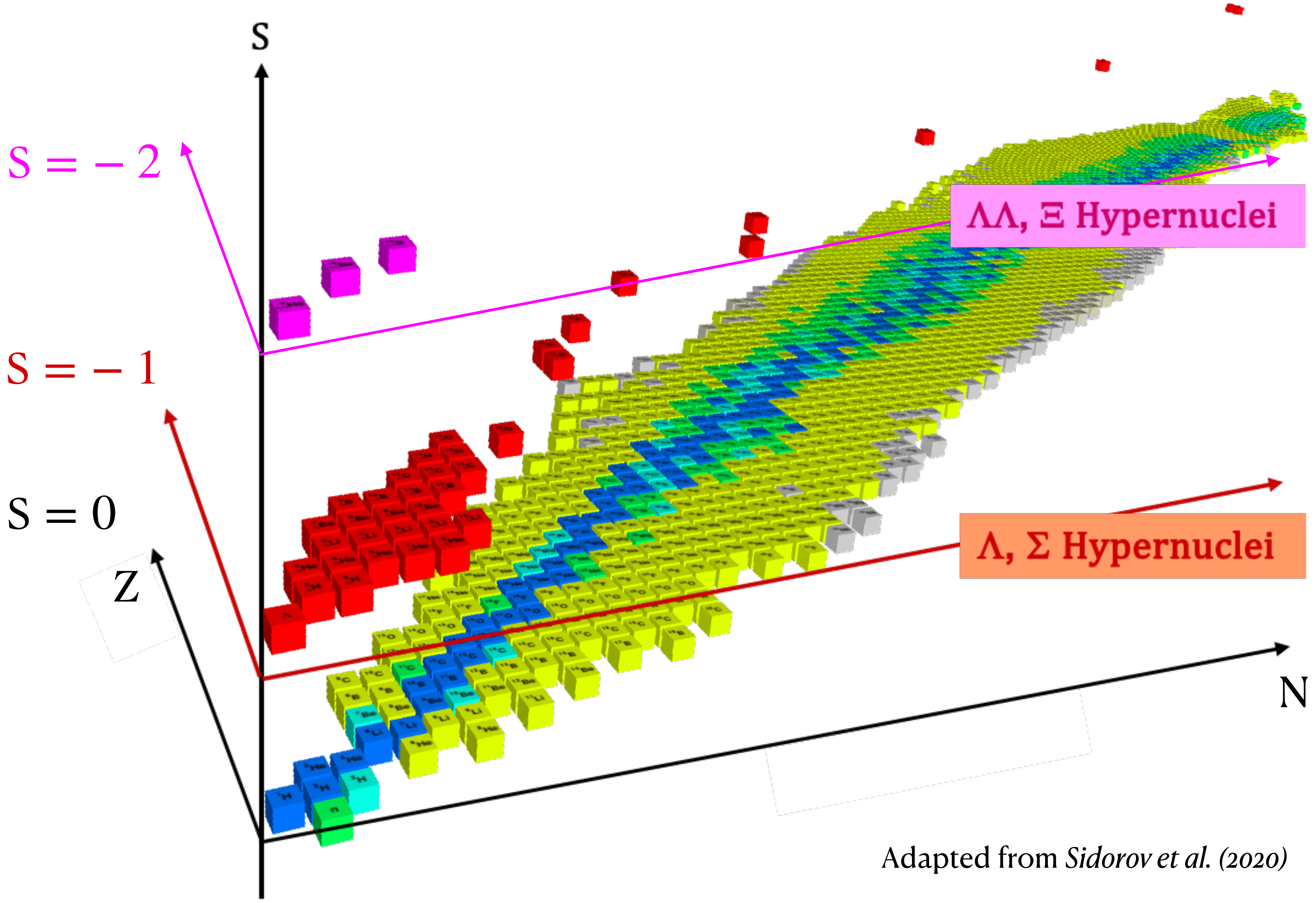
Introduction

Λ HYPERON

- Mass 1115.683 ± 0.006 MeV
- Lifetime $(2.617 \pm 0.010) \times 10^{-10}$ s
- Isospin, charge $= 0$

MOTIVATIONS

- Hyperon puzzle in Neutron Stars
- Hypernuclei studies



Adapted from *Sidorov et al. (2020)*

Potential model derivation

EFFECTIVE FIELD THEORY APPROACH

QCD

quarks, gluons

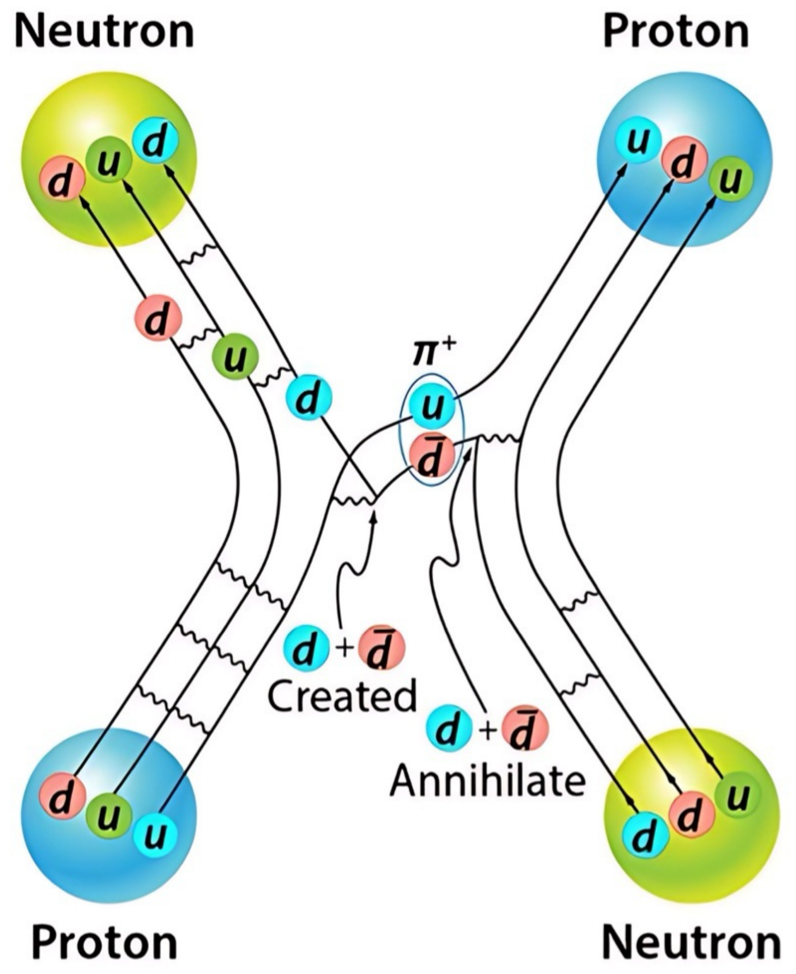
$\gtrsim$ 1 GeV

⇨

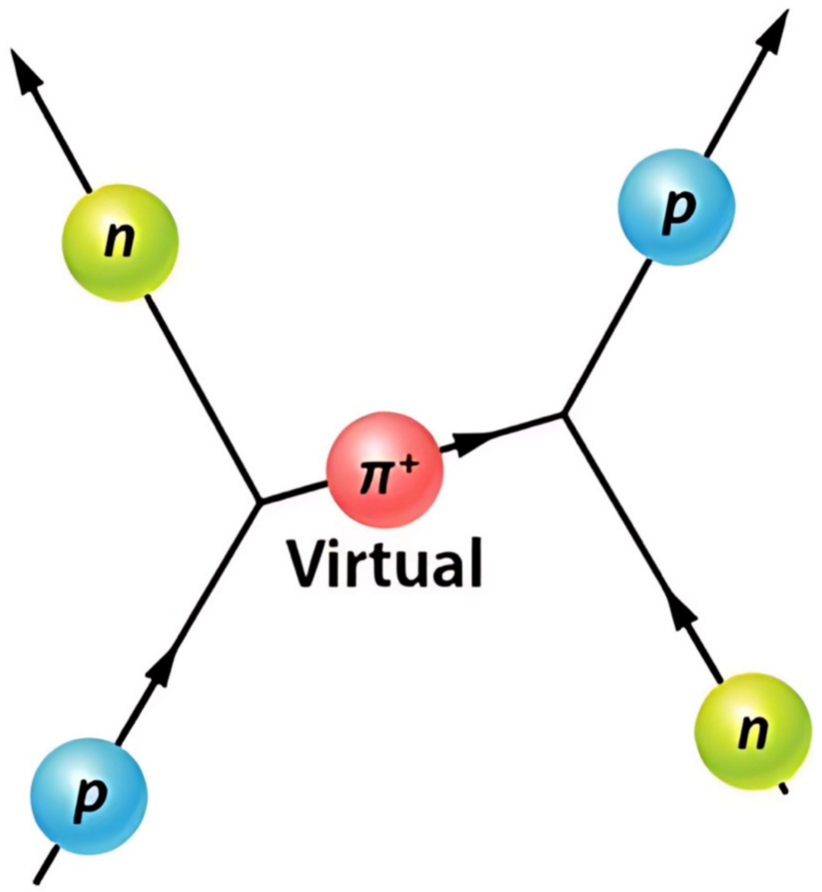
χEFT

nucleons, pions

$\sim$ 100 MeV



Adapted from M. Piarulli (2024)



- Interaction described by most general Lagrangian that respects the symmetries of QCD

- Cutoff scale (Λ_χ) → defines range of applicability of the theory

  ↓

  High energy effects included in contact terms that depend on low energy constants (LECs)

  ↓

  LECs determined through fit to experimental data

- Lagrangian expanded in powers of Q/Λ_χ

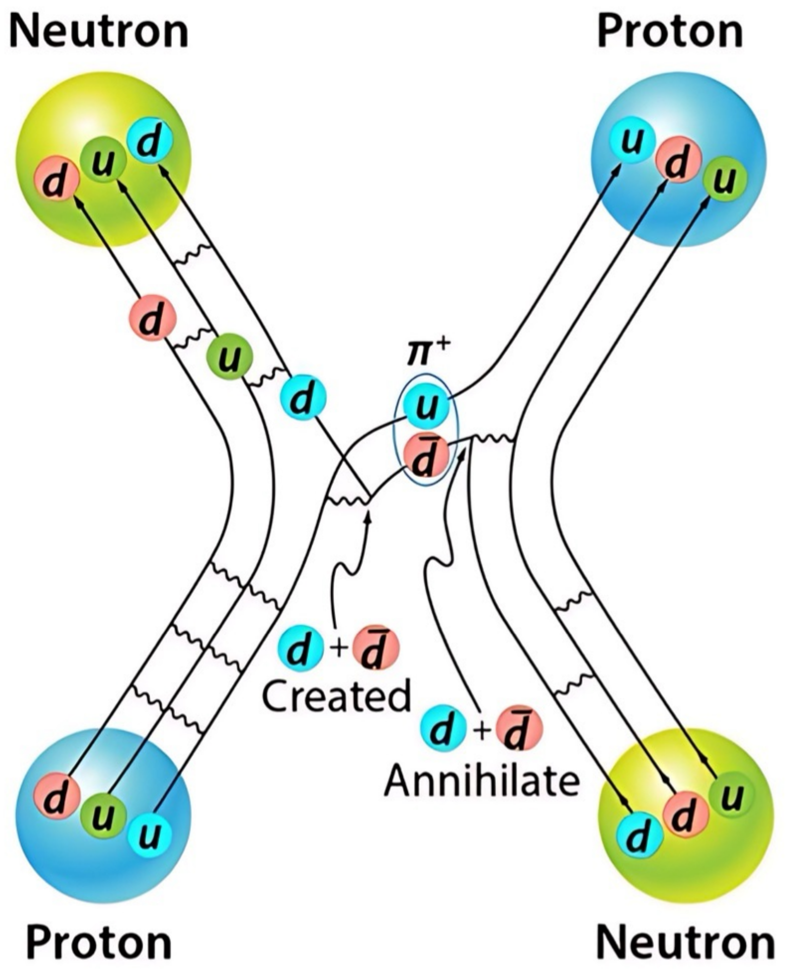
  ⇒ Organization in leading and sub-leading terms (LO, NLO, ...)

Potential model derivation

EFFECTIVE FIELD THEORY APPROACH

In $\not{\pi}$EFT even pions can be considered high-energy degrees of freedom $\Rightarrow$ interaction described only by contact terms

QCD	χEFT	$\not{\pi}$EFT
quarks, gluons	**nucleons, pions**	**nucleons**
$\gtrsim 1$ GeV	~ 100 MeV	~ 10 MeV



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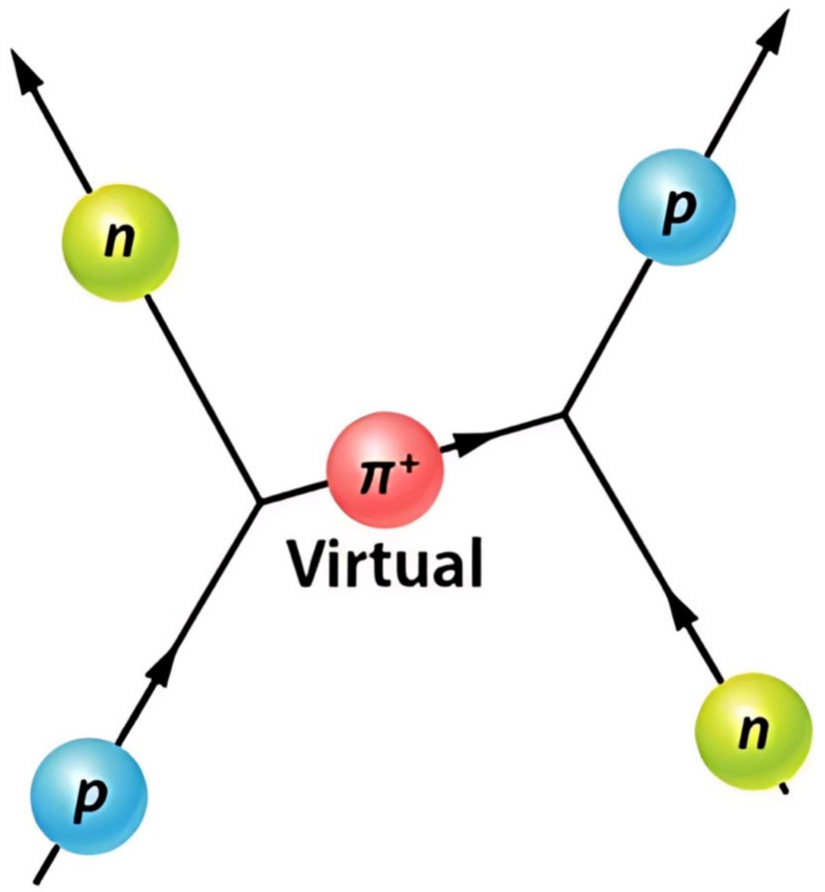

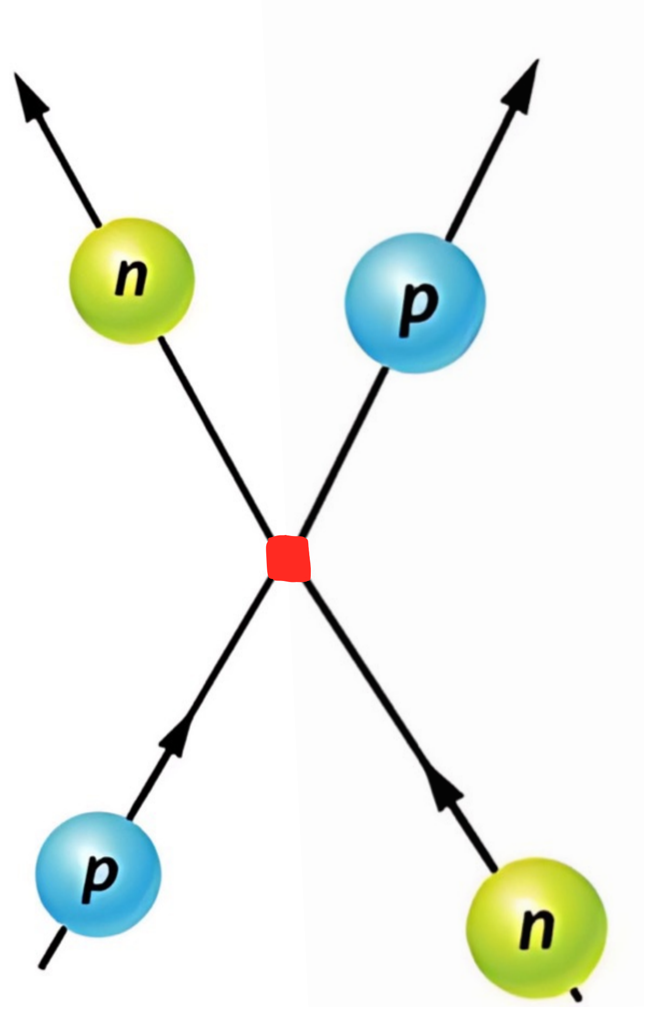

Potential model derivation

HYPERON-NUCLEON POTENTIAL MODEL – MOMENTUM SPACE

Aim of the project : develop a **local** potential model for the hyperon-nucleon interaction, in a **contact EFT** approach

Literature:

– *Haidenbauer et al. (2023)*: YN interaction in χEFT up to N2LO, momentum space

– *Schiavilla et al. (2021)*: NN interaction in contact EFT up to N3LO, coordinate space

↓

- Hyperon-nucleon interaction in momentum space, up to NLO, only contact terms:

$$V_{\Lambda N}^{\mathrm{LO}} = C_S + C_T\,(\sigma_\Lambda \cdot \sigma_N)\,,$$

$$V_{\Lambda N}^{\mathrm{NLO}} = C_1\mathbf{q}^2 + C_2\mathbf{q}^2(\sigma_\Lambda \cdot \sigma_N) + iC_3\mathbf{S}\cdot(\mathbf{k}\times\mathbf{q}) + C_4 S_{\Lambda N}(\mathbf{q}) + iC_5\mathbf{D}\cdot(\mathbf{k}\times\mathbf{q})$$

↓

7 LECs

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}\,,$$
$$\mathbf{k} = (\mathbf{p} + \mathbf{p}')/2$$
$$\mathbf{S} = (\sigma_\Lambda + \sigma_N)/2\,,$$
$$\mathbf{D} = (\sigma_\Lambda - \sigma_N)/2$$
$$S_{\Lambda N}(\mathbf{q}) = 3\,\sigma_\Lambda \cdot \mathbf{q}\,\sigma_N \cdot \mathbf{q} - q^2 \sigma_\Lambda \cdot \sigma_N\,.$$

Potential model derivation

HYPERON-NUCLEON POTENTIAL MODEL – MOMENTUM SPACE

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$\downarrow$

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$$\mathbf{q} = \mathbf{p}' - \mathbf{p}\,,$$
$$\mathbf{k} = (\mathbf{p} + \mathbf{p}')/2$$
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$$S_{\Lambda N}(\mathbf{q}) = 3\,\sigma_\Lambda\cdot\mathbf{q}\,\sigma_N\cdot\mathbf{q} - q^2\sigma_\Lambda\cdot\sigma_N\,.$$

Potential model derivation

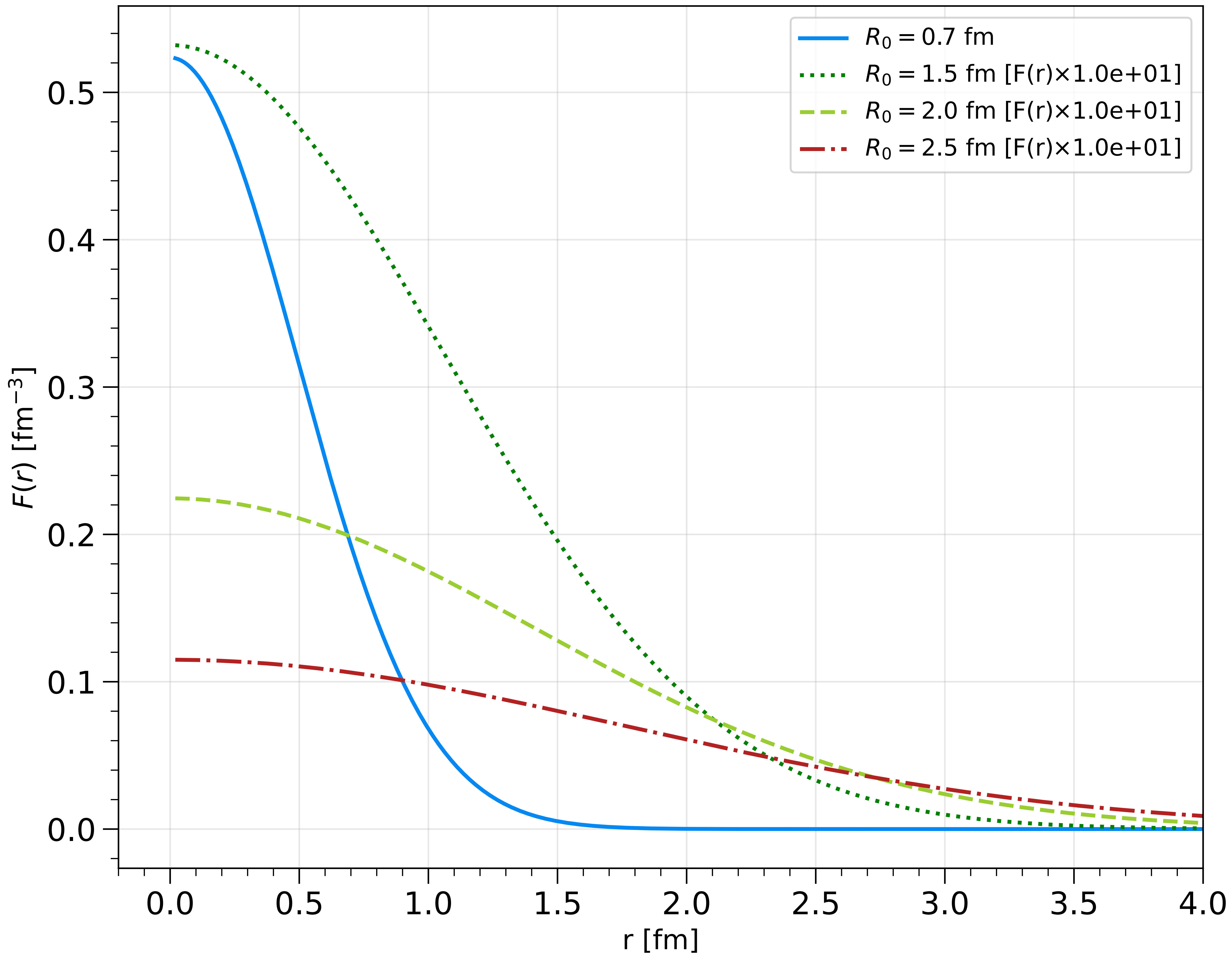
HYPERON-NUCLEON POTENTIAL MODEL – COORDINATE SPACE

- To obtain $V_{\Lambda N}$ in coordinate space → Fourier transform

- To regularize the interaction (avoid diverging integrals) ⟶ multiply $V_{\Lambda N}$ by a regulator function $F(r)$, as done in *Schiavilla et al. (2021)*

$$F(r) = \frac{1}{\pi^{3/2} R_0^3} \exp\left(-\frac{r^2}{R_0^2}\right)$$

- Investigated **cutoff parameter** values $R_0 \in [0.7, 2.5]$ fm



Potential model derivation

HYPERON-NUCLEON POTENTIAL MODEL – COORDINATE SPACE

- Fourier transform of the regularized potential in momentum space to obtain $V_{\Lambda N}$ in coordinate space

$$V_{\Lambda N}^{\mathrm{LO}} = \left[C_S + C_T\,(\sigma_\Lambda \cdot \sigma_N)\right] F(r)\,,$$

$$V_{\Lambda N}^{\mathrm{NLO}} = \sum_i \mathbf{v_i}\,(\mathbf{r})\,\mathcal{O}_i\,, \text{ with } \mathcal{O}_i = \mathbf{1}, \sigma_\Lambda \cdot \sigma_N, S_{\Lambda N}(\hat{\mathbf{r}}), \mathbf{L}\cdot\mathbf{S}, \mathbf{L}\cdot\mathbf{D}$$

Radial functions containing combinations of $F(r), F'(r), F''(r)$ and LECs $(C_1, \cdots, C_5)$

$$\downarrow$$

fixed through fitting procedure to experimental data $\rightarrow$ Λp elastic scattering cross section

$$S_{\Lambda N}(\hat{\mathbf{r}}) = 3\,\sigma_\Lambda \cdot \hat{\mathbf{r}}\,\sigma_N \cdot \hat{\mathbf{r}} - \sigma_\Lambda \cdot \sigma_N\,,$$

$$\mathbf{S} = (\sigma_\Lambda + \sigma_N)/2\,,$$

$$\mathbf{D} = (\sigma_\Lambda - \sigma_N)/2\,.$$

Cross section calculation

Available experimental data to perform the fit: Λp elastic scattering cross section

- Initial and final scattering states expanded in partial waves $\longrightarrow J^\pi$

- Decompose wave function $\longrightarrow$ **short-range (core)** $\psi_c^\alpha(q,\mathbf{r})$; nuclear potential is important, two particles close to each other $\longrightarrow$ $\psi_\alpha^c(q,r) = \sum_{i=1}^{M} d_{\alpha,i}(q) f_i(r)$

$\downarrow$

long-range (asymptotic) $\psi_a^\alpha(q,\mathbf{r})$; nuclear potential is negligible, two particles far apart $\longrightarrow$ expansion in regular and irregular spherical Bessel functions, with **R-matrix**

$$\psi_a^\alpha(q,\mathbf{r}) = \sum_\beta \left[\delta_{\alpha\beta}\,\Omega_\beta^R(q,\mathbf{r}) + R_{\alpha\beta}\,\Omega_\beta^I(q,\mathbf{r})\right]$$

Cross section calculation

- Kohn variational principle to compute the *R*-matrix and d-coefficients

$$\longrightarrow \quad [R_{\alpha\beta}(q)] = R_{\alpha\beta}(q) - \frac{2\mu}{\hbar^2}\langle \psi_\beta(q,\mathbf{r})\,|\,H - E\,|\,\psi_\alpha(q,\mathbf{r})\rangle$$

$$\downarrow$$

stationary with respect to the variation of any unknown coefficient ($d_{\alpha,i}(q), R_{\alpha\beta}(q)$ in this case)

Cross section calculation

- Kohn variational principle to compute the *R*-matrix and d-coefficients $\longrightarrow$ $[R_{\alpha\beta}(q)] = R_{\alpha\beta}(q) - \frac{2\mu}{\hbar^2}\langle\psi_\beta(q,\mathbf{r})\,|\,H - E\,|\,\psi_\alpha(q,\mathbf{r})\rangle$
  $\downarrow$
  stationary with respect to the variation of any unknown coefficient ($d_{\alpha,i}(q), R_{\alpha\beta}(q)$ in this case)

- Rewrite asymptotic wave function a sum of a **plane wave** and an **outgoing spherical wave**:

$$\psi_\alpha^a(q,\mathbf{r}) = -2i\left[\Omega_\alpha^R(q,\mathbf{r}) + \sum_\beta T_{\alpha\beta}\,\Omega_\alpha^+(q,\mathbf{r})\right], \qquad \Omega_\alpha^+(q,\mathbf{r}) = \Omega_\alpha^I(q,\mathbf{r}) + i\Omega_\alpha^R(q,\mathbf{r})$$

- The ***T*-matrix** is computed from the *R*-matrix: $T_{\alpha\beta} = i(R_{\alpha\beta} + i\delta_{\alpha\beta})^{-1}R_{\alpha\beta}$

- Total unpolarized cross section: $\sigma = \frac{4\pi}{q^2}\sum_{J,\alpha,\beta}\left|T_{\alpha\beta}\right|^2\frac{(2J+1)}{(2s_\Lambda+1)(2s_N+1)}$

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- χ^2 function using a constraint on **scattering length** obtained from potential model:

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th}(C_S, C_T, C_1, \ldots, C_5) - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

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Cross section experimental data from *Radboud University, NN online archive*

Fitting procedure

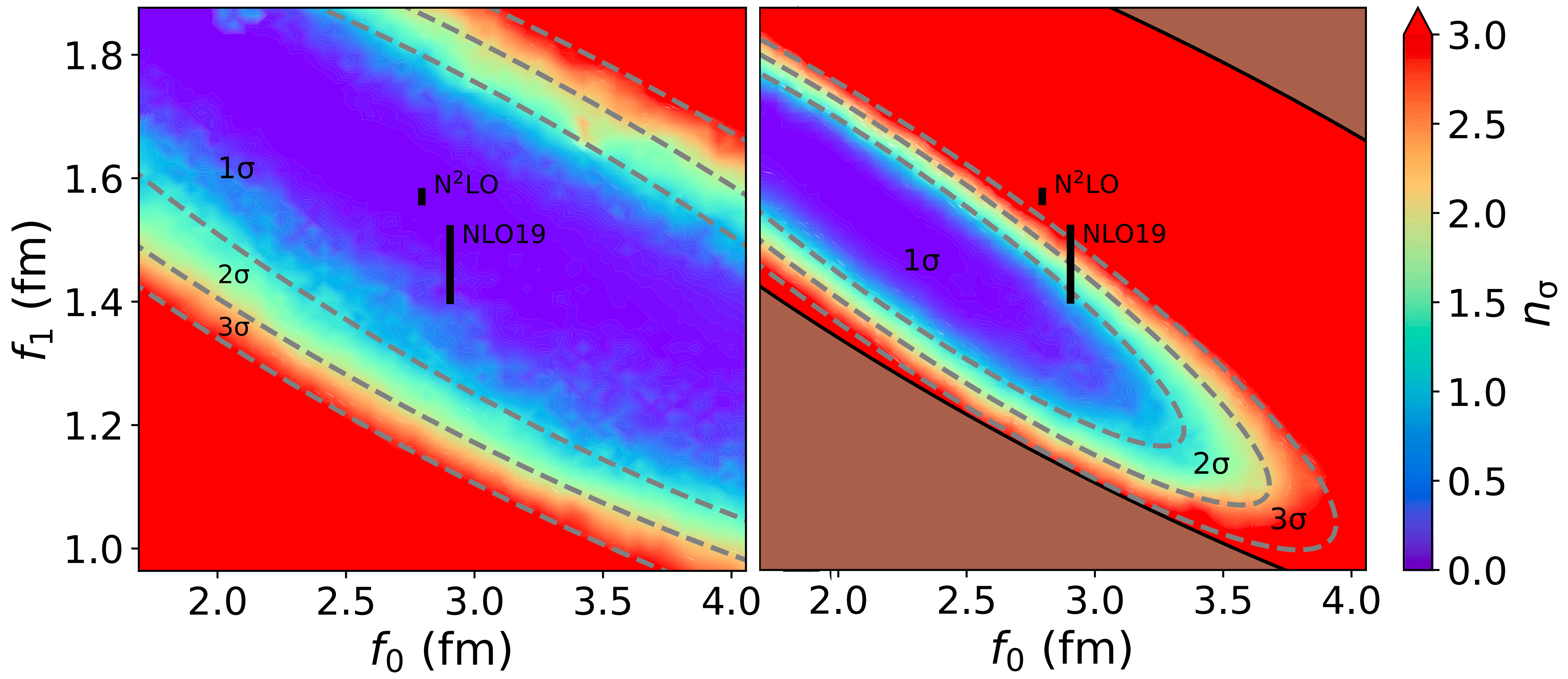
Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- χ^2 function using a constraint on **scattering length** obtained from potential model:

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th}(C_S, C_T, C_1, \ldots, C_5) - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

"Experimental" data for Λp scattering length from *Mihaylov et al. (2024)*

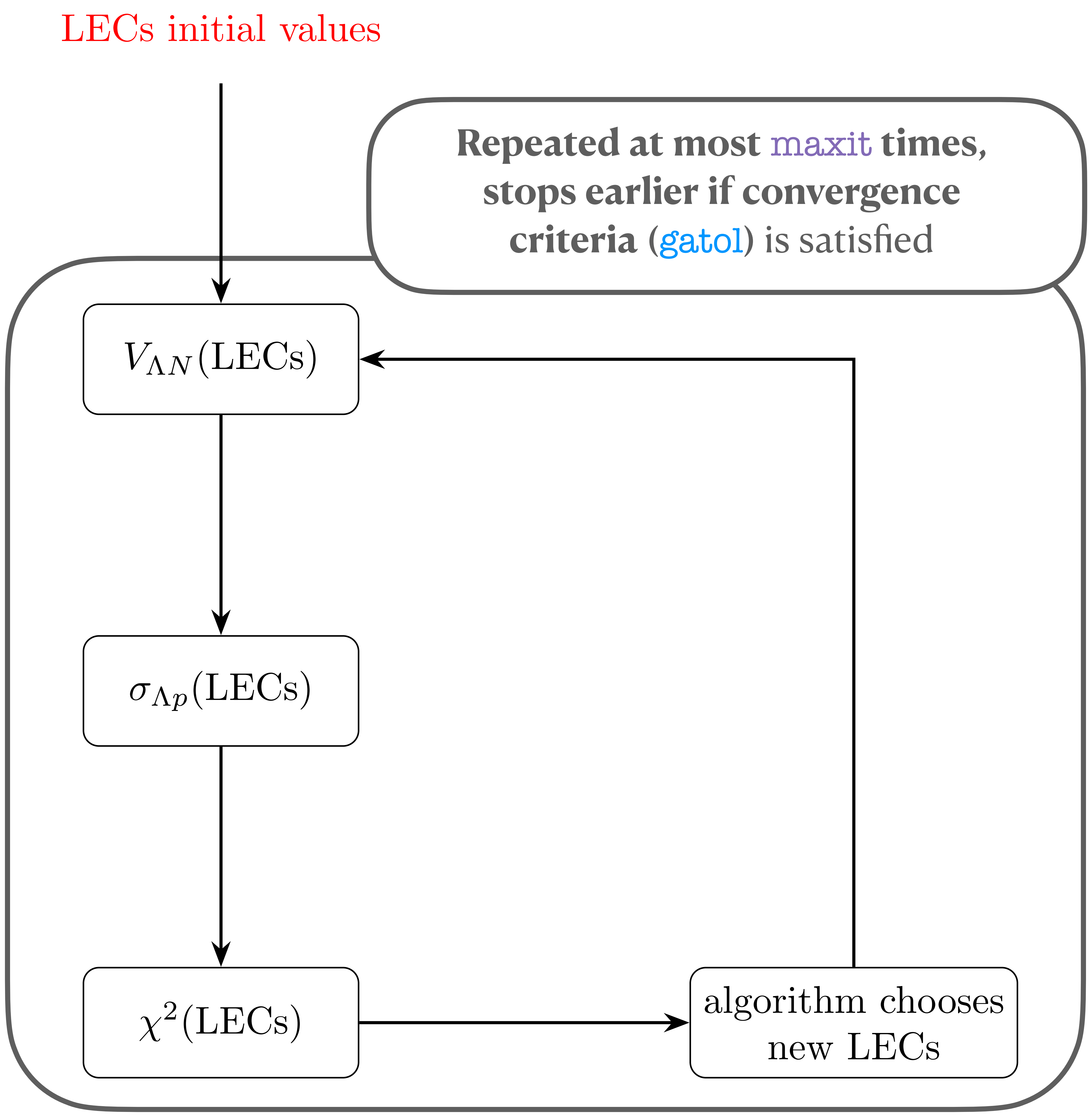
Cross section experimental data from *Radboud University, NN online archive*



Adapted from Mihaylov et al. (2024)

Fitting procedure

MINIMIZATION ALGORITHM



- TAOPOUNDERS, from PETSc + MPI to parallelise
- Adjustable parameters of the fitting procedure:
  - Cutoff parameter R_0
  - **Grid for initial values** of the LECs (max, min, step)
  - Parameter to define the threshold for converged optimization (gatol)
  - **Maximum number of calls** to optimization algorithm (maxit)
- Chosen values for algorithm parameters:
  - $R_0 \in [0.7, 2.5]$ fm
  - gatol $= 10^{-8}$
  - maxit $= 2 \times 10^3$

Fitting procedure

LO VS NLO FITTING STRATEGY

LO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 15$ MeV
  - $J = 0, 1$
  - Only positive parity
- All LECs chosen on a grid

NLO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 80$ MeV
  - $J = 0, 1$
  - Both parities
- LO LECs (i.e. C_s, C_T) fixed at LO best results
- NLO LECs (i.e. $C_1, \ldots, C_5$) chosen on a grid

Fitting procedure

LO VS NLO FITTING STRATEGY

LO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 15$ MeV
  - $J = 0, 1$
  - Only positive parity
- All LECs chosen on a grid
- Parameters for grid of initial values:
  - $\min = -15$
  - $\max = 15$ $\quad\Rightarrow\quad \sim 900$ points
  - step $= 1$

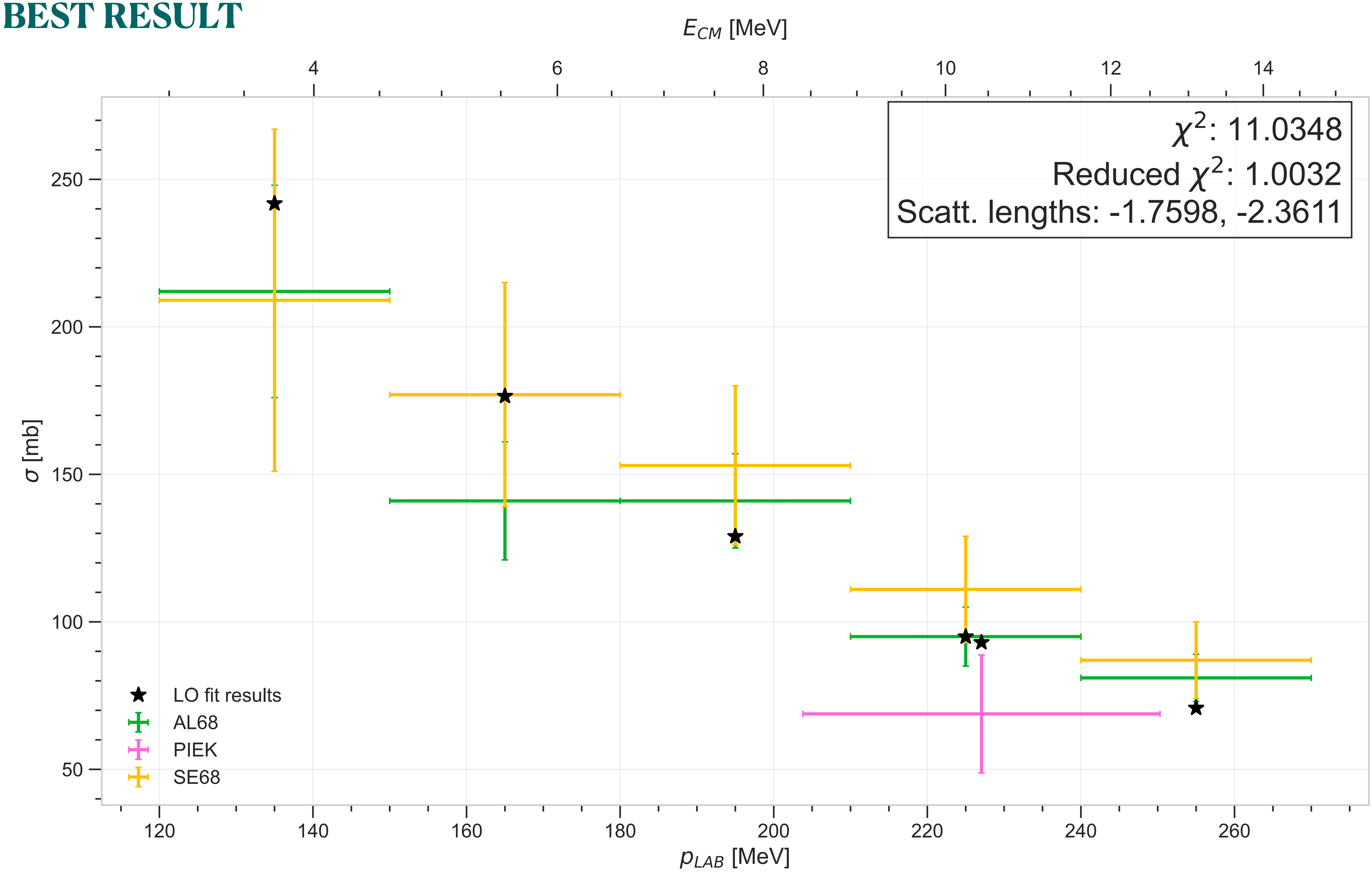
NLO

- Total angular momentum, energy and parity constraints:
  - $E_{CM} < 80$ MeV
  - $J = 0, 1$
  - Both parities
- LO LECs (i.e. C_s, C_T) fixed at LO best results
- NLO LECs (i.e. $C_1, \ldots, C_5$) chosen on a grid
- Parameters for grid of initial values:
  - $\min = -15$
  - $\max = 15$ $\quad\Rightarrow\quad \sim 10^5$ points
  - step $= 3$

Preliminary results

LEADING ORDER - BEST RESULT

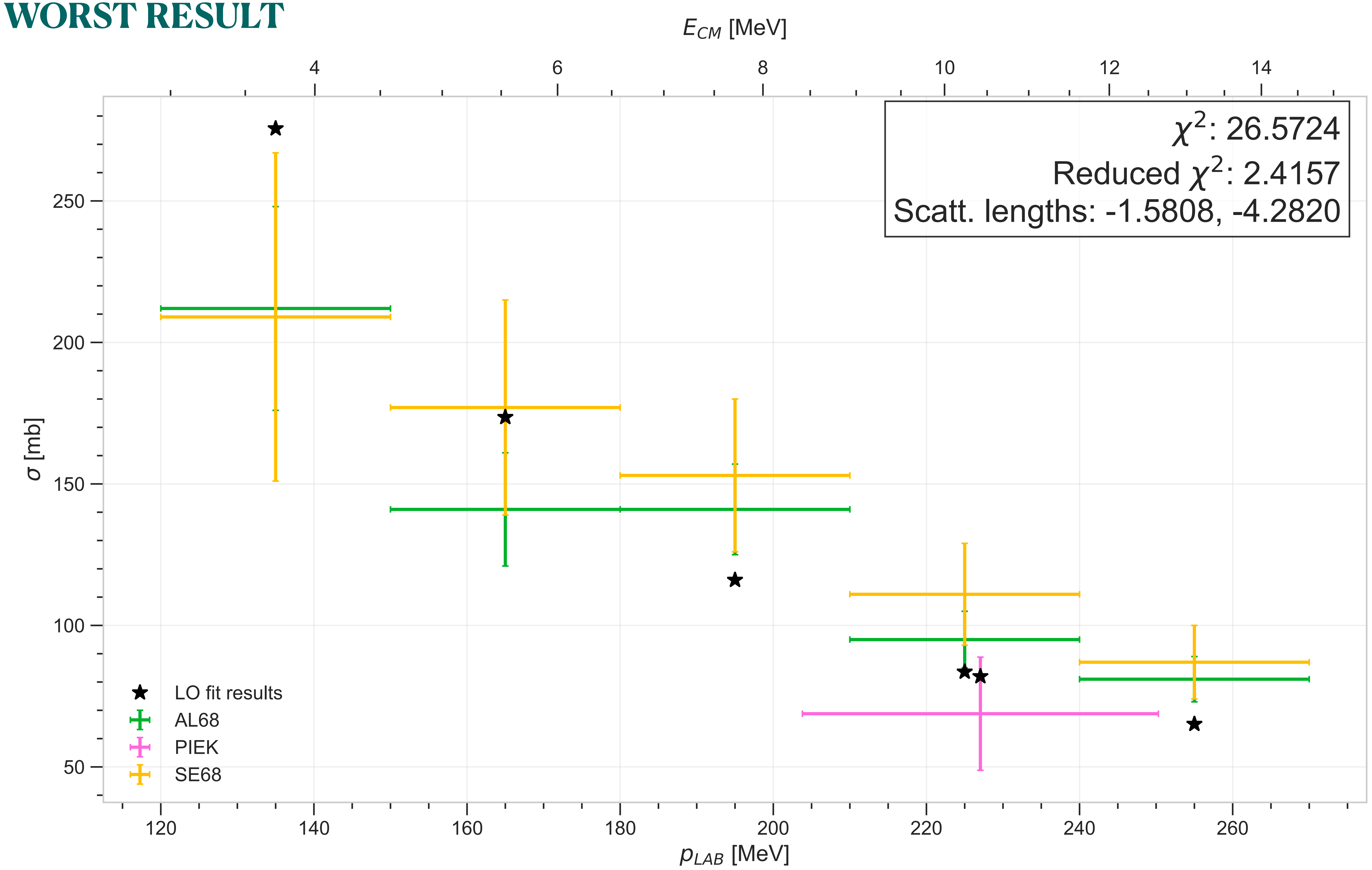
$R_0 = 1.5$ fm



Preliminary results

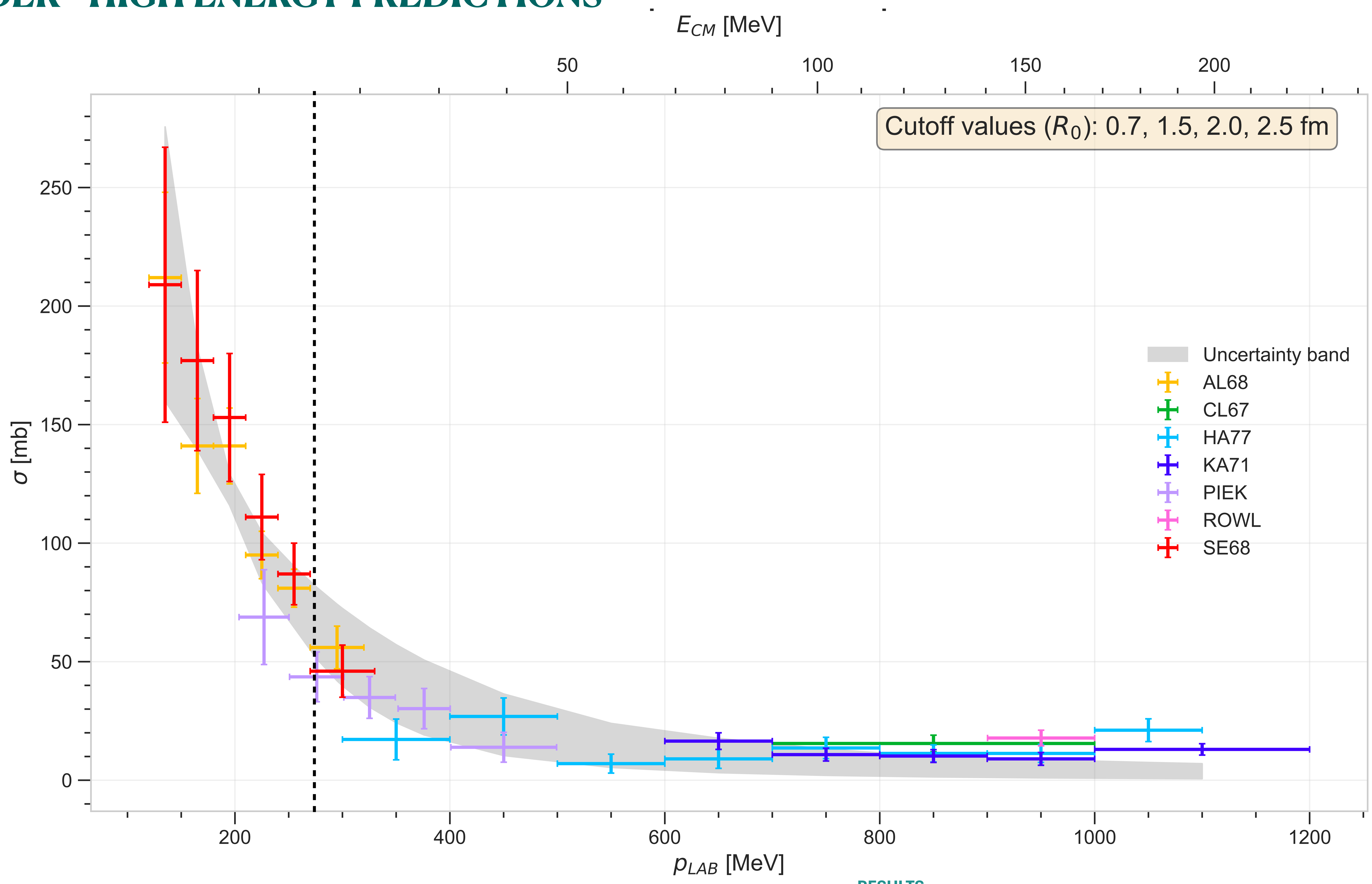
LEADING ORDER - WORST RESULT

$R_0 = 2.5$ fm



Preliminary results

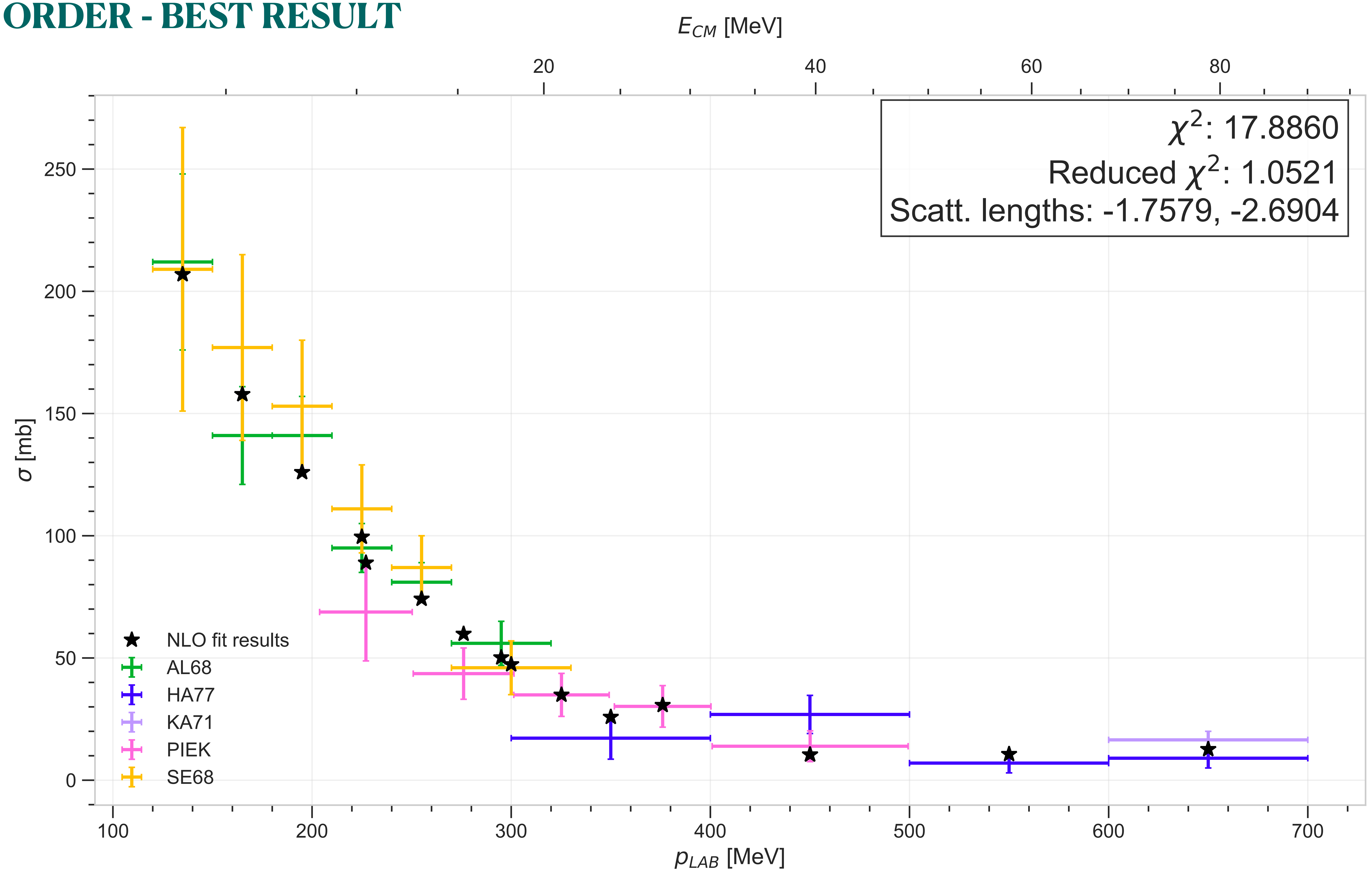
LEADING ORDER - HIGH ENERGY PREDICTIONS



Preliminary results

NEXT TO LEADING ORDER - BEST RESULT

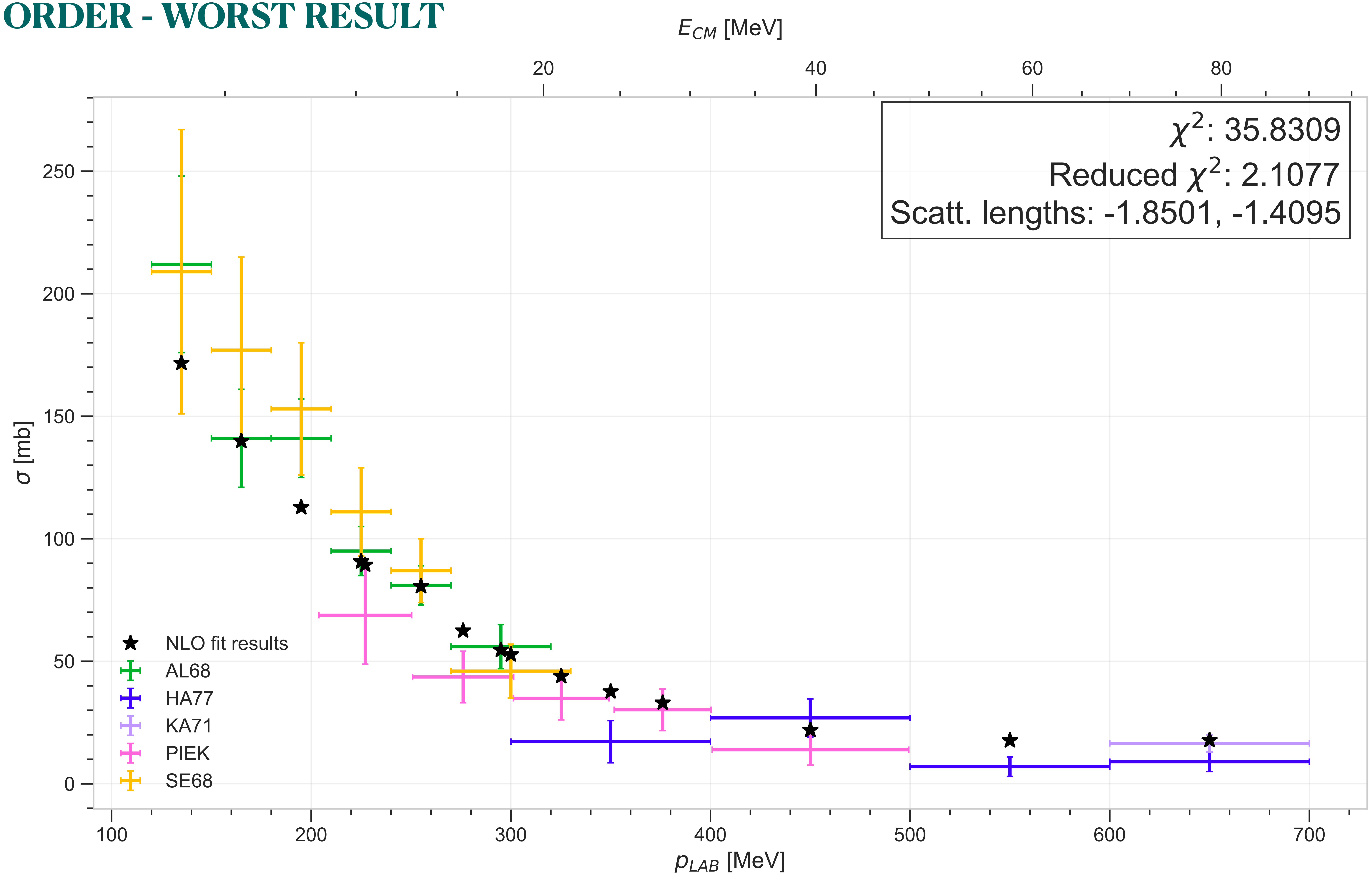
$R_0 = 2.0$ fm



Preliminary results

NEXT TO LEADING ORDER - WORST RESULT

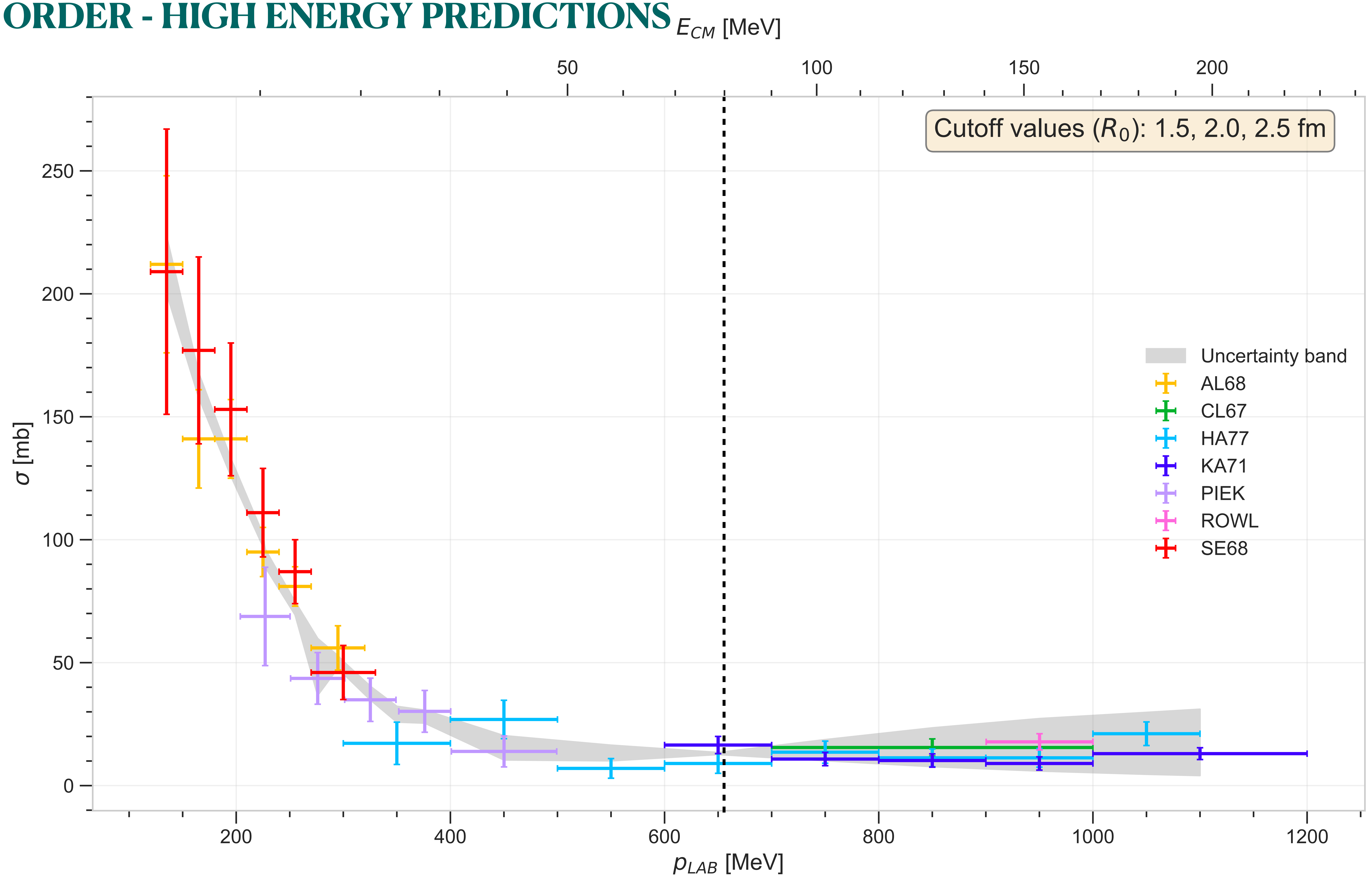
$R_0 = 0.7$ fm



Preliminary results

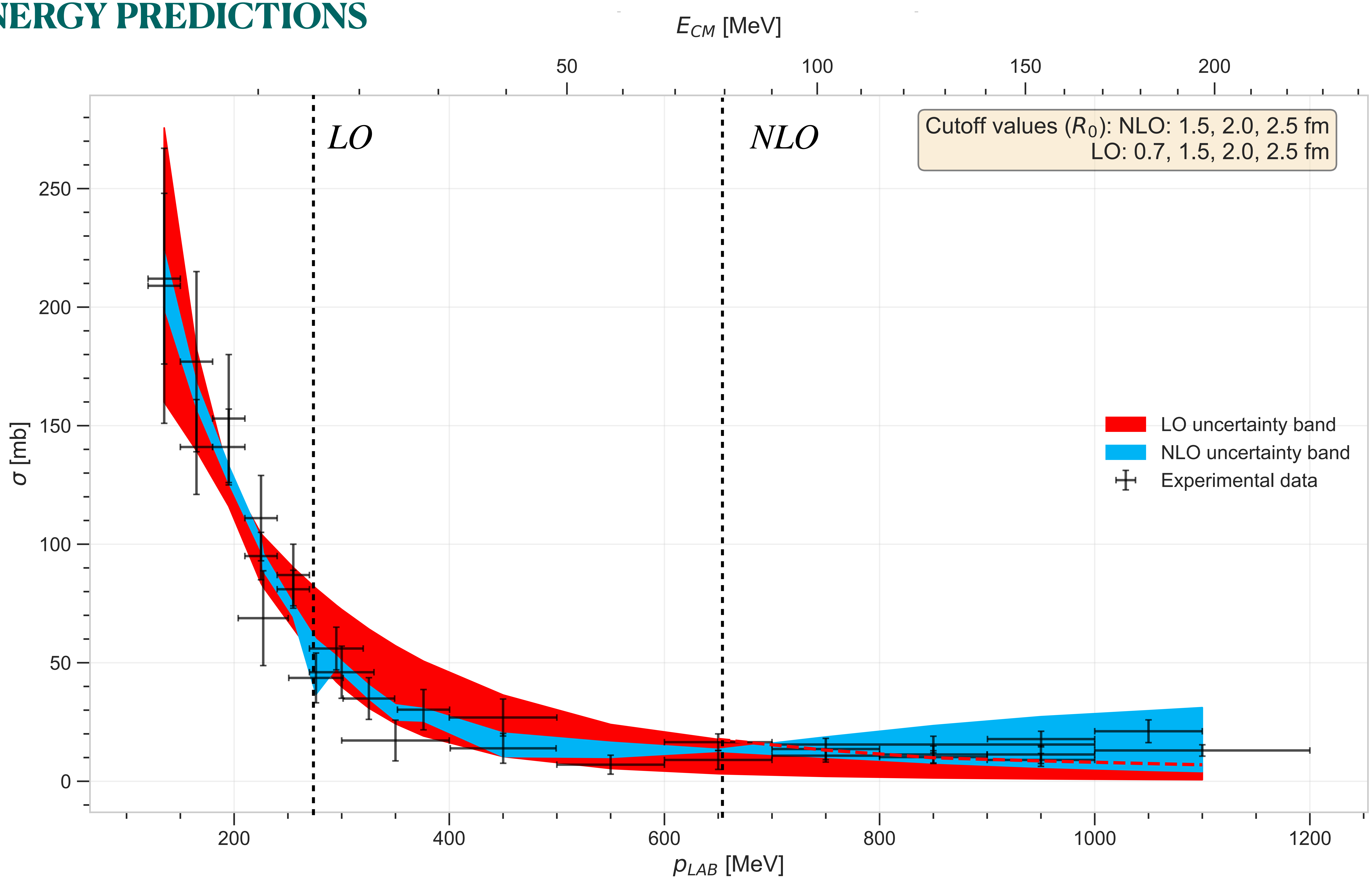
NEXT TO LEADING ORDER - HIGH ENERGY PREDICTIONS

- $R_0 = 0.7$ fm excluded



Preliminary results

NLO vs LO - HIGH ENERGY PREDICTIONS



Conclusions

In **summary**:

- Developed a local contact potential model for the ΛN interaction up to NLO
  - Sophisticated fitting procedure
  - Compatibility with scattering data and scattering lengths
- Results at LO quite accurate
- Results at NLO promising but still to be further analyzed

Future developments:

- Develop a local χEFT potential model, with $\Lambda N - \Sigma N$ coupling
- Three-body forces (YNN, YYN, YYY)
- Hypernuclei studies and $pp\Lambda$ correlation functions
- Studies on NS EoS

Thank you for your attention !

Backup slides

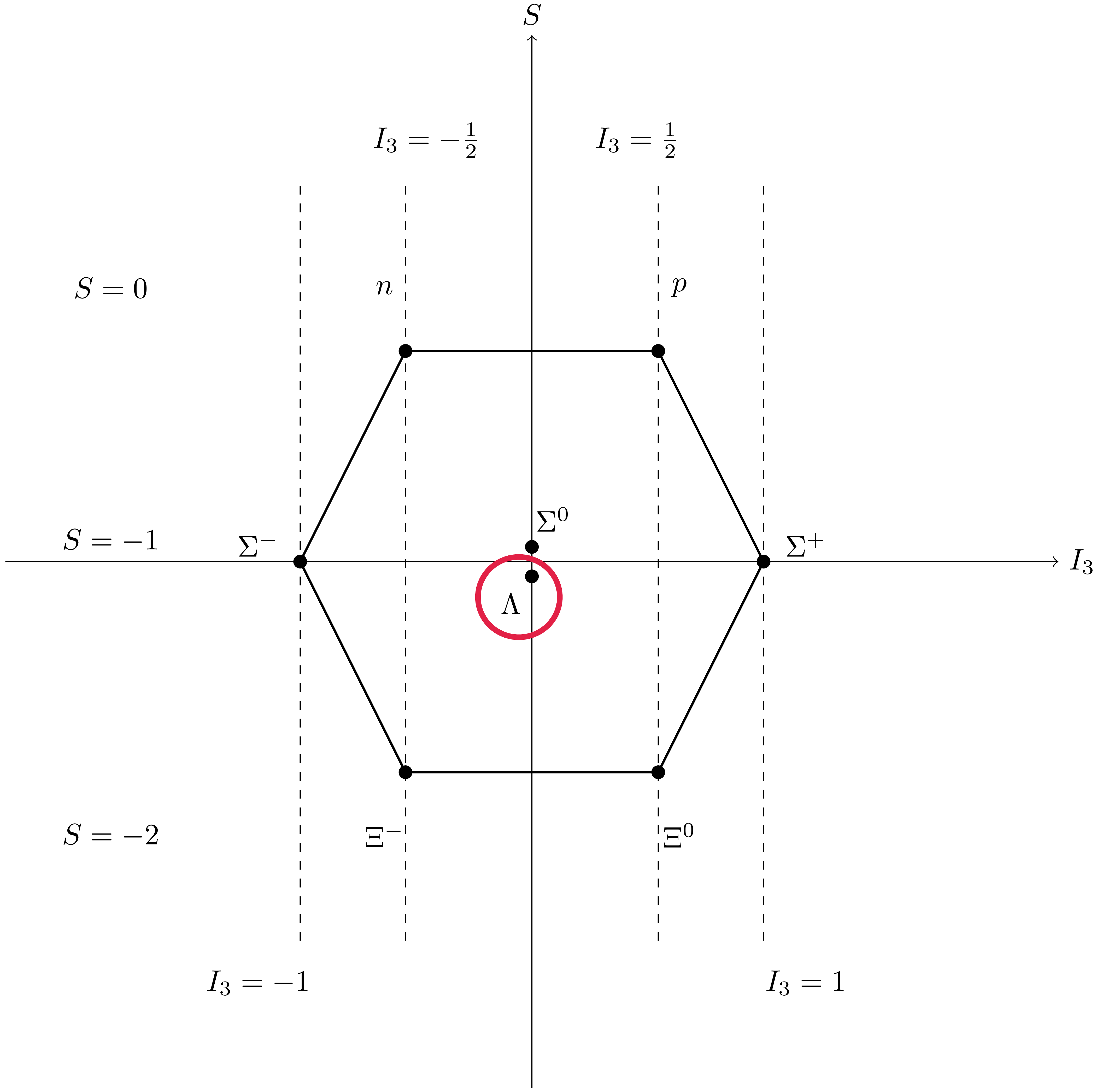
Introduction

HYPERONS

- Baryons containing at least one strange quark
- Fermions
- Lifetimes of the order of 10^{-10} s

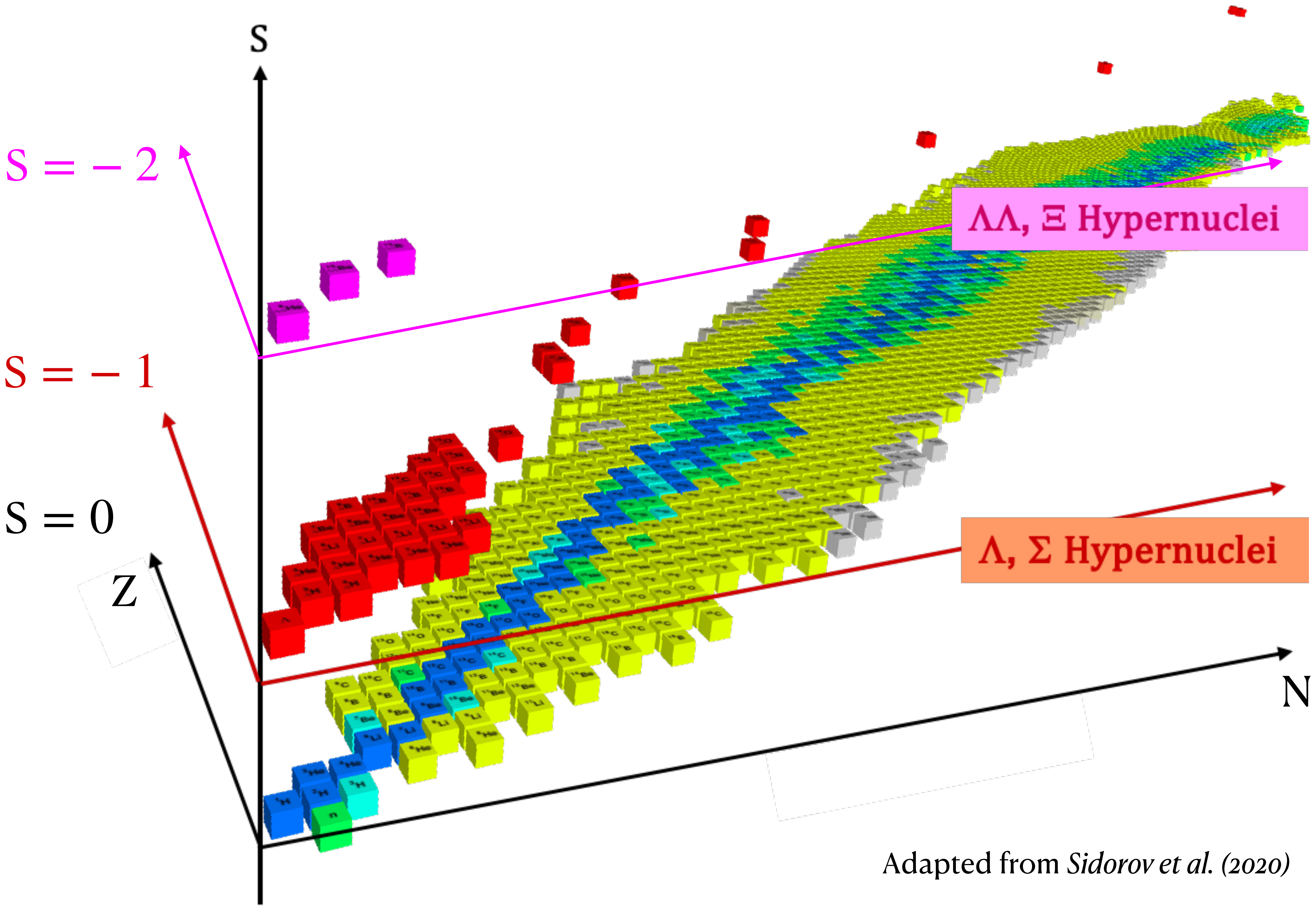
Λ HYPERON

- Mass 1115.683 ± 0.006 MeV
- Lifetime $(2.617 \pm 0.010) \times 10^{-10}$ s
- Isospin, charge $= 0$



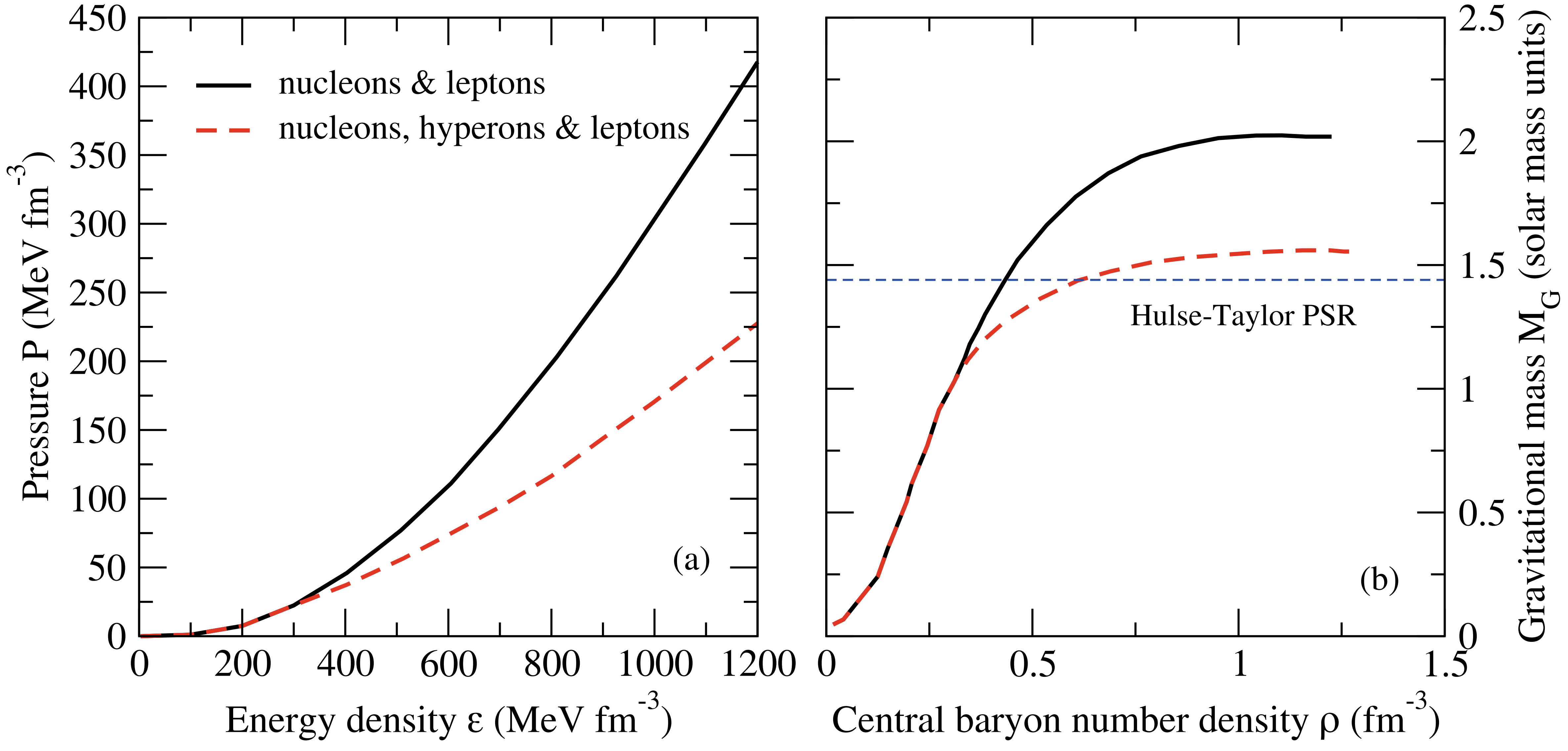
Hyperons can form hypernuclei, which are nuclei where a nucleon (n or p) is replaced by a hyperon

- Including hypernuclei in table of nuclides $\Rightarrow$ allow more comprehensive understanding of the strangeness properties
- Over 40 Λ-hypernuclei found, few double Λ-hypernuclei, some Ξ-hypernuclei, but no Σ-hypernuclei
- **Few experimental data available** for hypernuclei and hyperon-nucleon scattering
- **Theoretical predictions** to be **compared with experimental results**



Adapted from *Sidorov et al. (2020)*

Inside a neutron star (NS), hyperons can become stable particles $\Rightarrow$ changes in the NS equation of state (EoS)



I. Vidaña, (2016)

Hyperon puzzle

High density conditions in NS interior [$\rho = (2-3) \times 10^{13}$ g/cm^3]

$\Downarrow$

Increase in Fermi energy level of nucleons (Pauli exclusion principle)

$\Downarrow$

Conversion of nucleons into hyperons energetically favourable

$\Downarrow$

Decrease in energy

$\Downarrow$

Decrease in pressure $\longrightarrow$ softening of the EoS

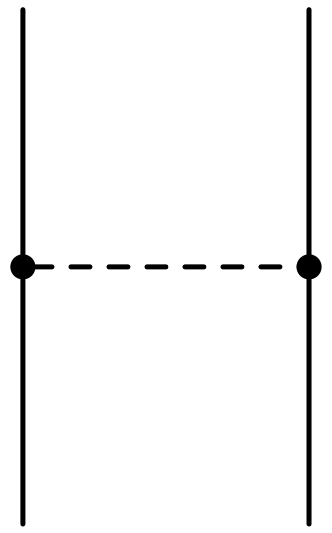
$\Downarrow$

Underestimation of **maximum mass** that can be reached in NS, which **contradicts experimental evidence** ($M_{NS} \sim 2.1$ M$_\odot$)

Potential model derivation

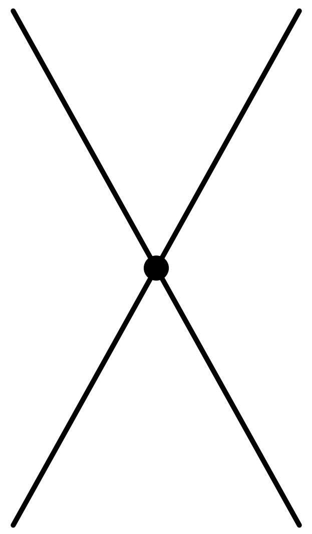
χEFT APPROACH OF *Haidenbauer et al. (2023)*

LO interaction



$$V_{YN}^{\mathrm{OBE}}(\mathbf{q}) = -f_{B_1B_3P}f_{B_2B_4P}\left(\frac{\sigma_N\cdot\mathbf{q}\;\sigma_Y\cdot\mathbf{q}}{\mathbf{q}^2+M_P^2} + C(M_P)\sigma_N\cdot\sigma_Y\right)\times\exp\left(-\frac{\mathbf{q}^2+M_P^2}{\Lambda^2}\right)I_{B_1B_2\to B_3B_4}$$

$$C(M_P) = -\left[\Lambda\left(\Lambda^2-2M_P^2\right)+2\sqrt{\pi}M_P^3\exp\left(\frac{M_P^2}{\Lambda^2}\right)\mathrm{erfc}\left(\frac{M_P}{\Lambda}\right)\right]/(3\Lambda^3)\,,\qquad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty dt\, e^{-t^2}$$

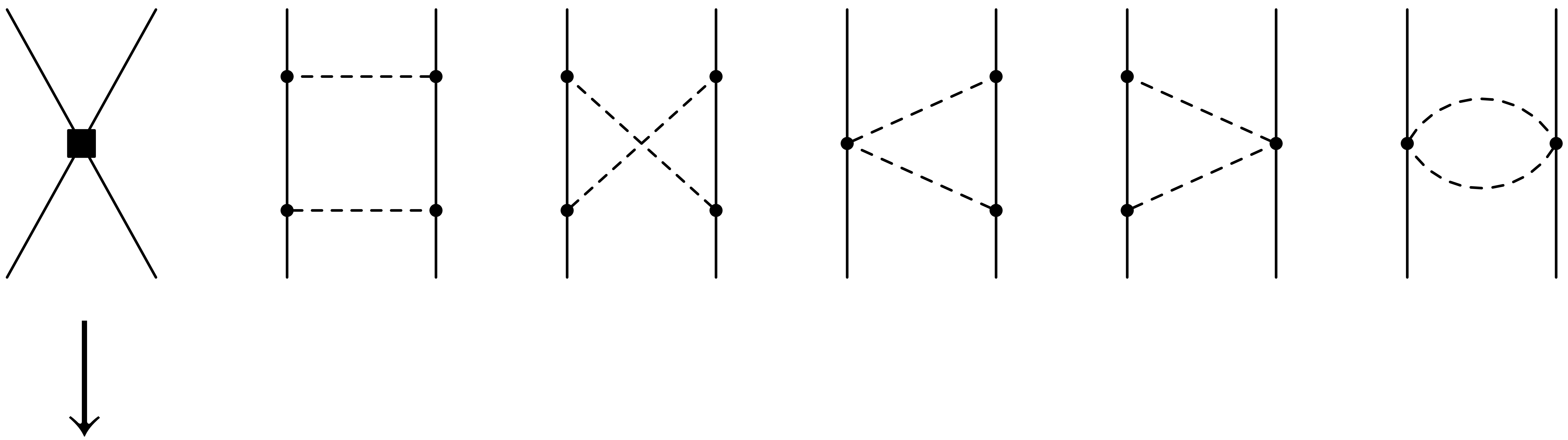


$$V_{YN}^{\mathrm{LO}} = C_S + C_T(\sigma_Y\cdot\sigma_N)$$

Potential model derivation

χEFT APPROACH OF *Haidenbauer et al. (2023)*

NLO interaction



$$V_{YN}^{\mathrm{NLO}} = C_1\mathbf{q}^2 + C_2\mathbf{k}^2 + (C_3\mathbf{q}^2 + C_4\mathbf{k}^2)\sigma_Y\cdot\sigma_N$$
$$+iC_5\frac{\sigma_Y+\sigma_N}{2}\cdot(\mathbf{q}\times\mathbf{k}) + C_6(\mathbf{q}\cdot\sigma_Y)(\mathbf{q}\cdot\sigma_N)$$
$$+C_7(\mathbf{k}\cdot\sigma_Y)(\mathbf{k}\cdot\sigma_N) + \frac{i}{2}C_8(\sigma_Y-\sigma_N)\cdot(\mathbf{q}\times\mathbf{k})$$

Potential model derivation

FOURIER TRANSFORM OF THE INTERACTION

The coordinate-space representation of a generic operator $\mathcal{O}(\mathbf{q},\mathbf{k})$ is

$$\mathcal{O}(\mathbf{r}) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\mathbf{k}}{(2\pi)^3}\, e^{i\mathbf{k}\cdot(\mathbf{r}'+\mathbf{r})/2}\, \mathcal{O}(\mathbf{q},\mathbf{k})\, e^{-\frac{R_0^2 q^2}{4}}\, e^{i\mathbf{q}\cdot(\mathbf{r}'-\mathbf{r})}\,,$$

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}\,,$$

$$\mathbf{k} = (\mathbf{p} + \mathbf{p}')/2$$

Cross section calculation

Available experimental data to perform the fit: Λp elastic scattering cross section

- Initial and final scattering states expanded in partial waves $\longrightarrow J^\pi$
- Included only partial waves with $J \leq 1$

J^π	α	L	S	${}^{2S+1}L_J$
0^+	1	0	0	${}^1S_0$
0^-	1	1	1	${}^3P_0$
1^+	1	0	1	${}^3S_1$
	2	2	1	${}^3D_1$
1^-	1	1	0	${}^1P_1$
	2	1	1	${}^3P_1$

Cross section calculation

Compute the total cross section of the Λp elastic scattering, starting from the potential model

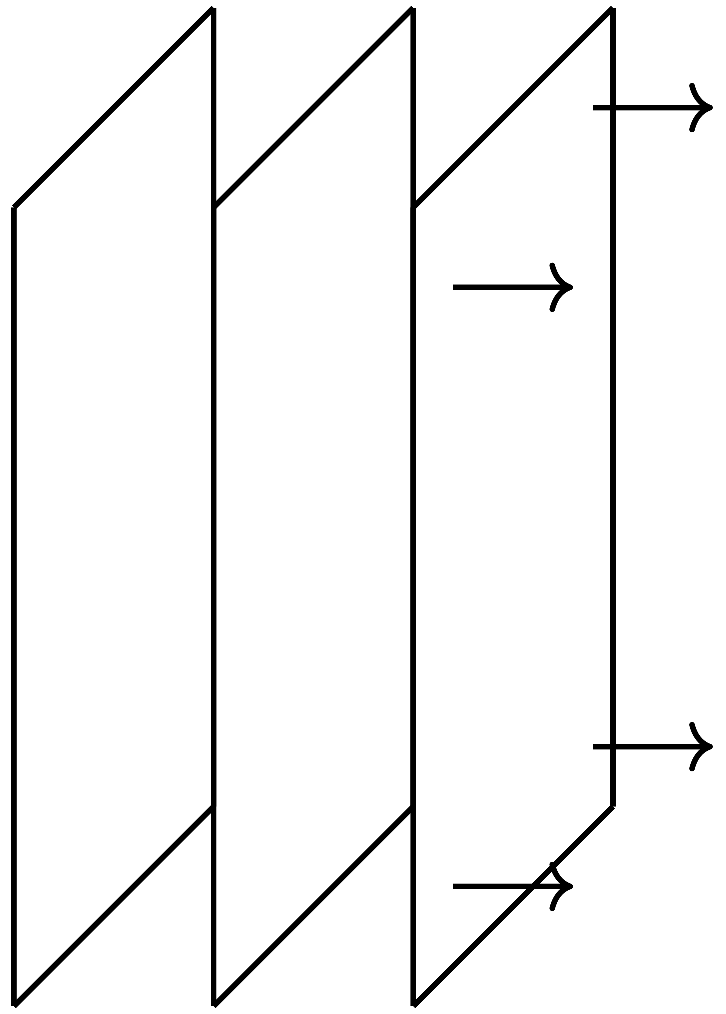
- Decompose wave function $\longrightarrow$ **short-range** wave function $\psi_c^\alpha(q,\mathbf{r})$ = "**core wave function**"; two particles close to each other

  $\searrow$ **long-range** $\psi_a^\alpha(q,\mathbf{r})$ = "**asymptotic wave function**"; potential is negligible, two particles far apart

  $\downarrow$

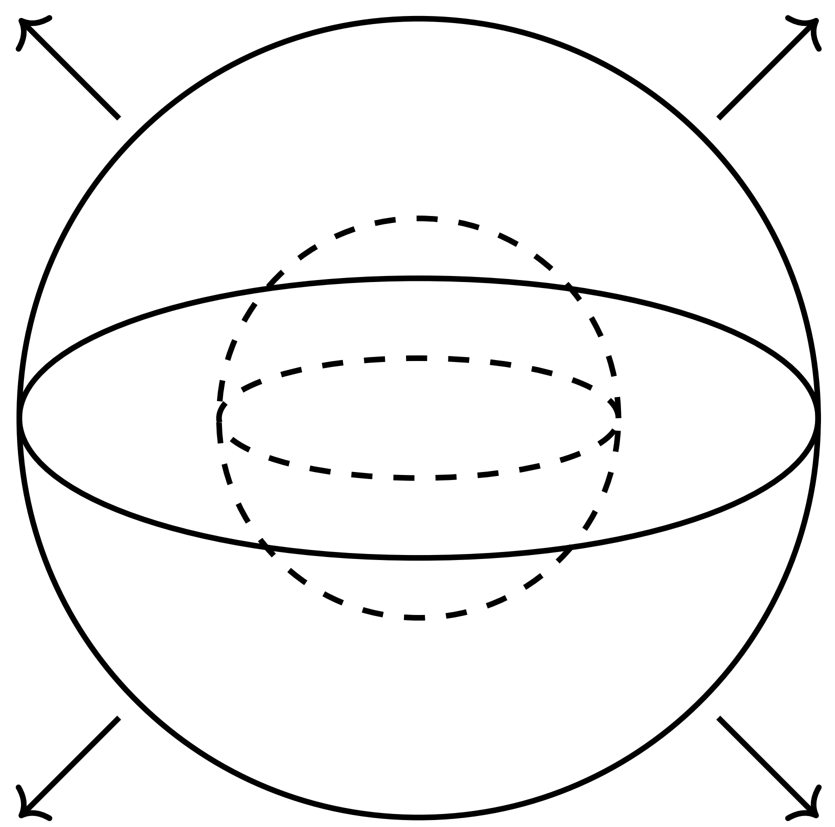
  rewrite as a sum of a **plane wave** and an **outgoing spherical wave**

$$\psi_\alpha^a(q,\mathbf{r}) = -2i\left[\Omega_\alpha^R(q,\mathbf{r}) + \sum_\beta T_{\alpha\beta}\,\Omega_\alpha^+(q,\mathbf{r})\right]$$



Incident plane waves

Scattered spherical waves



Cross section calculation

- Kohn variational principle statement $\Rightarrow$

$$\frac{\partial[R_{\alpha\beta}(q)]}{\partial d_{\rho,j}(q)} = 0\,,$$

$$\frac{\partial[R_{\alpha\beta}(q)]}{\partial R_{\rho\alpha}(q)} = 0$$

solution found numerically

$\downarrow$

***R*-matrix**

Fitting procedure

χ^2 FUNCTION

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

$$V_{\Lambda N}(\text{LECs}) \longrightarrow \sigma_{\Lambda p}(\text{LECs}) \longrightarrow \chi^2(\text{LECs})$$

- First attempt:

$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} \longrightarrow$$

Does not work:
$V_{\Lambda N} = 0$ at LO
when $S = 1$

- **Cross section experimental data** from *Radboud University, NN online archive*

Fitting procedure

χ^2 FUNCTION

Find an appropriate χ^2 function to be minimized, in order to fix LECs values

- χ^2 function using a constraint on **scattering length** obtained from potential model:

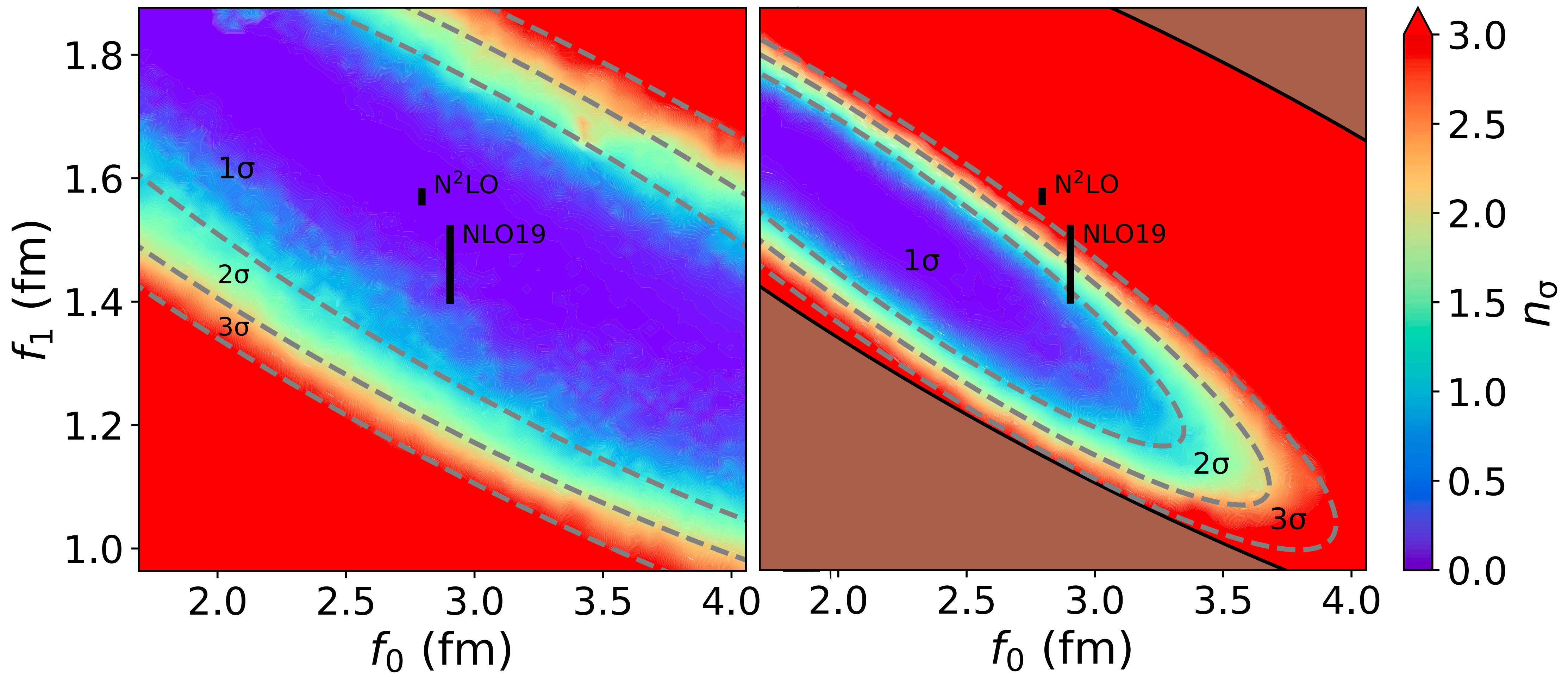
$$\chi^2 = \sum_i \frac{\left[\sigma_i^{th}(C_S, C_T, C_1, \ldots, C_5) - \sigma_i^{exp}\right]^2}{err(\sigma_i^{exp})^2} + \sum_{j=s,t} \frac{\left[a_j^{th} - a_j^{exp}\right]^2}{err(a_j^{exp})^2}$$

"Experimental" data for Λp scattering length from *Mihaylov et al. (2024)*

Numerical results

$a_s \in [2.10, 3.34]$ fm

$a_t \in [1.18, 1.56]$ fm



Adapted from Mihaylov et al. (2024)

Fitting procedure

DEFINING ENERGY LIMITS

- Partial wave projection of the potential, in momentum space

$$V(^1S_0) = 4\pi(C_S - 3C_T) + \pi(4C_1 + C_2 - 12C_3 - 3C_4 - 4C_6 - C_7)(p^2 + p'^2)\,,$$

$$V(^3S_1) = 4\pi(\underbrace{C_S + C_T}_{\text{LO LECs}}) + \pi\frac{3}{2}(12C_1 + 3C_2 + 12C_3 + 3C_4 + 4C_6 + C_7)(p^2 + p'^2)\,.$$

- Semi classical approach to compute p_{LAB} threshold for LO

$$\ell\,\hbar = p_{LAB}\,b$$

- $\ell = 1$ to exclude P-waves and higher-order partial waves
- $b \sim 1$ fm, $\hbar c = 197.33$ MeV fm

$$\left.\right\}\quad p_{LAB} \sim 200\text{ MeV} \Rightarrow E_{CM} \sim 15\text{ MeV}$$

Results

TEST WITH WOOD-SAXON POTENTIAL

- Expression for Wood-Saxon potential:

$$W_S = -\frac{V_0}{1 + e^{\frac{(x-R)}{a}}}$$

with $V_0 = 50$ Mev, $R = 1.25$ fm, $a = 0.5$ fm

- Wood-Saxon potential

$\downarrow$ code that computes δ_L with Numerov method

$\downarrow$ code that computes δ_L with Kohn variational principle

$\underbrace{\hphantom{xxxx}}$ comparison of the results

		δ_ℓ (deg)	
E_{CM} [Mev]	ℓ	Numerov	Kohn
5	0	26.180	26.182
5	1	0.522	0.516
100	0	19.930	19.931
100	1	10.460	10.370
1000	0	6.617	6.612
1000	1	6.252	6.224

Results

TEST WITH USMANI POTENTIAL

- Usmani potential model from *Bodmer* & *Usmani (1987)*

$$V_{\Lambda N} = \left[V_C(r) - (\bar{V} - \frac{1}{4}V_\sigma \sigma_\Lambda \cdot \sigma_\mathbf{N})T_\pi^2\right][1 - \epsilon + \epsilon P_x]$$

$$V_C(r) = W_C\left[1 + exp\left[\frac{r-R}{d}\right]\right]^{-1}$$

$$T_\pi(r) = \left[1 + \frac{3}{x} + \frac{3}{x^2}\right]\frac{e^{-x}}{x}(1 - e^{-cr^2})^2$$

with $R = 0.5$ fm, $d = 0.2$ fm, $\bar{V} = 6.15$ MeV,
$x = 0.7r, c = 2\ \text{fm}^{-2}, \epsilon \sim 0.25$

- Two free parameters, to fix with fitting procedure $\rightarrow W_c\ , V_\sigma$

Results

TEST WITH USMANI POTENTIAL

- Tested the implemented code with potential from *Bodmer & Usmani (1987),* with operatorial structure similar to V_{YN}^{LO} of present work

- Two free parameters, to fix with fitting procedure

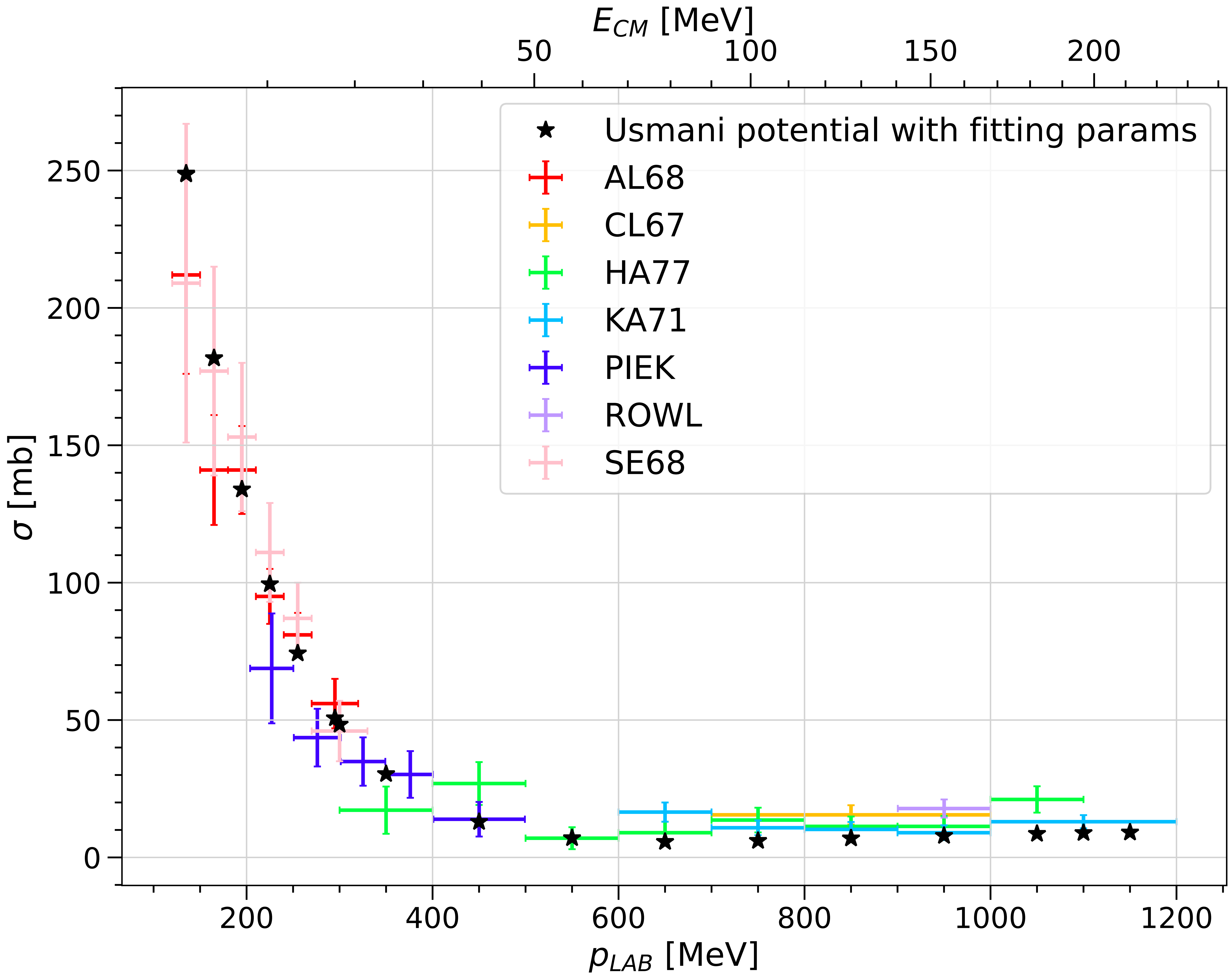
  $\rightarrow W_c\,,V_\sigma$

- Results:

$$W_c = 2142.777\ \ \mathrm{MeV}\,,$$
$$V_\sigma = 0.4857\ \ \mathrm{MeV}\,,$$
$$\chi^2/datum \sim 2.01$$

- Expected results from *Bodmer & Usmani (1987):*

$$W_c = 2137\ \ \mathrm{MeV},$$
$$0 \leq V_\sigma \leq 0.5\ \ \mathrm{MeV}$$



Results

TEST WITH USMANI POTENTIAL

- Result from *A.R. Bodmer and Q.N. Usmani (1987)* : $a_{unp}^{expected} \sim 1.9$ fm, $r_{unp}^{expected} \sim 3.4$ fm

- Usmani potential used to compute phase shifts, through code that exploits Kohn variational principle

- With δ_0 ($\ell = 0$), the scattering length for triplet and singlet (a_t, a_s) can be calculated, using $a = \lim_{k \to 0} \frac{sin\delta_0}{k}$

- Results for scattering length: $a_s \sim -2.88$ fm, $a_t \sim -1.66$ fm

- Unpolarized scattering length: $a_{unp} = \sqrt{\frac{a_s^2}{4} + 3\frac{a_t^2}{4}}$, obtained from $\sigma_{unp} = \frac{1}{4}\sigma_s + \frac{3}{4}\sigma_t$

$$\Downarrow$$

$$a_{unp} \sim 2.0 \text{ fm}$$

- Results for effective range: $r_0^s \sim 2.871$ fm, $r_0^t \sim 3.688$ fm

Results

NUMERICAL RESULTS FOR ENERGY BANDS PREDICTIONS

R_0	χ^2_{red} NLO	χ^2_{red} LO
0.7	165.618353	4.932156
1.5	7.388181	1.564845
2.0	1.214715	5.739532
2.5	2.709569	3.293418