**To the memory of excellent scientist and modest person,**
**Michael Shirokov**

**June 2025**

# Joint field theoretical description of the electron and neutrino scattering off nuclei

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# 1. Prelude

Despite the unpleasant current situation, our institute continues to work in a remote regime, allowing us to do some explorations at a distance. In any case, my young coworker, Yan Kostylenko, successfully defended his PhD yesteryear.

I order to understand better why I am here, let me remind you of several our papers in the 90s

{KorMelShe90} A. Korchin, Y. Mel'nik, A. Shebeko. Angular distributions and polarization of protons in the d(e, e′p)n reaction. Few-Body Syst. **9** (1990) 211;

{MelShe92} Y. Mel'nik, A. Shebeko. Calculation of proton polarization in deuteron disintegration with longitudinally polarized electrons. Few-Body Syst. **13** (1992) 59;

{MelShe93} Y. Mel'nik, A. Shebeko. Electrodisintegration of polarized deuterons. Phys. Rev. C **48** (1993) 1259;

{KotMelShe95} V. Kotlyar, Y. Mel'nik, A. Shebeko. Studies of polarization phenomena in photo- and electrodisintegration of the lightest nuclei at intermediate energies. PEPAN **26** (1995) 192 with English translation in the AIP Proc. 1995.

# 1. Prelude

Then, in this century, we have extended our explorations by applying field theoretical methods. In particular, working in the late 90s on the LTP (Dubna) in the papers by M.Shirokov and me

{SheShi00} A. Shebeko, M. Shirokov. Clothing procedure in relativistic quantum field theory and its applications to description of electromagnetic interactions with nuclei (bound systems). Progr. Part. Nucl. Phys. **44** (2000) 75;

{SheShi01} A. Shebeko, M. Shirokov. Unitary transformations in quantum field theory and bound states. Phys. Part. Nucl. **32** (2001) 15;

we developed the notion of the so-called clothed particles, i.e., particles with physical properties, put forward in the QFT by Greenberg and Schweber

{GreSch58} O. Greenberg, S. Schweber. Clothed particle operators in simple models of quantum field theory. Nuovo Cim. **8** (1958) 378.

{Sch61} S. Schweber. An Introduction to Relativistic Quantum Field Theory. New York: Row, Peterson & Co., 1961.

# 2. Clothed Particle Representation (CPR) in Action

within its basic idea to remove from the total Hamiltonian *H* for a system of interacting fields, e.g., meson and nucleon ones, undesirable (bad) terms that prevent one-body states to be *H* eigenvectors, viz., in the case of the nucleon, for instance,

$$H\,|\vec{p};\text{cloth}\rangle = E_{\vec{p}}\,|\vec{p};\text{cloth}\rangle, \quad E_{\vec{p}} = \sqrt{m^2 + \vec{p}^{\,2}} \tag{2.1}$$

for nucleon momentum $\vec{p}$ and mass $m^{(*)}$, instead of the bare particle representation (BPR), where bare one-particle states $|\vec{p};\text{bare}\rangle$ are not the $H$ eigenstates.

In Refs. {SheShi00, SheShi01} we have seen how one can go from the division $H = H_0 + V$ to $H = K_F + K_I$ using the unitary clothing transformations (UCTs). By the way, it means that $K_I\,|\vec{p};\text{cloth}\rangle = 0$.

$^{(*)}$ For brevity, its polarization index is omitted.

# 2. Clothed Particle Representation (CPR) in Action

An attractive feature of the UCT method is that it allows to build up both interaction operators responsible for physical processes between clothed particles (bosons and fermions)

$$\begin{aligned} H = K_F + K_I(ff \to ff) + K_I(\bar{f}\bar{f} \to \bar{f}\bar{f}) + K_I(f\bar{f} \to f\bar{f}) + K_I(bf \to bf) \\ + K_I(b\bar{f} \to b\bar{f}) + K_I(bb \to f\bar{f}) + K_I(f\bar{f} \to bb) + \cdots \end{aligned} \tag{2.2}$$

and opens a fresh look at finding the mass and charge shifts (key points in renormalization theories):
{KorShe04} V. Korda, A. Shebeko. Clothed particles representation in quantum field theory: mass renormalization. Phys. Rev. D. **70** (2004) 085011;
{My talk at FB18 conference, Santos, Brazil, 2006}
{KorCanShe07} V. Korda, L. Canton, A. Shebeko. Relativistic interactions for the meson-two-nucleon system in the clothed-particle unitary representation. Ann. Phys. **322** (2007) 736;
{SheDub10} I. Dubovyk, A. Shebeko. The method of unitary clothing transformations in the theory of nucleon-nucleon scattering. Few-Body Syst. **48** (2010) 109;
{My talk at FB20 conference, Fukuoka, 2012}
{KosShe23} Y. Kostylenko, A. Shebeko. Clothed particle representation in quantum field theory: Fermion mass renormalization due to vector boson exchange. Phys. Rev. D. **108** (2023) 125019.

# 3. Links between *in(out)* and clothed particle states in QFT

As well-known, when evaluating the $S$-matrix in the Heisenberg picture,

$$S_{if} = \langle f; out \mid i; in \rangle \tag{3.1}$$

one has to deal with the *in(out)* states (see, e.g., {GoldWat}), in particular, one-particle state

$$|\vec{p};\ in\,(out)\rangle = a^{\dagger}_{in\,(out)}(\vec{p})\big|\Omega\rangle, \tag{3.2}$$

where $|\Omega\rangle$ is the physical vacuum. The creation (destruction) *in(out)* operators $a^{\dagger}_{in(out)}$ ($a_{in(out)}$) meet canonical commutation relations for bosons and fermions. By definition, these states are the $H$ eigenstates

$$H|\vec{p}; in(out)\rangle = E_{\vec{p}}|\vec{p}; in(out)\rangle. \tag{3.3}$$

{GoldWat} M. Goldberger, K. Watson. Collision Theory. New York, London, Sydney: John Wiley & Sons, Inc., 1967.

# 3. Links between *in(out)* and clothed particle states in QFT

Omitting important details (see also Sec. 4 of {She04} and Sec. 1.4 of {KosThesis24}), one can prove the relations between states in the CPR and *in(out)* formalism for the one particle

$$|\vec{p}; in(out)\rangle \equiv a^{\dagger}_{in(out)}(\vec{p})\,|\,\Omega\rangle = a^{\dagger}_{c}(\vec{p})\,|\,\Omega\rangle, \tag{3.4}$$

and two particles

$$|\vec{p}_1\vec{p}_2; in\rangle \equiv a^{\dagger}_{in}(\vec{p}_1)a^{\dagger}_{in}(\vec{p}_2)\,|\,\Omega\rangle = \Omega^{(+)}_{c}a^{\dagger}_{c}(\vec{p}_1)a^{\dagger}_{c}(\vec{p}_2)\,|\,\Omega\rangle, \tag{3.5}$$

$$|\vec{p}_1\vec{p}_2; out\rangle \equiv a^{\dagger}_{out}(\vec{p}_1)a^{\dagger}_{out}(\vec{p}_2)\,|\,\Omega\rangle = \Omega^{(-)}_{c}a^{\dagger}_{c}(\vec{p}_1)a^{\dagger}_{c}(\vec{p}_2)\,|\,\Omega\rangle \tag{3.6}$$

with the Møller operators $\Omega^{(\pm)}_{c} \equiv \lim\limits_{t\to\mp\infty} \exp(iHt)\exp(-iK_F t)$, that hold under the condition

$$\lim_{t\to\pm\infty} W_D(t) = 1, \tag{3.7}$$

where the UCT in the D picture $W_D(t) = e^{iH_F t}We^{-iH_F t}$ and the limit is implied in the strong sense.
To some extent, relation (3.4) does not seem unexpected, since both one-particle clothed states and *in(out)* states, being equally normalized, are $H$ eigenvectors. Of course, it does not mean that $a_{in(out)}(\vec{p}) = a_c(\vec{p})$!


{She04} A. Shebeko, The S-matrix in the method of unitary clothing transformations, Nucl. Phys. A **737** (2004) 252;
{KosThesis24} Y. Kostylenko, Field-theoretical description of deuteron and positronium properties in the clothed-particle representation, phd thesis, NSC “Kharkiv Institute of Physics and Technology”, Kharkiv, Ukraine, 2024.

# 4. Two-body currents

Recall that by using the method of unitary clothing transformations, the total field Hamiltonian $H$ and other operators of great physical meaning (e.g., the Lorentz boosts and current density operators) are expressed through commutators of generators $R$ of UCTs $W = e^R$ with primary operators, e.g., when calculating the transition matrix elements $\langle f \,|\, J^\mu(0) \,|\, i \rangle$ between the initial $|\, i\rangle$ and final $|\, f\rangle$ states we will employ the Campbell–Hausdorff formula so

$$J^\mu(0) = e^R J_c^\mu(0) e^{-R} = J_c^\mu(0) + [R, J_c^\mu(0)] + \frac{1}{2}[R, [R, J_c^\mu(0)]] + \ldots, \qquad (4.1)$$

with primary Noether current $J_c^\mu(0)$ $^{(*)}$ in which ”bare” operators $\{\alpha\}$ are replaced by the clothed partners $\{\alpha_c = W^\dagger \alpha W\}$.

We have proposed in {KorCanShe07} a recursive technique for evaluating multiple commutators that inevitably appear along our guideline. This technique has been realized in the case of interacting boson and fermion fields with Yukawa-type couplings.

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$^{(*)}$ More exactly, operator $J_c^\mu(x)$ with $x = (t, \vec{x})$ taken at the point $(0, \vec{0})$.

# 4. Two-body currents

At this point, let me address the field theoretical description of electron scattering on the deuteron. The corresponding amplitude is proportional to the matrix element of the current density operator sandwiched between the initial deuteron state $|i\rangle = |d\rangle$ and final two-nucleon states ($|d\rangle$ for elastic scattering and $|np\rangle$ for breakup). It is the case where our task is reduced to

$$\langle \text{two-body} \,|\, J_\mu(0) \,|\, d\rangle = \langle \text{two-body} \,|\, [J_\mu^{[1]} + J_\mu^{[2]}] \,|\, d\rangle, \tag{4.2}$$

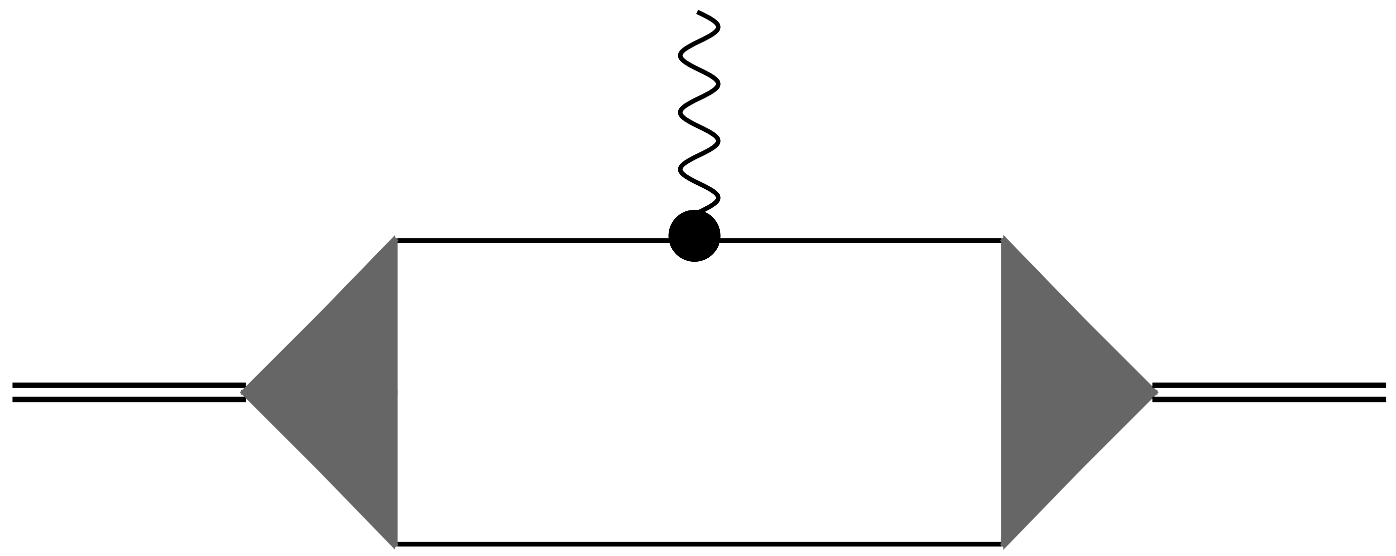
$$J^{[1]} = \sum\!\!\!\!\!\!\int d1'd1\, F(1',1) b_c^\dagger(1') b_c(1), \tag{4.3}$$

$$J^{[2]} = \sum\!\!\!\!\!\!\int d1'd2'd1d2\, F_{MEC}(1',2',1,2) b_c^\dagger(1') b_c^\dagger(2') b_c(1) b_c(2)\,. \tag{4.4}$$

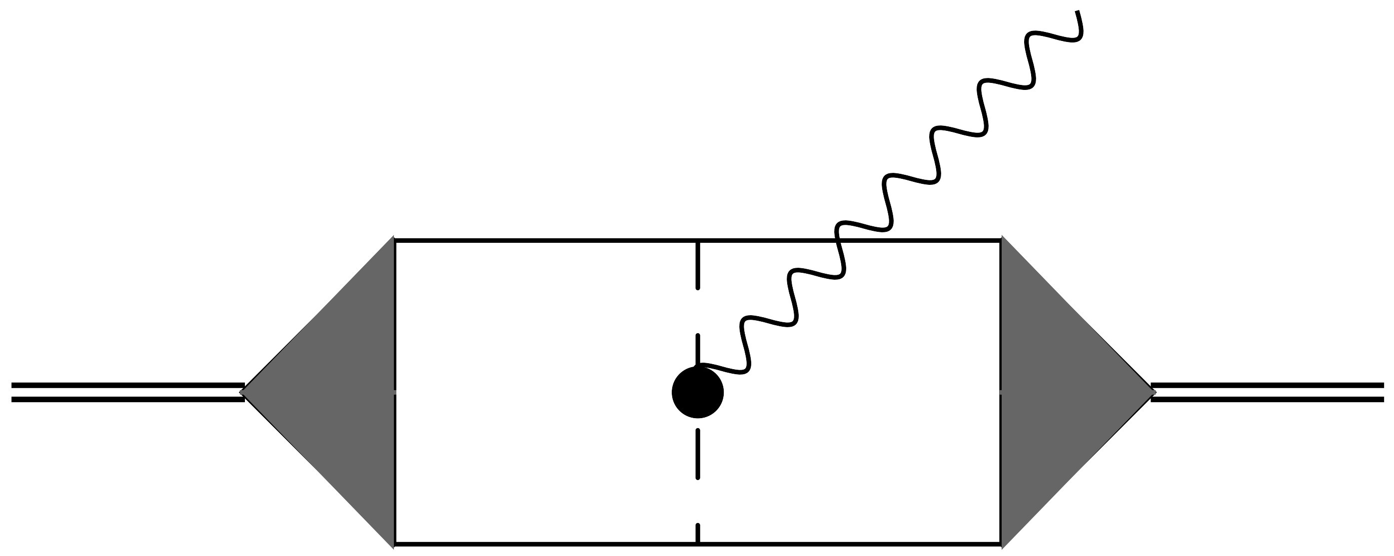
For brevity, the Lorentz label is omitted.

# 4. Two-body currents

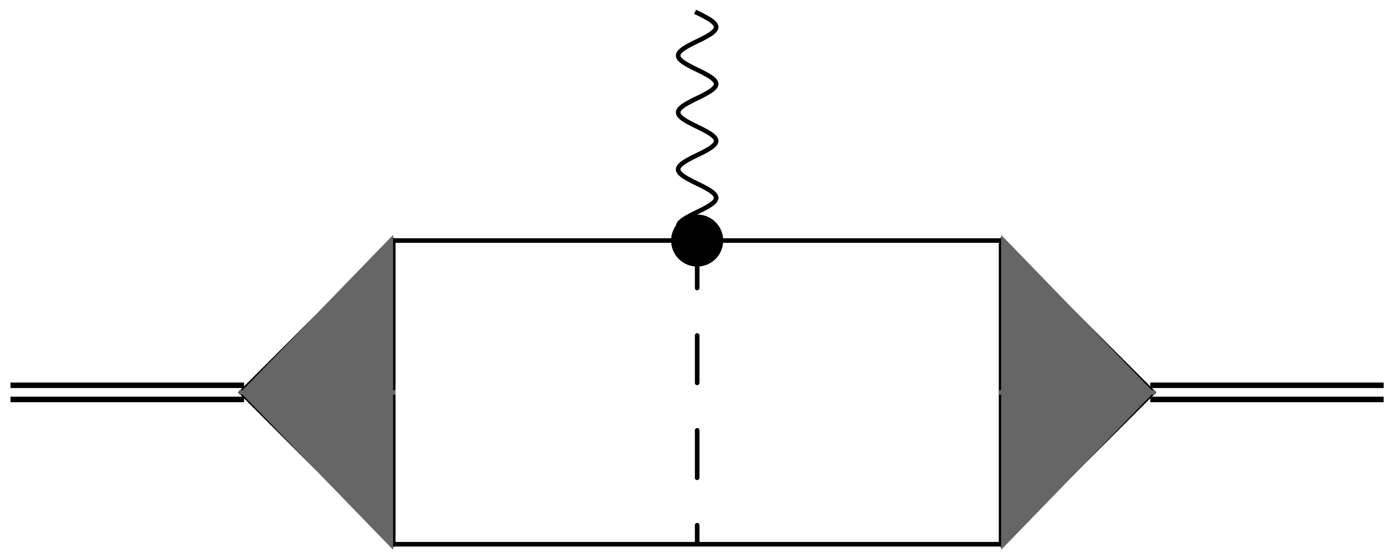
Schematically, this structure looks as



*one–body*



*mesonic*



*seagull*

# 4. Two-body currents

Here, the c-functions are determined by

$$F^{\mu}(1',1) = e\, m\, \bar{u}(1')\left\{ F_1[(p_1'-p_1)^2]\gamma^{\mu} + i\sigma^{\mu\nu}\frac{(p_1'-p_1)_{\nu}}{2m}F_2[(p_1'-p_1)^2] \right\}u(1), \tag{4.5}$$

with the Dirac (Pauli) form factor $F_1$ ($F_2$) and $F_{MEC} = F_{MCC} + F_{MNN}$, e.g., for $\pi$ mesons, looks as

$$F^{\mu}_{\pi CC}(1',2',1,2) = i\frac{eg_{\pi}^2 m^2}{2(2\pi)^6}[\vec{\tau}_1 \times \vec{\tau}_2]^z g_{11}^{\pi}(p_1'p_1)g_{11}^{\pi}(p_2'p_2)\frac{\bar{u}(1')\gamma_5 u(1)}{(p_1'-p_1)^2 - m_{\pi}^2}\frac{\bar{u}(2')\gamma_5 u(2)}{(p_2'-p_2)^2 - m_{\pi}^2}(p_2'-p_1'+p_1-p_2)^{\mu}, \tag{4.6}$$

$$F^{\mu}_{\pi NN}(1',2',1,2) = \frac{eg_{\pi}^2 m^2}{2(2\pi)^6}g_{11}^{\pi}(p_2'p_2)\frac{\bar{u}(2')\gamma_5 u(2)}{(p_2'-p_2)^2 - m_{\pi}^2}\left[\frac{1}{2}(\vec{\tau}_1\cdot\vec{\tau}_2 + \tau_2^z - i[\vec{\tau}_1\times\vec{\tau}_2]^z)g_{11}^{\pi}(p_1 s)\,\bar{u}(1')\gamma^{\mu}\Gamma(1',2',1,2)\gamma_5 u(1) \right.$$
$$\left. + \frac{1}{2}(\vec{\tau}_1\cdot\vec{\tau}_2 + \tau_2^z + i[\vec{\tau}_1\times\vec{\tau}_2]^z)g_{11}^{\pi}(p_1's')\,\bar{u}(1')\gamma_5\Gamma(1,2,1',2')\gamma^{\mu}u(1)\right], \tag{4.7}$$

$$\Gamma(1',2',1,2) = \frac{1}{2E_{\vec{s}}}\left[(\not{s}+m)\frac{E_{\vec{p}_1} - E_{\vec{p}_2} + E_{\vec{p}_2'} - E_{\vec{s}}}{(p_1-s)^2 - m_{\pi}^2} + (\not{s}_- - m)\frac{E_{\vec{p}_1} - E_{\vec{p}_2} + E_{\vec{p}_2'} + E_{\vec{s}}}{(p_1+s_-)^2 - m_{\pi}^2}\right],$$

where $s = (E_{\vec{s}}, \vec{s})$, $s' = (E_{\vec{s}'}, \vec{s}')$, $\vec{s} = \vec{p}_1 + \vec{p}_2 - \vec{p}_2'$, $\vec{s}' = \vec{p}_1' + \vec{p}_2' - \vec{p}_2$, $g^{\pi}$ the coupling constant and $g_{11}^{\pi}(p'p)$ the corresponding cutoff factors. Henceforth, we accept the abbreviation $\not{s} = s^{\mu}\gamma_{\mu}$.
Our calculations with such currents are underway.

# 4. Two-body currents

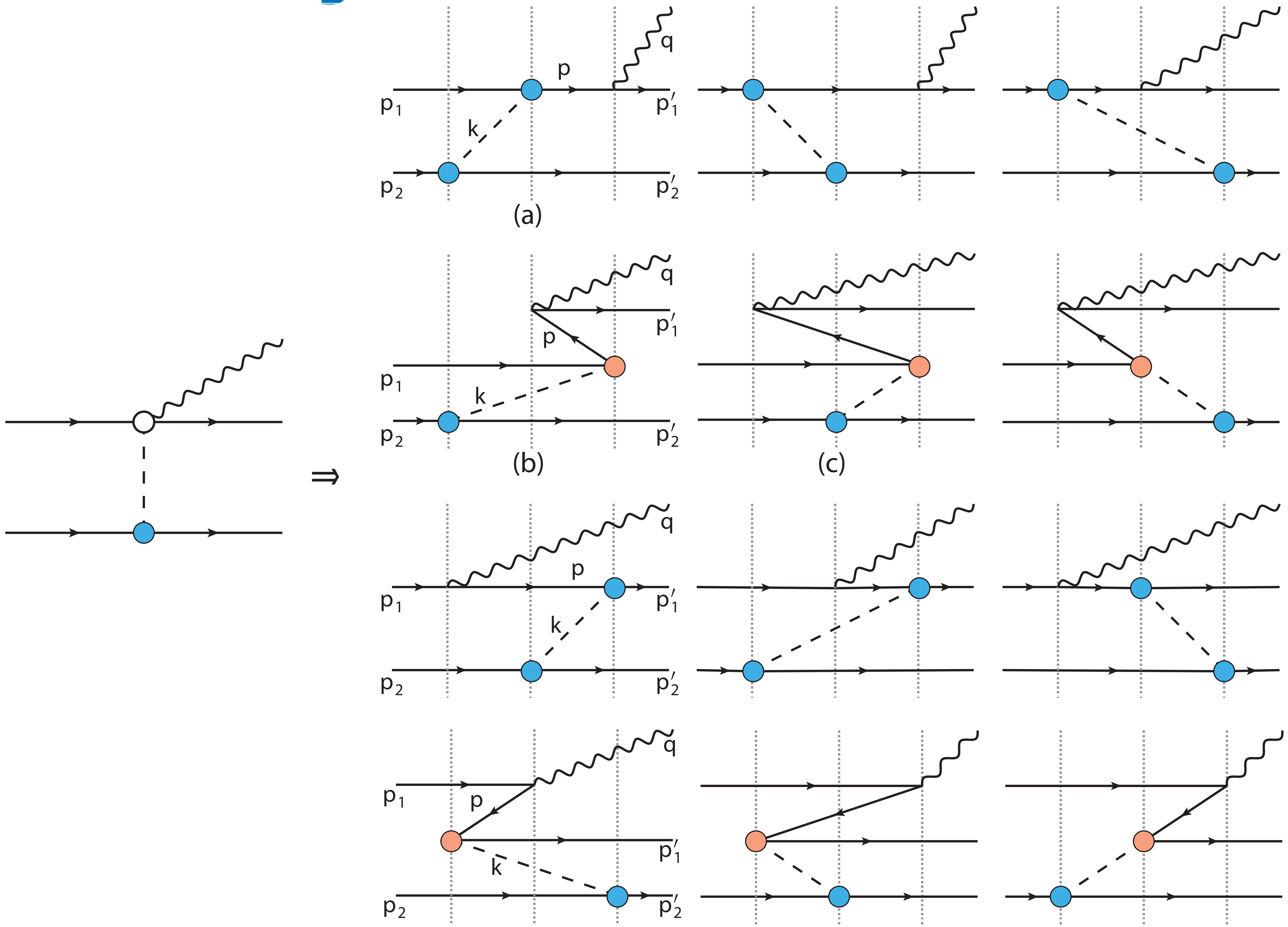



Illustration of the mechanisms that contribute to the seagull exchange current. Blue and orange circles correspond to the $g_{11}$ and $g_{12}$ cutoffs, respectively.

# 4. Two-body currents

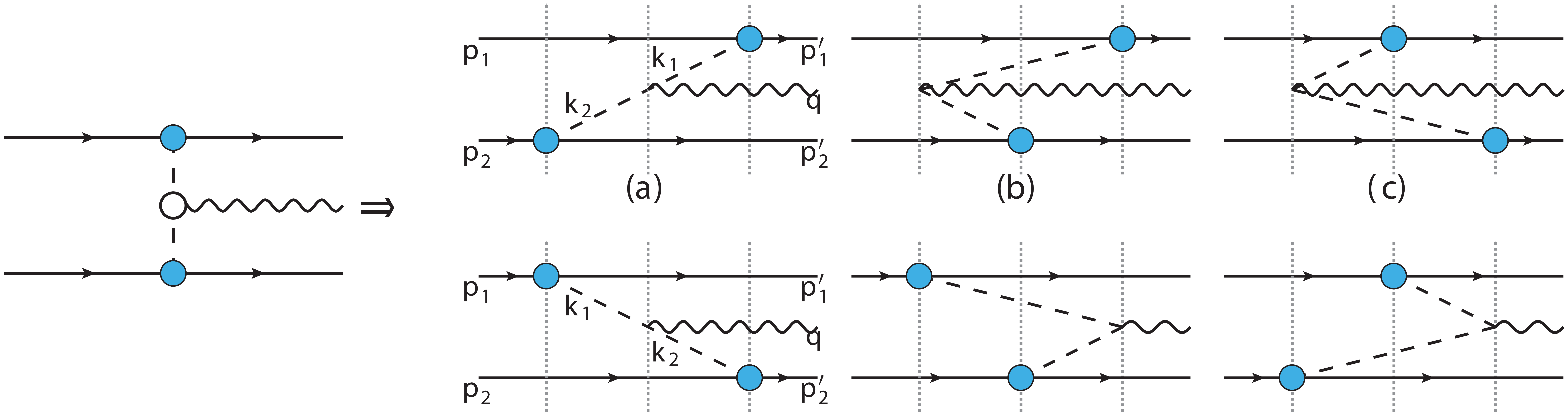



Illustration of the mechanisms that contribute to the mesonic meson exchange current. Blue circles correspond to the $g_{11}$ cutoff.

These figures are taken from {KosThesis24}.

# 4. Two-body currents

Many things under our consideration remain intact for describing the scattering of other leptons on few-nucleon systems.

## 5. Final state interactions in inclusive and semi-inclusive processes

Special attention in our studies is paid to the effects due to interactions between reaction products in processes induced by leptons off nucleons and nuclei below and above the pion production threshold. In this context, let me recall our collaborative research on the pion photoproduction off the deuterium $d(\gamma, \pi^+)nn$

V. Ganenko, A. Shebeko et al. (1973-1979)

and pion electroproduction in the reaction $d(e, e'\pi^+)nn$

L. Levchuk, A. Shebeko. Positive pion electroproduction on the deuteron near threshold. Yadernaya Fizika. **62** (1999) 263-271;

L. Levchuk, L. Canton and A. Shebeko. Nuclear effects in positive pion electroproduction on the deuteron near threshold. EPJA **21** (2004) 29–36.

## 5. Final state interactions in inclusive and semi-inclusive processes

These inclusive (in final states, only pions are detected) and semi-inclusive (pions are observable with scattered electrons) reactions are typical to illustrate a general idea, viz., we rewrite the expression

$$d\sigma_{\gamma\pi d} = (2\pi)^4 \sum\!\!\!\!\!\!\int_{nn} \delta(E_\pi - E_\gamma + E_{nn} - E_d)\,|\,\Gamma_{nn}\,|^2 d\vec{p}_\pi, \tag{5.1}$$

$$\Gamma_{nn} = \langle \psi_{nn}^{(-)}\,|\,F_{\gamma\pi}\,|\,d\rangle, \tag{5.2}$$

where $F_{\gamma\pi}$ the corresponding transition operator, $\langle \psi_{nn}^{(-)}\,|$ is the $H$ eigenvector that belongs to the energy $E_{nn} = E_{n_1} + E_{n_2}$ of the final $nn$-pair, in the form

$$d\sigma_{\gamma\pi d} = (2\pi)^4 \langle d\,|\,F_{\gamma\pi}^{\dagger}\delta(E - E_\pi - H)F_{\gamma\pi}\,|\,d\rangle d\vec{p}_\pi. \tag{5.3}$$

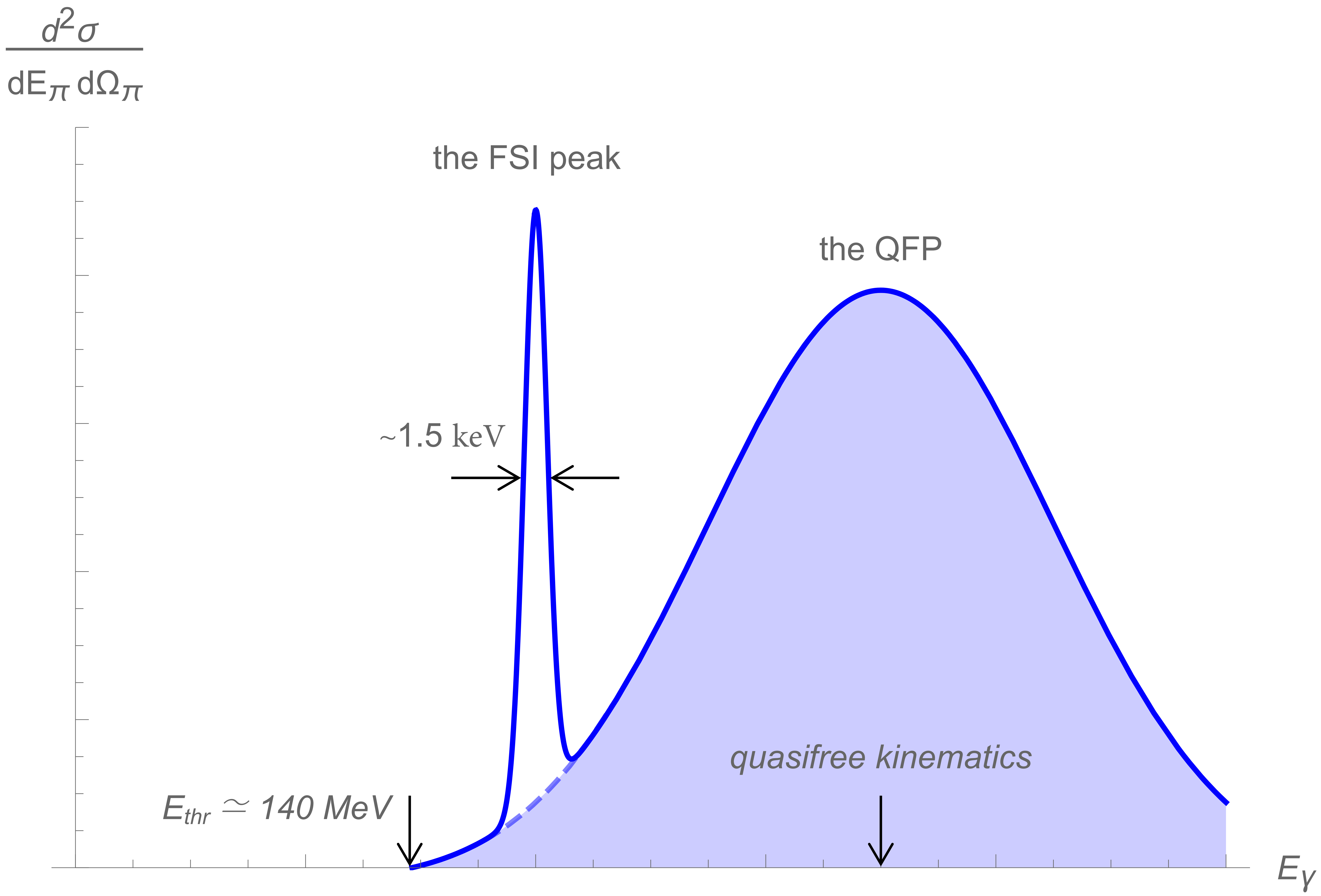
In its turn, the delta function $\delta(x - H) = -\,\pi^{-1}\mathrm{Im}(x + i0 - H)^{-1}$,

$$(x + i0 - H)^{-1} = g_0(x + i0) + g_0(x + i0)t_{nn}(x + i0)g_0(x + i0) \tag{5.4}$$

with the free resolvent $g_0(z) = (z - H_0)^{-1}$, so evaluation of our cross section reduces to plane wave contribution + FSI contribution **linear** in $t$-matrix of $nn$-scattering.

See details in my talk at Gordon Conference on photonuclear reactions, Tilton, 1976.

# 5. Final state interactions in inclusive and semi-inclusive processes




FSI – final state interaction peak (very sharp one separated from $E_{thr}$ at the distance $0.075\ MeV$; its height is proportional to the square of the $nn$ scattering length ($a_{nn}$) value and width $\approx 1.5\ KeV$).

The distinctive feature of the quasifree peak (QFP) (in general, such a wide bump in inclusive energy spectra) is that it is centered near the energy of the reaction $\gamma + p \to \pi^+ + n$ on free proton at rest.

# 6. Well-forgotten pages from past to future

First of all, we would like to recall that $^{(*)}$ the electroweak interaction is part of the Standard Model and based on a local $SU(2) \times U(1)$ gauge symmetry. After spontaneous symmetry breaking via the Higgs mechanism, we get for the interaction part of the Lagrangian
A.W. Thomas, W. Weise. The Structure of the Nucleon. Wiley-VCH, 2001

$$\mathscr{L}_{int} = -\frac{g}{2\sqrt{2}}\left(\mathscr{J}_\alpha^{CC} W^{\alpha\dagger} + \text{ h.c. }\right) - \frac{g}{2\cos\theta_W}\mathscr{J}_\alpha^{NC} Z^\alpha - e\mathscr{J}_\alpha^{EM} A^\alpha \tag{6.1}$$

The weak charged current (CC) $\mathscr{J}_\alpha^{CC}$, the weak neutral current (NC) $\mathscr{J}_\alpha^{NC}$ and the electromagnetic current (EM) $\mathscr{J}_\alpha^{EM}$ couple to the charged $W$-boson field $W^\alpha$, the neutral $Z$-boson field $Z^\alpha$ and the photon field $A^\alpha$, respectively. The currents can be separated into a leptonic part, denoted by $j_\alpha$, and a hadronic part $J_\alpha$:

$$\mathscr{J}_\alpha = j_\alpha + J_\alpha \tag{6.2}$$

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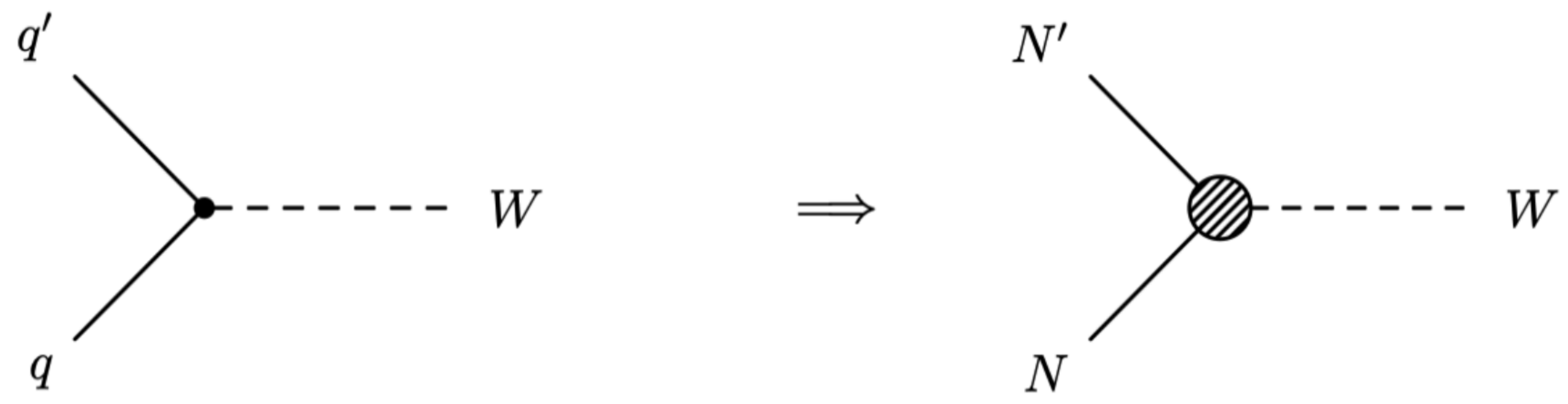
$^{(*)}$ Cited from Tina J. Leitner's master thesis.

# 6. Well-forgotten pages from past to future

*Ibid.* we encounter an essential simplification for further consideration where one uses the following transition from the point-like vertex to the vertex with composite particles




Then, for example, the amplitude of the neutrino-nucleon scattering amplitude can be expressed in terms of the matrix elements of the hadronic current density operator sandwiched between the nucleon initial state ($N$) and final system state ($X$),

$$\mathcal{M} = \left(\frac{g}{2\sqrt{2}}\right)^2 \bar{u}_l\left(k'\right)\gamma_\alpha\left(1-\gamma_5\right)u_\nu(k)\frac{i}{q^2-M_W^2}\left(-g^{\alpha\beta}+\frac{q^\alpha q^\beta}{M_W^2}\right)\langle X\left(p'\right)|\,J_\beta(0)|N(p)\rangle. \quad (6.3)$$

# 6. Well-forgotten pages from past to future

In this context, we could address the Fock-Weyl criterion and its consequences.

Gauge invariance and gauge independence of the S-matrix in nonrelativistic quantum mechanics and relativistic quantum field theories. Ann. Phys. **142** (1982) 80 by E. Kazes, T. E. Feuchtwang, P. H. Cutler and H.Grotch.

See also the survey

Historical roots of gauge invariance. Rev. Mod. Phys. **73** (2001) 663
by J. D. Jackson and L. B. Okun,

Clothing procedure in relativistic quantum field theory and its applications to description of electromagnetic interactions with nuclei (bound systems). Progr. Part. Nucl. Phys. **44** (2000) 75 by A. Shebeko, M. Shirokov.

# 6. Well-forgotten pages from past to future

Towards gauge-independent treatment of radiative capture in nuclear reactions, Phys. Atom. Nuclei **77**, 518–527 (2014) by Shebeko

An effective way of ensuring gauge independent treatment of single-photon processes on nuclei. Extension of the Siegert theorem

As shown in
L.G. Levchuk and A.V. Shebeko, Phys. At. Nucl. **56** (1993) 227
(cf. J. Friar and S. Fallieros, Phys. Rev. C **34** (1986) 2029,
J.L. Friar and W.C. Haxton, Phys. Rev. C **31** (1985) 2027 and
A.V. Shebeko, Sov. J. Nucl. Phys. **49** (1989) 30.),
the photonuclear reaction amplitude of interest (to be more definite for the photon emission with energy $E_\gamma$ and momentum $\vec{k}$), given in the conventional form:

$$T_{if} = \left[2(2\pi)^3 E_\gamma\right]^{-1/2} \left\langle \vec{P}_i - \vec{k}; f \left| \varepsilon^\mu \hat{J}_\mu(0) \right| \vec{P}_i; i \right\rangle \tag{6.4}$$

can be expressed through the electric ($\vec{E}(\vec{k})$) and magnetic ($\vec{H}(\vec{k})$) field strengths:

$$\vec{E}(k) = i\left[2(2\pi)^3 E_\gamma\right]^{-1/2}\left(E_\gamma \vec{\varepsilon}(\vec{k}) - \vec{k}\varepsilon_0(\vec{k})\right), \vec{H}(\vec{k}) = i\left[2(2\pi)^3 E_\gamma\right]^{-1/2} \vec{k}\times\vec{\varepsilon}(\vec{k})\,. \tag{6.5}$$

# 6. Well-forgotten pages from past to future

these manifestly gauge independent quantities, and the matrix elements $\overrightarrow{D}_{if}(\vec{k})$ and $\overrightarrow{M}_{if}(\vec{k})$of the so-called generalized electric and magnetic dipole moments of the nucleus:

$$T_{if} = \overrightarrow{E}(\vec{k}) \cdot \overrightarrow{D}_{if}(\vec{k}) + \overrightarrow{H}(\vec{k}) \cdot \overrightarrow{M}_{if}(\vec{k})\,. \tag{6.6}$$

Formulas for the matrix elements were first derived in

L. Levchuk and A. Shebeko, Phys. At. Nucl. **56** (1993) 227

without separation of the center-of-mass (CM) motion, and thus they can be used in relativistic nuclear models or in problems, where such a separation becomes hardly feasible as for photomeson processes on nuclei (see L. Levchuk, L. Canton and A. Shebeko, EPJA **21** (2004) 29 and refs. therein).

# 6. Well-forgotten pages from past to future

## Single-photon emission amplitude in terms of electric and magnetic field strengths

Using the nonrelativistic ansatz we will prove that

$$\left\langle \vec{P}_i + \vec{q}; f \left| \varepsilon^{\mu} \hat{J}_{\mu}(0) \right| \vec{P}_i; i \right\rangle = i \left[ \vec{q} \varepsilon_0(q) - q_0 \, \vec{\varepsilon}(q) \right] \vec{D} \left( q \right) - i \left[ \vec{q} \times \vec{\varepsilon}(q) \right] \vec{M}(q) \tag{6.7}$$

with

$$\vec{D}(q) = -\frac{1}{q_0} \int_0^1 \left( \vec{P}_i + \lambda \vec{q} \left| \hat{\vec{R}} [\hat{H}, \hat{\rho}(0)] \right| \vec{P}_i \right) d\lambda, \tag{6.8}$$

$$\vec{M}(q) = -\int_0^1 \left( \vec{P}_i + \lambda \vec{q} \left| \hat{\vec{R}} \times \hat{\vec{J}}(0) \right| \vec{P}_i \right) \lambda d\lambda \, . \tag{6.9}$$

Here, to comprise both the photon absorption and emission, the four-momentum transfer $q = \left( q_0, \vec{q} \right)$ is determined with $q_0 = E_f - E_i$ $(\vec{q} = \vec{P}_f - \vec{P}_i)$ for the photoabsorption and $q_0 = E_i - E_f$ $(\mathbf{q} = \mathbf{P}_i - \mathbf{P}_f)$ for the photoemission.

# 6. Well-forgotten pages from past to future

It is the case where the l.h.s. of Eq. (6.7) may be written as

$$\left\langle \vec{P}_i + \vec{q}; f \left| \varepsilon^\mu \hat{J}_\mu(0) \right| \vec{P}_i; i \right\rangle = \varepsilon_0(q)\langle f| \left( \vec{P}_i + \vec{q} \left| \hat{\rho}(0) \right| \vec{P}_i \right) |i\rangle - \vec{\varepsilon}(q)\langle f| \left( \vec{P}_i + \vec{q} \left| \hat{\vec{J}}(0) \right| \vec{P}_i \right) |i\rangle,$$

and it is convenient to employ the representation (6.10)

$$\vec{\varepsilon} e^{\vec{q}\cdot\hat{\vec{a}}} = \int_0^1 \left\{ \left[ \hat{\vec{b}}, (\vec{\varepsilon}\cdot\hat{\vec{a}}) e^{\lambda\vec{q}\cdot\hat{\vec{a}}} \right] + \lambda\hat{\vec{a}} \times \left[ \vec{\varepsilon} \times \vec{q} \right] e^{\lambda\vec{q}\cdot\hat{\vec{a}}} \right\} d\lambda \tag{6.11}$$

This equation with arbitrary $c$-vectors $\vec{\varepsilon}$ and $\vec{q}$ is valid for the two operators $\hat{\vec{a}} = \left(\hat{a}_1, \ldots, \hat{a}_n\right)$ and $\hat{\vec{b}} = \left(\hat{b}_1, \ldots, \hat{b}_n\right)$ that meet the commutation relations

$$\left[\hat{a}_j, \hat{b}_k\right] = \delta_{j,k} \qquad (j, k = 1, \ldots, n) \tag{6.12}$$

Of course, it is implied that each operator $a_i$ and $b_i$ is defined on an infinitely dimensional space.

# 6. Well-forgotten pages from past to future

With the help of (6.11) we get

$$\vec{\varepsilon}\cdot\left(\vec{P}_i+\vec{q}\,\middle|\,\hat{\vec{J}}(0)\,\middle|\,\vec{P}_i\right) = -\,i\int_0^1 d\lambda\left(\vec{P}_i+\lambda\vec{q}\,\middle|\,\vec{\varepsilon}\cdot\hat{\vec{R}}[\hat{H},\hat{\rho}(0)]\,\middle|\,\vec{P}_i\right)$$

$$-i\,[\vec{q}\times\vec{\varepsilon}]\int_0^1 \lambda d\lambda\left(\vec{P}_i+\lambda\vec{q}\,\middle|\,\hat{\vec{R}}\times\hat{\vec{J}}(0)\,\middle|\,\vec{P}_i\right) \tag{6.13}$$

In fact, one has

$$\left(\vec{P}_i+\vec{q}\,\right| = \left(\vec{P}_i\right|\exp\left(-i\vec{q}\hat{\vec{R}}\right), \tag{6.14}$$

where $\hat{\vec{R}}$ is the total CM coordinate operator. Now, putting in (A.3) $\hat{\vec{a}} = i\hat{\vec{R}}$, $\vec{q} = -\,\vec{q}$ and $\hat{\vec{b}} = -\,\hat{\vec{P}}$ we come to

$$\vec{\varepsilon}\cdot\left(\vec{P}_i+\vec{q}\,\middle|\,\hat{\vec{J}}(0)\,\middle|\,\vec{P}_i\right) = -\,i\int_0^1 d\lambda\left(\vec{P}_i\,\middle|\,\vec{\varepsilon}\cdot\hat{\vec{R}}e^{-i\lambda\vec{q}\cdot\hat{\vec{R}}}[\hat{\vec{P}},\hat{\vec{J}}(0)]\,\middle|\,\vec{P}_i\right)$$

$$-i\,[\vec{q}\times\vec{\varepsilon}]\int_0^1 \lambda d\lambda\left(\vec{P}_i\,\middle|\,e^{-i\lambda\vec{q}\cdot\hat{\vec{R}}}\hat{\vec{R}}\times\hat{\vec{J}}(0)\,\middle|\,\vec{P}_i\right) \tag{6.15}$$

# 6. Well-forgotten pages from past to future

Formula (6.11) works owing to the canonical relations (cf. (6.12))

$$\left[\hat{R}_j, \hat{P}_k\right] = i\delta_{j,k} \qquad \left(j, k = 1,2,3\right) \tag{6.16}$$

In addition, we have accounted for the equality

$$(A\left| \, \left[\hat{A}, \hat{B}\right] \, \right|A) = 0 \tag{6.17}$$

with the matrix elements between the eigenvectors $\left|A\right)$ of a given operator $\hat{A}$ (in our case $\hat{A} = \overrightarrow{P}$).

The conversion (6.15) is culminative in deriving Eq. (6.13) since it enables us to employ the continuity equation (CE) that results in (6.13).

Further,

$$\langle f| \left(\overrightarrow{P}_i + \vec{q}\left| \, \hat{\rho}(0) \right| \overrightarrow{P}_i\right) |i\rangle = \left(E_f - E_i\right)^{-1} \langle f| \left(\overrightarrow{P}_i + \vec{q}\left| [\hat{H}, \hat{\rho}(0)] \right| \overrightarrow{P}_i\right) |i\rangle \tag{6.18}$$

or

$$\langle f| \left(\overrightarrow{P}_i + \vec{q}\left| \, \hat{\rho}(0) \right| \overrightarrow{P}_i\right) |i\rangle = \left(E_f - E_i\right)^{-1} \langle f| \int_0^1 d\lambda \frac{d}{d\lambda} \left(\overrightarrow{P}_i + \lambda\vec{q}\left| [\hat{H}, \hat{\rho}(0)] \right| \overrightarrow{P}_i\right) |i\rangle. \tag{6.19}$$

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Once more, the CE in combination with Eq. (6.17) helps us to see that

$$\left(\vec{P}_i\middle|[\hat{H},\hat{\rho}(0)]\middle|\vec{P}_i\right) = \left(\vec{P}_i\middle|[\hat{\vec{P}},\hat{\vec{J}}(0)]\middle|\vec{P}_i\right) = 0 \tag{6.20}$$

At last, using the equation

$$\frac{d}{d\lambda}\left(\vec{P}_i + \lambda\vec{q}\right| = -i\vec{q}\left(\vec{P}_i + \lambda\vec{q}\right|\hat{\vec{R}} \tag{6.21}$$

we obtain

$$\langle f|\left(\vec{P}_i + \vec{q}\middle|\hat{\rho}(0)\middle|\vec{P}_i\right)|i\rangle = -i\frac{\vec{q}}{q_0}\langle f|\int_0^1 d\lambda\left(\vec{P}_i + \lambda\vec{q}\middle|\hat{\vec{R}}[\hat{H},\hat{\rho}(0)]\middle|\vec{P}_i\right)|i\rangle \tag{6.22}$$

Substituting expressions (6.13) and (6.22) into the r.h.s. of Eq. (6.10), we arrive at Eq. (6.7). The representation (6.6) follows from (6.7) at $q_0 = -E_\gamma,\ \vec{q} = -\vec{k}$.

# 6. Well-forgotten pages from past to future

## Gauge independent expression for the amplitude

## Nuclear effects in positive pion electroproduction on the deuteron near threshold. EPJA **21** (2004) 29–36 by L. Levchuk, L. Canton and A. Shebeko.

An incompleteness of the description may lead to results, which are not gauge independent. To restore the gauge independence (GI) of the treatment, one often adds an extra term to the amplitude making the subtraction

$$J_\mu \to J_\mu - q_\mu \, q \cdot J / q^2 \, . \qquad (6.23)$$

Of course, this procedure cannot reflect the complexity of the reaction mechanisms such as, e.g., the two-body processes. Moreover, it does not affect the transverse components of the transition matrix and is not unambiguous admitting extra subtraction of an arbitrary vector $X_\mu$ such that $q \cdot X = 0$.

# 6. Well-forgotten pages from past to future

In our consideration, to provide the GI of calculations, we make use of the extension

L. Foldy, Phys. Rev. **92** (1953) 178;
J. Friar and S. Fallieros, Phys. Rev. C **34** (1986) 2029;
A. Shebeko, Sov. J. Nucl. Phys. **49** (1989) 30;
L. Levchuk and A. Shebeko, Phys. At. Nuclei **56** (1993) 227;

of the Siegert theorem expressing the amplitude in an explicitly gauge independent way through the Fourier transforms of electric ($\overrightarrow{E}(\vec{q})$) and magnetic ($\overrightarrow{H}(\vec{q})$) field strengths,

$$T_{if} = \overrightarrow{E}(\vec{q})\overrightarrow{D}_{\mathrm{if}} + \overrightarrow{H}(\vec{q})\overrightarrow{M}_{\mathrm{if}}\,, \tag{6.24}$$

$$\overrightarrow{E}(\vec{q}) = i[2(2\pi)^3\omega]^{-\frac{1}{2}}\,(\omega\vec{\varepsilon} - \varepsilon_0\vec{q}), \tag{6.25}$$

$$\overrightarrow{H}(\vec{q}) = i[2(2\pi)^3\omega]^{-\frac{1}{2}}\,[\vec{q}\times\vec{\varepsilon}], \tag{6.26}$$

with $\overrightarrow{D}_{\mathrm{if}}$ and $\overrightarrow{M}_{\mathrm{if}}$ being matrix elements of generalized electric and magnetic dipole moments of the hadronic system containing the information on the nuclear dynamics.

# 6. Well-forgotten pages from past to future

To get representation (6.24), consider the expression

$$\delta(\overrightarrow{P}_i + \vec{q} - \overrightarrow{P}_f)\langle \overrightarrow{P}_f, f \mid J^\mu(0) \mid \overrightarrow{P}_i, i\rangle = (2\pi)^{-3}\int \exp(i\vec{q}\vec{s})\, j^\mu_{if}(\vec{s})\, \mathrm{d}\vec{s} \ , \tag{6.27}$$

$$j^\mu_{if}(\vec{s}) \equiv \left(\rho_{if}(\vec{s}), \vec{j}_{if}(\vec{s})\right) = \langle \overrightarrow{P}_f, f \mid J^\mu(\vec{s}) \mid \overrightarrow{P}_i, i\rangle = \langle \overrightarrow{P}_f, f \mid J^\mu(0) \mid \overrightarrow{P}_i, i\rangle \ \mathrm{e}^{-i(\overrightarrow{P}_f - \overrightarrow{P}_i)\vec{s}} \ . \tag{6.28}$$

Multiplying the space part of the matrix element (6.27) by an *arbitrary* vector $\vec{\varepsilon}(\vec{q})$ and applying the Foldy trick

L. Foldy, Phys. Rev. **92** (1953) 178;

$$\vec{\varepsilon}\mathrm{e}^{i\vec{q}\vec{s}} = \int_0^1 \{\, \nabla_{\vec{s}}(\vec{\varepsilon}\vec{s}\mathrm{e}^{i\lambda\vec{q}\vec{s}}) \ - \ i\lambda\vec{s} \times [\vec{q} \times \vec{\varepsilon}]\mathrm{e}^{i\lambda\vec{q}\vec{s}} \}\ \mathrm{d}\lambda \ , \tag{6.29}$$

with help of the GI condition $\mathrm{div}\vec{j}_{if}(\vec{s}) = -\,i(E_f - E_i)\rho_{if}(\vec{s})$, we get

$$\delta(\overrightarrow{P}_i + \vec{q} - \overrightarrow{P}_f)\langle \overrightarrow{P}_f, f \mid \vec{J}(0) \mid \overrightarrow{P}_i, i\rangle = i(E_f - E_i)\vec{d}_{if}(\vec{q}) - i[\vec{q} \times \vec{m}_{if}(\vec{q})] \ , \tag{6.30}$$

$$\vec{d}_{if}(\vec{q}) = (2\pi)^{-3}\int_0^1 \mathrm{d}\lambda \int \mathrm{e}^{i\lambda\vec{q}\vec{s}}\vec{s}\rho_{if}(\vec{s})\ \mathrm{d}\vec{s} \ , \qquad \vec{m}_{if}(\vec{q}) = (2\pi)^{-3}\int_0^1 \lambda\mathrm{d}\lambda \int \mathrm{e}^{i\lambda\vec{q}\vec{s}}\left[\vec{s} \times \vec{j}_{if}(\vec{s})\right]\ \mathrm{d}\vec{s} \ . \tag{6.31}$$

# 6. Well-forgotten pages from past to future

Then, due to charge conservation, one may write

$$\int i(E_f - E_i)\vec{q}\vec{d}_{if}(\vec{q})\ \mathrm{d}\vec{P}_i = \omega\langle \vec{P}_f, f \mid J^0(0) \mid \vec{P}_f - \vec{q}, i\rangle \tag{6.32}$$

$$-(E_f - E_i(\vec{P}_f))\langle \vec{P}_f, f \mid J^0(0) \mid \vec{P}_f, i\rangle = \omega\langle \vec{P}_f, f \mid J^0(0) \mid \vec{P}_f - \vec{q}, i\rangle \tag{6.33}$$

Integration of Eq. (6.30) over $\vec{P}_i$ with taking into account the relationship (6.33), results in representation (6.24), with quantities $\vec{D}_{if}$ and $\vec{M}_{if}$ being defined as

$$\vec{D}_{if} = -\int \frac{E_f - E_i}{\omega}\vec{d}_{if}(\vec{q})\ \mathrm{d}\vec{P}_i\ , \qquad \vec{M}_{if} = -\int \vec{m}_{if}(\vec{q})\ \mathrm{d}\vec{P}_i\ . \tag{6.34}$$

In case of reaction $e + d \to e' + \pi^+ + n + n$, one has

$$\vec{D}_{if} = i\omega^{-1}\int_0^1 \nabla_{\lambda\vec{q}}[\ (\ \omega + M_\mathrm{d} - \sqrt{M_\mathrm{d}^2 + (1-\lambda)^2\vec{q}^2}\ )\ J^0_{if}(\lambda\vec{q})\ ]\ \mathrm{d}\lambda \tag{6.35}$$

$$\vec{M}_{if} = i\int_0^1 \nabla_{\lambda\vec{q}} \times \vec{J}_{if}(\lambda\vec{q})\ \lambda\ \mathrm{d}\lambda \tag{6.36}$$

where $J^\nu_{if}(\lambda\vec{q})$ is obtained from $\langle \pi^+\mathrm{nn};\mathrm{out} \mid J^\nu(0) \mid \mathrm{d}\ \rangle$ replacing the deuteron momentum by $(1-\lambda)\vec{q}$.

# 6. Well-forgotten pages from past to future

It should be noted that, whereas quantities $\vec{d}_{if}$ and $\vec{m}_{if}$ in Eq. (6.30) are singular and not proportional to the delta function expressing the momentum conservation, the representation (6.24) is free of singularities. Furthermore, it has been derived here (cf. J. L. Friar and S. Fallieros, Phys. Rev. C 34 (1986) 2029) without decomposition of the e.m. current into the part associated with the motion of the hadronic system as a whole and the intrinsic current and, therefore, can be employed in relativistic calculations.

This representation generates a correction term additional to the "canonical" expression, which restores the GI of the amplitude in calculations that fail to satisfy the requirement $q_\mu J^\mu_{if}(\vec{q}) = 0$. However, when this condition does hold, this correction is equal to zero automatically. In the long-wave limit, Eq. (6.24) provides the fulfilment of the Siegert theorem

A. J. F. Siegert, Phys. Rev. **52** (1937) 787;

for electric transitions in reactions with nonmeson channels

A. V. Shebeko, Sov. J. Nucl. Phys. **49** (1989) 30; L. Levchuk and A. Shebeko, Phys. At. Nuclei **56** (1993) 227.

For pion photoproduction on the free nucleon at threshold, it leads (as shown in

L. G. Levchuk and A. V. Shebeko, Phys. At. Nuclei **58** (1995) 923.)

to the Kroll-Ruderman result

N. M. Kroll and M. A. Ruderman, Phys. Rev. **93** (1954) 233;

emerging here as a particular case of the Siegert theorem.

**Thank you very much for your attention!**