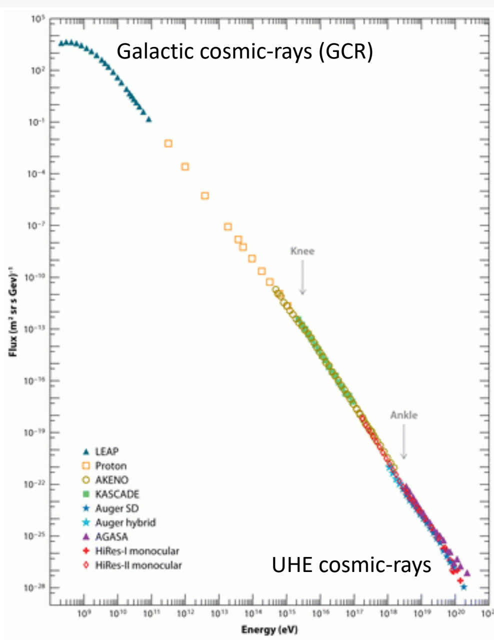
Cosmic ray -- Theory

Martin Lemoine
Astroparticule & Cosmologie (APC)
CNRS – Université Paris Cité

Outline:

Hands-on 2025 – PhD Summer School @ LNGS – Sept. 2025

The all-sky cosmic-ray spectrum



Beatty JJ, Westerhoff S. 2009.

→ cosmic rays: a flux of relativistic charged particles propagating through Galactic and extra-Galactic media

- spans ~12 orders of magnitude in energy

  ~10^4 particles/m^2/s at 1 GeV…
  ~1 particle/km^2/century at 10^{11}GeV…

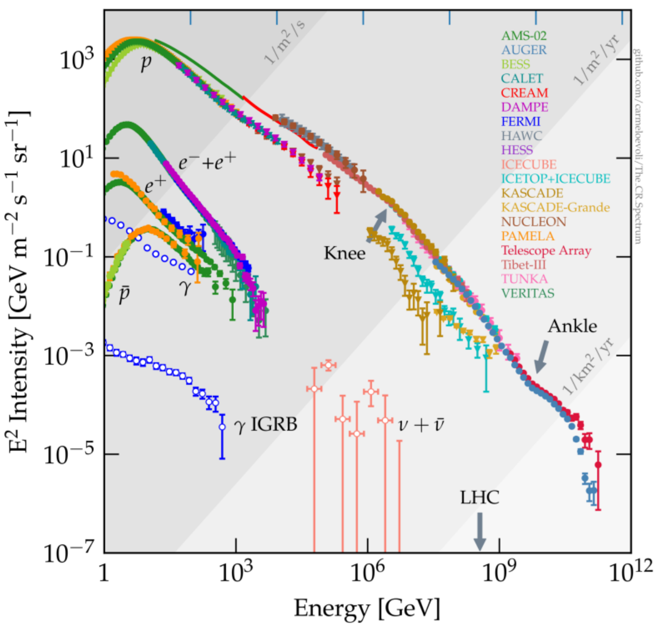
- an approximate powerlaw (with features!)

$$\frac{\mathrm{d}n}{\mathrm{d}\varepsilon} \propto \varepsilon^{-2.7\to -3.3}$$

→ origin of the bulk of cosmic rays: most likely acceleration at the external shock wave of supernovae remnants, through diffusive Fermi acceleration

→ origin of VHE – UHE cosmic rays: some other acceleration process ? in some yet unidentified source, e.g. relativistic supernovae, gamma-ray bursts, active galactic nuclei, radio-galaxies etc. ?

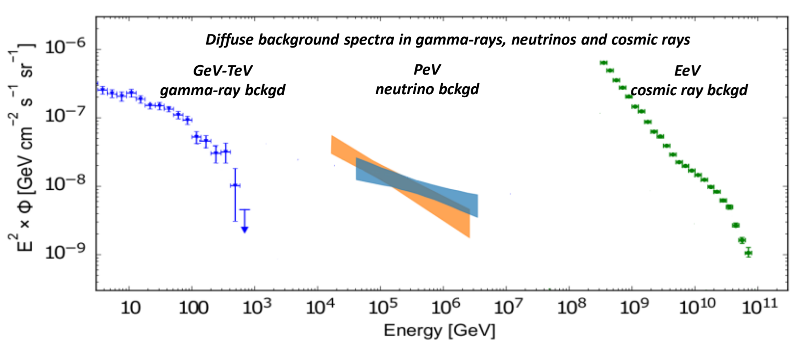
Detailed partition of cosmic ray + multi-messenger all-sky flux



Cosmic rays as high-energy particles: a connection to high-energy multi-messenger astrophysics

→ $\nu - \gamma$ − CR connection: acceleration of ions → cosmic rays, photons and neutrinos

→ *what are the accelerating machine(s) and the acceleration process(es) at work ?*



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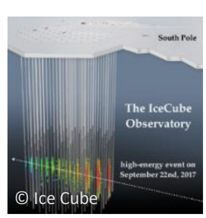


Particle radiation in HE astrophysics: generic processes

→ High energy electrons (and e+): $\gamma_e \equiv E_e/(m_e c^2)$

synchrotron radiation, with typical frequency:

$$\nu_{\rm syn} \sim 10\,{\rm Mhz}\,E_{\rm GeV}^2 B_{\mu{\rm G}}$$

inverse Compton on seed photon: $e+\gamma \rightarrow e+\gamma$

$$E'_\gamma \simeq 2\gamma_e^2 E_\gamma \sim 10\,{\rm keV}\,E_{\rm GeV}^2 E_{\gamma,{\rm CMB}}$$

SSC = synchrotron-self-Compton = IC on synchrotron photons
(+bremsstrahlung)

large loss rates imply that most radiation is of leptonic origin (in general!)

→ High energy protons:

p-p interaction: $p+p \rightarrow p+(p\,{\rm or}\,n)+\pi+\dots$

p-γ interaction: $p+\gamma \rightarrow (p\,{\rm or}\,n)+\pi+\dots$

hence neutrino production: $\pi^+ \rightarrow \mu^+ + \nu_\mu$
$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$

and gamma production: $\pi^0 \rightarrow \gamma+\gamma$

(+pair production p+γ→p+e+e with small inelasticity, possibly synchrotron radiation in strong magnetic fields, etc.)

astrophysical ν: unambiguous signature of hadron acceleration

Definitions

A cosmic ray ≡ a relativistic charged particle engaged in a long-term relationship with electromagnetic fields (E, B) ...

→ $\frac{{\rm d}\boldsymbol{p}}{{\rm d}t} = q\left(\mathbf{E}+\frac{\boldsymbol{v}}{c}\times\mathbf{B}\right)$

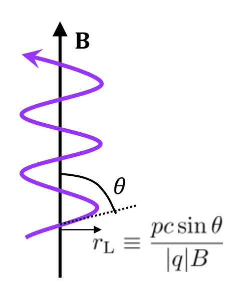
$\boldsymbol{p} \leftrightarrow \mathbf{E}$ describes acceleration because $\dot{\varepsilon} = q\,\boldsymbol{v}\cdot\mathbf{E}$
$\boldsymbol{p} \leftrightarrow \mathbf{B}$ describes transport because $|\mathbf{E}| \ll |\mathbf{B}|$ in general

→ defined by its mass m, charge q=eZ, energy $\varepsilon \approx pc$ / momentum $\boldsymbol{p}$ / Lorentz factor γ and gyroradius $r_g \simeq pc/|q|B$

note: use of Gaussian c.g.s. units

pitch-angle θ, cosine $\mu \equiv \cos\theta$ defines orientation wrt $\mathbf{B}$: $p_\parallel \equiv \boldsymbol{p}\cdot\mathbf{B}\;/\,p\,B = p\,\mu$
$p_\perp = p\,(1-\mu^2)^{1/2}$

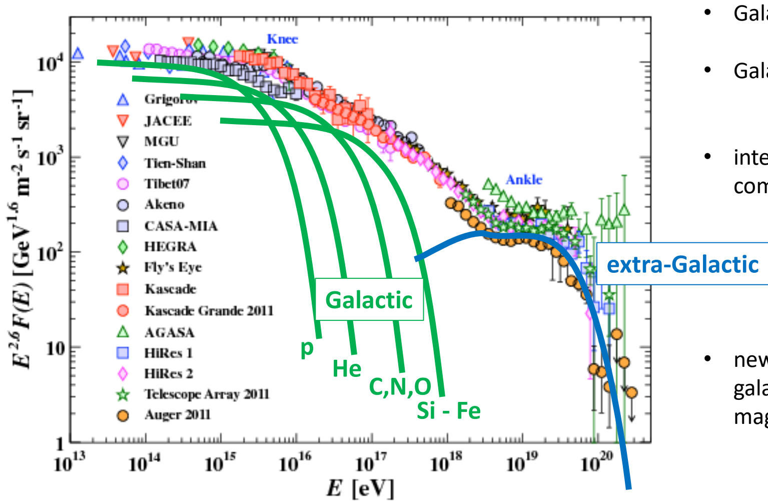
gyrofrequency: $\Omega \equiv |q|Bc\,/\,\varepsilon = |q|B/\gamma mc$, Larmor radius: $r_L \equiv p_\perp c/|q|B$



→ interactions with e.m. fields scale with ε/Ze (rigidity)

→ one-to-one relation between energy ε or momentum p and length scales (r_g): $r_g \simeq 1$ pc $\varepsilon_{\rm PeV}\,B_{\mu{\rm G}}^{-1}\,Z^{-1}$

Features and standard lore of all-sky cosmic-ray flux

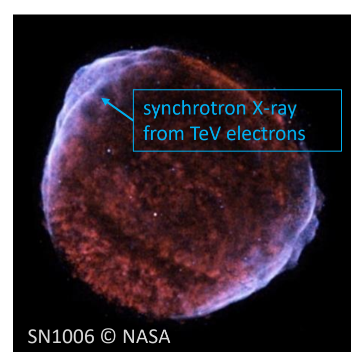


<u>Cosmic ray diffuse flux:</u>

- Galactic CR protons: maximum energy $E \sim$ PeV
- Galactic heavier species: max energy $E \sim Z$ PeV
- intermediate region: between 0.1 and 1 EeV, a new component (Galactic, extra-Galactic)?
- new component around 10^{18}eV = 1 EeV, extra-galactic: protons cannot be confined in Galactic magnetic field.

Origin of (the bulk of) Galactic cosmic rays: supernova remnants (SNR), most likely

Source: particles can be accelerated to high energies at the shock front of supernova remnants …



Order of magnitude (OoM) for energetics: assume that

- each supernova (Galactic rate $r_{SN} \sim 2/100$ yr) injects 10% of its shock kinetic energy ($E_{SN} \sim 10^{51}$ erg) in cosmic rays >GeV ⇒ $L_{CR} \sim 10^{41}$ erg/s
- CRs are confined in the Galaxy (volume $V \sim 300$ kpc^3 $\sim 10^{67}$ cm^3)(?) for a duration $\tau_{esc} \sim 10^7$ yrs(?)…

then, average energy density of CRs: $u_{\rm cr} \sim \frac{L_{\rm CR}\tau_{\rm esc}}{V} \sim 1\,{\rm eV\,cm^{-3}}$

Spectrum:
accelerated particle energy distribution inferred around SNR of the form $dn/d\varepsilon \propto \varepsilon^{-2.3}$ … if $\tau_{\rm esc} \propto \varepsilon^{-0.4}$, as observed(?), all-sky spectrum recovered… at least up to ~the knee…

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Outline:

1. Introductory remarks
2. Cosmic ray transport
   2.1 General overview
   2.2 Transport physics
   + current topics / emerging trends
3. Cosmic ray acceleration
   3.1 General overview
   3.2 Acceleration scenarios
   + current topics / emerging trends

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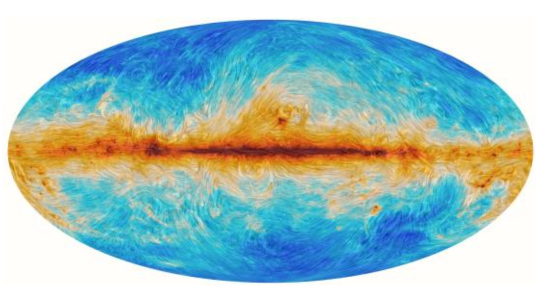
Cosmic-ray confinement in the Galaxy

The magnetized Milky way:



Polarized emission from dust © Planck

… a mean magnetic field of strength $B \sim O(\mu\mathrm{G})$ with a complex morphology, incl. spiral pattern in disk, halo component etc, with typical curvature scale $\sim O(\mathrm{kpc})$

… an irregular component associated with turbulence, characterized (at least) by outer scale $\ell_c \sim O(100\mathrm{pc})$ and amplitude $\delta B/B \sim O(1)$ …

$\Rightarrow$ magnetic fluctuations on all scales, from $\sim O(1\mathrm{AU})$ to $\sim O(100\mathrm{pc})$, with characteristic scaling $\delta B_l \sim \delta B\ (l/\ell_c)^{0.3}$

OoM for transport:

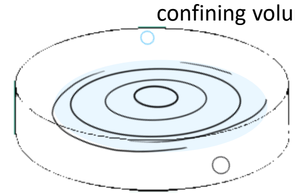
$r_g \sim 1\mathrm{pc}\ \varepsilon_{\mathrm{PeV}} B_{\mu\mathrm{G}}^{-1} Z^{-1} \ll \ell_c$ $\Rightarrow$ Galactic CRs (GCRs) are "magnetized"… diffuse in Galactic turbulence

$r_g \sim 1\mathrm{kpc}\ \varepsilon_{\mathrm{EeV}} B_{\mu\mathrm{G}}^{-1} Z^{-1} \gg \ell_c$ $\Rightarrow$ UHECRs are not confined in the Galaxy ($\Rightarrow$ extra-galactic origin)

Diffusive transport: random walk with mean free path λ (isotropic?), diffusion coefficient $\kappa \sim \lambda c/3$ (isotropic?)
$\Rightarrow$ confinement time in the Galaxy $\tau_{\mathrm{esc}} \sim H^2/\kappa$ where $H \sim$ characteristic size of confining volume

Cosmic-ray transport in the Galaxy

confining volume



Phenomenological model: ``leaky-box''
↔ CRs are produced in disk (SNRs + …), diffuse in disk and halo, to eventually escape on timescale $\tau_{\rm esc}(\rm p)$…
… during transport, suffer interactions → photons, neutrinos, other nuclei through spallation and nuclear decay, etc.

Methods:

- Monte Carlo: track many individual CRs, modelling transport as a random walk (Brownian motion)
- Solve a transport equation describing the evolution of the CR distribution function

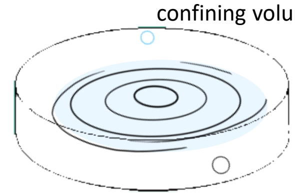
Transport equation: in its simplest form, for species s

$$\partial_t f_s \ = \partial_{x^i} D_{ij} \partial_{x^j} f_s \ + \ Q_s(\boldsymbol{r}, t, p) + \dots$$

f_s: distribution function (density $n_s = \int d^3p\, f_s$)
Q_s: injection term, production by sources in Galactic disk
D_{ij}: spatial diffusion coefficient

Cosmic-ray transport in the Galaxy

confining volume



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Improved transport equation: for species s

$$\partial_t f_s + \boldsymbol{v}_{\mathbf{p}} \cdot \nabla_{\boldsymbol{r}} f_s - \frac{1}{3} \nabla_{\boldsymbol{r}} \cdot \boldsymbol{v}_{\mathbf{p}}\, p \partial_p f_s \ = \partial_{x^i} D_{ij} \partial_{x^j} f_s + \frac{1}{p^2} \partial_p\, p^2 D_{pp} \partial_p f_s + Q_s(\boldsymbol{r}, t, p) - \frac{1}{p^2} \partial_p \left(\dot{p} p^2 f_s \right) - \frac{f_s}{\tau_s} + \sum_{s' \to s} \frac{f_{s'}}{\tau_{s' \to s}}$$

advection with gas

adiabatic term

turbulent acceleration

source term: acceleration by SNe

energy losses (e.g. ionization, synchrotron…)

radioactive decay or inelastic collisions with interstellar gas

nuclear decay or inelastic collisions from heavier nuclei

Standard solutions of leaky box… a successful model

Stationary solutions:

→ for « primary » CRs (produced at source):
assuming that sources inject powerlaw spectra with slope s

$$\frac{\mathrm{d}n^I}{\mathrm{d}\varepsilon} \sim Q_{\rm inj}\tau_{\rm esc}\frac{V_{\rm disk}}{V_{\rm conf.}} \propto \varepsilon^{-s}\tau_{\rm esc}$$

→ for « secondary » CRs (produced by interactions of primaries):

$$\frac{\mathrm{d}n^{II}}{\mathrm{d}\varepsilon} \sim \frac{\mathrm{d}n^I}{\mathrm{d}\varepsilon}\frac{1}{\tau_{\rm int}}\tau_{\rm esc}\frac{V_{\rm disk}}{V_{\rm conf.}} \propto \varepsilon^{-s}\tau_{\rm esc}^2$$

→ escape timescale: $\tau_{\rm esc} \sim \frac{H^2}{\kappa}$ H scale height for escape, κ diffusion coefficient

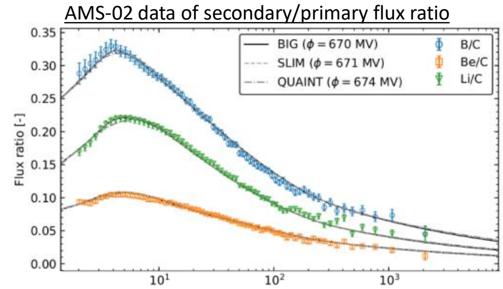
if $\tau_{\rm esc} \propto \varepsilon^{-\alpha}$ then $dn^I/d\varepsilon \propto \varepsilon^{-s-\alpha}$ … vs observations: $s \simeq 2.3,\ s+\alpha \simeq 2.7 \Rightarrow \alpha \simeq 0.4$

Consistency test: measured spectra of secondary species (e.g. Li, Be, B), produced by spallation interactions of primaries with ISM gas, and take ratio to primary spectra to extract $\tau_{\rm esc}$

→ $\tau_{\rm esc} \sim 10^7\mathrm{yrs}\,(\varepsilon/1\mathrm{GeV})^{-0.4}$

… suggesting an average $\kappa(\varepsilon) \propto \varepsilon^{0.4}$ at > 10 GeV

AMS-02 data of secondary/primary flux ratio



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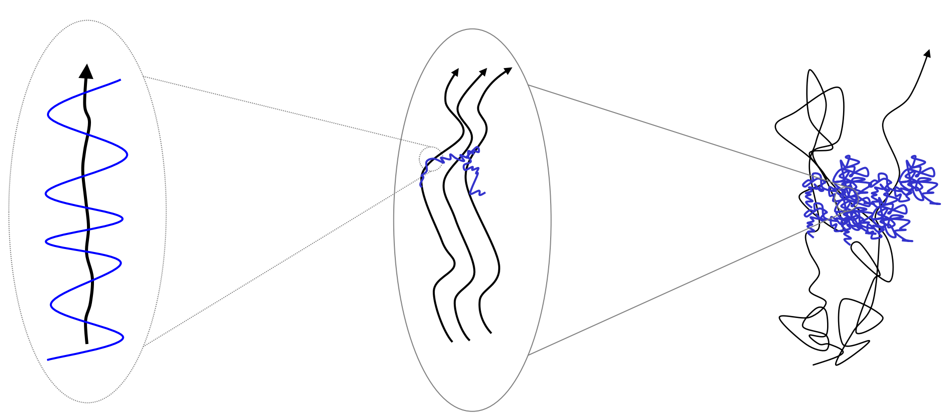
Cosmic-ray transport in magnetized turbulence: toward a microphysical picture(?)

High-energy particle transport in astrophysical plasmas: in collisionless regime, transport regulated by wandering with large-scale field combined with scattering on magnetic perturbations and perpendicular transport...

on short length scales ($< \ell_c$), CRs follow field lines, their pitch angle undergoes a random walk with mean free path:
$\lambda_\parallel(p) = c/\nu_{\rm scatt}$, with $\nu_{\rm scatt} \equiv D_{\mu\mu} = \langle\Delta\mu^2\rangle/2\Delta t$
spatial diffusion coefficient: $\kappa_\parallel(p) \sim c\lambda_s$

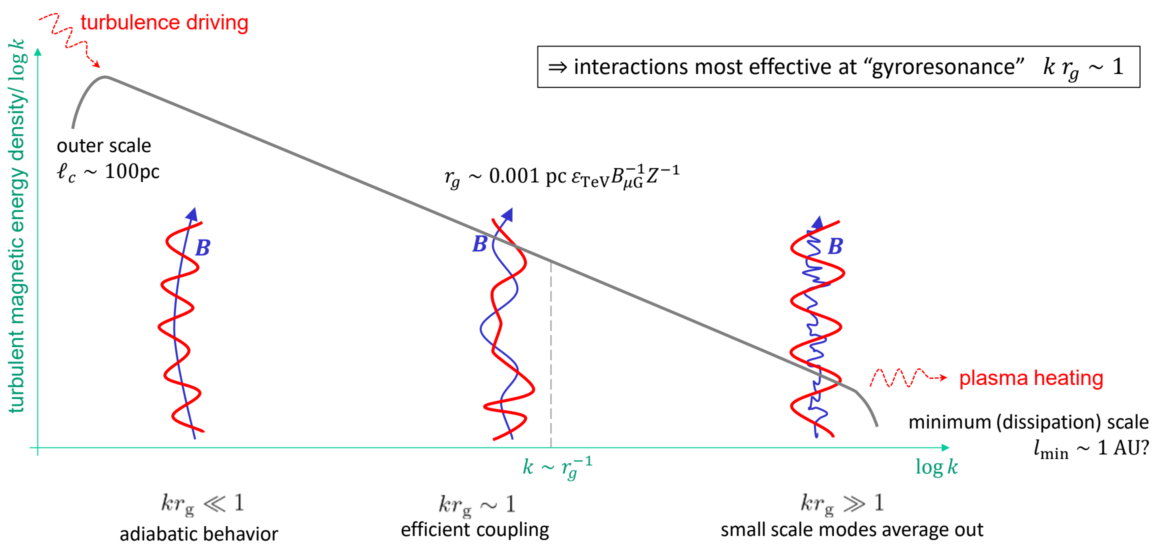
on intermediate scales, CRs diffuse across B…
associated diffusion coefficient $D_\perp$?

field lines themselves diffuse and bend on large scales…
overall diffusion coefficient κ?



Notion of scattering resonance between gyromotion and perturbation wavelength

Particle scattering in a turbulent plasma: particles with r_g in inertial range of turbulence interact with a broad range of modes (i.e. scales defined by length l or wavenumber $k \sim l^{-1}$)



Spatial diffusion coefficient κ: heuristics and classical quasilinear theory

Heuristic calculation: if every step of length $l \sim r_g$, particles experience random deflection $\delta\theta \sim \pm O(\delta B_l/\mathrm{B})$, then after $c\Delta t/l$ steps, variation $\langle \Delta\theta^2 \rangle \sim \langle \delta B_l^2 \rangle / B^2\ c\Delta t / l$ i.e. scattering rate $\nu_{\text{scatt}} \simeq \langle \Delta\theta^2 \rangle / \Delta t \propto \left(r_g/\ell_c\right)^{-0.5\ \ldots -0.3}$

… i.e., a spatial diffusion coefficient $\kappa_{\|} = \lambda_{\|} c = c^2/\nu_{\text{scatt}} \approx (\delta B/B)^{-2}\, \ell_c\, (r_g/\ell_c)^{0.3\ldots0.5} \propto \varepsilon^{0.3\ldots0.5}$

Formal calculation: in the context of perturbative quasi-linear theory, assume (1) that particles gyrate around a fixed background magnetic field and collect the influence of perturbations along their motion; (2) that the turbulence is composed of a large number of uncorrelated plane waves…

$$\nu_{\text{scatt}} \simeq \int \mathrm{d}\boldsymbol{k}\ \ldots\ \delta\left(k_{\|}\mu c - \omega_k + nc/r_g\right)\ \mathcal{P}_{\boldsymbol{k}}$$

↑ picks up gyroresonant modes out of power spectrum of magnetic fluctuations P_k

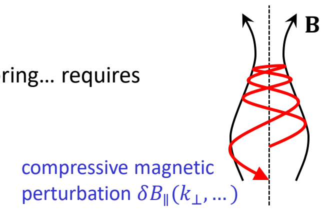
Perpendicular transport: dominated by field line wandering combined with parallel transport ("compound diffusion"), giving $\kappa_{\perp} \sim (\delta B/B)^4 \kappa_{\|}$ (4 = approximate exponent)

⇒ theory complete and OK?

Spatial diffusion coefficient κ: from mirrors and anisotropy to confusion…

Issues with the theory:

- large-scale compressive perturbations can trap particles through mirroring… requires finite-amplitude perturbations (not predicted by theory)…

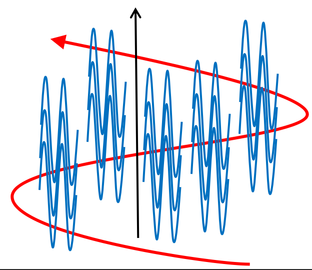


- turbulence assumed isotropic, i.e. equal shape and power along and perpendicular to magnetic field… in modern theories (Goldreich-Sridhar) for Alfvén turbulence, eddies elongated along mean field → destroys resonances

… eddies become progressively more elongated along mean magnetic field at smaller length scales… $k_\parallel \sim k_\perp^{2/3} l_c^{-1/3} \Rightarrow k_\parallel \ll k_\perp$

… resonances disappear: if $k_\parallel \sim r_g^{-1}$ then $k_\perp r_g \gg 1$, which implies that during a gyration, the particle explores many uncorrelated modes in the transverse direction…

⇒ inefficient scattering …

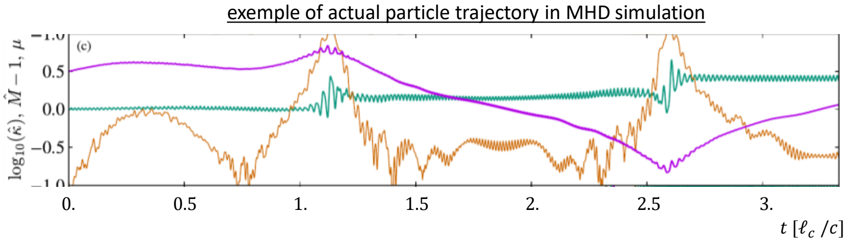


Spatial diffusion coefficient κ: structures rather than plane waves?

Issues with the theory:

- assumption "turbulence is composed of a large number of uncorrelated plane waves" known to be wrong… likely a major impact on diffusion coefficient, affected by interactions with localized structures rather than a continuous bath of waves

exemple of actual particle trajectory in MHD simulation



Definitions:

$\mu = \boldsymbol{p}\cdot\boldsymbol{b}/p$ pitch-angle cos

$M(t)$: magnetic moment

$\hat{M} \equiv M(t)/M(0)$

$\hat{\kappa} \equiv \kappa\,\langle B\rangle/B$

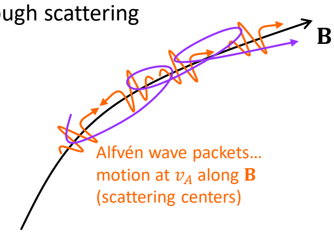
note: $\hat{\kappa}\langle r_g\rangle \sim \max(\kappa r_g)$

… pitch-angle evolves "adiabatically" between large-scale mirrors
… scattering is seen as jumps of M, localized events associated with sharp curvature bends of the field

Spatial diffusion coefficient κ: accounting for CR feedback on the turbulence

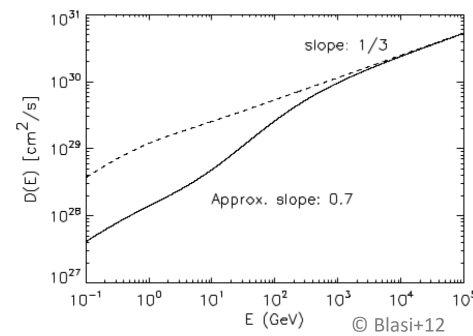
CR feedback on turbulence: CRs exchange energy and momentum with turbulence through scattering

… cosmic ray w/ mean velocity $\langle v_\parallel \rangle$ larger than v_A excite Alfvén waves, which in turn scatter CRs!



CR streaming instability: net motion along $\mathbf{B}$ ↔ anisotropy…
→ weak for CRs on average, but pronounced if no scattering mechanism
… instability growth dominant at $k\, r_g \sim 1 \Rightarrow$ only $n_{\rm CR}(>\varepsilon)$ with $r_g = \varepsilon/eB$ contributes to exciting waves at k …

→ in principle, more effective for low-energy CRs (more numerous)



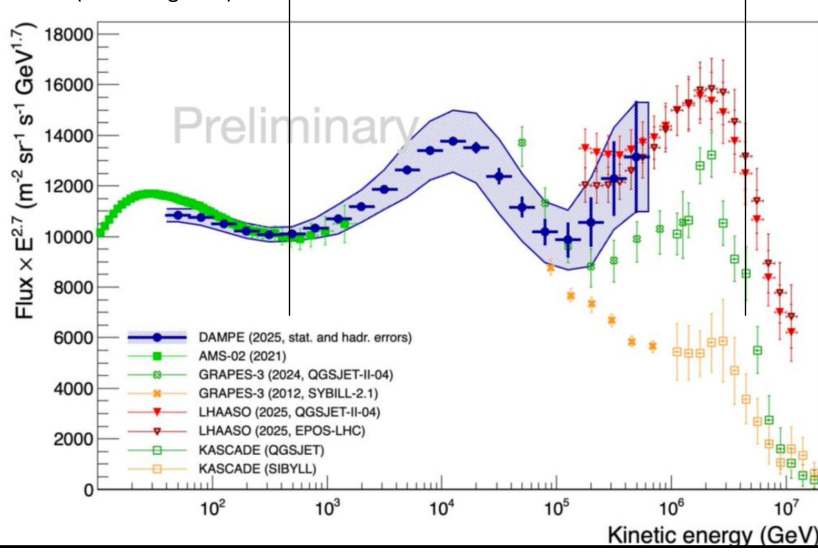
Self-regulation of transport:
… amplified waves can be damped by turbulent cascade, or by other channels (e.g. ion-neutral friction) …
→ equilibrium: growth rate balanced by damping … determines κ

… in practice, relevant for CRs with energy $\lesssim$ 300 GeV: stream along field lines at mean velocity v_A + diffuse with self-regulated κ

… for $\varepsilon \gtrsim 300\,{\rm GeV}$, growth rate too slow … no self-regulation

Transport properties of Galactic CRs as a function of energy

CR diffusion in self-generated turbulence (streaming inst.)? | CR diffusion in pre-existing (external) turbulence of disk+halo? | above "knee", transition to near ballistic regime, decoupling from turbulence?



(figure from ICRC-2025 conference, courtesy C. Evoli)

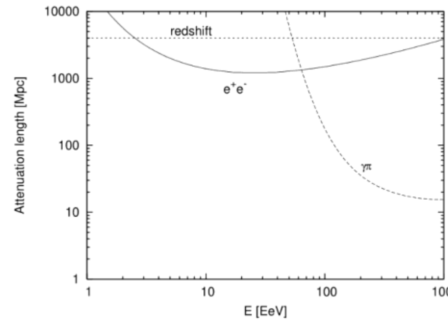
Transport of UHE cosmic rays: catastrophic photo-hadronic losses

Greisen-Zatsepin-Kuzmin suppression:

… at UHE, CMB photons are seen as ~gamma-rays in the rest frame of the proton…

… $p + \gamma \to \cdots + \pi$ leads to catastrophic on length scales $\sim$ 50Mpc for $\varepsilon \gtrsim$50EeV (+production of cosmogenic neutrinos)

… protons can also pair-produce on CMB (Bethe-Heitler)



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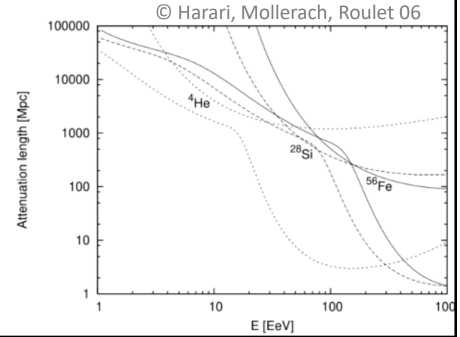
Interactions for nuclei:

… for nuclei, interactions dominated by photo-dissociation (ripping off nucleons or α nuclei) on CMB + infrared photon backgrounds …

… threshold $\propto \gamma$ while $\varepsilon \propto \gamma A\, m_p$ implies lighter nuclei have lower threshold energy …



→ much more this afternoon in Hands On session!

Transport for VHE to UHE CRs: semi-ballistic transport, $\lambda_\parallel \gg \ell_c$

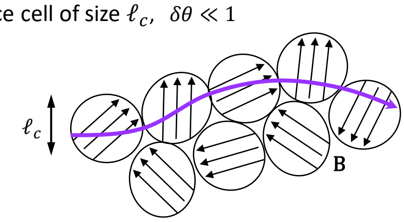
Small-angle deflection regime at high energy: if, when crossing a coherence cell of size ℓ_c, $\delta\theta \ll 1$

… through crossing a given cell, deflection $\delta\theta \sim \pm O(\ell_c/r_g)$

… after $c\Delta t/\ell_c$ steps, accumulated $\langle\Delta\theta^2\rangle \sim \delta\theta^2\, c\Delta t/\ell_c$

… scattering rate/frequency: $\nu_{\rm scatt} = \langle\Delta\theta^2\rangle/\Delta t \sim c\,\ell_c/r_g^2 \propto \varepsilon^{-2}$

… mean free path to deflection: $\boxed{\lambda_\parallel = c/\nu_{\rm scatt} \sim r_g^2/\ell_c \propto \varepsilon^2}$ (if no mean field: $\lambda_\parallel \to \lambda_{\rm scatt}$)



Application to UHE cosmic rays:

… at VHE in Galaxy: for $B \sim\ \mu$G, $\ell_c \sim 100$pc, $\lambda_{\rm scatt} \sim 10$ kpc $\varepsilon_{\rm EeV}\, Z^{-1}$

… extra-galactic rms B field unknown, but assuming $B \sim 1\ n$G with coherence length $\ell_c \sim 100$kpc gives $\lambda_{\rm scatt} \sim 1$ Gpc $\varepsilon_{10\rm EeV}\, Z^{-1}$!

… to determine transport regime, compare to source distance:
ballistic for close-by sources / high rigidity ($\ll$ Gpc, $\varepsilon \gtrsim 10$ EeV, or $Z \sim 1$)
diffusive for far-away sources / low rigidity ($\gtrsim$ Gpc, $\varepsilon \lesssim 10$ EeV, or $Z \gg 1$)

Transport of UHE cosmic rays in the magnetized Universe

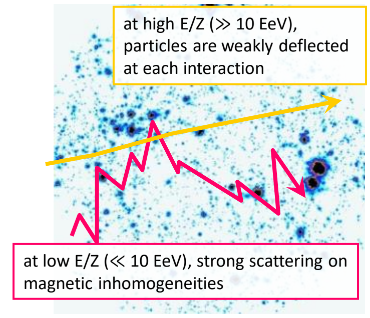
More detailed picture:

- most of scattering likely associated with localized regions of enhanced B field, e.g. galaxies, clusters, filaments, superclusters… as in a billiard

- sources of highest energy CRs must be close-by → expect weak deflection at UHE

  note: 1. GZK effect implies that max source distances ↘ with ε ↗

  2. most of flux comes from farthest sources (Olber's paradox)

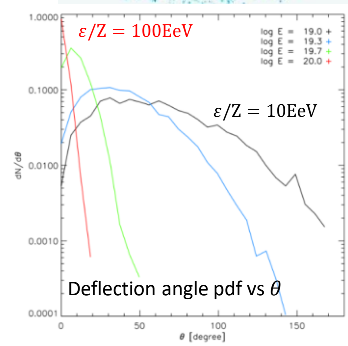


Different regimes of transport:

- diffusive at low energy, ballistic at high-energy…
- + possible diffusion in local supercluster

- possible existence of a energy-dependent magnetic horizon, introducing low-energy cut-off around $0.1 - 1\ Z$ EeV

  note: horizon from $t(E) = D^2/\lambda \leq H^{-1}$ (D source distance, H^{-1} age of Universe)



Some (old or new) pressing questions and emerging topics

- What is the physical mechanism controlling particle transport in a turbulent plasma !?
  - → relation between turbulence anisotropy, compressible modes down to small length scales $\sim r_g(\mathrm{TeV})$ …
  - → role of coherent turbulent structures on transport (mirrors and sharp curvature bends)?
  - → detailed physics of the streaming instability in a realistic turbulent context?

- How to interpret the breaks in the all-particle or individual CR spectra?
  - → distinguish effects resulting from different transport regimes from other causes?
  - → are there different types of sources? e.g. do recent sources (microquasars) detected at PeV contribute?

- Cosmic rays as tools in cosmology and particle physics?
  - → use of cosmic rays as a laboratory for beyond-standard-model physics?
  - → importance of cosmic rays in Galactic dynamics: the cosmic-ray feedback and hydrodynamics picture

- At UHE, similar questions, but a lot less data…
  - → interpretation of breaks seen in data, signatures of evolving chemical composition?
  - → transport of UHE cosmic rays in the local environment, in the large-scale structure?
  - → understanding the chemical composition: origin of UHECRs as a nucleosynthesis problem!

Cosmic ray -- Theory

Martin Lemoine
Astroparticule & Cosmologie (APC)
CNRS – Université Paris Cité

Outline:

Hands-on 2025 – PhD Summer School @ LNGS – Sept. 2025

Basic notions about particle acceleration…

→ Recall: $\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = q\left(\mathbf{E} + \frac{\boldsymbol{v}}{c}\times\mathbf{B}\right)$ but $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = q\,\boldsymbol{v}\cdot\mathbf{E}$

… yet, $\mathbf{E} \longleftrightarrow \mathbf{B}$ in a change of reference frame: beware of illusions!

Fundamental example: MHD flows ↔ plasmas of "infinite" conductivity, $\mathbf{E} \simeq -\boldsymbol{v_E}\times\mathbf{B}/c$ with $\boldsymbol{v_E}$ drift velocity…
… $\boldsymbol{v_E}$ = velocity of magnetic field lines frozen into plasma motion …

… MHD approximation: relevant on large scales (≫ skin depth and thermal gyroradius), in practice relevant in most astrophysical media except: 1. magnetospheres of compact objects; 2. small spatial scales

ion skin depth: $c/\omega_{\rm pi} \sim 10^7\,{\rm cm}\;\; n_0^{-1/2}$

ion thermal gyroradius: $r_{\rm g,\,th} \sim 10^8\,{\rm cm}\;\; T_4^{1/2}B_{\mu{\rm G}}^{-1}$

… here, all throughout: assume MHD flows → Fermi acceleration scenarios

… note: particle acceleration takes place in collisionless systems (collision m.f.p. ≫ relevant scales)!

Particle acceleration schemes in HE astrophysics… Fermi's model (1949, 1954)

→ Fermi picture of acceleration:
high conductivity implies small electric fields ↔ in the plasma rest frame $\boldsymbol{E} \sim 0$ everywhere on length/time scales of interest…

… however, in a plasma moving at velocity $\boldsymbol{v_E}$, $\mathbf{E} = -\boldsymbol{v_E} \times \mathbf{B}/c \Rightarrow$ particle acceleration in inductive electric fields associated with plasma motion!

… Issue 1: $\mathbf{E} \perp \mathbf{B} \Rightarrow$ need a mechanism to ensure cross-field transport… turbulence!
… Issue 2: if $\boldsymbol{v_E}$ uniform in space and time, no E field anywhere anytime in plasma frame ⇒ no acceleration

→ original Fermi picture: particles gain (or lose) energy by scattering on randomly moving magnetized inhomogeneities

→ Application: the Hillas bound on maximal energy

… at most, particles crossing a length scale L (< size of source) reach energy $\varepsilon_{max} = Z\, e\, E\, L$
… with $E \sim B\, v_E/c$,

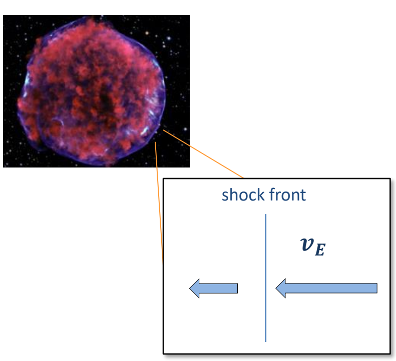
$$\varepsilon_{\rm max} \sim Ze\, B\, L\, v_E/c$$

… e.g., for SNR $L \sim 1\text{pc}$, $B \sim 1\mu\text{G}$, $v_E/c \sim 0.1$: $\varepsilon_{\rm max} \sim 0.1\ \text{PeV}$ … a strict upper limit → issue with the knee!

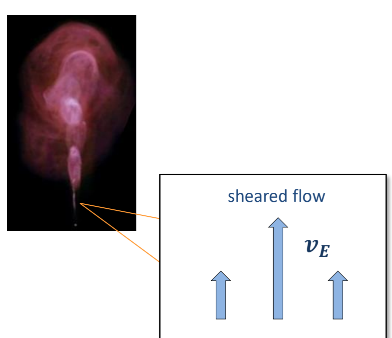
The family tree of Fermi acceleration scenarios

→ acceleration is controlled by the spatial (or time) variation of the velocity flow v_E

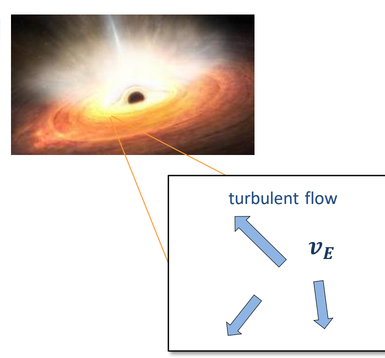
… recall: ideal MHD conditions → $\mathbf{E}$ vanishes in frame moving at $\boldsymbol{v_E} = c\, \mathbf{E} \times \mathbf{B}/B^2 \Rightarrow$ no acceleration in absence of shear



→ at a shock: $\partial_x v_E^x \neq 0$

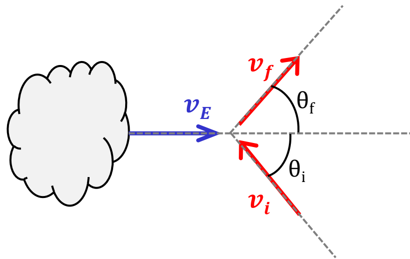


→ in a sheared flow: $\partial_x v_E^y \neq 0$



→ in a turbulent flow: $\partial_i v_E^j \neq 0$

The fundamental idea of Fermi acceleration



a particle, with initial velocity $\boldsymbol{v_i}$ interacts with a scattering center (mass M) moving at $\boldsymbol{v_E}$ to achieve final velocity $\boldsymbol{v_f}$
key assumption: interaction is elastic in reference frame of scattering center

→ kinematical view:

conservation of energy, momentum: $$\frac{E_f - E_i}{E_i} = \frac{\boldsymbol{v_E} \cdot \Delta \boldsymbol{v}}{c^2}$$

(assumes $v_E \ll c$)

$\Delta\boldsymbol{v} = \boldsymbol{v_f} - \boldsymbol{v_i}$
if oriented toward same half-plane as $\boldsymbol{v_E}$ then particle has taken energy from scattering center, otherwise particle has given energy

in detail:

$$\left.\begin{aligned} M\boldsymbol{v_{Ef}} + \boldsymbol{p_f} &= M\boldsymbol{v_E} + \boldsymbol{p_i} \\ \frac{1}{2}M{v_{Ef}}^2 + E_f &= \frac{1}{2}M{v_E}^2 + E_i \end{aligned}\right\} \quad \frac{E_f - E_i}{E_i} = \frac{\boldsymbol{v_E} \cdot (\boldsymbol{p_f} - \boldsymbol{p_i})}{E_i} + \mathcal{O}\left(\frac{(\boldsymbol{p_f} - \boldsymbol{p_i})^2}{M E_i}\right)$$

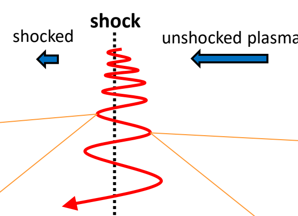
(test-particle: $M \to \infty$)

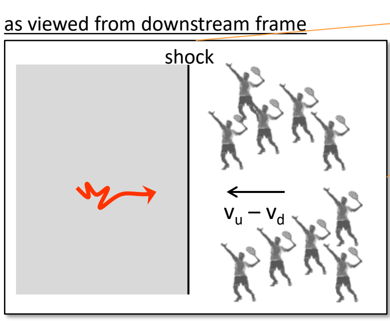
Shock acceleration: particles gain energy through head-on collisions with the moving plasma

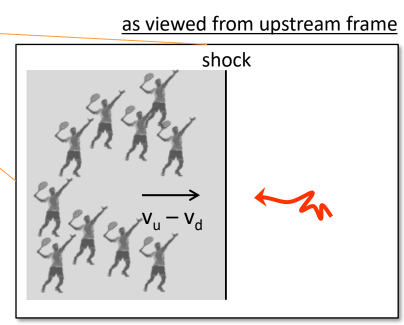
→ **systematic energy gain:** whether downstream or upstream, a particle returning to the shock sees a converging flow, thus suffers head-on collisions with energy gain, if it further returns to the shock…
i.e. as long as the particle bounces back and forth across the shock, it gains energy…

shocked ↔ downstream: outflowing from shock

unshocked ↔ upstream: inflowing into shock







→ at each cycle around the shock (up – down – up):

$$\langle \Delta p \rangle \simeq \frac{4}{3}\frac{v_u - v_d}{v}\, p > 0$$

positive on average ↔ systematic acceleration

escape downstream with prob. $\langle P_{esc} \rangle = \frac{4}{r}\frac{v_{sh}}{v}$ $\qquad r = v_u/v_d \simeq 4$

Microscopic picture of shock acceleration: some questions

→ One cycle: shock → downstream → shock → upstream → shock $\left\langle \frac{\Delta p}{p} \right\rangle = +\frac{4}{3}\frac{v_\mathrm{u} - v_\mathrm{d}}{v_\mathrm{i}} + \mathcal{O}\left(v_\mathrm{u}^2/c^2\right)$

→ where does the energy come from?

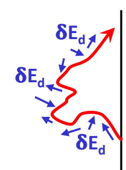
→ as viewed in the shock frame, both upstream and downstream plasmas carry motional turbulent electric fields:

$$\boldsymbol{\delta E}_\mathbf{d} = -\boldsymbol{v}_\mathbf{d} \times \boldsymbol{\delta B}_\mathbf{d}/c \qquad \boldsymbol{\delta E}_\mathbf{u} = -\boldsymbol{v}_\mathbf{u} \times \boldsymbol{\delta B}_\mathbf{u}/c$$

→ in calculations, the electric field was "illusioned away" by a trick: Lorentz transform to and from the frame in which it vanishes, in which particles undergo elastic pitch angle scattering…

… as viewed from the shock frame, the actual trajectory:

… turbulent $\boldsymbol{\delta B_d}$ ensures pitch-angle scattering, associated $\boldsymbol{\delta E_d}$ leads to energy gain/loss, cumulative effect of $\boldsymbol{\delta E_d}$ fields experienced along trajectory captured by trick of Lorentz transform!



→ in MHD approximation, $\boldsymbol{\delta B}$ field is tied to the plasma: energy reservoir is bulk plasma motion!

Diffusive shock acceleration: standard results in the test-particle limit

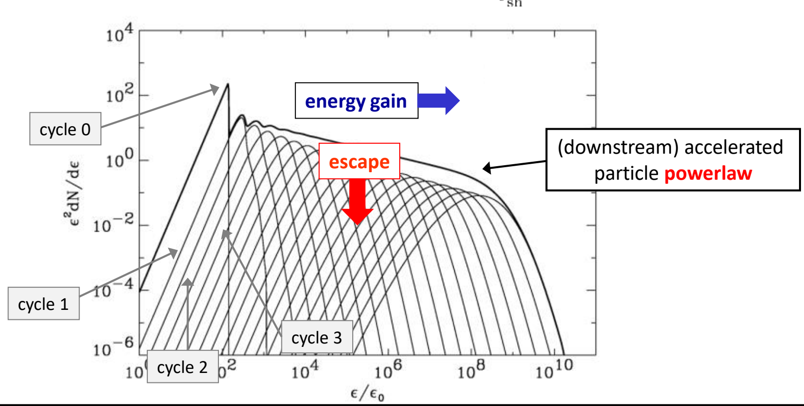
in short: gain in momentum per cycle $\langle \Delta \ln p \rangle \simeq +\frac{4}{3}\frac{v_\mathrm{sh}}{v}\left(\frac{r-1}{r}\right)$

probability of escape per cycle $\langle P_\mathrm{esc} \rangle = \frac{4}{r}\frac{v_\mathrm{sh}}{v}$ $(r \sim 4)$

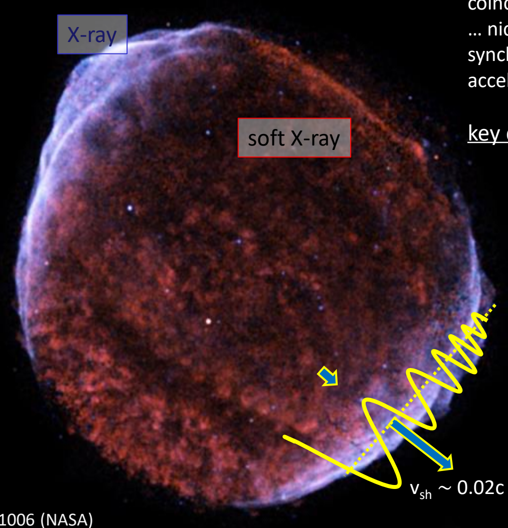
… shapes a powerlaw spectrum: $\frac{\mathrm{d}N}{\mathrm{d}p} \propto \left(\frac{p}{p_0}\right)^{-s}$, $s = 1 + \frac{\ln\langle P_\mathrm{esc}\rangle}{\langle \Delta \ln p\rangle} \simeq 2$

} competition between energy gain and escape leads to powerlaw

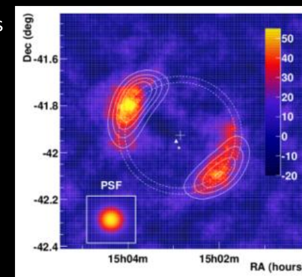
… with a characteristic acceleration timescale: $t_\mathrm{acc} \simeq \frac{\kappa}{v_\mathrm{sh}^2}$ (κ spatial diffusion coefficient)



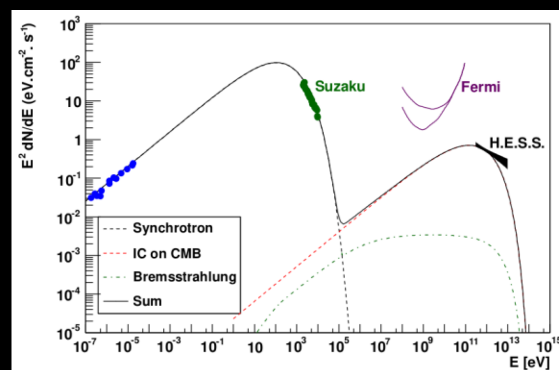
Evidence for particle acceleration at supernovae remnant shock waves



H.E.S.S (2010): detection of TeV gamma-rays coincident with X-rays…
… nice reconstruction of spectrum with synchrotron + inverse Compton of electrons accelerated to powerlaw with s ~ 2.0 – 2.2



key questions:
→ hadronic output?
→ maximum energy?



Diffusive shock acceleration at SNR shock waves: searching for Pevatrons

Observational constraints on the maximum energy: no evidence so far for a SNR Pevatron …

… comparing acceleration timescale to age of SNR:

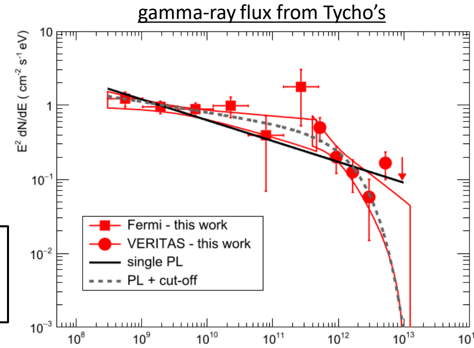
$$t_{\rm acc} \simeq \frac{\kappa}{v_{\rm sh}^2} \sim \frac{Ec}{eB}\frac{1}{v_{\rm sh}^2} \quad \text{vs} \quad t_{\rm age} \simeq \frac{R}{v_{\rm sh}}$$

``Bohm estimate'': $\kappa \sim r_g c$

$$t_{\rm acc}(E_{\rm max}) \sim t_{\rm age} \Leftrightarrow \boxed{E_{\rm max} \sim ZeBRv_{\rm sh} \simeq 30\,{\rm TeV}\,\frac{B}{1\mu{\rm G}}\,\frac{R}{1\,{\rm pc}}\,\frac{v_{\rm sh}}{0.03c}}$$

``Hillas limit''

gamma-ray flux from Tycho's



⇒ need for amplification of magnetic field to large values,
\+ need to search for young (=fast) SNR (Sedov-Taylor: few hundred yrs)

Evidence for magnetic field amplification at supernovae remnant shock waves

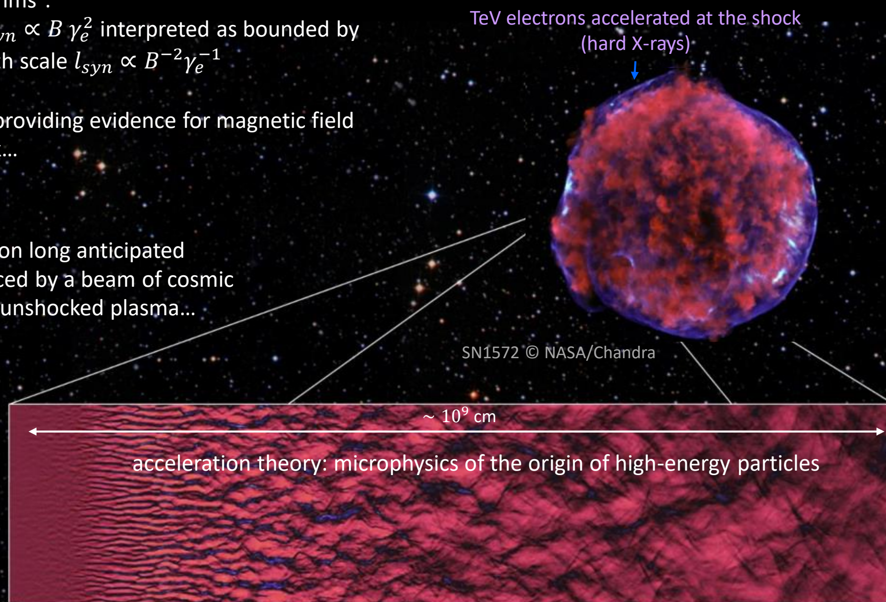
→ interpretation of thin X-ray rims[1]:

X-ray rims observed at $\nu_{syn} \propto B\,\gamma_e^2$ interpreted as bounded by electrons cooling on length scale $l_{syn} \propto B^{-2}\gamma_e^{-1}$

… implies: $B \sim 100\mu$G , providing evidence for magnetic field amplification at the shock…

→ microphysical picture:

magnetic field amplification long anticipated through instabilities induced by a beam of cosmic rays interpenetrating the unshocked plasma…



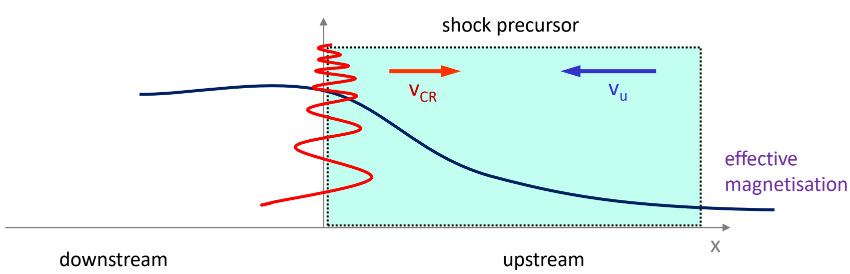
Ref.: 1. Völk+04, …

Feedback of cosmic-rays on the shock structure: the kinetic picture…

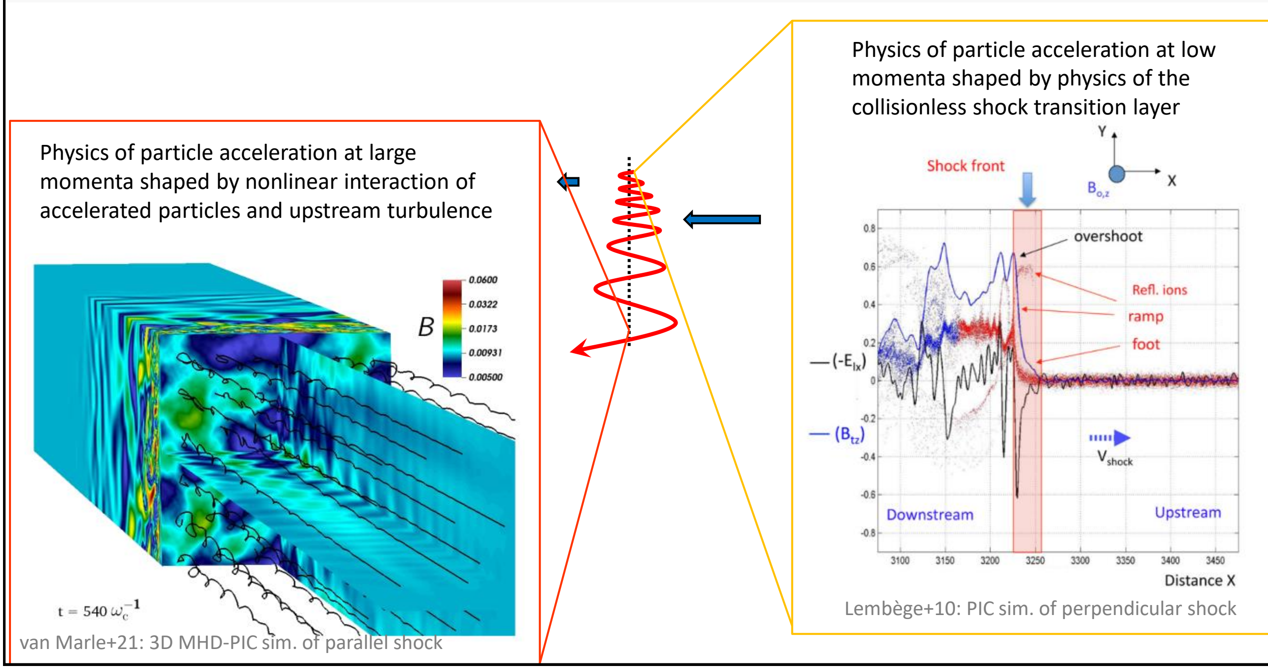
cosmic-ray precursor: accelerated particles outrun the shock and create a cosmic-ray precursor…

… cosmic rays + incoming background plasma + magnetic field = system prone to electromagnetic instabilities…

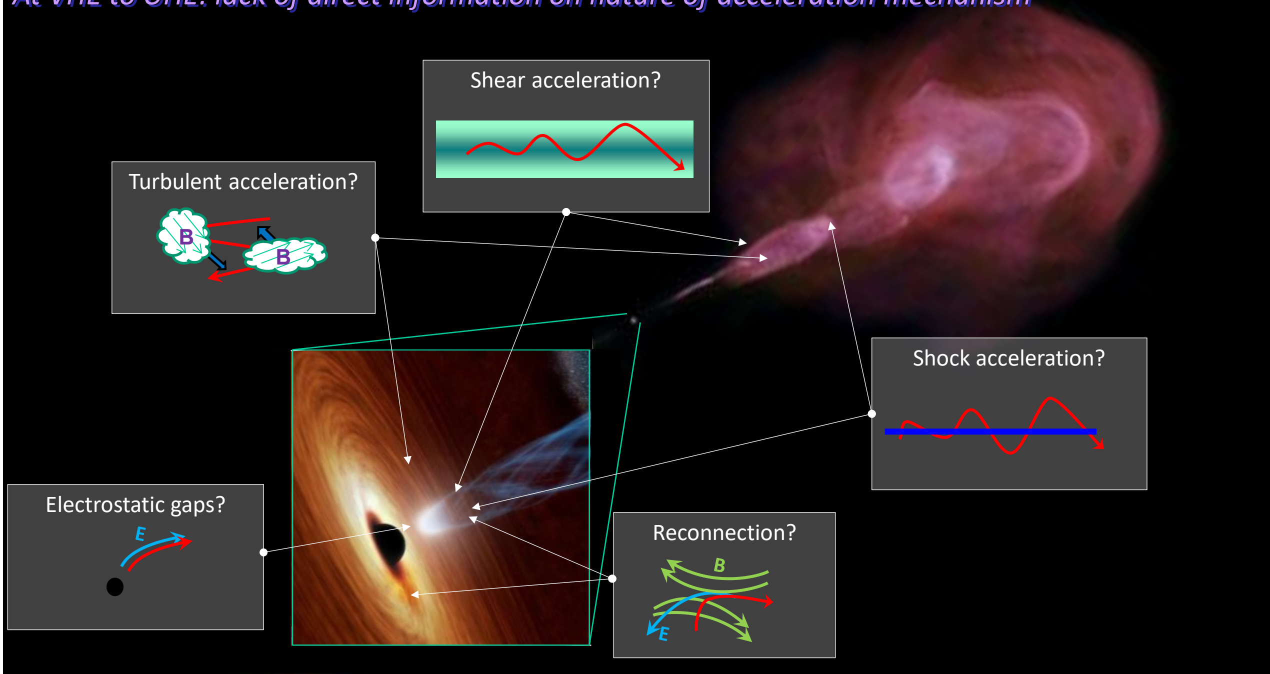
δB fluctuations scatter cosmic rays and plasma and mediates coupling between fluids



Shock acceleration: turning to numerical HPC simulations (+lab experiments)



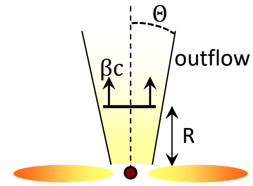
At VHE to UHE: lack of direct information on nature of acceleration mechanism



Particle acceleration beyond the SNR paradigm: from PeV to EeV's

Going relativistic: as $E \sim B\, v_E/c$, particle acceleration to very high energies generally requires $v_E \sim c$… i.e., relativistic shock waves, relativistic turbulence, relativistic shear acceleration, relativistic reconnection etc.
… key question for phenomenology: $t_{\rm acc}$ acceleration timescale (vs ε)

Improved Hillas bound for acceleration in (relativistic) outflow:



… write acceleration timescale (comoving plasma frame) as: $t_{\rm acc} = \mathcal{A}\, r_{\rm g}/c$

e.g. shock waves: $t_{\rm acc} \sim \kappa/v_{\rm sh}^2 > r_g c/v_{\rm sh}^2 \Rightarrow A > (c/v_{\rm sh})^2 \gg 1$

… compare $t_{\rm acc}$ and age $t_{\rm dyn} = R/(\beta\Gamma c)$: $t_{\rm acc} \leq t_{\rm dyn} \Rightarrow \varepsilon_{\rm obs} \leq \mathcal{A}^{-1} ZeBR/\beta$

… relate BR to the magnetic luminosity of the source: $L_B = 2\pi R^2 \Theta^2 \frac{B^2}{8\pi}\Gamma^2 \beta c$

… invert to write **a lower bound on L_B to reach $\varepsilon_{\rm obs}$**:

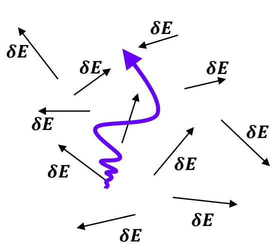
$$L_{\rm tot} \geq L_B \gtrsim 10^{43} \ \ldots\ \mathcal{A}^2 \left(\frac{\varepsilon}{Z\times 10^{19}\,{\rm eV}}\right)^2 \ {\rm erg/s}$$
$$\gtrsim 10^{35} \ \ldots\ \mathcal{A}^2 \left(\frac{\varepsilon}{1\,{\rm PeV}}\right)^2 \ {\rm erg/s}$$

… for references: Eddington luminosity $L_{\rm edd} \sim 10^{38} (M_{\rm BH}/M_\odot)\,{\rm erg/s}$

gamma-ray burst luminosity $L_{\rm GRB} \sim 10^{51}\,{\rm erg/s}$

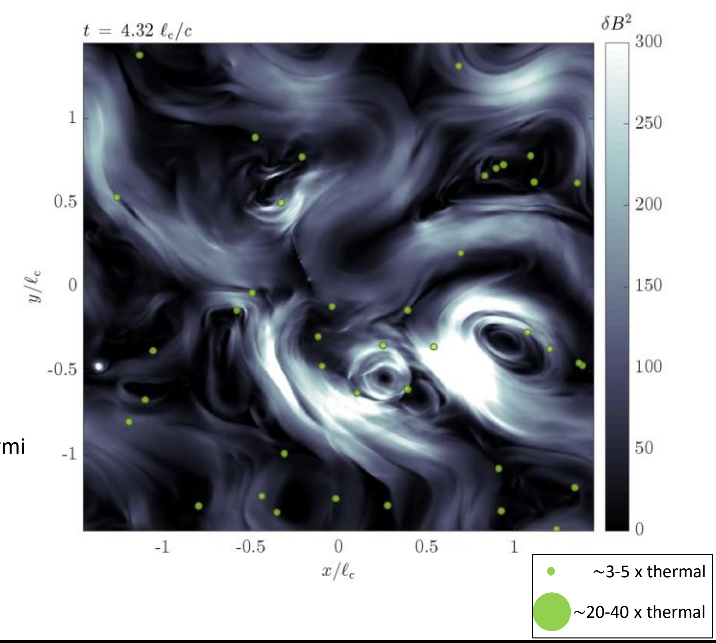
Particle acceleration in turbulence (aka « stochastic » or « Fermi type-II »)

… a key question: how to describe stochastic acceleration in random electric fields…



… at each "interaction", particles can gain or lose energy: a diffusive process in energy space (aka Fermi II, vs Fermi I at shock)

⇒ acceleration characterized (?) by advection and diffusion coefficients (vs ε)

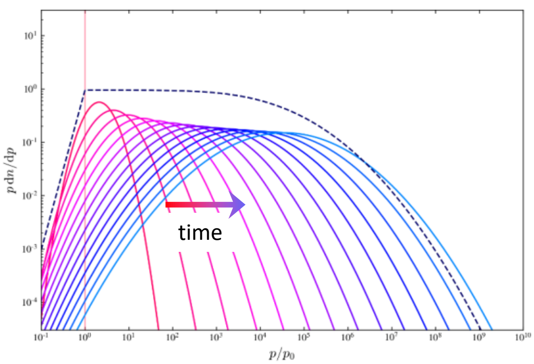


The standard Fokker-Planck scheme for modeling turbulent acceleration

→ Transport equation: $$\frac{\partial}{\partial t} f(p,t) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp}\frac{\partial}{\partial p} f(p,t)\right] - \frac{1}{p^2}\partial_p\left[|\dot{p}_{\rm loss}|\, p^2 f(p,t)\right] - \frac{f(p,t)}{t_{\rm esc}} + \dots$$

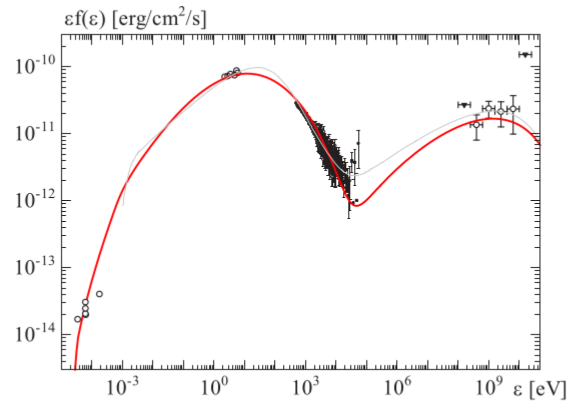
(1) no loss, no escape, $D_{pp} = \nu\, p^2$:

$$\frac{{\rm d}n}{{\rm d}p} = 4\pi p^2 f(p,t) = \sqrt{\frac{4\pi}{\nu p^2 t}}\, e^{-[\ln(p/p_0)-3\nu t]^2/(4\nu t)}$$



(2) w/ loss, escape, $D_{pp} = \nu\, p^2$:

e.g., modeling of SED of blazar PKS2155-304



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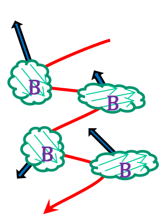
Standard schemes to model particle acceleration in turbulence

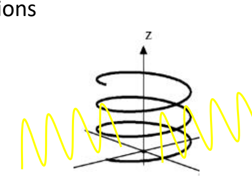
→ **Original Fermi (49,54) acceleration**: scattering off **discrete** magnetic scatterers, with **E=0** in local rest frame



→ kinematics: two-body collision, isotropic + elastic scattering in scattering center rest frame
⇒ $\Delta p > 0$ for head-on, $\Delta p < 0$ tail-on

$$D_{pp} \equiv \frac{\langle \Delta p^2\rangle}{2\Delta t} = \frac{1}{3}\left(\frac{v_E}{c}\right)^2 \frac{p^2}{t_{\rm int}}$$

→ **Quasilinear theory**: transport in a **bath of linear waves** (e.g. Alfvén, magnetosonic)… energy gain through resonant interactions



… interactions dominated by resonances, e.g. $k\, r_g \sim 1$ …

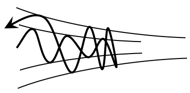
$$D_{pp} \sim \frac{\langle \delta B^2\rangle}{B^2}\left(\frac{v_{\rm A}}{c}\right)^2 \frac{p^2}{\ell_c/c}\left(\frac{r_{\rm g}}{\ell_c}\right)^{q-2}$$

ℓ_c coherence scale of turbulence
r_g gyroradius of particles
q index of δB spectrum (~5/3)

→ transport equation = Fokker-Planck: $$\frac{\partial}{\partial t} f(p,t) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp}\frac{\partial}{\partial p} f(p,t)\right]$$

The Fermi picture for particle acceleration (1949, 1954)

→ assumption: perfectly conducting magnetized plasma composed of moving scattering centers…
particle acceleration on motional electric fields $\boldsymbol{E} = -\boldsymbol{v_E} \times \boldsymbol{B}/c$



Fermi type A reflection of a cosmic-ray particle

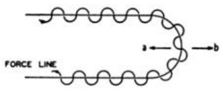
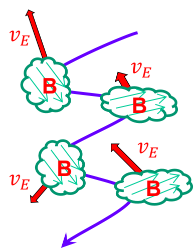


Fig. 1. Type *B* reflection of a cosmic-ray particle.



→ sequence of discrete interactions with point-like scattering centers… in each scattering center rest frame: elastic collision (ideal MHD ⇒ $\boldsymbol{E} = 0$ in rest frame)

→ kinematics: same as two-body collision, or gravitational assist! energy gain if $\boldsymbol{\Delta p} \cdot \boldsymbol{v_E} > 0$, energy loss otherwise

→ stochastic acceleration (diffusion in momentum space)… e.g. Fokker-Planck equation:

$$\frac{\partial}{\partial t} f(p,t) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp}\frac{\partial}{\partial p} f(p,t)\right]$$

momentum diffusion coefficient: $D_{pp} \sim \frac{v_E^2}{c^2}\frac{p^2}{t_{\rm int}}$

→ an issue: implementing stochastic acceleration in turbulence?

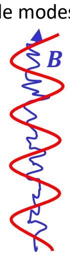
Quasilinear calculations of momentum diffusion coefficient

→ Momentum diffusion coefficient: $D_{pp} = \left(\frac{v_{\rm A}}{c}\right)^2 \sum_n \int {\rm d}\boldsymbol{k}\,[\dots]\,\mathcal{S}_{\boldsymbol{k}}\,\delta\left(k_\parallel v_\parallel - \omega_{\boldsymbol{k}} + n\omega_{\rm g}\right) J_n\left(k_\perp v_\perp r_{\rm g}/c\right)$

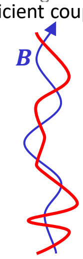
gyroresonances $\boldsymbol{n \neq 0}$: along background magnetic field, phase of electric field changes by $\Delta\phi = k_\parallel \Delta z - \omega_{\boldsymbol{k}}\Delta t$ in time Δt; in one gyro-orbit $\Delta t = 2\pi r_g/c$, particle has come back to same location, displaced by $\Delta z = v_\parallel\,\Delta t$… At gyroresonance $n \neq 0$, phase change $\Delta\phi = 2n\pi$: **particle in phase with $\boldsymbol{E}$ field**

in practice, $\omega_k \ll k$ for $v_A \ll c$ hence gyroresonance means $k\,r_g \sim n \sim 1$

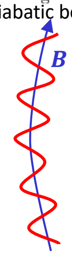
$kr_{\rm g} \gg 1$
small scale modes average out



$kr_{\rm g} \sim 1$
efficient coupling



$kr_{\rm g} \ll 1$
adiabatic behavior



… recall that Fermi picture ↔ angular deflection + moving wave = energy gain/loss

Phhenomenological bounds on the sources of UHECRs

Main critical properties of UHECR sources:

→ a large (apparent) source density: $n_{\rm UHECR} \gtrsim 10^{-5}\,/{\rm Mpc}^3$

→ a high output of cosmic rays: $\dot{e}_{\rm UHECR} \sim 10^{44}\,{\rm erg/Mpc^3/yr}$

… a non-trivial constraint:
e.g. $L_{UHE}/L_{\gamma} \sim 10$ for HL GRBs…
e.g. $L_{UHE}/L \sim \mathcal{O}(1\%)$ for radio-galaxies…

→ large/magnetized to confine UHECRs: $r_{\rm g} \le L \Rightarrow E \le 10^{20}\,{\rm eV}\,Z B_{\mu{\rm G}} L_{100\,{\rm kpc}}$

→ a high magnetic luminosity: $L_{\rm tot} \gtrsim 10^{45}\,{\rm erg/s}\,\ldots \left(\frac{t_{\rm acc}}{t_{\rm g}}\right)^2 \left(\frac{E/Z}{10^{20}\,{\rm eV}}\right)^2$

… leading contenders, for accelerating intermediate nuclei (Z ~ 10):
→ powerful radio-galaxies, L ~ 10^{44} erg/s, mildly relativistic outflows u ~ c …
→ relativistic supernovae, L ~ 10^{44} erg/s, mildly relativistic outflows u ~ c …

… need extreme sources for accelerating light nuclei (Z ~ 1) to highest energies:
→ gamma-ray bursts, fast-spinning magnetar/pulsar wind nebulae…
→ most powerful FRII like radio-galaxies

Some (old or new) pressing questions and emerging topics

- To make progress from the theoretical side: a view from first principles?
  → how do accelerated particles backreact on the shock and modify acceleration?
  → what is the detailed mechanism for particle acceleration in a turbulent plasma?
  → what is reconnection on large astrophysical scales?

- To make progress using numerical simulations: new schemes?
  → current self-consistent simulations are limited in dynamic range, starting from the smallest scales…
  → bring simulations closer to reality: including losses, radiative feedback etc.

- How to connect acceleration with observations?
  → need to develop theoretical models that extrapolate results of simulations to realistic scales
  → develop improved macroscopic transport equations to model particle acceleration
  → include acceleration theory into MHD simulations of sources on large scales

- At UHE, similar questions, but a lot less data…
  → a window on most extreme accelerators: get information from multi-messenger channels (ν, γ)
  → chemical composition: a clue on the accelerator?

A tiny bit of bibliography

→ Transport & Phenomenology:

V. Berezinsky, S. V. Bulanov, V. A. Dogiel, V. L Ginzburg, V. S. Ptsukin, « Astrophysics of cosmic rays » (1990, North-Holland)

+ ask the world experts in l'Aquila: R. Aloisio, P. Blasi, D. Boncioli, C. Evoli, and others

→ The founding papers on acceleration:

E. Fermi, "On the origin of cosmic radiation", Phys. Rev. 75, 1169 (1949)

E. Fermi, "Galactic magnetic fields and the origin of cosmic radiation", Astrophys. J. 119, 1 (1954)

→ Shock acceleration:

L. O'C. Drury, "An introduction to the theory of diffusive shock acceleration of energetic particles in tenuous plasmas", Rep. Prog. Phys. 46, 973 (1983)

R. Blandford & D. Eichler, "Particle acceleration at astrophysical shock waves: a theory of cosmic ray origin", Phys. Rep. 154, 1 (1987)

R. Schlickeiser "Cosmic-ray astrophysics", Springer-Berlin (2002)

→ Fermi-type acceleration:

G. Webb, "Relativistic transport theory for cosmic rays", Astrophys. J. 296, 319 (1985)

M. Lemoine, "Generalized Fermi acceleration", Phys. Rev. D 99, 083006 (2019)