Strangeness S=-1 and -2 hypernuclei based on chiral interactions

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Workshop on "Lepton Interactions with Nucleons and Nuclei", Marciana Marina, Italy

- Motivation
- YN & YY interactions
- SRG evolution of (hyper-)nuclear interactions
- Determination of CSB contact interactions and Λn scattering length
  & application to $A = 7$ and 8 hypernuclei
- Uncertainty of Λ separation energies & chiral YNN interactions
- $S = -2$ hypernuclei: predictions for $A \leq 6$
- Conclusions & Outlook

in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

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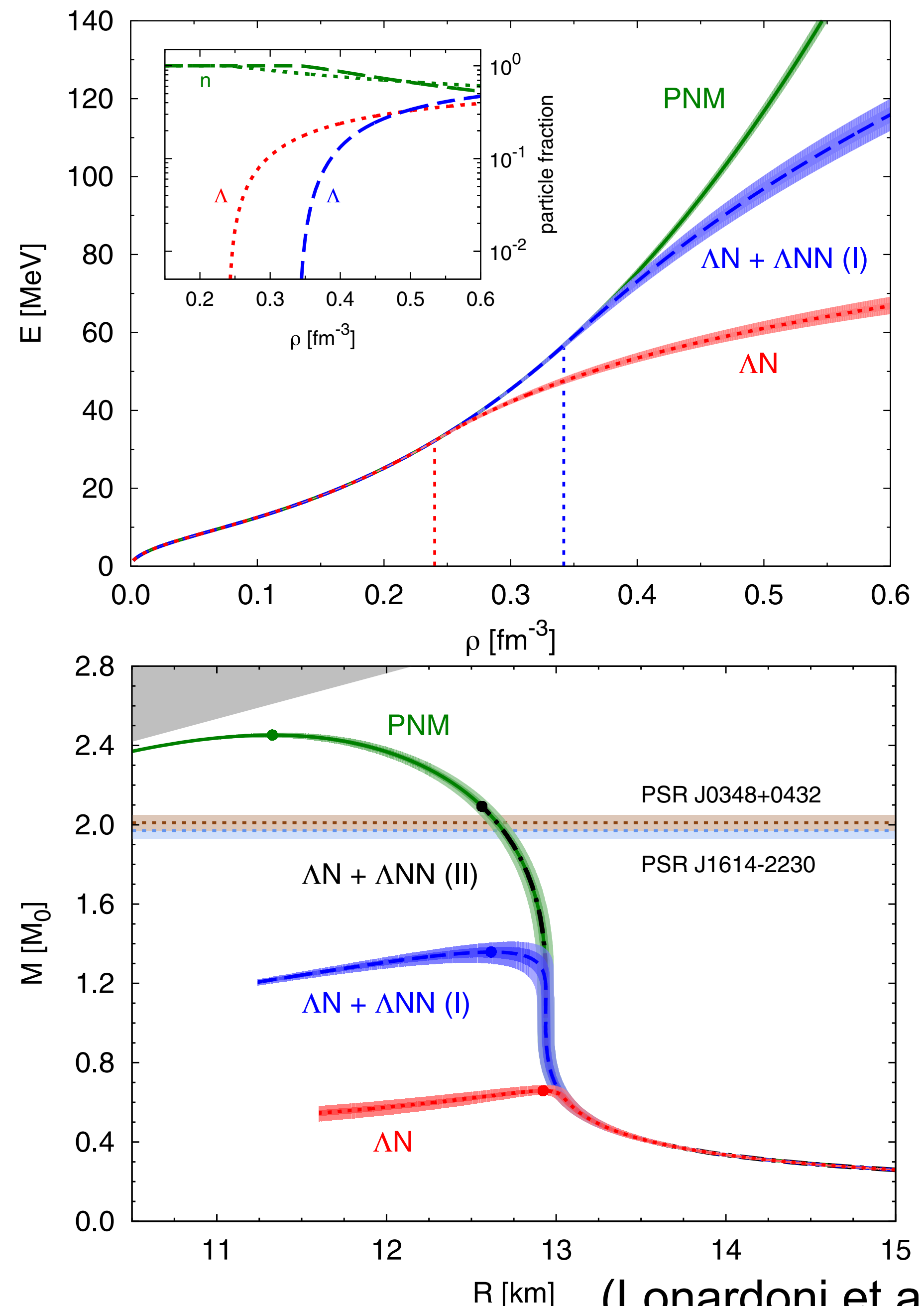
Mitglied der Helmholtz-Gemeinschaft

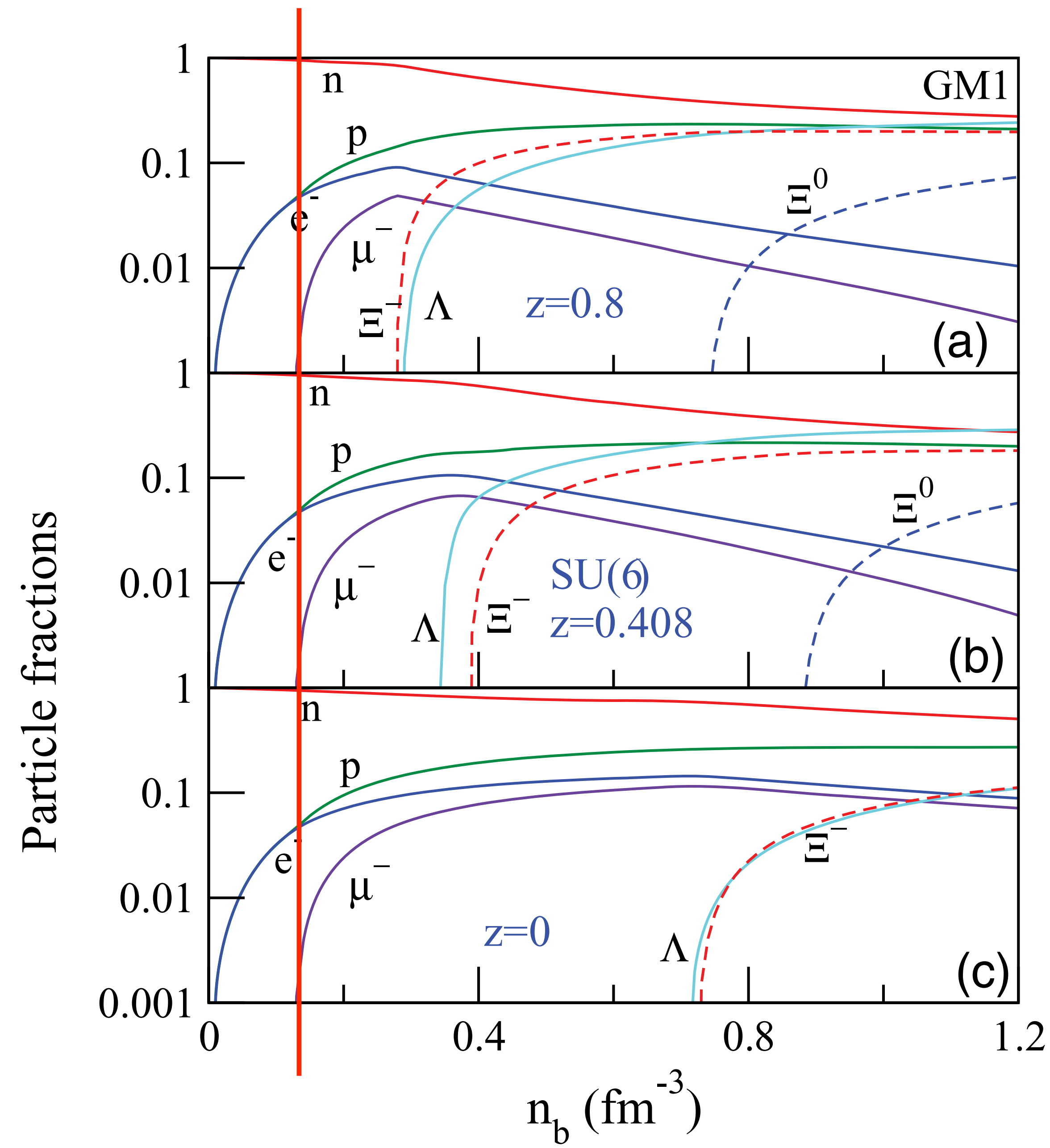
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

- *hyperon contribution to the EOS, neutron stars, supernovae: "hyperon puzzle"*
- ***flavor dependence of baryon-baryon interactions: explicit chiral symmetry breaking***
- *Λ as probe to nuclear structure*



(Lonardoni et al. (2015))

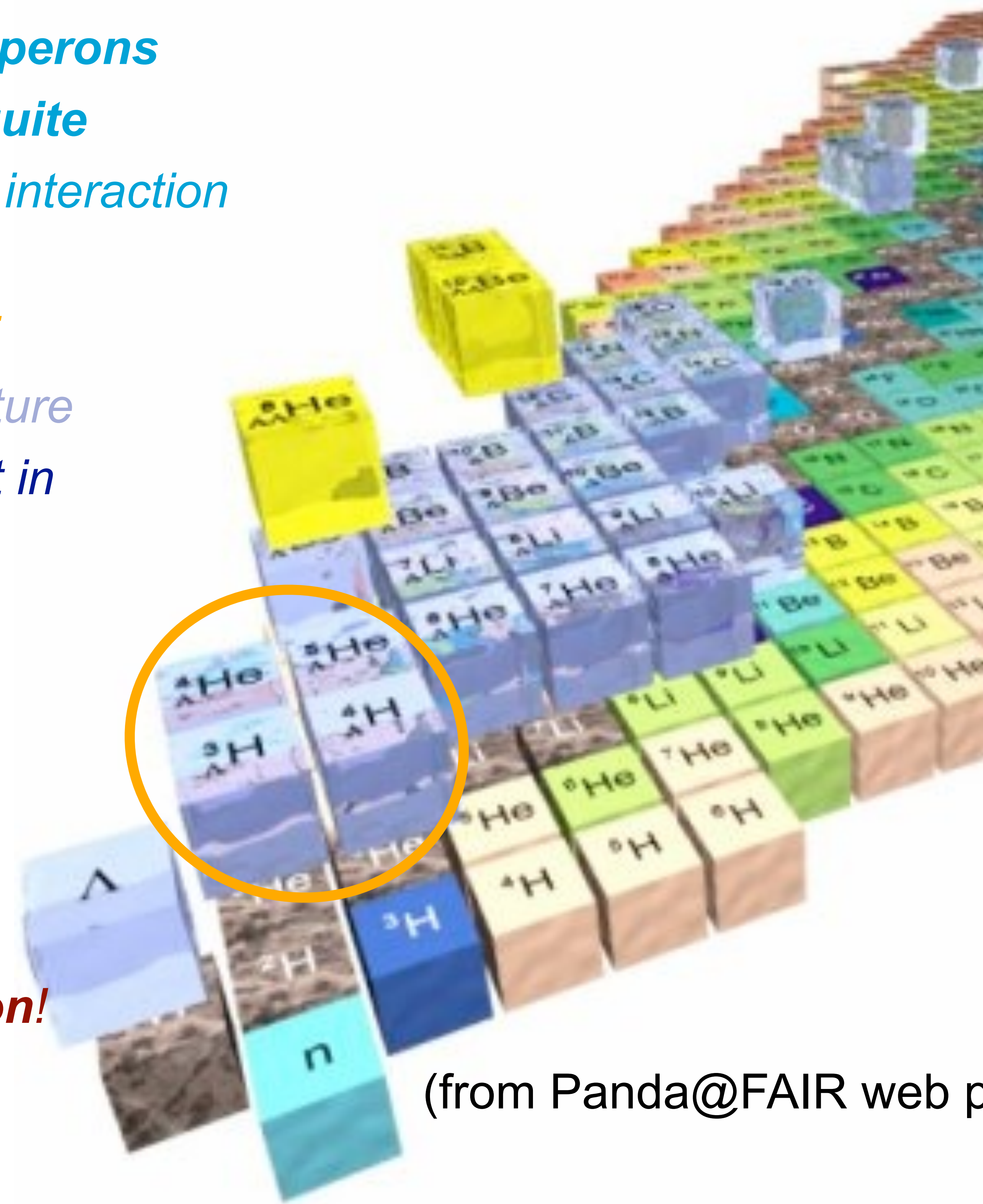


(Weissenborn et al. 2012)

Hypernuclei

Only few YN (YY) data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies“ are* ***quite*** *independent from NN(+3N) interaction*
- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***
- ***non-trivial constraints*** *on the YN interaction even from lightest ones*
- *size of* ***YNN*** *interactions? need to include* ***Λ-Σ conversion****!*



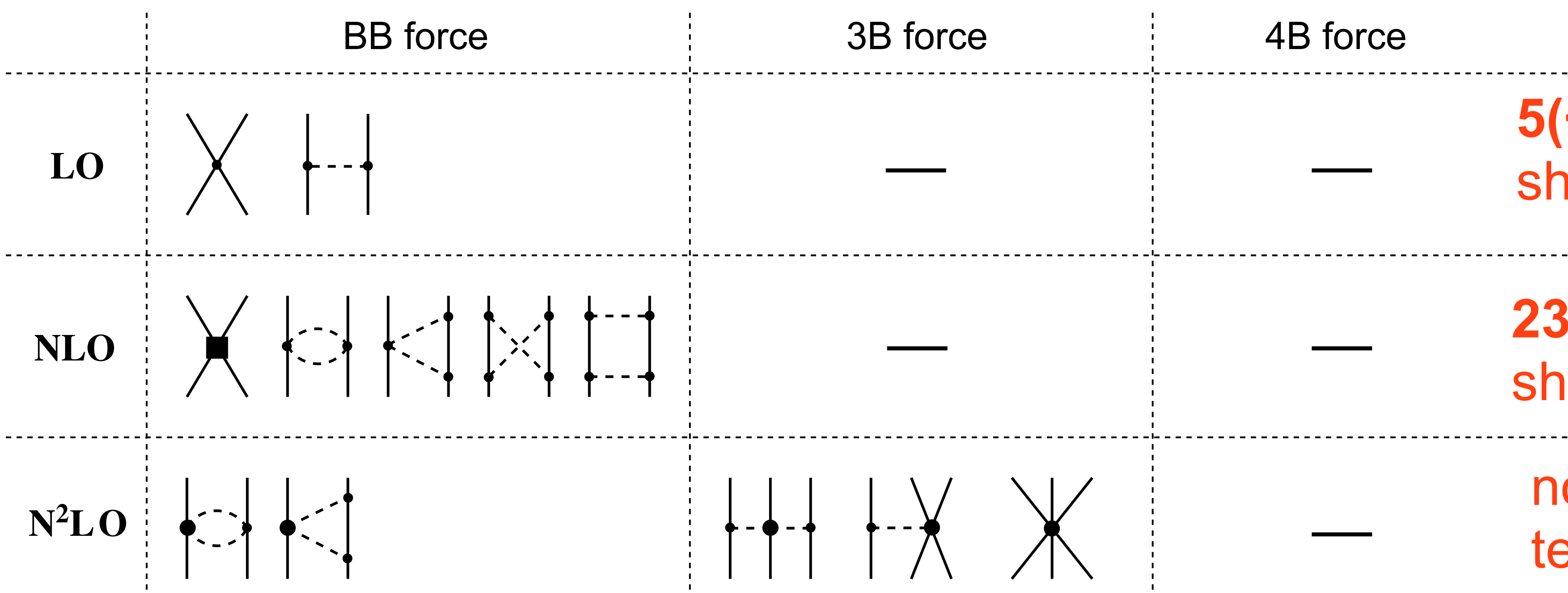
(from Panda@FAIR web page)

Chiral NN & YN & YY interactions

EFT based approaches

Chiral EFT implements **chiral symmetry of QCD**

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N²LO (for YN)



5(+1) NN/YN (YY) short range parameters

23(+5) NN/YN (YY) short range parameters

no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

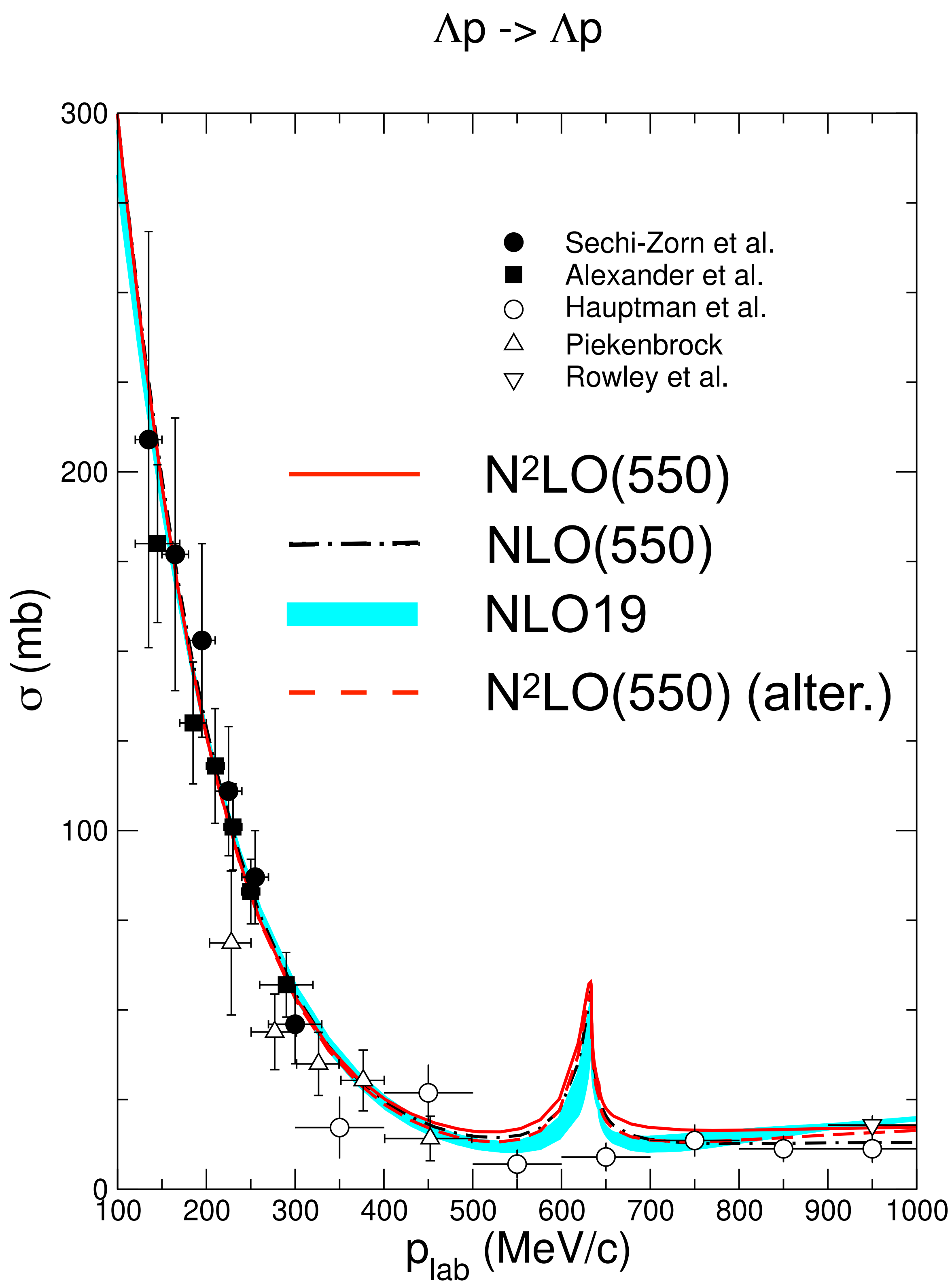
Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

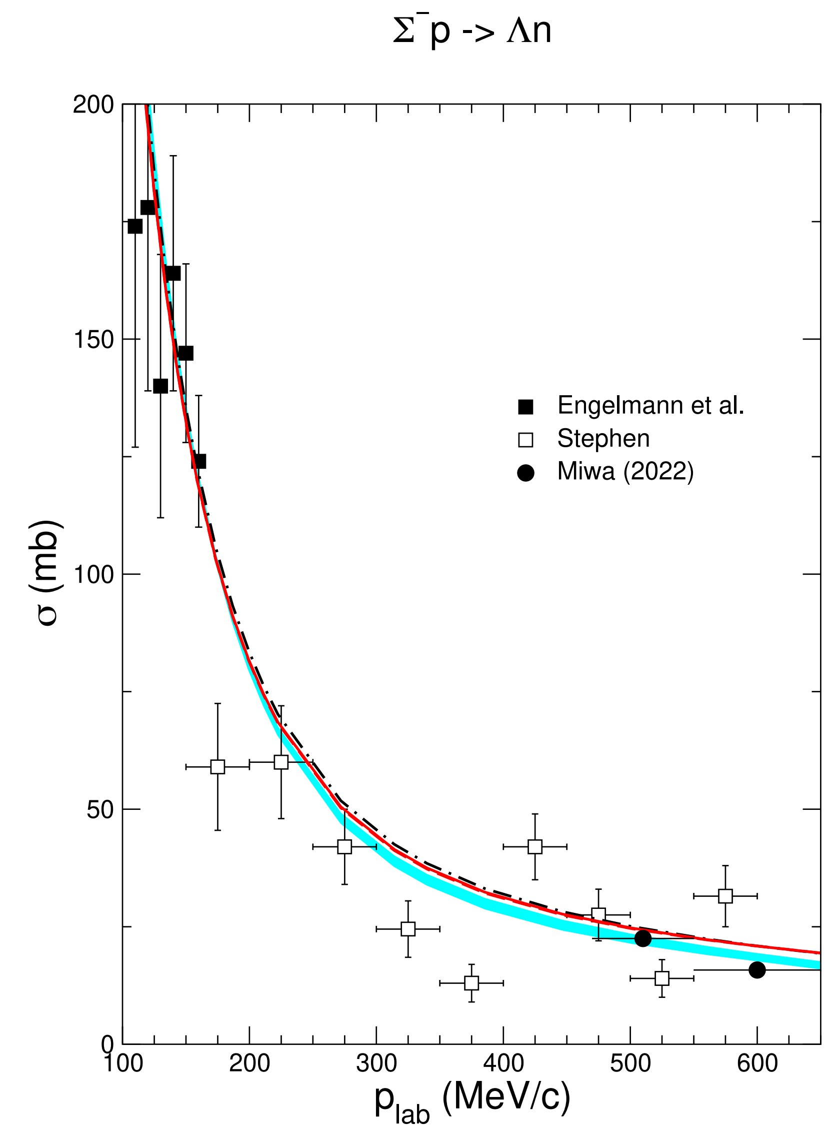
$\Lambda - \Sigma$ ($\Lambda\Lambda - \Sigma\Sigma - \Xi N$) **conversion** is explicitly included (3BFs only in N²LO)

SMS NLO/N²LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



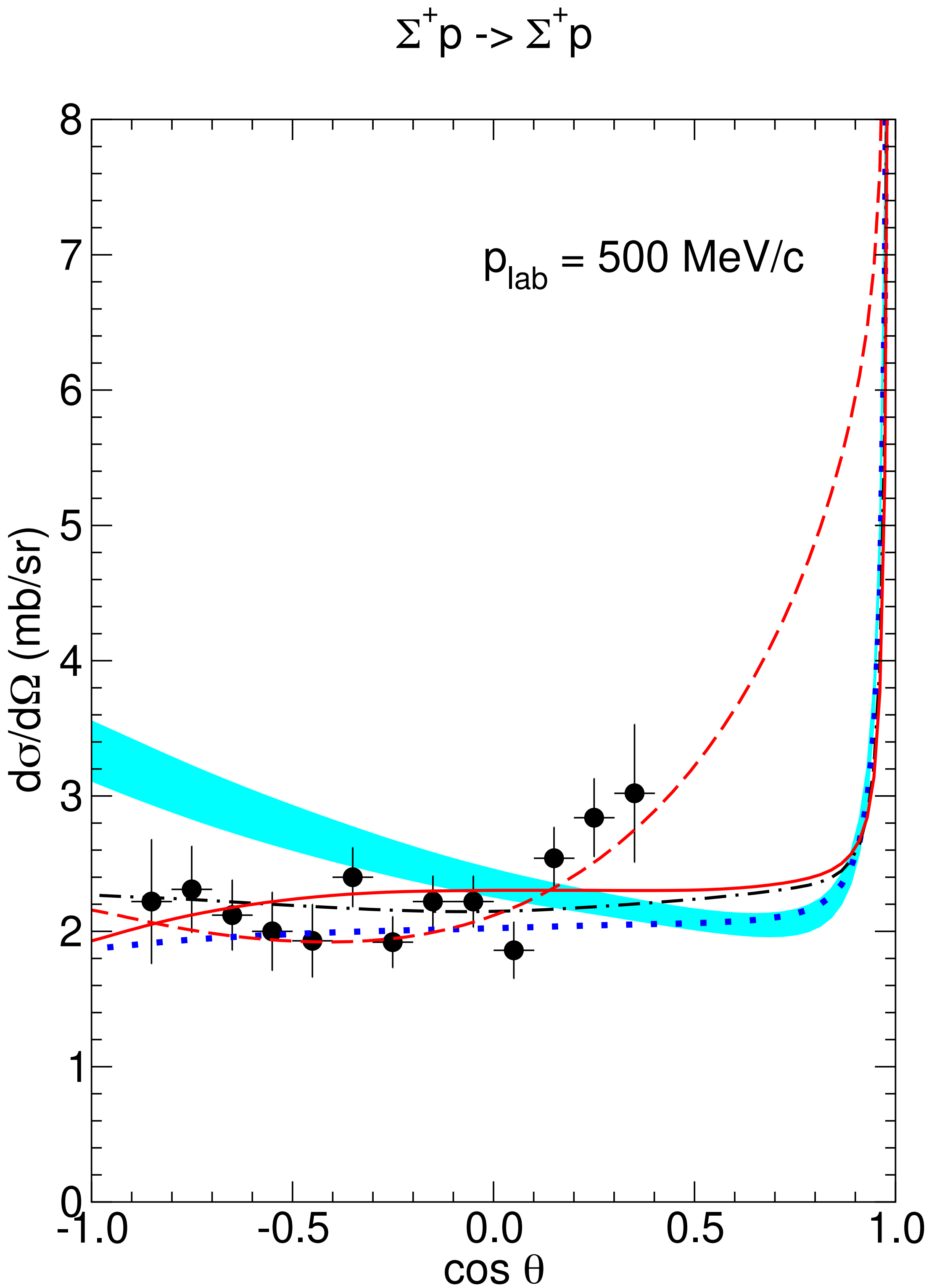
- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)

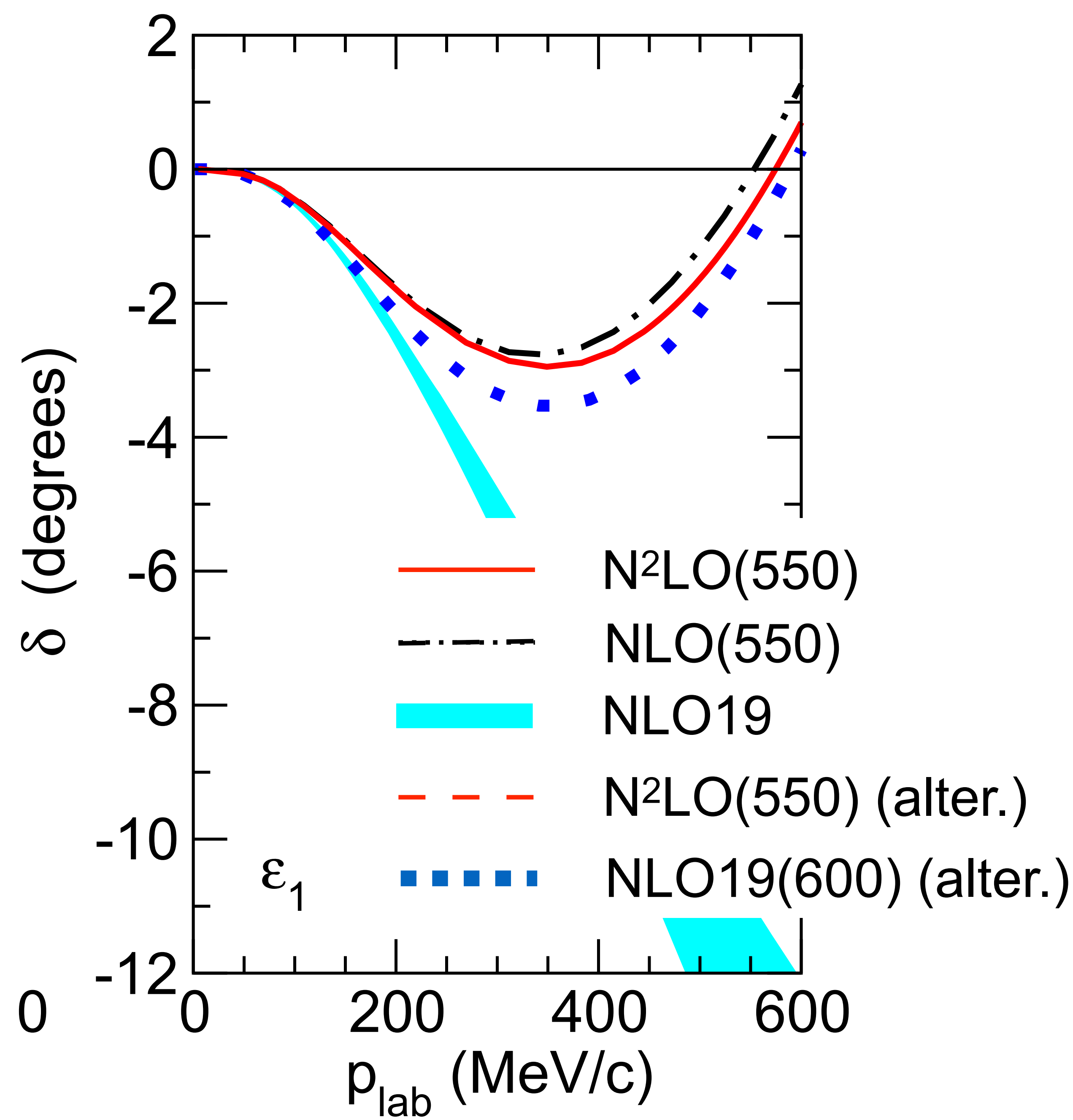


J. Haidenbauer et al. EPJ A 59, 63 (2023)

SMS NLO/N²LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!





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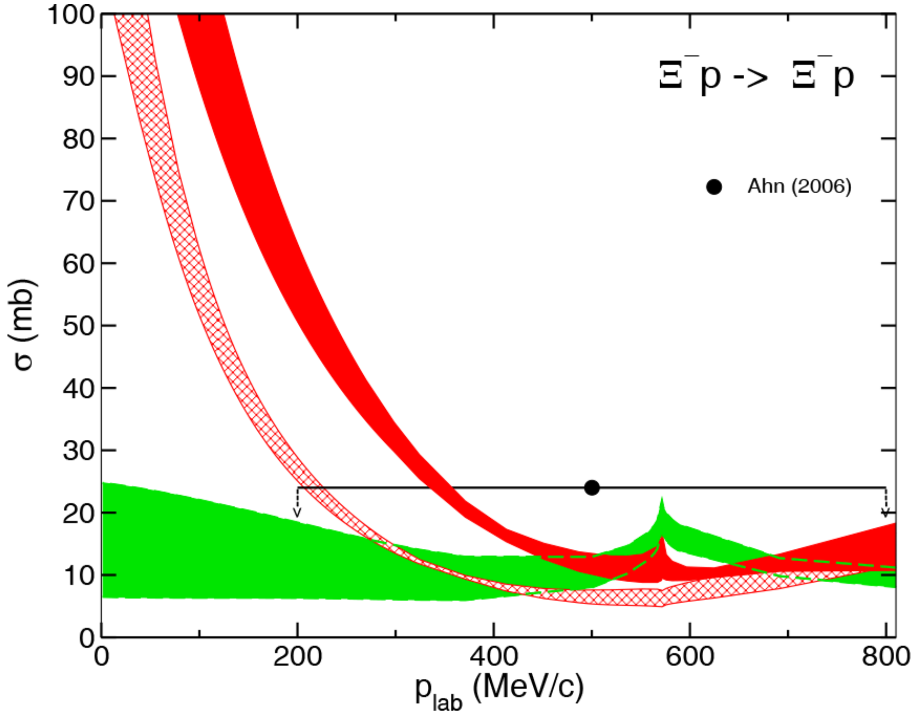
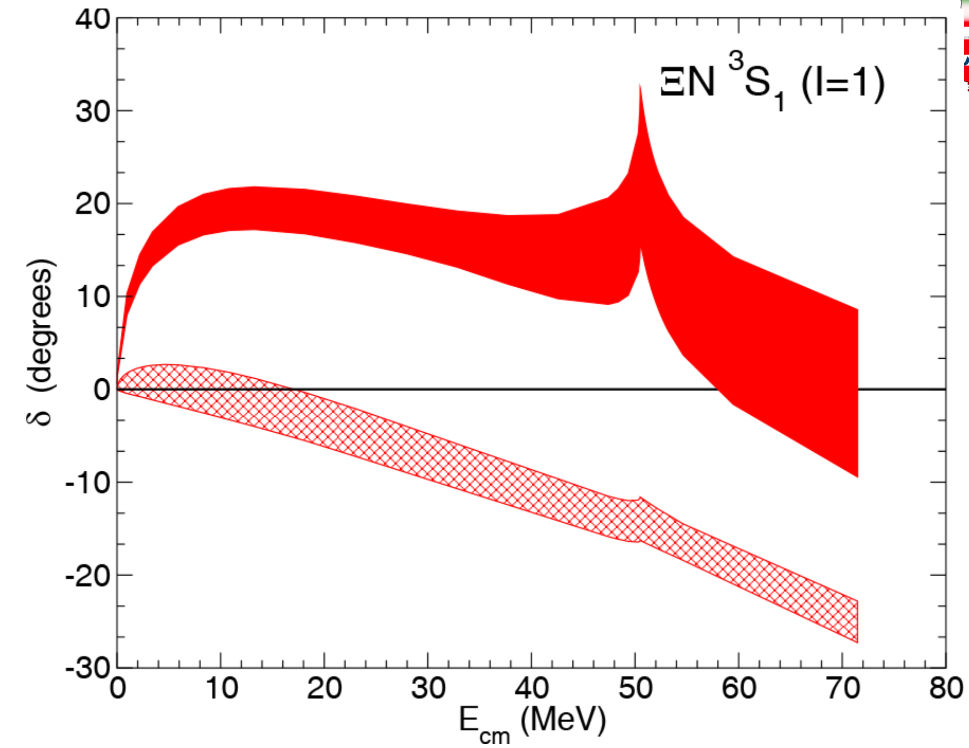
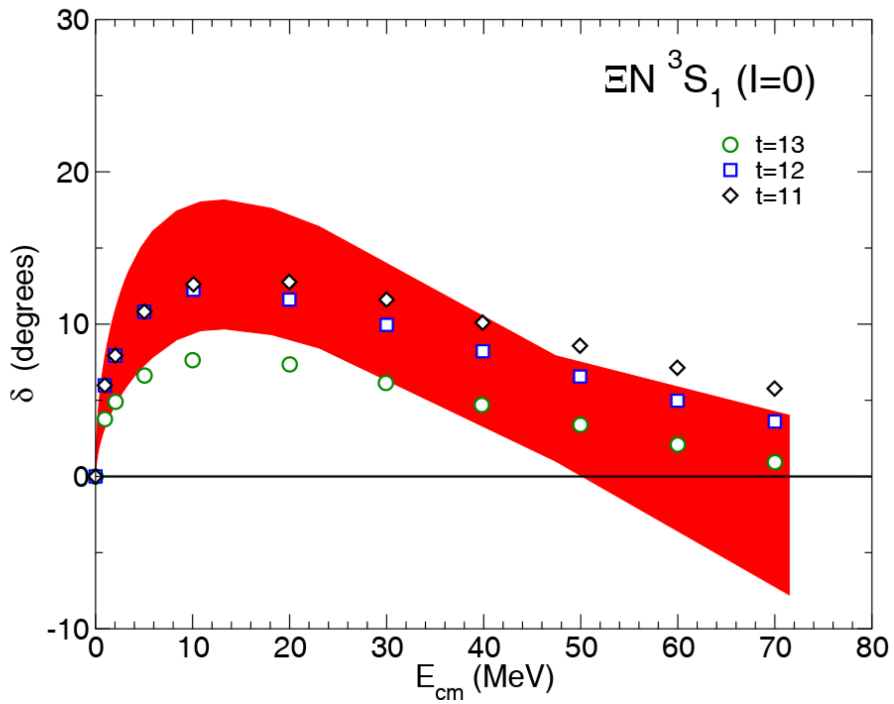
YY interaction







LO

NLO

NLO(2016)

(Haidenbauer at al., 2019)

→ adjusted to data & LQCD (HAL QCD)

updated version consistent with Ξ-nuclei (only change in $\Xi N\ ^3S_1$)

Tools

Need reliable predictions for hypernuclei to further constrain interactions

Faddeev-Yakubovsky (FY) equations for $A = 3$ and 4 (momentum space)

- long distance tails of wave functions can be well represented
- uses Jacobi coordinates separating off CM motion
- chiral interactions can be directly used
- hugh linear eigenvalue problem (dimension 10^9x10^9) even for A=4 systems
- is feasible only for A ≤ 4

(see AN, Glöckle, Kamada, 2002))

Jacobi-no core shell model (J-NCSM) for $A \geq 4$ (HO space)

- smaller dimensions allow to tackle p-shell nuclei
- exact antisymmetrization of wave functions can be prepared
- uses Jacobi coordinates separating off CM motion
- chiral interactions require similarity renormalization group (SRG) evolution
- long distance wave functions require **correction**/large HO model spaces

(see Liebig et al., 2016; Le et al., 2020 & 2021)

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches
(NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

(Bogner et al. PRC 75,061001 (2007))

$$\frac{dH_s}{ds} = \big[\, \underbrace{[T\,,\,H(s)]}_{\equiv \eta(s)}\,,\,H(s)\,\big] \qquad H(s) = T + V(s)$$

this choice of generator drives *V(s)* into a diagonal form in momentum space

- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation!)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

SRG dependence: BB & 3B level

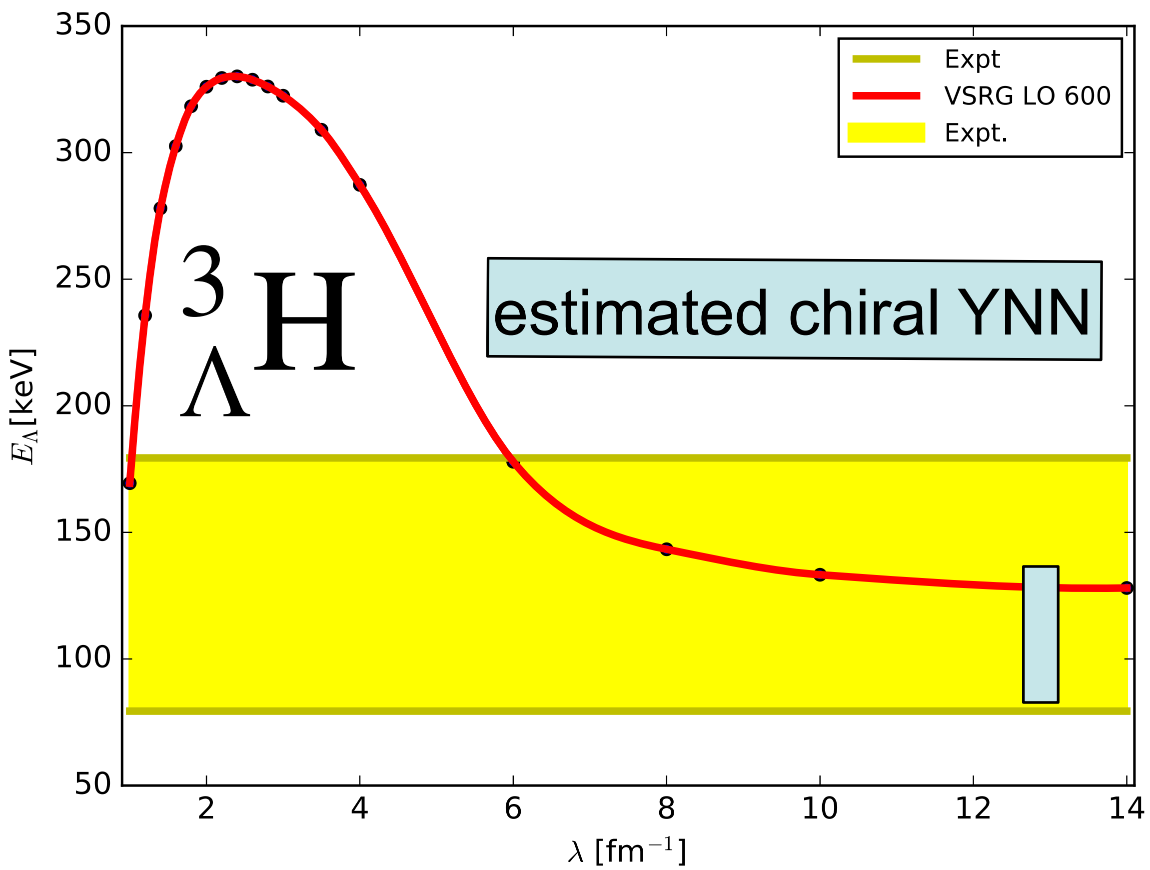
Λ separation energies: $E_\Lambda = E\left({}^{A-1}Z\right) - E\left({}^{A}_{\Lambda}Z\right)$

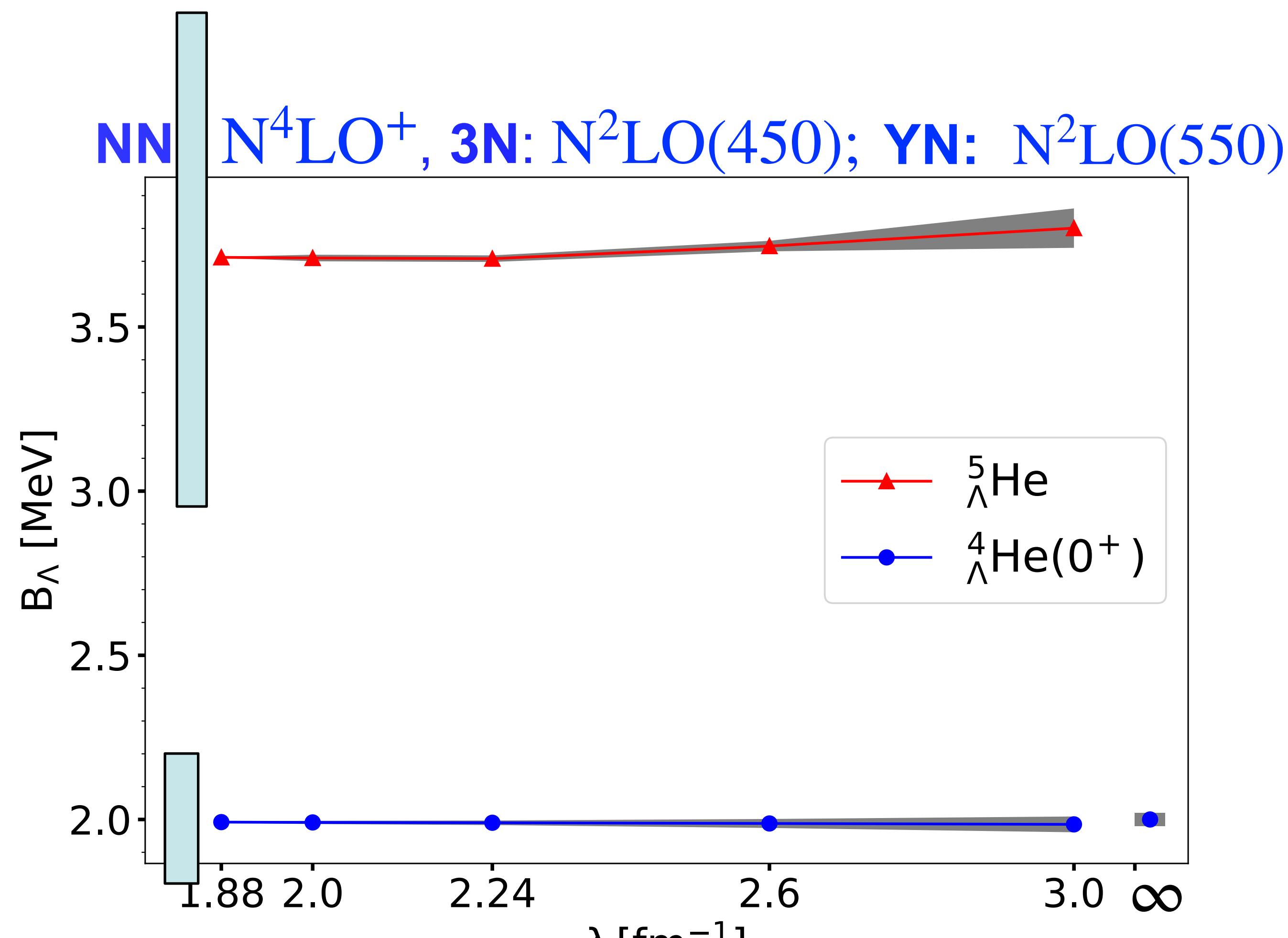
- NN only: SRG dependence comparable to chiral 3NF
- YN only: SRG dependence much larger (Λ–Σ conversion, Wirth et al. (2016))
- 3N/YNN: SRG dependence smaller than 3BF contribution

→ state of the art calculations include SRG-induced NN, YN, 3N and YNN forces

hypernuclei: SRG induced BB and 3B interactions are sufficient

no SRG induced 3BF for $S = -2$ taken into account (yet)

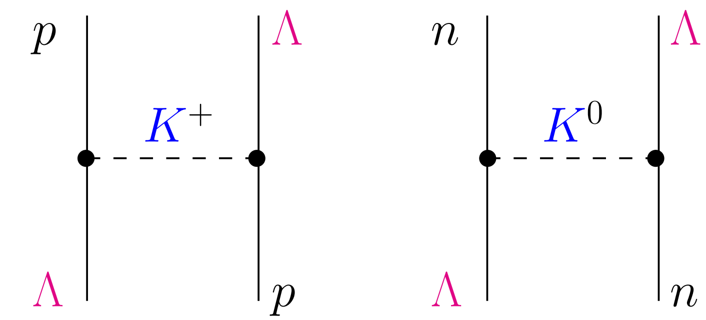




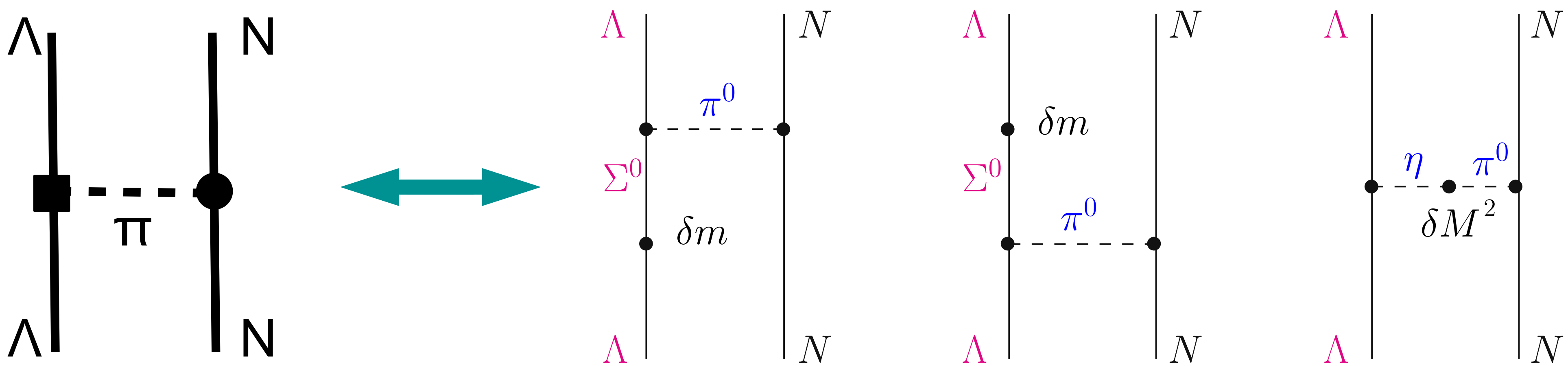
(Le (2023))

CSB contributions to YN interactions

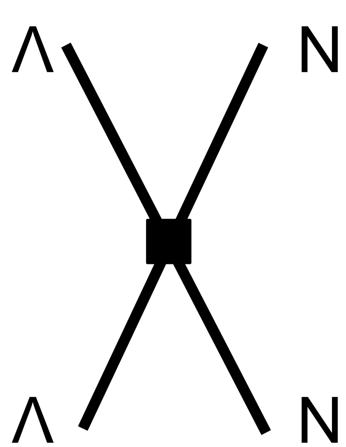
- **formally leading** contributions:
  Goldstone boson mass difference
  — very small due to the small relative difference of kaon masses



- **subleading but most important**
  — effective CSB ΛΛπ coupling constant (Dalitz, von Hippel, 1964)



$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$

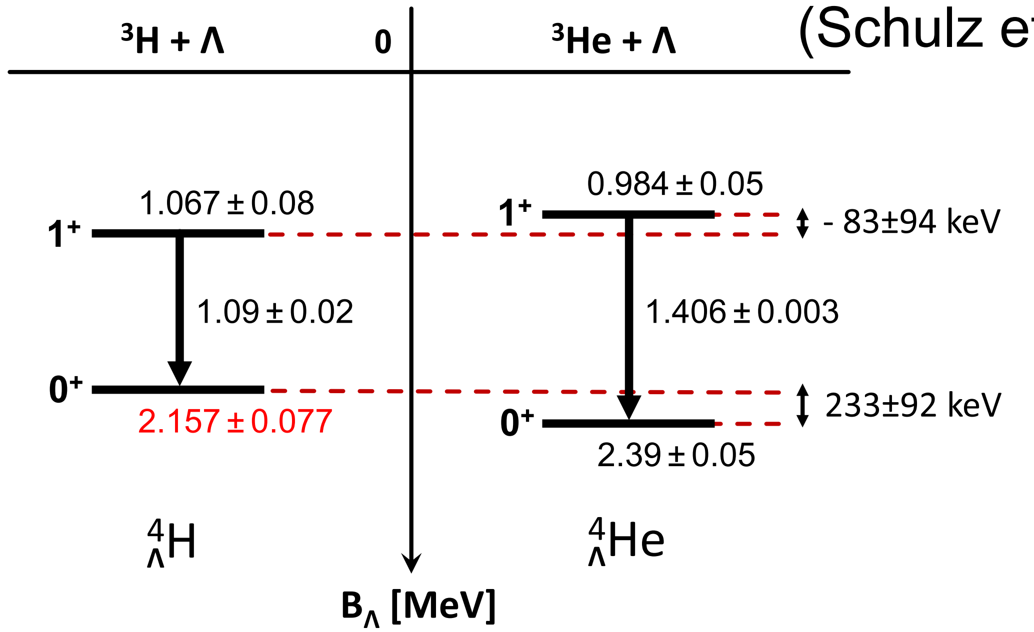


CSB contact interactions (for singlet and triplet):
often not considered but necessary for proper renormalization

Need to determine two unknown CSB LECs and predict Λn scattering

Fit of contact interactions to ${}^4_\Lambda\mathrm{He}/{}^4_\Lambda\mathrm{He}$

JÜLICH Forschungszentrum



(Schulz et al.,2016; Yamamoto, 2015)

- CSB in $A = 4$: 233 ± 92 keV / -83 ± 94 keV
- update: Mainz average including new star data: 178 ± 55 keV/ -139 ± 58 keV (**not** used here)

(https://hypernuclei.kph.uni-mainz.de)

- CSB LECs as expected by power counting
- here only **fit to central values** to test theoretical uncertainties
- $a_{s,t}^{\Lambda n}$ independent of interaction / $a_{s,t}^{\Lambda n}$ depends on CSB input
- CSB / CSB* fits ("standard"/STAR data only)

	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$
NLO13(500)	-2.604	-1.647	-3.267	-1.561
NLO13(550)	-2.586	-1.551	-3.291	-1.469
NLO13(600)	-2.588	-1.573	-3.291	-1.487
NLO13(650)	-2.592	-1.538	-3.271	-1.452
NLO19(500)	-2.649	-1.580	-3.202	-1.467
NLO19(550)	-2.640	-1.524	-3.205	-1.407
NLO19(600)	-2.632	-1.473	-3.227	-1.362
NLO19(650)	-2.620	-1.464	-3.225	-1.365

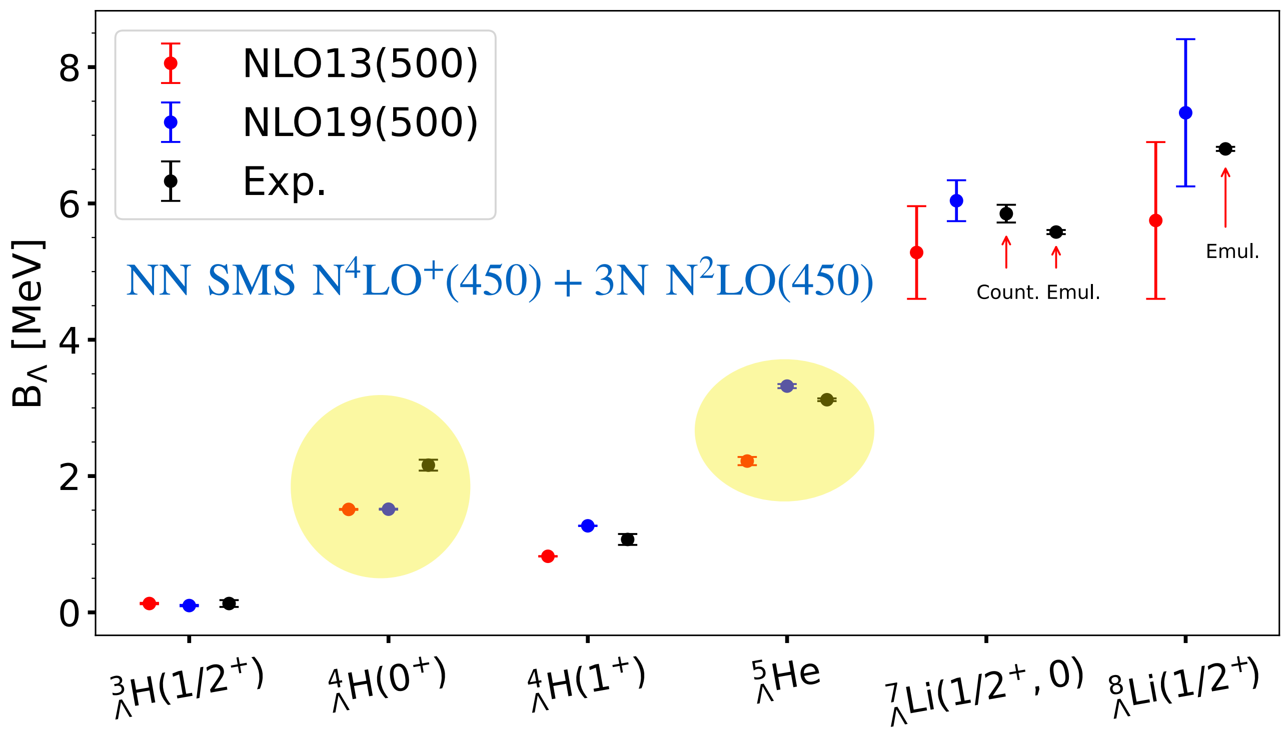
	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

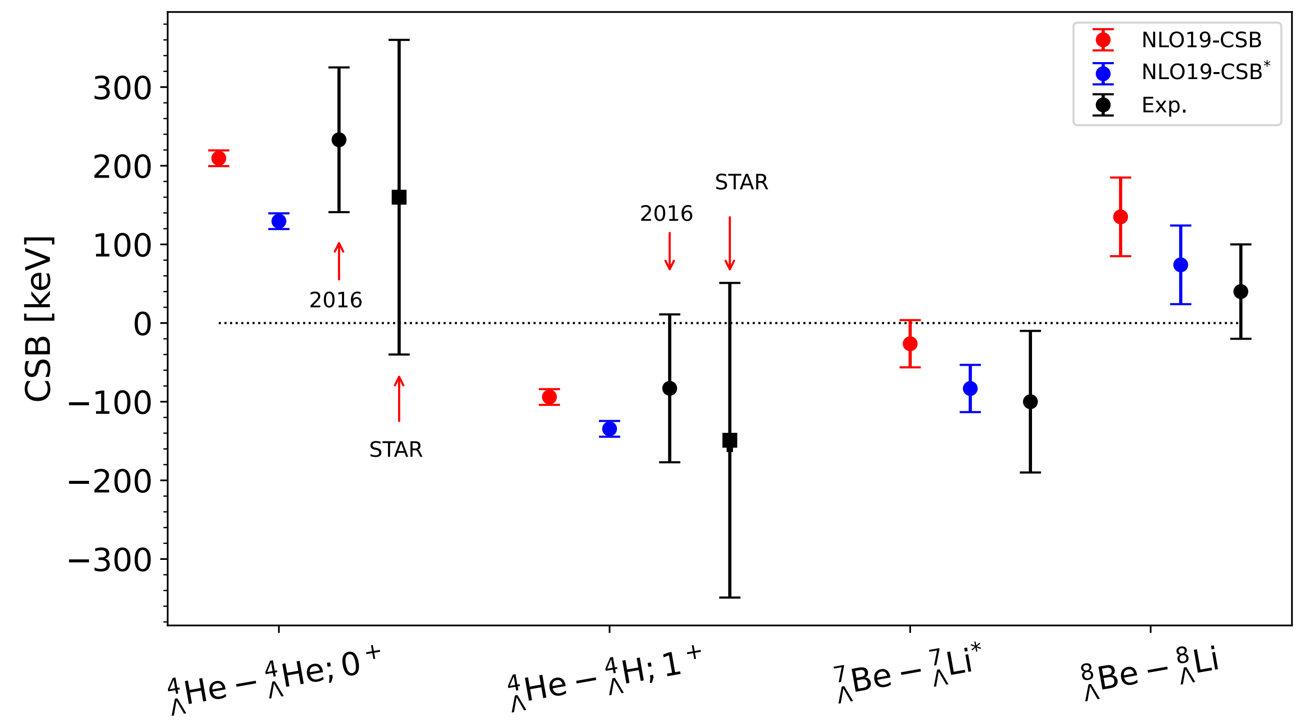
(see Haidenbauer et al. FBS 62,105 (2021))

Predictions for $A = 7$ and 8

- mostly good description for $A = 7{,}8$ hypernuclei
- NLO13/NLO19 differ mainly for $A = 5$
- NLO13/NLO19 fail for 0^+ state of $A = 4$
- uncertainties are numerical only — no estimate of chiral uncertainties

- "standard" scenario only **marginally consistent** with CSB in $A = 8$
- "STAR" scenario better for $A = 7{,}8$





(Le et al. PRC 102,024002 (2023))

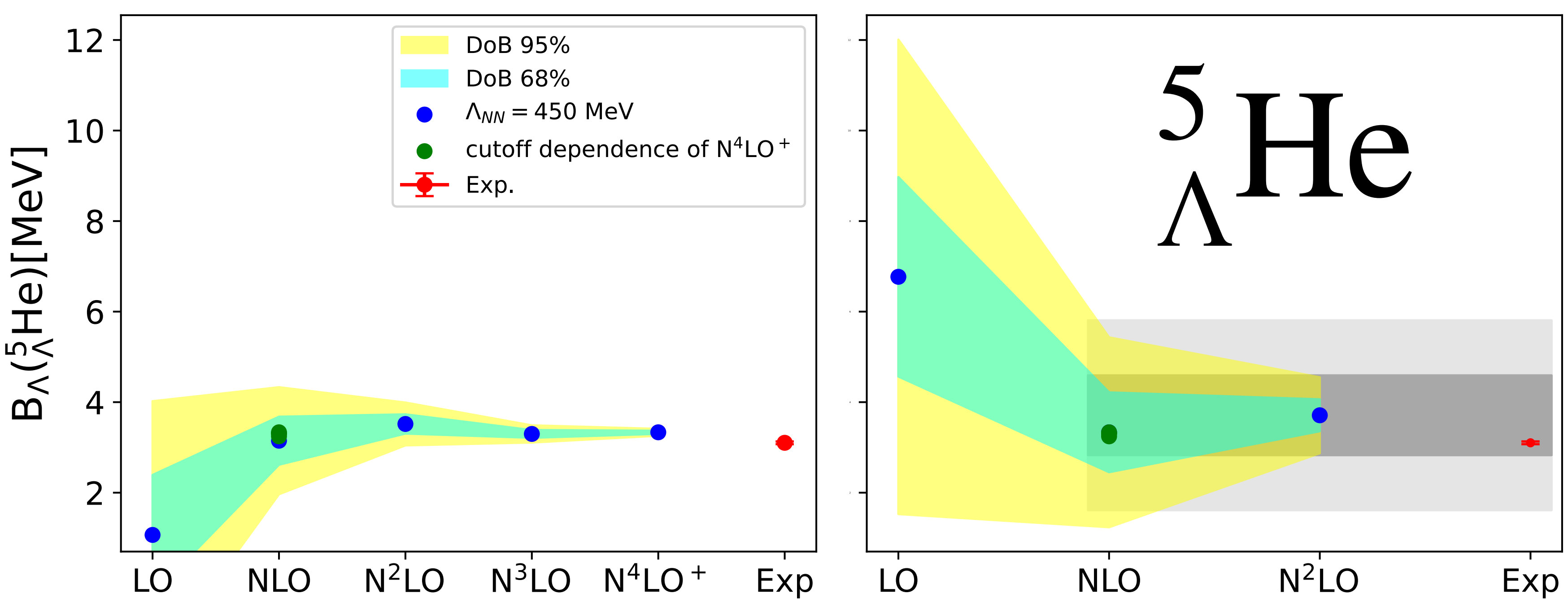
Uncertainty analysis for $A = 3$ to 5

Bayesian analysis of order by order convergence (Melendez et al. 2017,2019)

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, …})$$

→ **probability distributions** for c_k → uncertainty estimate of observable

- **without YNN force:** NLO uncertainty relevant
- sizable uncertainties at $A = 4$ and 5 / $A = 3$ **sufficiently** accurate
- NLO uncertainty is estimate of YNN force
- chiral uncertainty more relevant than numerical uncertainty!



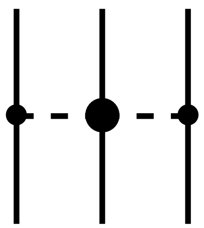
nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
$^3_\Lambda$H	0.011	0.015
$^4_\Lambda$He (0^+)	0.157	0.239
$^4_\Lambda$He (1^+)	0.114	0.214
$^5_\Lambda$He	0.529	0.881

(Le et al. EPJ A 60, 3 (2024))

YNN (ΛNN) interactions

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

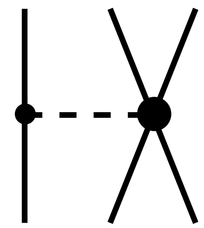
ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN (up to 10)

$\propto C^2$

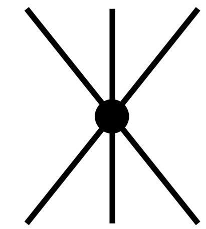
For ΛNN: $\propto C^2$



2 LECs in ΛNN (up to 14)

$\propto CG_1, CG_2$

$\propto C(G_1 + 3G_2)$



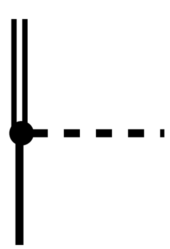
3 LECs in ΛNN ($C'_{1,2,3}$)

5 LECs in ΣNN + 1 Λ-Σ transition

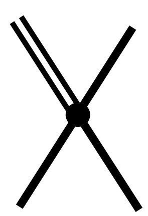
$\propto (G_1)^2, (G_2)^2, G_1G_2$ 2 LECs

$\propto (G_1 + 3G_2)^2$ 1 LEC

only few data → reduce **# of LECs** → decuplet saturation "YNN(Δ)"



$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$

(Petschauer et al., NPA 957, 347 (2017))

***ad hoc* choice:** add non-zero C'_2:

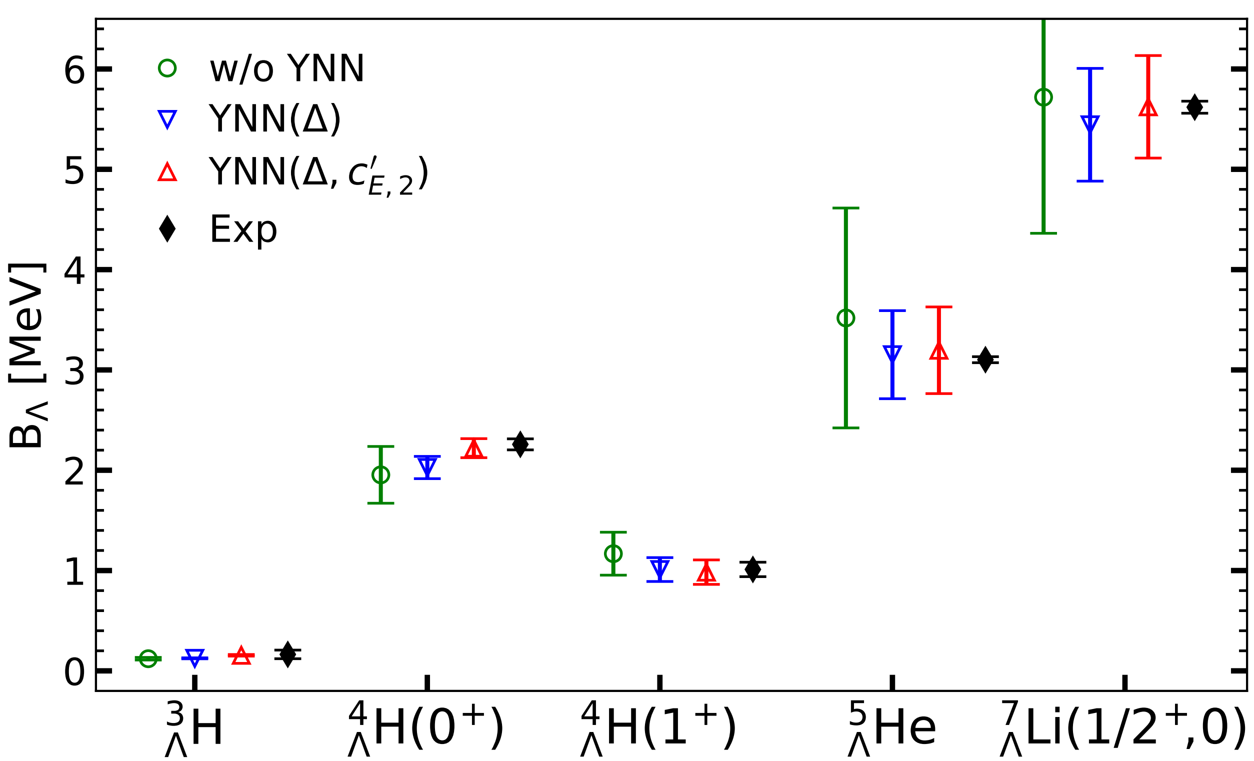
$$C'_1 = C'_3 = \frac{(G_1 + 3G_2)^2}{72\Delta} \qquad C'_2 = 0 \quad \longrightarrow \quad V_{\Lambda NN} = C_2{}' \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 + \vec{\sigma}_3) \, (1 - \vec{\tau}_2 \cdot \vec{\tau}_3)$$

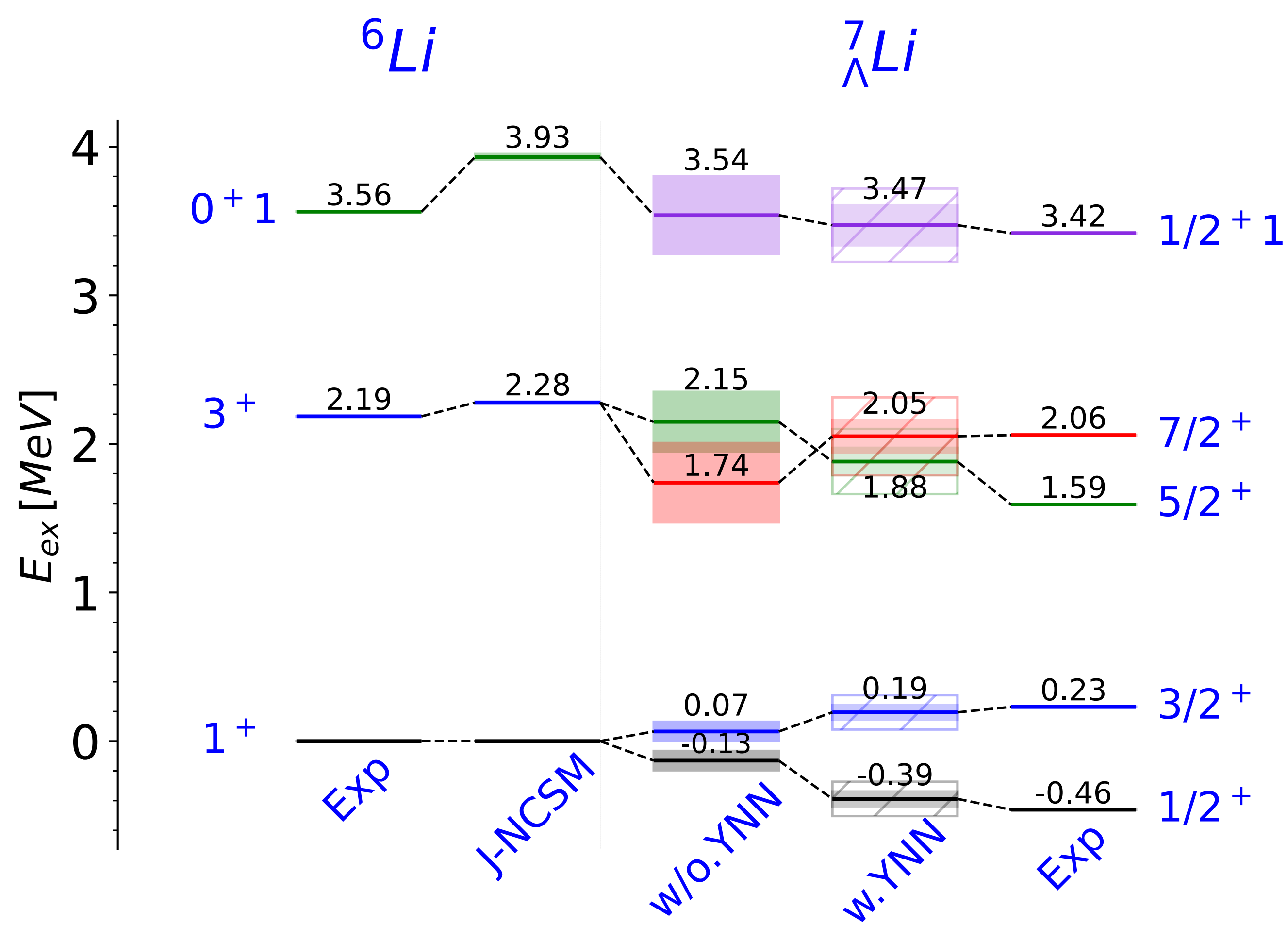
"YNN(Δ, $C'_{E,2}$)"

YNN fit / predictions

- fit to 0^+ and 1^+ state of ${}^4_\Lambda\mathrm{He}$ and/or ${}^5_\Lambda\mathrm{He}$
- spin-dependence in $A = 4$ not well explained by decuplet saturation
- C_2' term improves mainly 0^+ state of ${}^4_\Lambda\mathrm{He}$
- improved spin splittings for ${}^7_\Lambda\mathrm{Li}$ (C_2' term not important!)

- agreement generally much better than $\mathrm{N^2LO}$ uncertainty

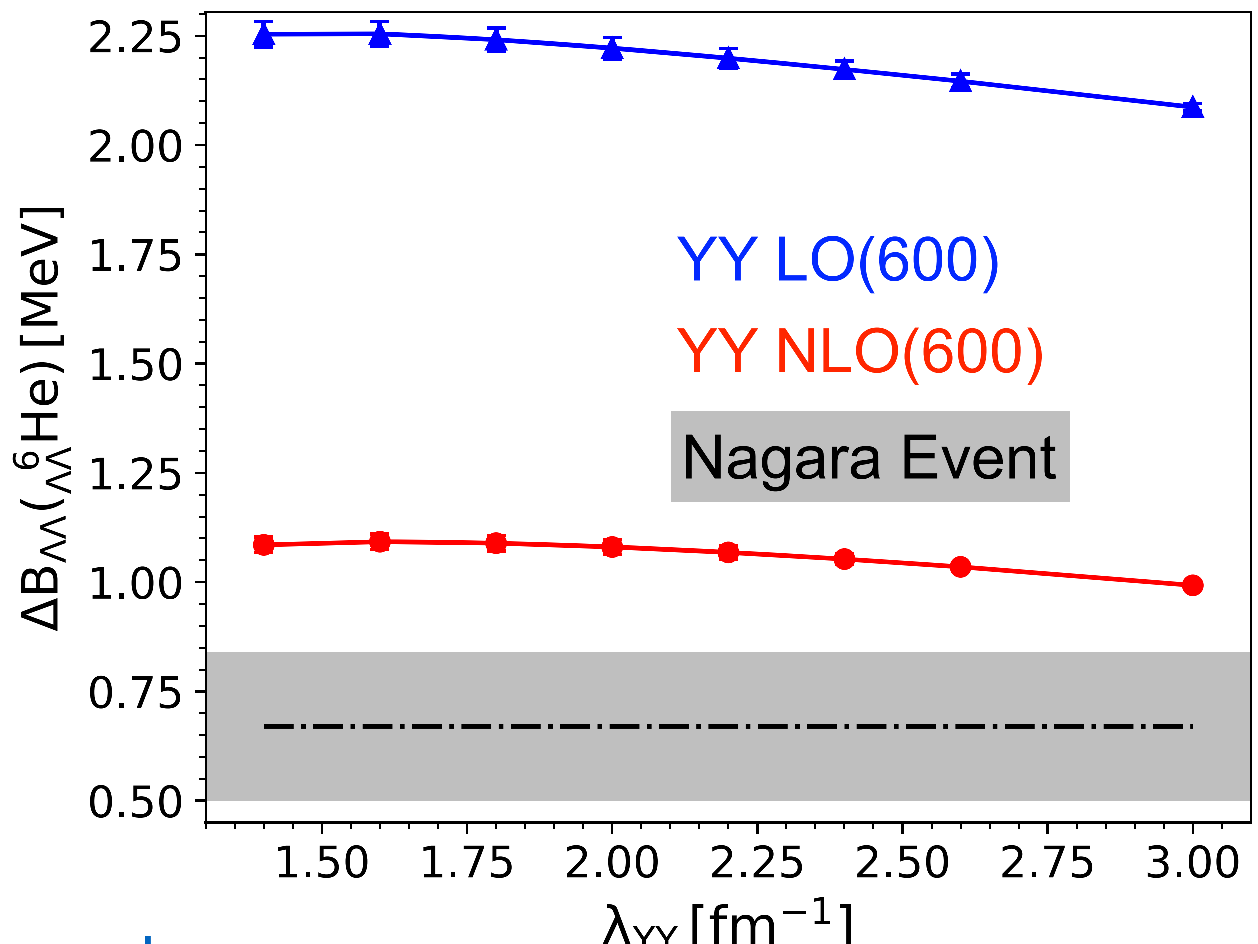




$S = -2$ hypernuclei — ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

$\Lambda\Lambda$ excess binding energy: $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda} = 2E\left({}^{A-1}_{\Lambda}X\right) - E\left({}^{A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$

- NN, YN and YY interactions contribute
- 3NF and YNN forces not included here
- use NN and YN that describe nuclei and single Λ hypernuclei
- small λ_{YY} dependence
- LO too attractive
- NLO predicts binding **fairly** well



NN SMS N4LO+(450) $\lambda_{NN} = 1.6\,\mathrm{fm}^{-1}$

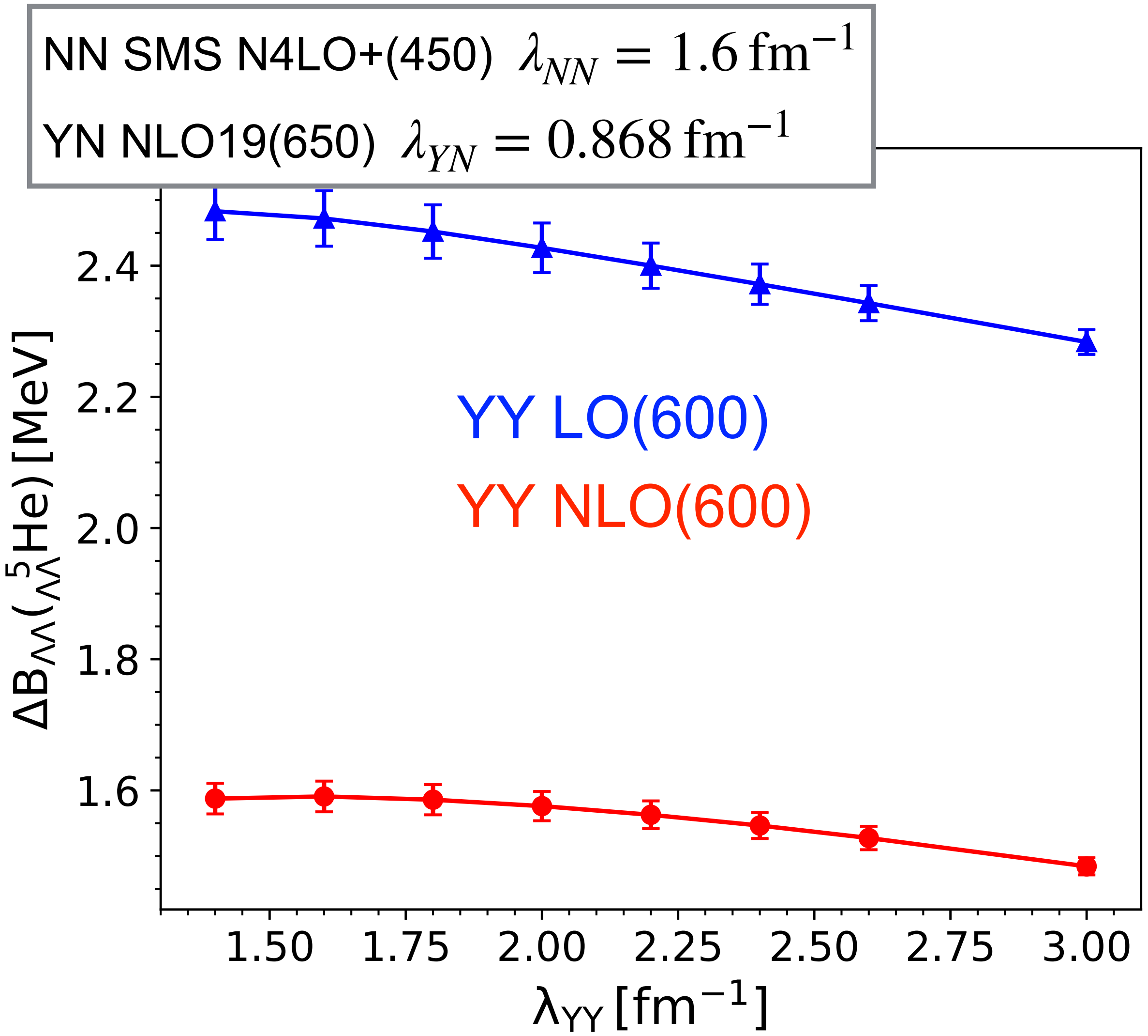
YN NLO19(650) $\lambda_{YN} = 0.868\,\mathrm{fm}^{-1}$

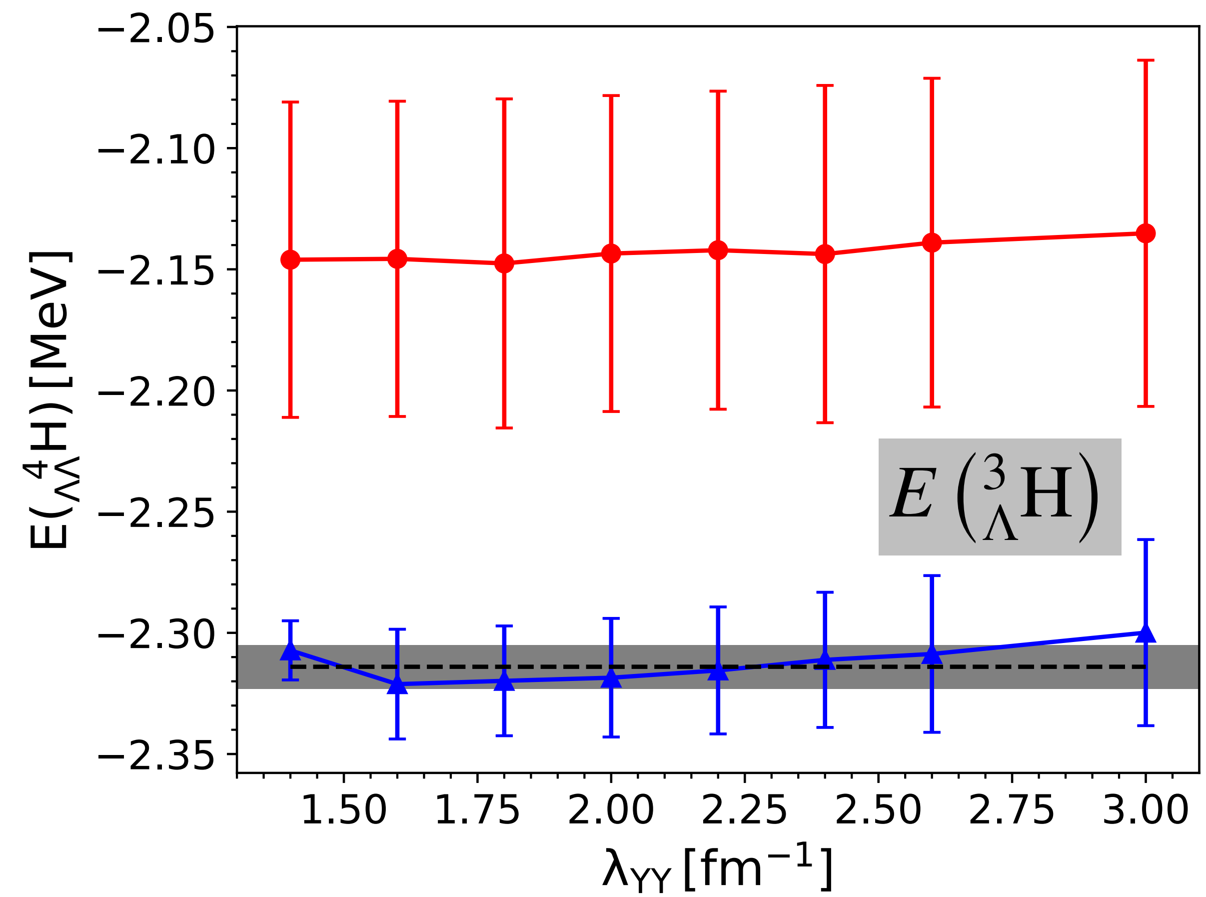
More systems will provide information on spin dependence and contribution $\mathrm{YY}-\Xi\mathrm{N}$ conversion

Can an $S = -2$ bound state for $A = 4{,}5$ be expected?

(Le et al., 2021)

$S = -2$ hypernuclei — $^{5}_{\Lambda\Lambda}\mathrm{He}$ & $^{4}_{\Lambda\Lambda}\mathrm{H}$





- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., 2019)
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., 2002!)

→ **$S = -2$ bound state for $A = 5$ can be expected,**

for $A = 4$ less likely but not ruled out!

Conclusions & Outlook

- **YN & YY interactions not well understood**
  - *scarce YN/YY data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - $^1S_0$ ΛN *scattering length &* $^3_\Lambda\mathrm{H}$
  - $^1S_0$ $\Lambda\Lambda$ *scattering length &* $^{\;\,6}_{\Lambda\Lambda}\mathrm{He}$ *& predictions for A=4,5*
  - *CSB of* ΛN *scattering &* $^4_\Lambda\mathrm{He}$ / $^4_\Lambda\mathrm{H}$
- **New SMS YN interactions**
  - *order LO, NLO and N²LO allow uncertainty quantification*
  - *have a **non-unique** determination of contact interactions (data necessary)*
- **Chiral 3BF**
  - *decuplet saturation improve description of data*
  - *spin-dependent* $\Lambda\mathrm{NN}$ *leads to further improvement*
  - *study cutoff dependence / application to more p-shell hypernuclei*
  - *contribution to hypernuclear matter*
- **Decay of hypernuclei**
  - *decuplet saturation decay provides new insights to structure*
  - *currents required, final state interactions*