Neutrino Nature & Mass: An Introduction

A theoretical lecture in preparation of the activities at Hands-On 2025, Gran Sasso Lab

Francesco Vissani, INFN Gran Sasso

how to measure neutrino mass

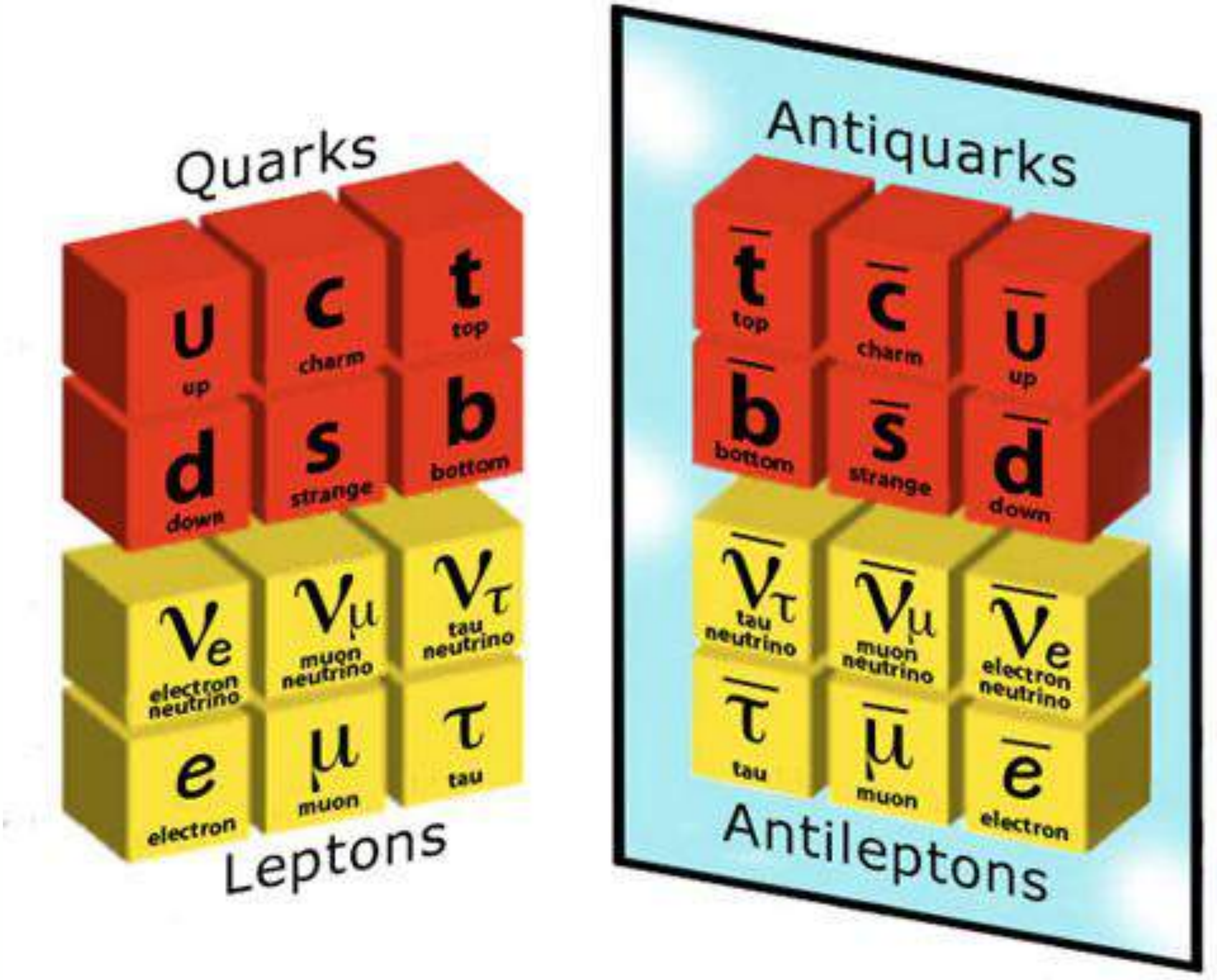
- ⭐ **kinematics** (Pauli 1930) neutrino versus photon race / *time of flight* measurement
- ⭐ **spectral distortions** (Fermi 1933) *end-point* of electron spectrum in the β decay
- ⭐ **neutrinoless $\beta\beta$ decay** (Furry; Grueling Whitten 1939-60) *Majorana neutrino* & $\mathbf{\Delta L = 2}$
- ⭐ **neutrino transmutation** (Pontecorvo; Sakata et al 1957-1967) "*oscillations*"
- ⭐ **observational cosmology** (Gershtein Zeldovich 1966) *distortions* of the cosmic distributions
- ⭐ **neutrino capture** (Cocco, Mangano, Messina 2007) direct observation of *big-bang neutrinos*

the 4th method has provided results, proving that the SM is incomplete

Nobel 2015

from the press announcement

*For particle physics this was a **historic** discovery. Its Standard Model of the innermost workings of matter had been incredibly successful, having resisted all experimental challenges for more than 20 years.*



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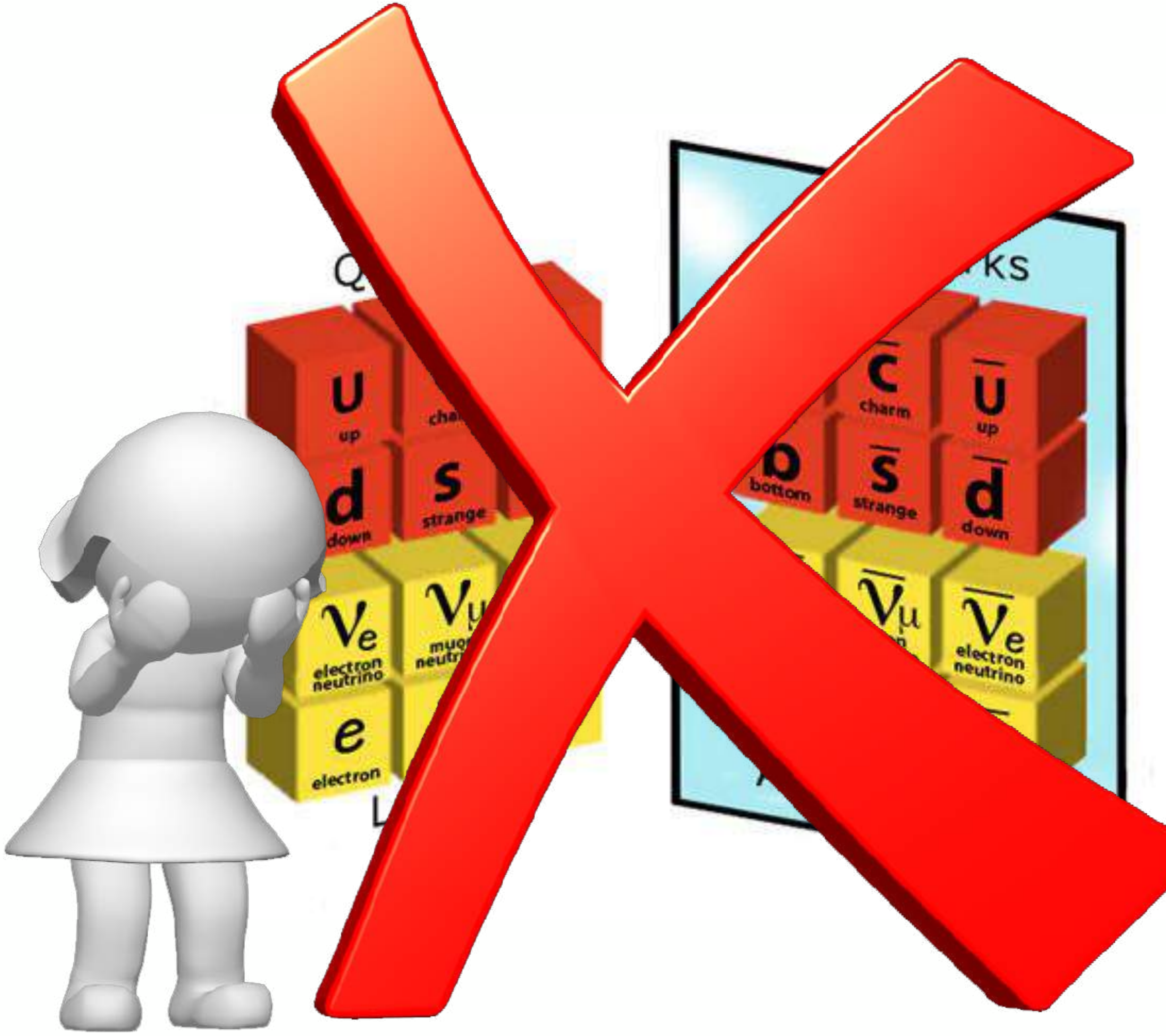
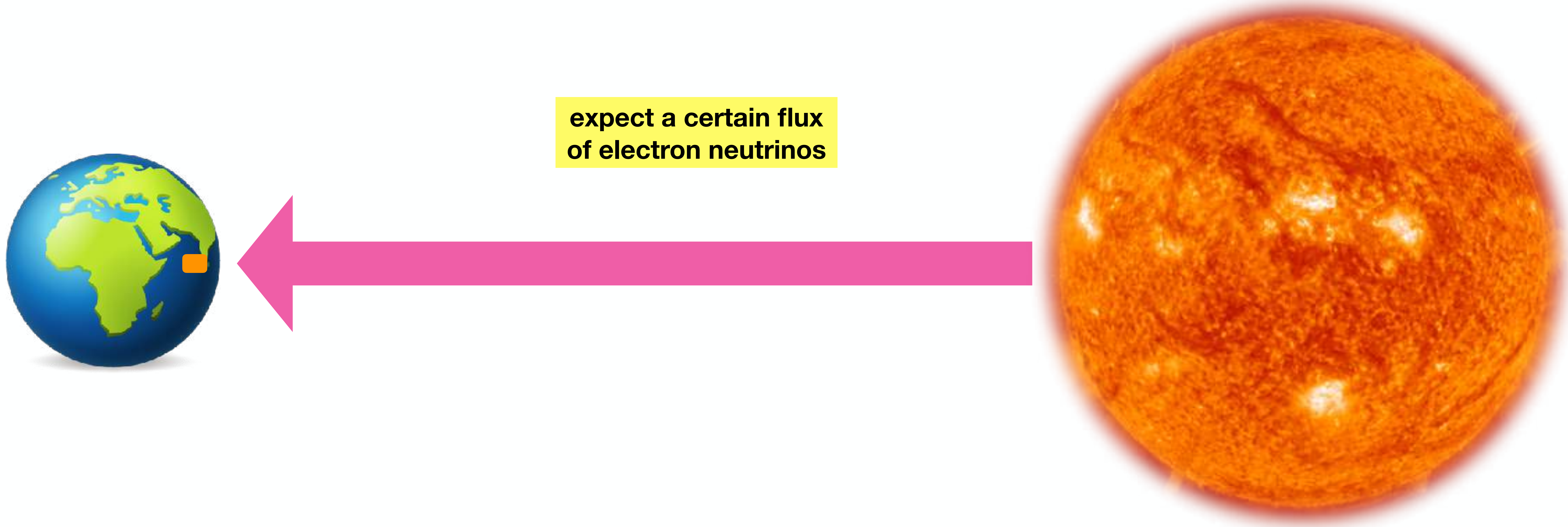
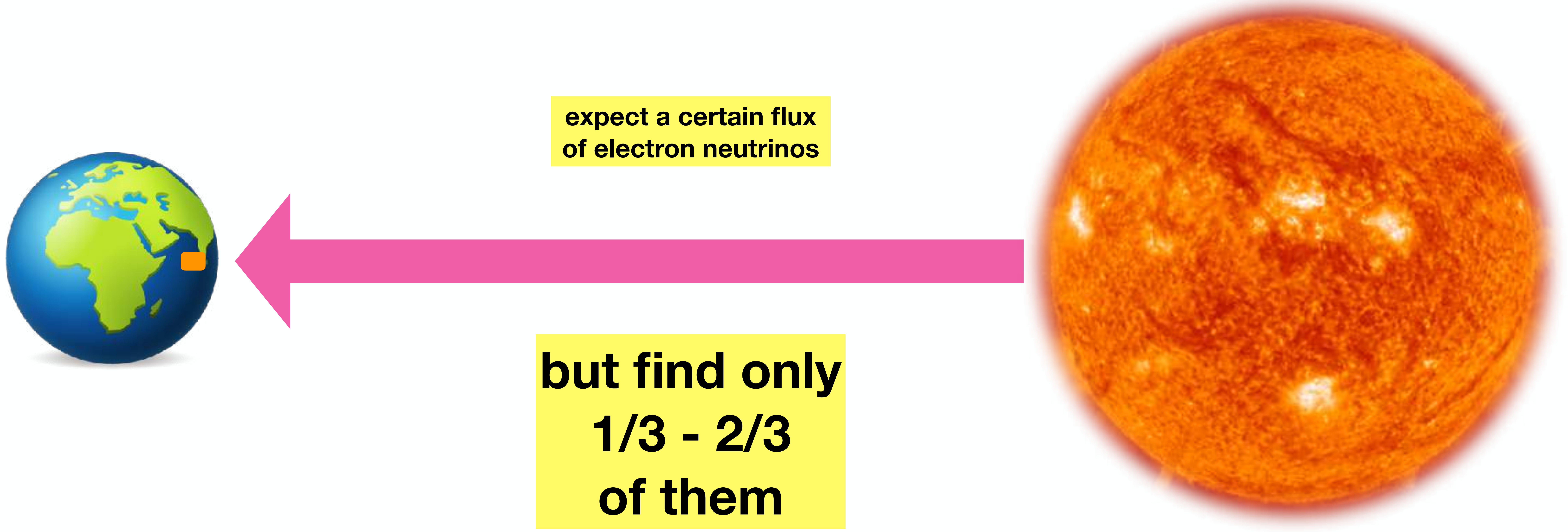
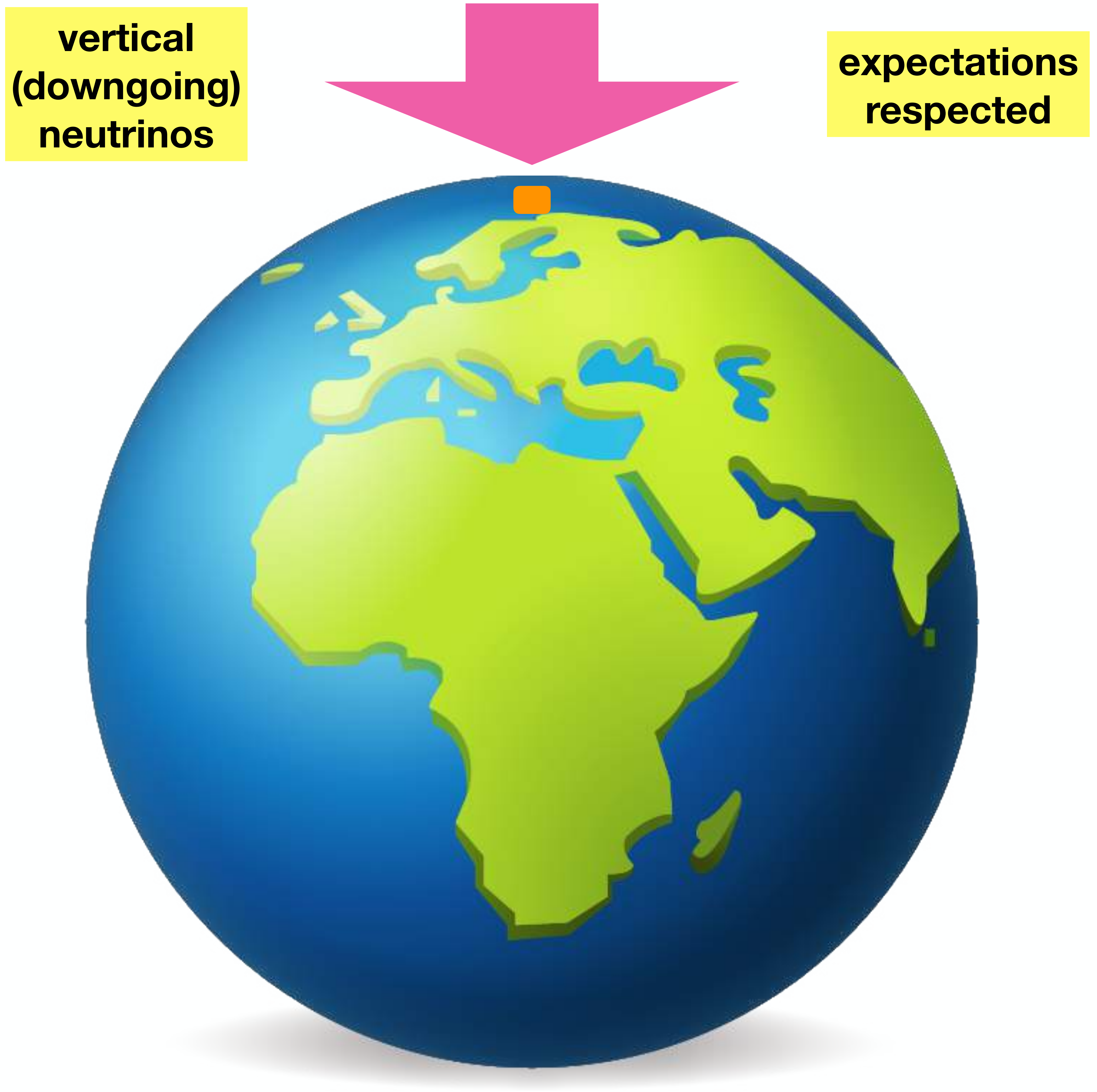
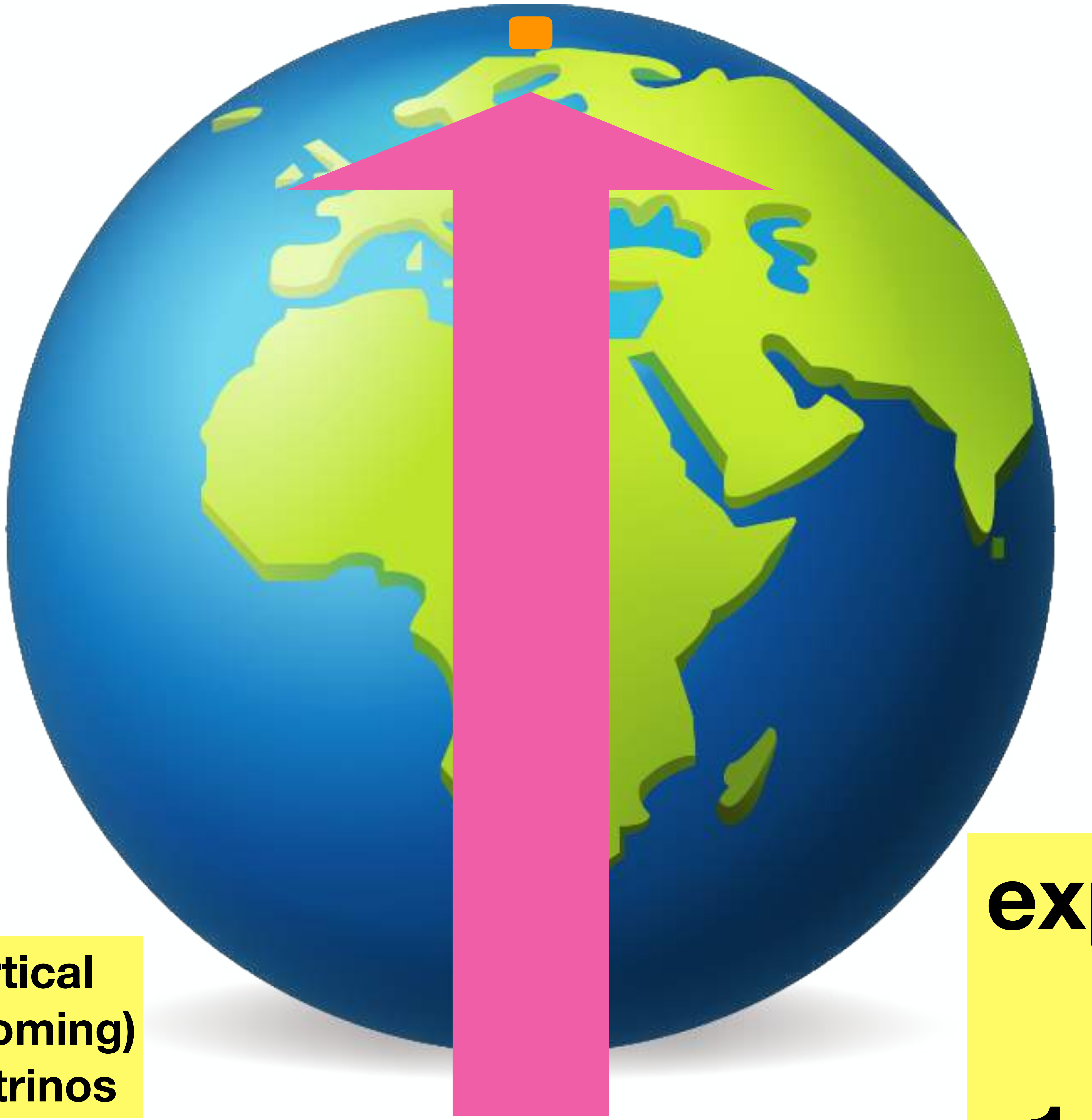
However, as it requires neutrinos to be massless, the new observations had clearly showed that the Standard Model ***cannot be*** *the complete theory of the fundamental constituents of the universe.*




expect a certain flux
of electron neutrinos


expect a certain flux
of electron neutrinos
but find only
1/3 - 2/3
of them

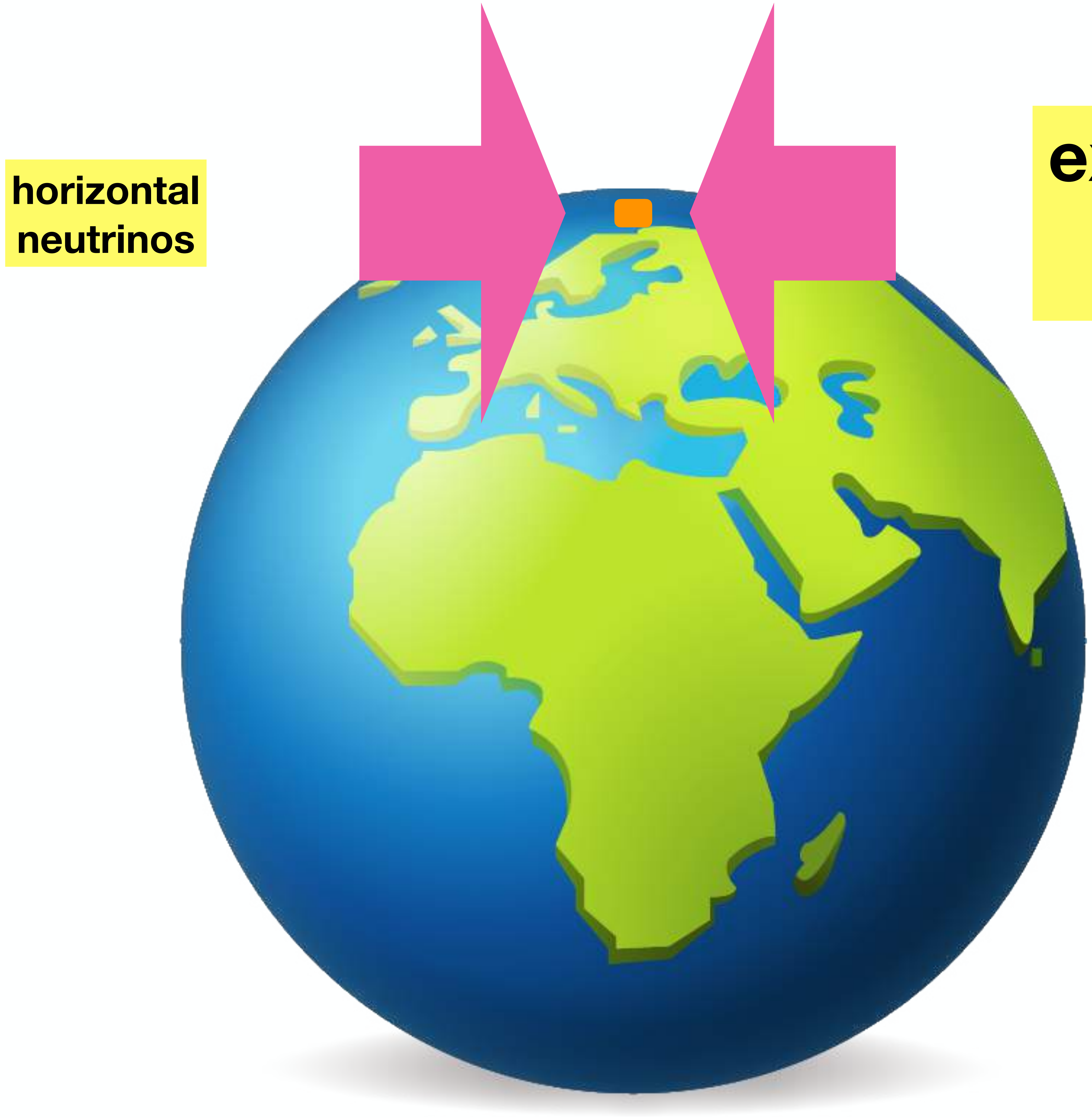

vertical
(downgoing)
neutrinos
expectations
respected



vertical (upcoming) neutrinos

expectations violated

1/2 ν_μ seen



expectations violated, some ν_μ missing

organisation of this lecture:

- ⭐ **theoretical background notes**
- ⭐ **discussions of the six methods**

a few references

GRAN SASSO SUMMER INSTITUTE 2014

HANDS-ON EXPERIMENTAL UNDERGROUND PHYSICS AT LNGS

SEPTEMBER 22 - OCTOBER 3, 2014
LABORATORI NAZIONALI DEL GRAN SASSO
ASSERGI, ITALY

Neutrino Sources and Properties

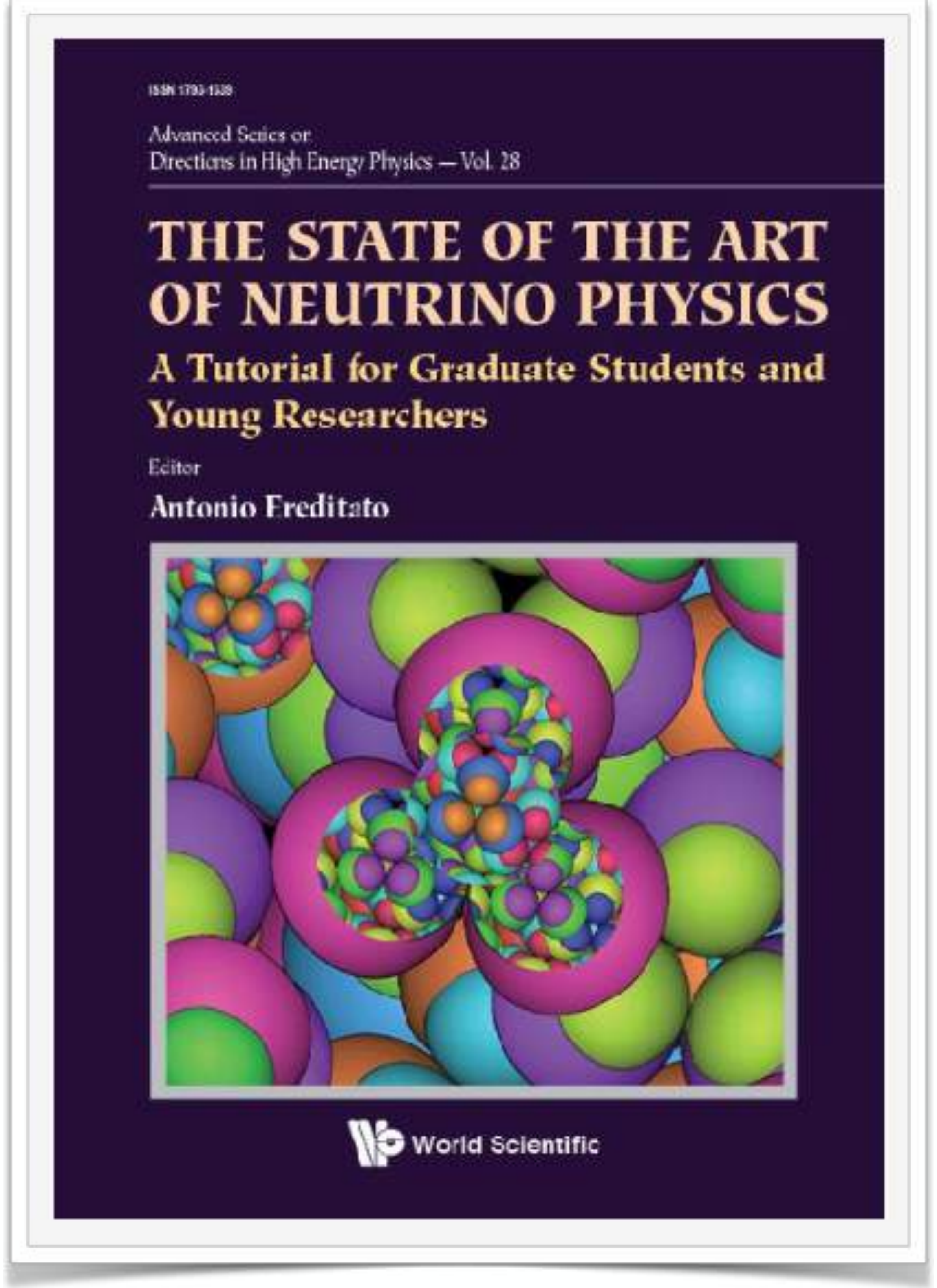
Francesco Vissani*
INFN, Laboratori Nazionali del Gran Sasso
E-mail: vissani@lngs.infn.it

In this lecture, prepared for PhD students, basic considerations on neutrino interactions, properties and sites of production are overviewed. The detailed content is as follows: *Sect. 1, Weak interactions and neutrinos:* Fermi coupling; definition of neutrinos; global numbers. *Sect. 2, A list of neutrino sources:* Explanatory note and examples (solar pp- and supernova-neutrinos). *Sect. 3, Neutrinos oscillations:* Basic formalism (Pontecorvo); matter effect (Mikheev, Smirnov, Wolfenstein); status of neutrino masses and mixings. *Sect. 4, Modifying the standard model to include neutrinos masses:* The fermions of the standard model; one additional operator in the standard model (Weinberg); implications. One summary table and several exercises offer the students occasions to check, consolidate and extend their understanding; the brief reference list includes historical and review papers and some entry points to active research in neutrino physics.

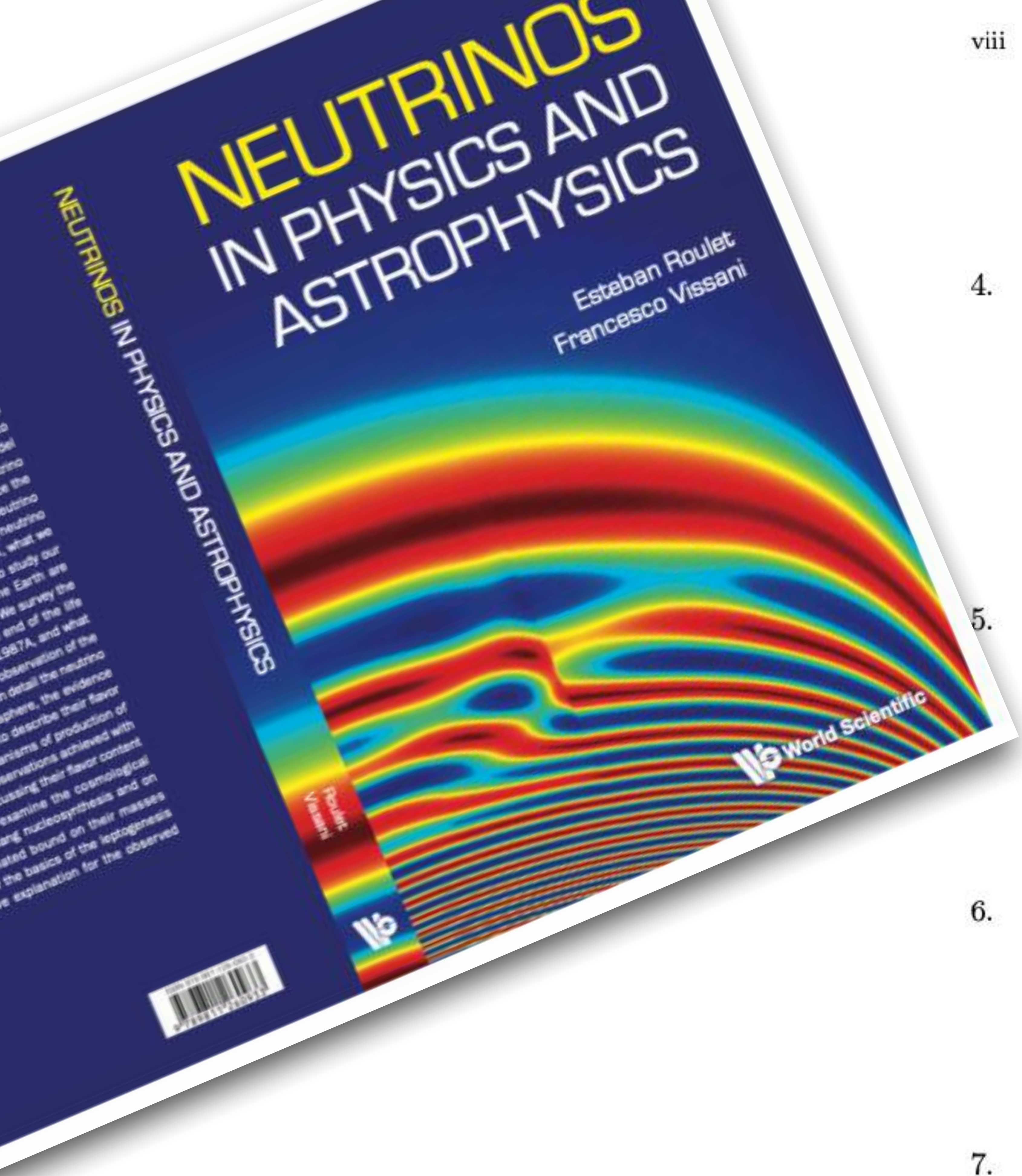
Advanced Series on Directions in High Energy Physics
| The State of the Art of Neutrino Physics, pp. 37-119 (2018)

Chapter 2: Introduction to the Formalism of Neutrino Oscillations

G. Fantini, A. Gallo Rosso, V. Zema and F. Vissani



also
https://arxiv.org/pdf/1802.05781.pdf



Neutrinos in Physics and Astrophysics

https://doi.org/10.1142/12982 | October 2022
Pages: 250

By (author): Esteban Roulet (*CONICET, Argentina*) and Francesco Vissani (*INFN, Italy*)

ISBN: 978-981-126-093-3 (hardcover)

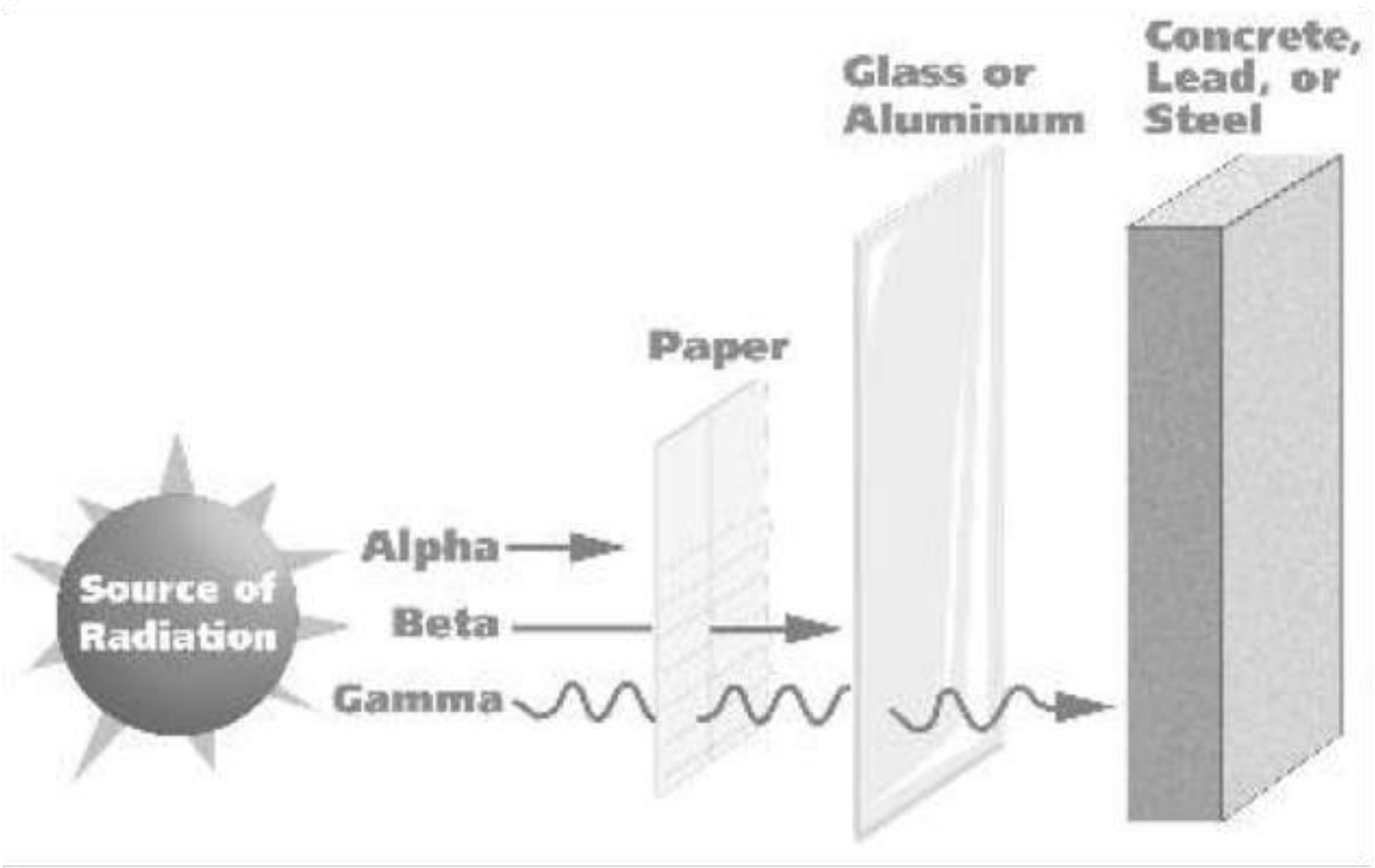
recall

birth of the neutrino

(born while updating a theory of the nucleus)

high energy radiation

- high energy emission from certain substances is discovered 100 yr ago
- generically called *"radioactivity"*
- energy is much larger than the atomic emission, till some MeV
- penetration power characterises the different type of rays (see figure)

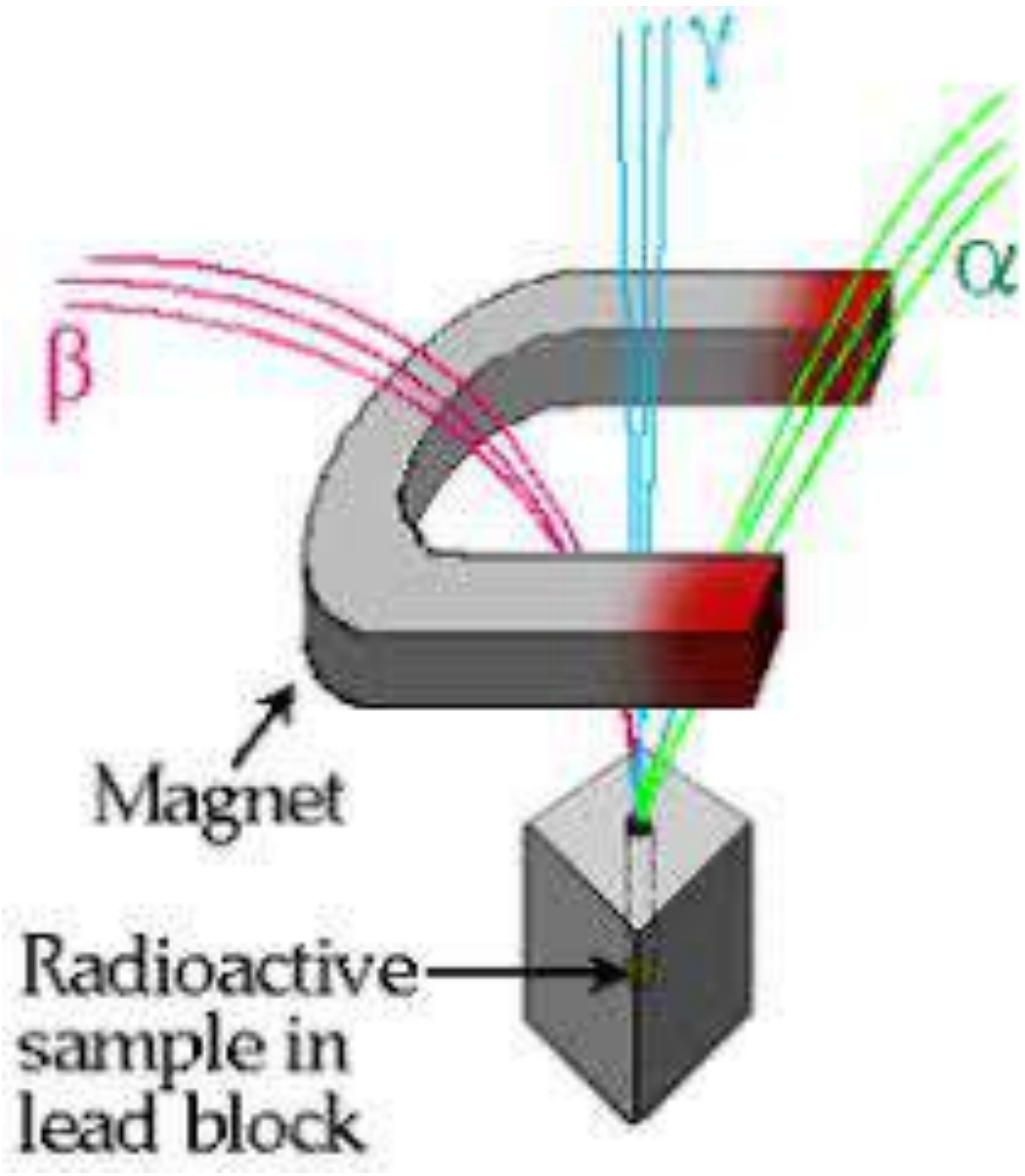


high energy radiation

- α are nuclei of helium 4 nuclei
- β are high energy electrons
- γ is high energy e.m. radiation

The traditional names assigned by Rutherford and Villard maintain their aura of mystery

(while for Maxwell theory we use terms such as "e.m. radiation")



the prevailing theory of the nucleus till 1932

van der Broek, Bohr

- if the nuclei emit fragments of matter, **they contain these fragments**;
- *(*forget gamma ray, this is just radiation, not matter)
- in a few years, the general consensus is that nuclei are assemblies of two particles:

electron & protons

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BUT why β radiation doesn't obey the same rule?

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BUT why β radiation doesn't obey the same rule?

In fact, some energy is systematically missing from the expectations.

maybe there are excited nuclei in final state?

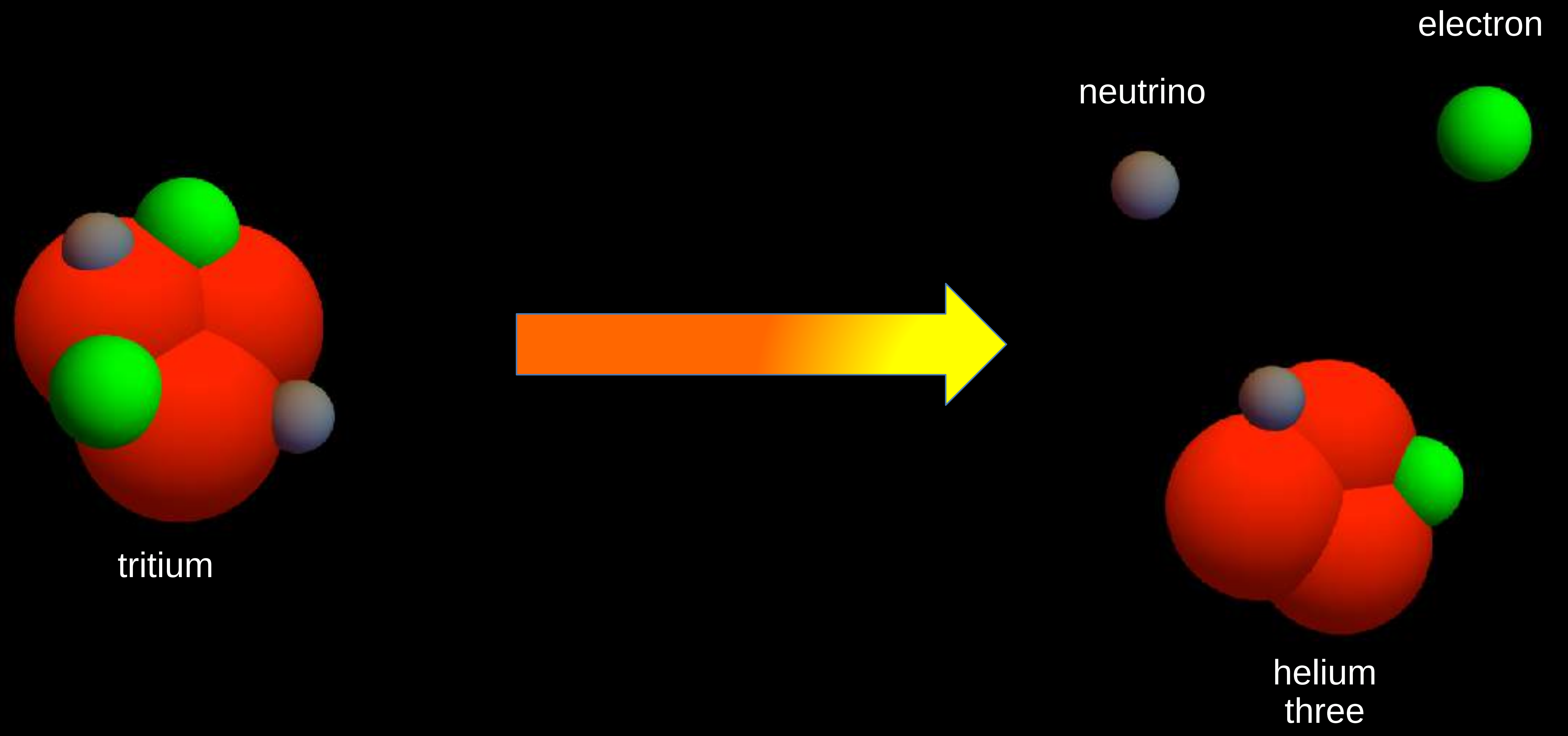
maybe energy is not conserved in nuclear physics?

or the model of the nucleus needs revision?

(as proposed by Goeppert-Mayer, Bohr and Pauli, respectively)

none of these is evidently wrong

Pauli's answer saves energy conservation



nucleus with electrons, protons and <u>neutrinos</u>.
the latter subtracts (steals) energy in the β decay

Original - Photocopie of PLC 0393

Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

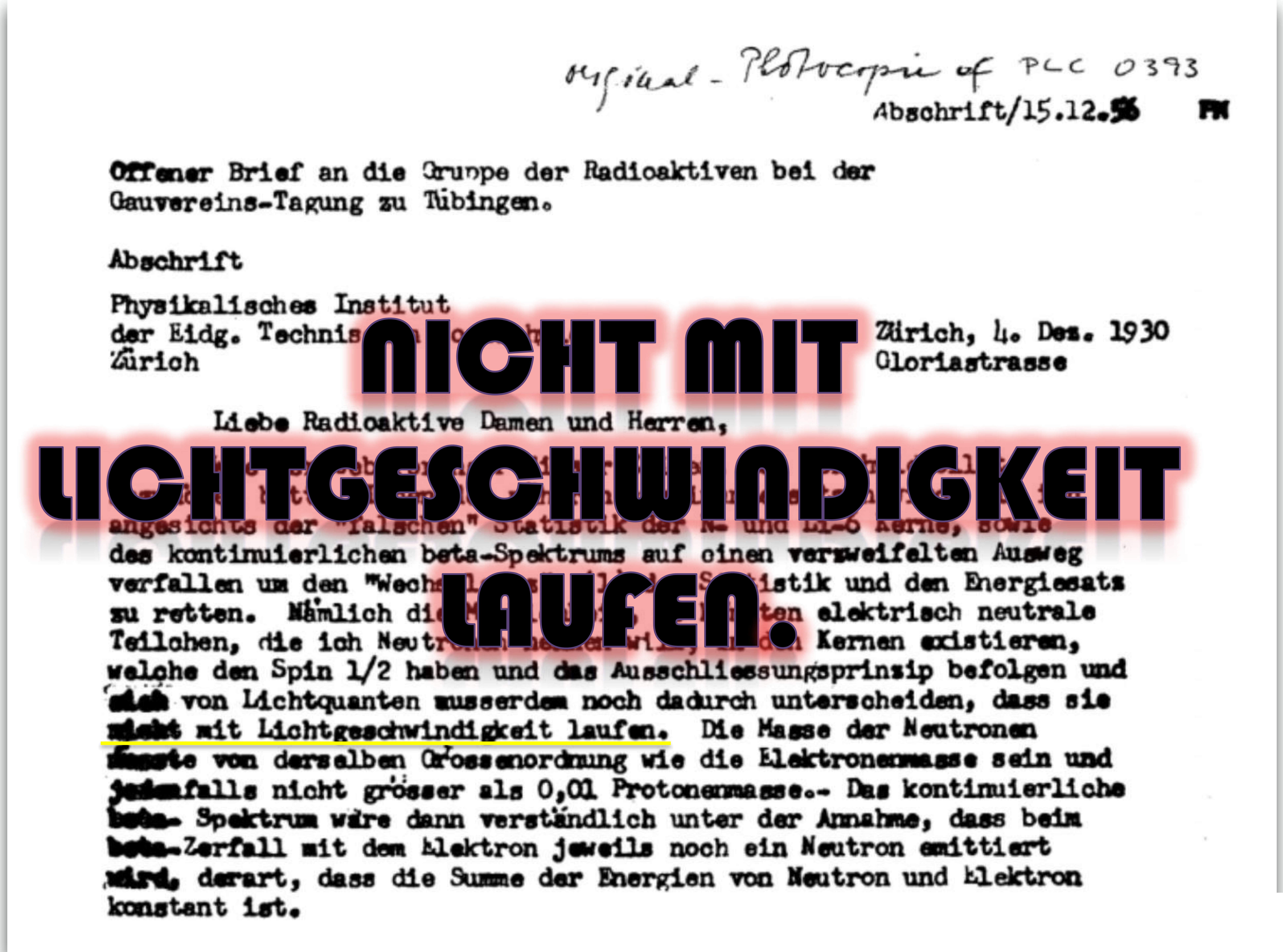
Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

method 1

the neutrino race method

(Pauli … Zatsepin)

NICHT MIT LICHTGESCHWINDIGKEIT LAUFEN.



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due to their mass, neutrinos are slower than photons

a race of neutrinos and photons

The arrival of a cosmic neutrino rush

57 years later, a supernova was seen at 170,000 light year. Astronomers knew that neutrinos precede light and asked to check neutrino telescope data. A burst of events of ~10 s, with energies 7 to 40 MeV, was found 3-4 hours before the light

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observations allow to argue that light and neutrino arrived at the same time within 0.5 h, but there is more...

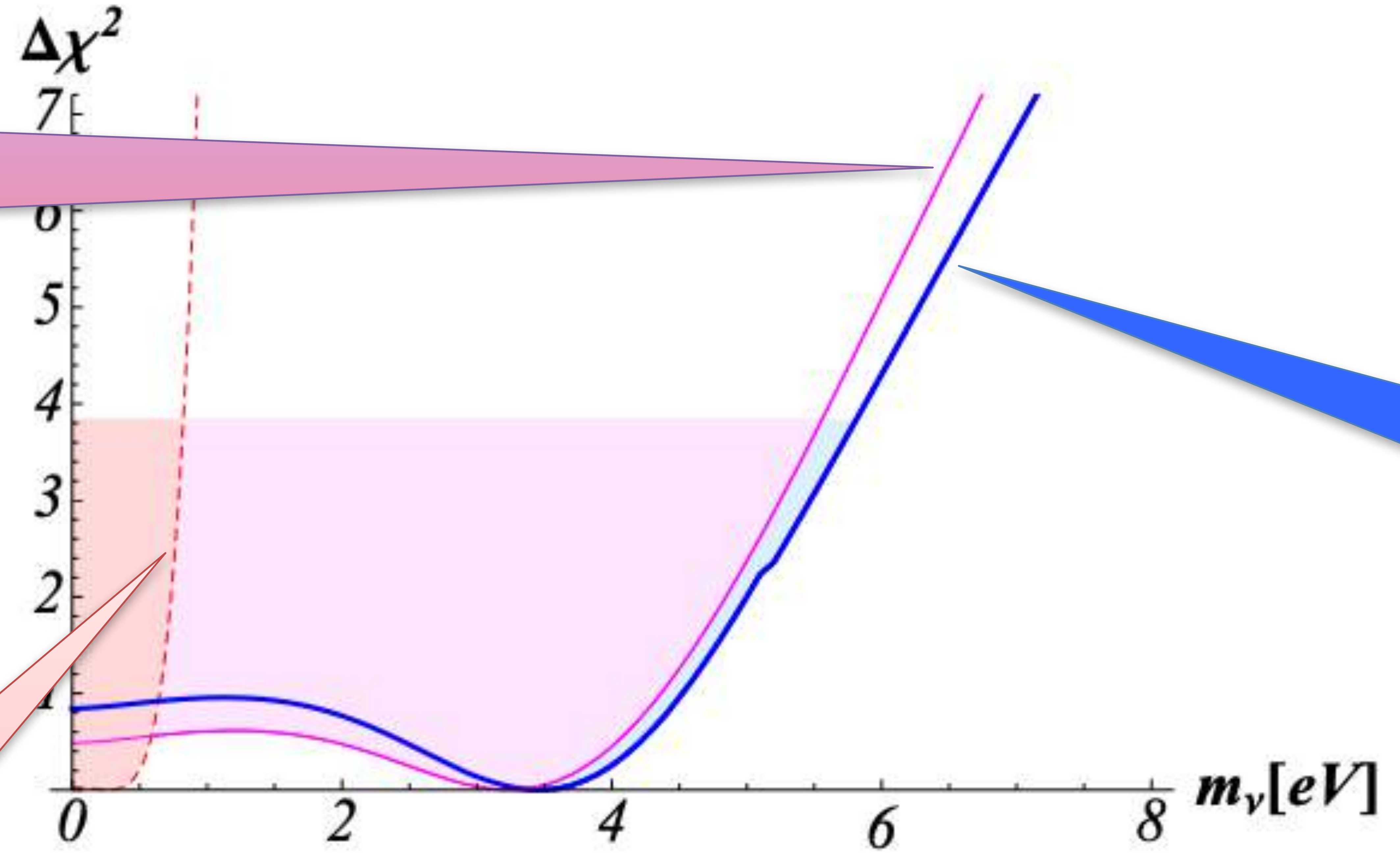
a better method

- the original idea of Zatsepin was based on the theoretical expectation (now considered outdated) that the supernova neutrino emission happens in a ms burst
- the modern method is based instead on a sort of neutrino "acromaticity": neutrino arrival time depends upon neutrino energy

$$t = \frac{L}{v} \approx \frac{L}{c}\left(1 + \frac{1}{2}\left(\frac{mc^2}{E}\right)^2\right)$$

less energetic neutrinos are slower and arrive later

result



$\bar{\nu}$ MASS (electron based)

Those limits given below are for the square root of $m^{2(\text{eff})}_{\nu_e} \equiv \sum_i |U_{ei}|^2 m^2_{\nu_i}$. Limits that come from the kinematics of $^3\text{H}\beta^-\bar{\nu}$ decay are the square roots of the limits for $m^{2(\text{eff})}_{\nu_e}$. Obtained from the measurements reported in the Listings for "$\bar{\nu}$ Mass Squared," below.

VALUE (eV)	CL%	DOCUMENT ID		TECN	COMMENT
< 2	**OUR EVALUATION**				
< 2.3	95	[1] KRAUS	05	SPEC	^{3}H β decay
< 2.5	95	[2] LOBASHEV	99	SPEC	^{3}H β decay
• • • We do not use the following data for averages, fits, limits, etc. • • •					
< 5.8	95	[3] PAGLIAROLI	10	ASTR	SN1987A
<21.7	90	[4] ARNABOLDI	03A	BOLO	^{187}Re β-decay

recall

understanding matter particles

(from wave equations to early quantum fields)

particles and waves

till 100 years ago (prehistory)

Einstein's relation for *quanta of light* (1905): $E = h \times \nu$

Bohr's generalizes it to matter particles, *atomic electrons* (1913)

de Broglie's (1924): $\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = \sin\left[\frac{xp - tE}{\hbar}\right]$

twenties have been the era of electron wave equations

electron wave equations

- **Schrodinger (1925):** electron wave in an external potential propagates *analogously* to light wave in a non-uniform medium.

- **Klein Gordon (1926):** extension to a relativistic electron… w/o spin.

- **Pauli (1927):** Spin in Schrodinger hamiltonian; wavefunction is 2-D.

- **Dirac (1928):** a new relativistic equation explains the existence of the spin and the coupling to the magnetic fields. Wavefunction becomes 4-D.

- **Weyl (1929):** if the mass of the electron is neglected, Dirac's equation splits into two simpler (2-D) equations, apparently useless.

Dirac & Anderson

(1931 - 1932)

What about the negative energy solutions of Dirac's ***wave equation?***

Dirac & Anderson

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What about the negative energy solutions of Dirac's ***wave equation?***

- HP 1: states with negative solutions **exist**
- HP 2: they are **occupied** & due to the exclusion principle inaccessible

This is the **Dirac sea.**

Thus atoms are stable; moreover, if one photon extract one of the states, one sees a regular (positive energy) electron and the hole, that will be interpreted as an increased energy and increased electric charge of the sea, namely an ***anti-electron***.

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— - —

1932: ***positrons*** are observed by Anderson and identified with **anti-electrons** by Dirac.

Fermi

(1933)

Fermi implements a revolutionary idea: Electron & neutrino are generated in β decay

$$n \to p + e + \nu$$

For the first time, **matter particles (electrons) are not** assumed to be **eternal.**

This revolutionary idea is somewhat facilitated by de Broglie conception of matter waves; see Iwanenko & Ambartsumian (1930) Perrin (1933).

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In order to do so, Fermi accepts 1) Pauli's neutrino 2) Dirac's sea 3) Jordan/Wigner/Klein/Fock formalism for relativistic fermions, with $\langle 0 | \mathbf{a}_s | 1_s \rangle = \langle 1_s | \mathbf{a}_s^\dagger | 0 \rangle = 1.$

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$$\boldsymbol{\Psi} = \sum_s \mathbf{a}_s \, \psi_s \qquad \text{where } s = \text{helicity, momentum, energy with sign}$$

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recall

few words on Fermi heritage

(an excuse to recall weak interactions)

after Fermi theory of β ray emission

after Fermi theory of β ray emission

a lot of results follow immediately:

β^+emission; electron capture; cross sections; Yukawa's improvements; variants of the hamiltonian; neutral currents; etc.

connected processes

Processes	Discussed	Observed
(A,Z) —> (A,Z+1) e- bar-v	1933	1899
(A,Z) —> (A,Z-1) e+ v	1934	1942
e- (A,Z) —> (A,Z-1) v	1934	1939-42
e+ (A,Z) —> (A,Z+1) bar-v	1938? 1955?	—
bar-v (A,Z) —> (A,Z-1) e+	1934	1956
v (A,Z) —> (A,Z+1) e-	1942	1969

Fermi theory of β ray emission: debate

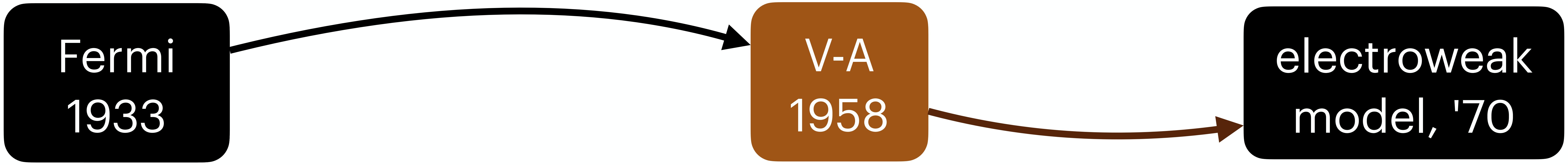
- *Pauli:* *the theory* ***ceases to hold*** *at sufficiently high energies.*
- *Gamow & Teller:* *currents* ***are not purely vectorial****. Starting from Lee & Yang, we will arrive at the V-A theory and shortly thereafter at Cabibbo's theory.*
- *Majorana* *introduces the* ***modern quantization*** *of fermionic fields, superior to the Dirac-Jordan-Klein formalism based on the Dirac sea (more later).*

Fermi theory of β ray emission: debate

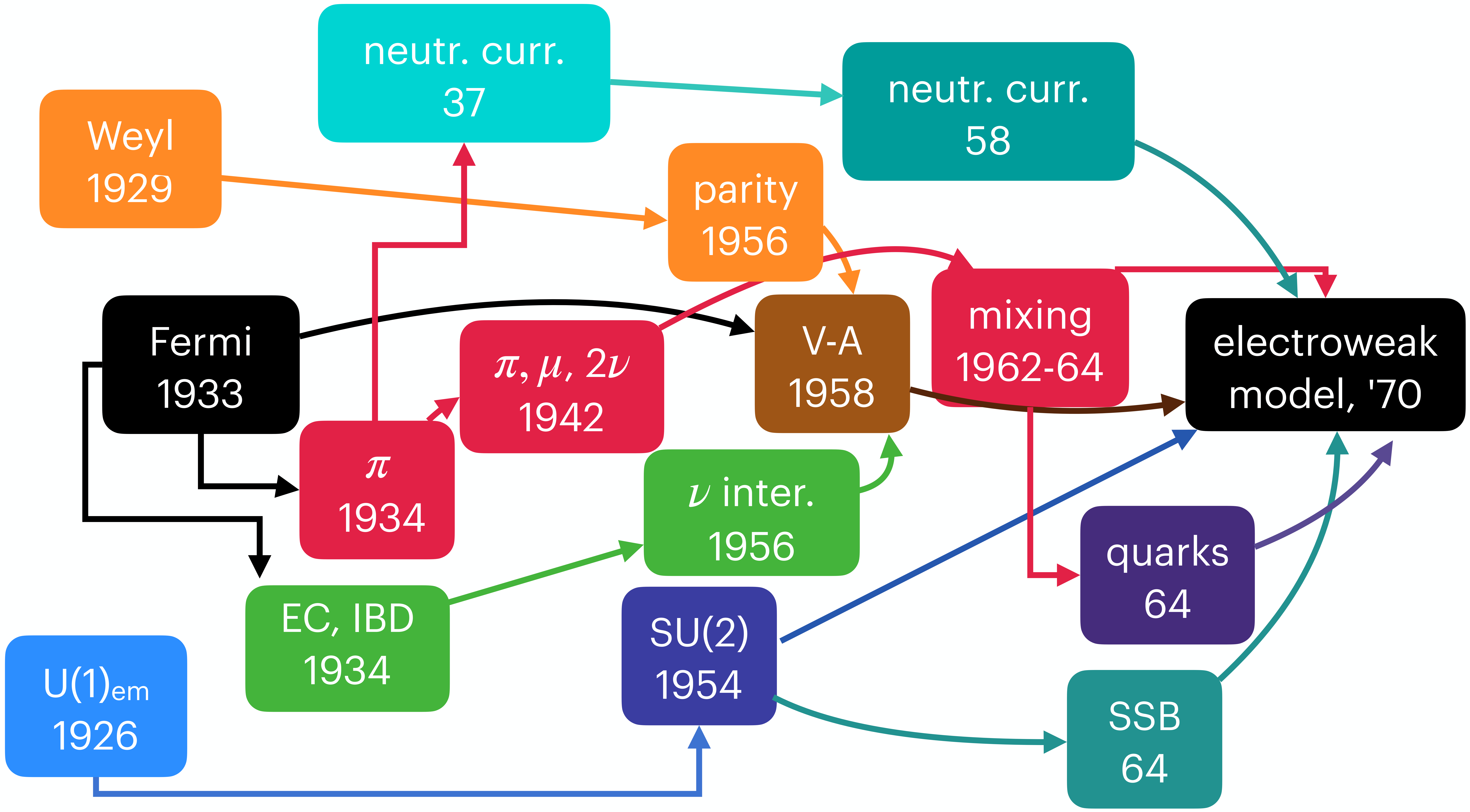
- ***Pauli:*** *the theory **ceases to***
- ***Gamo*** … *Yang, we will* … *ory.*

… ***ization** of fermionic fields, superior to the Dirac-* … *on the Dirac sea (more later).*

None of these criticisms have undermined Fermi's theory: they have refined it!



neutrino interactions from Fermi to SM (easier version)



neutrino interactions from Fermi to SM (better version)

method 2

endpoint

(Fermi method)

structure of Fermi hamiltonian

to describe beta ray emission

$$\mathbf{H} = g \cdot \mathbf{T}^{+} \cdot \mathbf{J}^{-}$$

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$\langle p \,|\, \mathbf{T}^{+} \,|\, n \rangle \neq 0$ describes the change of isospin, harmless

$\langle e\nu \,|\, \mathbf{J}^{-} \,|\, 0 \rangle \neq 0$ describes the **creation** of two (matter) particles

structure of Fermi hamiltonian

to describe beta ray emission

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of course g is the coupling, with dimensions energy×volume

$$\Gamma_{i \to f} = \frac{2\pi}{\hbar} \left| \langle f | \mathbf{H} | i \rangle \right|^2 \rho(E_f)$$

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shape is due to the phase space

$$d^3p_\nu = 4\pi\, p_\nu^2\ dp_\nu = 4\pi\, p_\nu E_\nu\ dE_\nu = 4\pi\sqrt{(Q - E_e)^2 - \mu^2}\ (Q - E_e)\ dE_e$$

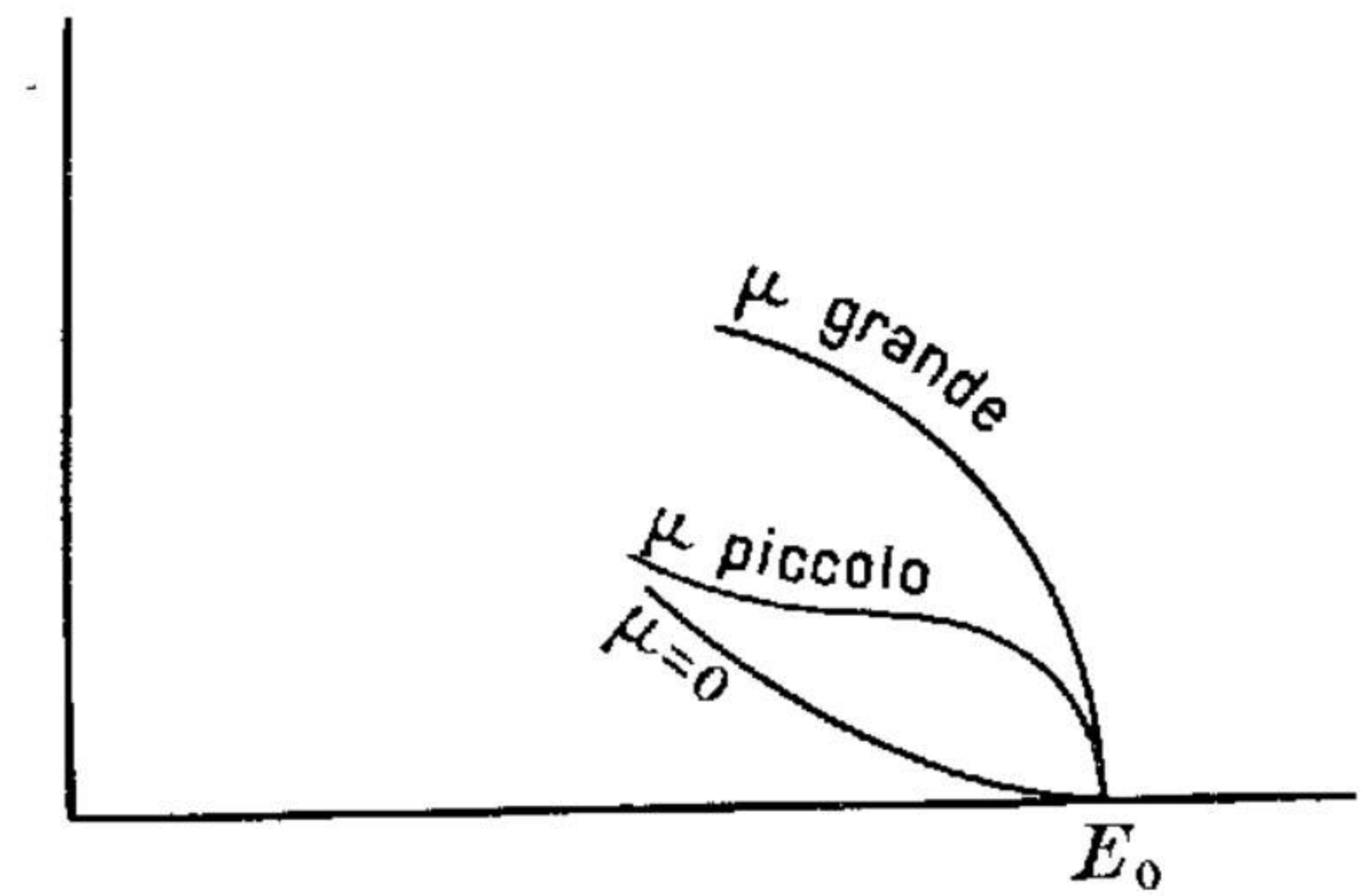


Fig. 1

Science

HOME > SCIENCE > VOL. 388, NO. 6743 > DIRECT NEUTRINO-MASS MEASUREMENT BASED ON 259 DAYS OF KATRIN DATA

RESEARCH ARTICLE | PARTICLE PHYSICS

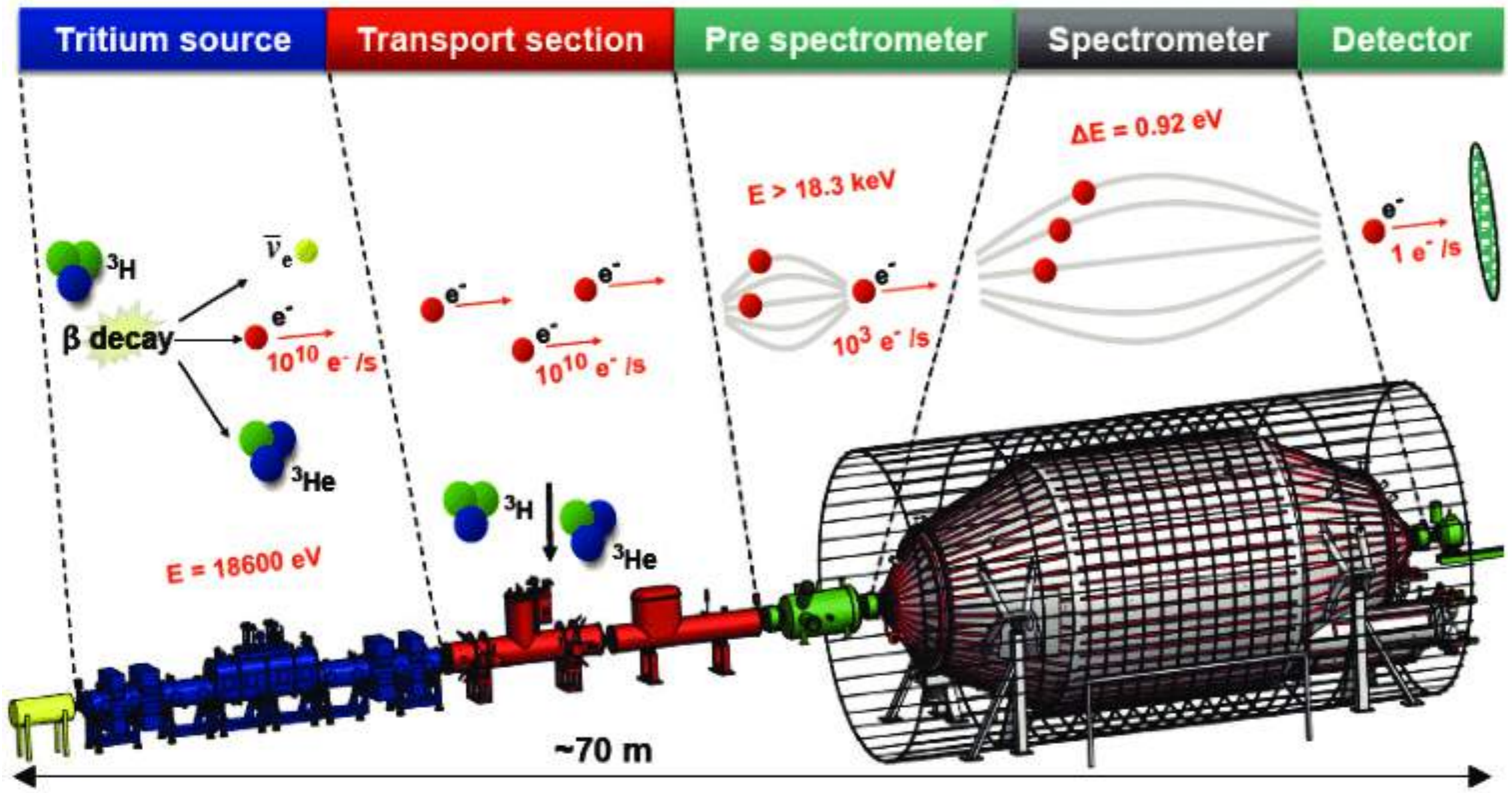
Direct neutrino-mass measurement based on 259 days of KATRIN data

KATRIN COLLABORATION, MAX AKER, DOMINIC BATZLER, ARMEN BEGLARIAN, JAN BEHRENS, JUSTUS BEISENKÖTTER, MATTEO BIASSONI, BENEDIKT BIERINGER, YANINA BIONDI, [...], AND GENRICH ZELLER +140 authors Authors Info & Affiliations

SCIENCE · 10 Apr 2025 · Vol 388, Issue 6743 · pp. 180-185 · DOI: 10.1126/science.adq9592

Abstract

That neutrinos carry a nonvanishing rest mass is evidence of physics beyond the Standard Model of elementary particles. Their absolute mass holds relevance in fields from particle physics to cosmology. We report on the search for the effective electron antineutrino mass with the KATRIN experiment. KATRIN performs precision spectroscopy of the tritium β-decay close to the kinematic endpoint. On the basis of the first five measurement campaigns, we derived a best-fit value of $m_\nu^2 = -0.14^{+0.13}_{-0.15}$ eV^2, resulting in an upper limit of $m_\nu < 0.45$ eV at 90% confidence level. Stemming from 36 million electrons collected in 259 measurement days, a substantial reduction of the background level, and improved systematic uncertainties, this result tightens KATRIN's previous bound by a factor of almost two.



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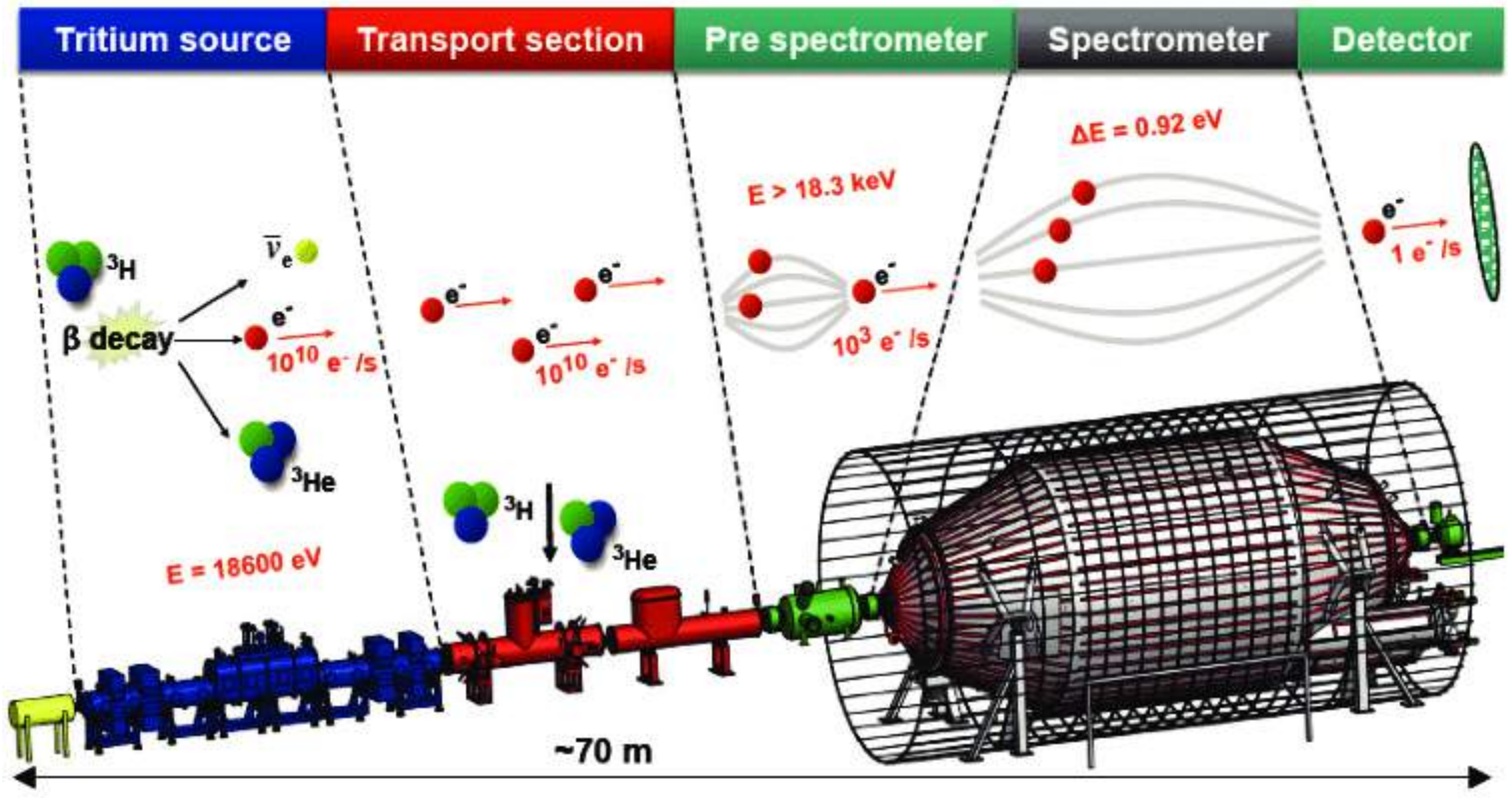

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recall

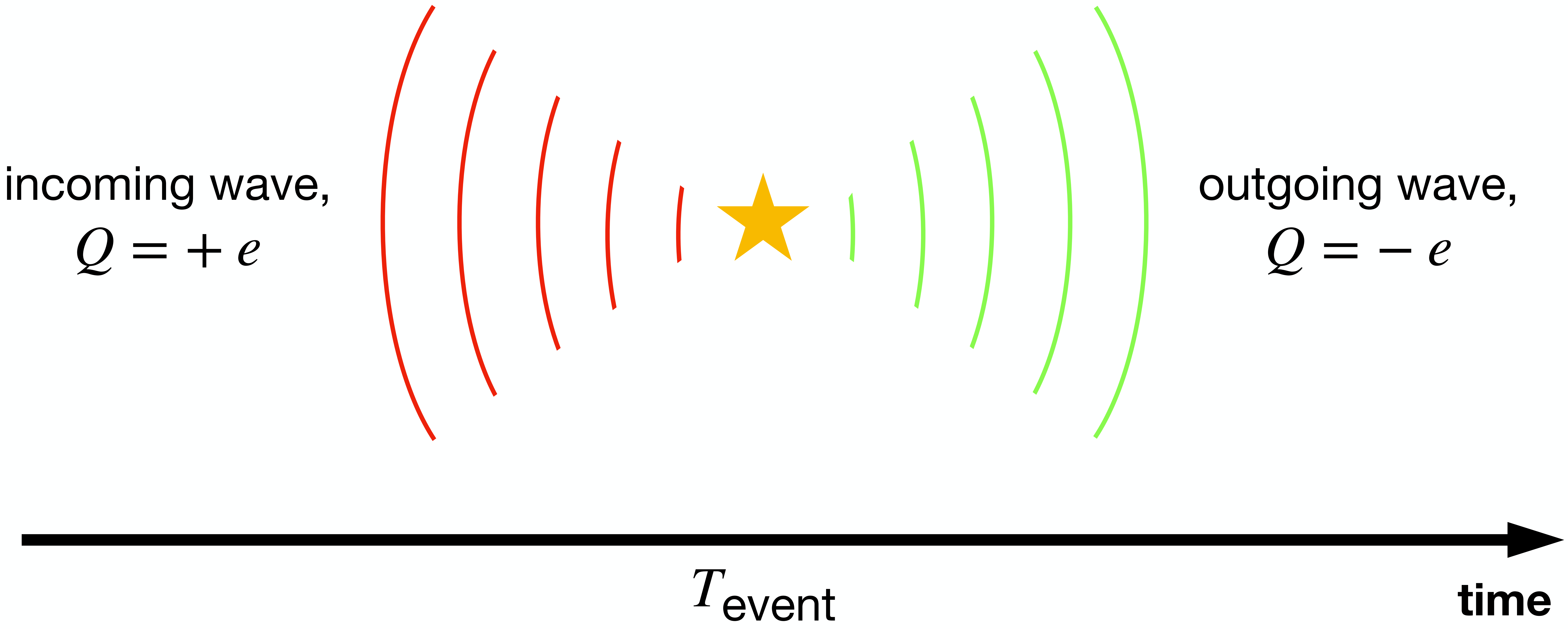
a new theory of the neutrinos

(from Majorana to Furry)

the modern conception of antimatter emerges

(Fock; Furry & Oppenheimer; Heisenberg; Pauli & Weisskopf; Majorana; Stueckelberg; 1933 - 1941)

The Dirac sea is not needed / does not exist / can be thought in a very different way.



Quantised field obeying Dirac equation (Majorana)

(1937)

The Dirac sea is not needed / does not exist / can be thought in a very different way.

The electrons are described as operators (quantum fields). They obey Heisenberg equation with Dirac hamiltonian. This implies **fermionic character**.

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$$\boldsymbol{\Psi} = \sum_{s, E_s > 0} \left(\mathbf{a}_s \, \psi_s + \mathbf{a}_s^{\dagger} \, \psi_s^{*} \right)$$

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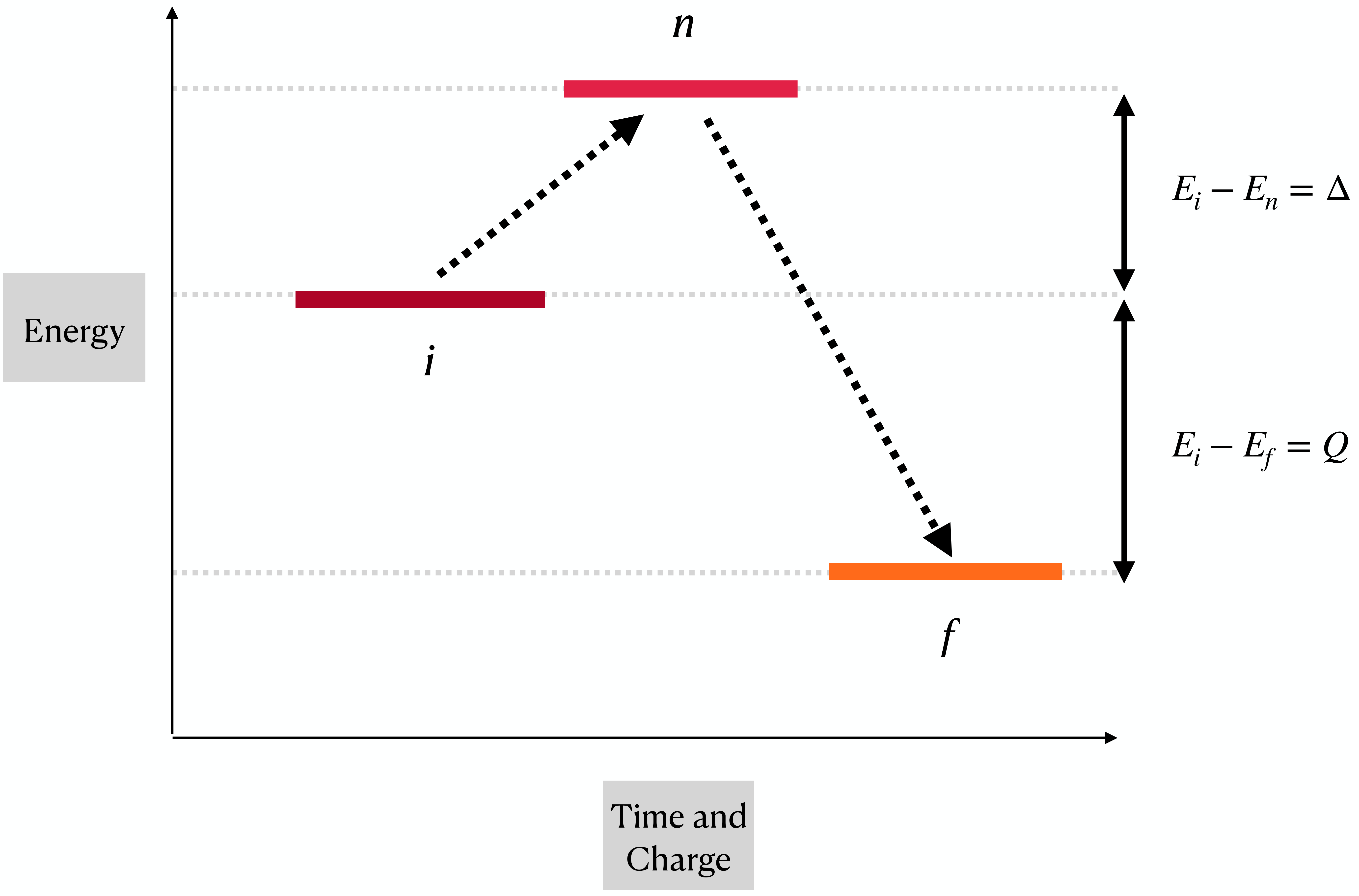
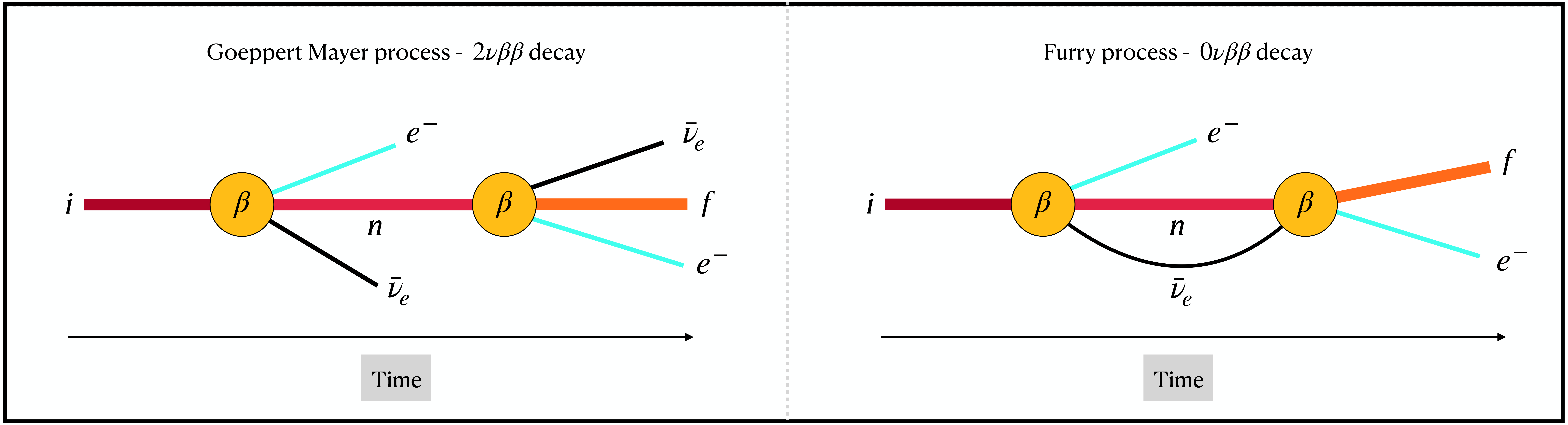
Apart from abbreviation & use of $(\gamma_\mu)^* = -\gamma_\mu$ that we'll follow, this is a **modern quantum field**

test: the "double beta" decay



two types of "double beta" decay



estimating the width of "double beta" decay

$$\Gamma = \frac{2\pi}{\hbar} \, |H^{\beta\beta}_{i\to f}|^2 \, \rho_f$$

$\rho_f \sim \{ V(p/h)^3 \}^{N_\ell}/Q$ where there are $N_\ell = 4$ e 2 leptons

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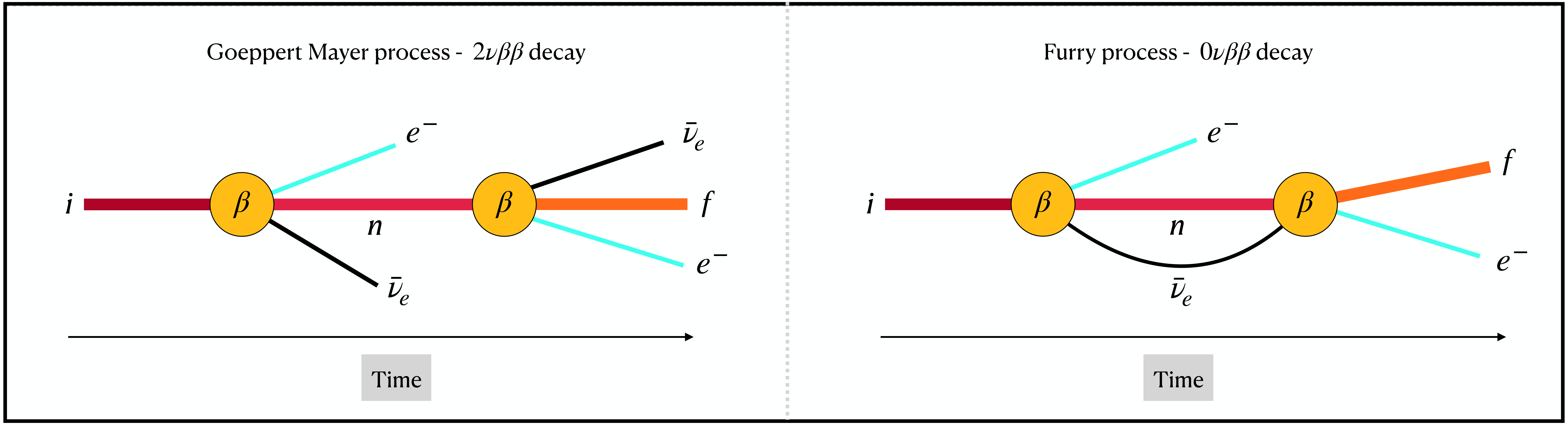
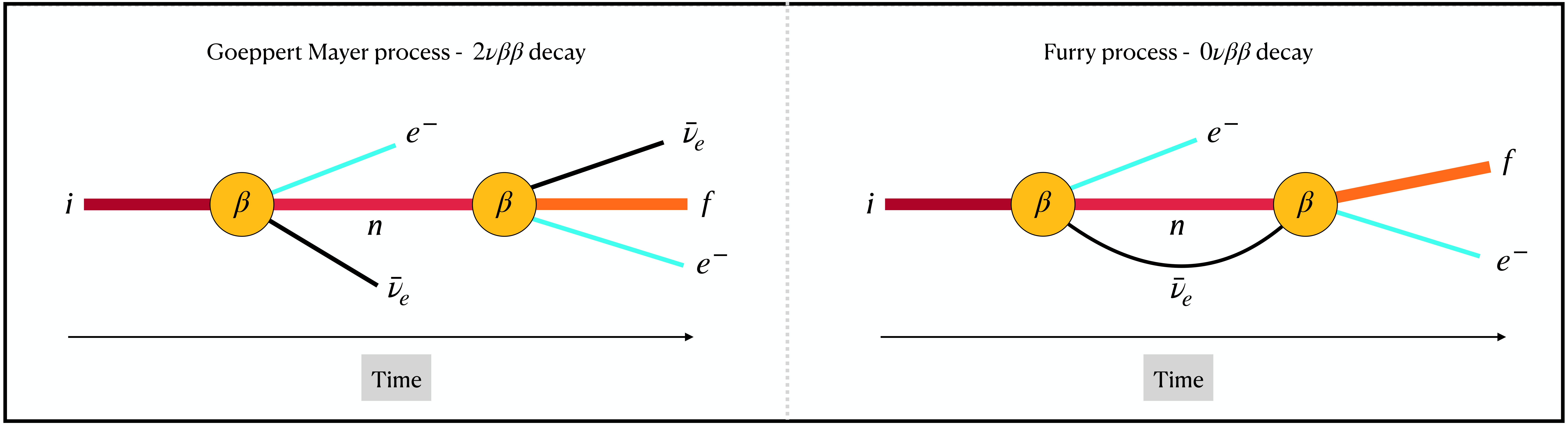
$$H^{\beta\beta}_{i\to f} = \sum_n \frac{H^{\beta}_{i\to n} H^{\beta}_{n\to f}}{E_i - E_n} \sim \left(\frac{G_F}{V}\right)^2 \begin{cases} \dfrac{1}{\Delta} & \text{Goeppert Mayer process} \\ \dfrac{1}{E_\nu} \times \dfrac{V}{R^3} & \text{Furry process (original)} \end{cases}$$

$$E_\nu \sim \frac{hc}{R} \sim 100 \text{ MeV}$$

$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}} \sim \frac{\Delta^2\ E_\nu^4}{Q^6} >> 1 \qquad \text{[old estimate]}$$



$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}} \sim \frac{\Delta^2\ E_\nu^4}{Q^6} >> 1\ ???\ \text{[old estimate]}$$



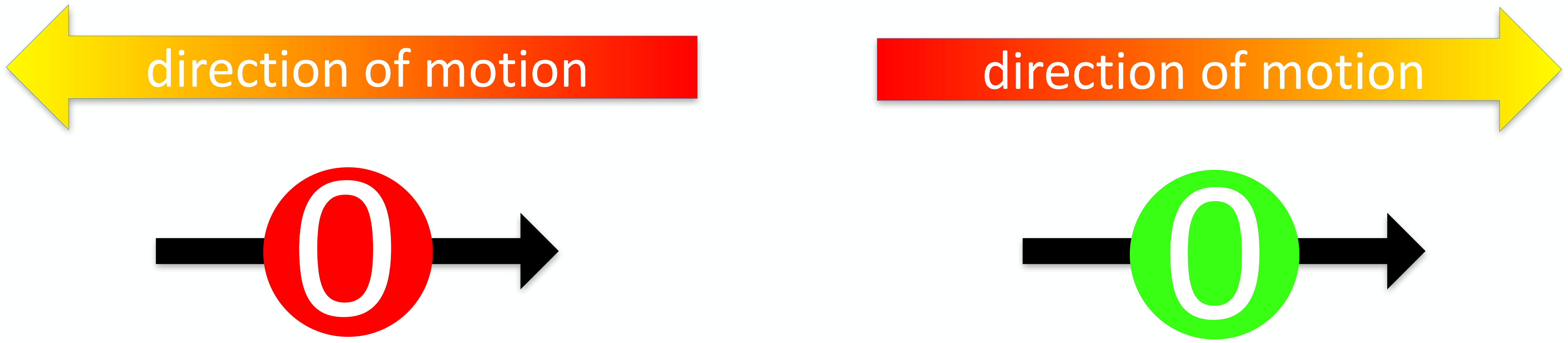
$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}} \sim \frac{\Delta^2\ E_\nu^4}{Q^6} >> 1 \ ??? \ \text{[old estimate]}$$

method 4

neutrinoless double beta decay

(Majorana, V-A interactions & Greuling-Whitten)

Majorana neutrinos in V-A context

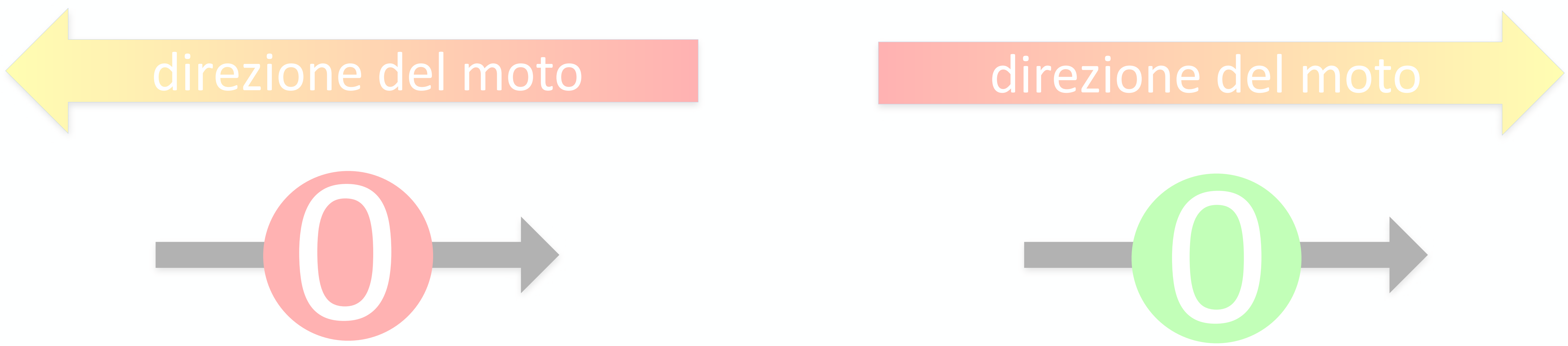


parallel/antiparallel means neutrino/antineutrino

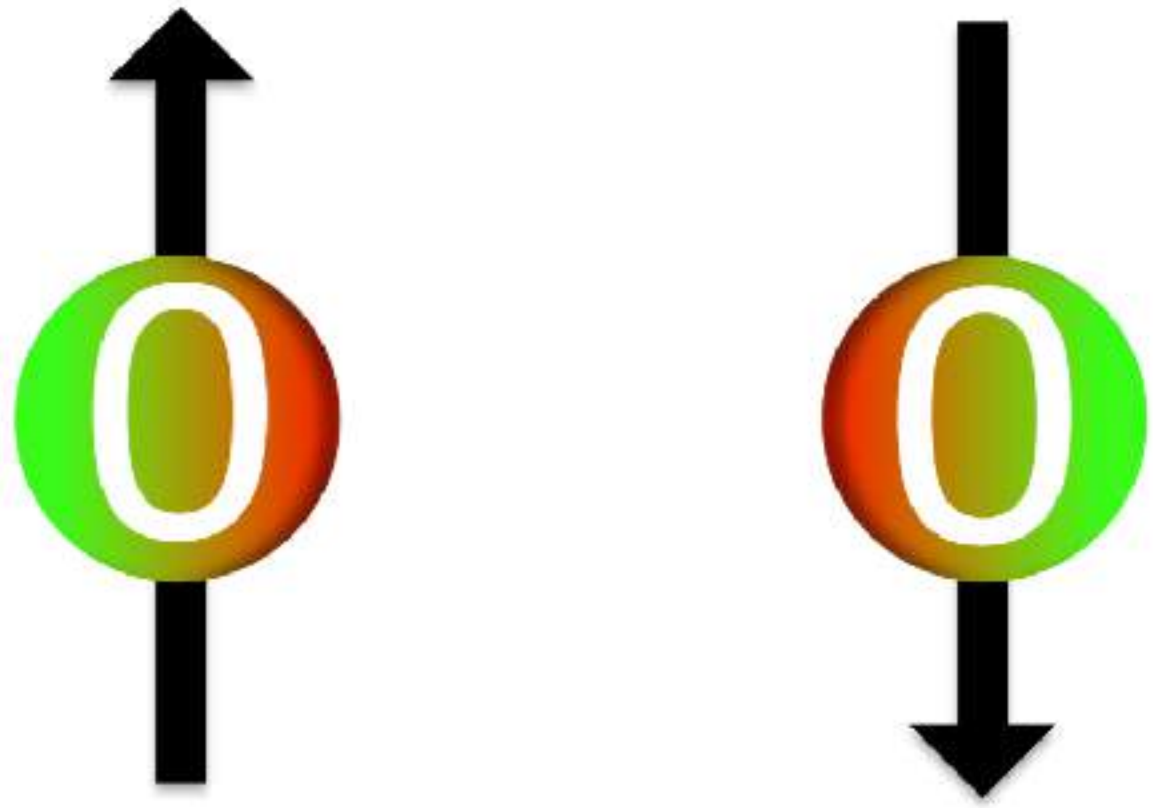
Majorana neutrinos in V-A context

parallel/antiparallel means neutrino/antineutrino

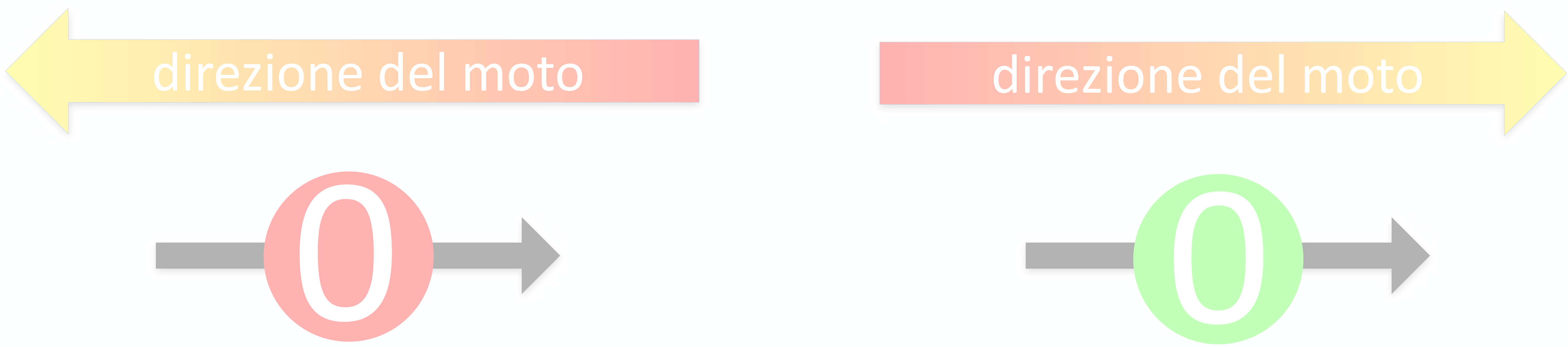
Majorana neutrinos in V-A context



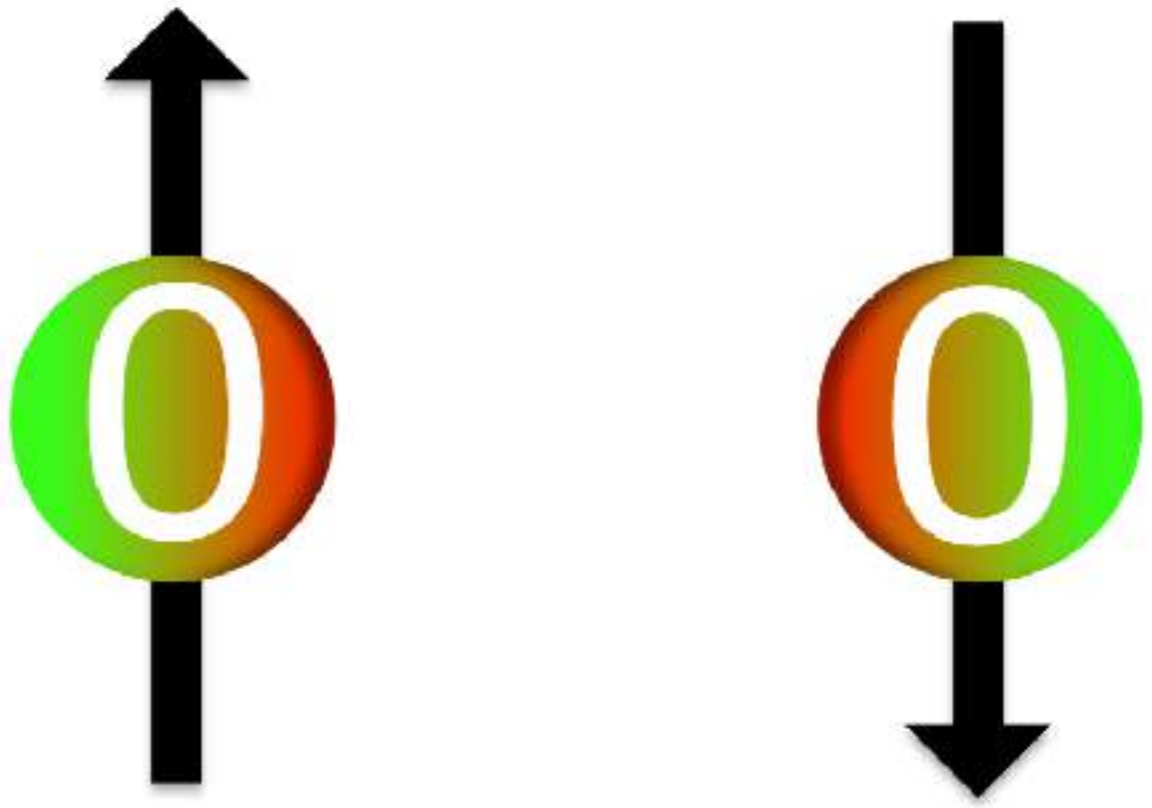
but in the rest system they seem to be the same!



Majorana neutrinos in V-A context

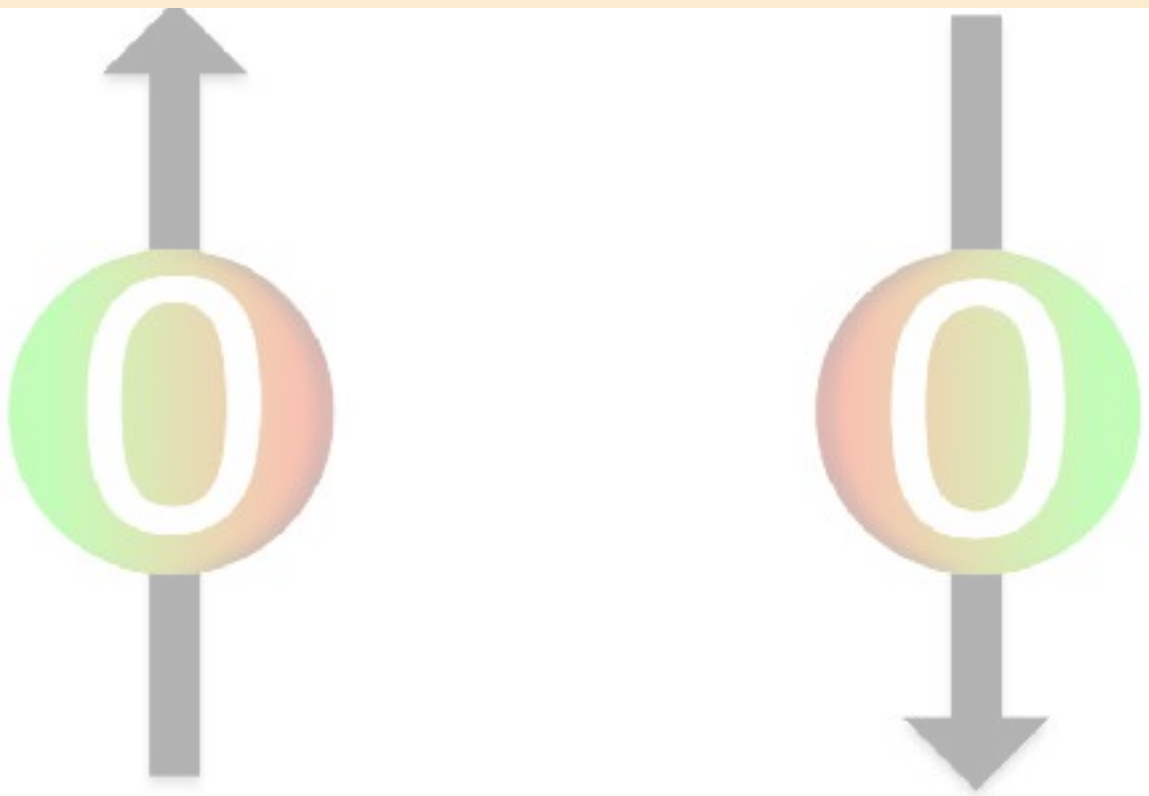


Majorana hypothesis: neutrino is matter and antimatter

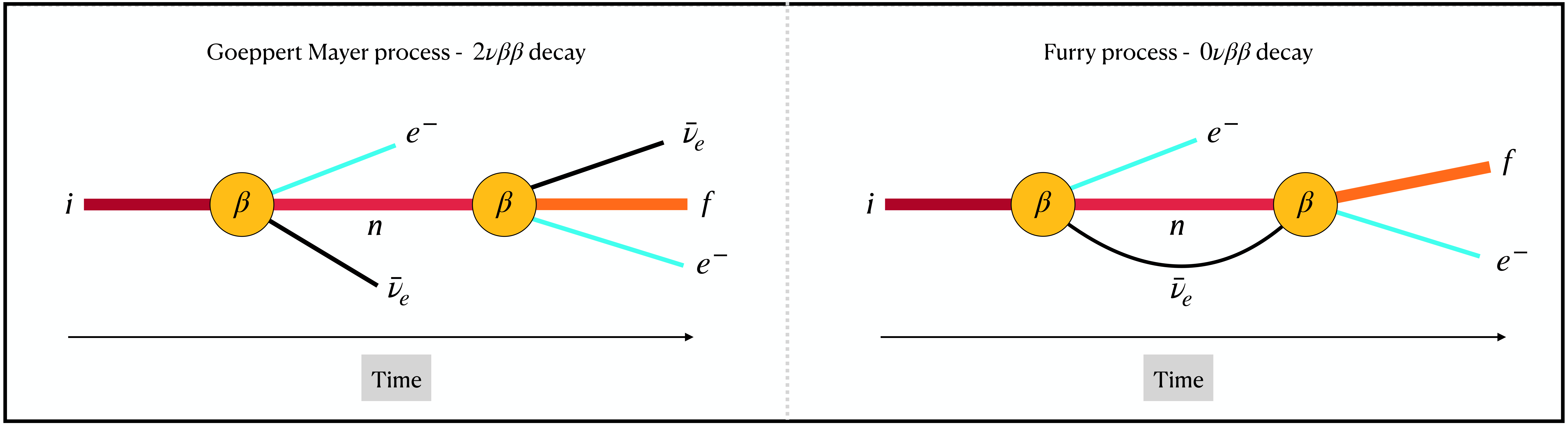


Majorana neutrinos in V-A context

all lepton number violating effects have to be $\propto m_\nu$

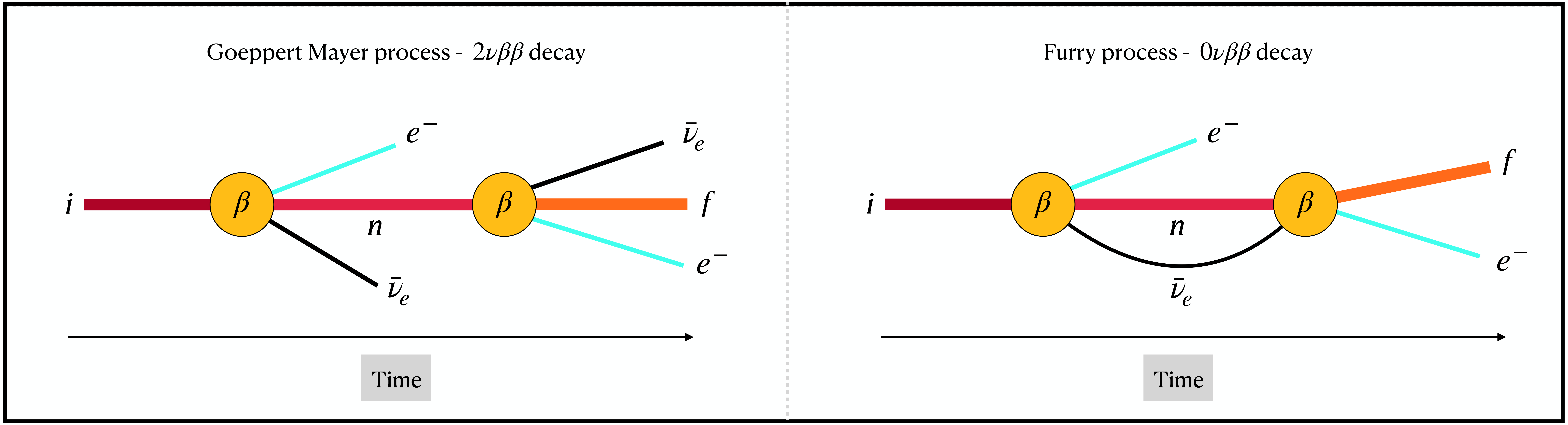


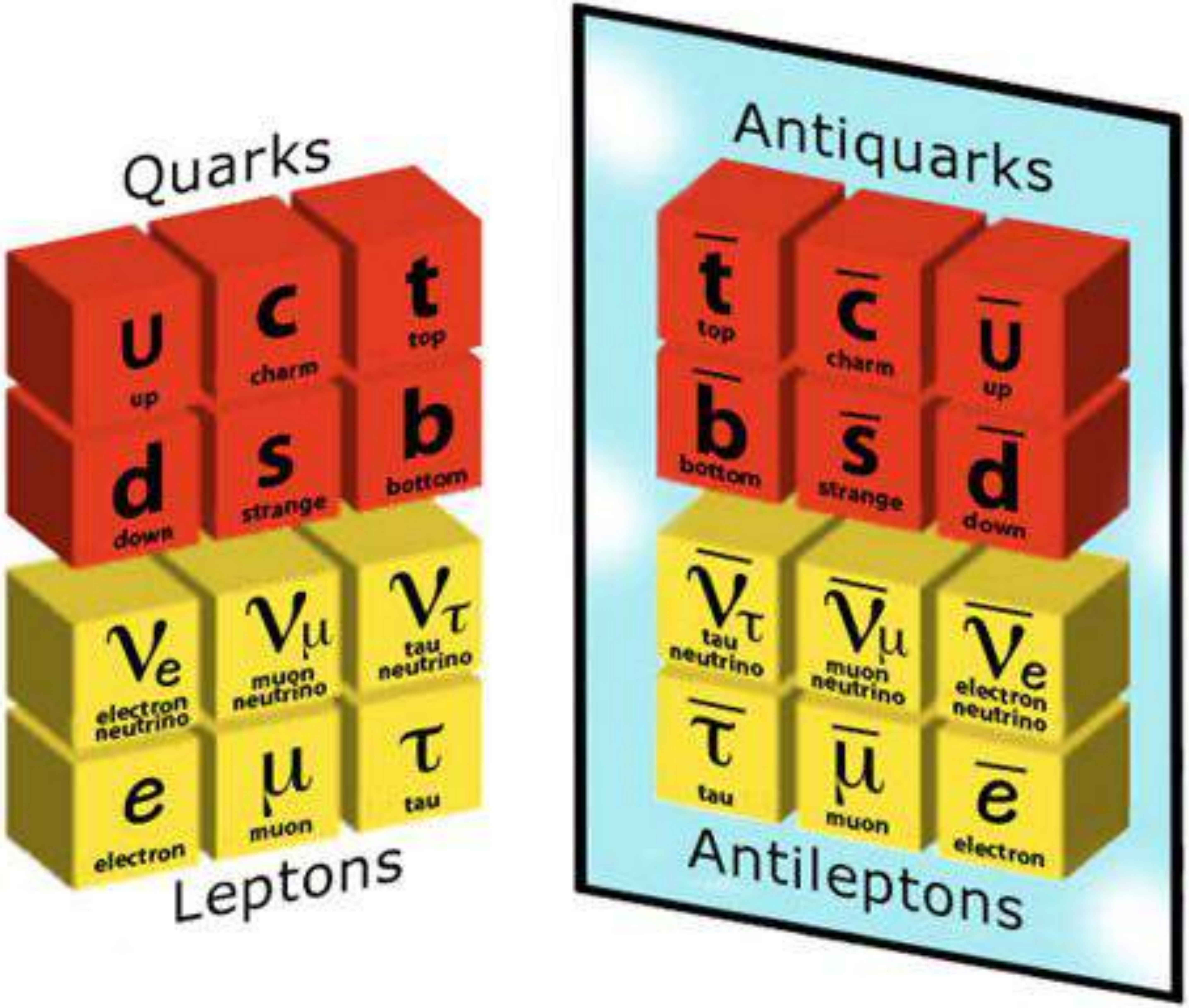
$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}} \sim \frac{\Delta^2 \; E_\nu^2 \; (m_\nu c^2)^2}{Q^6} \quad \text{[current estimate]}$$



$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}} \sim \frac{\Delta^2 \; E_\nu^2 \; (m_\nu c^2)^2}{Q^6} \quad \text{[current estimate]}$$

<< **1 !!!**





Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
ν_e
electron neutrino
ν_μ
muon neutrino
ν_τ
tau neutrino
e
electron
μ
muon
τ
tau
Leptons
Antiquarks
$\bar{t}$
top
$\bar{c}$
charm
$\bar{u}$
up
$\bar{b}$
bottom
$\bar{s}$
strange
$\bar{d}$
down
$\bar{\nu}_\tau$
tau neutrino
$\bar{\nu}_\mu$
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$\bar{\nu}_e$
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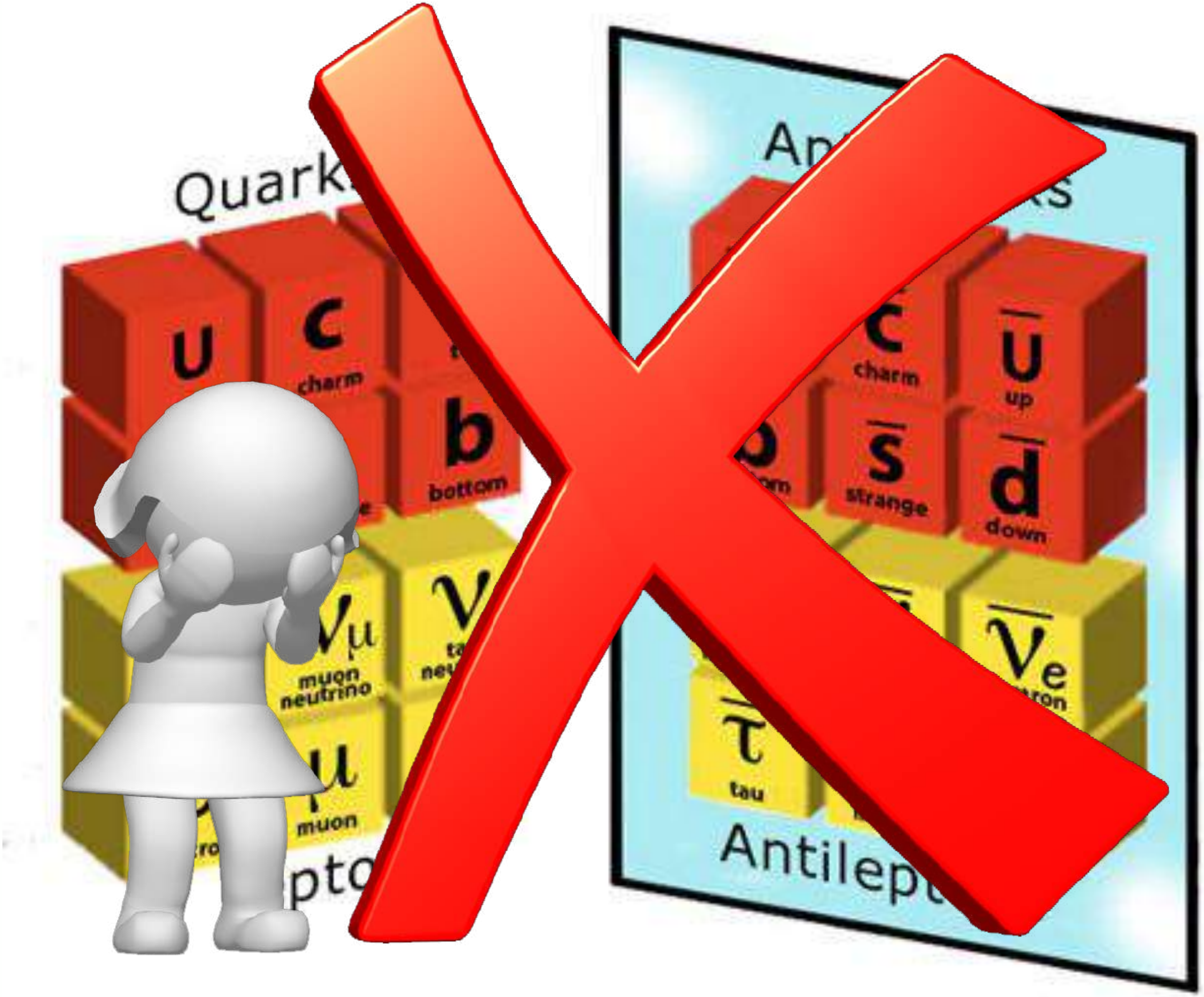
$\bar{\mu}$
muon
$\bar{e}$
electron
Antileptons


u
c
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tau

recall

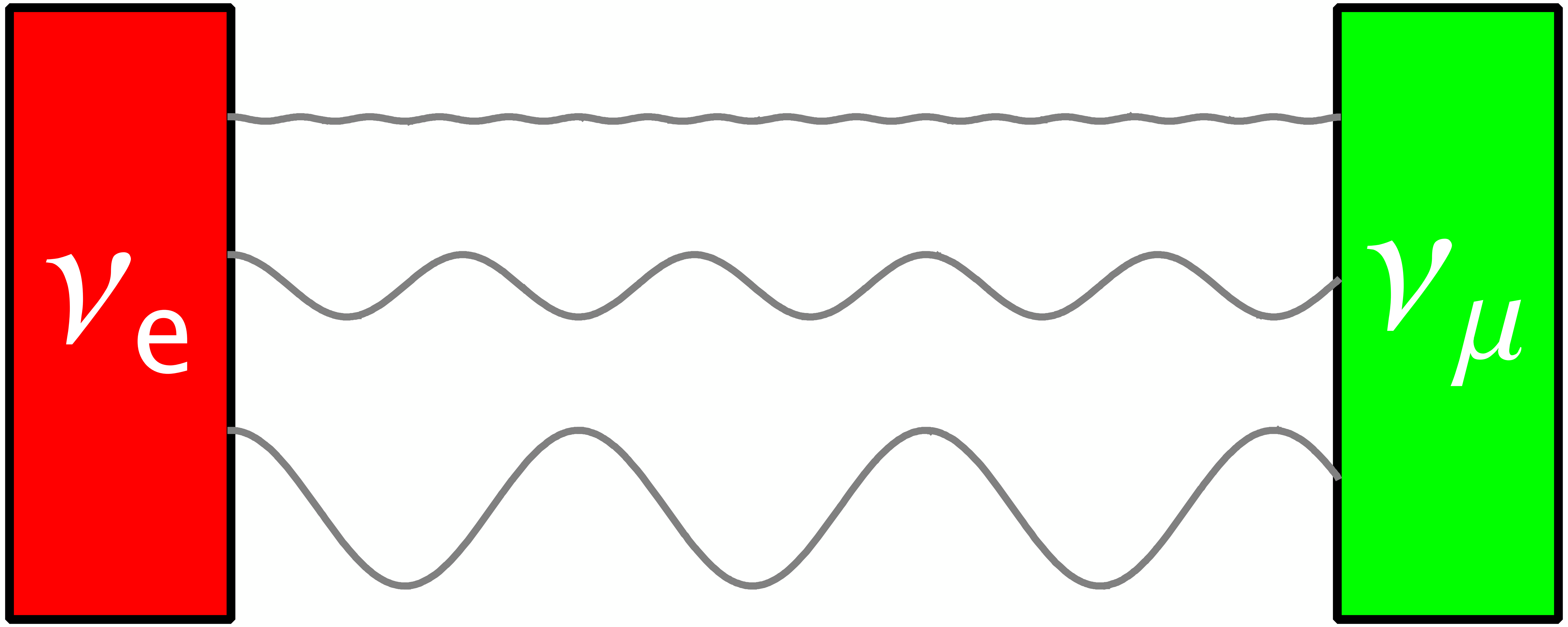
idea and formalism of neutrino trasmutation

(Pontecorvo Sakata method 1957-1967)

Neutrino oscillations (Pontecorvo 57, Sakata et al 62)

Each neutrino is produced as a mixture of 3 waves, each describing a particle with a different mass.

Their different phase velocities force the neutrino to change nature during propagation.



the hypothesis of leptonic mixing

$$\nu_\ell = \mathbf{U}_{\ell i}\, \nu_i \text{ with } \ell = e, \mu, \tau$$

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$$\mathscr{L}_{int} \ni \frac{g}{\sqrt{2}} W^a\ \bar{\ell}\gamma_a \nu_{\ell \mathrm{L}} = \frac{g}{\sqrt{2}} \mathbf{U}_{\ell i}\ W^a\ \bar{\ell}\gamma_a \chi_{i\mathrm{L}}$$

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$$\mathscr{L}_{mass} = -\frac{m_i}{2}\left[\nu_i^t \gamma^0 \nu_i + \nu_i^\dagger \gamma^0 \nu_i^*\right] = -\frac{1}{2}\left[\nu_\ell^t \mathbf{m}_{\ell\ell'} \gamma^0 \nu_{\ell'} + \nu_\ell^\dagger \mathbf{m}^*_{\ell\ell'} \gamma^0 \nu^*_{\ell'}\right]$$

$$\mathbf{m}_{\ell\ell'} = \sum_i \mathbf{U}^*_{\ell i}\, m_i\, \mathbf{U}^*_{\ell' i}$$

the standard leptonic mixing matrix

Pontecorvo-Maki-Nakagawa-Sakata's

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \quad (2.38)$$

where $s_{ij}, c_{ij} \equiv \sin\theta_{ij}, \cos\theta_{ij}$ and where the angles lie in the first quadrant whereas the phase δ is generic, $\delta \in [0, 2\pi)$. Note the usage of the same phase convention and parameterization of the quark (CKM) mixing matrix even if, of course, the values of the parameters are different.

fields and states for ultrarelativistic motions

the case of neutrino oscillations

$$\nu_\ell(x) = U_{\ell i}\, \nu_i(x)$$

$$|\nu_\ell\rangle = U^*_{\ell i} |\nu_i\rangle$$

$$|\bar{\nu}_\ell\rangle = U_{\ell i} |\bar{\nu}_i\rangle$$

proof

the mass states (index i) are also summed

$$\nu_\ell = \mathbf{U}_{\ell i}\ P_\mathrm{L} \sum_s \left[\mathbf{a}_{i,s} \psi_{i,s}(\overrightarrow{x}) + \mathbf{a}^*_{i,s} \psi^*_{i,s}(\overrightarrow{x}) \right]$$

proof

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$$\approx \mathbf{U}_{\ell i} \sum_{\overrightarrow{p}} \left[\mathbf{a}_{i,\overrightarrow{p},-}\psi_{\overrightarrow{p},-}(\overrightarrow{x}) + \mathbf{a}^*_{i,\overrightarrow{p},+}\psi^*_{\overrightarrow{p},+}(\overrightarrow{x})\right]$$

proof

the mass states (index i) are also summed

$$\nu_\ell = \mathbf{U}_{\ell i}\, P_L \sum_s \left[\mathbf{a}_{i,s}\psi_{i,s}(\vec{x}) + \mathbf{a}^*_{i,s}\psi^*_{i,s}(\vec{x})\right]$$

$$\approx \mathbf{U}_{\ell i} \sum_{\vec{p}} \left[\mathbf{a}_{i,\vec{p},-}\psi_{\vec{p},-}(\vec{x}) + \mathbf{a}^*_{i,\vec{p},+}\psi^*_{\vec{p},+}(\vec{x})\right]$$

the mass is negligible in the relativistic wavefunctions

proof

the case of neutrino oscillations

thus, we extract easily the oscillators

$$a_{\ell,\vec{p},-} \approx \mathbf{U}_{\ell i}\, a_{i,\vec{p},-} \Rightarrow |\nu_\ell\rangle = U^*_{\ell i}|\nu_i\rangle$$

$$a^*_{\ell,\vec{p},+} \approx \mathbf{U}_{\ell i}\, a^*_{i,\vec{p},+} \Rightarrow |\bar{\nu}_\ell\rangle = U_{\ell i}|\bar{\nu}_i\rangle$$

recall

neutrino trasmutation: applications

(Pontecorvo Sakata method)

an explicit formula for the case $m_3 \gg m_1, m_2$

("single mass dominance")

state

$$|\nu_\mu\rangle = U^*_{\mu i}|\nu_i\rangle \quad \text{and} \quad |\nu_\tau\rangle = U^*_{\tau i}|\nu_i\rangle$$

amplitude

$$\langle\nu_\tau|\nu_\mu, t\rangle = [\, U_{\tau 1}U^*_{\mu 1} + U_{\tau 2}U^*_{\mu 2}\,]e^{-itE_1/\hbar} + U_{\tau 3}U^*_{\mu 3}e^{-itE_3/\hbar}$$

$$= -\,U_{\tau 3}U^*_{\mu 3}e^{-itE_1/\hbar} + U_{\tau 3}U^*_{\mu 3}e^{-itE_3/\hbar}$$

probability

$$|\langle\nu_\tau|\nu_\mu, t\rangle|^2 = 4|U^2_{\tau 3}||U^2_{\mu 3}| \times \sin^2\left[\frac{E_3 - E_1}{2\hbar}t\right]$$

an explicit formula for the case $m_3 \gg m_1, m_2$

("single mass dominance")

$$|\langle \nu_\tau | \nu_\mu, t\rangle|^2 = 4\,|U_{\tau 3}^2|\,|U_{\mu 3}^2| \times \sin^2\left[\frac{E_3 - E_1}{2\hbar} t\right]$$

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("single mass dominance")

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$$= c_{13}^4 \sin^2(2\theta_{23}) \times \sin^2\left[\frac{E_3 - E_1}{2\hbar} t\right]$$

where we have used the standard parameterization

$$(U_{e3},\ U_{\mu_3},\ U_{\tau 3}) = (s_{13} e^{-i\delta},\ c_{13}\, s_{23},\ c_{13}\, c_{23})$$

an explicit formula for the case $m_3 \gg m_1, m_2$

("single mass dominance")

$$|\langle \nu_\tau | \nu_\mu, t\rangle|^2 = 4\,|U_{\tau 3}^2|\,|U_{\mu 3}^2| \times \sin^2\left[\frac{E_3 - E_1}{2\hbar}t\right]$$

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where we have used the standard parameterization

$$(U_{e3},\ U_{\mu_3},\ U_{\tau 3}) = (s_{13}e^{-i\delta},\ c_{13}\,s_{23},\ c_{13}\,c_{23})$$

further assume that θ_{13} is in first approximation negligible, $c_{13} \to 1$

$$P_{\mu\to\tau} \approx \sin^2(2\theta_{23}) \times \sin^2\left[\frac{E_3 - E_1}{2\hbar}t\right]$$

$$E_2 - E_1 = \frac{E_2^2 - E_1^2}{E_2 + E_1} \approx \frac{(m_2c^2)^2 - (m_1c^2)^2}{2pc}$$

$p \gg mc!$

$$E_2 - E_1 = \frac{E_2^2 - E_1^2}{E_2 + E_1} \approx \frac{(m_2c^2)^2 - (m_1c^2)^2}{2pc}$$

in natural units

$$\frac{E_2 - E_1}{2} \approx \frac{\Delta m_{21}^2}{4E}$$

$$E_2 - E_1 = \frac{E_2^2 - E_1^2}{E_2 + E_1} \approx \frac{(m_2c^2)^2 - (m_1c^2)^2}{2pc}$$

$$\frac{E_2 - E_1}{2} \approx \frac{\Delta m_{21}^2}{4E}$$

in practical units

$$\frac{\Delta m_{21}^2 \times L}{4E} \approx 1.27 \frac{\Delta m_{21}^2}{\mathrm{eV}^2} \times \frac{L}{\mathrm{km}} \times \frac{\mathrm{GeV}}{E}$$

$$P_{\mu\to\tau} \approx \sin^2(2\theta_{23}) \times \sin^2\left[1.27\frac{\Delta m^2 L}{E}\right]$$

where L is in km, E in GeV, Δm^2 in eV^2

method 3

oscillations

(the only experimental proof of neutrino mass & observational evidence of BSM)

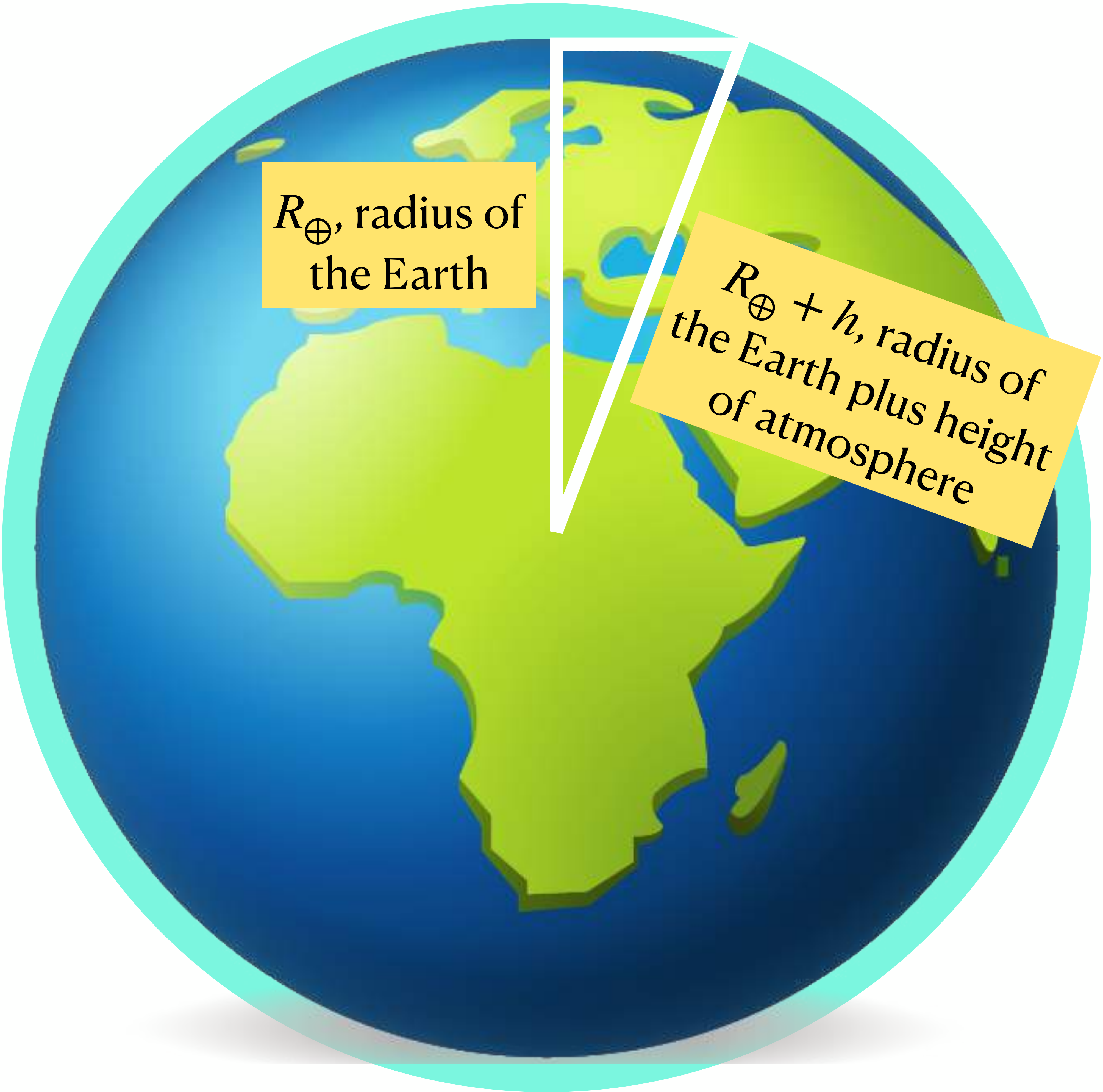
cosmic rays collide on Earth's atmosphere producing pions and secondary neutrinos of energy ~ 1 GeV

they are observed in the KamiokaNDE detector and tagged as muonic or electronic

when muon neutrinos travel long enough, say from horizontal direction, something interesting happens

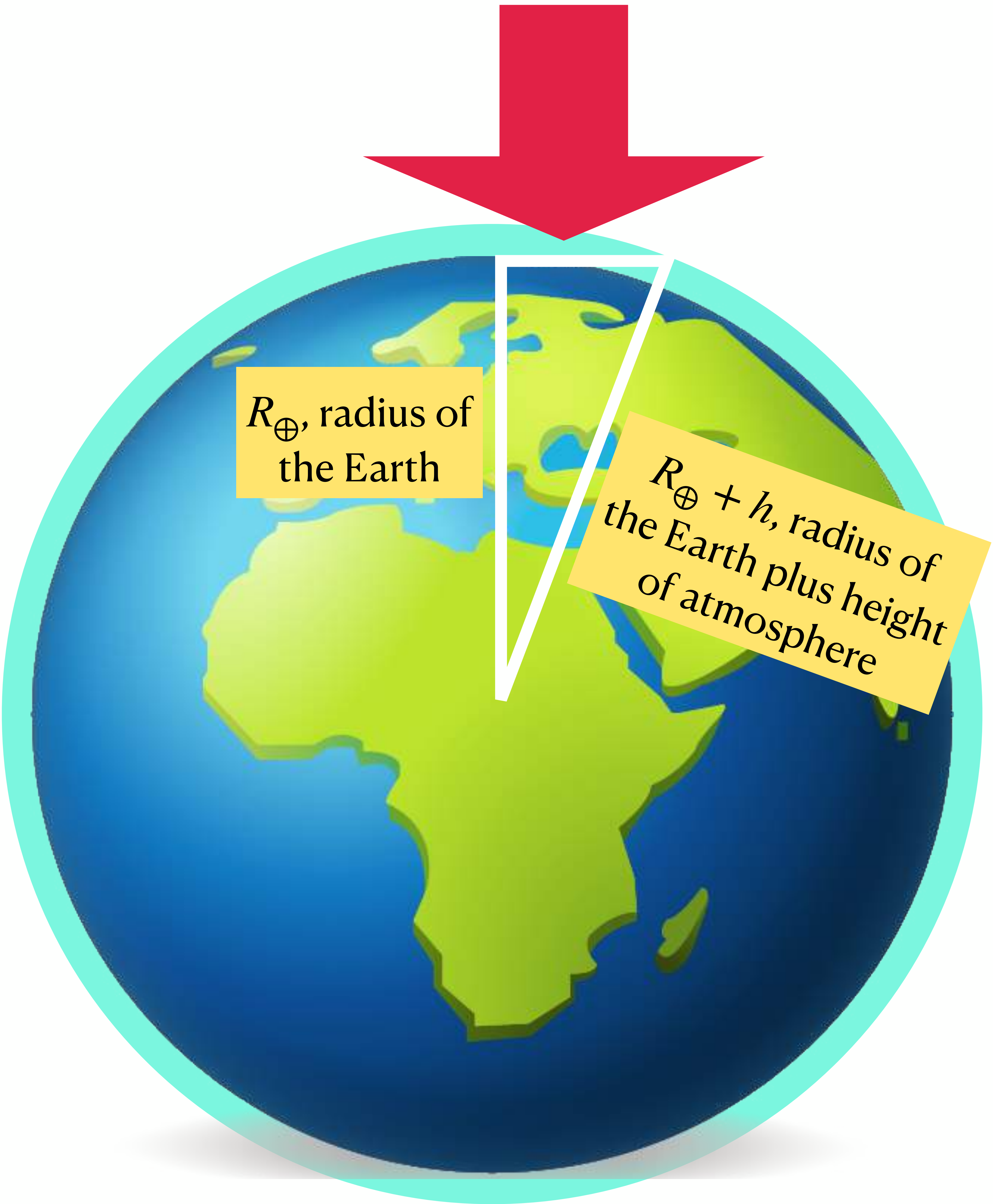
path length

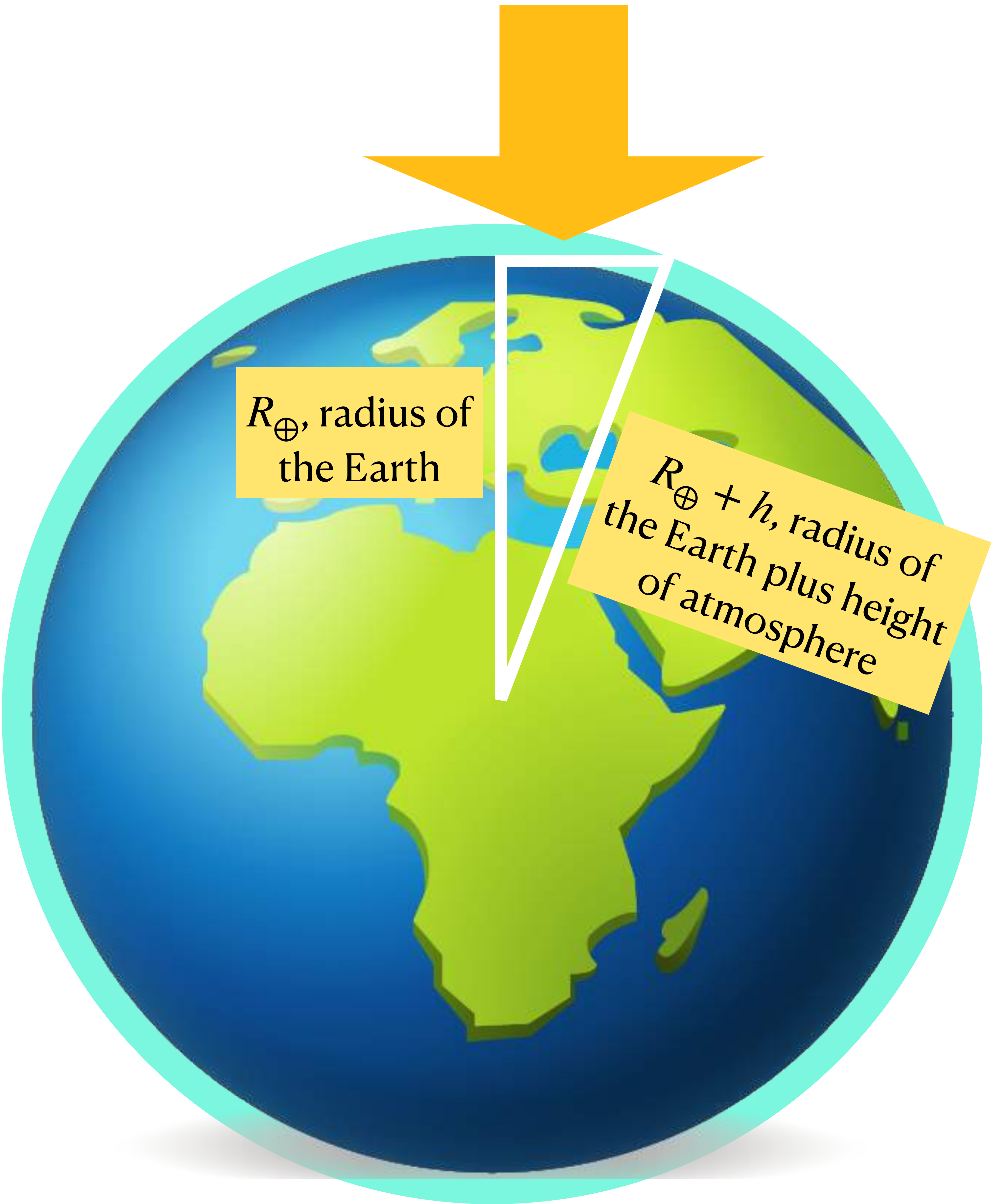
for horizontal atmospheric neutrinos



path length

for horizontal atmospheric neutrinos



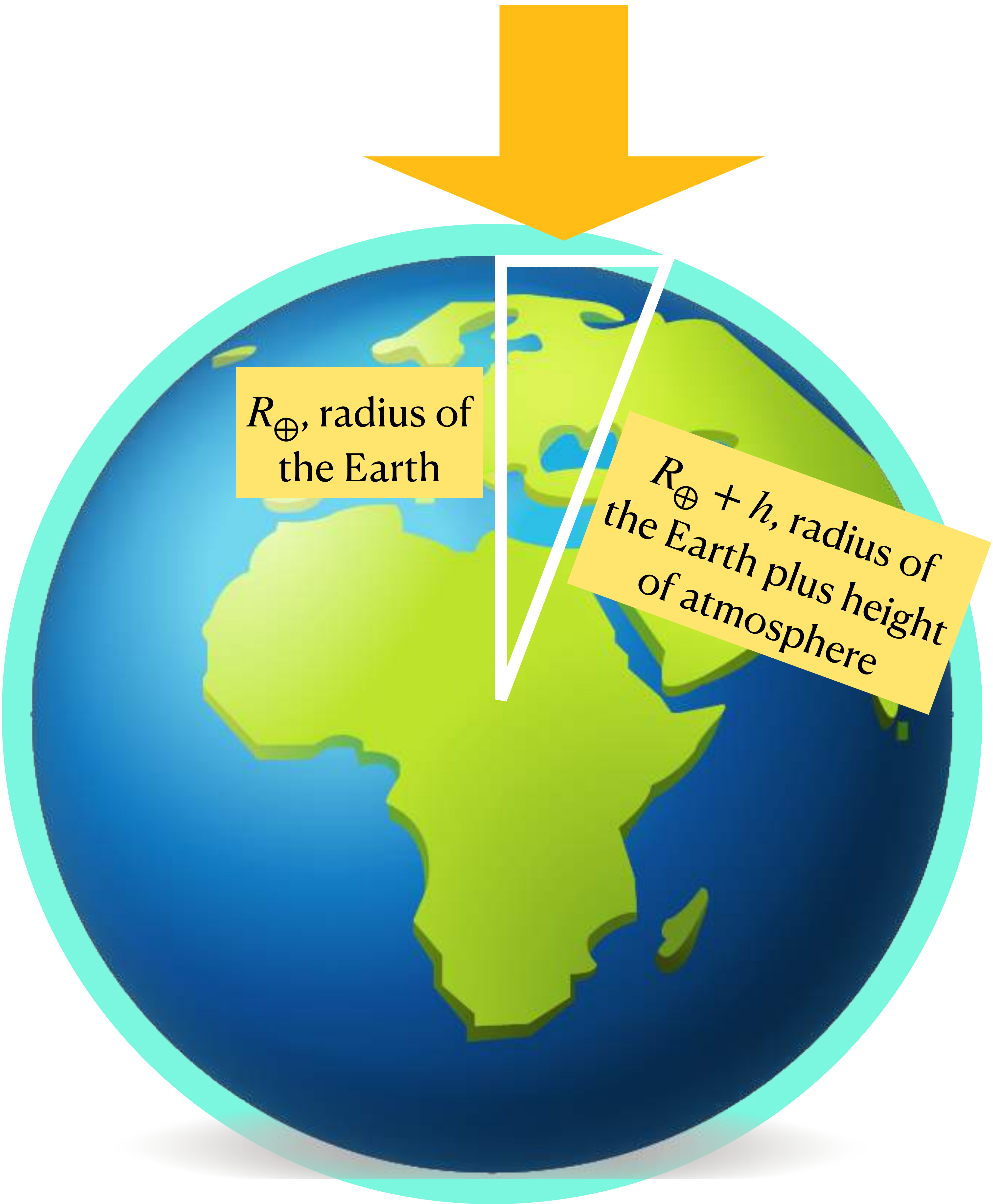


path length

for horizontal atmospheric neutrinos

- Pythagoras says

$$\sqrt{(R_\oplus + h)^2 - R_\oplus^2} \approx \sqrt{2R_\oplus h} \approx \sqrt{7{,}000 \times 30} \approx 500\ \mathrm{km}$$



path length

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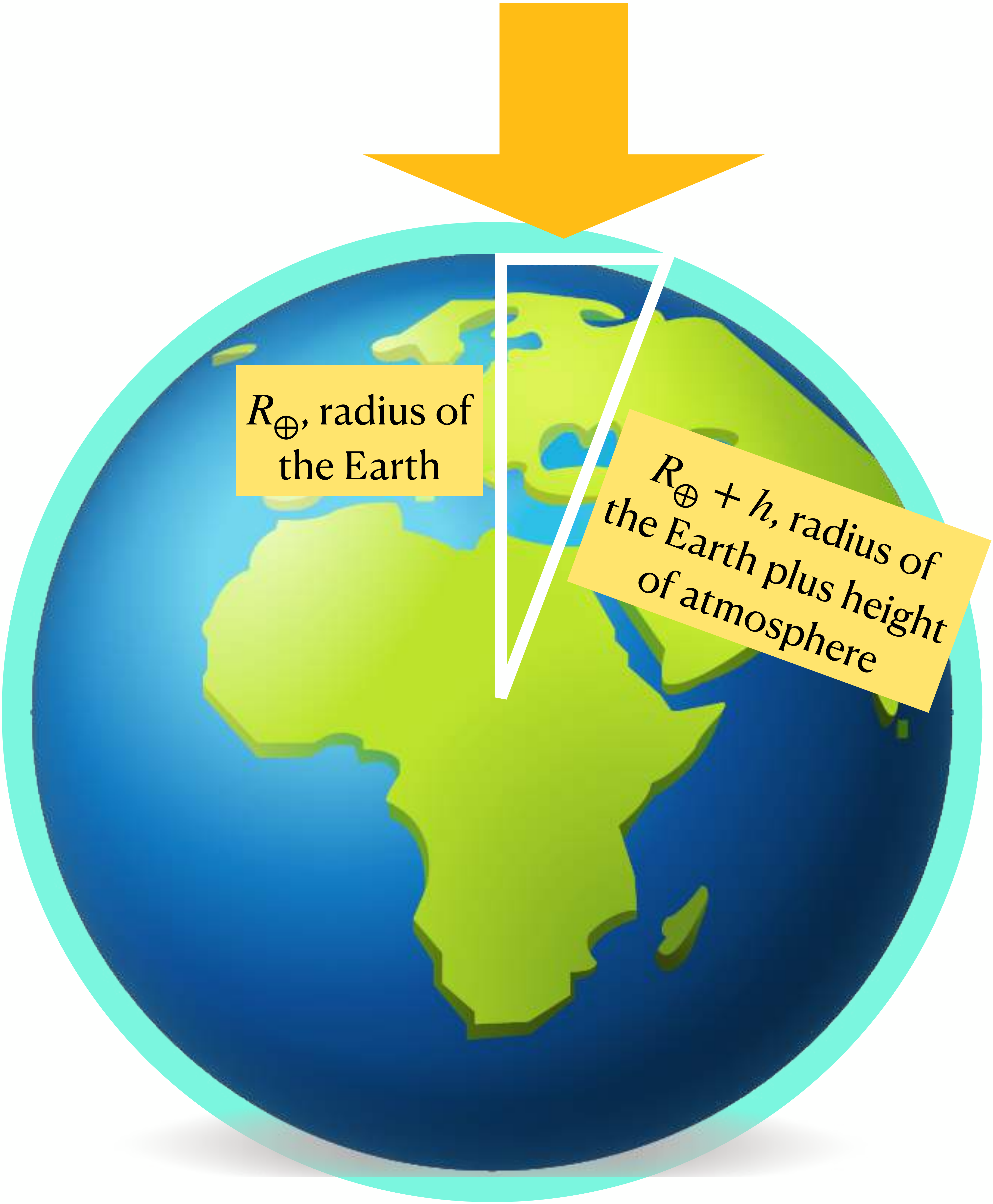
$$1.27\frac{\Delta m^2}{1\text{GeV}}500\text{ km} \sim \frac{\pi}{2}$$



path length

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- Previous formulae suggests that

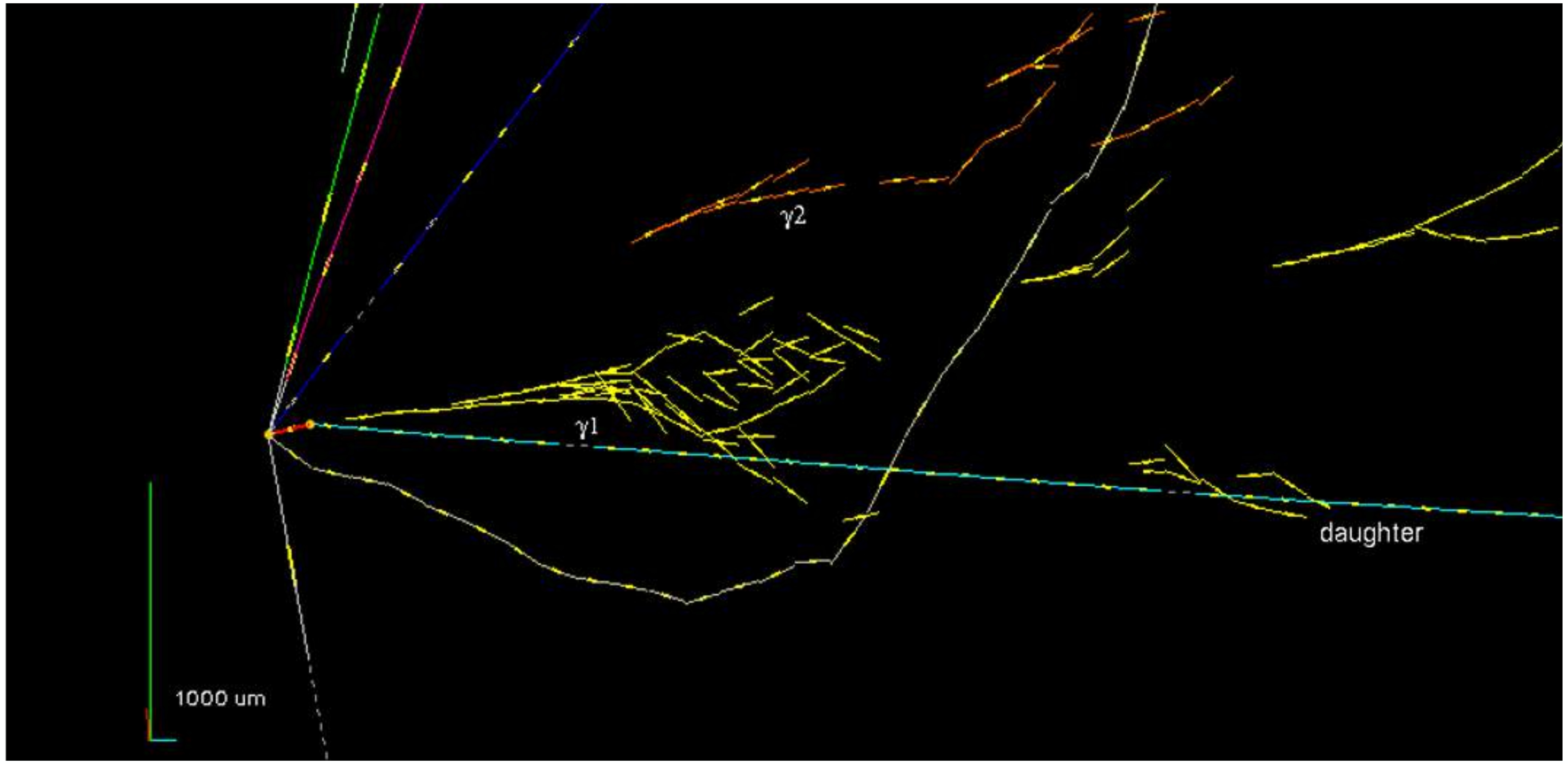
$$1.27 \frac{\Delta m^2}{1\mathrm{GeV}} 500\ \mathrm{km} \sim \frac{\pi}{2}$$

- This leads us to conclude that

$$\Delta m^2 \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2 \ \ \& \ \ \theta_{23} \sim 45^\circ$$

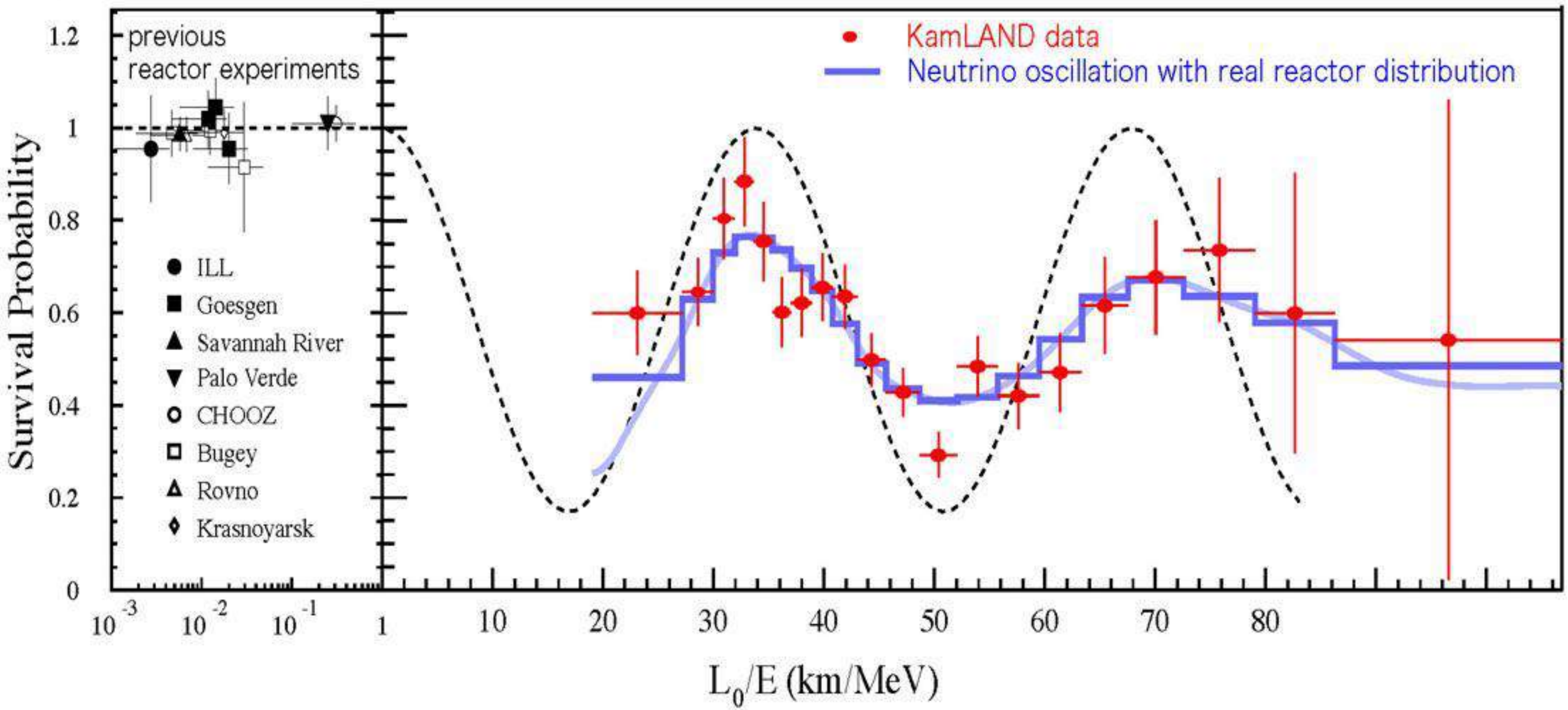
tests and results

OPERA experiment at Gran Sasso lab, planned to test the hypothesis of $\nu_\mu \to \nu_\tau$ oscillation using the CERN beam with $E_\nu \sim 20$ GeV, observed τ's after a distance of $L \sim 730$ km, supporting the interpretation of atmospheric neutrinos (2010 onward)



$$1.27 \times 8 \times 10^{-5}\ \mathrm{eV}^2 \times \frac{50\ \mathrm{km}}{3 \times 10^{-3}\ \mathrm{GeV}} \approx \frac{\pi}{2}$$

(equivalent to: $L_0/E \sim 15$ km/MeV)



global analysis

(meaning: take theory as seriously as you can, avoid biases, cherry picking & alike)

- ☑ Consider a set of **relevant experiments** (=informative on neutrino oscillation)
- ☑ Choose a **hypothesis** - e.g., oscillations with 3,4,5... νs; non standard effects; etc
- ☑ **Model the experiments,** their features & responses (background, efficiency, etc)
- ☑ Include the effect of **oscillations**, assigning free parameters such as Δm^2, θ_{ij}, δ_{CP}
- ☑ Build some **statistical indicator** (likelihood or χ^2) to compare parameterised models & data. Minimize the χ^2 and find the error budgets. Report the outcomes.

result of the newest global analyses

NuFit-6.0: updated global analysis of three-flavor neutrino oscillations

Regular Article - Theoretical Physics | Open access | Published: 30 December 2024

Ivan Esteban, M. C. Gonzalez-Garcia, Michele Maltoni, Ivan Martinez-Soler, João Paulo Pinheiro & Thomas Schwetz

Abstract

We present an updated global analysis of neutrino oscillation data as of September 2024. The parameters θ_{12}, θ_{13}, Δm^2_{21}, and $|\Delta m^2_{3\ell}|$ ($\ell = 1, 2$) are well-determined with relative precision at 3σ of about 13%, 8%, 15%, and 6%, respectively. The third mixing angle θ_{23} still suffers from the octant ambiguity, with no clear indication of whether it is larger or smaller than 45°. The determination of the leptonic CP phase δ_{CP} depends on the neutrino mass ordering: for normal ordering the global fit is consistent with CP conservation within 1σ, whereas for inverted ordering CP-violating values of δ_{CP} around 270° are favored against CP conservation at more than 3.6σ. While the present data has in principle $2.5-3\sigma$ sensitivity to the neutrino mass ordering, there are different tendencies in the global data that reduce the discrimination power: T2K and NOvA appearance data individually favor normal ordering, but they are more consistent with each other for inverted ordering. Conversely, the joint determination of $|\Delta m^2_{3\ell}|$ from global disappearance data prefers normal ordering. Altogether, the global fit including long-baseline, reactor and IceCube atmospheric data results into an almost equally good fit for both orderings. Only when the χ^2 table for atmospheric neutrino data from Super-Kamiokande is added to our χ^2, the global fit prefers normal ordering with $\Delta\chi^2 = 6.1$. We provide also updated ranges and correlations for the effective parameters sensitive to the absolute neutrino mass from β-decay, neutrinoless double-beta decay, and cosmology.

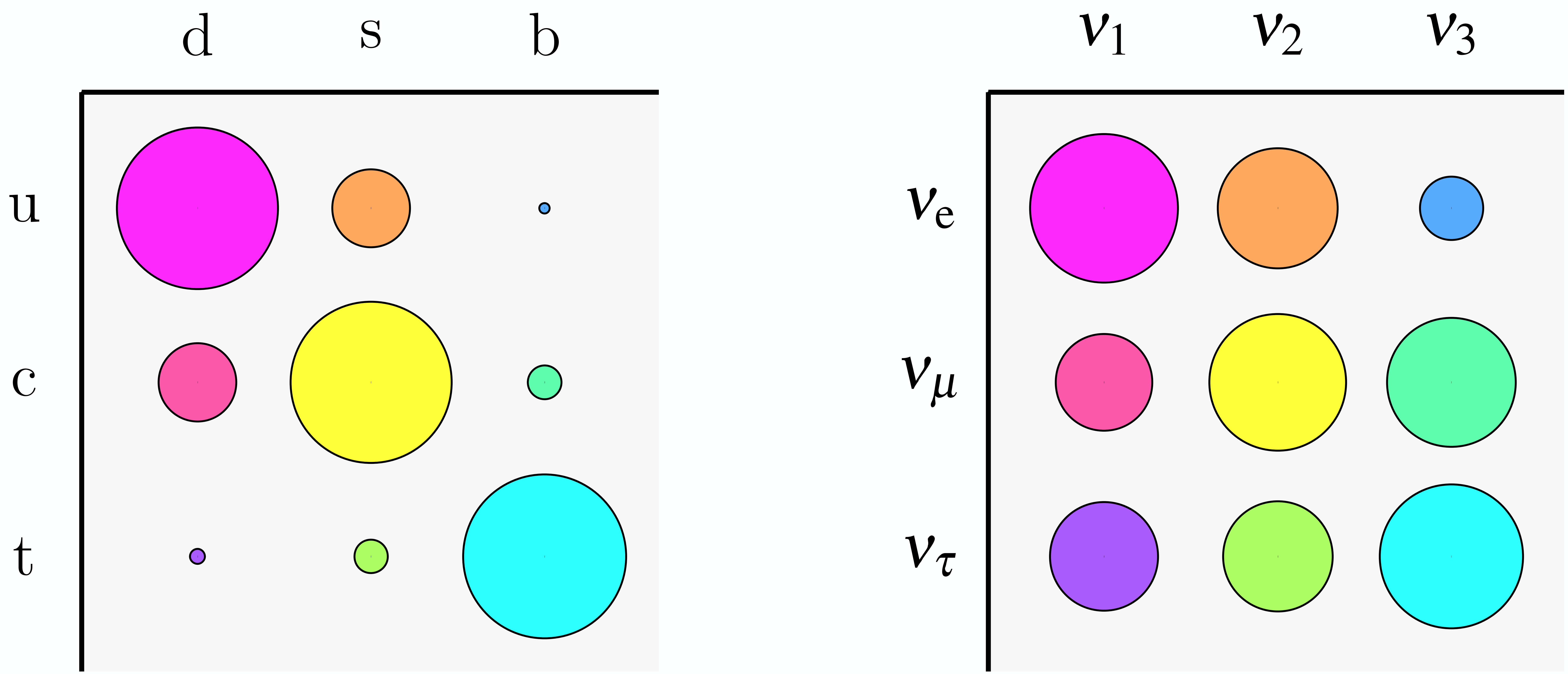
- THE "NORMAL" SPECTRUM IS KNOWN ONLY WITH 2.2σ CONFIDENCE AS
  - T2K & NOvA DO NOT AGREE PERFECTLY
  - ATMOSPHERIC χ^2 FROM SUPER-KAMIOKANDE HAS A RELEVANT WEIGTH FOR THE INFERENCE
- $\theta_{23} - 45°$ & δ_{CP} ARE ONLY POORLY DETERMINED
- A LOT OF SPACE TO PROGRESS

Neutrino masses and mixing: Entering the era of subpercent precision

Francesco Capozzi, William Giarè, Eligio Lisi, Antonio Marrone, Alessandro Melchiorri, Antonio Palazzo

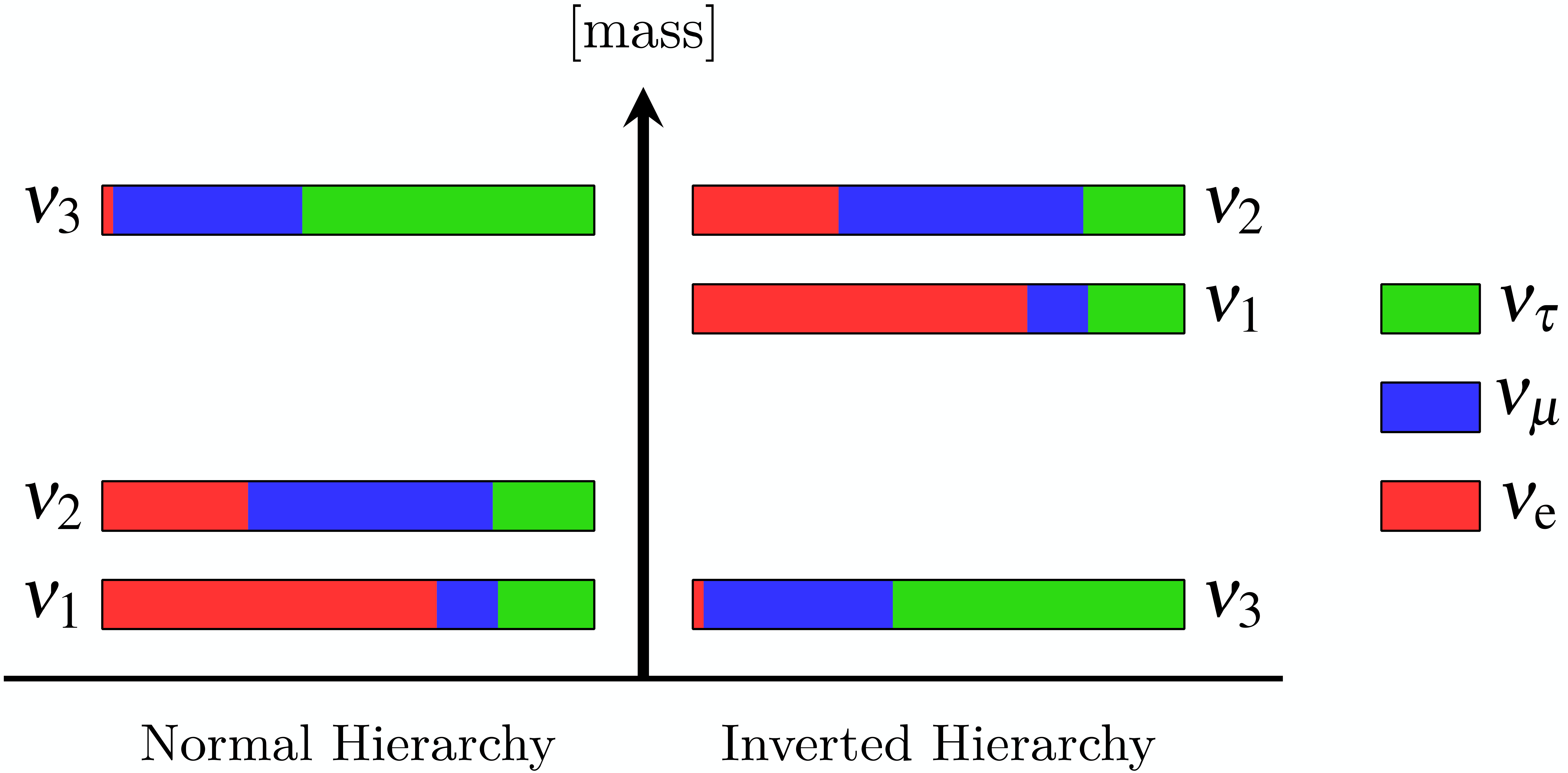
We perform an updated global analysis of the known and unknown parameters of the standard 3ν framework as of 2025. The known oscillation parameters include three mixing angles (θ_{12}, θ_{23}, θ_{13}) and two squared mass gaps, chosen as $\delta m^2 = m_2^2 - m_1^2 > 0$ and $\Delta m^2 = m_3^2 - \frac{1}{2}(m_1^2 + m_2^2)$, where $\alpha = \mathrm{sign}(\Delta m^2)$ distinguishes normal ordering (NO, $\alpha = +1$) from inverted ordering (IO, $\alpha = -1$). With respect to our previous 2021 update, the combination of oscillation data leads to appreciably reduced uncertainties for θ_{23}, θ_{13} and $|\Delta m^2|$. In particular, $|\Delta m^2|$ is the first 3ν parameter to enter the domain of subpercent precision (0.8\% at 1σ). We underline some issues about systematics, that might affect this error estimate. Concerning oscillation unknowns, we find a relatively weak preference for NO versus IO (at 2.2σ), for CP violation versus conservation in NO (1.3σ) and for the first θ_{23} octant versus the second in NO (1.1σ). We discuss the status and qualitative prospects of the mass ordering hint in the plane (δm^2, Δm^2_{ee}), where $\Delta m^2_{ee} = |\Delta m^2| + \frac{1}{2}\alpha(\cos^2\theta_{12} - \sin^2\theta_{12})\delta m^2$, to be measured by the JUNO experiment with subpercent precision. We also discuss upper bounds on nonoscillation observables. We report $m_\beta < 0.50$~eV and $m_{\beta\beta} < 0.086$~eV (2σ). Concerning the sum of neutrino masses Σ, we discuss representative combinations of data, with or without augmenting the ΛCDM model with extra parameters accounting for possible systematics or new physics. The resulting 2σ upper limits are roughly spread around the bound $\Sigma < 0.2$~eV within a factor of three. [Abridged]

overall summary on mass and mixing



areas of the circles $\propto |V_{qq'}|$ & $|U_{\ell i}|$

overall summary on mass and mixing



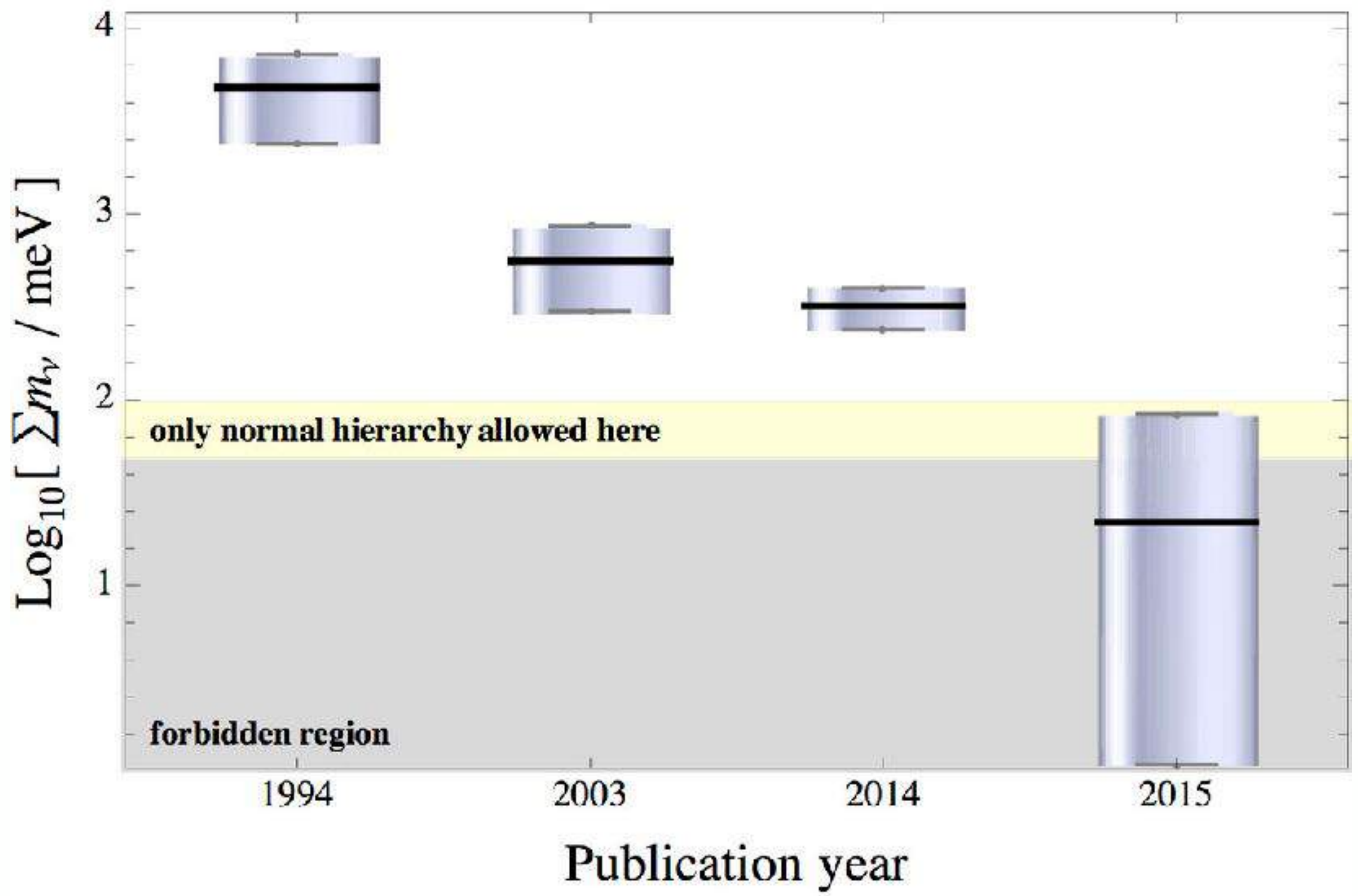
Fantini, Gallo Rosso et al, 2018

jargonic terminology: hierarchy=spectrum type=order/ordering

method 5

observational cosmology

(Gamow, Harrison-Peebles-Yu methods)



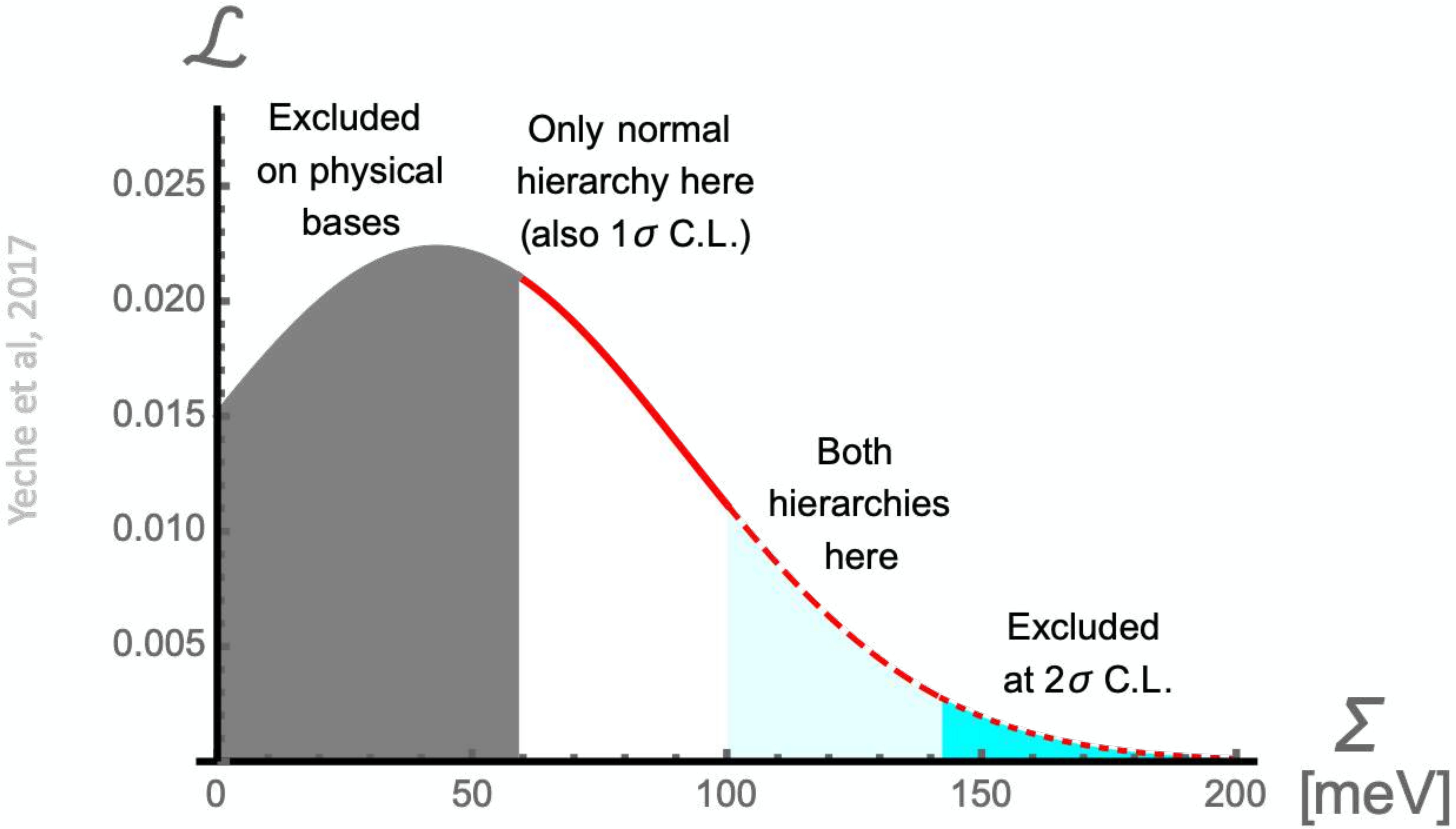
The 2015 Neutrino Mass Crash

Francesco Vissani, PhD
Laboratori Nazionali del Gran Sasso at INFN

May 27, 2015

What happened to the stock market in 1929 is happening to the values of neutrino masses right now! Indeed, the newest analyses of cosmological data point toward very small neutrino masses. If this result is correct, it will have an important impact on the interpretation of the experiments under way.

CMB is sensitive to $\Sigma = m_1 + m_2 + m_3$



DESI 2025

bounds from CMB, BAO, SNIa...

- ΛCDM+observations imply
  $\Sigma < 64$ meV at 95% near to $\sqrt{\Delta m^2_{osc}}$
- The fact that the prior $\Sigma \geq 0$ is so crucial put cosmologist on notice
- If $p/\rho = w_0 + w_a(1-a)$, bound weaken to $\Sigma < 163$ meV at 95%

[Submitted on 18 Mar 2025]

Constraints on Neutrino Physics from DESI DR2 BAO and DR1 Full Shape

The Dark Energy Spectroscopic Instrument (DESI) Collaboration has obtained robust measurements of baryon acoustic oscillations (BAO) in the redshift range, $0.1 < z < 4.2$, based on the Lyman-α forest and galaxies from Data Release 2 (DR2). We combine these measurements with external cosmic microwave background (CMB) data from Planck and ACT to place our tightest constraints yet on the sum of neutrino masses. Assuming the cosmological ΛCDM model and three degenerate neutrino states, we find $\sum m_\nu < 0.0642$ eV (95%). When accounting for neutrino oscillation constraints, we find a preference for the normal mass ordering and an upper bound of $m_l < 0.023$ eV (95%) on the lightest neutrino mass. However, we determine using frequentist and Bayesian methods that our constraints are in moderate tension with the lower limits derived from neutrino oscillations. Correcting for the physical boundary at zero mass, we report a 95% Feldman-Cousins upper bound of $\sum m_\nu < 0.053$ eV, breaching the lower limit from neutrino oscillations. Considering a more general Bayesian analysis with an effective cosmological neutrino mass parameter, $\sum m_{\nu,\mathrm{eff}}$, that allows for negative energy densities and removes unsatisfactory prior weight effects, we derive constraints that are in 3σ tension with the same oscillation limit. In the absence of unknown systematics, this finding could be interpreted as a hint of new physics not necessarily related to neutrinos. The preference of DESI and CMB data for an evolving dark energy model offers one possible solution. In the w_0w_aCDM model, we find $\sum m_\nu < 0.163$ eV (95%), resolving the neutrino tension. [Abridged]

other important results

Big Bang nucleosynthesis determines ***abundance of light elements*** such as **D & He.** This depends on the expansion rate, and therefore on the number of neutrinos.

Even the ***distribution of cosmic inhomogeneities*** at the time of decoupling, on which the observable ones depend, is modified by the number of neutrinos in equilibrium.

other important results

Big Bang nucleosynthesis determines ***abundance of light elements*** such as **D & He.** This depends on the expansion rate, and therefore on the number of neutrinos.

$$N_{\nu,\mathrm{eff}}^{\mathrm{BBN}} = 2.8 \pm 0.3$$

Even the ***distribution of cosmic inhomogeneities*** at the time of decoupling, on which the observable ones depend, is modified by the number of neutrinos in equilibrium.

$$N_{\nu,\mathrm{eff}}^{\mathrm{CMB}} = 3.0 \pm 0.2$$

LUNA 2020 & DESI 2025 agree with SM expectation. A 4^{th} neutrino is unwelcome.

method 6

relic (big-bang) neutrinos

(Weinberg, Cocco, Mangano, Messina method)

is it possible to see big bang neutrinos in lab?

neutrinos can be absorbed by a radioactive target!

(Weinberg 1962 discuss the principle; Cocco Mangano & Messina 2007 decide to try it)

$$\nu_e + {}^3\mathrm{H} \rightarrow e^- + {}^3\mathrm{He}$$

(neutrino capture on tritium)

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1. neutrino at big-bang times are

   $|\,\vec{p}\,-\rangle = \mathbf{a}^{\dagger}_{\vec{p}\,-}\,|0\rangle$ for Dirac or Majorana

2. $\Psi_{\mathrm{Dirac}} = \sum_{\vec{p}\,\lambda} \left(\mathbf{a}_{\vec{p}\,\lambda}\,\psi_{\vec{p}\,\lambda} + \mathbf{b}^{\dagger}_{\vec{p}\,\lambda}\,\psi^{*}_{\vec{p}\,\lambda} \right)$

3. $\langle 0\,|\,\mathbf{P_L}\Psi_{\mathrm{Dirac}}\,|\,\vec{p}\,-\rangle = \psi_{\vec{p}\,-}\,;$

4. $\int d^3x\,\left|\psi_{\vec{p}\,-}\right|^2 = \dfrac{1+\beta}{2}$

   same for Dirac and Majorana

$$\nu_e + {}^3\mathrm{H} \to e^- + {}^3\mathrm{He}$$

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1. neutrino at big-bang times are
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4. $\int d^3x \left|\psi_{\vec{p}-}\right|^2 = \frac{1+\beta}{2}$

   same for Dirac and Majorana

1. antineutrinos are also present
   $|\vec{p}+\rangle = \mathbf{a}^\dagger_{\vec{p}+}|0\rangle$ or $|\vec{p}+\rangle = \mathbf{b}^\dagger_{\vec{p}+}|0\rangle$
2. $\Psi_{\text{Majorana}} = \sum_{\vec{p}\lambda}\left(\mathbf{a}_{\vec{p}\lambda}\psi_{\vec{p}\lambda} + \mathbf{a}^\dagger_{\vec{p}\lambda}\psi^*_{\vec{p}\lambda}\right)$
3. $\langle 0|\mathbf{P_L}\Psi_{\text{Majorana}}|\vec{p}+\rangle = \psi_{\vec{p}+}$;
4. $\int d^3x \left|\psi_{\vec{p}+}\right|^2 = \frac{1-\beta}{2}$ but only for

   Majorana; for Dirac it is forbidden

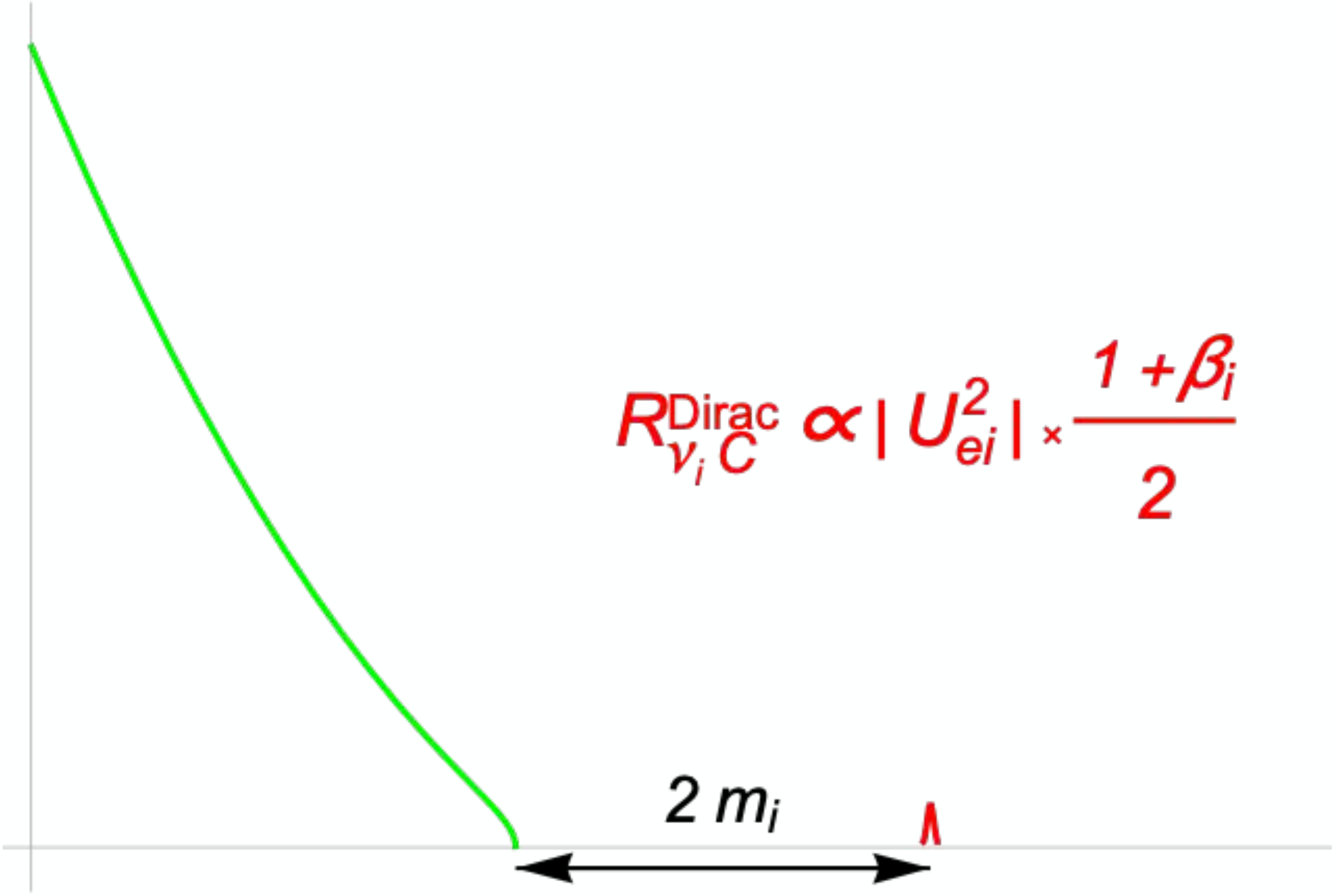
cosmic neutrino capture

and why the mass matters

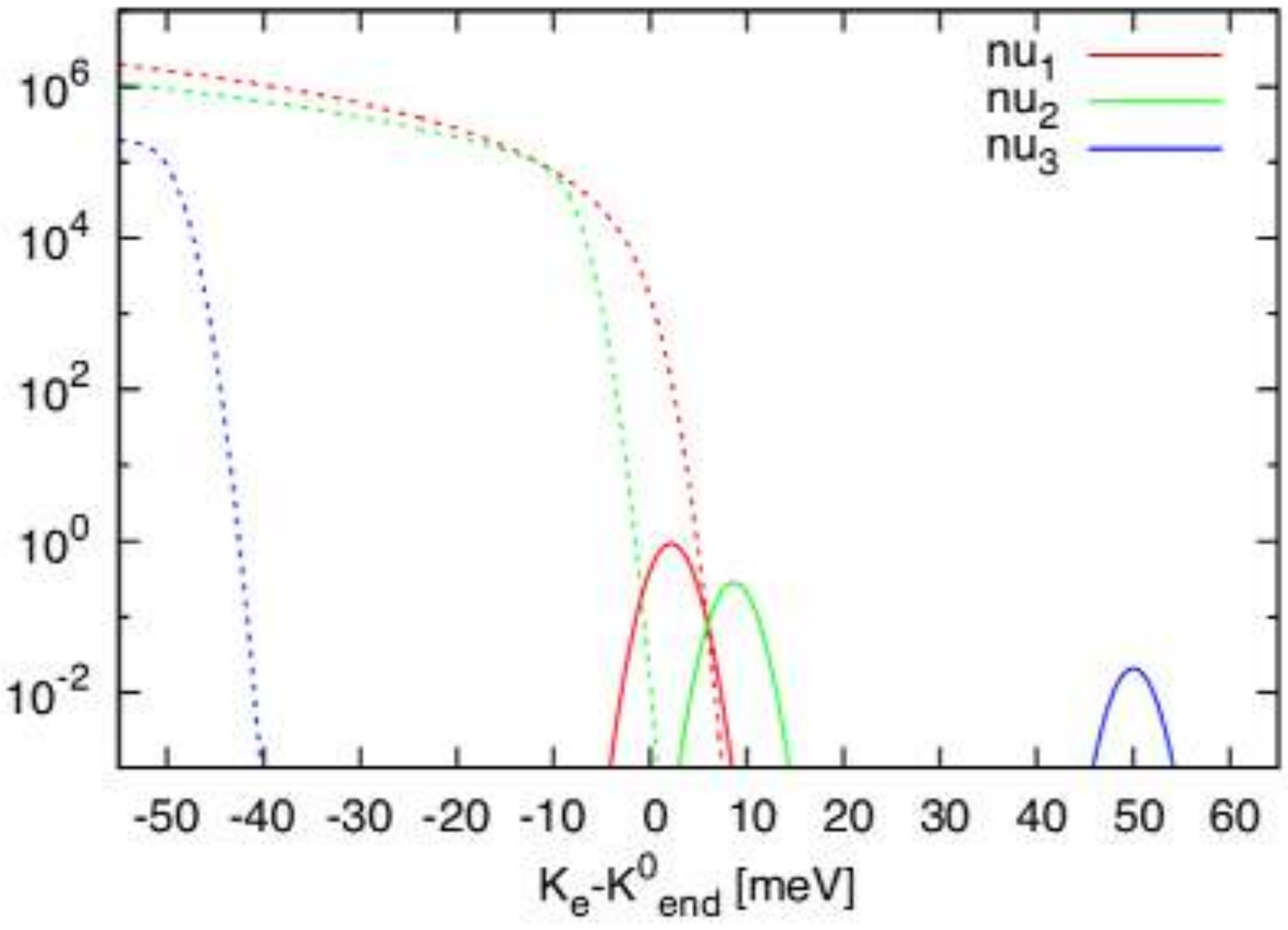
Three lines: positions depend upon m_i
intensity depends upon $|U_{ei}{}^2|$

Thus with normal hierarchy the most intense line is due to the lightest neutrino

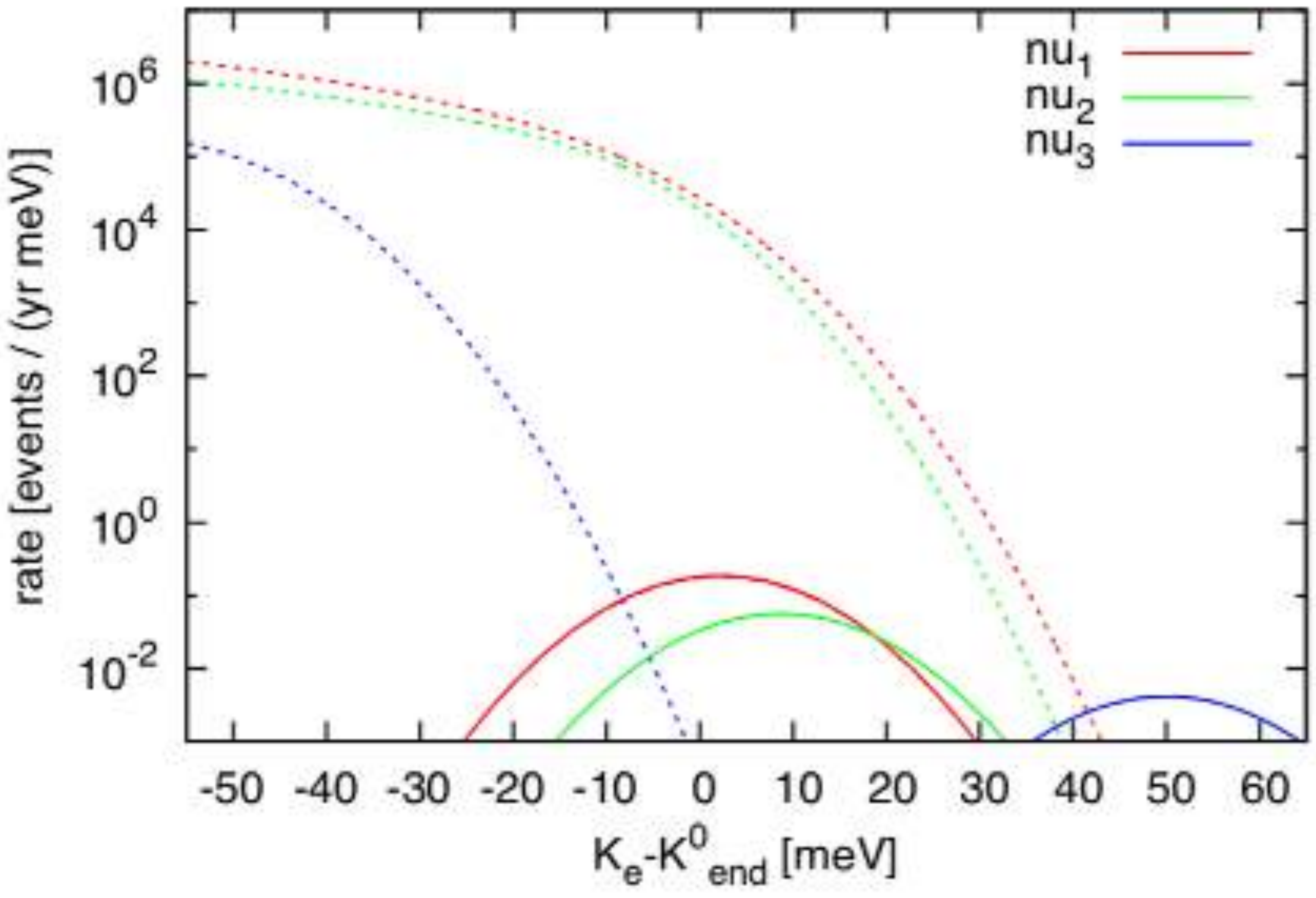
Issue of energy resolution especially with small m_i and normal hierarchy



expected spectra



$m_1 = 1$ meV and $\Delta = 4$ meV



$m_1 = 1$ meV and $\Delta = 20$ meV

let's stop here!
thanks & enjoy...


LEFT
EXIT 12
more theory
hands on
PhD student

more slides

V-A, chirality & helicity

the structure of weak (c.c.) interactions

(1956-1958)

In the mid-50s many important developments occur; let's recall the main ones

- the *conservation of parity* is challenged (Lee & Yang 56)
- the ideas that *lepton number is conserved* and *neutrinos are massless* is proposed (Salam; Landau; Lee&Yang 57) this simplifies the discussion, <u>but it is not necessary</u> (Pauli; Touschek&Radicati 57)
- Goldhaber *et al 1959* experiment supports the simpler (S.M.'s) position
- Weak interactions have **V-A** / chiral structure (Sudarshan&Marshak; Feynman&Gell-Mann 58)

first of all & very important

there is always a

$$P_L = \frac{1 - \gamma_5}{2}$$

projector in c.c. weak int.

$$H = \overrightarrow{\alpha}\ \overrightarrow{p}\, c + \beta m c^2$$

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$$\vec{\alpha} \overset{!}{=} \gamma^5 \vec{\Sigma}\ \text{(check it)} \Rightarrow$$

$$H = \vec{\alpha}\;\vec{p}\,c + \beta m c^2$$

$$\vec{\alpha} \overset{!}{=} \gamma^5 \vec{\Sigma} \text{ (check it)} \Rightarrow$$

Recall the definitions:

Dirac matrices: $\alpha^i = \gamma^0 \gamma^i$;

Chirality: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$;

Spin: $\Sigma^1 = \frac{i}{2}[\gamma^2, \gamma^3]$ and cyclic.

$$H = \vec{\alpha}\,\vec{p}\,c + \beta mc^2$$

$$\vec{\alpha} = \gamma^5 \vec{\Sigma} \text{ (check it)} \Rightarrow$$

$$H \approx \gamma^5\, \vec{\Sigma}\,\vec{p}\,c \Rightarrow$$

$$H = \vec{\alpha}\,\vec{p}\,c + \beta m c^2$$

$$\vec{\alpha} = \gamma^5 \vec{\Sigma} \text{ (check it)} \Rightarrow$$

$$H \approx \gamma^5\,\vec{\Sigma}\,\vec{p}\,c \Rightarrow$$

$$\begin{cases} P_{\mathrm{L}} H \approx -\,\vec{\Sigma}\,\vec{p}\,c \\ P_{\mathrm{R}} H \approx +\,\vec{\Sigma}\,\vec{p}\,c \end{cases}$$

Theorem:

there is tight connection between

helicity *(spin projected on momentum)*

and

chirality *(γ^5 projectors)*

$$\begin{cases} P_{\mathrm{L}} H = - \vec{\Sigma}\,\vec{p}\,c \\ P_{\mathrm{R}} H = + \vec{\Sigma}\,\vec{p}\,c \end{cases}$$

$$H = \pm \vec{\sigma} \cdot \vec{p} c$$

Weyl's Hamiltonian for massless electrons (1929), revived in 1957 for neutrinos

more on helicity-chirality connection

consider the wavefunction in **Dirac** representation:

$$\psi_\lambda(\overrightarrow{x}) = \frac{e^{i(\overrightarrow{x},\overrightarrow{p})}}{\sqrt{2V}}\, u_\lambda$$

with

$$\lambda = \pm 1 \;\; ; \;\; u_\lambda = \begin{pmatrix} \sqrt{1+\varepsilon}\;\varphi_\lambda \\ \lambda\sqrt{1-\varepsilon}\;\varphi_\lambda \end{pmatrix} \; ; \; \varepsilon = \frac{mc^2}{E}$$

more on helicity-chirality connection

evaluate the amount of "wrong" chirality

$$P_L\, u_+ \qquad \text{where} \quad P_L = \frac{1}{2}\begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix}$$

we find easily

$$P_L\, u_+ = \frac{\varepsilon}{\sqrt{1+\varepsilon}+\sqrt{1-\varepsilon}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \varphi_+$$

which is small when $p \gg mc$, being $\propto \varepsilon = (mc^2)/E$

long distance formulae (Gribov-Pontecorvo limit)

an example: what happens after long distances

$$|\nu_\ell, t\rangle = \sum_j U^*_{\ell j} |\nu_j, t\rangle = \sum_j e^{-i\frac{tE_j}{\hbar}} U^*_{\ell j} |\nu_j\rangle$$

an example: what happens after long distances

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$$\langle \nu_{\ell'} | \nu_\ell, t\rangle = \sum_j U^*_{\ell' j} e^{-i\frac{tE_j}{\hbar}} U^*_{\ell j}$$

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$$|\langle \nu_{\ell'} | \nu_\ell, t\rangle|^2 = \sum_j \left| U_{\ell' j} U^*_{\ell j} \right|^2 + \text{rapidly oscillating terms}$$

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$$P_{\ell \to \ell'} \approx \sum_{i=1}^{n} |U^2_{\ell i}|\,|U^2_{\ell' i}|$$

matter term, idea & formalism, numerics

formal amplitude in vacuum:

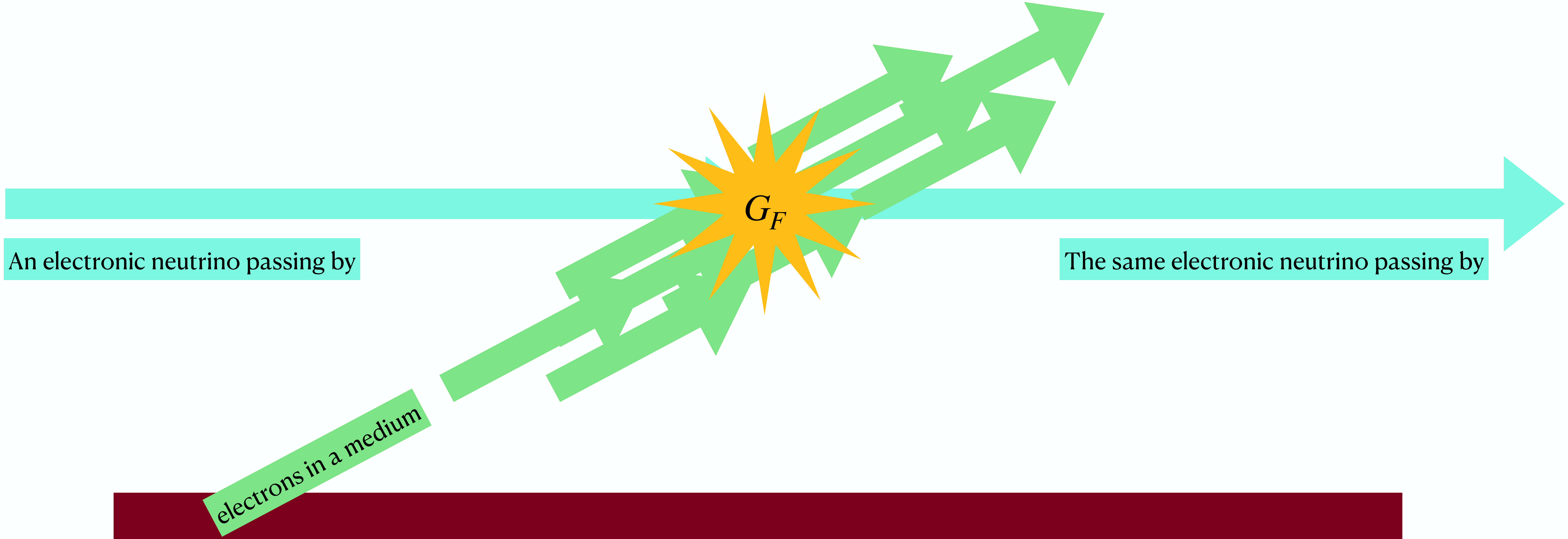
$$\mathscr{A}_{\nu_\ell \to \nu'_\ell} = U_{\ell' i} \, \mathrm{diag}\left[e^{-i\frac{tE_i}{\hbar}} \right] \, U^*_{\ell i}$$

formal amplitude in vacuum:

$$\mathscr{A}_{\nu_\ell \to \nu'_\ell} = U_{\ell' i} \,\mathrm{diag}\left[e^{-i\frac{tE_i}{\hbar}} \right] U^*_{\ell i}$$

corresponding to the hamiltonian

$$i\hbar\frac{\partial}{\partial t} = \mathbf{H}^{\mathrm{vac}}_{\ell'\ell} = U_{\ell' i} \,\mathrm{diag}\left[E_i \right] U^*_{\ell i}$$



even in absence of scattering - i.e., of momentum transfer - the hamiltonian of charged current weak interactions *modifies the overall phase of propagation of electronic neutrinos* by a term proportional to G_F and to the density of electrons in a medium

a new term for oscillations in matter

$$\mathbf{H}_{prop} = \mathbf{H}_{vac.} + \sqrt{2}G_F n_e(\overrightarrow{x})\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

a new term for oscillations in matter

$$\mathbf{H}_{prop} = \mathbf{H}^{*}_{vac.} - \sqrt{2} G_F n_e(\overrightarrow{x}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

for antineutrinos the matter term changes sign, as
the charge $J^0_{\nu_e} = \mathbf{Q} = \nu^\dagger \nu$ counts the number of particles:
$\langle \nu | \mathbf{Q} | \nu \rangle = +1$ and $\langle \bar{\nu} | \mathbf{Q} | \bar{\nu} \rangle = -1$

vacuum wavenumber

$$k = \frac{\Delta m^2}{2E} \approx \frac{2.5}{\mathrm{m}} \times \frac{\Delta m^2}{\mathrm{eV}^2} \times \frac{\mathrm{MeV}}{E}$$

vacuum wavenumber

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matter term wavenumber

$$V = \sqrt{2}\, G_\text{F}\, n_e \approx \frac{4 \times 10^{-7}}{\text{m}} \times \frac{n_e}{\text{mol/cm}^3}$$

vacuum wavenumber

$$k = \frac{\Delta m^2}{2E} \approx \frac{2.5}{\mathrm{m}} \times \frac{\Delta m^2}{\mathrm{eV}^2} \times \frac{\mathrm{MeV}}{E}$$

matter term wavenumber

$$V = \sqrt{2}\, G_{\mathrm{F}}\, n_e \approx \frac{4 \times 10^{-7}}{\mathrm{m}} \times \frac{n_e}{\mathrm{mol/cm^3}}$$

their ratio [ν1, for solar neutrinos]

$$\frac{V}{k} \approx \frac{n_e}{100\ \mathrm{mol/cm^3}} \times \frac{8 \times 10^{-5}\ \mathrm{eV}^2}{\Delta m^2} \times \frac{E}{5\ \mathrm{MeV}}$$

vacuum wavenumber

$$k = \frac{\Delta m^2}{2E} \approx \frac{2.5}{\mathrm{m}} \times \frac{\Delta m^2}{\mathrm{eV}^2} \times \frac{\mathrm{MeV}}{E}$$

matter term wavenumber

$$V = \sqrt{2}\, G_{\mathrm{F}}\, n_e \approx \frac{4 \times 10^{-7}}{\mathrm{m}} \times \frac{n_e}{\mathrm{mol/cm}^3}$$

their ratio [v2, for atmospheric neutrinos]

$$\frac{V}{k} \approx \frac{n_e}{3\ \mathrm{mol/cm}^3} \times \frac{2.5 \times 10^{-3}\ \mathrm{eV}^2}{\Delta m^2} \times \frac{E}{5\ \mathrm{GeV}}$$

a list of possibilities

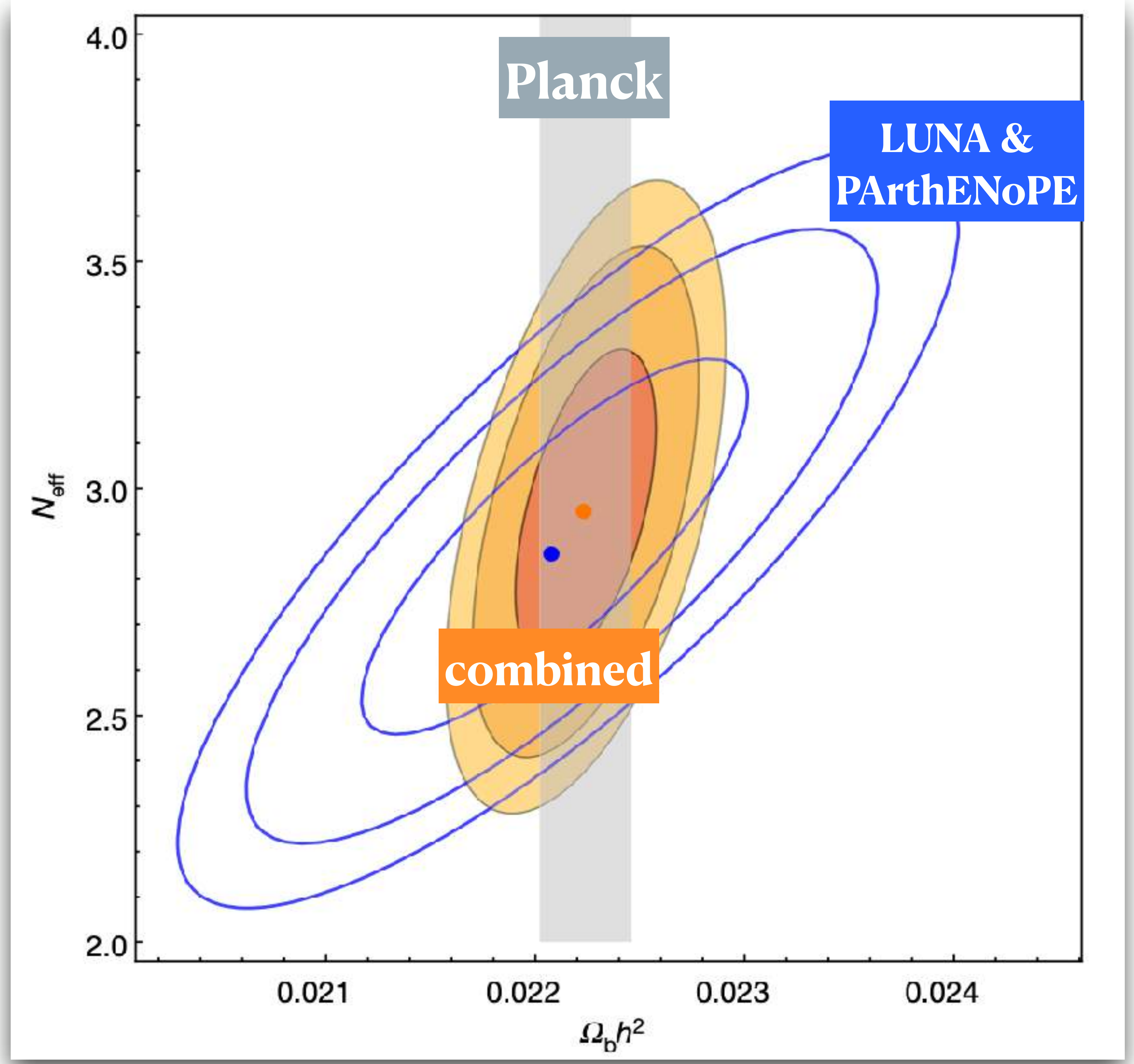
- ***survival probability of electron neutrinos from the Sun is energy dependent*** (big effect, connected to θ_{12} and Δm^2_{12}, observed)
- ***day-night effect***: the solar electron neutrinos that arrive to the detector on night are more abundant (small effect, connected to θ_{12} and Δm^2_{12}, first observations)
- ***atmospheric neutrino fluxes that pass through the Earth at some special angles*** are modified (small effect, connected to θ_{13} and Δm^2_{13}, not-so-clear evidence)

more on $N_{\nu,\mathrm{eff}}$

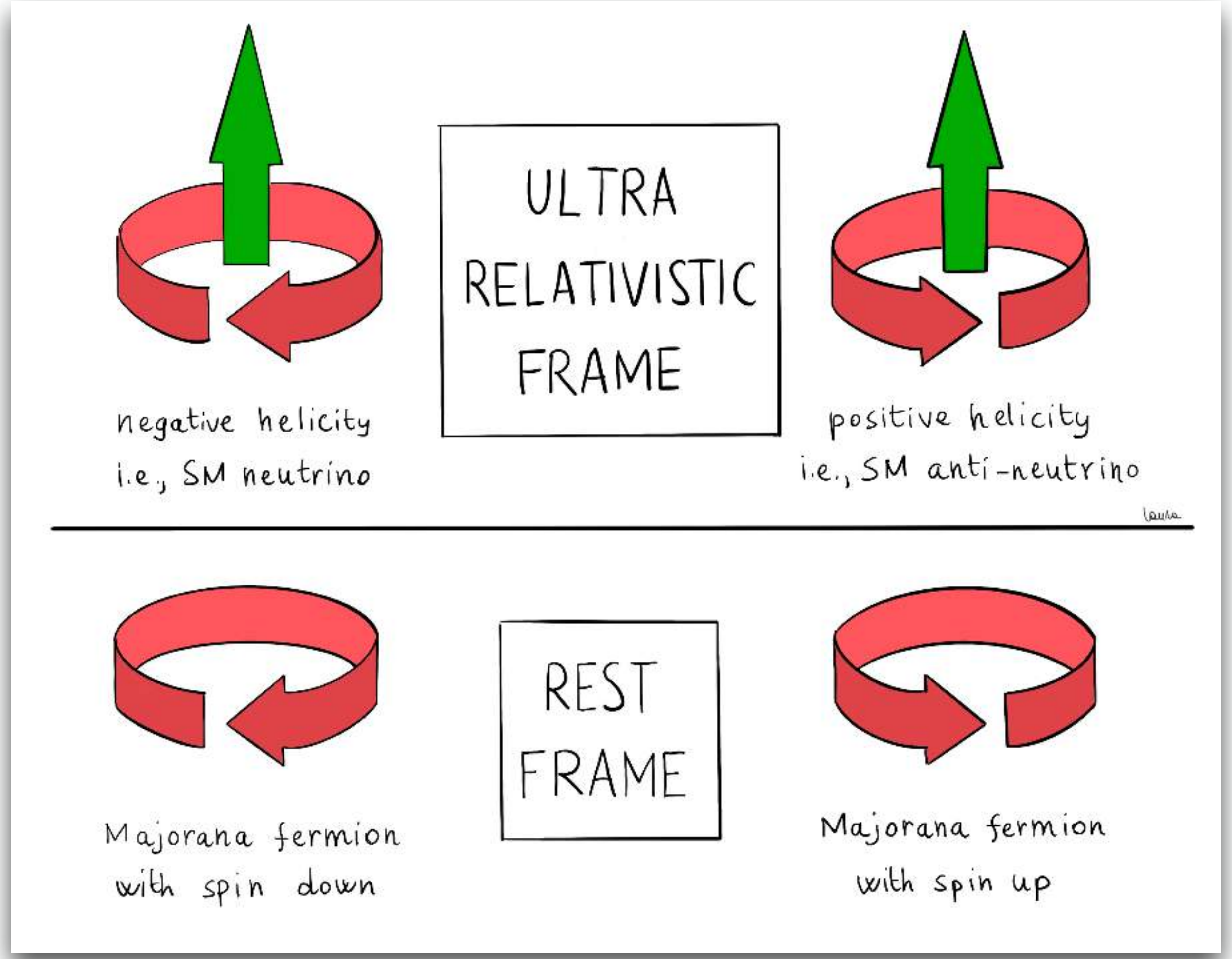
LUNA & PArthENoPE

- an improved description of deuterium dynamics in early universe conditions was obtained by LUNA
- this allowed the BBN simulations of PArthENoPE, based on Gamow's ideas, to claim a good agreement with **baryon mass fraction** $\Omega_b h^2$
- moreover this has implications on the **number of neutrinos** N_{eff} see figure

from Nature 2020



on V-A & $0\nu\beta\beta$



FV, 2023

in the fast-moving system $\nu \neq \bar{\nu}$ **and** in the rest system $\nu = \bar{\nu}$ in V-A model

Electron creation and the parameter m_{ee}

Consider the semi-leptonic Hamiltonian density leading to the emission of an electron $\mathscr{H} = \sqrt{2} G_F \boldsymbol{J}^+_\mu \boldsymbol{j}^{-\mu}$, where the leptonic current is

$$\boldsymbol{j}^-_\mu = \bar{e} \gamma_\mu \boldsymbol{\nu}_{Le} = \sum_{j=1}^{3} U_{ej}\, \bar{e} \gamma_\mu P_L \boldsymbol{\chi}_j \text{ with } \boldsymbol{\chi}_j = \boldsymbol{\chi}_j^* \tag{33}$$

where we have postulated that the neutrino mass eingestates are Majorana fields. The leptonic part of the amplitude, that describes the creation of a couple of electrons, is $\langle ee|T[\boldsymbol{j}^-_\nu(x) \boldsymbol{j}^-_\mu(y)]|0\rangle$ and it requires to evaluate the contraction

$$\langle 0|T[\boldsymbol{\nu}_{Le}(x)\, \boldsymbol{\nu}^t_{Le}(y)]|0\rangle$$

namely, an unusual type of propagator, that however is non-zero in Majorana's theory. In fact, from $\boldsymbol{\nu}_{Le} = U_{ej} P_L \boldsymbol{\chi}_j$, used above, and its transpose, written as $\boldsymbol{\nu}^t_{Le} = U_{ej}\, \bar{\boldsymbol{\chi}}_j P_L \gamma^0$, the core of the problem reduces to the calculation of an ordinary propagator, namely $\langle 0|T[\boldsymbol{\chi}_j(x)\, \overline{\boldsymbol{\chi}}_j(y)]|0\rangle$. The result is

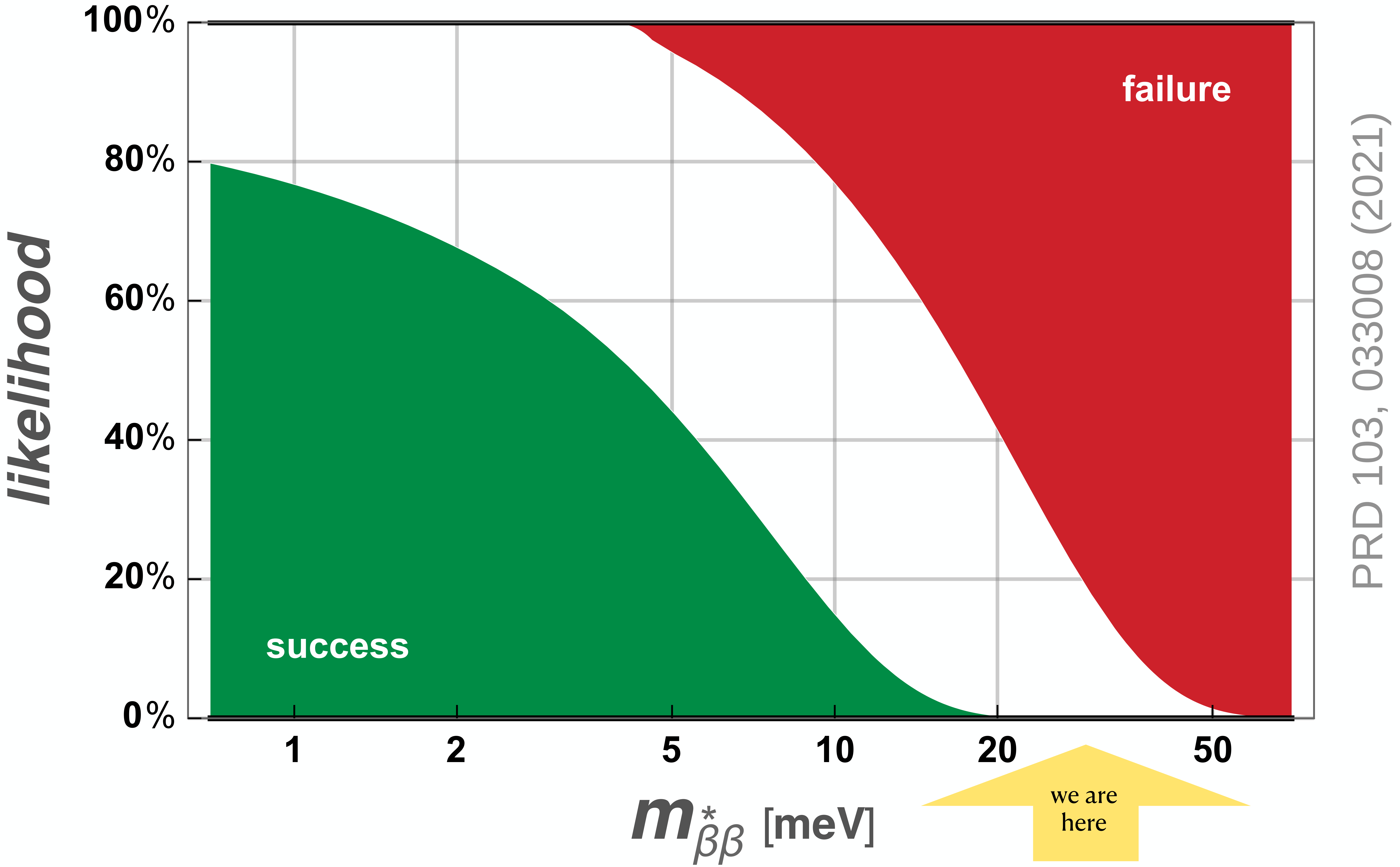
$$\langle 0|T[\boldsymbol{\nu}_{Le}(x)\, \boldsymbol{\nu}^t_{Le}(y)]|0\rangle = P_L \gamma^0 \int \frac{d^4p}{(2\pi)^4} \frac{i U^2_{ej} m_j\, e^{-ip(x-y)}}{p^2 - m_j^2 + i0^+} \tag{34}$$

The virtual momentum in the denominator has a small time component due to kinematical constraints, whereas the spatial component is of the order of the radius $|\vec{p}| \sim 1/R_0$; therefore, the masses of the light neutrinos $m_j \ll 100$ MeV are absolutely negligible in the denominator, and the lifetime will depend upon neutrino masses and mixing only through

$$m_{ee} = \left| \sum_{j=1}^{3} U^2_{ej} m_j \right| = m_{\beta\beta} \tag{35}$$

on $0\nu\beta\beta$ & cosmology

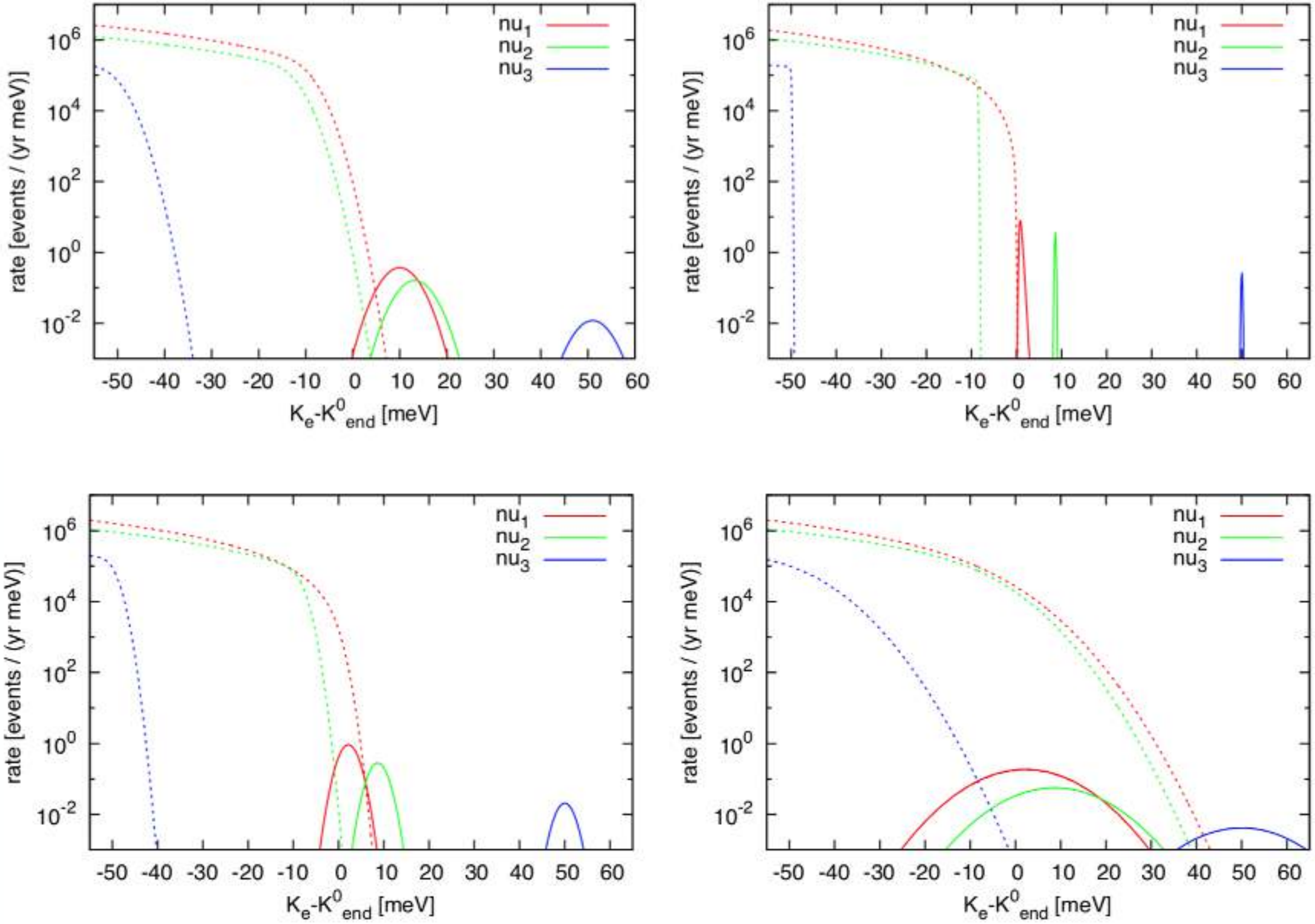
discovery potential for normal spectrum



again on neutrino absorption

$m_1 = 10$ meV
$\Delta = 7$ meV

$m_1 = 0.3$ meV
$\Delta = 0.3$ meV



$m_1 = 1$ meV
$\Delta = 4$ meV

$m_1 = 1$ meV
$\Delta = 20$ meV

SM and neutrinos

neutrino masses in modern language

(extending the lagrangian density of the standard model)

$$\mathscr{L} = i\,\bar{\nu}_L\,\partial_a\gamma^a\,\nu_L$$

Lepton # conserving	*Lepton # breaking*

neutrino masses in modern language

(extending the lagrangian density of the standard model)

$$\mathscr{L} = i\,\bar{\nu}_L\,\partial_a \gamma^a\,\nu_L - (m_{LL}\,\bar{\nu}_L C \bar{\nu}_L^t + \text{h.c.})/2$$

Lepton # conserving	*Lepton # breaking*

neutrino masses in modern language

(extending the lagrangian density of the standard model)

	Lepton # conserving	Lepton # breaking
$\mathscr{L}$	$= \ i\, \bar{\nu}_L\, \partial_a \gamma^a\, \nu_L$	$-(m_{LL}\, \bar{\nu}_L C \bar{\nu}_L^t + \text{h.c.})/2$
	$-(m_{LR}\, \bar{\nu}_R \nu_L + \text{h.c.})$	
	$+ i\, \bar{\nu}_R\, \partial_a \gamma^a\, \nu_R$	$-(m_{RR}\, \bar{\nu}_R C \bar{\nu}_R^t + \text{h.c.})/2$