The photodisintegration cross section of ^{9}Be and ^{12}C within Cluster Effective Field Theory

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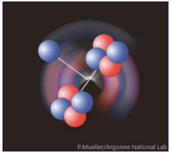
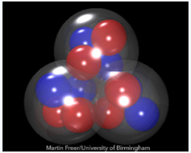
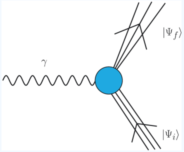
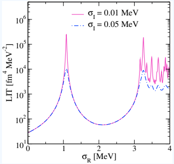


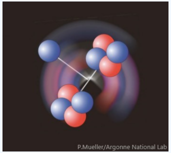


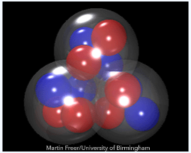




INTRODUCTION and OUTLINE

The photodisintegration cross section of ^{9}Be and ^{12}C within **Cluster Effective Field Theory**

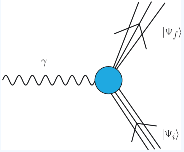




MODEL

- **Effective particles**
  - nucleons and α-particles ← **see Wed 22/10 session talks**
- **Interaction**
  - potential models from Effective Field Theory (EFT)

The **photodisintegration cross section of ^{9}Be and ^{12}C** within Cluster Effective Field Theory

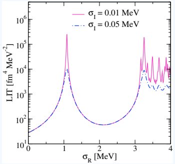


$$\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$$

$$\gamma + {}^{12}\text{C} \rightarrow \alpha + \alpha + \alpha$$

- Study of the reactions of *astrophysical relevance* in a **3-body *ab initio*** approach and in the **low-energy regime**
  - 3-body binding energies
  - cross sections
- Comparison of the results with the **experimental data**

The **photodisintegration cross section of ^{9}Be and ^{12}C** within **Cluster Effective Field Theory**



METHOD

- **Bound-state problem**
  - variational method
  - Non-Symmetrized Hyperspherical Harmonics (NSHH) method
- **Continuum-states problem**
  - integral transform approach: Lorentz Integral Transform (LIT) method

Cluster Effective Field Theory (EFT) approach

[Hammer *et al.* JPG **44** 103002 (2017), Hammer *et al.* RMP **92** 025004 (2020)]

$$^{6}\text{He} \sim \alpha n n\,,\, ^{9}\text{Be} \sim \alpha\alpha n\,,\, ^{10}\text{Be} \sim \alpha\alpha n n\,,\, ^{12}\text{C} \sim \alpha\alpha\alpha\,,\, ^{16}\text{O} \sim \alpha\alpha\alpha\alpha\,, \ldots$$

Borromean systems:

$$^{9}\text{Be} \sim \alpha\alpha n \quad S_3 \approx 1.573 \text{ MeV} \ll S_p(^4\text{He}) \approx 19.81 \text{ MeV}$$

$$^{12}\text{C} \sim \alpha\alpha\alpha \quad S_3 \approx 7.275 \text{ MeV} < S_p(^4\text{He}) \approx 19.81 \text{ MeV}$$

$\Rightarrow$ 3-body (or 4-body) *effective clustering* systems in the **low-energy regime**

Separation of energy scales $\rightarrow$ **Halo/Cluster EFT approach**

momentum scales: M_{low}, M_{high}

$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^{\nu}$

$\downarrow$

error estimate

Cluster Effective Field Theory (EFT) approach

[Hammer *et al.* JPG **44** 103002 (2017), Hammer *et al.* RMP **92** 025004 (2020)]

$$^{6}\mathrm{He} \sim \alpha nn\,,\, ^{9}\mathrm{Be} \sim \alpha\alpha n\,,\, ^{10}\mathrm{Be} \sim \alpha\alpha nn\,,\, ^{12}\mathrm{C} \sim \alpha\alpha\alpha\,,\, ^{16}\mathrm{O} \sim \alpha\alpha\alpha\alpha\,,\ldots$$

Borromean systems:

$^{9}\mathrm{Be} \sim \alpha\alpha n \quad S_3 \approx 1.573$ MeV $\ll S_p(^{4}\mathrm{He}) \approx 19.81$ MeV

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momentum scales: M_{low}, M_{high}

$\downarrow$

EFT expansion in $\left(\frac{M_{low}}{M_{high}}\right)^{\nu}$

$\downarrow$

error estimate

Chiral EFT

$\downarrow$

Pionless EFT

$\downarrow$

Halo/Cluster EFT

$\mathcal{L}^{\mathrm{eff}} \rightarrow \mathcal{V}^{\mathrm{eff}}$

2-body effective potentials

[Hammer *et al.* JPG **44** 103002 (2017), Ji]

Effective potential in momentum space and in the partial wave ℓ:

$$\mathcal{V}_\ell(p,p') = p^\ell p'^\ell \left[\lambda_0 + \lambda_1 (p^2 + p'^2) + \dots \right] g(p;\Lambda)\; g(p';\Lambda)$$

- sum of **contact terms** parametrized by the LECs
- **momentum-regulator function** : $g(p;\Lambda) = e^{-\left(\frac{p}{\Lambda}\right)^{2m}}, \quad m = 1, 2$

①

We calculate the $\mathcal{T}$-matrix by solving the Lippmann-Schwinger eq.
(on-shell: $\boldsymbol{p} = \boldsymbol{p}' = \boldsymbol{k}$)

α–n : $\mathcal{T}_\ell(k)$
α–α : $\mathcal{T}_\ell^{SC}(k)$ Coulomb-distorted strong term

②

We compare the calculated low-energy $\mathcal{T}_\ell$ with its ERE or Coulomb-modified ERE up to terms $O(k^2)$

$$\lambda_i = \lambda_i(a_\ell, r_\ell, \Lambda)$$

③

The LECs are fixed on the experimental values of scattering length a_ℓ^{exp} and effective range r_ℓ^{exp}

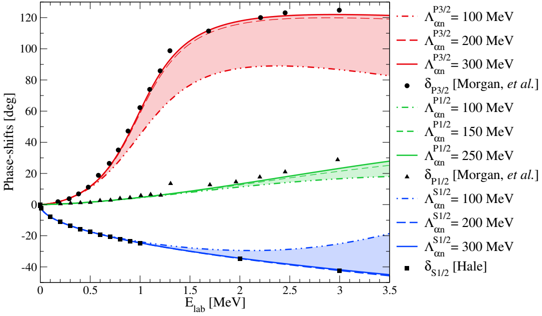
$$\lambda_i = \lambda_i(a_\ell^{\text{exp}}, r_\ell^{\text{exp}}, \Lambda)$$

The effective potentials $\mathcal{V}_\ell^{\alpha n}$ and $\mathcal{V}_\ell^{\alpha\alpha}$ reproduce the experimental low-energy α–n and α–α phase-shifts ➠

Low-energy phase-shifts

Calculated phase-shifts for different two-body cut-offs $\Lambda < \Lambda^{max}$ ($\Leftarrow$ Wigner bound)

$\alpha - n$

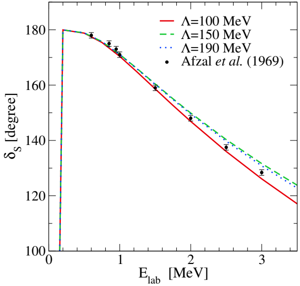


$P_{3/2}$
Resonance

$P_{1/2}$

$S_{1/2}$

$\alpha - \alpha$



S_0
Resonance

Resonant waves are promoted ➠

2-body potentials

^{9}Be

LO $$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}}) + \mathcal{V}^{\alpha n}_{P_{1/2}}(\Lambda^{\alpha n}_{P_{1/2}})$$

2-body potentials

^{9}Be

LO $$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}}) + \boxed{\mathcal{V}_\Gamma} + \mathcal{V}^{\alpha n}_{P_{1/2}}(\Lambda^{\alpha n}_{P_{1/2}})$$

- $\mathcal{V}^{\alpha n}_{P_{1/2}}$ $\rightarrow$ subleading
- $\mathcal{V}^{\alpha n}_{S_{1/2}}$ $\rightarrow$ **subleading** in bound-state calculations
  $\rightarrow$ **LO effect** in cross section calculations

  $\rightarrow$ αn deep bound state

To "project out" the deep bound states we add a **projection potential**

$$\mathcal{V}_\Gamma(p', p) \propto \Gamma \psi^*_{S_{1/2}}(p')\psi_{S_{1/2}}(p)$$

Theoretically: $\Gamma \rightarrow \infty$

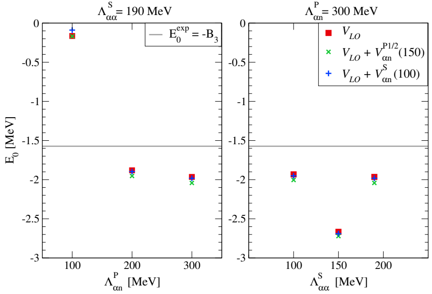
In practice: Γ-independence of the three-body results

$\Rightarrow$ Γ is **NOT** a free parameter

2-body + 3-body potentials

^{9}Be

LO $$\mathcal{V}^{\alpha\alpha}_{S_0}(\Lambda^{\alpha\alpha}_{S_0}) + \mathcal{V}^{\alpha n}_{P_{3/2}}(\Lambda^{\alpha n}_{P_{3/2}}) + \mathcal{V}^{\alpha n}_{S_{1/2}}(\Lambda^{\alpha n}_{S_{1/2}}) + \mathcal{V}_\Gamma + \mathcal{V}_3$$



To avoid the dependence of the 3-body results on the 2-body cutoffs, we add a *state-dependent* 3-body potential:

$$\mathcal{V}_3(Q, Q') = \lambda_3 \, g(Q;\Lambda_3) g(Q';\Lambda_3)$$

- we choose the 2-body cutoffs $\Lambda^{\alpha n}, \Lambda^{\alpha\alpha}$
- for each value of the 3-body cutoff Λ_3, the LEC λ_3 is fixed on a 3-body observable (e.g.: ^{9}Be binding energy for bound-state calculations)

Electromagnetic inclusive reactions

Cross section $\sigma_{\text{EM}} \propto \mathcal{R}(\omega)$

Response function $\mathcal{R}(\omega) = \int df \, |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$

Electromagnetic inclusive reactions

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Response function $\mathcal{R}(\omega) = \int df \; |\langle\Psi_f|\hat{\mathcal{O}}|\Psi_0\rangle|^2 \; \delta(E_f - E_0 - \omega)$

❶ Calculation of the initial state $|\Psi_0\rangle$ ➛ **Variational method + Non-Symmetrized HH basis**

[Gattobigio, *et al.* PRC **83** (2011), Deflorian, *et al.* FBS **54** (2013)]

- $\hat{\mathcal{H}}$ is represented on a Non-Symmetrized **basis in momentum space**

$$\Psi \equiv \sum_{m\{K\}} c_{m\{K\}} \, f_m(Q) \, \mathcal{Y}_{\{K\}}(\Omega_Q)$$

$f_m(Q) \propto$ Laguerre polynomials basis ($m = 1, \ldots, N_{\text{Lag}}$)

$\mathcal{Y}_{\{K\}}(\Omega_Q) =$ complete basis of the HH functions ($K = 1, \ldots, K^{\text{max}}$)

- $\hat{\mathcal{H}}$ is diagonalized

$$\sum_{\nu'} \langle\Psi_\nu|\hat{\mathcal{H}}|\Psi_{\nu'}\rangle \, c_{\nu'} = E \, c_\nu \qquad E_0, \{ c^0_\nu \} \Rightarrow \Psi_0$$

Electromagnetic inclusive reactions

Cross section $\sigma_{\text{EM}} \propto \mathcal{R}(\omega)$

Response function $\mathcal{R}(\omega) = \int df \, |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$

❷ calculation of the final states $|\Psi_f\rangle$ in the continuum ➛ integral transform approach

Lorentz Integral Transform (LIT) method

[Efros, *et al.* JPG **34** (2007)]

- Definition of an **Integral Transform** $\mathcal{L}(\sigma)$ of the response function $\mathcal{R}(\omega)$ with a **Lorentzian kernel**

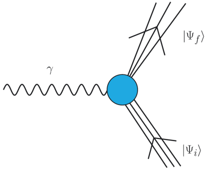
$$\mathcal{L}(\sigma) = \int d\omega \frac{1}{(\omega - E_0 + \sigma_R)^2 + \sigma_I^2} \mathcal{R}(\omega) \,, \qquad \mathcal{L}(\sigma) \xrightarrow{\text{INVERSION}} \mathcal{R}(\omega)$$

Electromagnetic inclusive reactions

Cross section $\sigma_{\mathrm{EM}} \propto \mathcal{R}(\omega)$

Response function $\mathcal{R}(\omega) = \int df \; |\langle \Psi_f | \hat{\mathcal{O}} | \Psi_0 \rangle|^2 \; \delta(E_f - E_0 - \omega)$

❸ determination of the operator $\hat{\mathcal{O}}$ ➛ photodisintegration processes [Bacca and Pastore JPG **41** (2014)]



$\omega = |\boldsymbol{q}|$ $\quad |\Psi_0\rangle \to |\Psi_f\rangle$

transition

Photon: $\boldsymbol{A}(\boldsymbol{x})$, $\hat{\epsilon}_{\boldsymbol{q},\lambda}$, $\boldsymbol{q} \parallel \hat{\boldsymbol{z}}$

Nucleus: $\rho(\boldsymbol{x})$ nuclear charge, $\boldsymbol{J}(\boldsymbol{x})$ nuclear current

$$\mathcal{R}_\gamma(\omega) \sim \left| \langle \Psi_f | \, \hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) \, | \Psi_0 \rangle \right|^2$$

Nuclear current matrix element

Twofold calculation

[Siegert PR **52** (1937)]

1. Siegert operator $\mathcal{T}^{el,S}_{J\lambda}(q;\rho)$

- Multipole decomposition: $\hat{\epsilon}_{\boldsymbol{q},\lambda} \cdot \boldsymbol{J}(\boldsymbol{q}) = -\sum_J \sqrt{2\pi(2J+1)}\left[\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) + \lambda\mathcal{T}^{mag}_{J\lambda}(q;\boldsymbol{J})\right]$

$$\mathcal{T}^{el}_{J\lambda}(q;\boldsymbol{J}) = \mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) + \mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J})$$

Dominant: $EJ = E1, E2$

- Siegert theorem (**continuity equation**)

$$\mathcal{T}^{el,I}_{J\lambda}(q;\boldsymbol{J}) \propto \int \mathrm{d}\hat{q}'\, \boldsymbol{q}' \cdot \boldsymbol{J}(\boldsymbol{q}')\, Y_{J\lambda}(\hat{q}') \qquad \leftarrow \quad \omega\rho(\boldsymbol{q}) - \boldsymbol{q}\cdot\boldsymbol{J}(\boldsymbol{q}) = 0$$

$$\propto \int \mathrm{d}^3\boldsymbol{x}\, j_J(qx)\,\rho(\boldsymbol{x})\,Y_{J\lambda}(\hat{x}) \;\propto \mathcal{T}^{el,S}_{J\lambda}(q;\rho)$$

- Long-wavelength approximation ($qR \ll 1$)

$$\mathcal{T}^{el,S}_{J\lambda}(q;\rho) \propto \omega^J \int \mathrm{d}^3\boldsymbol{x}\, x^J \rho(\boldsymbol{x}) Y_{J\lambda}(\hat{x})$$ ← dipole $\boldsymbol{d}_\lambda$ or quadrupole $\boldsymbol{u}_\lambda$ op.

$$\mathcal{T}^{el,II}_{J\lambda}(q;\boldsymbol{J}) \propto \omega^{J+1}$$ ← correction

Twofold calculation

[Bacca and Pastore JPG **41** (2014)]

2. One-body current $J_\lambda^{[1]}(q)$

- The nuclear current operator is a sum of terms

$$\boldsymbol{J} = \boxed{\boldsymbol{J}^{[1]}} + \boldsymbol{J}^{[2]} + \boldsymbol{J}^{[3]} + \dots$$

← the nuclear current m.e. can be calculated
by using only the **one-body term**
i.e. the nuclear convection current

- Specifically with our EFT, the continuity equation is **NOT** fully satisfied

continuity equation: $\boldsymbol{q} \cdot \boldsymbol{J}(\boldsymbol{q}) = [\mathcal{H}, \rho(\boldsymbol{q})]\,, \qquad \mathcal{H} = T + \mathcal{V}_{2B} + \dots$

$\boldsymbol{q} \cdot \boldsymbol{J}^{[1]}(\boldsymbol{q}) = [T, \rho(\boldsymbol{q})]$ always satisfied

$[\mathcal{V}_{2B}, \rho(\boldsymbol{q})] \neq 0 \;\Rightarrow\; \exists\ \boldsymbol{J}^{[2]}(\boldsymbol{q})$ such that $\boldsymbol{q} \cdot \boldsymbol{J}^{[2]}(\boldsymbol{q}) = [\mathcal{V}_{2B}, \rho(\boldsymbol{q})]$

$\dots$

Twofold calculation

1. Siegert operator $\mathcal{T}^{el,S}_{J\lambda}(q;\rho)$

2. One-body current $J^{[1]}_{\lambda}(q)$

- With the **Siegert theorem**, since the *continuity equation* is used explicitly, the contribution due to the many-body current operators is implicitly included in the calculation

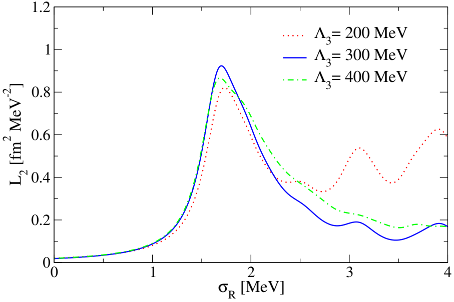
- With the **one-body current**, the *continuity equation* is not fully satisfied already at LO

- Comparison between the two calculations
$\Rightarrow$ the contribution due to the many-body currents can be quantified

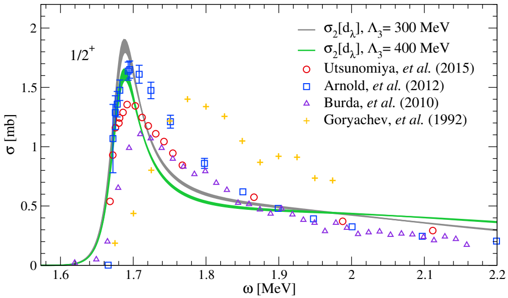
$$\gamma + {}^{9}\text{Be} \to \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \to |f\rangle : J^{\pi} = 1/2^{+}$$

$1/2^{+}$ LIT and cross section

[YC *et al.*, arXiv:2506.05040]



LIT: *slight dependence* on the 3-body cutoff Λ_3



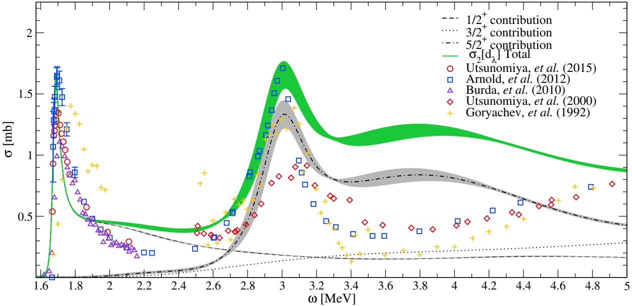
CS: bands $\to$ *error* due to the inversion procedure

$\mathcal{L}(\sigma_R) \to \mathcal{R}(\omega)$

$$\gamma + {}^{9}\mathrm{Be} \to \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \;\to\; |f\rangle : J^{\pi} = 1/2^{+}, 5/2^{+}, 3/2^{+}$$

Total photodisintegration cross section

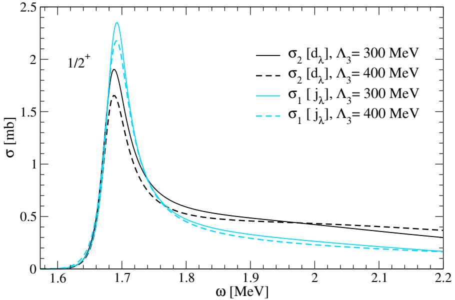
[arXiv:2506.05040]



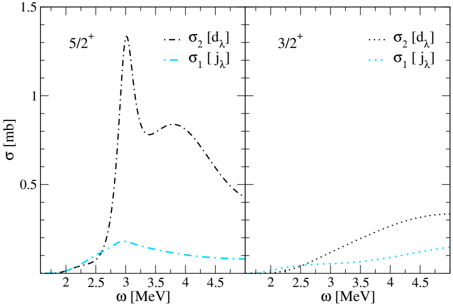
- $\Lambda_3 = 400$ MeV
- 3-body LEC $\lambda_3^{1/2^+}$, $\lambda_3^{5/2^+}$, $\lambda_3^{3/2^+}$ chosen to locate the resonance at the experimental energy
- $1/2^+$ band: error due to the inversion procedure
- $5/2^+$ band: error from the model-space convergence
- $3/2^+$ resonance: broad
- inclusion of the $P_{3/2}$ shape parameter $\to$ improvement at higher energies

$$\gamma + {}^9\text{Be} \to \alpha + \alpha + n \qquad |0\rangle : J^\pi = 3/2^- \to |f\rangle : J^\pi = 1/2^+, 5/2^+, 3/2^+$$

$1/2^+, 5/2^+, 3/2^+$ cross section: j_λ (1-body current) – d_λ (dipole) comparison



$\Rightarrow$ *non-negligible* contribution
of the many-body currents

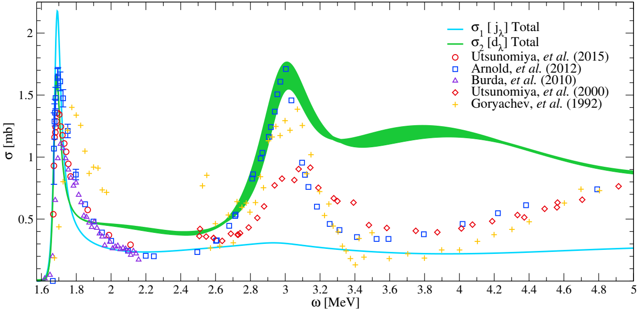


$\Rightarrow$ *dominant* effect
of the many-body currents

$$\gamma + {}^{9}\text{Be} \to \alpha + \alpha + n \qquad |0\rangle : J^{\pi} = 3/2^{-} \to |f\rangle : J^{\pi} = 1/2^{+}, 5/2^{+}, 3/2^{+}$$

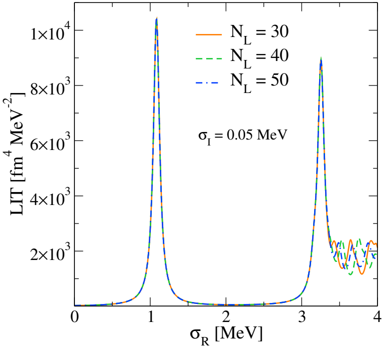
Total photodisintegration cross section: j_λ (1-body current) – d_λ (dipole) comparison

[arXiv:2506.05040]



$\gamma + {}^{12}\mathrm{C} \to \alpha + \alpha + \alpha$ $\qquad |0\rangle : J_n^\pi = 2_1^+$ (bound state) $\to |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

0^+ LIT (LO)



- Quadrupole operator
- $\mathcal{V}_{LO+3} = \mathcal{V}_{S_0}^{\alpha\alpha} + \mathcal{V}_\mathbf{3}(\Lambda_3, \lambda_3)$

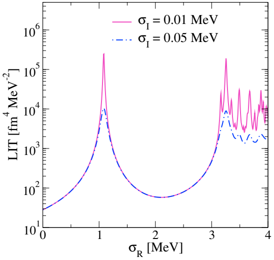
  $\lambda_3^{2^+}$ chosen to reproduce $E^{\mathrm{exp}}(2_1^+) = -2.875$ MeV,

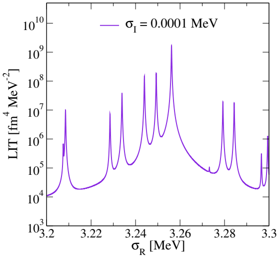
  $\lambda_3^{0^+}$ chosen to locate the resonance peak at the experimental energy ≈ 3.25 MeV
- 0_1^+ (ground state) and 0_2^+ are **NOT** simultaneously reproduced

  $E(0_1^+) = -1.792$ MeV $\neq E^{\mathrm{exp}}(0_1^+) = -7.275$ MeV

$\gamma + {}^{12}\mathrm{C} \rightarrow \alpha + \alpha + \alpha$ $\quad |0\rangle : J_n^\pi = 2_1^+$ (bound state) $\rightarrow |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

0^+ LIT (LO)





Construction of the response function:

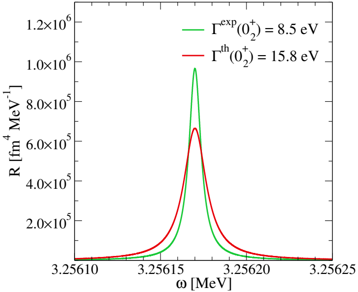
- we take the highest peak as a single "LIT state"

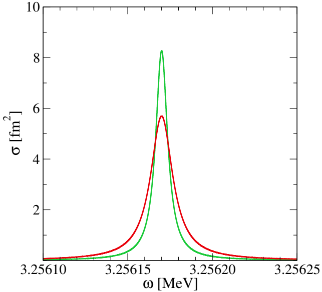
1. we impose the experimental width of the resonance
   ($\Gamma^{\exp}(0_2^+) \sim$ eV, very narrow!)
2. we impose a width Γ^{th} from Suno, *et al.* PRC **94** (2016)

$\gamma + {}^{12}\mathrm{C} \to \alpha + \alpha + \alpha$ $\quad |0\rangle : J_n^\pi = 2_1^+$ (bound state) $\to |f\rangle : J_n^\pi = 0_2^+$ (Hoyle state)

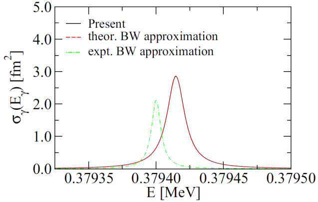
0^+ cross section (LO)

[Γ^{th} from Suno, *et al.* (2016)]





[Suno, *et al.* PRC **94**(2016)]



Comparison

$\Rightarrow$ factor ~ 4 or ~ 2

Conclusions and outlook

Photodisintegration reactions of 3-body cluster nuclei *at low energy*

- potentials from Cluster EFT
- LIT method: cross sections
- study of the effect of the many-body currents

^{9}Be: [Submitted to PRC, arXiv:2506.05040]

- total low-energy cross section: agreement with the experimental data
- non-vanishing effect of the many-body currents, dominant in the $5/2^{+}$ channel

☞ Calculation of the magnetic transition $M1$

^{12}C: [YC PhD Thesis (2024)]

- LO calculation (early stage!): overestimation of the experimental data

☞ Addition of the D-wave α–α potential

- Four-body calculations
- NN interaction from EFT (^{10}Be,...)

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In collaboration with:

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thank you for your attention

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