The ppp and $pp\Lambda$ correlation functions

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Introduction

- When a high-energy pp or p–nucleus collision occurs, particles are produced and emitted at relative distances of the order of the nuclear force
- The effect of the mutual interaction between hadrons is reflected as a correlation signal in the momentum distributions of the detected particles which can be studied using correlation functions
- The correlation function incorporate information on the emission process as well as on the final state interaction of the emitted pairs
- By measuring correlated particle pairs or triplets at low relative energies and comparing the yields to theoretical predictions, it is possible to study the hadron dynamics.

The two-particle correlation function

- The two-particle correlation function is defined as the ratio of the yield of a particle pair to the product of the single-particle yields.

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{\mathcal{P}\left(\vec{p}_1, \vec{p}_2\right)}{\mathcal{P}\left(\vec{p}_1\right)\mathcal{P}\left(\vec{p}_2\right)}$$

- $\mathcal{P}(\vec{p}_1, \vec{p}_2)$ is the probability of finding a pair with momenta $\vec{p}_1$ and $\vec{p}_2$
- $\mathcal{P}(\vec{p}_i)$ is the probability of finding each particle with momentum $\vec{p}_i$.
- In absence of correlations, the two-particle probability factorizes, $\mathcal{P}(\vec{p}_1, \vec{p}_2) = \mathcal{P}(\vec{p}_1)\mathcal{P}(\vec{p}_2)$, and the correlation function is equal to unity.

The two-particle correlation function

- The correlation between the pair is related to the particle emission and the subsequent interaction of the pair

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{1}{\Gamma} \sum_{m_1, m_2} \int d^3 r_1 \, d^3 r_2 S_1\left(r_1\right) S_1\left(r_2\right) \times \left|\Psi_{m_1, m_2}(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2)\right|^2$$

- $S_1(r)$ describes the spatial shape of the source for single-particle emissions. It can be approximated as a Gaussian probability distribution with a width R_M

$$S_1(r) = \frac{1}{(2\pi R_M^2)^{\frac{3}{2}}} e^{-r^2/2R_M^2}$$

- The integration on the CM coordinates leads to the Koonin-Pratt relation for two-particle correlation function

$$C(k) = \frac{1}{\Gamma} \int d^3 r \, S(r) |\psi_k\left(\vec{r}\right)|^2$$

The pp correlation function

- $S(r)$ is the two-particle emission source, given by

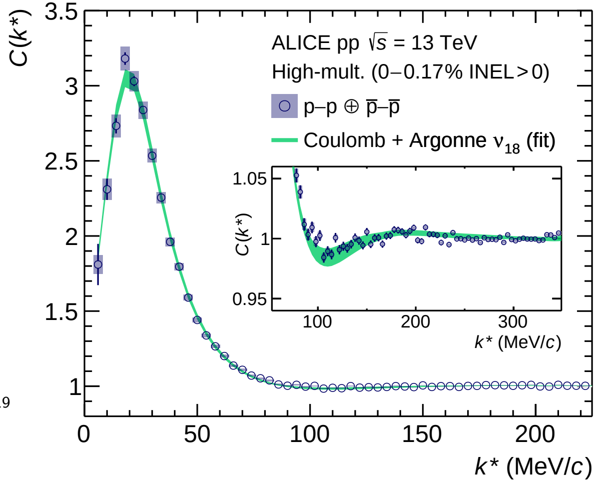
$$S(r) = \left(\frac{1}{4\pi R_{\rm M}^2}\right)^{3/2} e^{-\frac{r^2}{4R_{\rm M}^2}}$$

- $\psi_k(\vec{r})$ is the two-particle scattering wave function at $E = \hbar^2 k^2/2\mu$
- The scattering wave function is expanded in partial waves

$$\psi_k = 4\pi \sum_{JJ_z} \sum_{\ell m S S_z} i^\ell (kr)^{-1} u_\ell(kr) \mathcal{Y}_{[\ell S]}(\hat{r})(\ell m S S_z | J J_z) Y^*_{\ell m}(\hat{k})$$

- In the case of two protons $u_\ell(kr) \to F_\ell(\eta, kr) + T_{\ell\ell}\mathcal{O}_\ell(kr)$

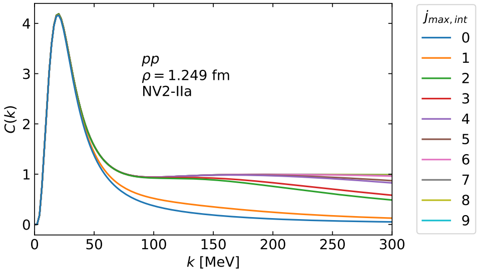
The pp Correlation Function



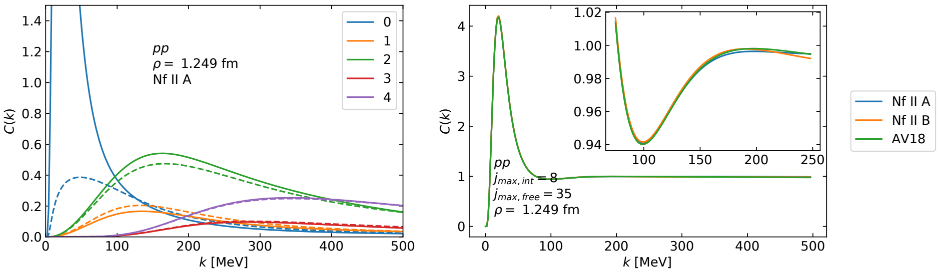
ALICE collaboration
Phys. Lett. B 805, (2020) 135419

The *pp* correlation function

$$C(k) = \frac{1}{N}\left[\sum_{j\le j_m}\sum_{l,l',s,t}\int dr\, r^2 S(r)\left|\Psi^{(l')}_{k;l,s,j,t}(r)\right|^2 + \sum_{j_m<j\le j_M}\sum_{l,s,t}\int dr\, r^2 Sr\left|\Psi_{\mathrm{free};k;l,s,j,t}(r)\right|^2\right]$$



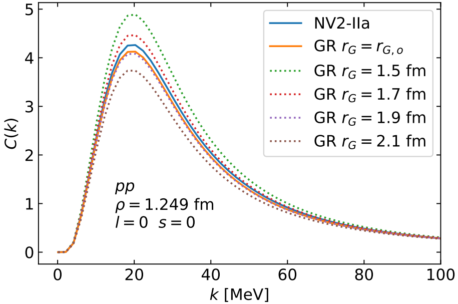
The *pp* correlation function



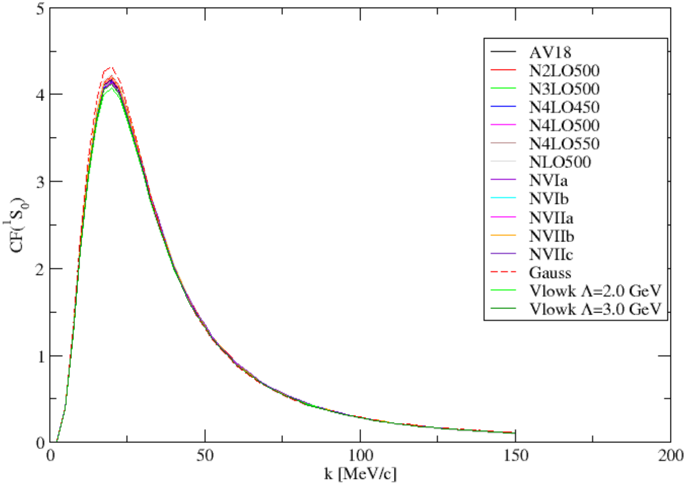
The *pp* correlation function from a Gaussian representation

$$V_{pp}(^1S_0) = V_0 e^{-(r/r_G)^2} + \frac{e^2}{r}$$

with V_0 fixed to reproduce the *pp* scattering length. When $r_G = r_{G,o}$ the *pp* effective range is described too.



The *pp* correlation function using several potentials



Considering the Coulomb interaction

The *pp* case

We consider two cases

$a) V_{pp}(r) = V_0\, e^{-(r/r_0)^2}\mathcal{P}_0 + \frac{e^2}{r}$

$b) V_{pp}^{sc}(r) = V_0\, e^{-(r/r_0)^2}\mathcal{P}_0 + \frac{e^2}{r} e^{-(r/r_{sc})^n}$

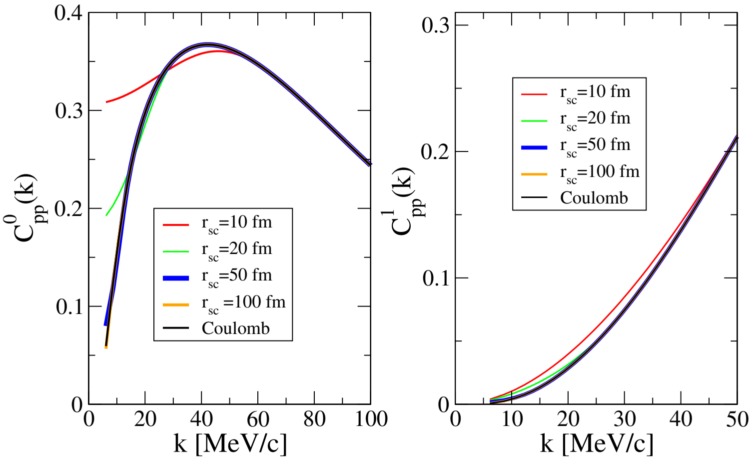
The correlation function is

$$C_{pp}(k) = \sum_\ell C_{pp}^\ell = \int dr \frac{e^{-(r^2/4R^2)}}{4\sqrt{\pi}R^3 k^2} \left(\sum_{\ell\equiv \mathrm{even}} u_\ell^2(kr)(2\ell+1) + 3 \sum_{\ell\equiv \mathrm{odd}} u_\ell^2(kr)(2\ell+1) \right)$$

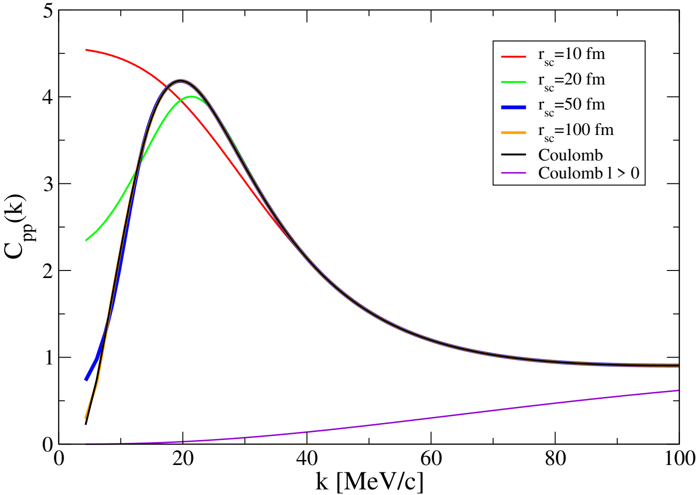
$a) u_\ell(kr \to \infty) \longrightarrow [F_\ell(\nu, kr) + T_{\ell\ell}\mathcal{O}(\nu, kr)]$

$b) u_\ell(kr \to \infty) \longrightarrow kr[j_\ell(kr) + T_{\ell\ell}\mathcal{O}(kr)]$

Considering the Coulomb interaction



Considering the Coulomb plus the short-range interaction



The pd Correlation Function

- We now consider the pd correlation function:

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int d^3r_1 d^3r_2 d^3r_3 \; S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2,m_1}|^2$$

- the probability of deuteron formation

$$A_d = \frac{1}{3} \sum_{m_2} \int d^3r_1 d^3r_2 \; S_1(r_1) S_1(r_2) |\phi_{m_2}|^2$$

- the single particle source function

$$S_1(r) = \frac{1}{(2\pi R_{\mathrm{M}}^2)^{\frac{3}{2}}} e^{-r^2/2R_{\mathrm{M}}^2}$$

The pd Correlation Function

- the pd correlation function results

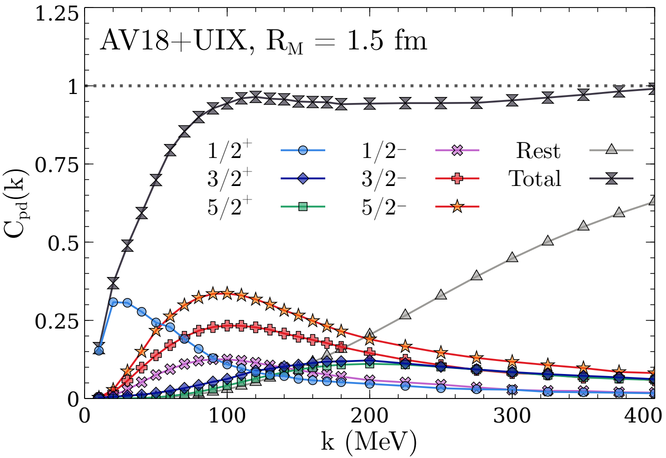
$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi_{m_2,m_1}|^2$$

$$\Psi_{m_2,m_1} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 \mid S J_z)(L 0 S J_z \mid J J_z) \Psi_{LSJJ_z}$$

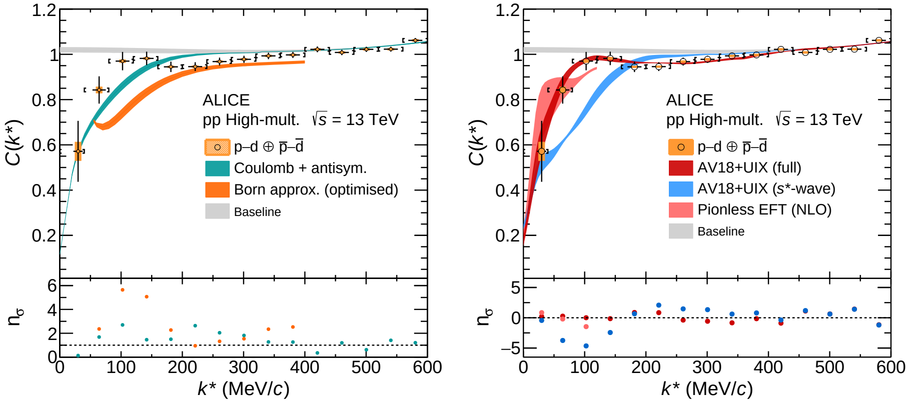
- the Jacobi coordinates: $\mathbf{x}_\ell = \mathbf{r}_j - \mathbf{r}_i$, $\mathbf{y}_\ell = \mathbf{r}_\ell - \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$
- the hyperspherical coordinates $\rho = \sqrt{x_1^2 + (4/3) y_1^2}$, $\Omega \equiv [\alpha_1, \hat{x}_1, \hat{y}_1]$
- The scattering wave function is expanded in partial waves using the HH basis

$$\Psi_{LSJJ_z} = \rho^{-5/2} \sum_{[K]} u_{[K]}(\rho) \mathcal{Y}_{[K]}^{LSJJ_z}(\Omega)$$

The pd Correlation Function: partial-wave contributions



The pd Correlation Function: comparison to experiment



M. Viviani, S. König, A. Kievsky, L.E. Marcucci, B. Singh, O. Vázquez Doce, Phys. Rev. C 108, 064002 (2023)
ALICE collaboration, Physical Review X 14, 031051 (2024)

The ppp correlation function

- Now we consider the ppp correlation function:

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega\, S_{\rho_0}(\rho)|\Psi_{ppp}|^2$$

with Q the hyper-momentum, S_{ρ_0} the source function defined as

$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Ψ_{ppp} is the ppp scattering wave function

$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho)\mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,[K]}^{\overline{J},\overline{K}} \Psi^J_{[K]}$$

To be noticed that Ψ^0 is not well known. In $\Psi^J_{[K]}$ the interaction has been considered up to $\overline{J}$ and $\overline{K}$

Considering the Coulomb interaction

The *ppp* case

As a preliminary step we introduce the hypercentral Coulomb force obtained after averaging the bare Coulomb force on the hyperangles

$$V_{\mathrm{Coul}}(\rho) = \frac{1}{\pi^3} \int d\Omega_\rho \sum_{i<j} \frac{e^2}{r_{ij}} = \frac{3(4\pi)^2}{\pi^3} \int d\alpha \sin^2\alpha \cos^2\alpha \frac{e^2}{\rho\cos\alpha} = \frac{16}{\pi}\frac{e^2}{\rho}$$

Now the asymptotic solution is a regular Coulomb function with order $K + \frac{3}{2}$, the Sommerfeld parameter $\eta = 16me^2/(\pi\hbar^2 Q)$. The norm of the continuum wave function is formally equal to the non Coulomb case replacing $J_{K+2}(z) \longrightarrow \sqrt{\frac{2}{\pi z}} F_{K+\frac{3}{2}}(z)$:

$$|\Psi^0_s|^2_\Omega = \frac{96}{\pi}\frac{1}{(Q\rho)^5}\sum_K F^2_{K+3/2}(Q\rho)(N^m_{ST}(K) + 4N^a_{ST}(K))$$

Considering the Coulomb interaction: asymptotic behavior

Without considering long-range interaction, the correlation function is

$$C^0_{ppp}(Q) = \frac{6}{8}\frac{2^6}{Q^4\rho_0^6}\int \rho\, d\rho\, e^{-\frac{\rho^2}{\rho_0^2}} \sum_K J^2_{K+2}(Q\rho)\,[N^m_{ST}(K) + 4N^a_{ST}(K)]$$

When the Coulomb force is considered in its hypercentral approximation it results

$$C^{0,c}_{ppp}(Q) = \frac{96}{\pi}\frac{1}{Q^5\rho_0^6}\int \rho\, d\rho\, e^{-\frac{\rho^2}{\rho_0^2}} \sum_K F^2_{K+3/2}(Q\rho)\,[N^m_{ST}(K) + 4N^a_{ST}(K)]$$

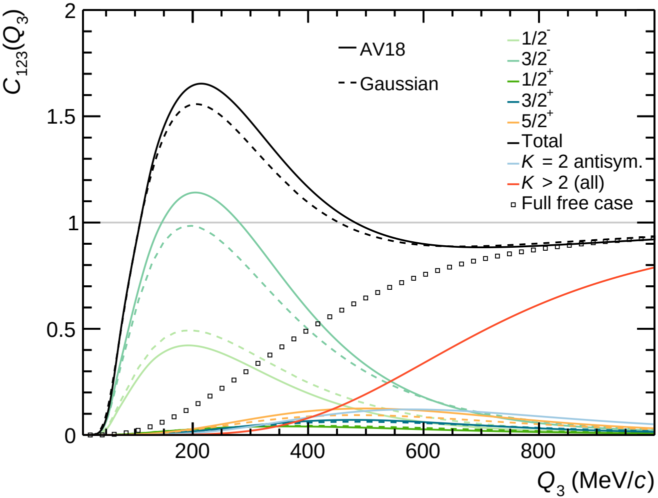
We now introduce the screened hypercentral Coulomb potential

$$V^{sc}_{\rm Coul}(\rho) = \frac{16}{\pi}\frac{e^2}{\rho}e^{-(\rho/\rho_{sc})^n}$$

The correlation function results ($\epsilon = c, sc$)

$$C^{0,\epsilon}_{ppp}(Q) = \sum_K C^{0,\epsilon}_K = \frac{6}{8}\frac{2^6}{Q^4\rho_0^6}\int \rho\, d\rho\, e^{-\frac{\rho^2}{\rho_0^2}} \sum_K |u_K|^2(Q\rho)\,[N^m_{ST}(K) + 4N^a_{ST}(K)]$$

The ppp correlation function



Some remarks

- To compare the experimental and the theoretical correlation functions some corrections have been considered
- For the *pp* case the corrected correlation function is defiend as

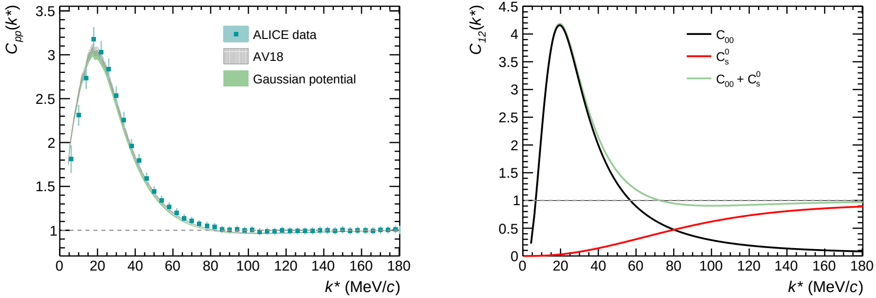
$$C(k) = \lambda_{pp} C_{pp}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\rm X} C_{\rm X}(k)$$

- primary protons $\lambda_{pp} = 0.67$, secondary protons produced mainly in the decay of the Λ, $\lambda_{pp_\Lambda} = 0.203$, misidentification contributions $\lambda_x = 0.127$
- For the *ppp* case the corrected correlation function is defiend as

$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{ppp_\Lambda} C_{ppp_\Lambda}(Q_3) + \lambda_{\rm X} C_{\rm X}(Q_3)$$

- primary protons $\lambda_{ppp} = 0.618$, secondary protons produced mainly in the decay of the Λ, $\lambda_{ppp_\Lambda} = 0.196$, misidentification contributions $\lambda_x = 0.186$

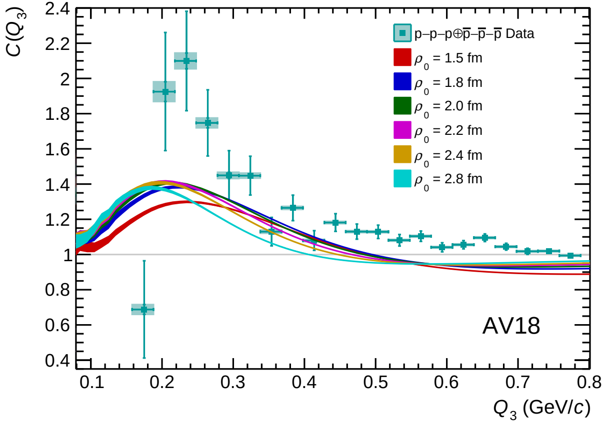
The pp correlation function



$$C_{12}(k) = C_s^0 + C_{00} = \int d\mathbf{r} S_{12}(r) \left[|\Psi_s^0|^2_\Omega - \frac{1}{2}\left(\frac{F_0(\eta, kr)}{kr}\right)^2 + \frac{1}{2}\left(\frac{u_0(kr)}{kr}\right)^2 \right]$$

$$C_{pp}(k) = \lambda_{pp} C_{12}(k) + \lambda_{p p_\Lambda} C_{p p_\Lambda}(k) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(k)$$

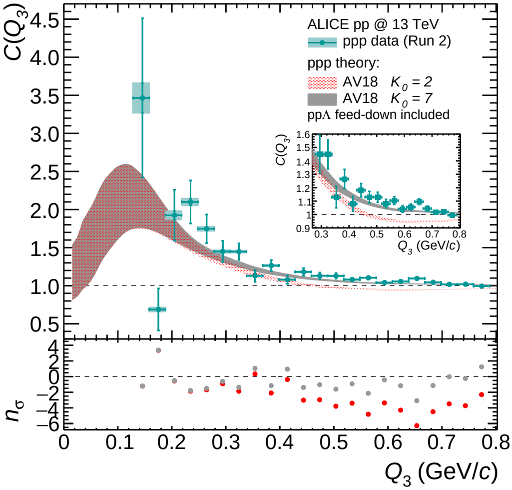
The *ppp* correlation function



$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{ppp_\Lambda} C_{ppp_\Lambda}(Q_3) + \lambda_{\rm X} C_{\rm X}(Q_3)$$

A. Kievsky, E. Garrido, M. Viviani, L. E. Marcucci, L. Šerkšnytė, and R. Del Grande Phys. Rev. C 109, 034006 (2024)

The *ppp* correlation function



E. Garrido et al. PLB, submitted

The $p\Lambda$ and $pp\Lambda$ correlation functions

- The $p\Lambda$ correlation function is defined as

$$C(k) = \int d^3r\, S(r) |\psi_{p\Lambda}(\vec{r})|^2$$

- $\psi_{p\Lambda}$ is the scattering $p\Lambda$ wave function. It is governed by the $p\Lambda$ interaction which is not very well kown
- The few $p\Lambda$ scattering data can be described in the context of the EFT at different orders (see for example J. Haidenbauer et al. Eur. Phys. J. A 59 (2023) 63)
- At different cutoffs different sets of low-energy scattering parameters appear

	NLO13						NLO19				SMS N2LO		
C(MeV)	450	500	550	600	650	700	500	550	600	650	500	550	600
a_0 (fm)	-2.90	-2.91	-2.91	-2.91	-2.90	-2.90	-2.91	-2.90	-2.91	-2.90	-2.80	-2.79	-2.80
r_e^0 (fm)	2.64	2.86	2.84	2.78	2.65	2.56	3.10	2.93	2.78	2.65	2.82	2.89	2.68
a_1 (fm)	-1.70	-1.61	-1.52	-1.54	-1.51	-1.48	-1.52	-1.46	-1.41	-1.40	-1.56	-1.58	-1.56
r_e^1 (fm)	3.44	3.05	2.83	2.72	2.64	2.62	2.62	2.61	2.53	2.59	3.16	3.09	3.17

Introduction to universal physics

- In the 40's Bargmann and Hulthén, and before Eckart, studied potentials that give specific representations of the S-matrix
- Here we are interested in the S-matrix representing one shallow state, virtual or bound

$$S(k) = \frac{k + i/a_B}{k - i/a_B} \frac{k + i/r_B}{k - i/r_B}$$

  with the energy of the system $E = \hbar^2 k^2/m$
- The energy pole is described by the energy length $a_B \longrightarrow E_2 = -\hbar^2/ma_B^2$
- E_2 is a bound or virtual state when $a_B > 0$ or $a_B < 0$
- the superfluous pole is described by the length $r_B = a - a_B$, with a the scattering length. It ensures the correct asymptotic behavior of the Jost function and it is always positive.
- When $r_B = 0$ implies a contact interaction and the superfluous pole goes to ∞

Shallow states: definition

- The given S-matrix is equivalent to the effective range expansion

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_e k^2$$

- The physical pole verifies

$$\frac{1}{a_B} = \frac{1}{a} + \frac{1}{2} \frac{r_e}{a_B^2}$$

- The degree of validity of this relation defines the shallow characteristic of the state
- In real systems there are small corrections in the effective range expansion

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \sum_{n=2} v_n k^{2n}$$

with v_n the shape parameters. Inside the region of interest, the universal window, they are small and the state is shallow

Effective description

- The S-matrix

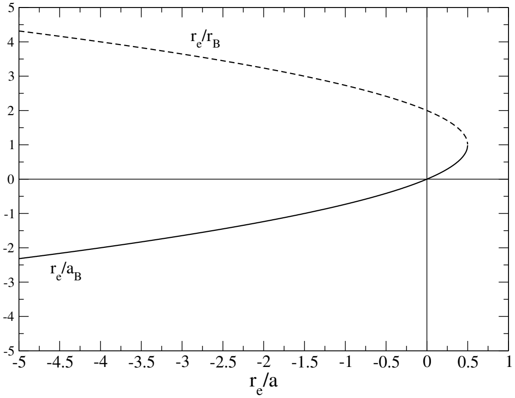
$$S(k) = \frac{k + i/a_B}{k - i/a_B}\frac{k + i/r_B}{k - i/r_B}$$

is exactly represented by the Eckart potential:

$$V(r) = -2\frac{\hbar^2}{mr_0^2}\frac{\beta e^{-r/r_0}}{(1+\beta e^{-r/r_0})^2}$$

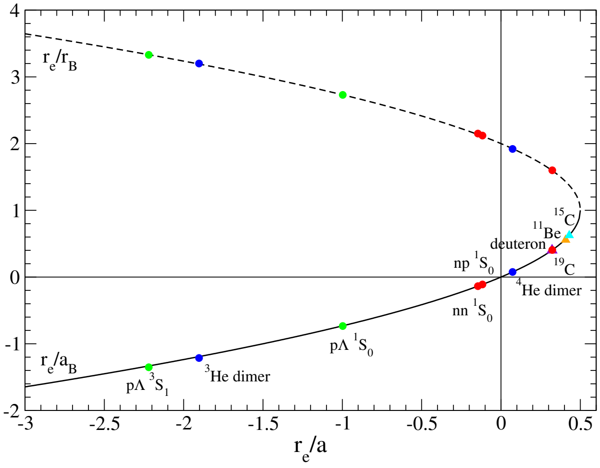
$$\begin{cases} a = 4r_0\frac{\beta}{\beta-1} \\ a_B = 2r_0\frac{\beta+1}{\beta-1} \end{cases} \qquad \begin{cases} r_e = 2r_0\frac{\beta+1}{\beta} \\ r_B = 2r_0 \end{cases}$$

Effective description in the plane $[r_e/a, r_e/a_b]$



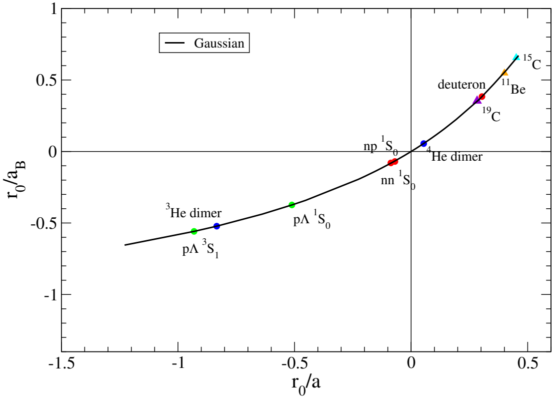
in terms of the potential parameters $r_e/a = \frac{1}{2}\frac{\beta^2-1}{\beta^2}$, $r_e/a_B = \frac{\beta-1}{\beta}$ and $r_e/r_B = \frac{\beta+1}{\beta}$

Physical systems inside the Eckart window



Systems are put on the figure by their experimental data

The universal window in terms of the Gaussian parameters



$$V(r) = -\frac{\hbar^2}{m r_0^2}\beta e^{-(r/r_0)^2}$$

Effective description

- System inside the window have been described using different EFT frameworks
- The nuclear system is currently described using chiral potentials or using pionless EFT
- Atomic helium has been extensively studied using potentials models (Aziz potentials, TTY potential, etc) and also using contact EFT
- Halo nuclei are currently studied using potential models and also Halo EFT
- Hadron systems as $N - \Lambda$ and hypernuclei are studied using potential models and also using chiral or contact EFT
- The above discussion suggests an effective description of a system inside the universal window based on the Eckart or Gaussian potential

$$V_{LO} = V[\beta(a, a_B, r_e), r_0(a, a_B, r_e)]$$

- We consider this description a optimized LO description

The $p\Lambda$ effective interaction

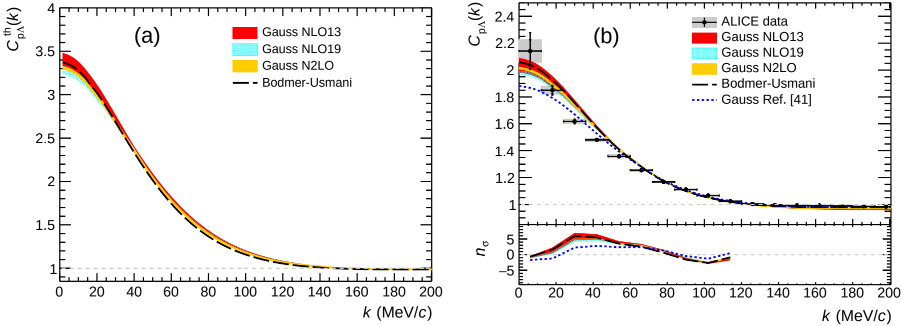
Using the Gaussian form, we define the effective $p\Lambda$ interaction as

$$V_{p\Lambda}(r) = \sum_{S=0,1} V_S e^{-(r/r_S)^2} \mathcal{P}_S$$

	NLO13				NLO19				SMS N2LO		
C(MeV)	500	550	600	650	500	550	600	650	500	550	600
V_0 (MeV)	-30.180	-30.574	-31.851	-34.831	-25.954	-28.817	-31.851	-34.831	-31.140	-29.753	-34.273
r_0 (fm)	1.467	1.459	1.434	1.380	1.563	1.495	1.434	1.380	1.439	1.466	1.382
V_1 (MeV)	-29.205	-33.839	-36.258	-38.455	-38.984	-39.470	-42.055	-40.373	-27.544	-28.609	-27.392
r_1 (fm)	1.338	1.247	1.216	1.183	1.178	1.163	1.126	1.143	1.361	1.344	1.364
$B(^3_\Lambda\mathrm{H})$ (MeV)	2.8729	2.87956	2.92508	2.98499	2.79212	2.83929	2.90455	3.25522	2.81932	2.79875	2.8785
W_3 (MeV)	11.83	11.733	12.32	12.873	10.545	11.056	11.795	12.294	10.65	10.375	11.4
ρ_3 (fm)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

The $p\Lambda$ correlation function: $C_{p\Lambda}(k) = \int d^3r\ S(r)\ |\Psi_{p\Lambda}|^2$

$$C_{p\Lambda}(k) = \lambda_{p\Lambda} C^{\rm th}_{p\Lambda}(k) + \lambda_{{\rm p}\Lambda_{\Sigma^0}} C_{p\Lambda_{\Sigma^0}}(k) + \lambda_{p\Lambda_{\Xi}} C_{p\Lambda_{\Xi}}(k) + \lambda_{\rm flat}$$



$V^{BU}_{p\Lambda} = V_C(r)(1 - \epsilon + \epsilon P_x) + 0.25\, V_G\ T^2_\pi(r)\, \sigma_\Lambda \cdot \sigma_p$

$a_0 = -2.88$ fm, $r^0_e = 2.87$ fm

$a_1 = -1.66$ fm, $r^1_e = 3.67$ fm

$B(^3_\Lambda{\rm H}) = 2.73$ MeV

$V_{p\Lambda}(r) = \sum_{S=0,1} V_S\, e^{-(r/r_S)^2} \mathcal{P}_S$

$a_0 = -2.10$ fm, $r^0_e = 3.21$ fm

$a_1 = -1.54$ fm, $r^1_e = 3.16$ fm

$B(^3_\Lambda{\rm H}) = 2.40$ MeV

The $pp\Lambda$ system

Jacobi coordinates for two nucleons of mass m and the Λ of mass M in $\mathbf{r}_3$

$r-$space | $q-$space

$$\begin{cases} \mathbf{x} = \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} = \sqrt{\frac{4}{(1+2m/M)}}\,(\mathbf{r}_3 - \frac{\mathbf{r}_1+\mathbf{r}_2}{2}) \end{cases} \qquad \begin{cases} \mathbf{k} = \frac{1}{2}(\mathbf{p}_2 - \mathbf{p}_1) \\ \mathbf{q} = \sqrt{\frac{m}{M}}\sqrt{\frac{m}{2m+M}}\,(\mathbf{p}_3 - \frac{M}{m}\frac{\mathbf{p}_1+\mathbf{p}_2}{2}) \end{cases}$$

The hyperradius $\rho = (x^2 + y^2)^{1/2}$ the hypermomentum $Q = (k^2 + q^2)^{1/2}$

$[\Omega_\rho \equiv \hat{x}, \hat{y}, \alpha = \arctan(x/y)]$ $[\Omega_Q \equiv \hat{k}, \hat{q}, \tilde{\alpha} = \arctan(k/q)]$

In terms of the particle distances $\frac{\rho^2}{2} = r_1^2 + r_2^2 + \frac{M}{m} r_3^2 - \frac{M+2m}{m} R^2$

The hypermomentum is related to the total energy $E = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2M} = \frac{Q^2}{m}$

The $pp\Lambda$ source function

The correlation function for three particles is given by

$$C_{123}(Q) = \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \; S_1(\mathbf{r}_1) S_2(\mathbf{r}_2) S_3(\mathbf{r}_3) |\Psi_s|^2$$

The source function $S_i(\mathbf{r}_i)$ is approximated by a Gaussian probability distribution. The widths of the proton and Λ distributions as R_m and R_M, respectively.

$$S_1(\mathbf{r}_1) S_1(\mathbf{r}_2) S_1(\mathbf{r}_3) = \frac{e^{-(\frac{\rho^2}{2} - (\frac{R_m^2}{R_M^2} - \frac{M}{m}) r_3^2 + \frac{M+2m}{m} R^2)/2R_m^2}}{(2\pi R_m^2)^3 (2\pi R_M^2)^{\frac{3}{2}}}$$

with the condition $R_m^2/R_M^2 = M/m$ after integrating our the center of mass

$$S_{123}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

with $\rho_0 = 2R_m$

The $pp\Lambda$ correlation function

$$C_{123}(Q) = \frac{1}{\pi^3 \rho_0^6} \int e^{-(\rho/\rho_0)^2} |\Psi_s|^2 \rho^5 d\rho d\Omega_\rho$$

With the three-body scattering wave function

$$\Psi_s = \frac{1}{\sqrt{N_S}} \frac{(2\pi)^3}{(Q\rho)^{5/2}} \sum_{JJ_z} \sum_{K\gamma} \Psi^{JJ_z}_{K\gamma} \sum_{M_L M_S} (LM_L SM_S|JJ_z) \mathcal{Y}^{\ell_x \ell_y}_{KLM_L}(\Omega_Q)^*$$

N_S is the number of spin states and $\gamma \equiv \{\ell_x, \ell_y, L, s_x, S\}$. The coordinate wave functions, $\Psi^{JJ_z}_{K\gamma}$, in the HH formalism take the general form

$$\Psi^{JJ_z}_{K\gamma} = \sum_{K'\gamma'} \Psi^{K'\gamma'}_{K\gamma}(Q,\rho) \Upsilon^{K'\gamma'}_{JJ_z}(\Omega_\rho)$$

$$\Upsilon^{K\gamma}_{JJ_z}(\Omega_\rho) = \sum_{M_L M_S} (LM_L SM_S|JJ_z) \mathcal{Y}^{\ell_x \ell_y}_{KLM_L}(\Omega_\rho) \chi^{s_x}_{SM_S}.$$

The $pp\Lambda$ correlation function

$$|\Psi_s|^2_\Omega = \frac{1}{\pi^6}\int d\Omega_\rho \int d\Omega_Q |\Psi_s|^2$$

For non-interacting particles $\Psi^{K'\gamma'}_{K\gamma}(Q,\rho) = i^K\sqrt{Q\rho}J_{K+2}(Q\rho)\delta_{KK'}\delta\gamma\gamma'$
and the norm results, with N_{ST} the number of states for a given K

$$|\Psi^0_s|^2_\Omega = \frac{2}{N_S}\frac{2^6}{(Q\rho)^4}\sum_K J^2_{K+2}(Q\rho)N_{ST}(K)$$

$$C_{pp\Lambda}(Q) = \frac{1}{4}\frac{2^6}{Q^4\rho_0^6}\int \rho\, d\rho\, e^{-\frac{\rho^2}{\rho_0^2}}\left(\sum_J (2J+1)\left|\frac{u^J_{n_0}}{\sqrt{Q\rho}}\right|^2 + \sum_{K>1} J^2_{K+2}(Q\rho)N_{ST}(K)\right)$$

where the sum over J includes the states $J^\pi = 1/2^+, 1/2^-, 3/2^-, 5/2^-$ with $u^J_{n_0}$ the corresponding wave function.

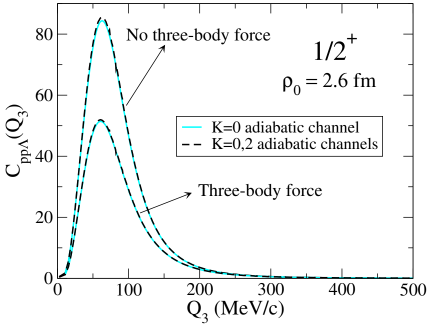
The $pp\Lambda$ three-body force

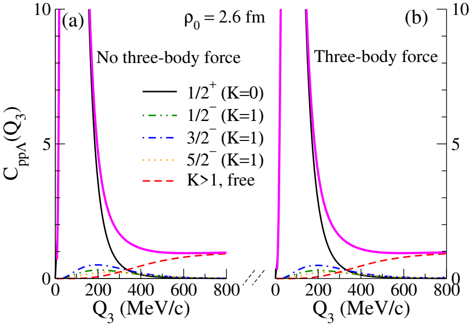
- The optimized LO $p\Lambda$ potential hase been constructed to describe the scattering length and effective range in the two spin channels
- Going to $NN\Lambda$ system this description has to completed including a three-body force
- This is related to what is called The three-body parameter as in pion-less EFT
- Accordingly, when describing the $pp\Lambda$ system we consider the following three-body force

$$W(r_{12}, r_{13}) = W_0 e^{-(r_{12}/\rho_0)^2-(r_{13}/\rho_0)^2}$$

- W_0, ρ_0 fixed to describe the hypertriton and if possible the $N = 4, 5$ hypernuclei

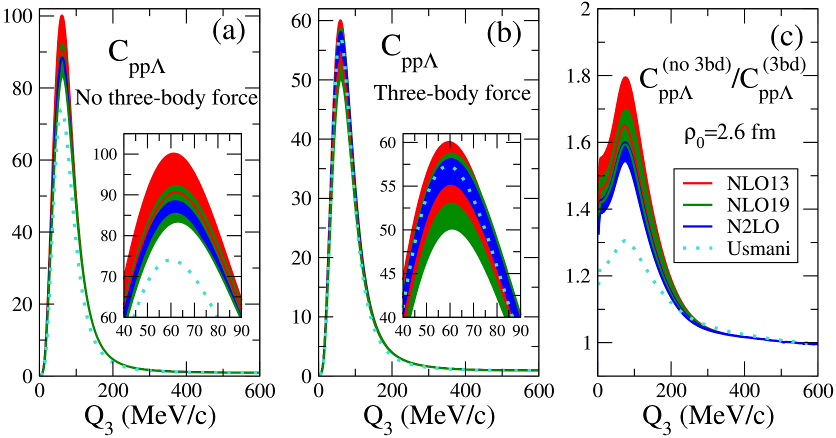
The $pp\Lambda$ correlation function



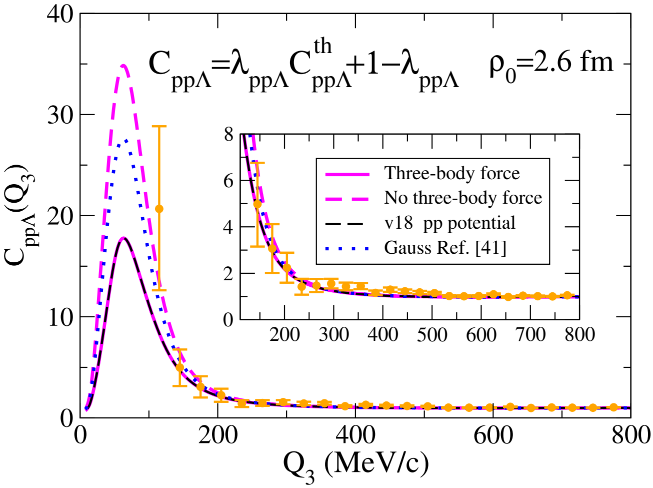


Contribution of the different partial waves

The $pp\Lambda$ correlation function



The $pp\Lambda$ correlation function



- A $NN\Lambda$ three-body force is included fixed to describe the $B(^3_\Lambda H)$

Summary

- Although its apparent simplicity, the three-body problem is of great complexity
- Measurements of the correlation function allow for new tests of the NN, NNN, $N\Lambda$, $NN\Lambda$,... interactions
- In the ppp case the Coulomb interaction couples the asymptotic dynamics increasing the difficulties of the numerical treatment
- The corrections of the computed pp and ppp correlation functions needs the kwonledge of the $p\Lambda$ and $pp\Lambda$ correlation functions
- The $N\Lambda$ and $NN\Lambda$ interactions are not very well known
- The universal window could help to link the correlation function data and the potential

- Studies on the $p\Lambda$ and $pp\Lambda$ correlation functions have been started
- The $pp\Lambda$ correlation functions could be sensitive to the $NN\Lambda$ three-body force, an important ingredient in the studies of compact systems