Gravitational waves in astrophysics

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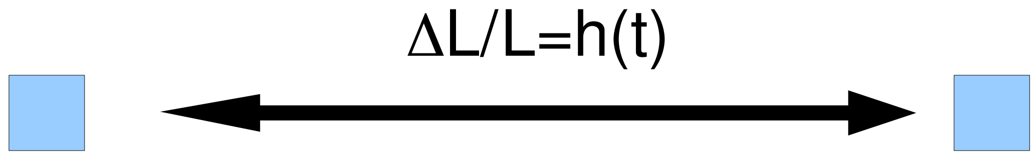
WARSZAWSKIEGO

Basics



- The waves affect free floating bodies
- Space is curved
- There is no global flat coordinate system

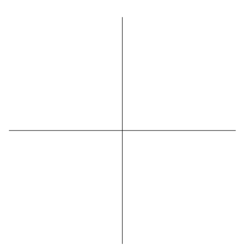
Physical basics

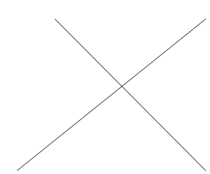
- Spacetime oscillations
- Measurement with light

- Curvature – tensor of 4th order: R

Physical basics

- Transverse wave
- Distortions of perpendicular geometry
- Symmetry to rotation by 180 deg
- For the EM case – 360 deg
- Photon spin =1, graviton =2
- Two polarizations: EM 90 deg; GW 45deg





Multipole expansion

- Waves - amplitude goes as 1/r
- Oscillating masses
- Multipole moments – mass, momentum
- Dimensional analysis
- Mass quadrupole moment
- Momentum quadrupole moment

The waves

- Source: mass M, size L, period P
- Quadrupole moment ML^2

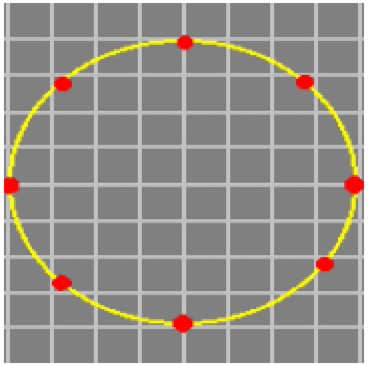
$$h = (G/c^4)(ML^2/P^2)/r$$

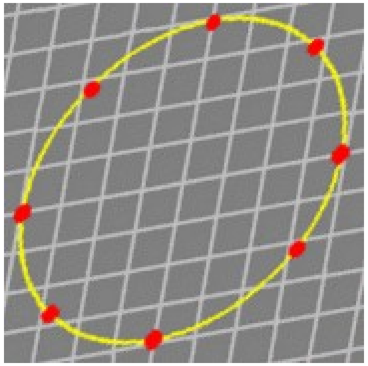
- Newtonian potential of the kinetic energy
- Higher moments – factors of (v/c)
- Maximally:

$$h \approx \frac{R_g}{D}\left(\frac{v}{c}\right)^2$$

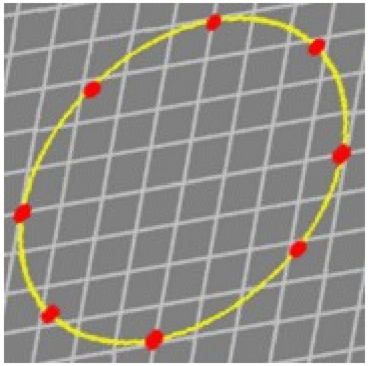
Gravitational wave

Linear polarization



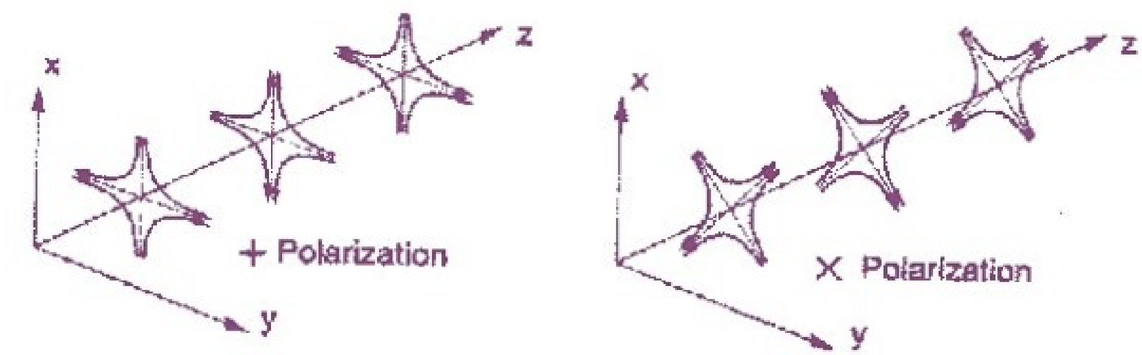


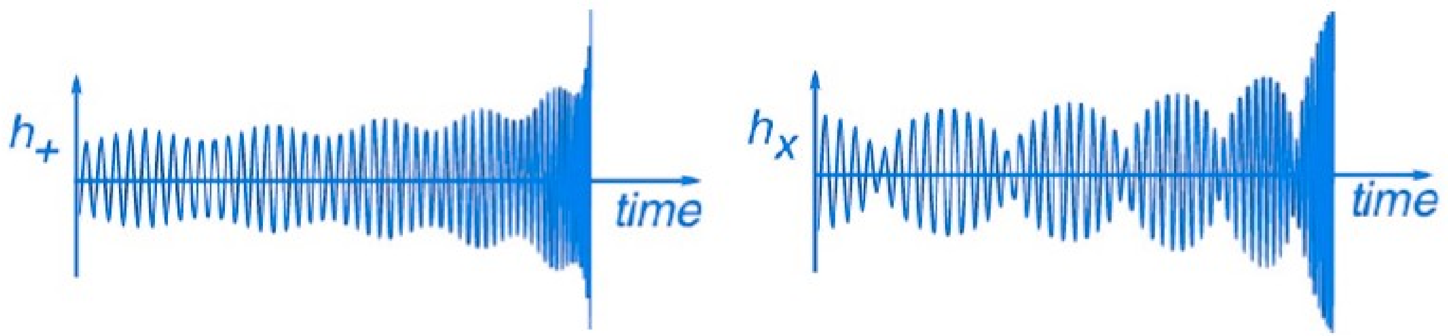
Circular polarization



Basics

- Two independent polarisations



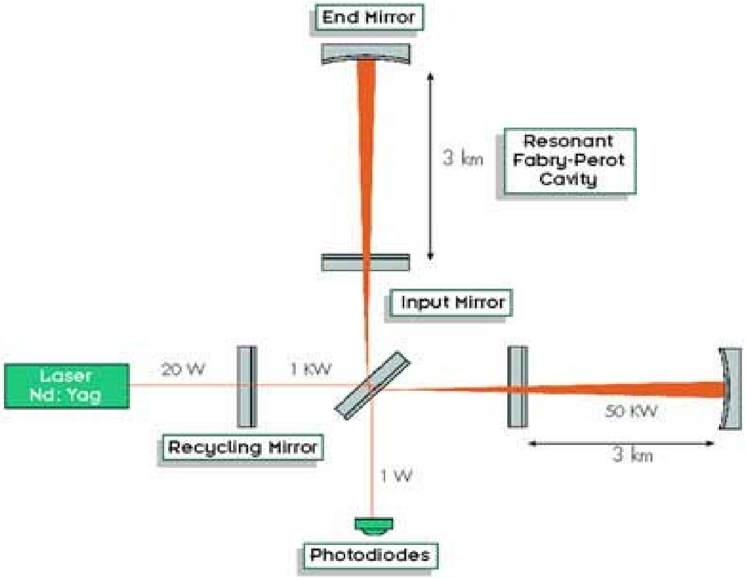


Wave propagation

- High frequency waves (wavelength smaller than curvature radius move along straight (geodetic) lines –geometrical optics
- Zero mass of graviton
- Redshift
- If wavelength comparable to curvature - scattering
- Dispersion: Negliglible- less than one wavelength in propagation through the Universe

Spectrum

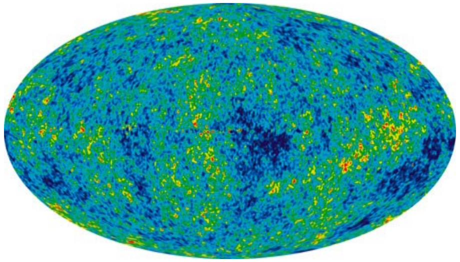
- 20 orders of magnitude
- Basics 4 ranges:
  - ELF log f = -16 do -10
  - VLF log f= -10 do -6
  - LF log f = -6 do 0
  - HF log f = 0 do 6



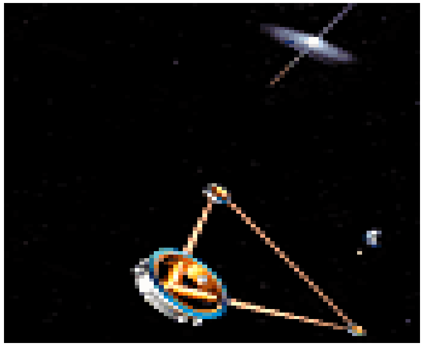
Cosmic observatories
LISA

Backgroud
radiation

Interferometers



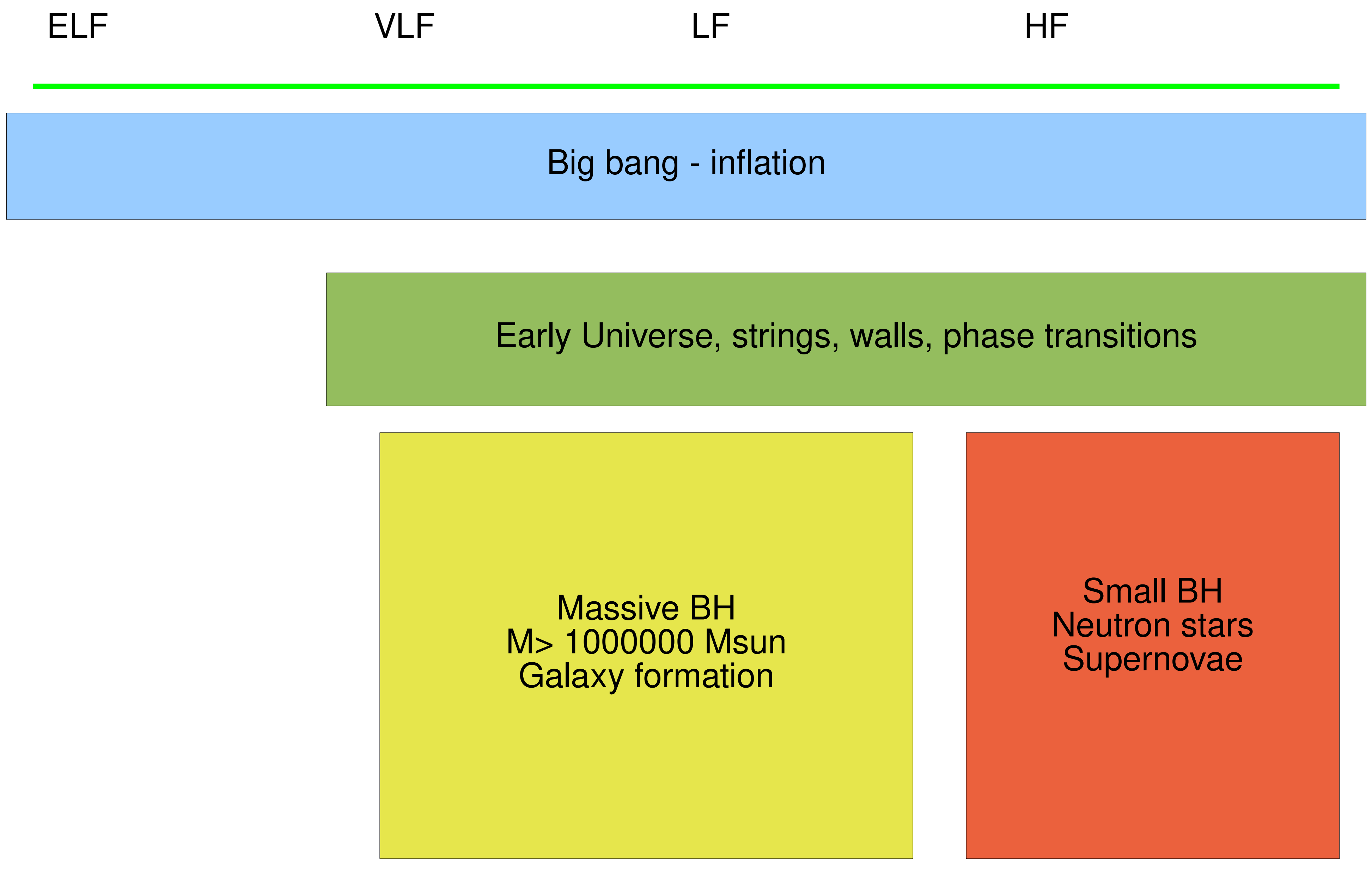
Pulsar timing



Resonsnt
detectors

-16 -8 0 8 log f

Sources


ELF
VLF
LF
HF
Big bang - inflation
Early Universe, strings, walls, phase transitions
Massive BH
M> 1000000 Msun
Galaxy formation
Small BH
Neutron stars
Supernovae

Basic wave generation

Look for perpendicular part

$$I_{jk} = \sum_A m_A (x_j^A x_k^A - \frac{1}{3}\delta_{jk} x_m^A x_m^A)$$

$$P_{ij} = \delta_{ij} - n_i n_j$$

$$I_{jk}^{TT} = P_{jk} P_{km} I_{lm} - \frac{1}{2} P_{jk}(P_{lm} I_{lm})$$

$$h_{jk}^{TT}(t, \vec{x}) = \frac{2}{r}\frac{G}{c^4}\ddot{I}_{jk}^{TT}(t - \frac{r}{c})$$

Expected amplitude

$$R_g = 2\frac{GM}{c^2}$$

Gravitational radius

$$h \approx \frac{R_g}{D}\frac{v^2}{c^2}$$

For a fiducial source: two 30 solar mass black holes at 100Mpc

$$h \approx \frac{R_g}{D}\frac{v^2}{c^2} \approx 2 \times 10^{-21}$$

Energy and angular momentum

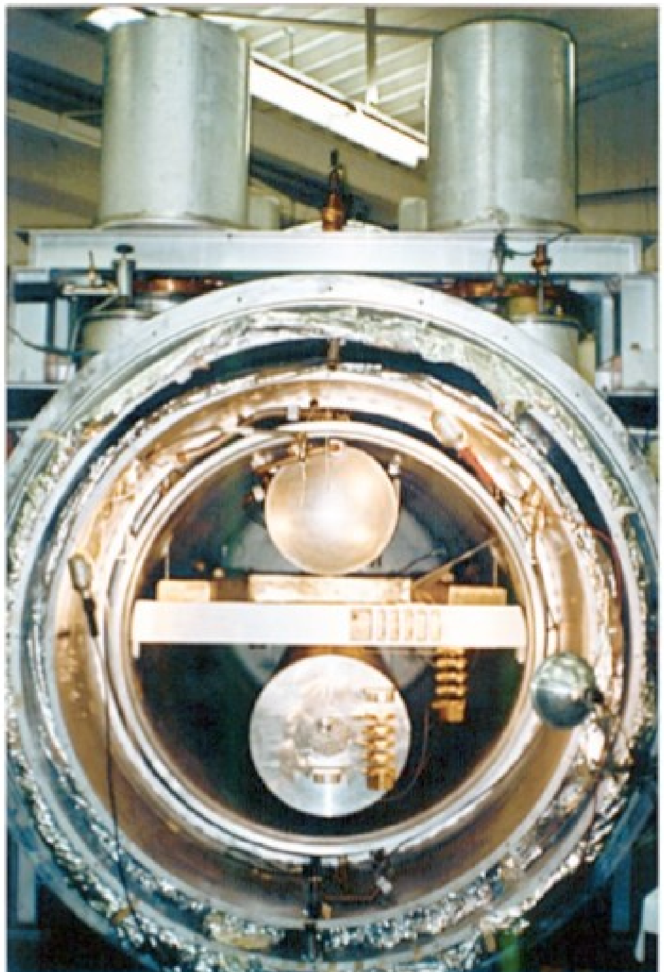
$$L_{GW} = \frac{1}{5}\frac{G}{c^5}\frac{d^3\, I_{jk}}{dt^3}\frac{d^3\, I_{jk}}{dt^3}$$

$$\frac{dJ_i}{dt} = -\frac{2}{5}\frac{G}{c^5}\epsilon_{ijk}\frac{d^2\, I_{jm}}{dt^2}\frac{d^3\, I_{km}}{dt^3}$$

Detection

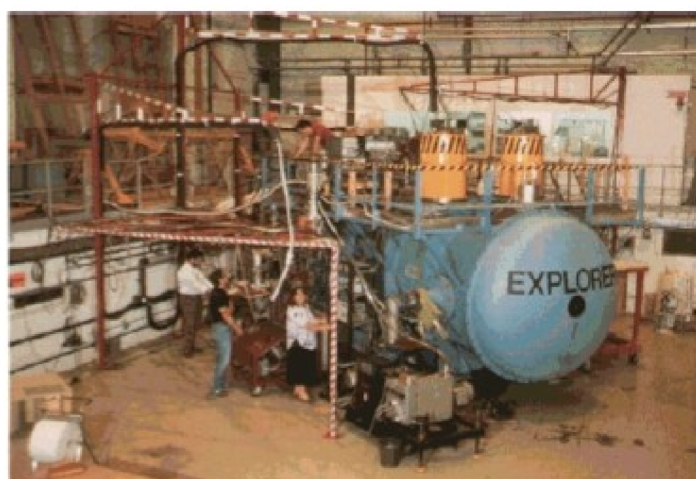
- Test masses
- Measuring distances
- Need a network of detectors

First attempts: resonant detectors

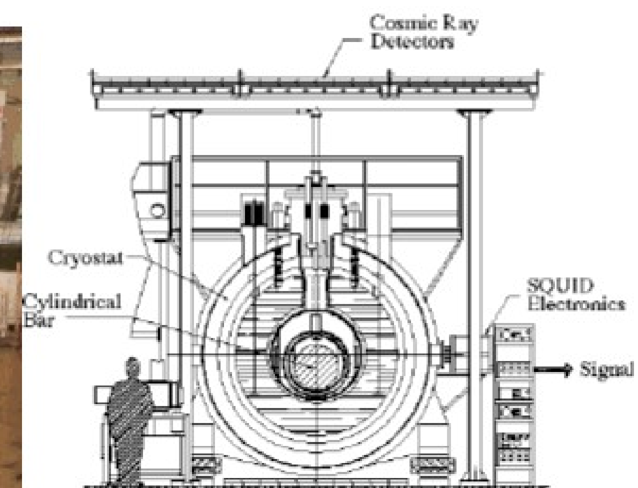

Louisiana State U. - Allegro


U. West Australia - Niobe


U. Padova - Auriga

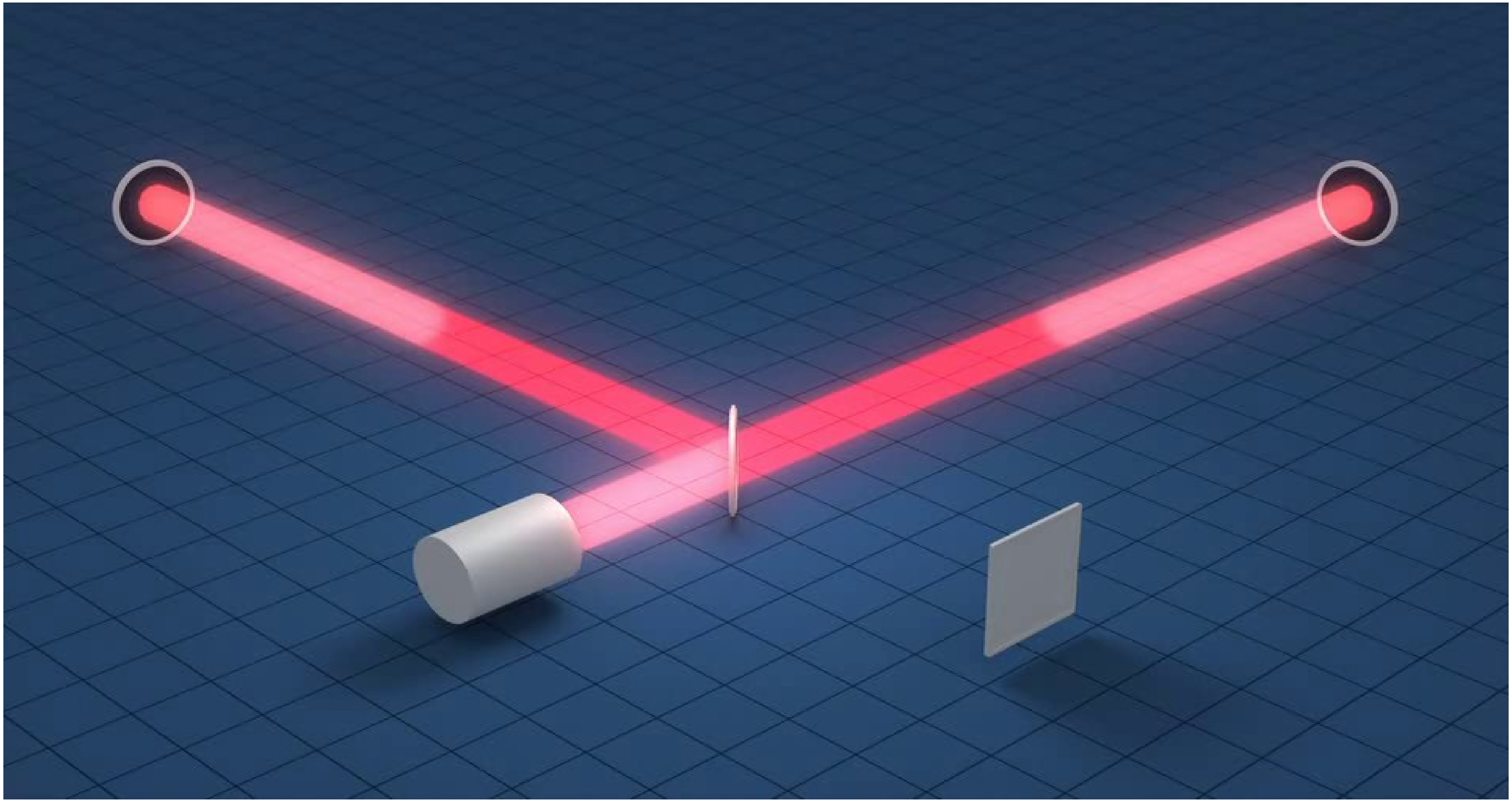


CERN - Explorer

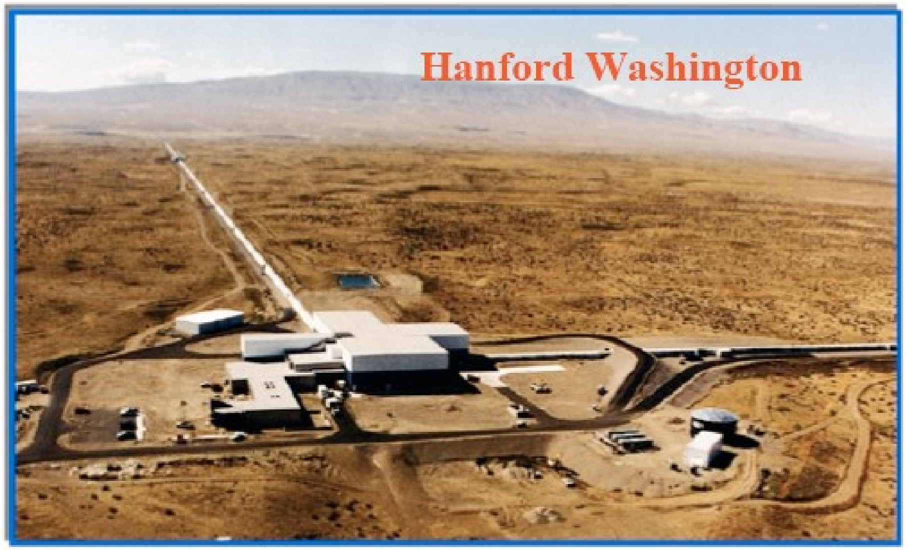


U. Rome - Nautilus

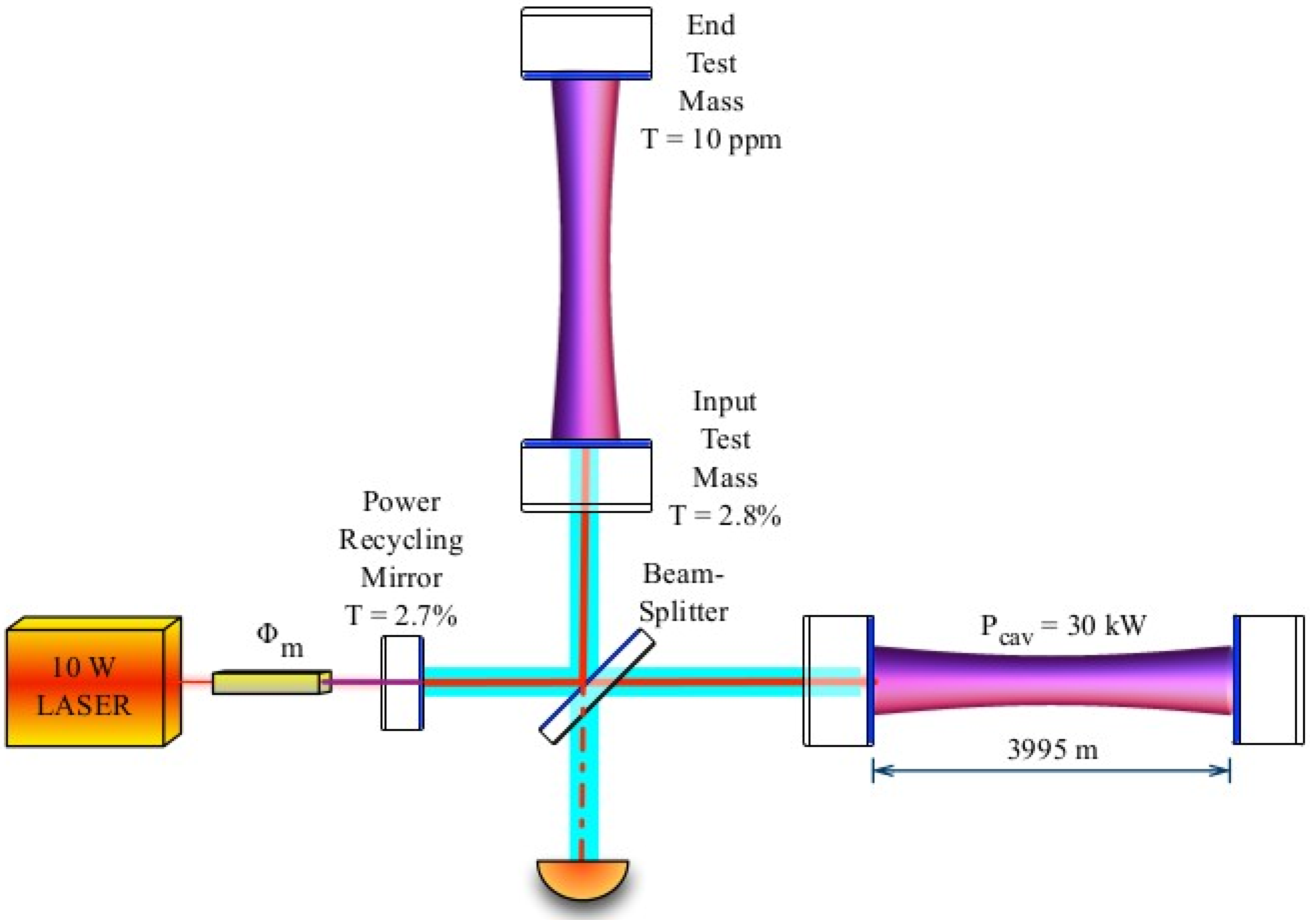
Interferometers



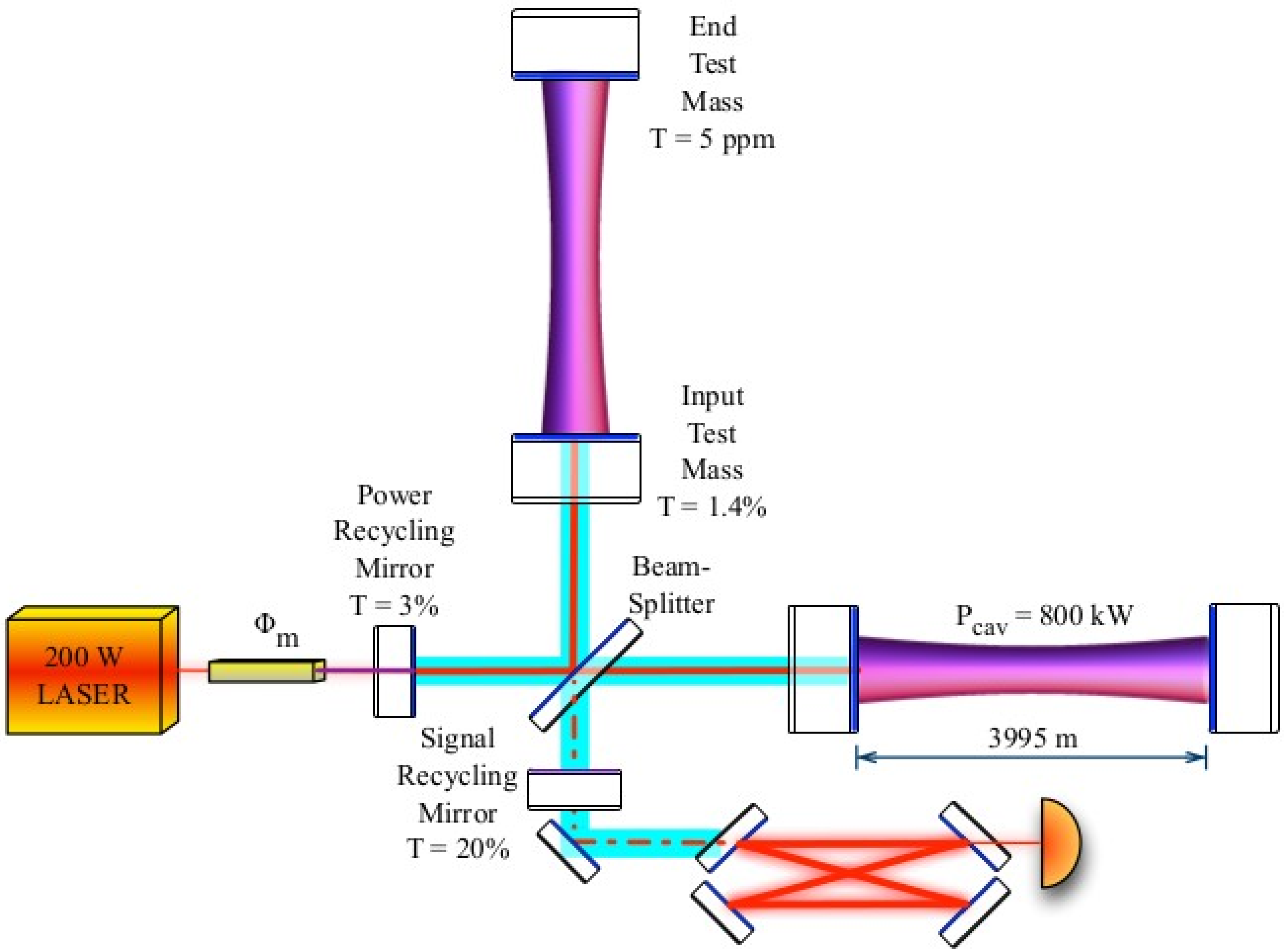
Interferometers - HF



Interferometers: initial



Second generation interferometers



Noise reduction

- Seismic noise quenched by 10 orders of magnitude
- Thermal noise – heavy mass, smart coating
- Quantum shot noise - laser power increase – about 100kW in the cavity
- But – note – fundamental quantum limit:
  - Low vs. high frequency

Order of magnitude estimates

$$\Delta L \approx 10^{-18} m$$

500 round trips, expected delay:

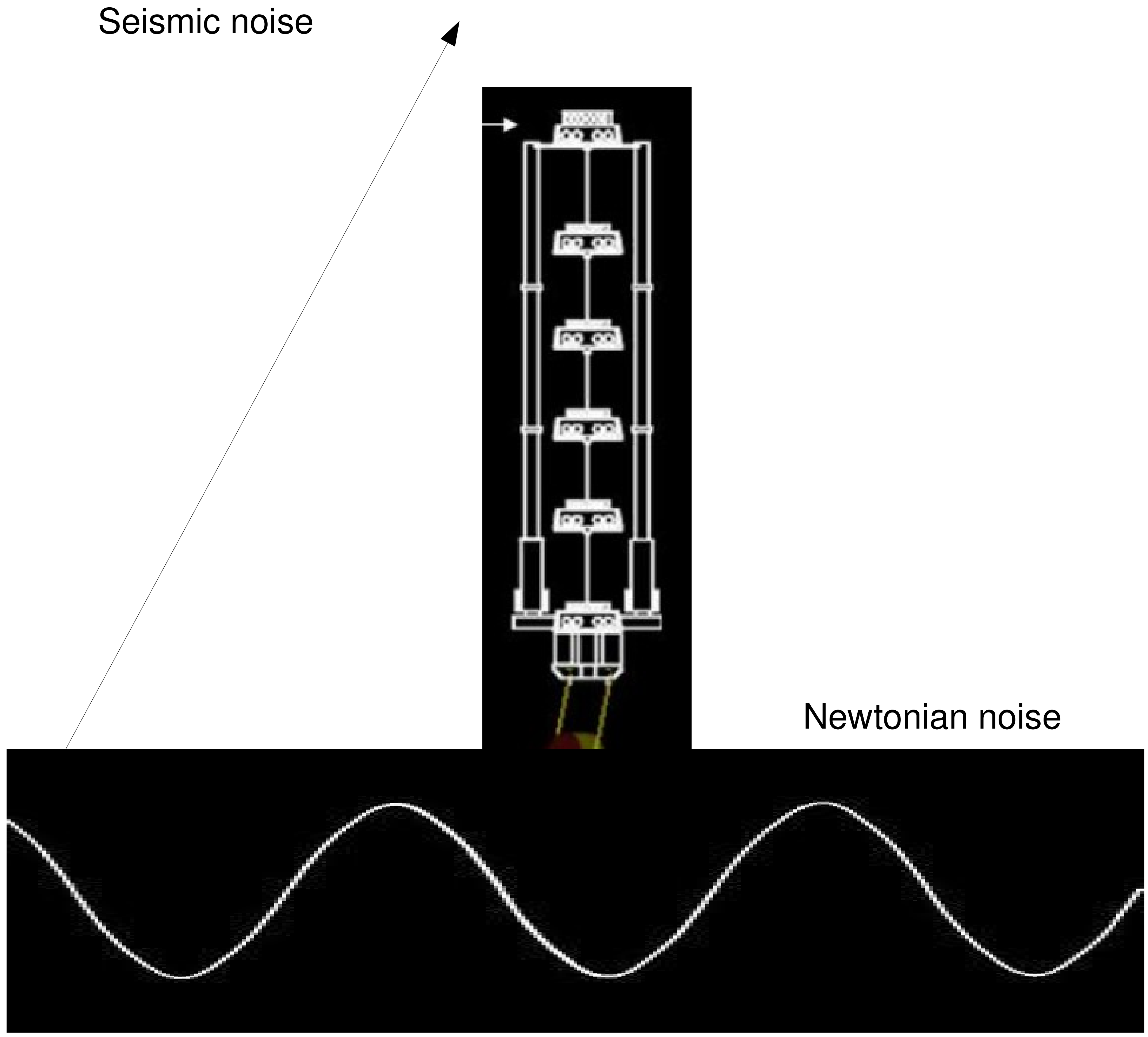
$$\Delta L_{eff} = 500 \times 10^{-18}\mathrm{m} \approx 5 \times 10^{-16}\mathrm{m}$$

The laser wavelength is 1064nm. Expected phase amplitude:

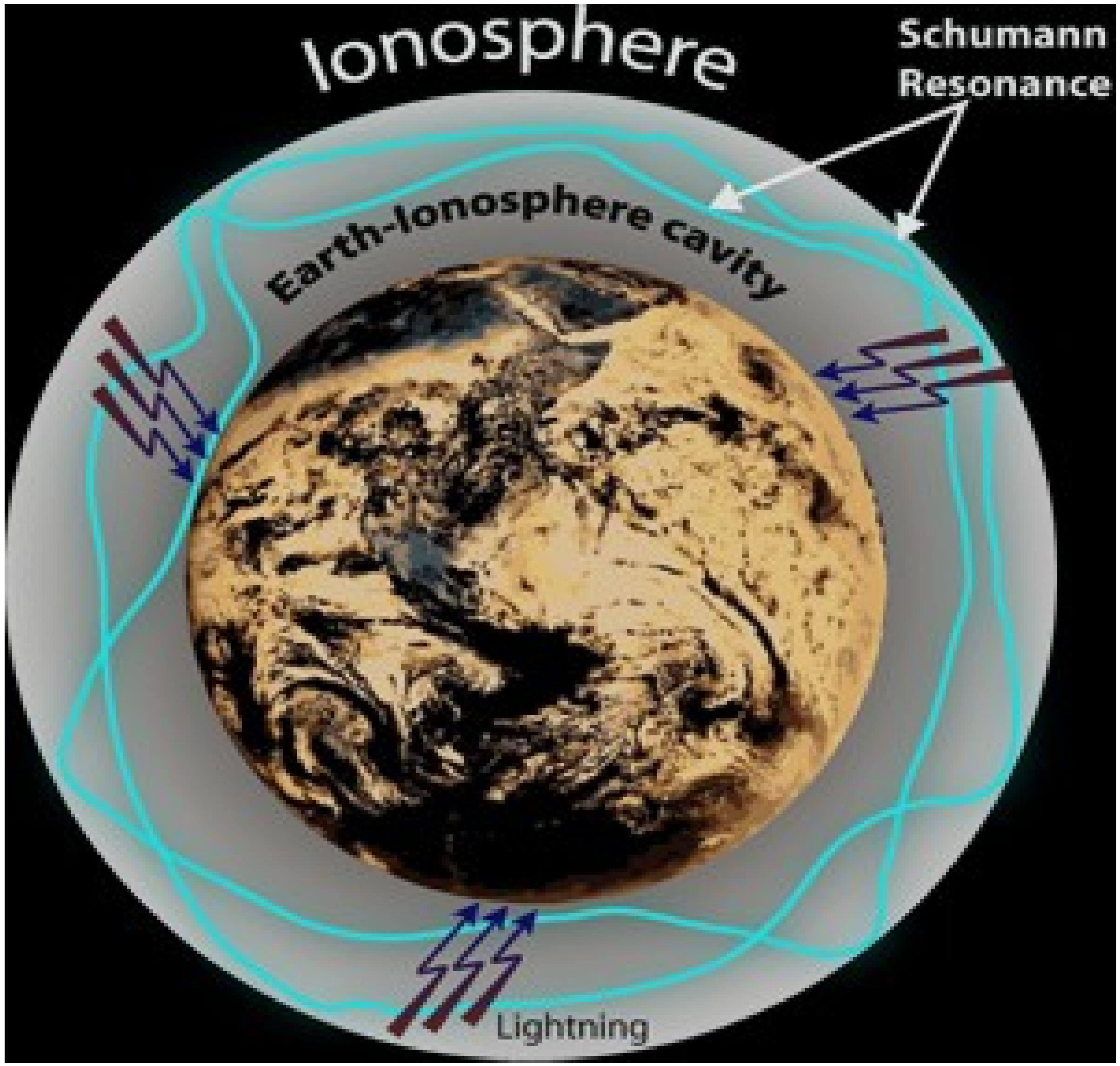
$$\Delta\phi \approx 5 \times 10^{-16}/10^{-6} = 5 \times 10^{-10}$$

With 100kW power the Poisson noise fluctuations are

$$\delta\phi = N^{-1/2} = \sqrt{\frac{hc}{\lambda P \Delta T}} = \sqrt{\frac{10^{-19}\mathrm{J}}{10^{5}\mathrm{W}10^{-2}\mathrm{s}}} \approx 10^{-11}$$

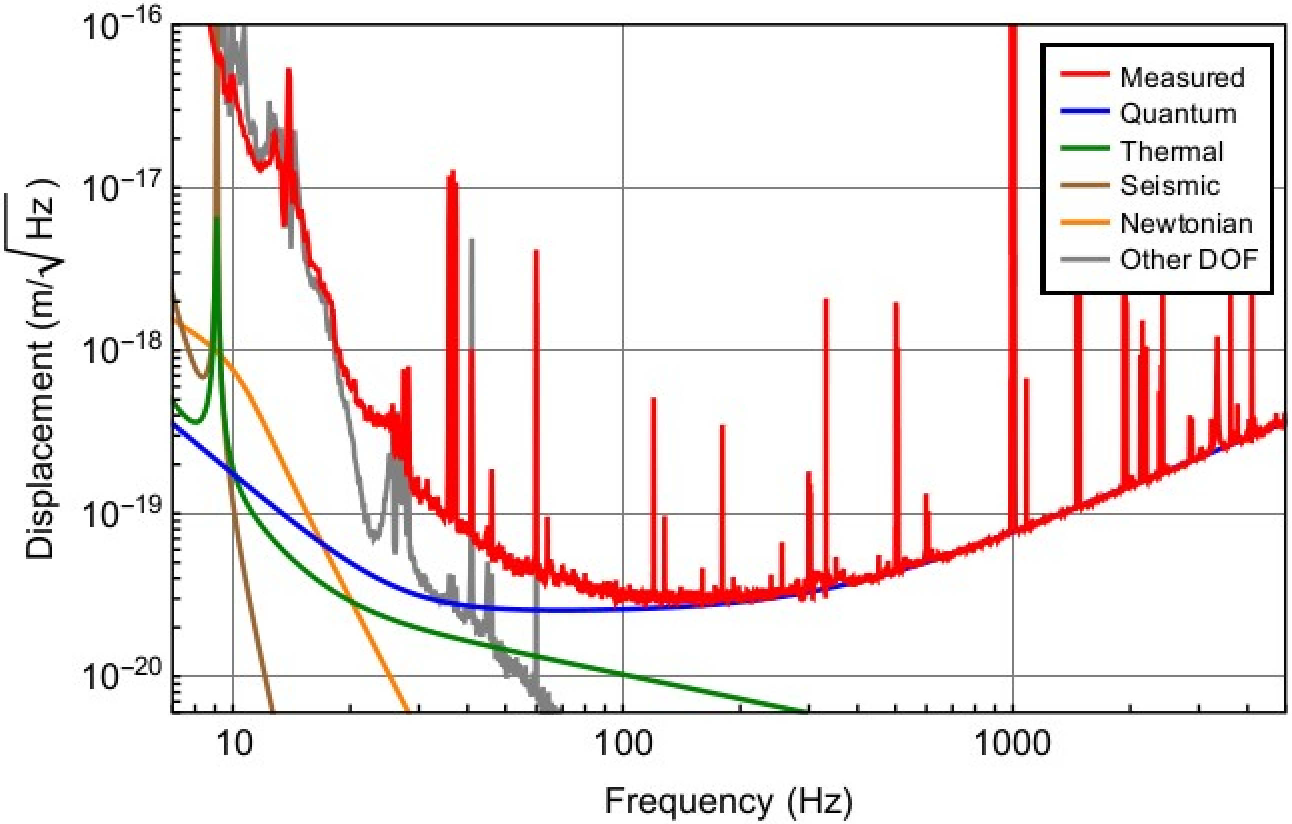

Seismic noise
Newtonian noise

Magnetic noise



Technical challenges

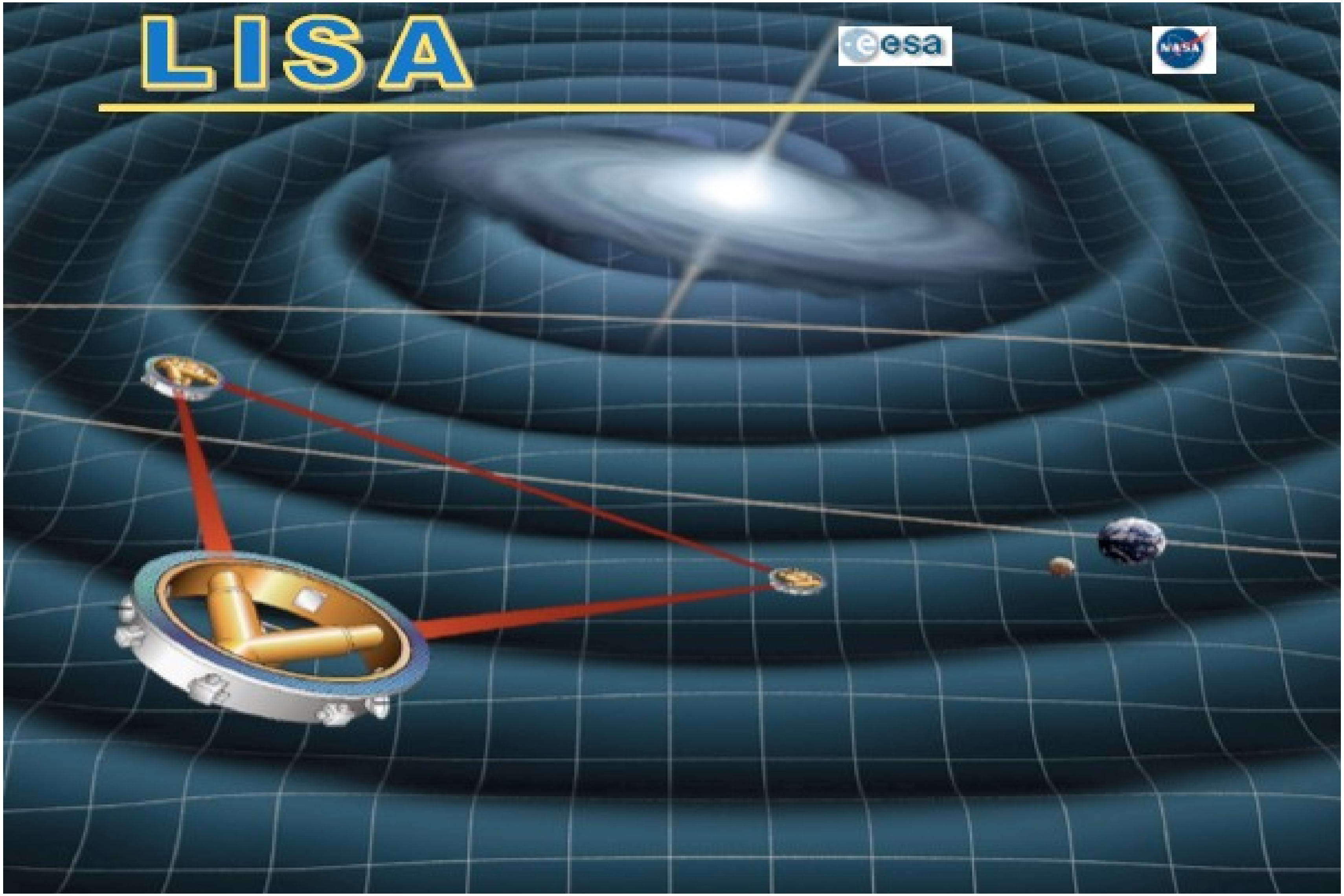
- Seismic isolation
- Vacuum system
- High power lasers
- Thermal noise
- Quantum fluctuations



Abbott et al 2016

Also – newtonian noise, magnetic noise

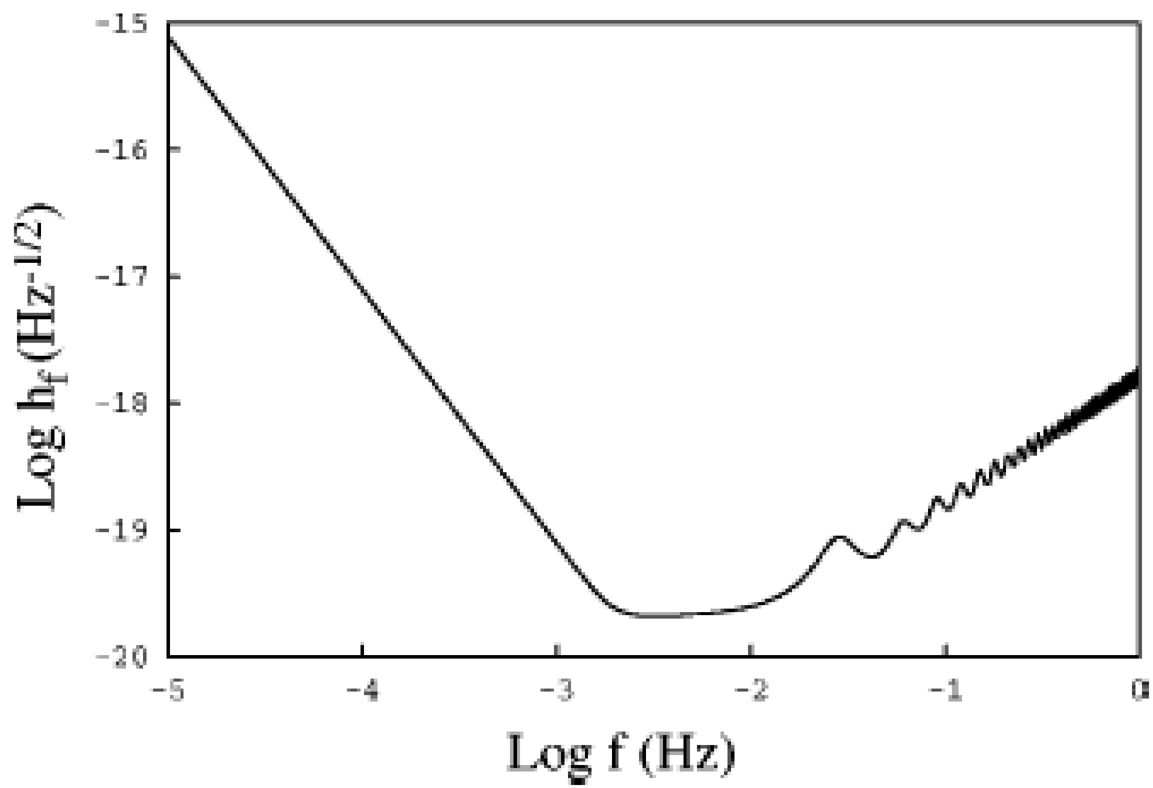
LF instruments



LISA

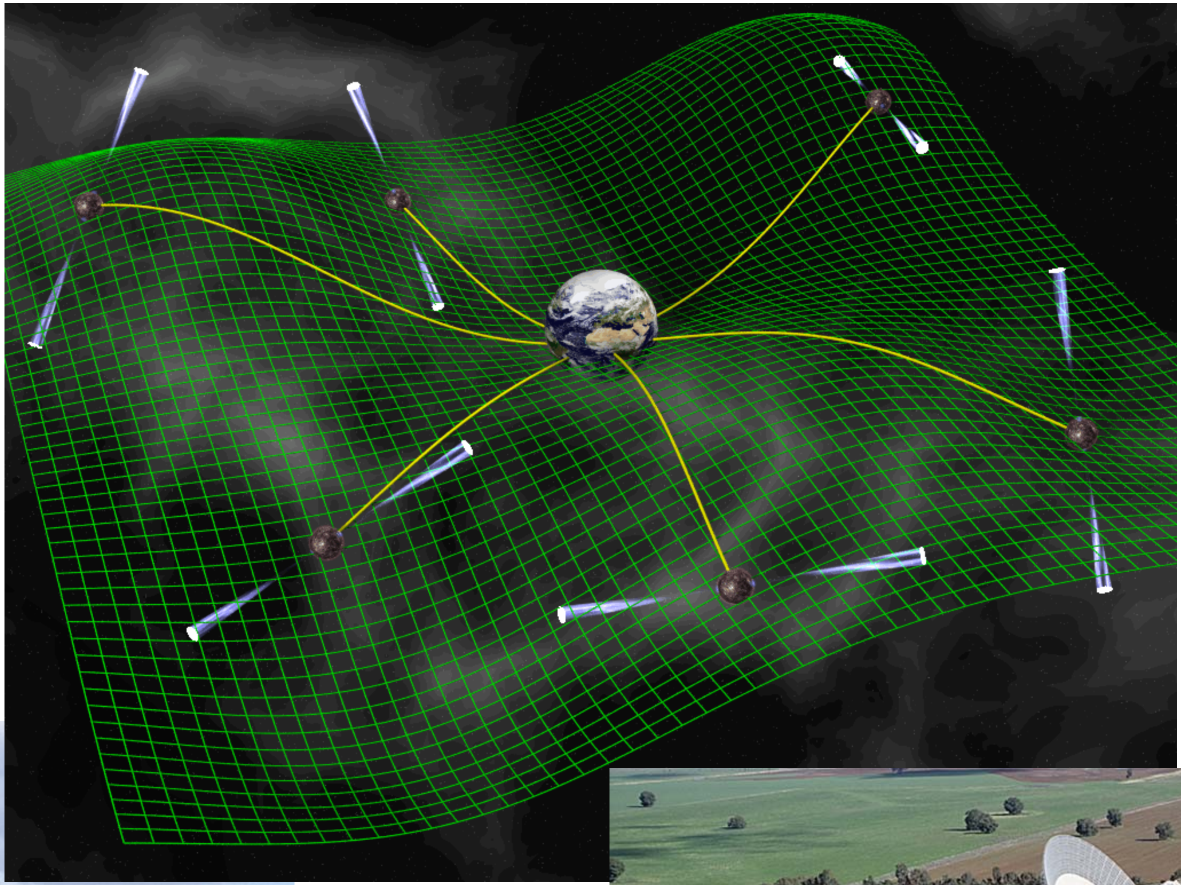
- Three satellites 5 Gm apart
- Lasers 1W
- Telescopes 30 cm
- Satellite motion

- Launch 2037
- DECIGO, BBO, AL



Pulsar Timing Arrays

- PPTA
- EPTA
- SKA

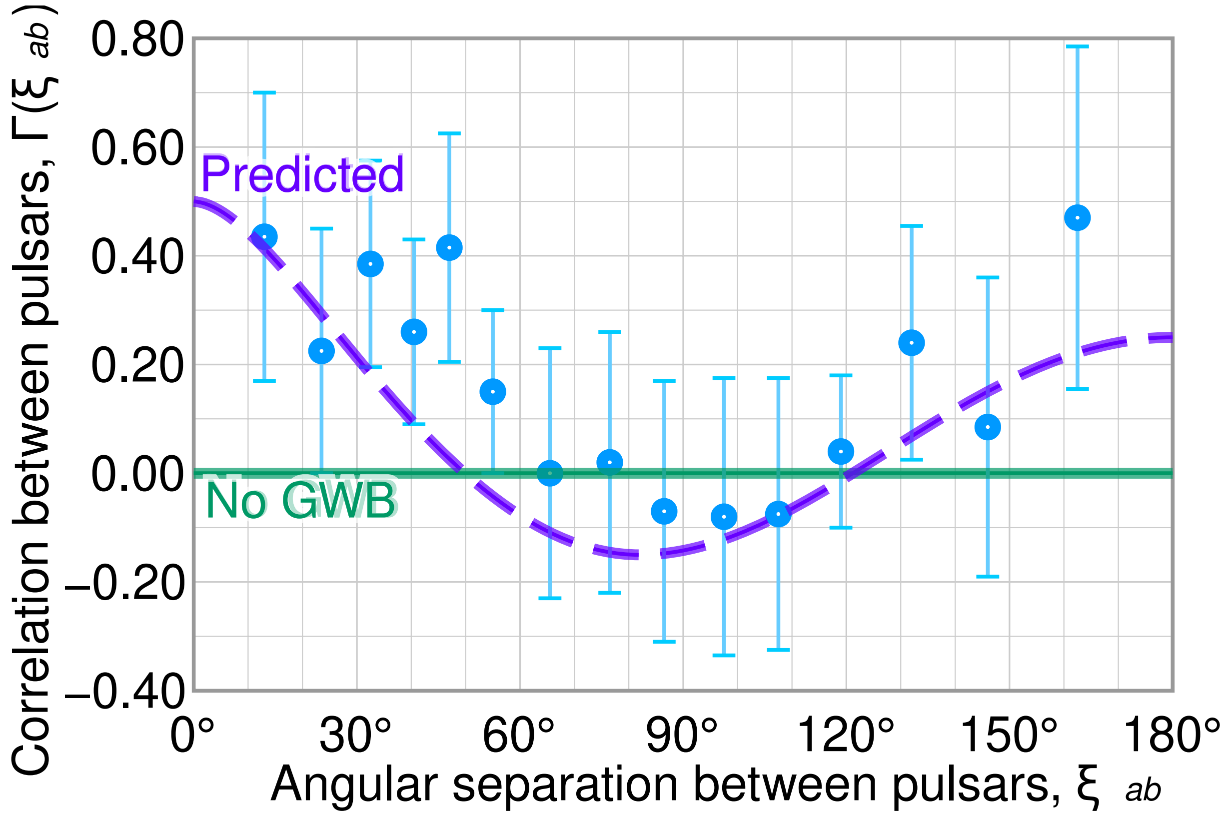


PTA search and detection

Correlation between timing residuals of distant pulsars

Helling Downs curve.

Expect more in the coming months!



Detector sensitivities and readout

Detector provides a digitized output – a string of numbers.

h1 h2 h3 ... hn

$$\tilde{h}(f) = \mathcal{F}(x(t))(f) = \int_{-\infty}^{\infty} h(t)\exp(2\pi i f t)dt$$

$$h(t) = \mathcal{F}^{-1}(\tilde{h}(f))(t) = \int_{-\infty}^{\infty} \tilde{h}(f)\exp(-2\pi i f t)df$$

Power spectral density

$$\left\langle \tilde{h}(f)\tilde{h}^*(f') \right\rangle = \frac{1}{2}\delta(f-f')S_c(f)$$

Units!

Detector noise curve

Characteristic strain

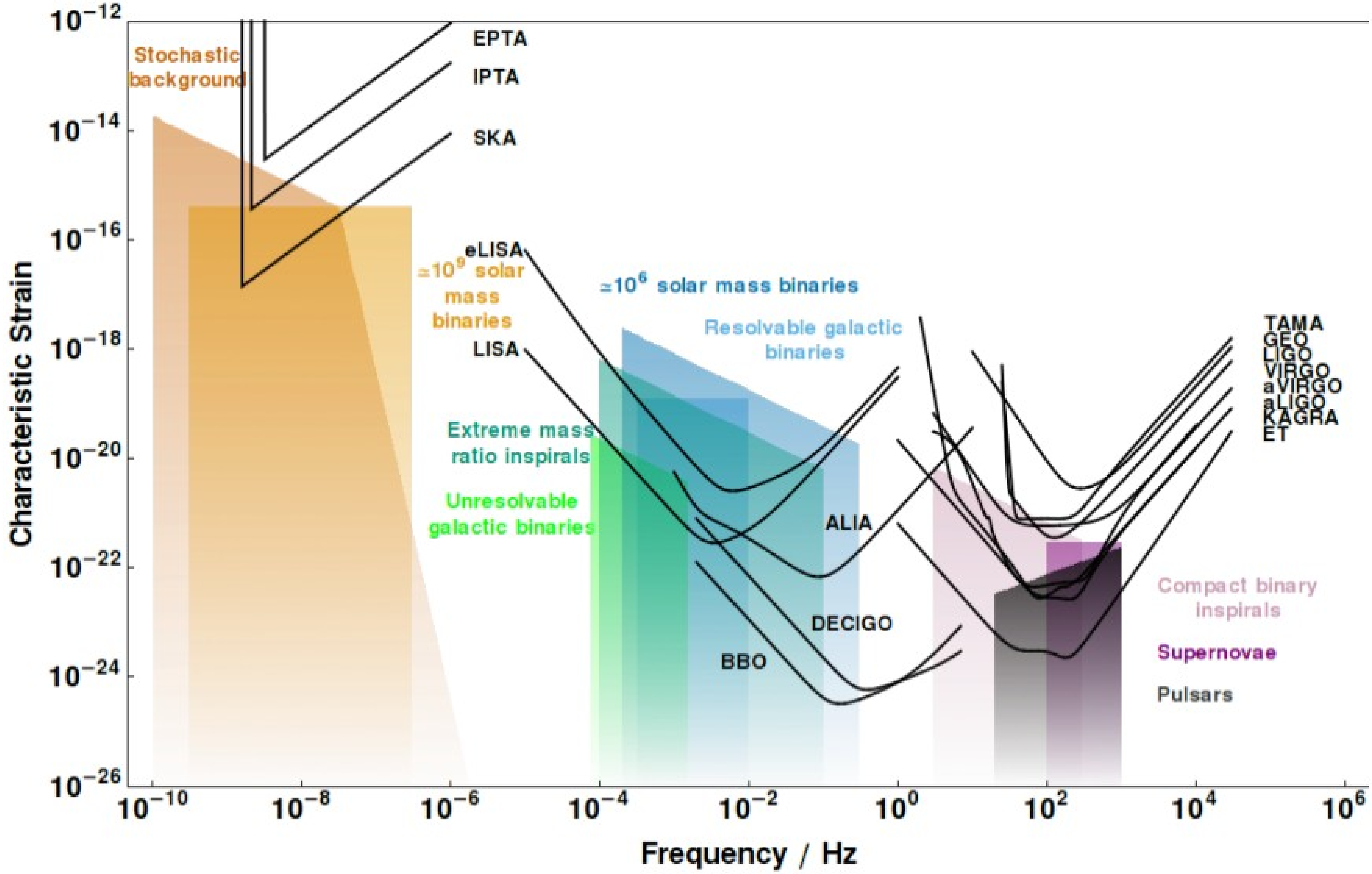
$$h_c(f) = \sqrt{f S_c(f)}$$

Signal to noise $$\rho^2 = \int d(\log f) \frac{|h_c(f)|^2}{|h_n(f)|^2}$$

+ Easy to read the S/N off graphs

- does not correspond to signal amplitude

+ shows the rms in a given band



Characteristic Strain
Frequency / Hz
Stochastic background
EPTA
IPTA
SKA
eLISA
LISA
≃10^9 solar mass binaries
≃10^6 solar mass binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
ALIA
DECIGO
BBO
TAMA
GEO
LIGO
VIRGO
aVIRGO
aLIGO
KAGRA
ET
Compact binary inspirals
Supernovae
Pulsars

Detector noise curves

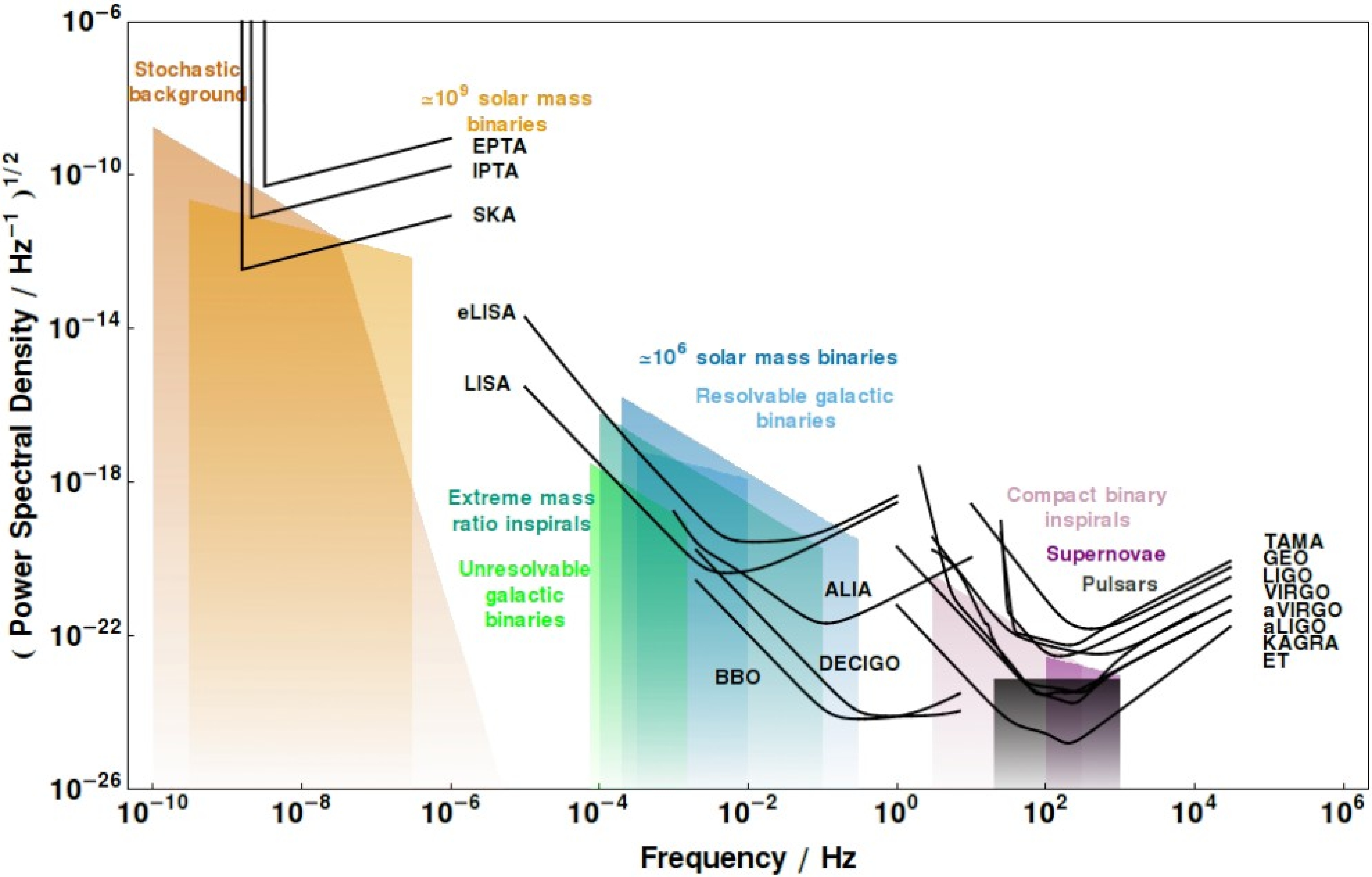
Amplitude spectral density (or root PSD)

$$\mathrm{ASD}(f) = \sqrt{S_h(f)} = h(f) f^{-1/2}$$

- units

- mean square amplitude of the signal from PSD

- most commonly used



Stochastic background
≃10^9 solar mass binaries
EPTA
IPTA
SKA
eLISA
LISA
≃10^6 solar mass binaries
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KAGRA
ET
(Power Spectral Density / Hz^-1)^1/2
Frequency / Hz

Detector noise curves

Energy density

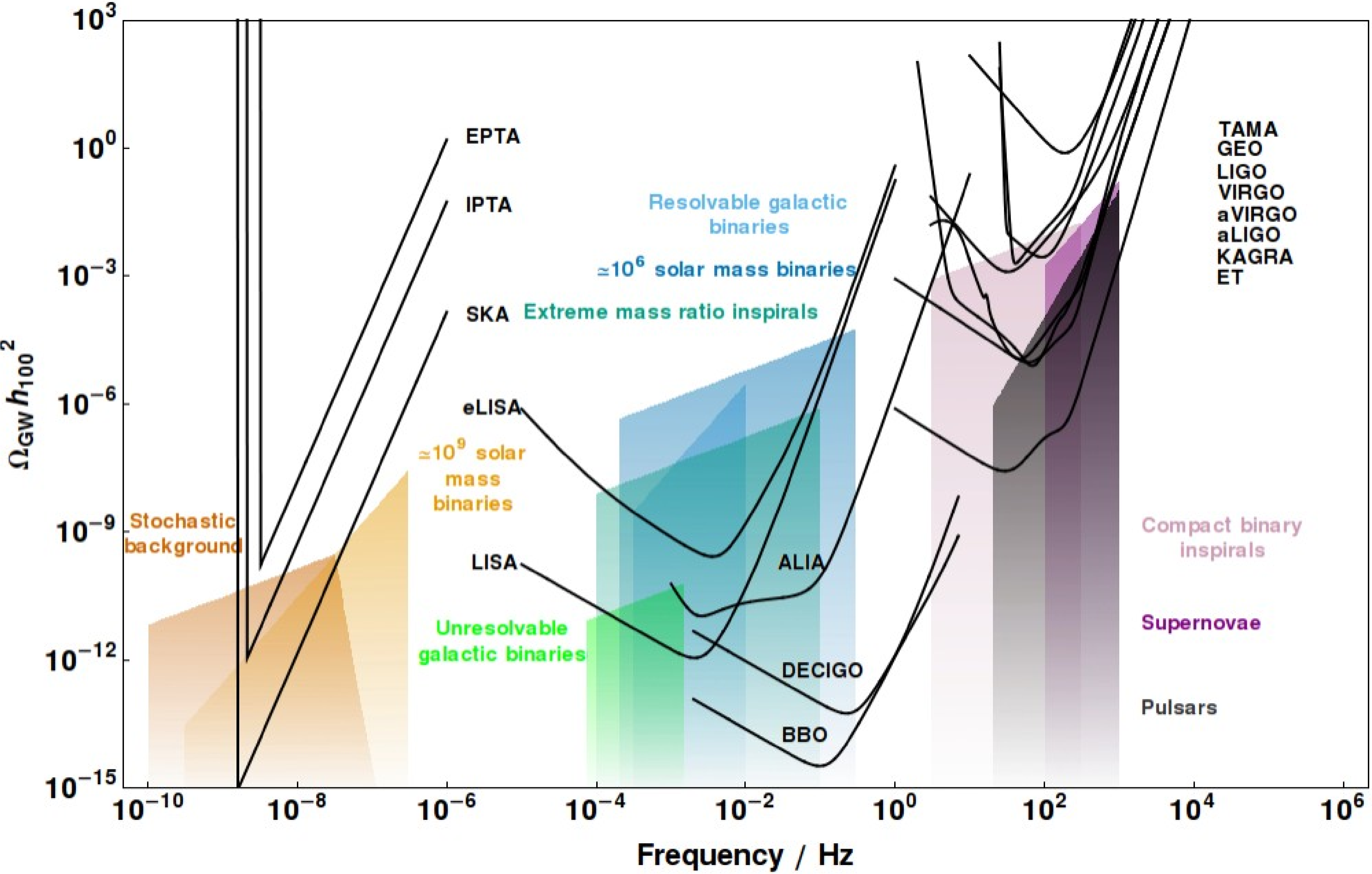
$$T_{\mu\nu} = \frac{c^4}{32\pi G} \langle \partial_\mu h_{\alpha\beta} \partial_\nu h_{\alpha\beta} \rangle$$

$$\rho c^2 = \frac{c^2}{16\pi G} \int_0^\infty df (2\pi f)^2 \tilde{h}(f) \tilde{h}^*(f) = \int_0^\infty df \frac{\pi c^2}{4G} f^2 S_h(f)$$

$$S_E(f) = \frac{\pi c^2}{4G} f^2 S_h(f)$$

Energy density per logarithmic frequency interval

$$\Omega_{GW}(f) = \frac{f S_E(f)}{\rho_c c^2} = \frac{8\pi G}{3 H_0^2 c^2} f S_E(f)$$



10^{3}
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
$\Omega_{GW} h_{100}^{2}$
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Frequency / Hz
EPTA
IPTA
SKA
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LISA
Resolvable galactic binaries
$\simeq 10^{6}$ solar mass binaries
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Conversions

$$\Omega_{GW}(f) = \frac{8\pi G}{3H_0^2 c^2} f S_E(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f) = \frac{2\pi^2}{3H_0^2} f^2 [h_c(f)]^2 = \frac{8\pi^2}{3H_0^2} f^4 \left|\tilde{h}(f)\right|^2$$

$$\frac{3H_0^2 c^2}{8\pi G} \frac{\Omega_{GW}(f)}{f} = S_E(f) = \frac{c^2 \pi}{4G} f^2 S_h(f) = \frac{c^2 \pi}{4G} f [h_c(f)]^2 = \frac{\pi c^2}{G} f^3 \left|\tilde{h}(f)\right|^2$$

$$ASD(f) = S_h(f)^{1/2} = \frac{|h_c(f)|}{f} = 2f|\tilde{h}(f)| = \sqrt{\frac{4G S_E(f)}{\pi c^2 f^2}} = \sqrt{\frac{3H_0^2 \Omega_{GW}}{2\pi^2 f^3}}$$

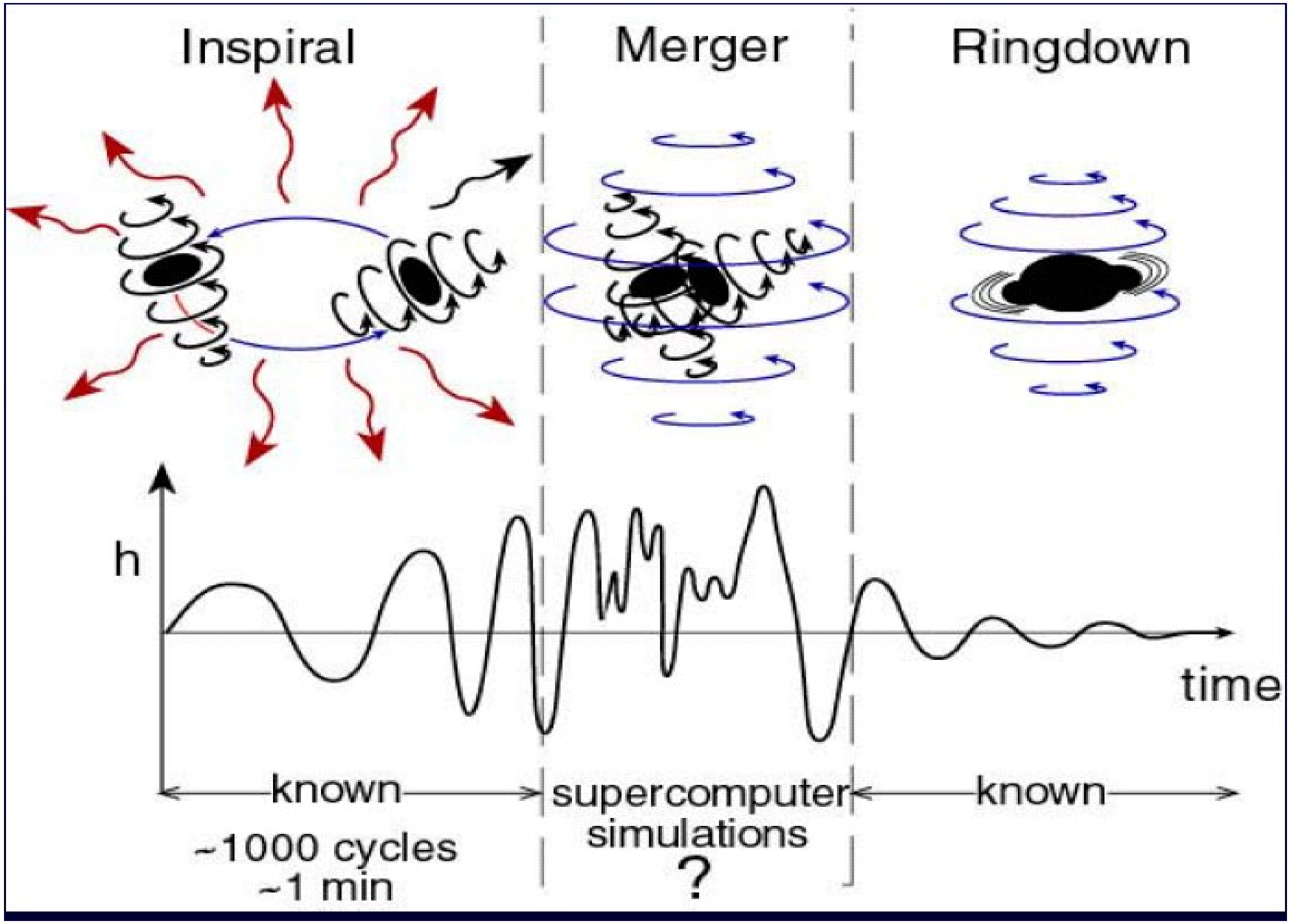
$$h_c(f) = f ASD(f)$$

Sources -HF gravitational waves

- Binaries
  - BBH
  - BNS
  - BHNS
- Pulsars
- Supernovae
- Bursts

Three phases of coalescence

- "inspiral"
- "merger"
- "ringdown"



A simple binary

$$I_{xx} = (m_1 a_1^2 + m_2 a_2^2)\cos^2\phi + \mathrm{const}$$
$$I_{yy} = (m_1 a_1^2 + m_2 a_2^2)\sin^2\phi + \mathrm{const}$$
$$I_{xy} = (m_1 a_1^2 + m_2 a_2^2)\sin\phi\cos\phi + \mathrm{const}$$

$$I_{xx} = -I_{yy} = \frac{1}{2}\mu a^2 \cos 2\phi + \mathrm{const}$$
$$I_{xy} = I_{yx} = \frac{1}{2}\mu a^2 \sin 2\phi$$

Amplitude of the wave

$$h_{jk}^{TT}(t,\vec{x}) = \frac{2}{r}\frac{G}{c^4}\ddot{I}_{jk}^{TT}(t-\frac{r}{c})$$

Let us consider the wave in the direction x

$$h = \frac{4}{r}\left(\frac{G\mathcal{M}}{c^2}\right)^{5/3}\left(\frac{\Omega}{c}\right)^{2/3} = 4\frac{G\mathcal{M}}{rc^2}\left(\frac{G\mathcal{M}\Omega}{c^3}\right)^{2/3}$$

$$M_{chirp} = \mu^{3/5}M^{2/5}$$

Luminosity

$$L_{GW} = \frac{1}{5}\frac{G}{c^5}\dddot{I}_{jk}\dddot{I}_{jk}$$

$$L_{GW} = \frac{1}{5}\frac{G}{c^5}(2\Omega)^6\left(\frac{1}{2}\mu a^2\right)^2(\sin^2 2\Omega t + \sin^2 2\Omega t + 2\cos^2 2\Omega t)$$

$$\Omega^2 = \frac{GM}{a^3}$$

$$\boxed{L_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}}$$

More on emitted power

$$L_0 = \frac{c^5}{G}$$

$$L_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}$$

$$L_{GW} = \frac{16}{10}L_0\left(\frac{r_g^{\mu}}{a}\right)^2\left(\frac{v}{c}\right)^6 \approx L_0\left(\frac{v}{c}\right)^{10}$$

Orbit contraction

Energy

$$E = -\frac{1}{2}\frac{G\mu M}{a}$$

$$\frac{1}{P}\frac{dP}{dt} = \frac{3}{2}\frac{1}{a}\frac{da}{dt} = -\frac{3}{2}\frac{1}{E}\frac{dE}{dt} = -\frac{96}{5}\frac{G^3}{c^5}\frac{M^2\mu}{a^4}$$

Time to merger

$$t = \frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{M^2\mu}$$

Orbit evolution

$$\frac{1}{P}\frac{dP}{dt} = -\frac{96}{5}\frac{G^3}{c^5}\frac{M^2\mu}{a^4}$$

$$\frac{dP}{dt} = -\frac{96}{5}2\pi\left(\frac{2\pi G M_{chirp}}{c^3 P}\right)^{5/3}$$

Chirp mass again:

$$\boxed{M_{chirp} = \mu^{3/5} M^{2/5}}$$

Orbit change

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3\mu M^2}{c^5a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$\frac{de}{dt} = -\frac{304}{15}e\frac{G\mu M^2}{c^5a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

Peters 1964

Peters and Matthews 1963

Note on luminosity

- Amplitude and period change depend on the chirp mass
- Both are measurable
- Standard sirens!
- Angular factors play a role....

- Physical reason

GW spectrum

$$\frac{dE}{df} = \frac{dE}{dt}\frac{dt}{df}$$

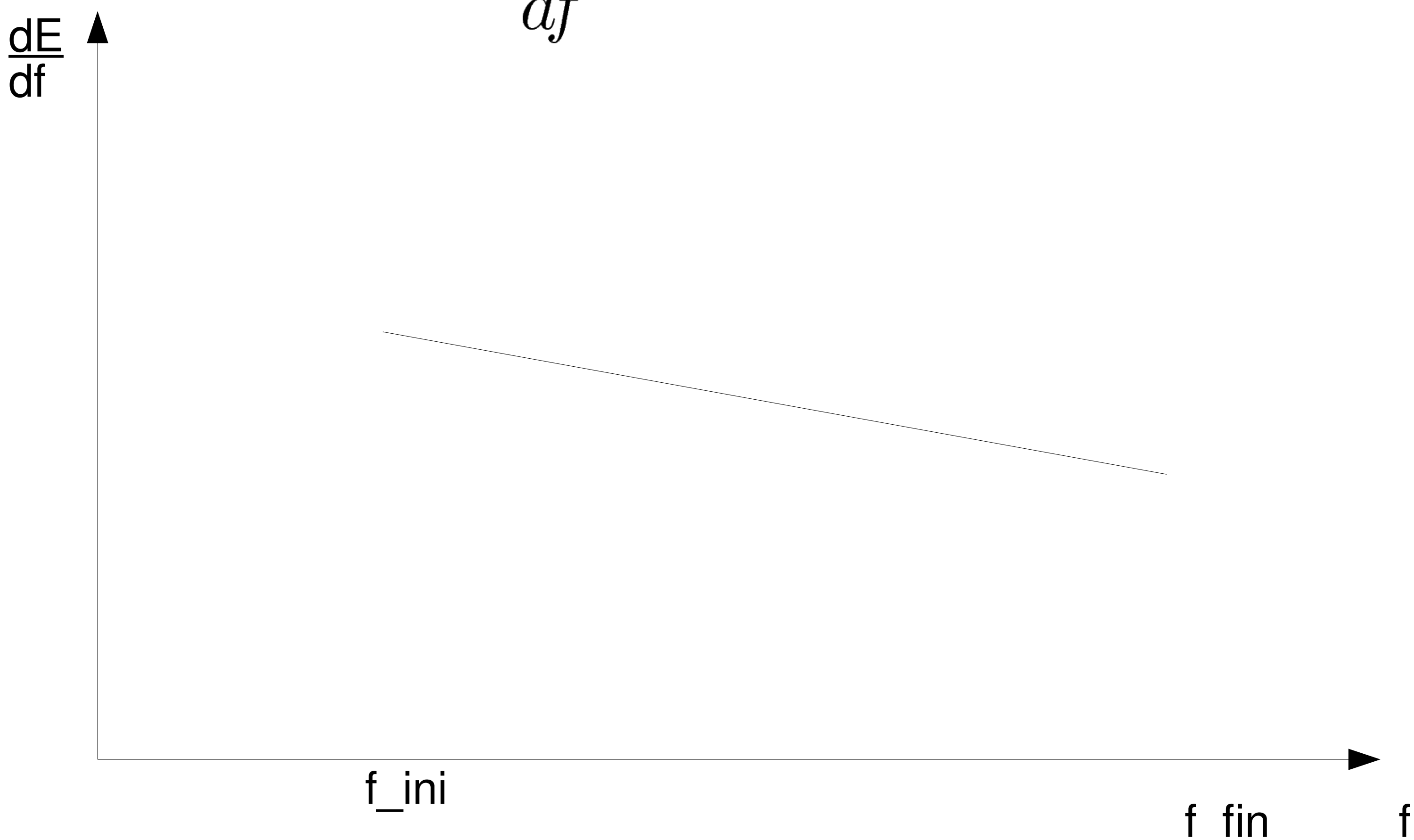
$$f = \frac{2}{P}$$

$$\frac{dP}{dt} = -\frac{96}{5}2\pi\left(\frac{2\pi G M_{chirp}}{c^3 P}\right)^{5/3}$$

$$L_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}$$

Spectrum from a binary

$$\frac{dE}{df} \propto f^{-1/3}$$



Pulsars

- Asymmetry needed
- Instabilities
- Stellar oscillations

Rotating neutron stars

$$L_{GW} = \frac{32}{5}\frac{G}{c^5}\Omega^6(I_1 - I_2)^2$$

$$I_1 - I_2 = I\epsilon$$

Emission at f and 2f

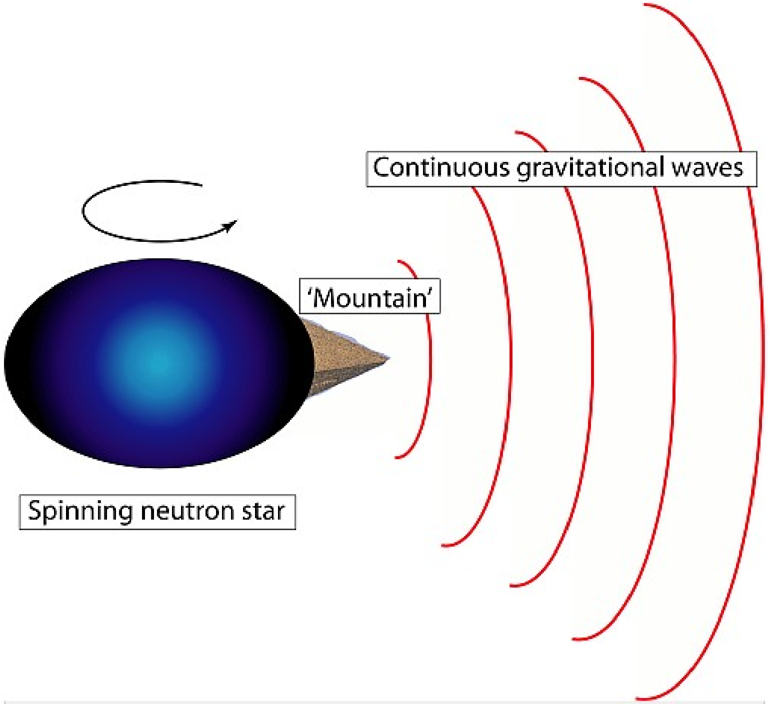
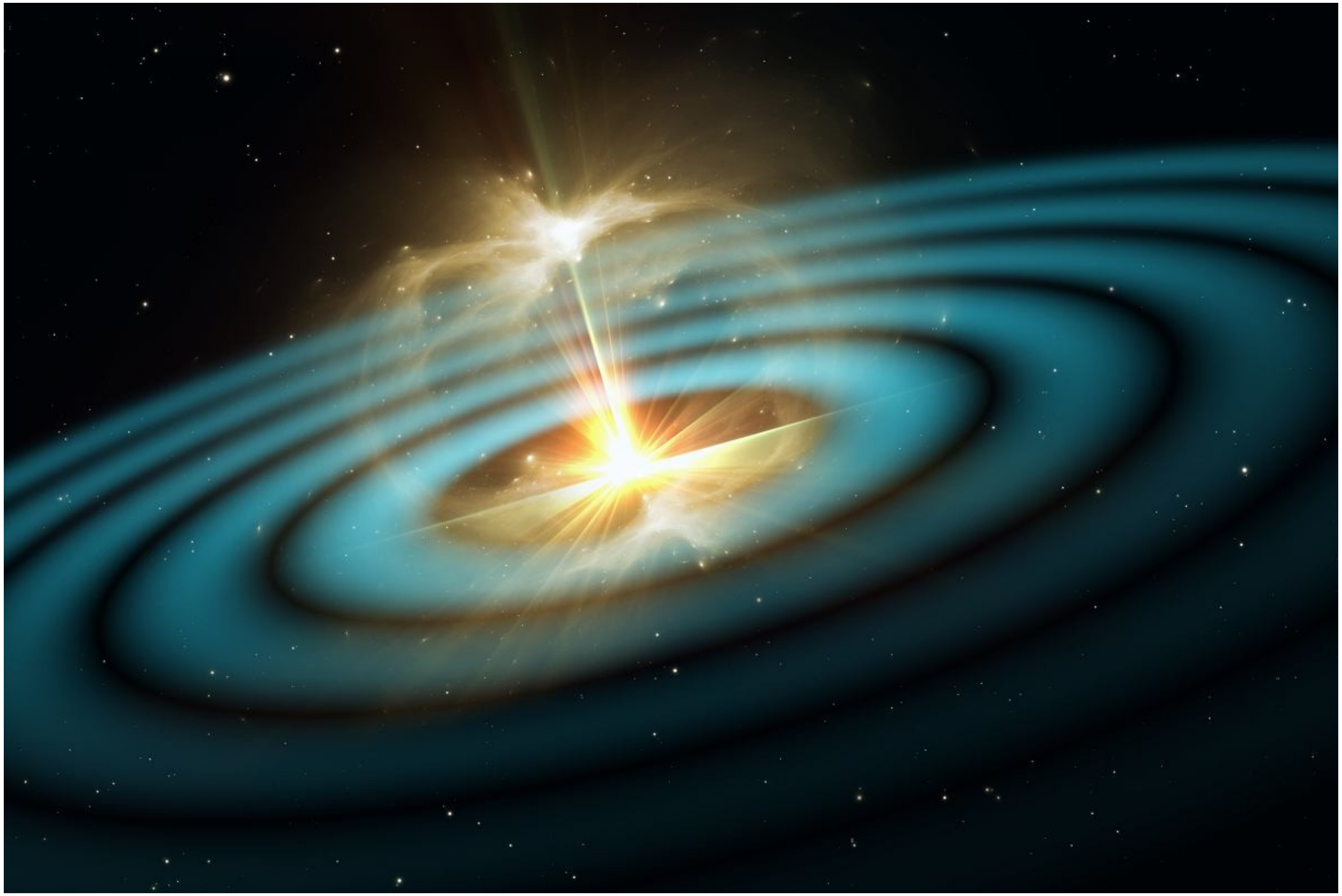


Image Credit: https://astrobites.org/2018/01/29/hunting-for-gravitational-waves-from-spinning-neutron-stars/

Supernovae

- Asymmetry
- Range
- Numbers



Summary

- Baseline of physics
- Detectors
- Sources

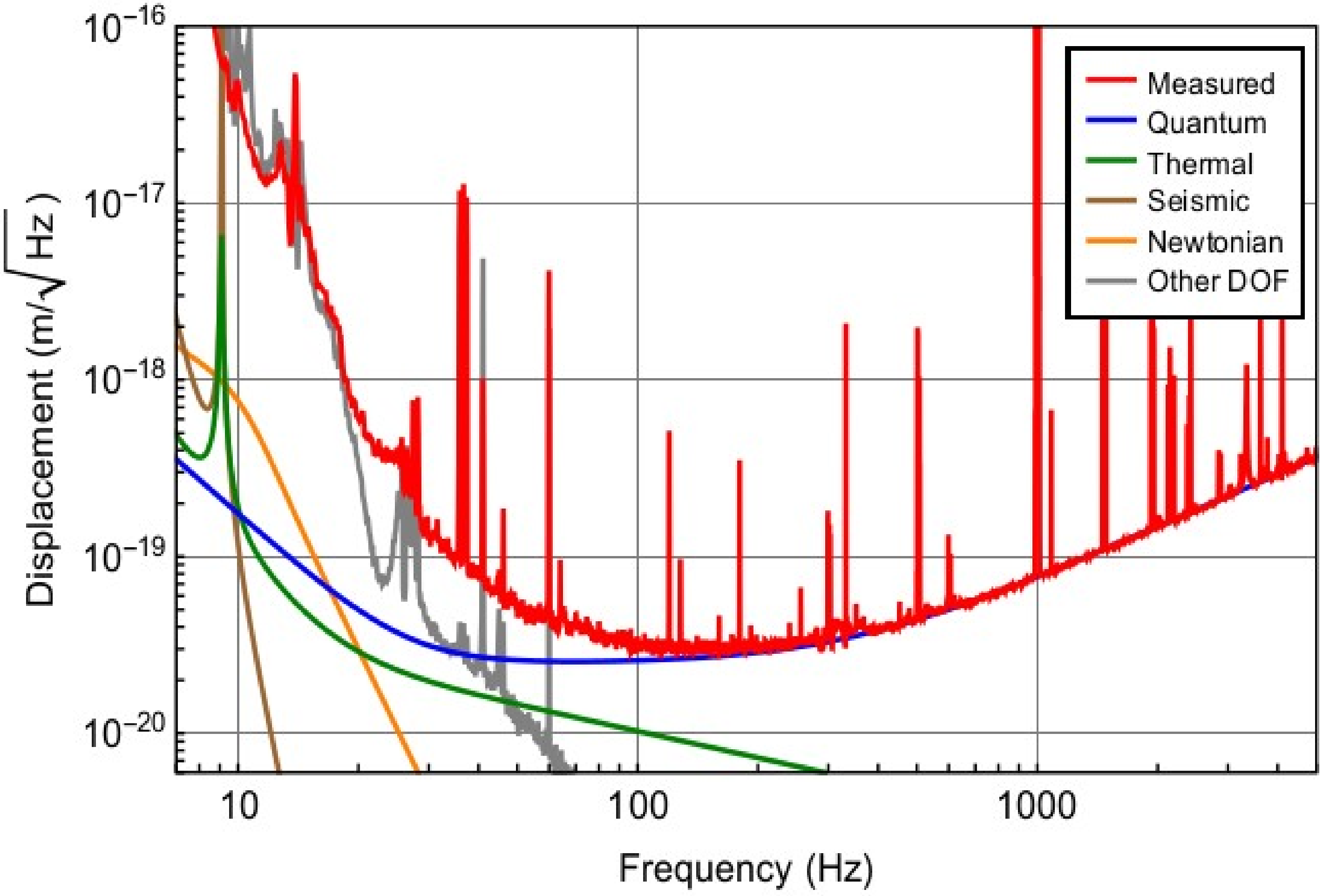
First detection

LIGO detectors

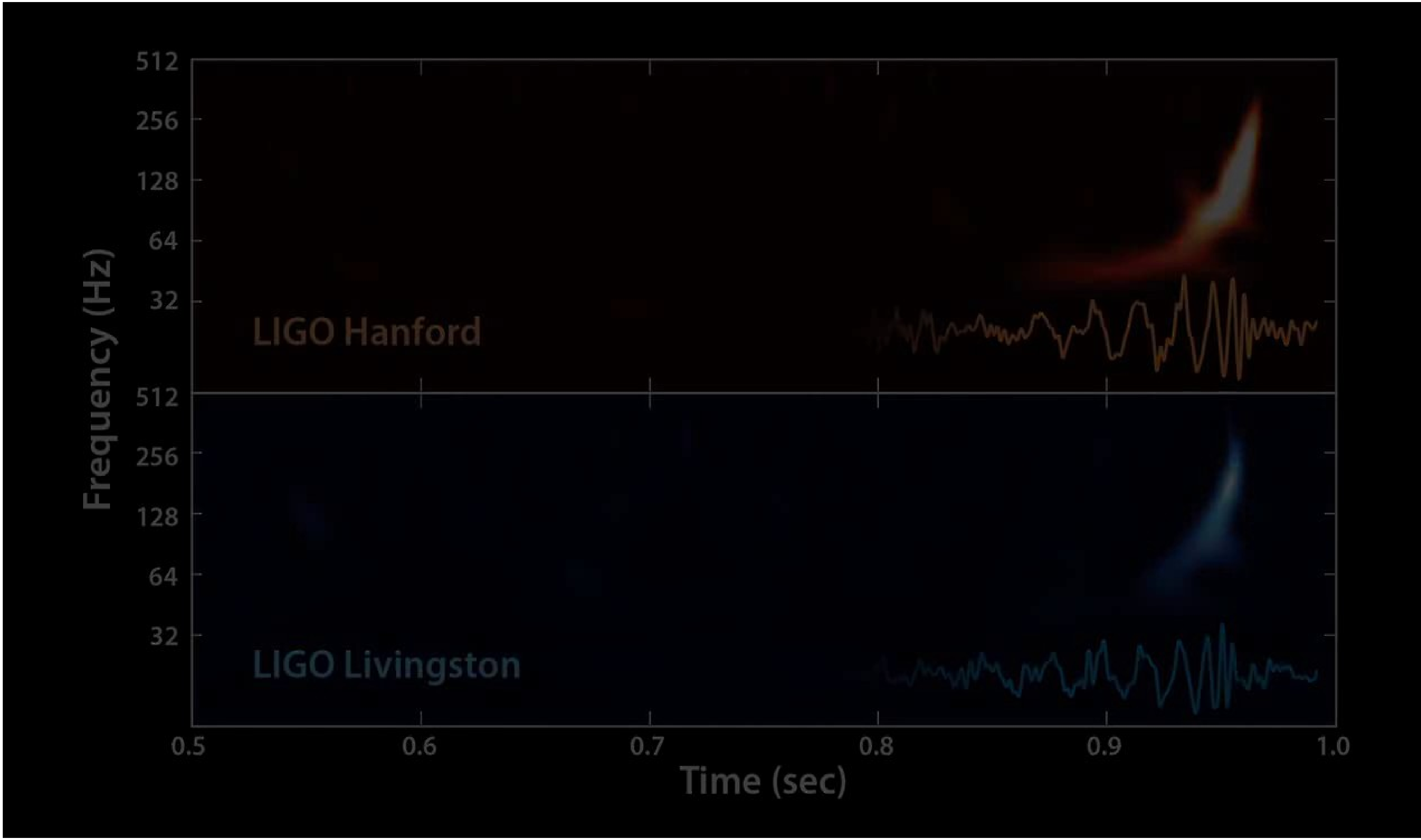
Livingstone, LA

Hanford, WA

Sensitivity in 2015

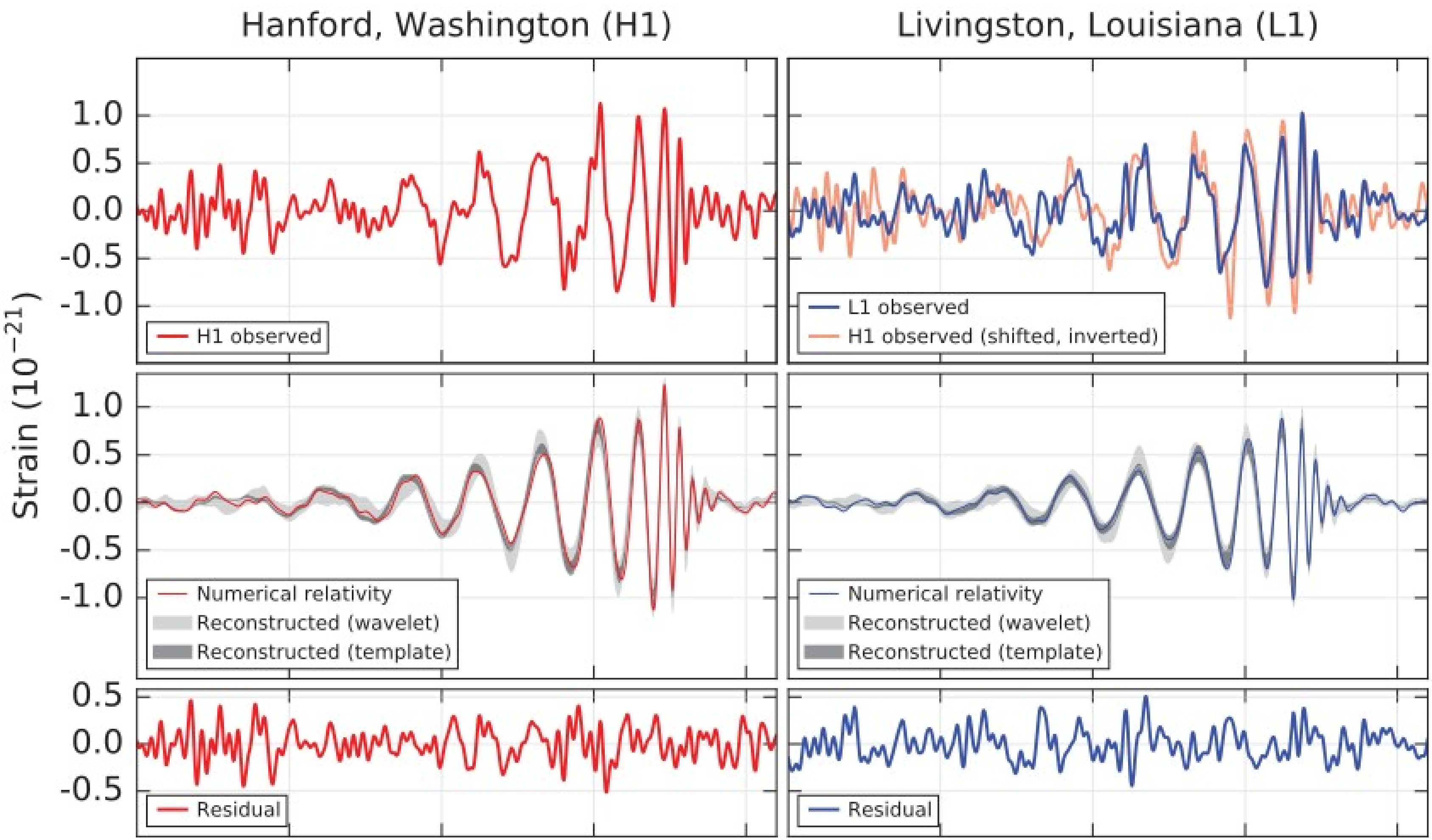


$$h_{min}(100\mathrm{Hz}) \approx 10^{-22}$$



512
256
128
64
32
LIGO Hanford
512
256
128
64
32
LIGO Livingston
Frequency (Hz)
0.5
0.6
0.7
0.8
0.9
1.0
Time (sec)

14 September 2015

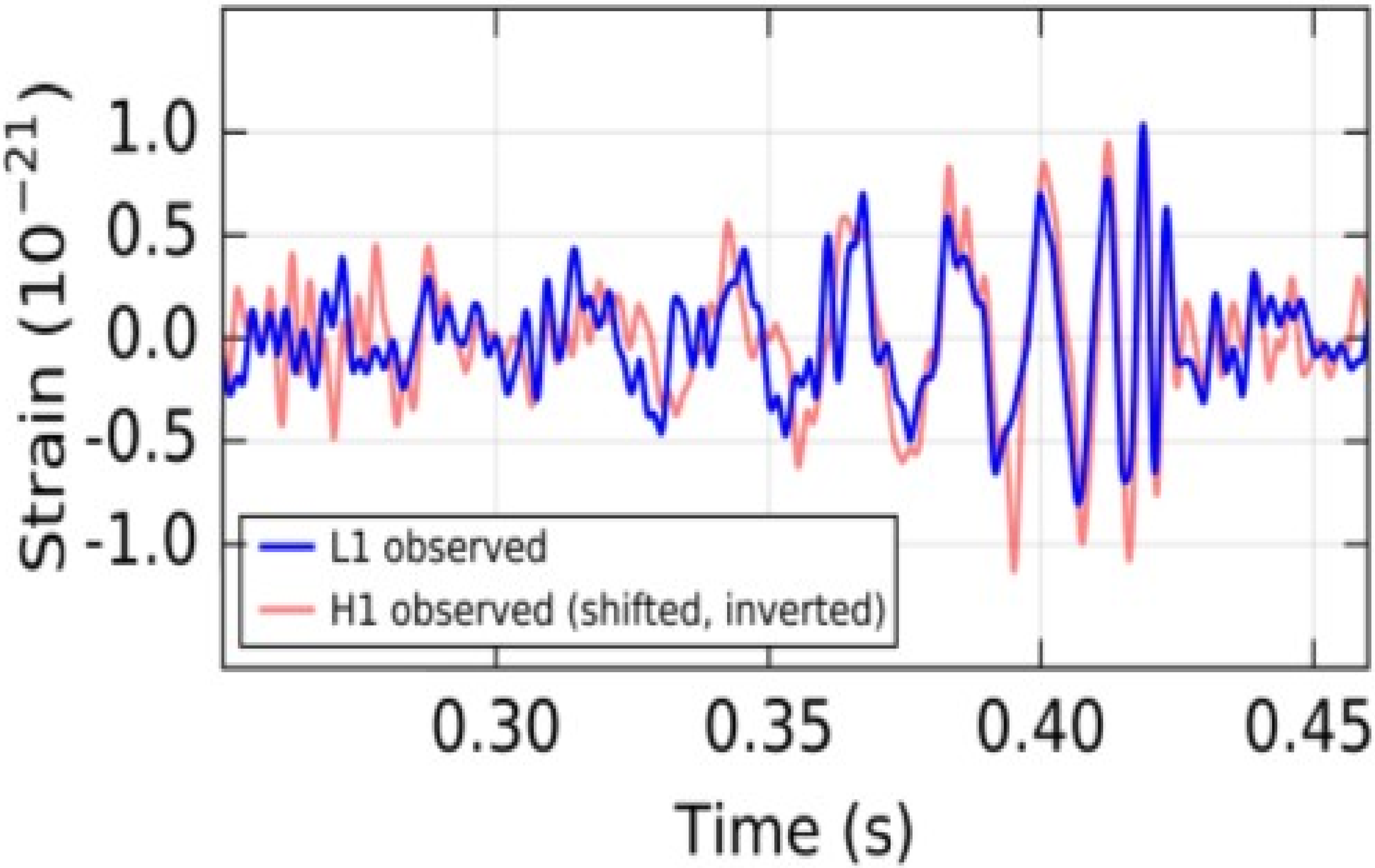


GW150914

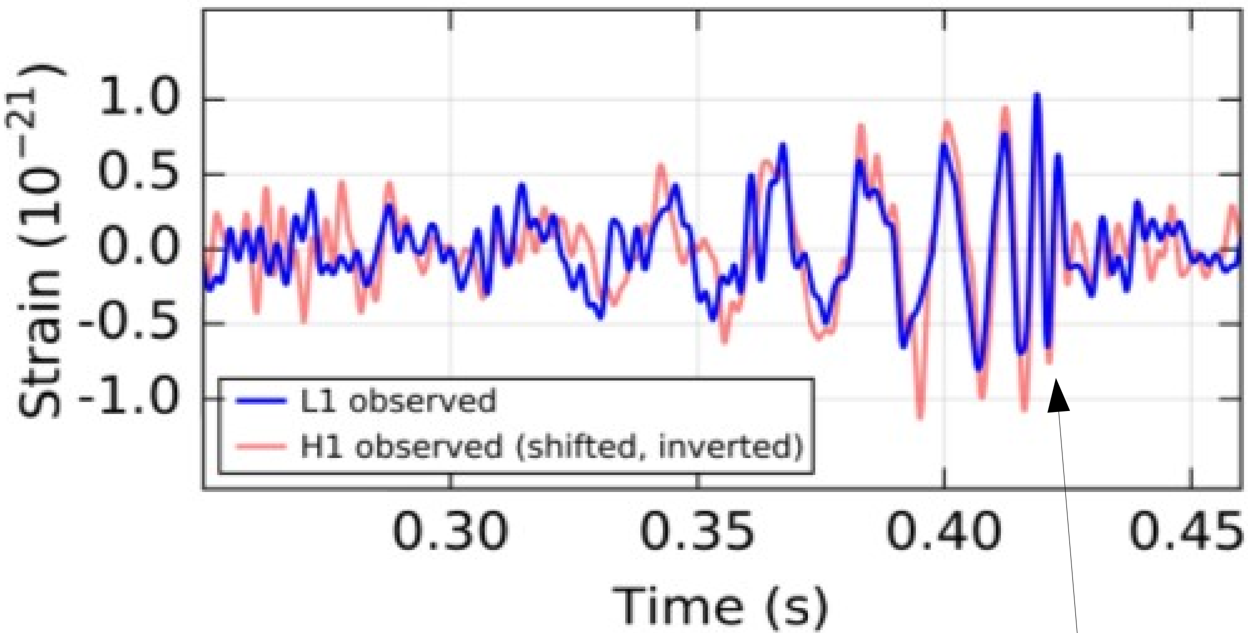
- Significant signal in two detectors
- Coherent signal
- 6.9 ms delay with the distance of 3000km (10ms)

- Real astrophysical signal!

What is it?


Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (s)
L1 observed
H1 observed (shifted, inverted)

Maximum frequency



$$P_{min} \approx 7\text{ms} \qquad f_{GW,max} = 150\text{Hz}$$

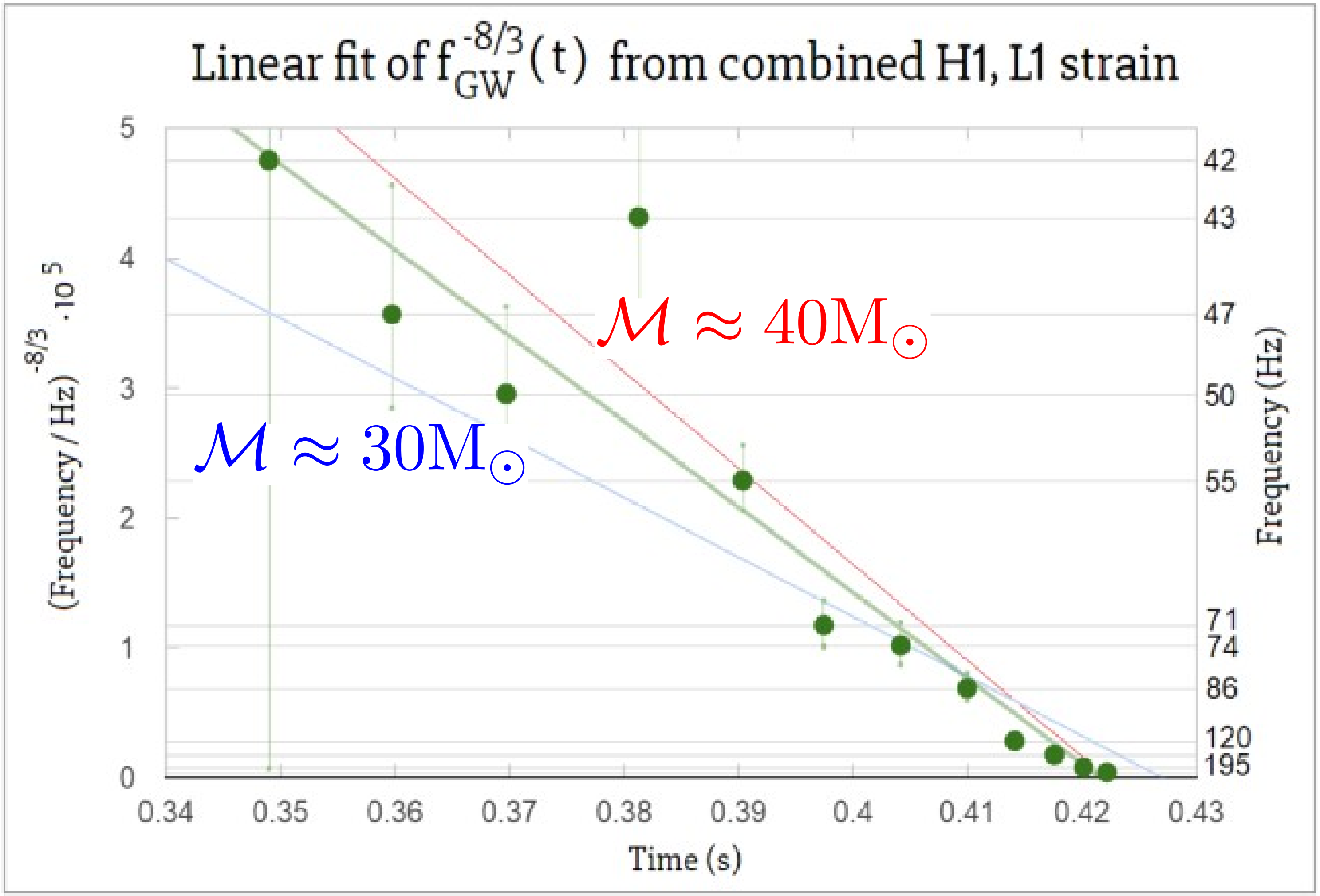
$$\omega_{Kep,max} = 2\pi \frac{f_{GW,max}}{2} = 2\pi \times 75\text{Hz} \qquad \omega = \sqrt{\frac{GM}{A^3}}$$

Rate of frequency change

$$\mathcal{M}^5 = \frac{c^3}{G}\left(\frac{5}{96}\right)^3 \pi^{-8} f_{GW}^{-11} \dot{f}_{GW}^3$$

Chirp mass: $$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$f_{GW}^{-8/3} = \frac{(8\pi)^{8/3}}{5}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}(t_c - t)$$



Chirp mas: $\mathcal{M} \approx 37\mathrm{M}_\odot$

If $m_1 = m_2$ then $m_1 = m_2 = 2^{1/5}\mathcal{M} = 42.5\mathrm{M}_\odot$

Linear sizes

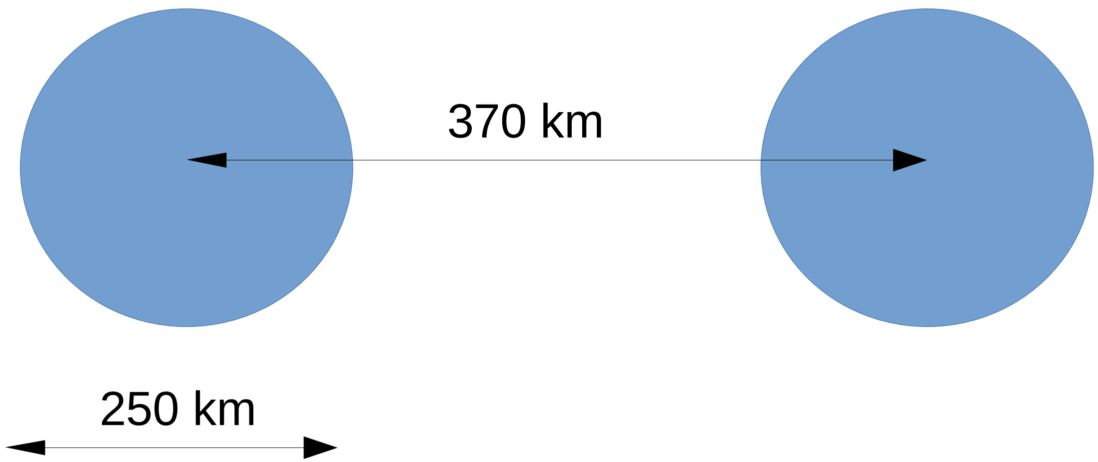
$$R_{Sch} = \frac{2GM}{c^2} = 2.95\,\frac{M}{M_\odot}\,\mathrm{km}$$

$$R_{sch}(42.5M_\odot) = 125\mathrm{km}$$

Orbit size:

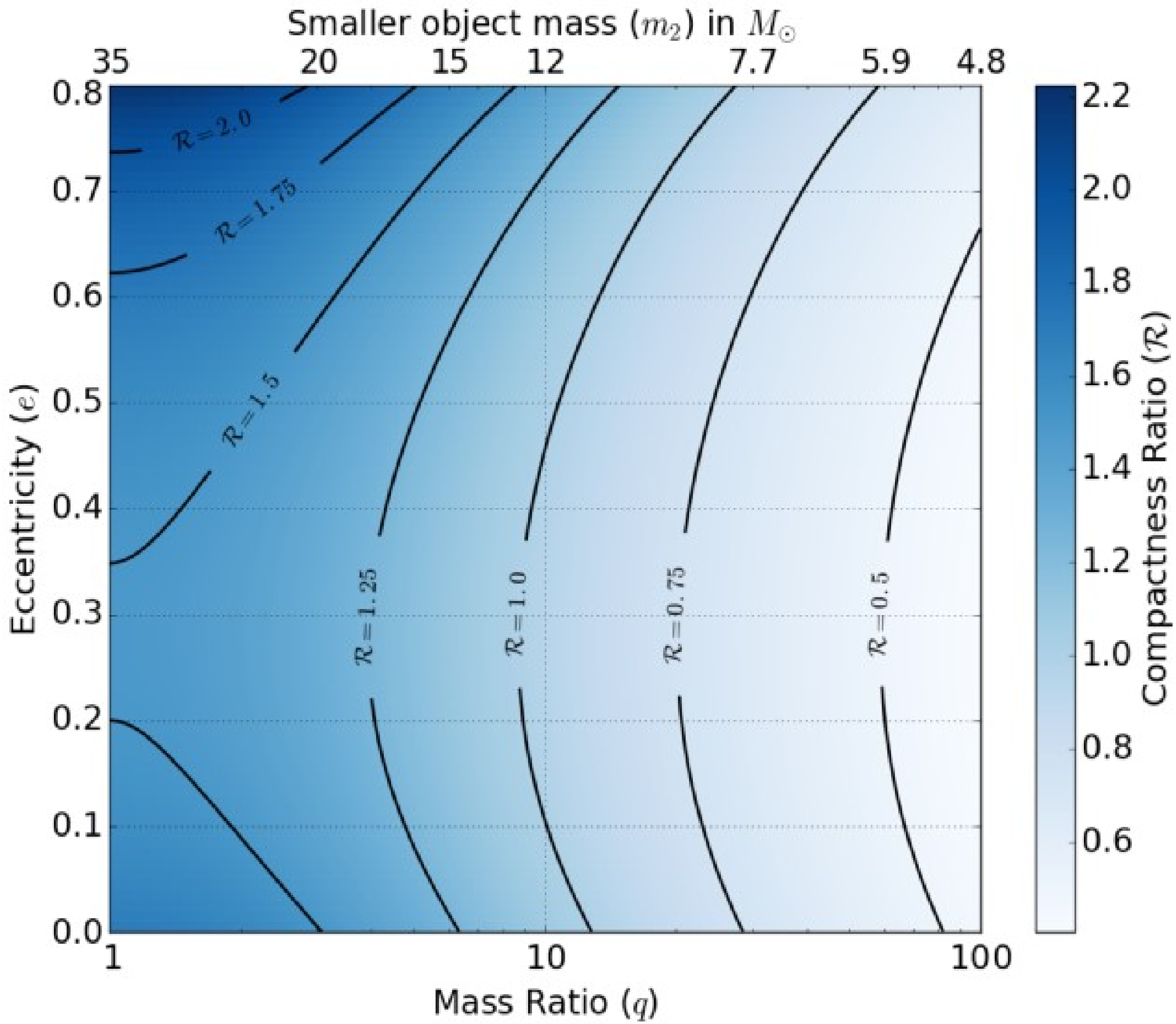
$$A = \left(\frac{GM}{\omega^2}\right)^{1/3} = 370\,\mathrm{km}\left(\frac{M}{85M_\odot}\right)^{1/3}\left(\frac{f}{150\mathrm{Hz}}\right)^{-2/3}$$

Compactness

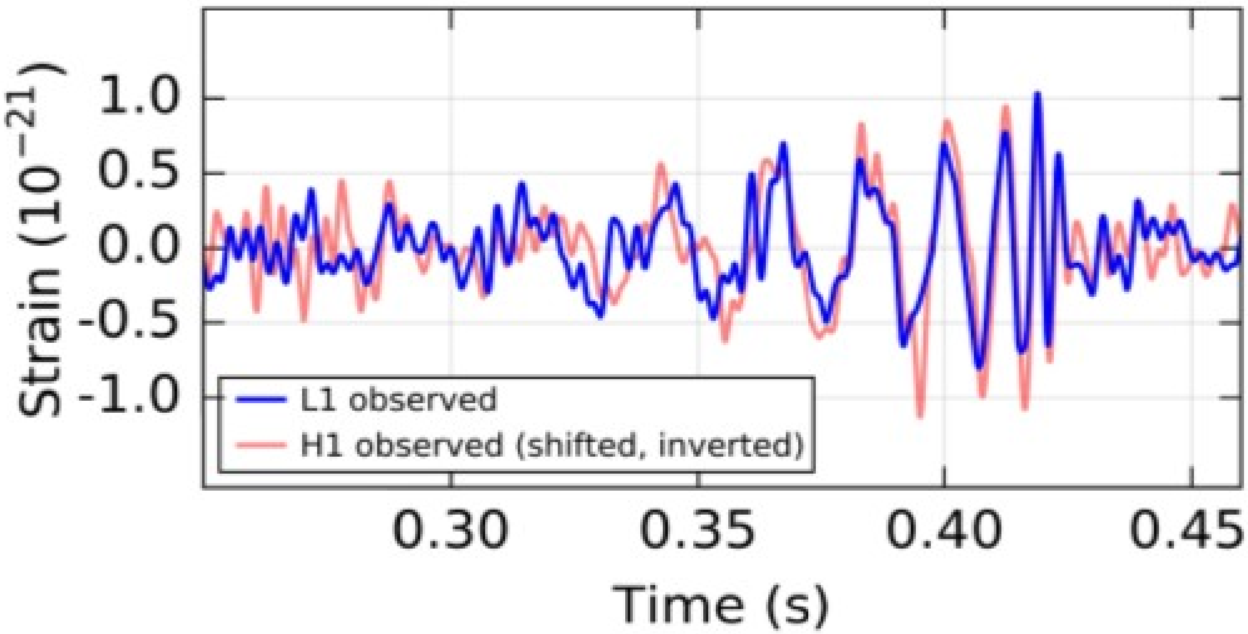


$$\mathcal{R} = \frac{R}{2R_{Sch}} \approx 1.7$$

No equal masses?



Energy flux



Amplitude:

$$h_{max} \approx 10^{-21}$$

$$L_{GW} = \frac{c^3}{16\pi G} \int\int |\dot{h}|^2 dS \approx \frac{c^5}{G} \frac{d_L^2 h^2 \omega_{GW}^2}{4c^2}$$

$$F_{GW} \approx \frac{c^5}{G} \frac{h^2 \omega_{GW}^2}{16\pi c^2} \approx 4 \frac{\mathrm{erg}}{\mathrm{cm}^2 \mathrm{s}}$$

Maximum luminosity

$$L_{GW} = \frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6$$

We assume:

$$v = \omega r = 0.5c$$

$$r = r_{ISCO} = 6\frac{GM}{c^2}$$

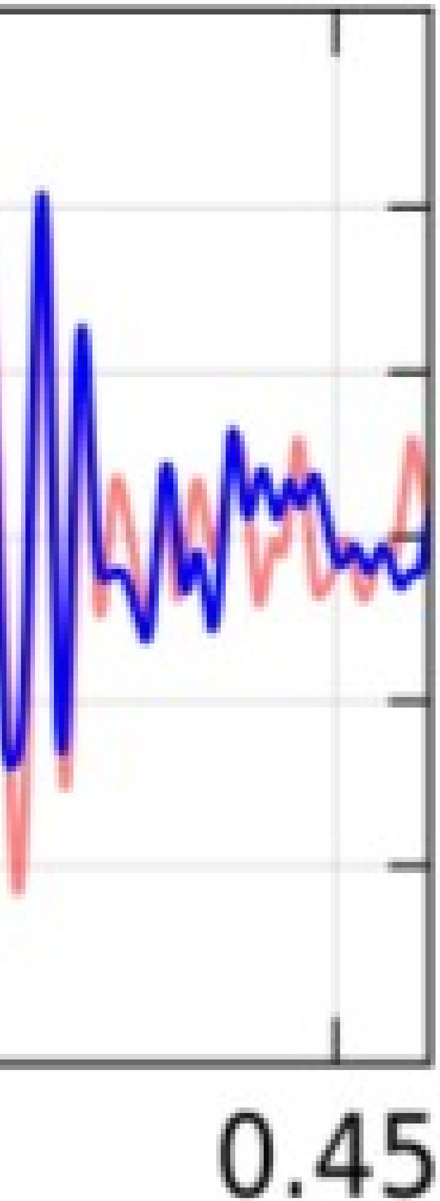
$$L_{GW} = 0.2 \times 10^{-3}\frac{c^5}{G}$$

$$L_{Planck} = \frac{c^5}{G} = 3.6 \times 10^{52}\mathrm{W}$$

Distance – standard sirens

Comparing the theoretical and observed flux

$$d_L = 300\mathrm{Mpc}\left(\frac{f_{GW}}{150\mathrm{Hz}}\right)^{-1}\left(\frac{h_{max}}{10^{-21}}\right)^{-1}$$



Ringdown

New BH reaches the Kerr solution:

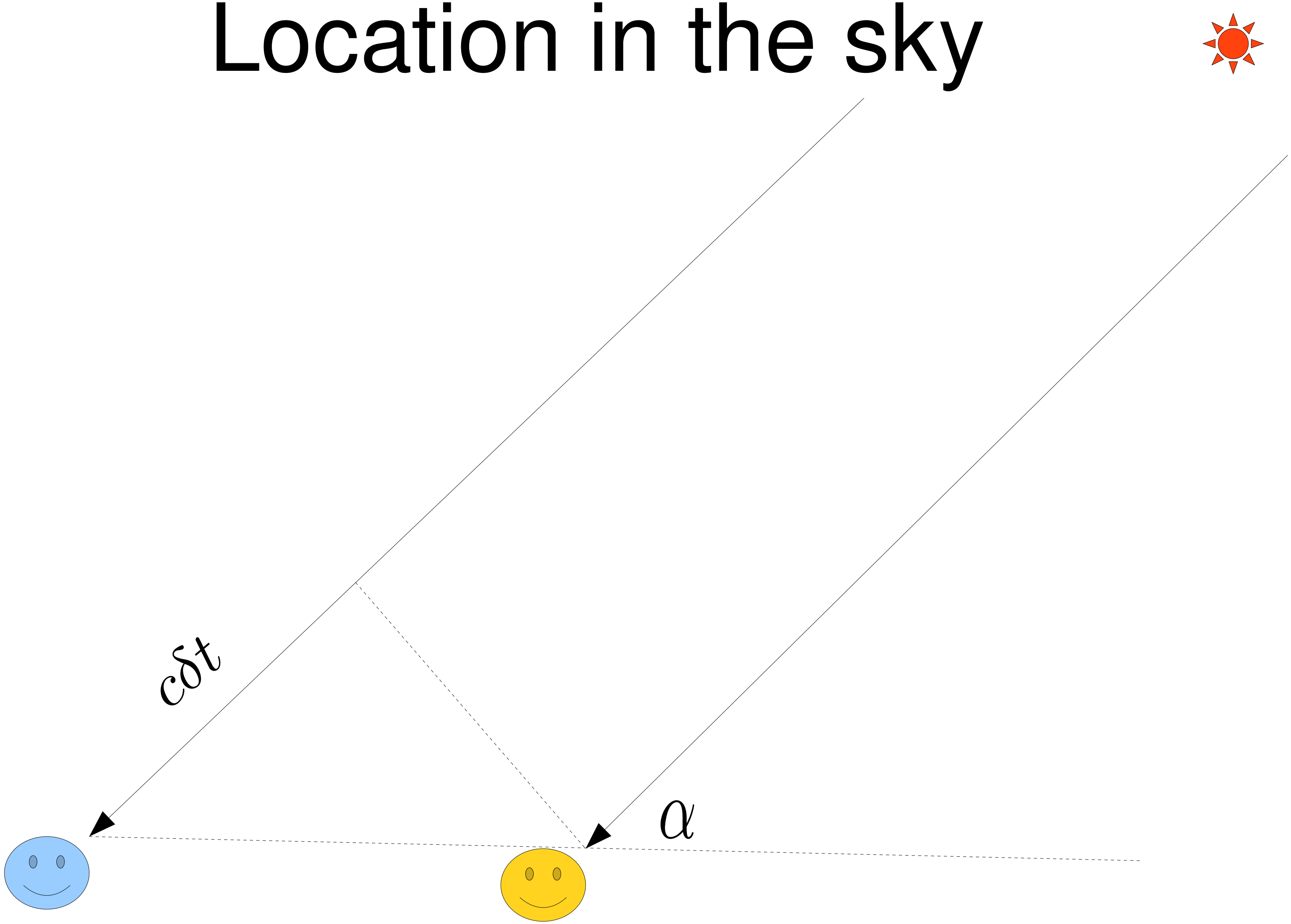
$$\frac{GM}{c^3}\omega_R = x + iy = 0.53 + 0.081i$$

Mod l=2 m=2 n=0, for BH with spin 0.7

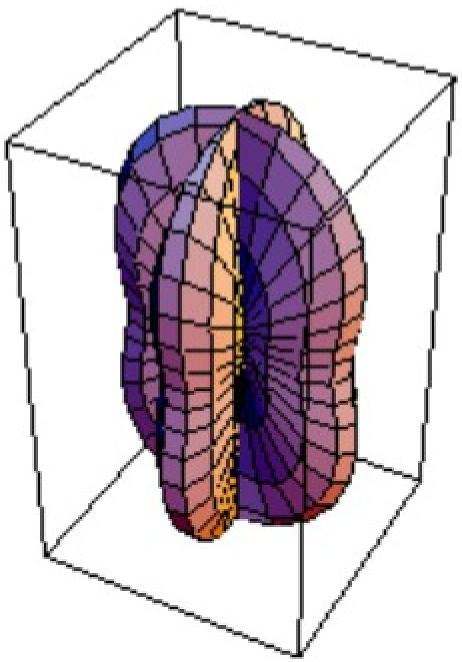
$$f_{GW} = 210Hz\left(\frac{80M_\odot}{M}\right)$$

$$\tau_{damp} = 5ms\left(\frac{M}{80M_\odot}\right)$$

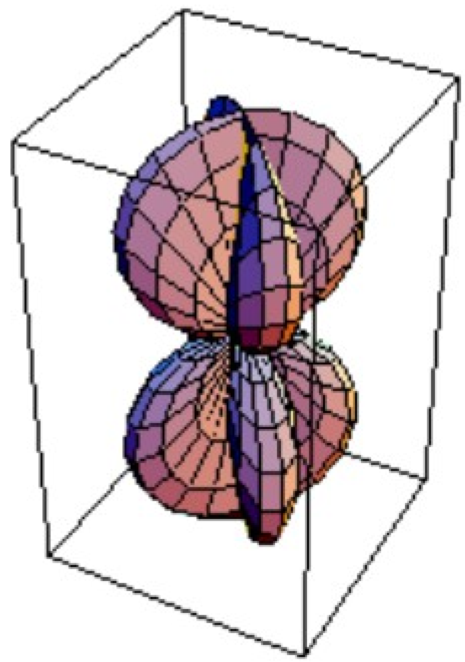
Location in the sky



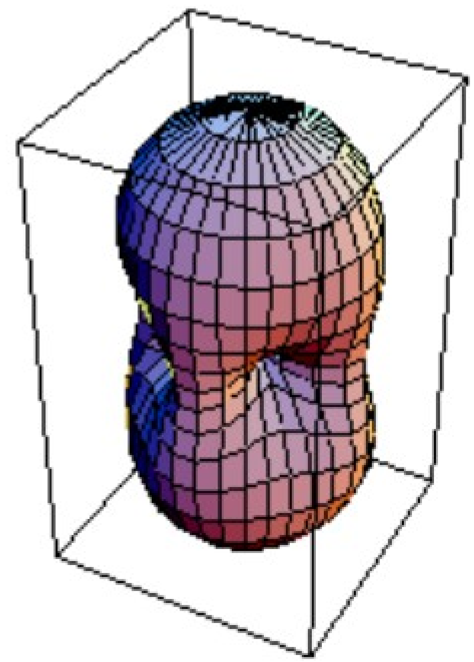
Antenna pattern



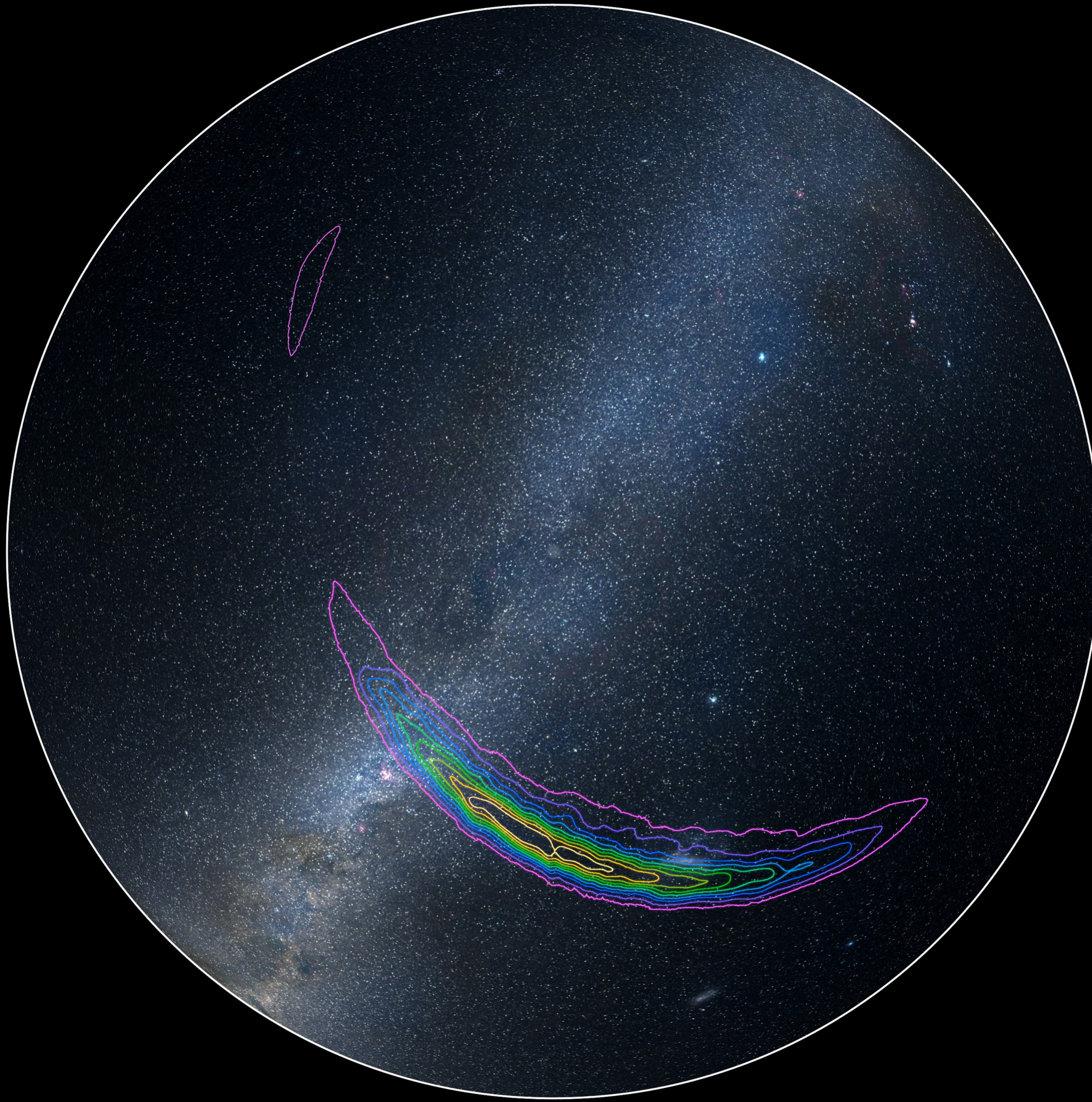
+ polarization



× polarization

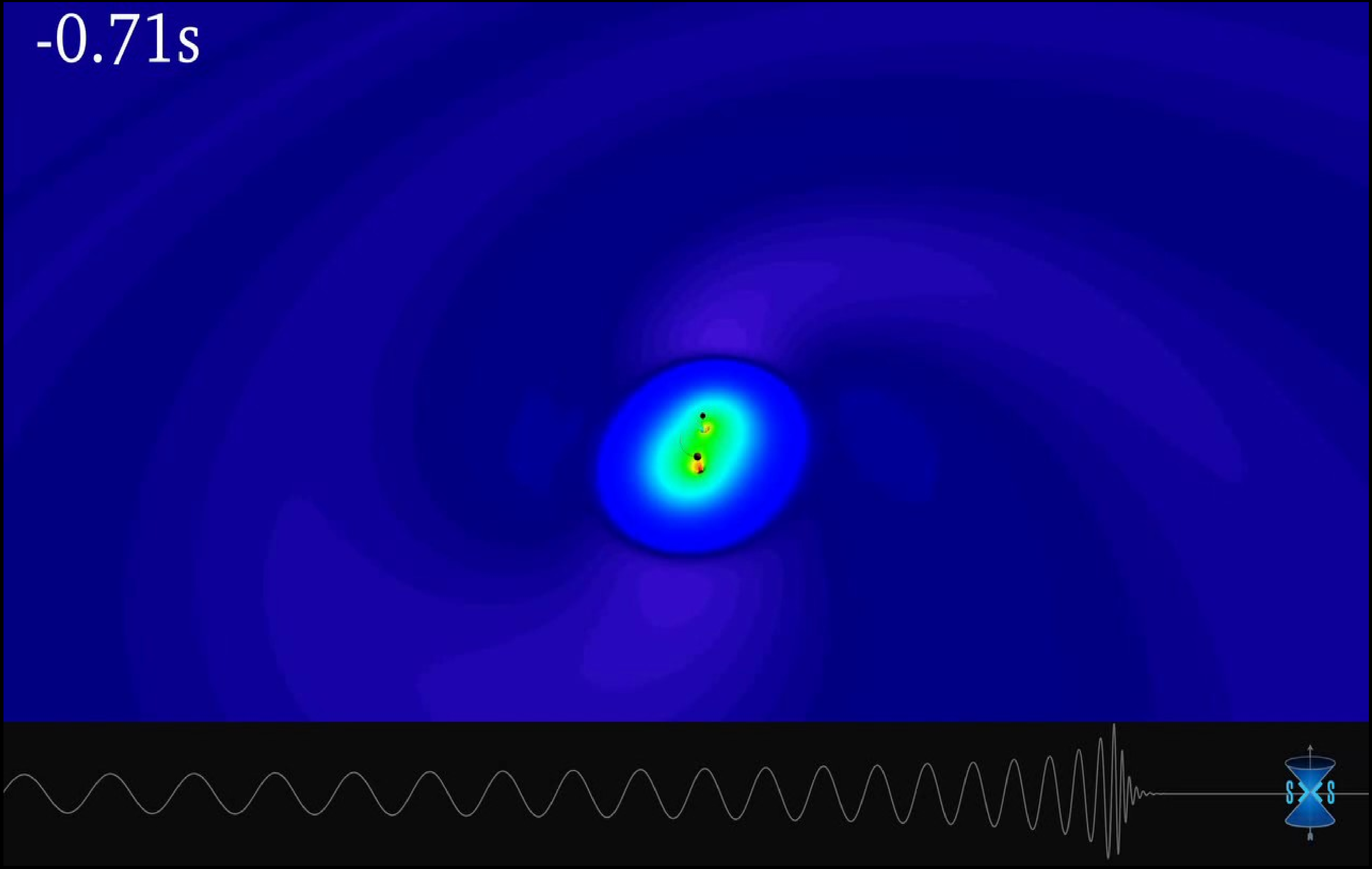


unpolarized



Results of the detailed analysis:

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$


-0.71s

What is new?

- GW detection
- Direct detection of BH formation
- Detection of the first BH binary
- Black holes with masses 30 i 60 solar mass
- Testing GR
- The brightest source ever seen

$$L_{GW} = 200^{+30}_{-20} \mathrm{M}_{\odot} s^{-1} = 3.6^{+0.5}_{-0.4} \times 10^{49} \mathrm{W}$$

Summary

- GW – astronomy – 10 years of discoveries
- Mainly high frequency (LVK), but
  - PTAs are exciting
  - LISA is coming
- Future observatories in the making
  - Einstein telescope
  - Cosmic Explorer

Further reading

- M. Maggiore, “Gravitational Waves”, Oxford University Press
- Misner, Thorn and Wheeler: “Gravitation”, Freeman
- Sensitivities; GWPlotter, http://gwplotter.com/
- LVK publications