

Ab Initio Structure and Reactions (in Milan)

NUSDAF

Catania, ott. 22-24

Carlo Barbieri

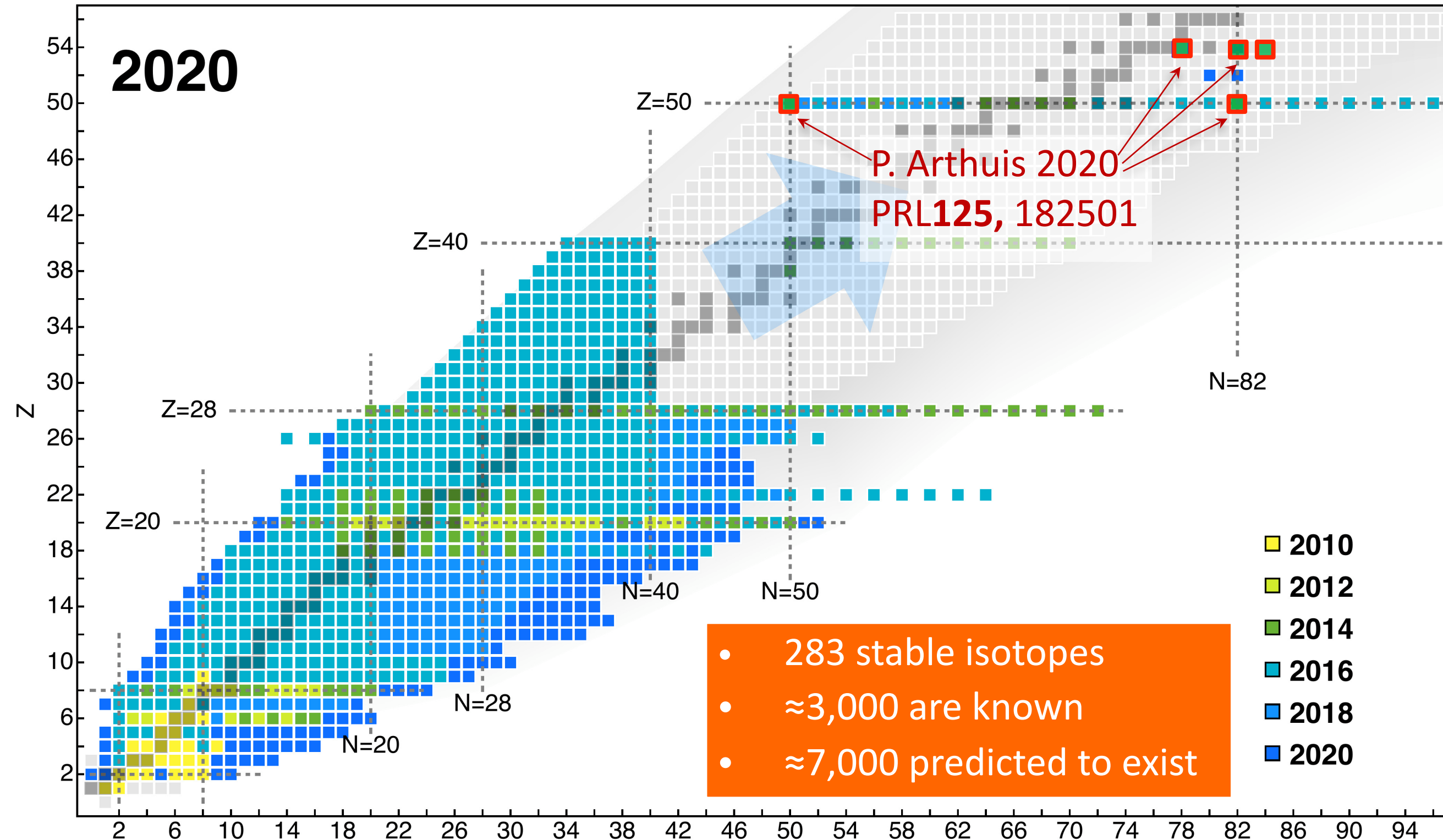
Università degli Studi di Milano & INFN



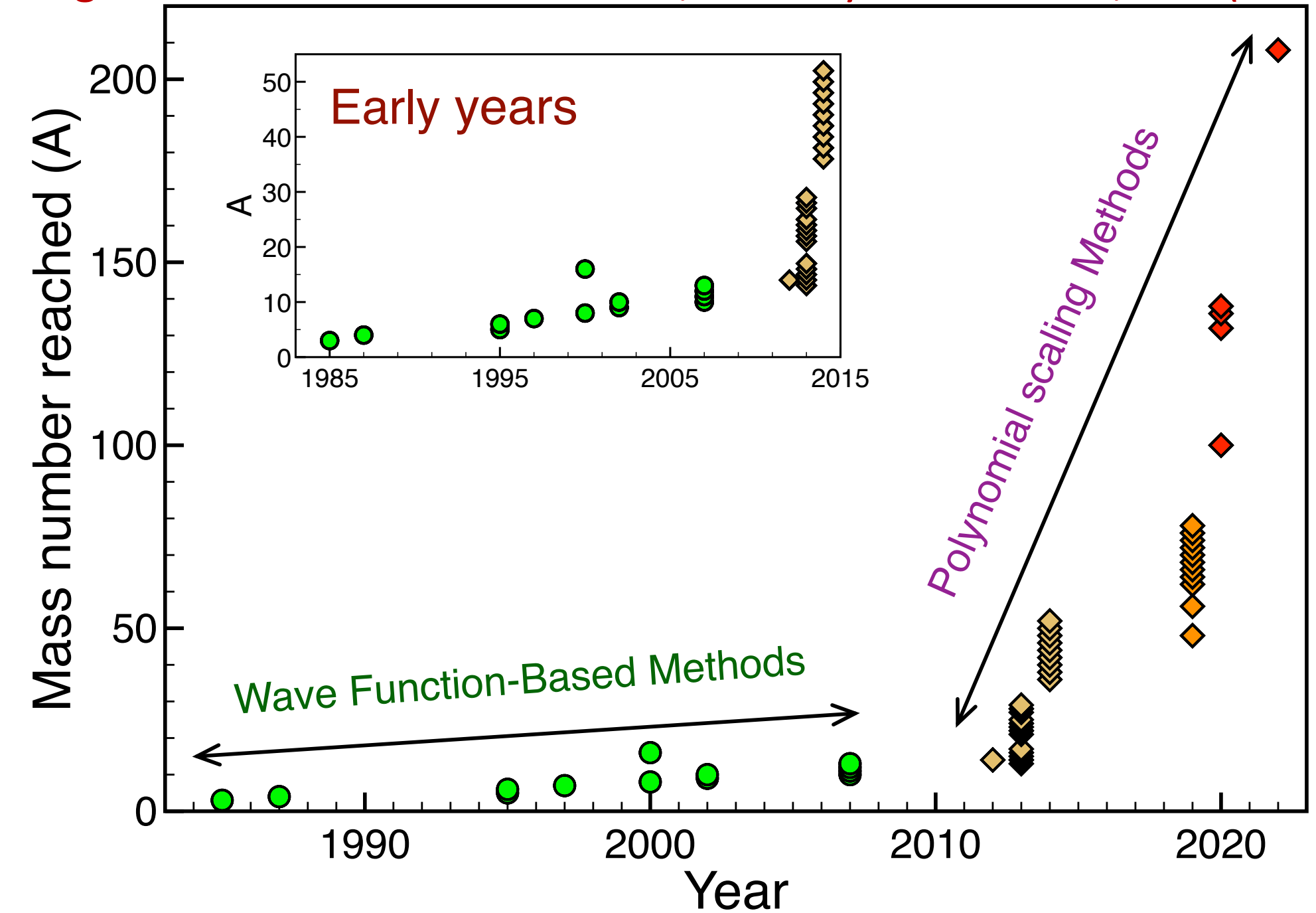
Reach of ab initio methods across the nuclear chart

Extension beyond few-nucleons thanks to:

- Soft (nearly perturbative) effective nuclear forces
- Diagrammatic many-body approaches



Legnaro Natl' Lab Mid Term Plan; Eur. Phys. J. Plus **138**, 709 (2023)



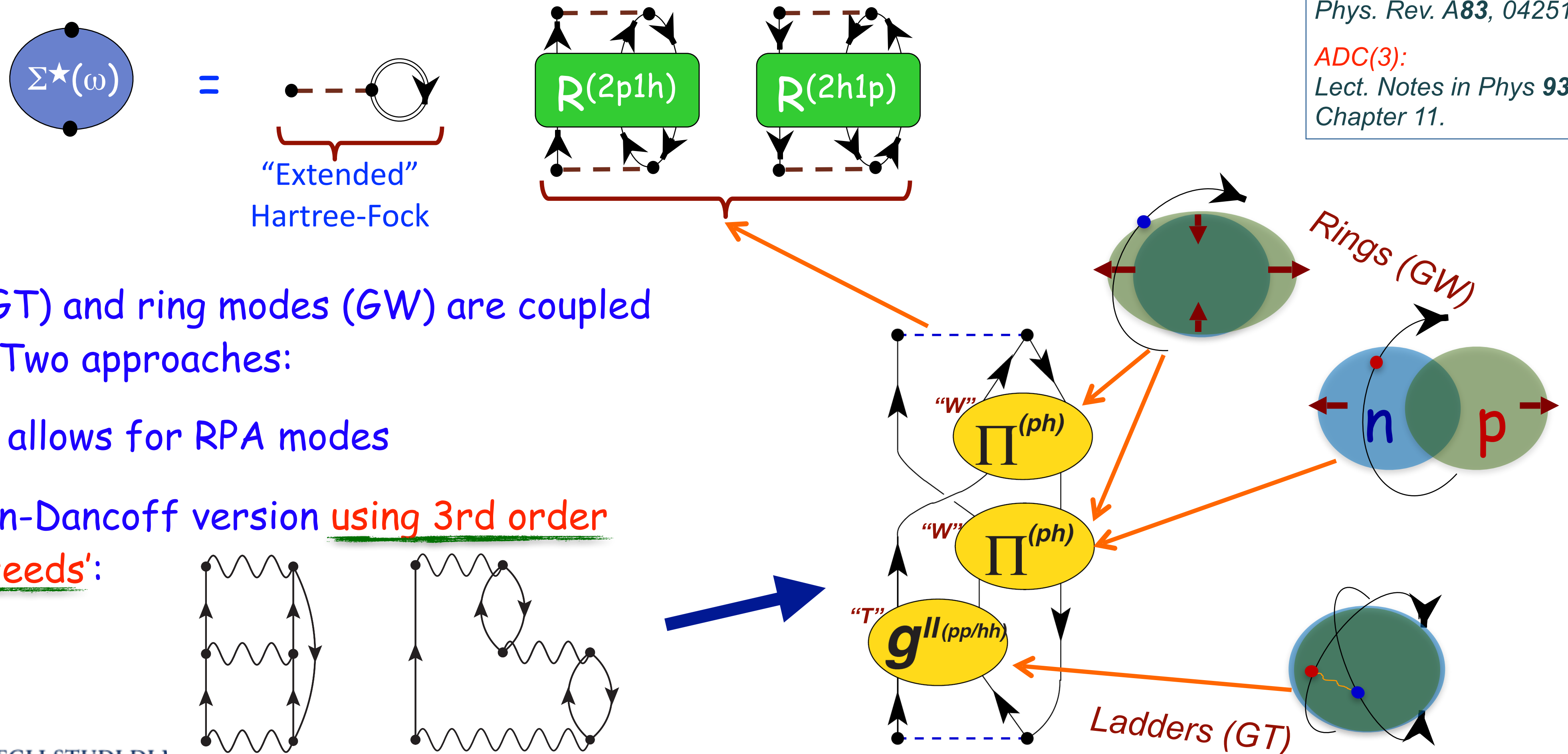
Open challenges:

- Accuracy (better theory of nuclear forces)
- Mass number limit (optimised model spaces)
- Precision & scattering (high-order diag. resummations)

The Faddev-RPA and ADC(3) methods in a few words

Compute the nuclear self energy to extract both scattering (optical potential) and spectroscopy.

Both ladders and rings are needed for atomi nuclei:



F-RPA:

Phys. Rev. C **63**, 034313 (2001)

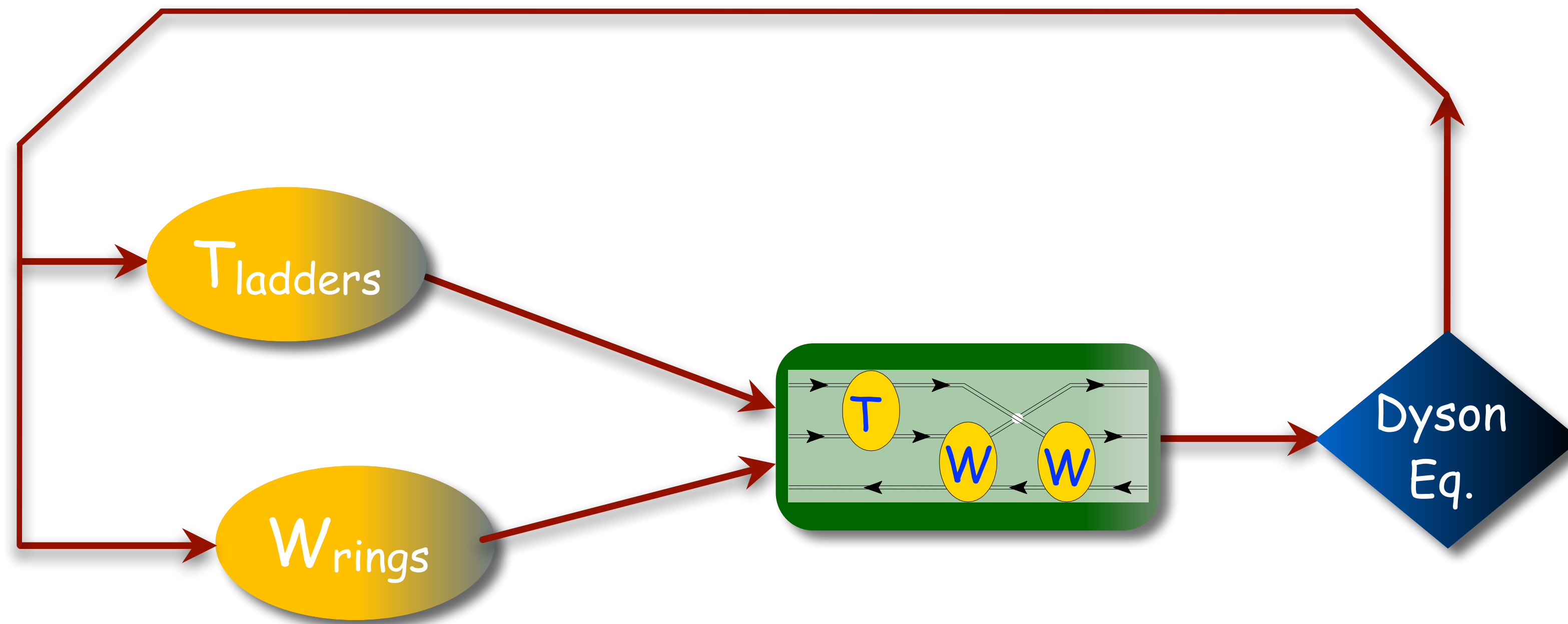
Phys. Rev. A **76**, 052503 (2007)

Phys. Rev. A **83**, 042517 (2011)

ADC(3):

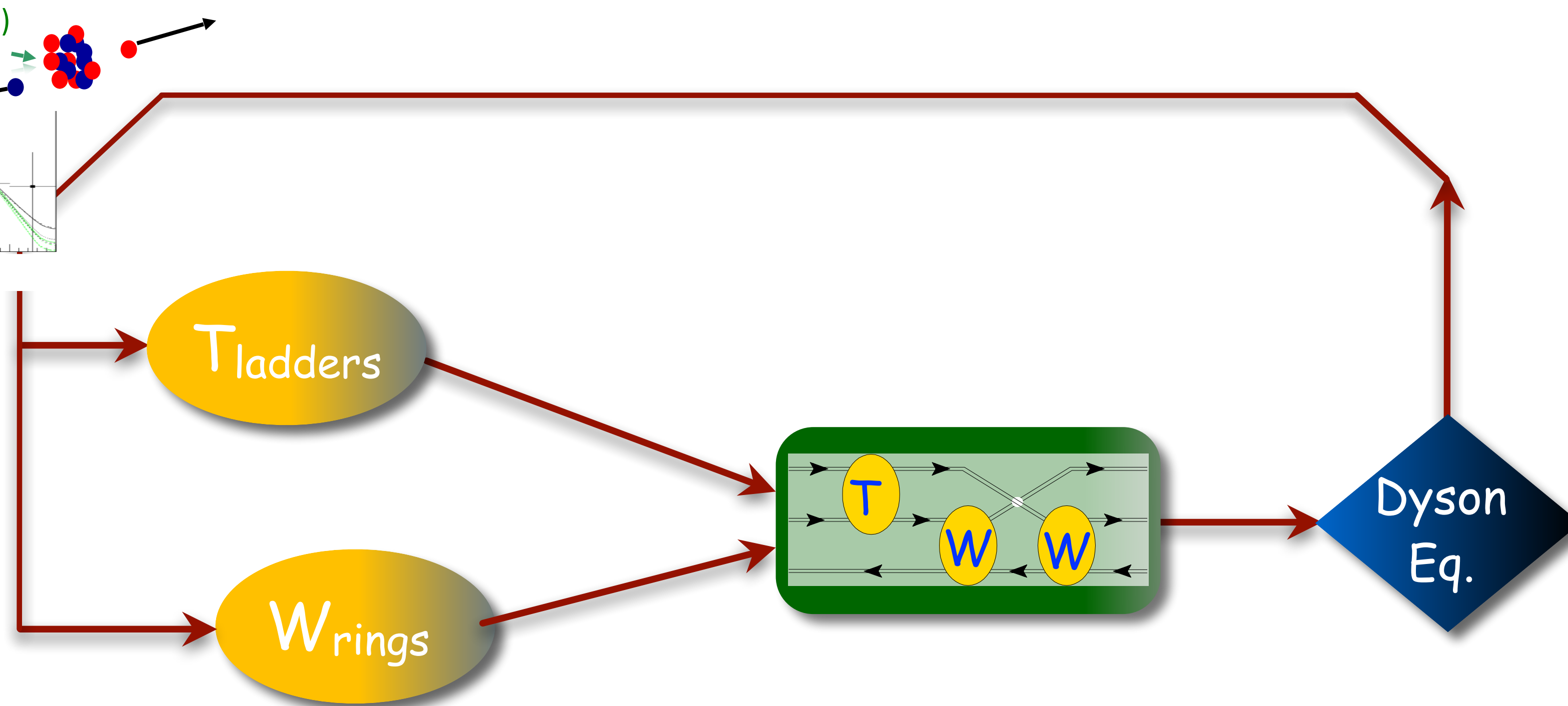
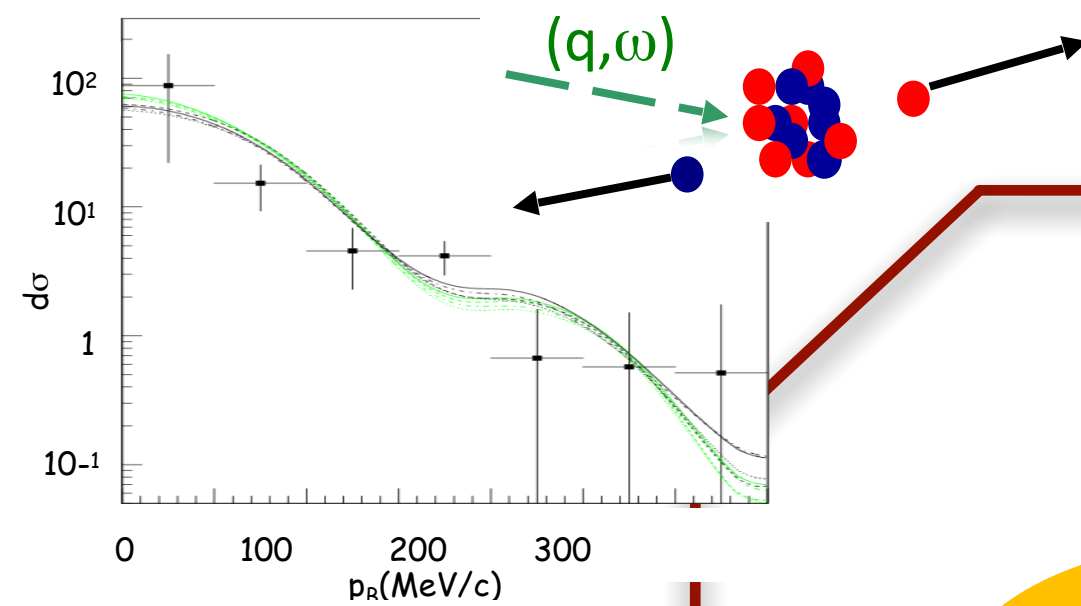
Lect. Notes in Phys **936** (2017)-
Chapter 11.

The Self-Consistent Green's Function with Faddeev-RPA



The Self-Consistent Green's Function with Faddeev-RPA

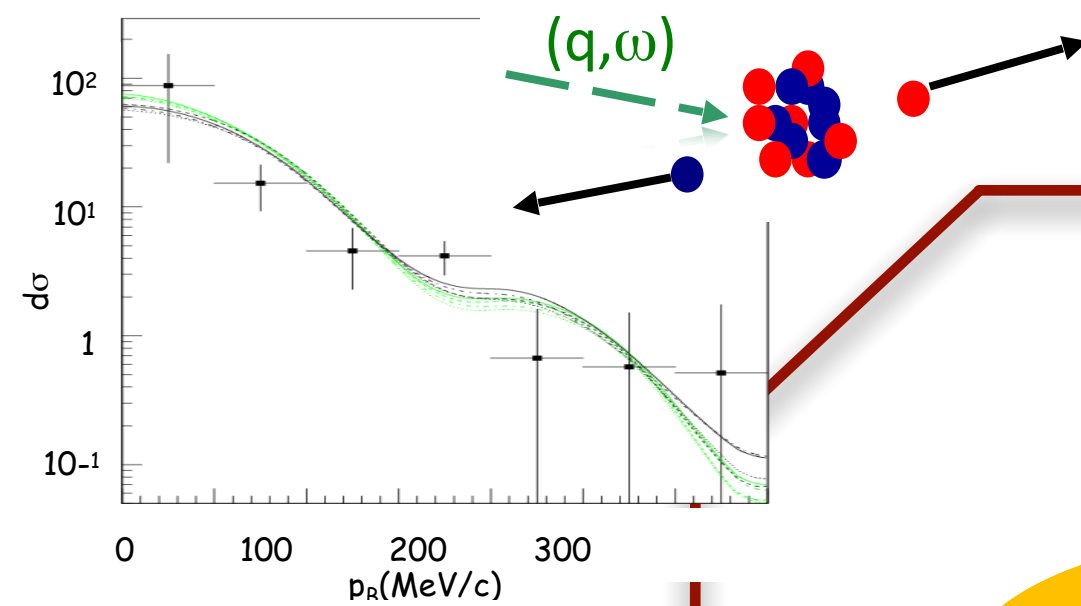
Two-nucleon emission: $^{16}\text{O}(e,e'pn)^{14}\text{N}$
[Eur. Phys. J. A43, 137 (2010)]



The Self-Consistent Green's Function with Faddeev-RPA

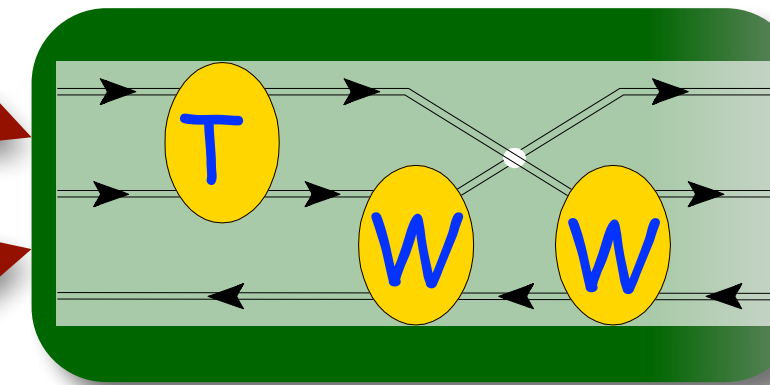
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T_{ladders}

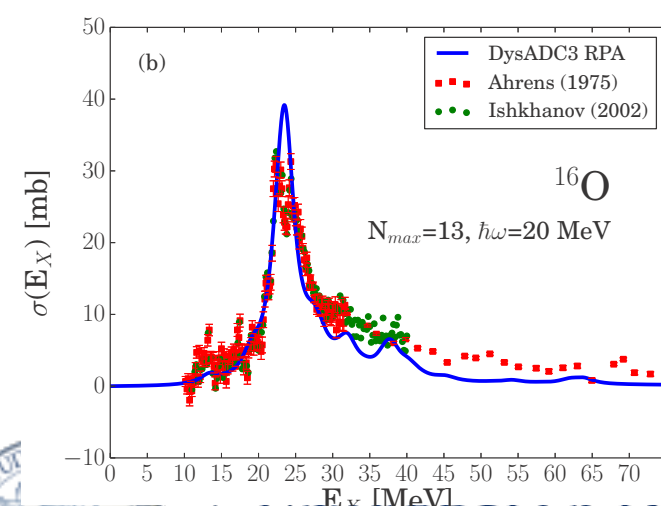
W_{rings}



Dyson Eq.

Nuclear ELM response and dipole polarisability, α_D

[Phys Rev. C77, 024304 (2008)]



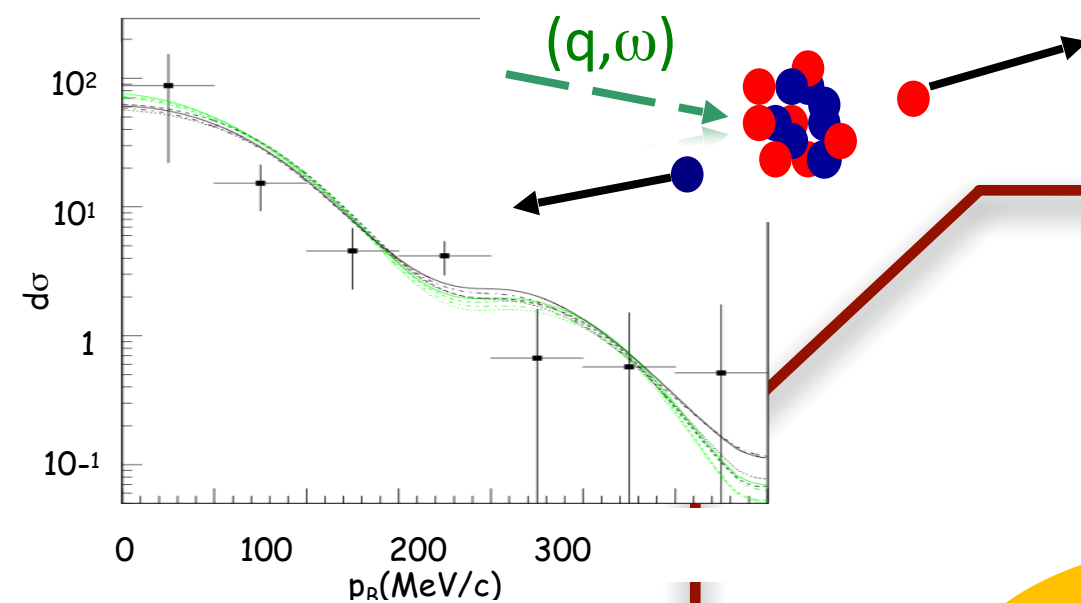
^{68}Ni :

	SCGF	Exp
$E_{\text{PDR}} \text{ (MeV)}$	10.68 10.92	9.55(17)
$E_{\text{GDR}} \text{ (MeV)}$	18.1	17.1(2)
$\alpha_D \text{ (fm}^3\text{)}$	3.60	3.40(23) 3.88(31)

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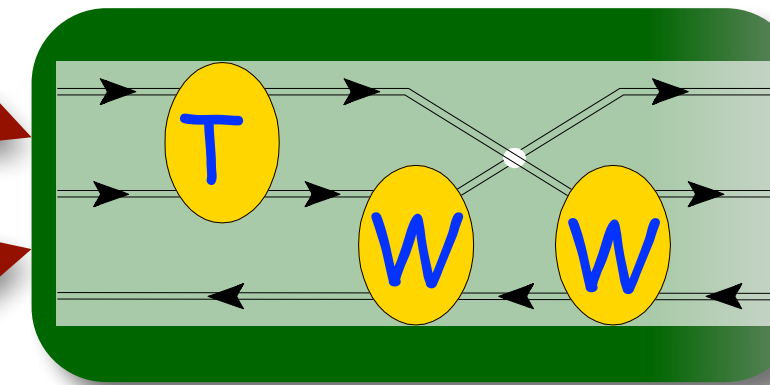
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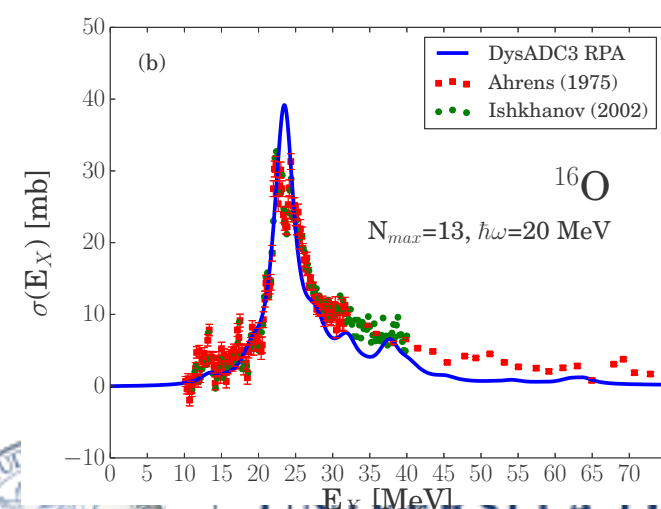
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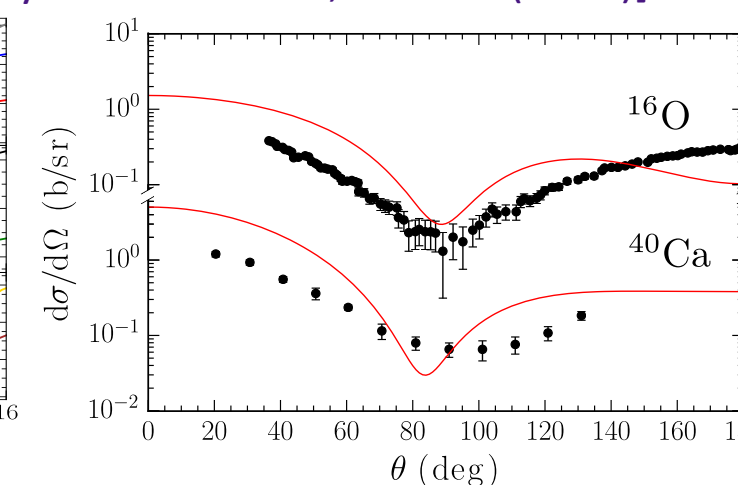
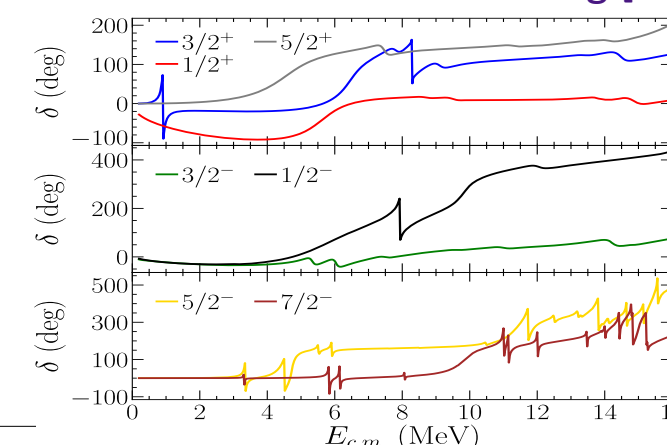


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Optical potential

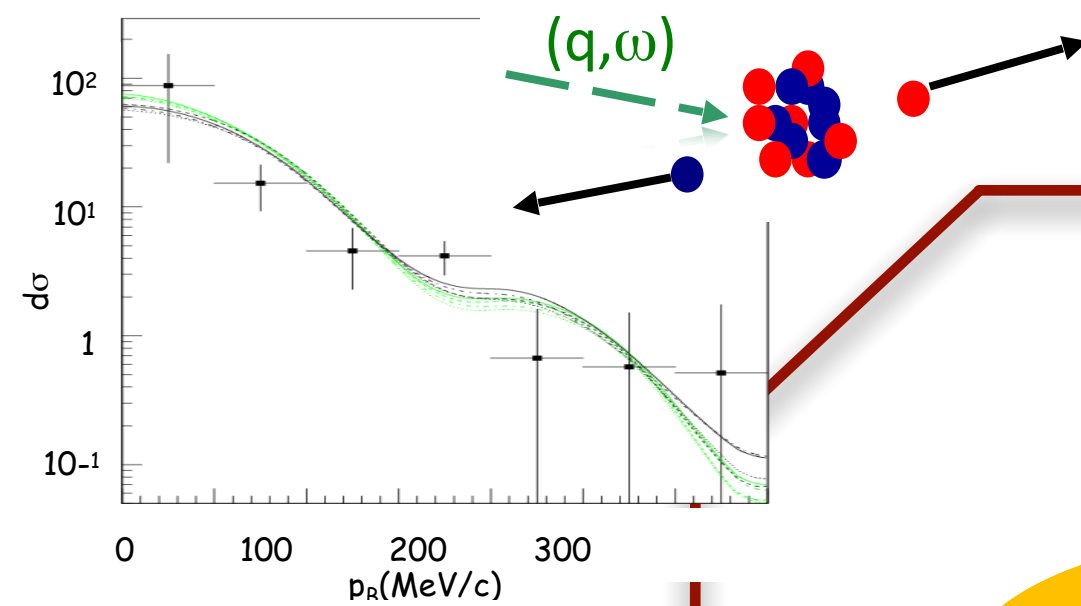
Elastic neutron scattering [Phys. Rev. Lett. 123, 092501 (2013)]



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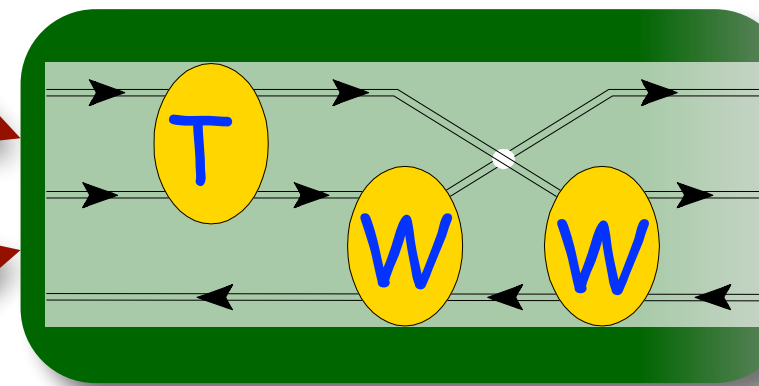
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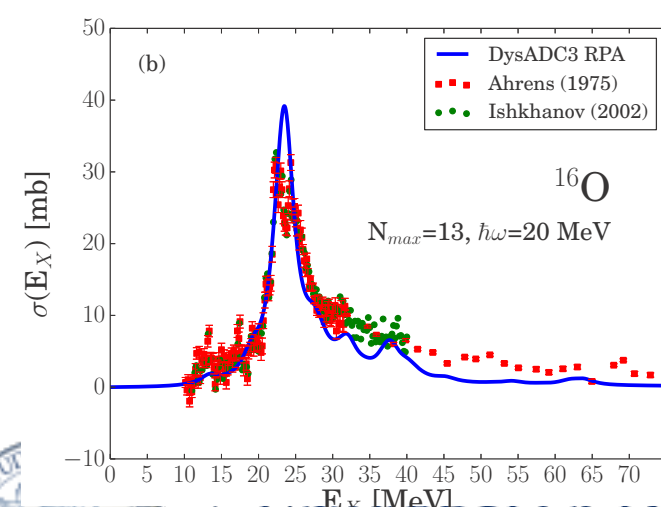
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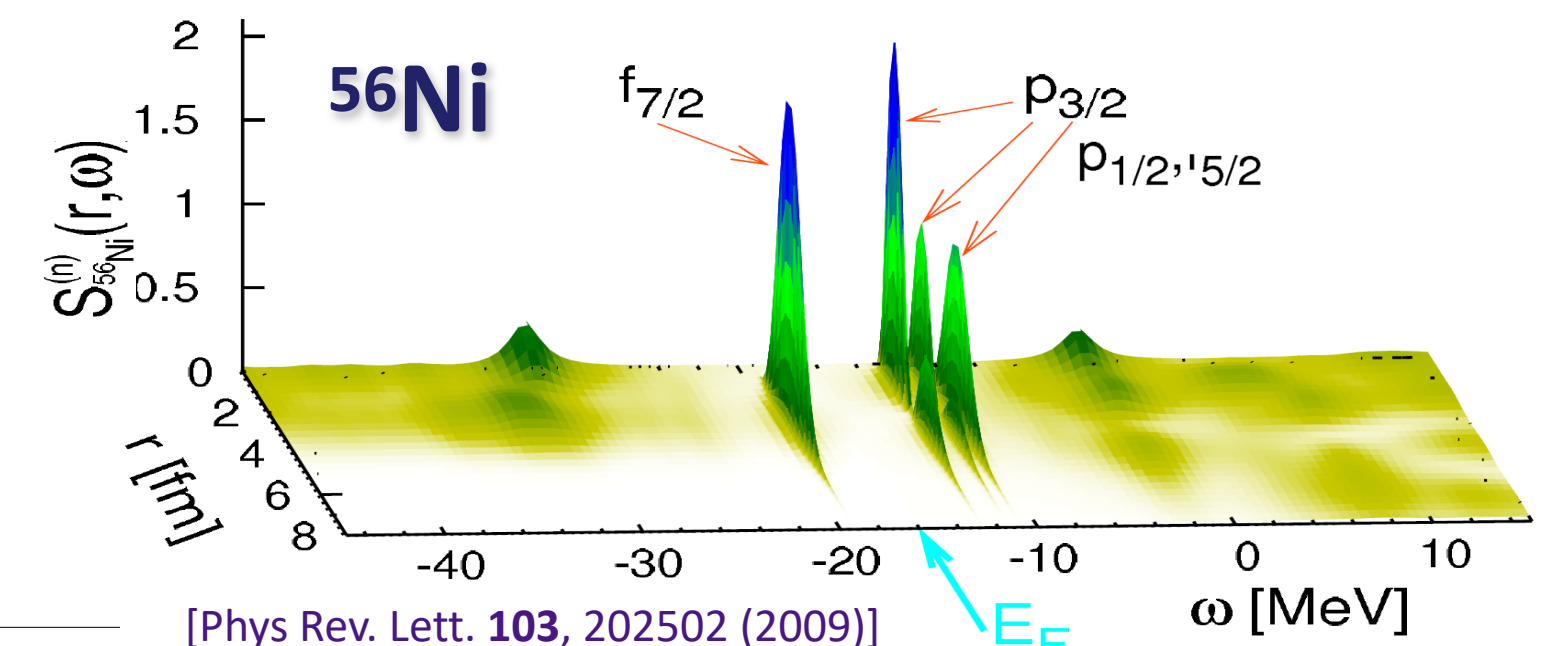
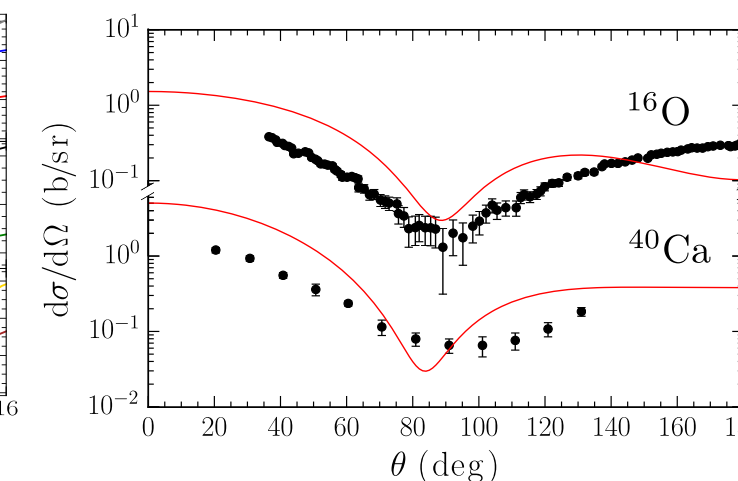
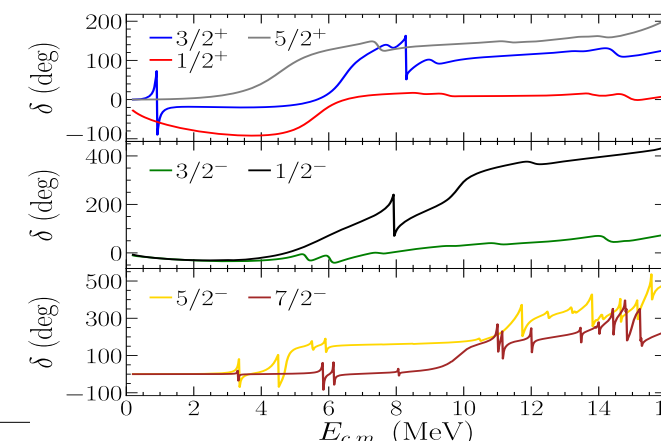


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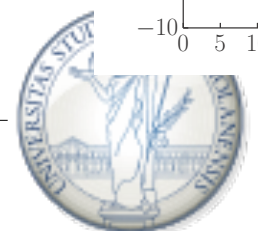
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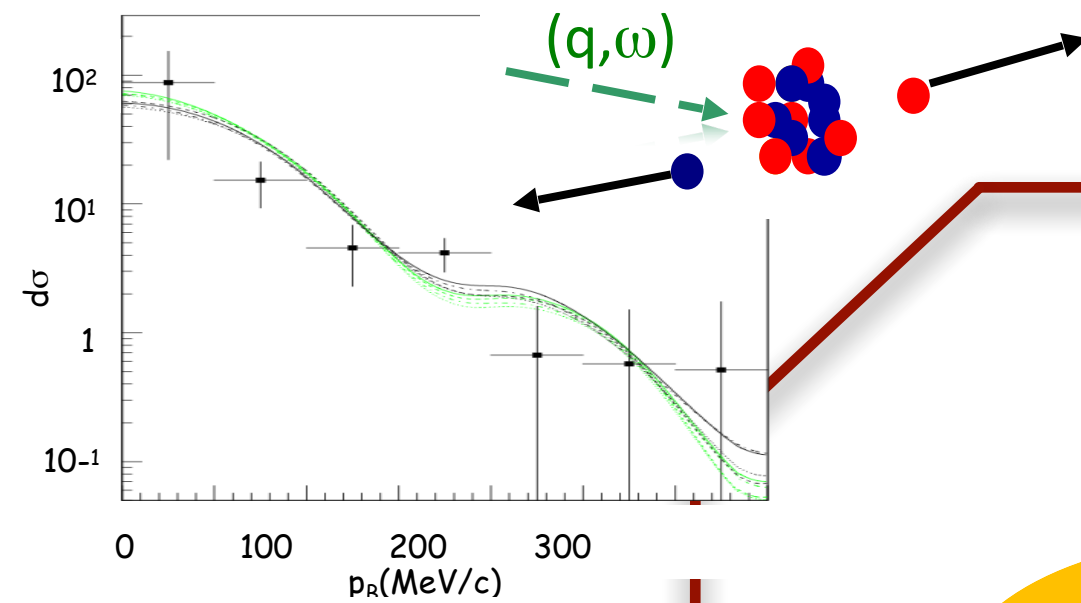
[Phys. Rev. Lett. 103, 202502 (2009)]



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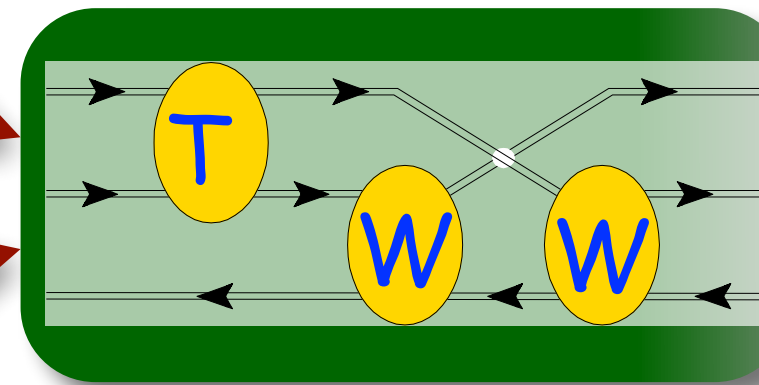
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T_{ladders}

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Dyson Eq.

Spectroscopy

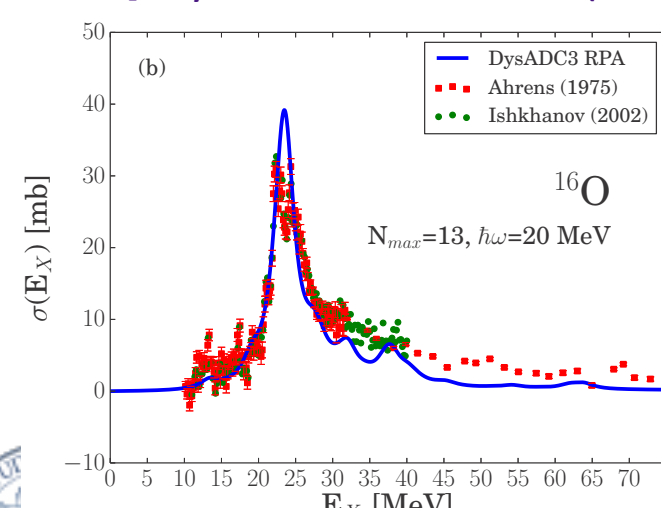
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[Phys. Rev. A. 83, 042517 (2011); 85, 012501 (2012)]

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	4σ	20.19	19.46	19.59	19.72
H ₂ O	1b ₁	12.86	12.62	12.67	12.62
	3a ₁	15.15	14.91	14.98	14.74
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Nuclear ELM response and dipole polarisability, α_D

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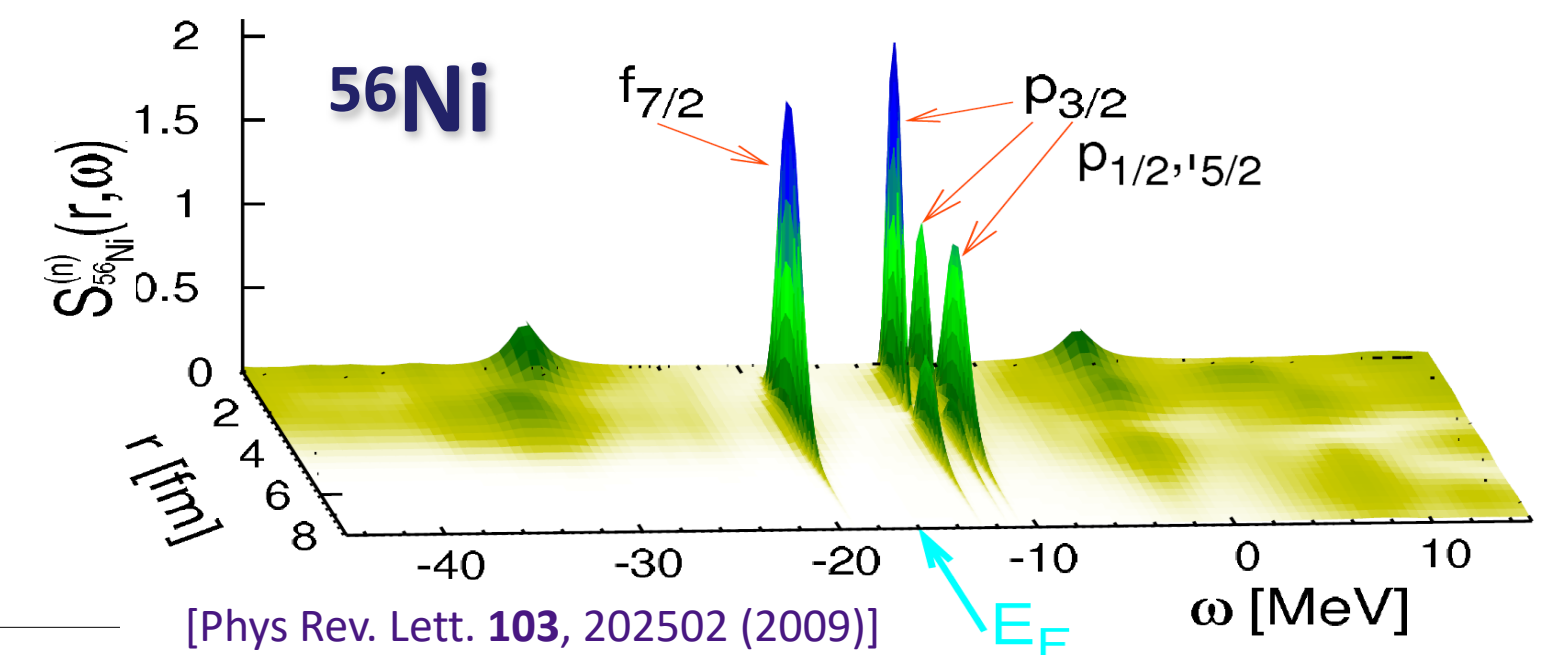
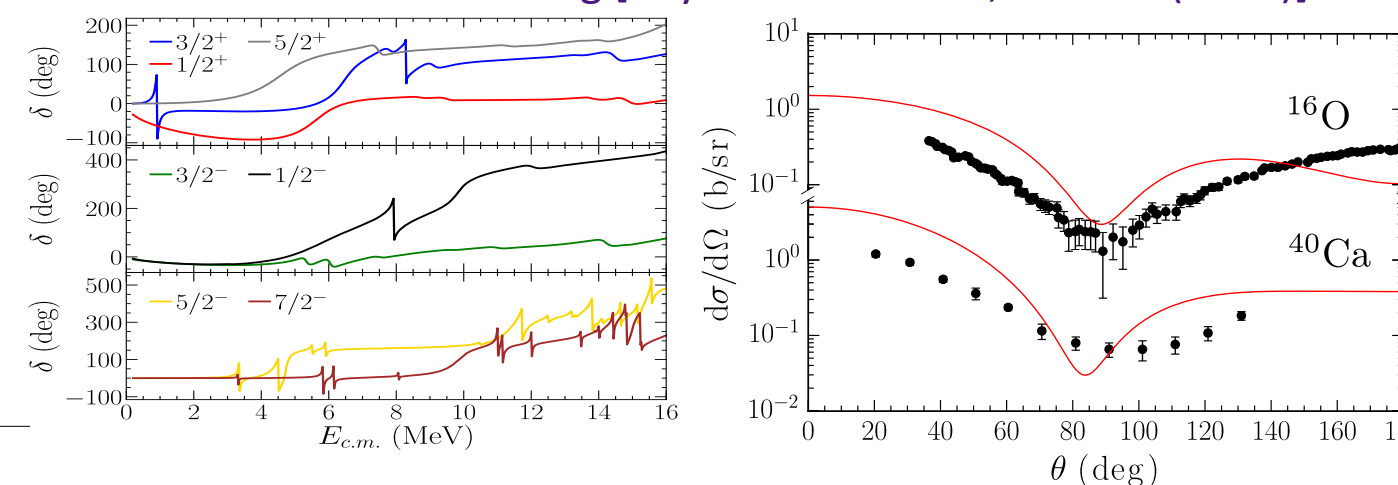


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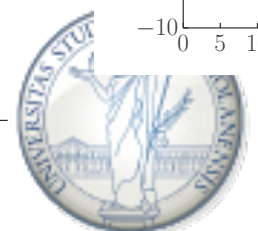
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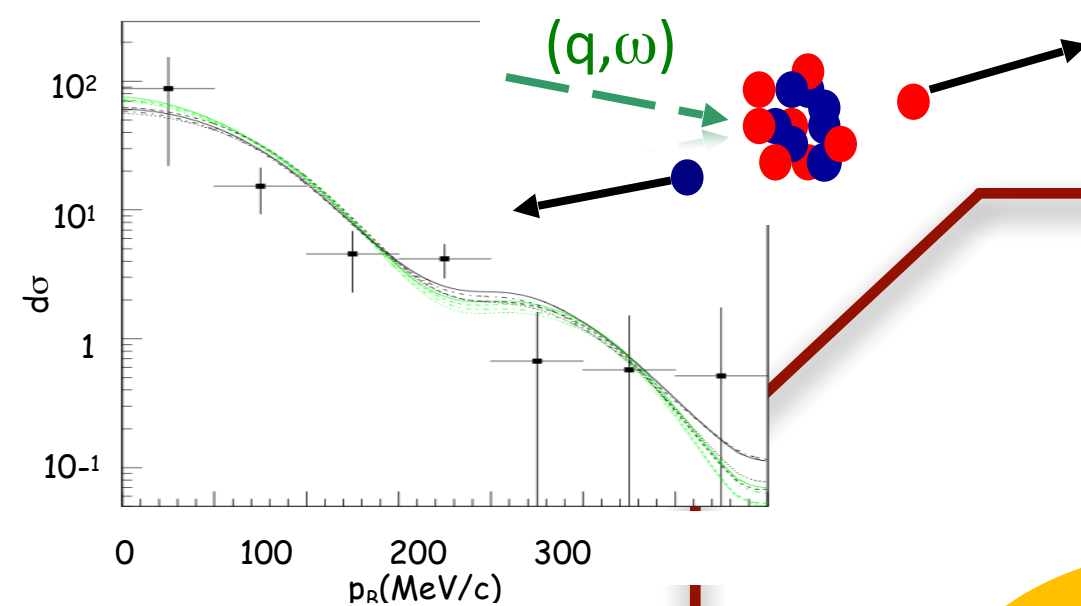
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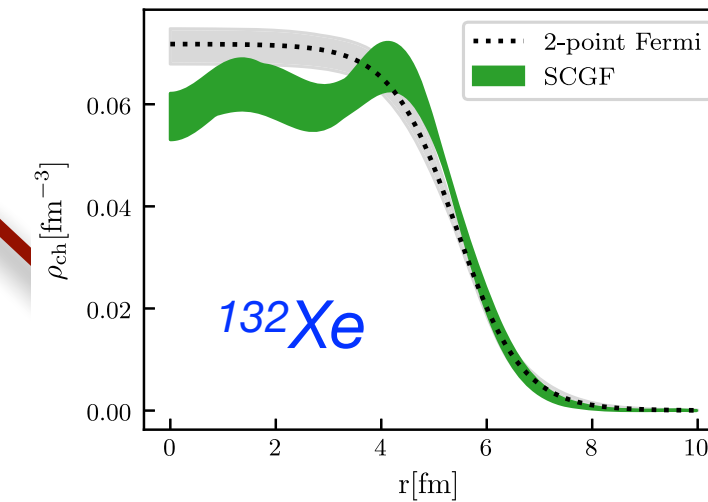
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Charge & matter distribution

Neutron skins [Phys. Rev. Lett. 125, 182501 (2020)]



	SCGF	Exp.
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^{132}Sn	4.725 – 4.956	4.7093
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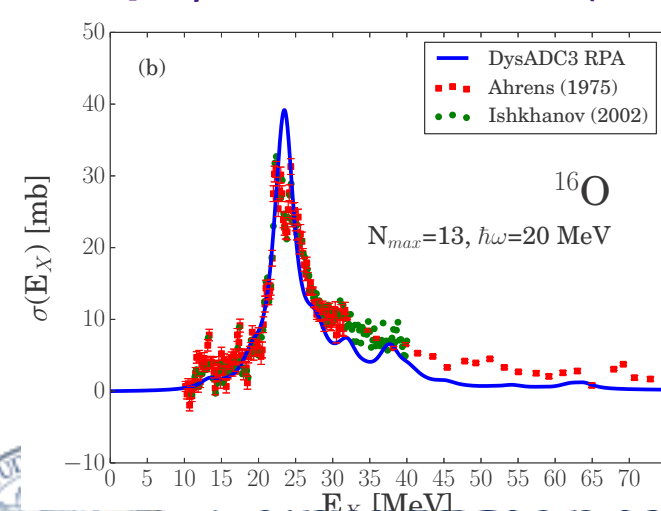
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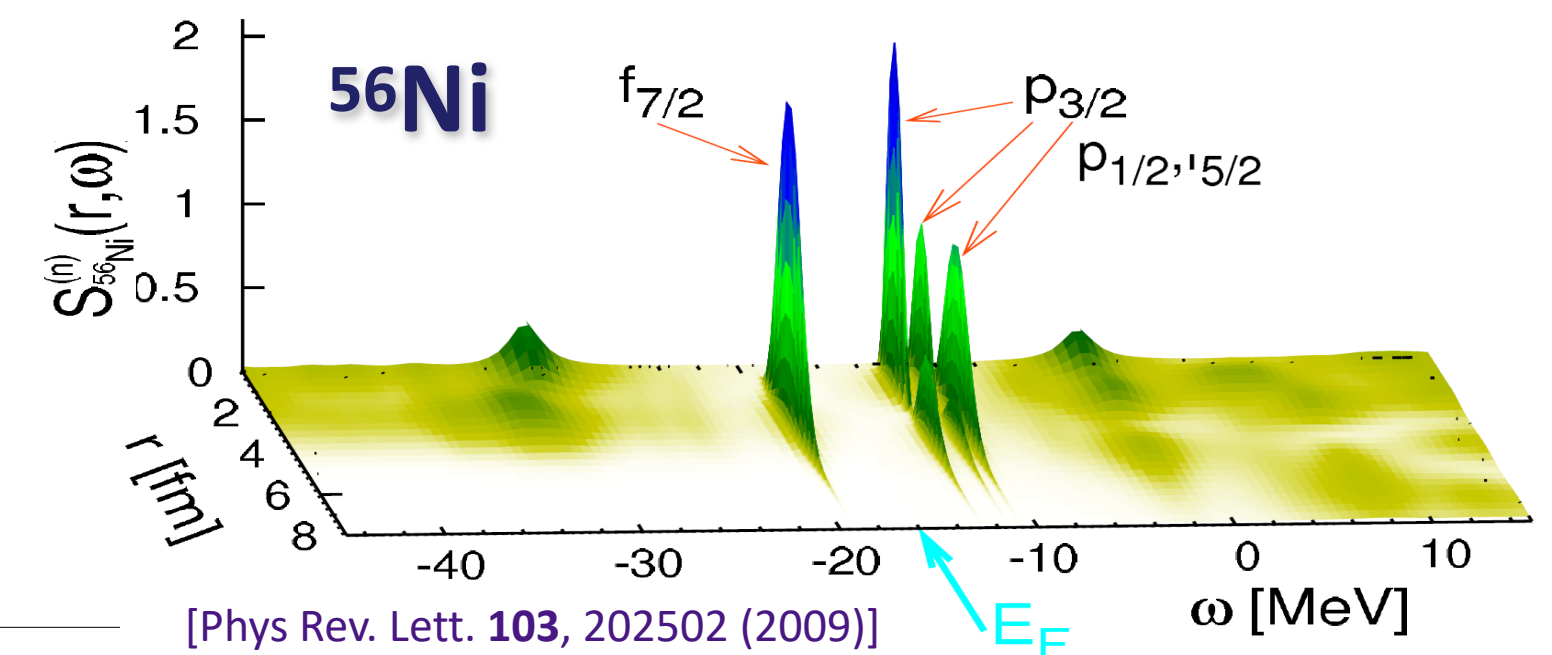
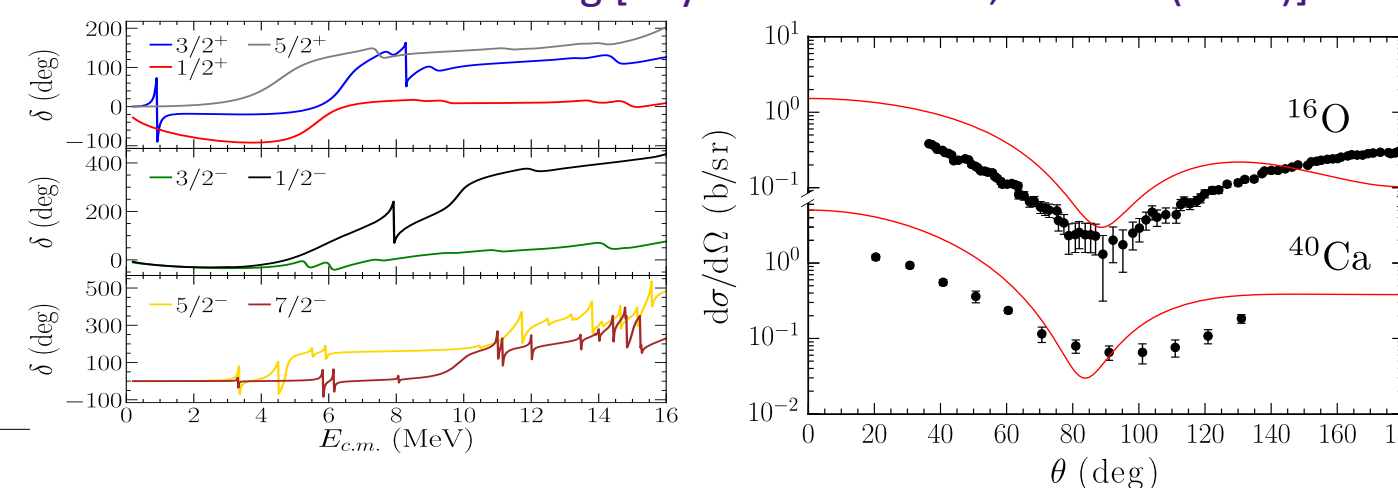


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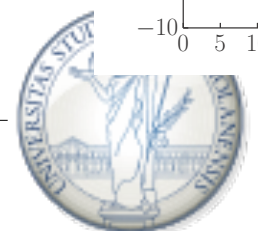
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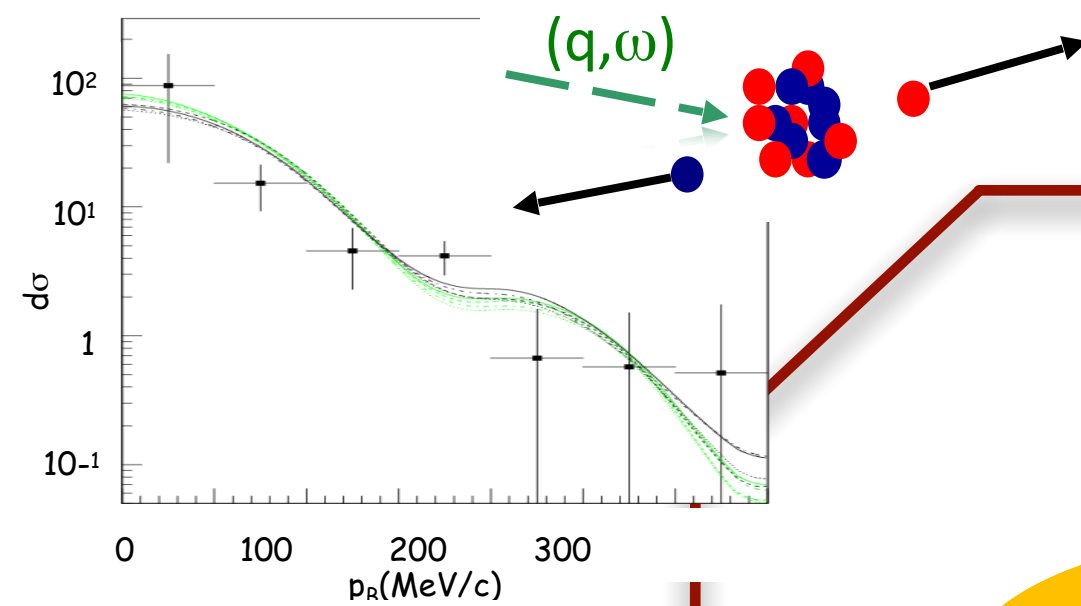
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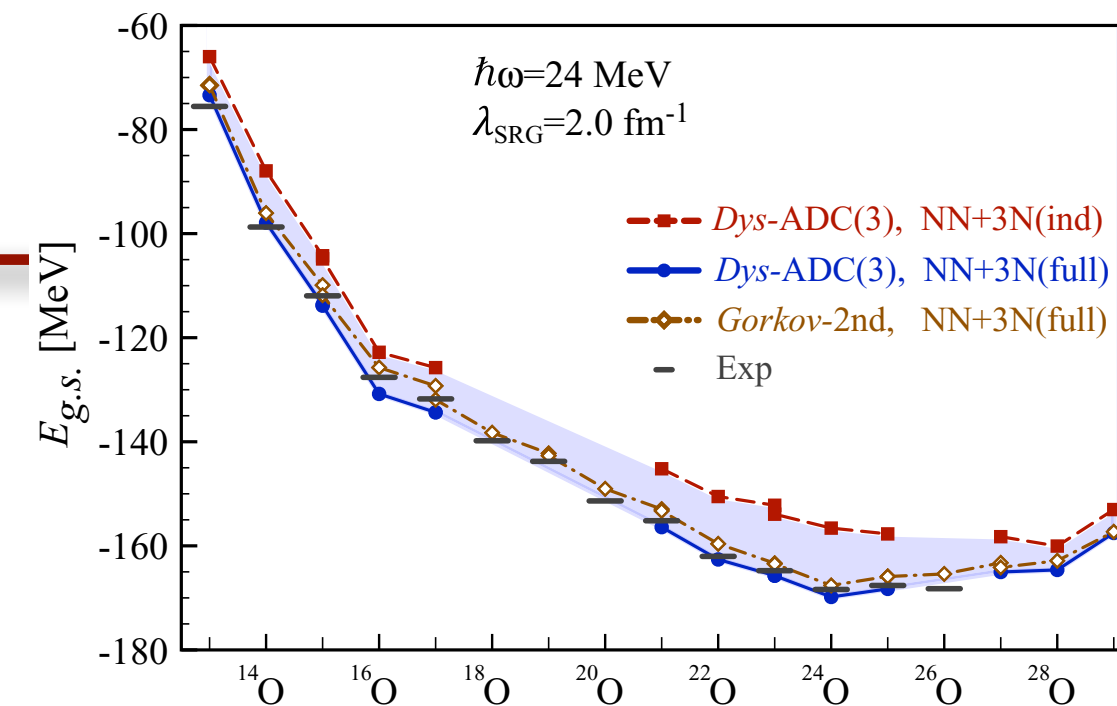
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Binding energies

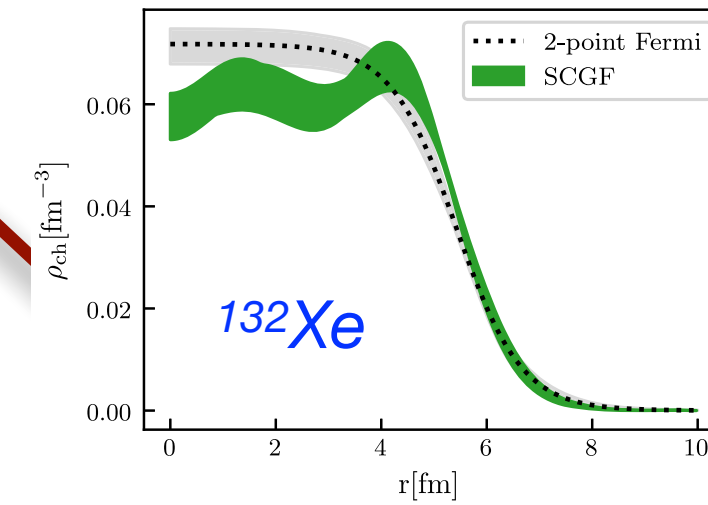
Oxygen drip line

[Phys. Rev. Lett. 111, 062501 (2013)]



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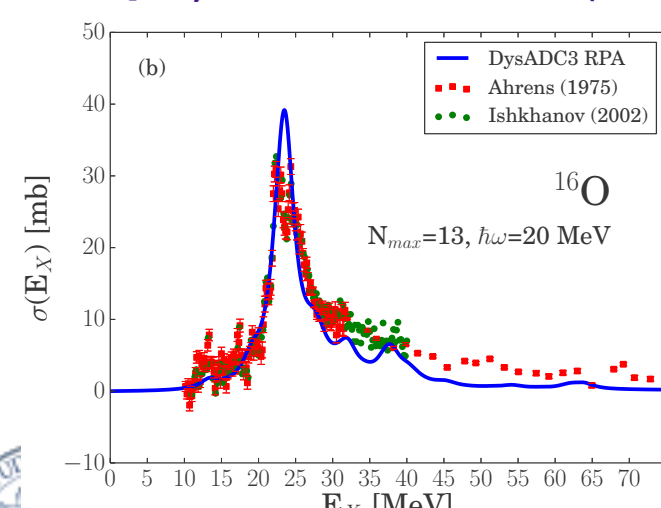
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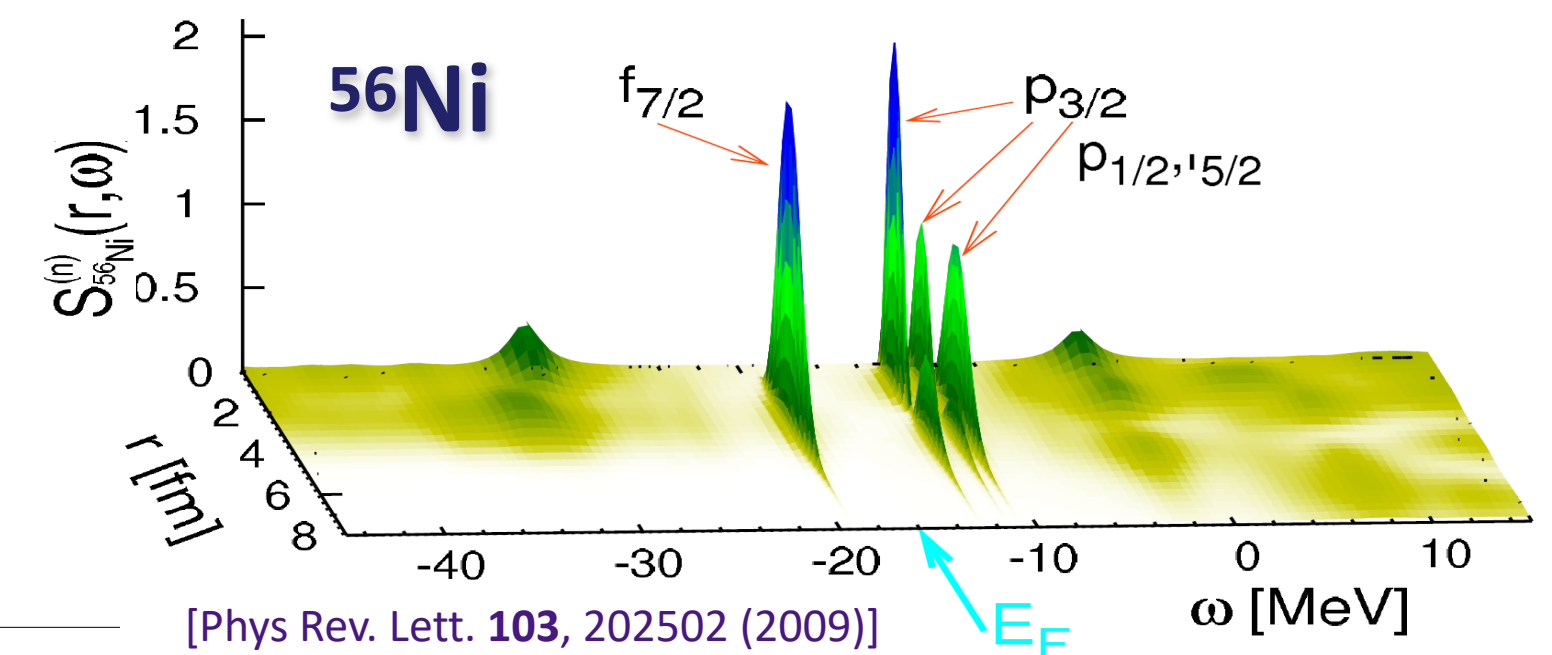
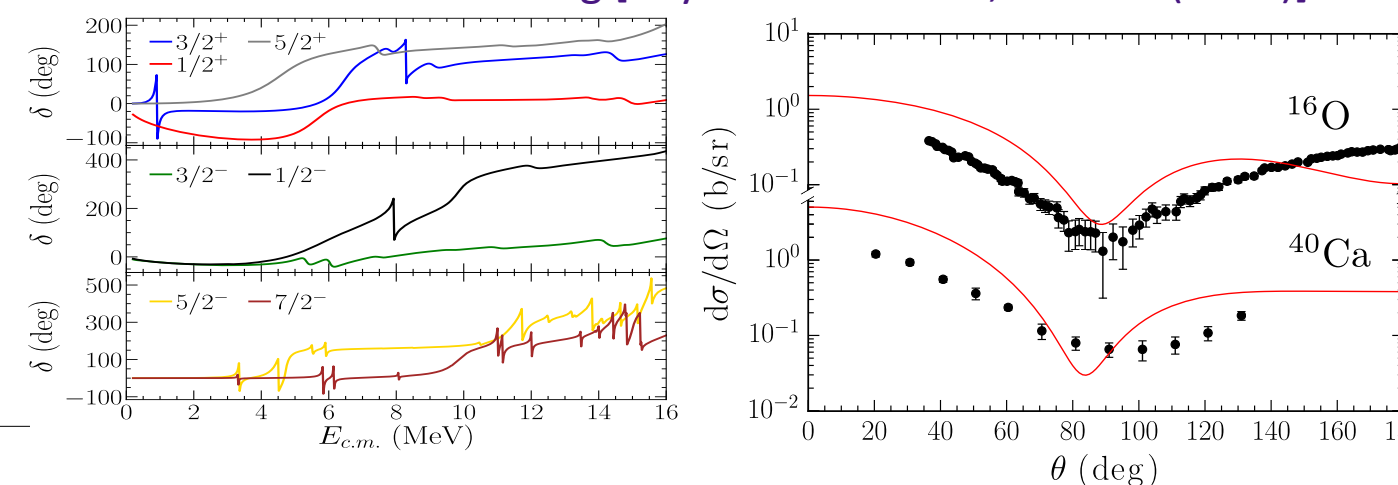


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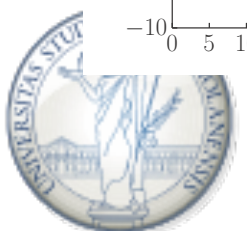
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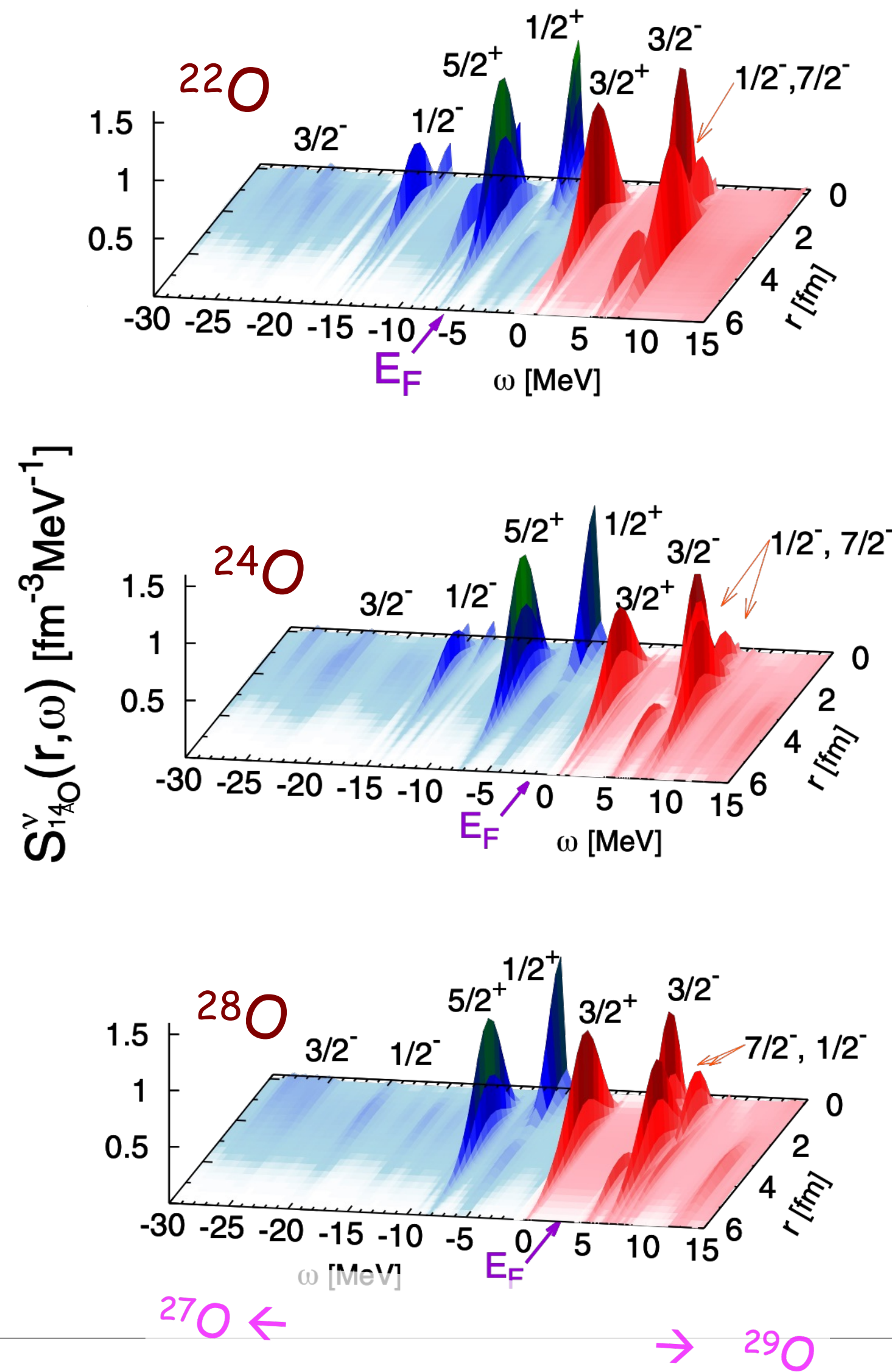
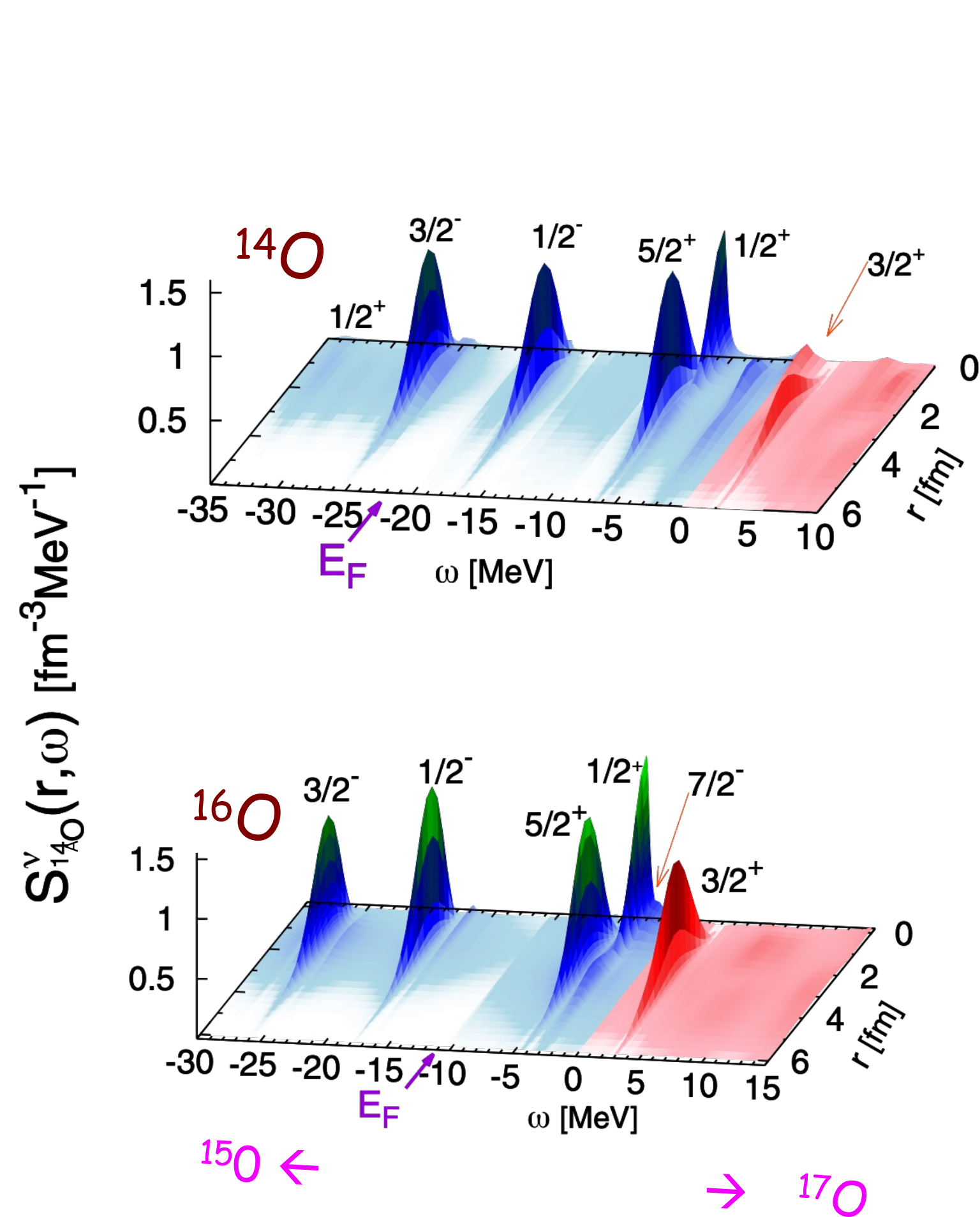


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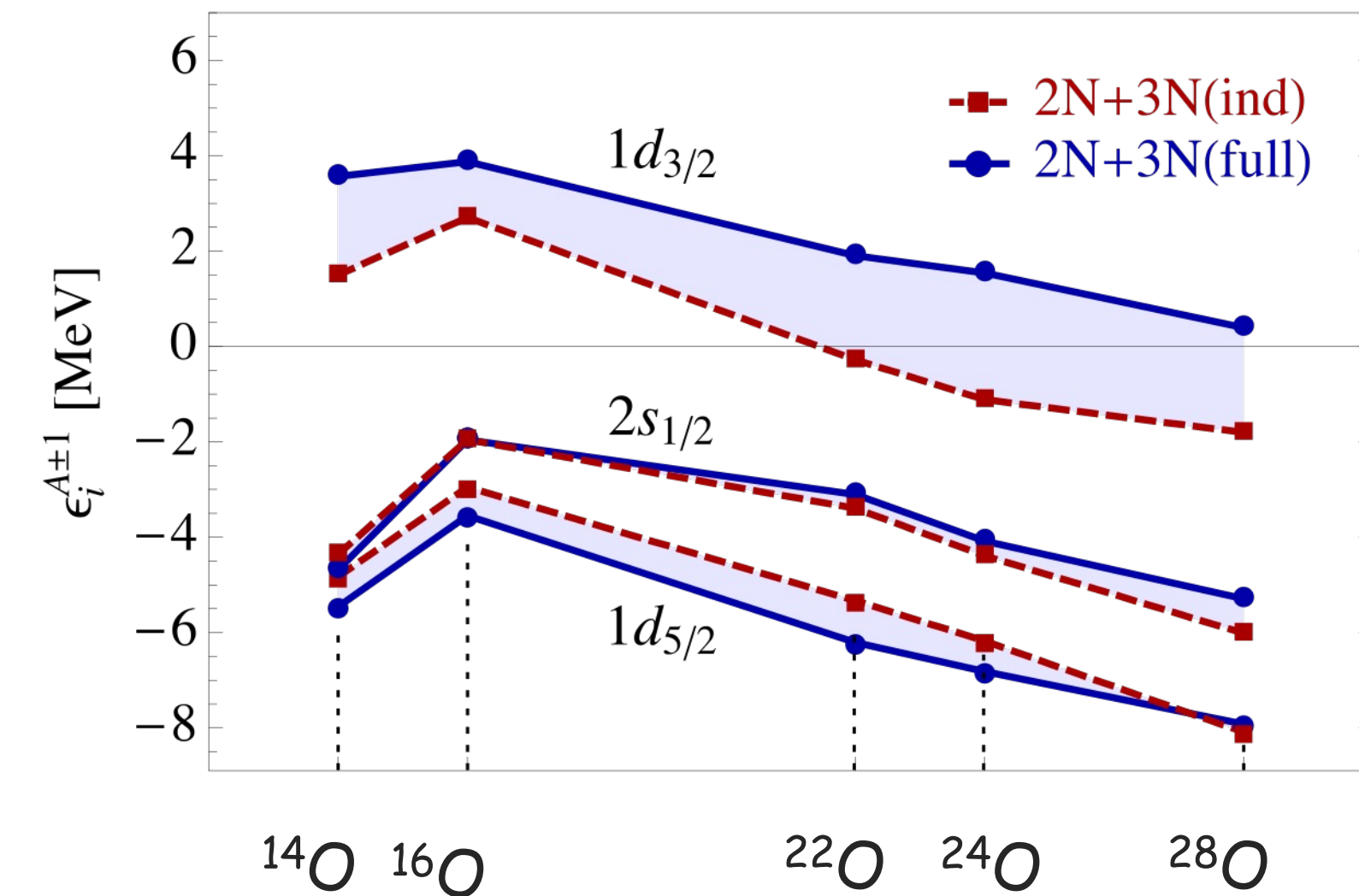


Neutron spectral function for oxygen isotopes

A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* **92**, 014306 (2015)

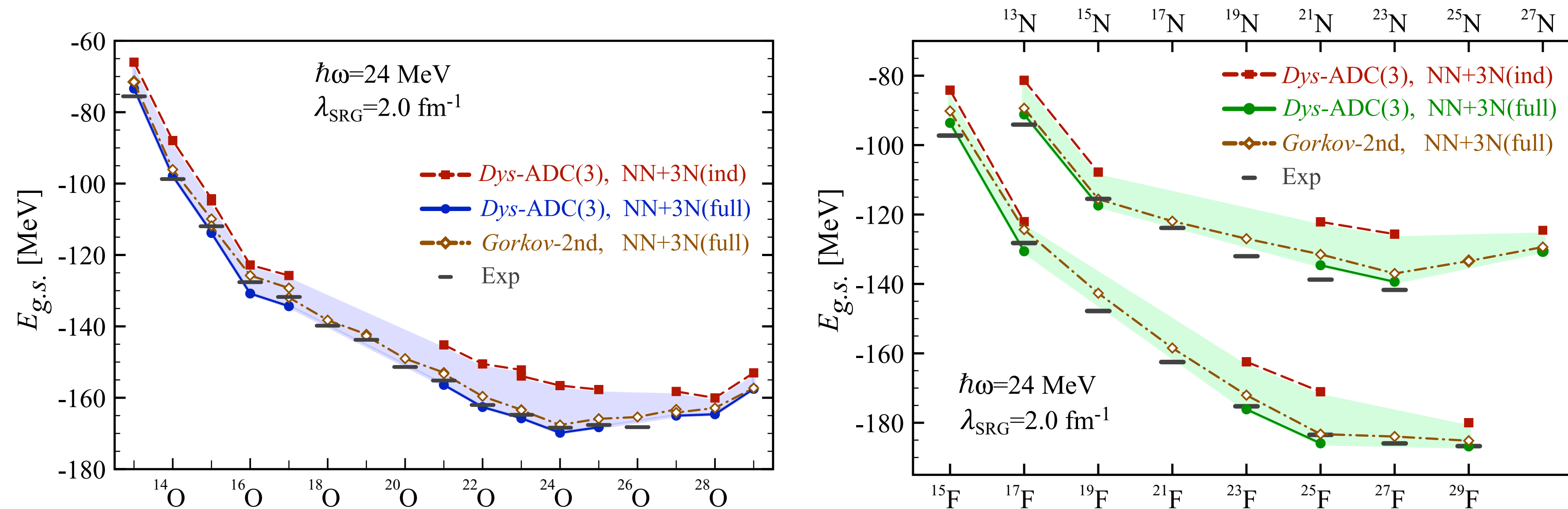


Neutron quasiparticle energies



Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)

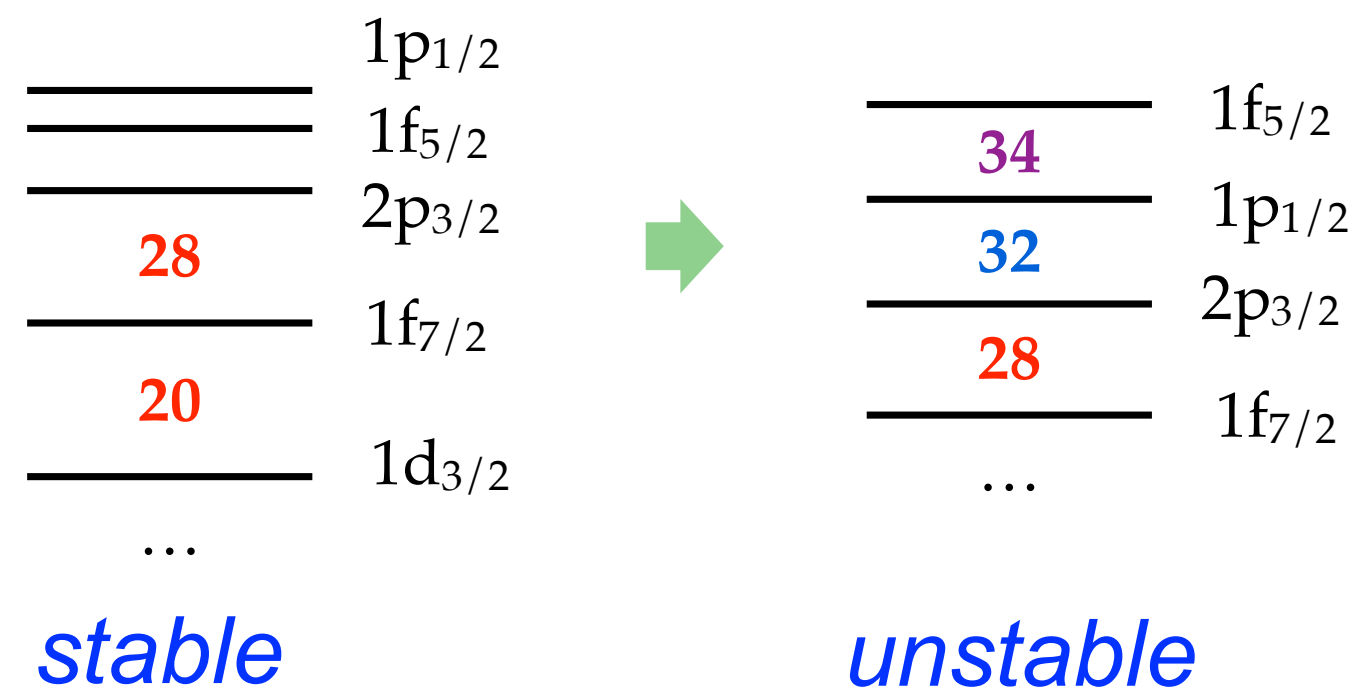


→ 3NF crucial for reproducing binding energies and driplines around oxygen

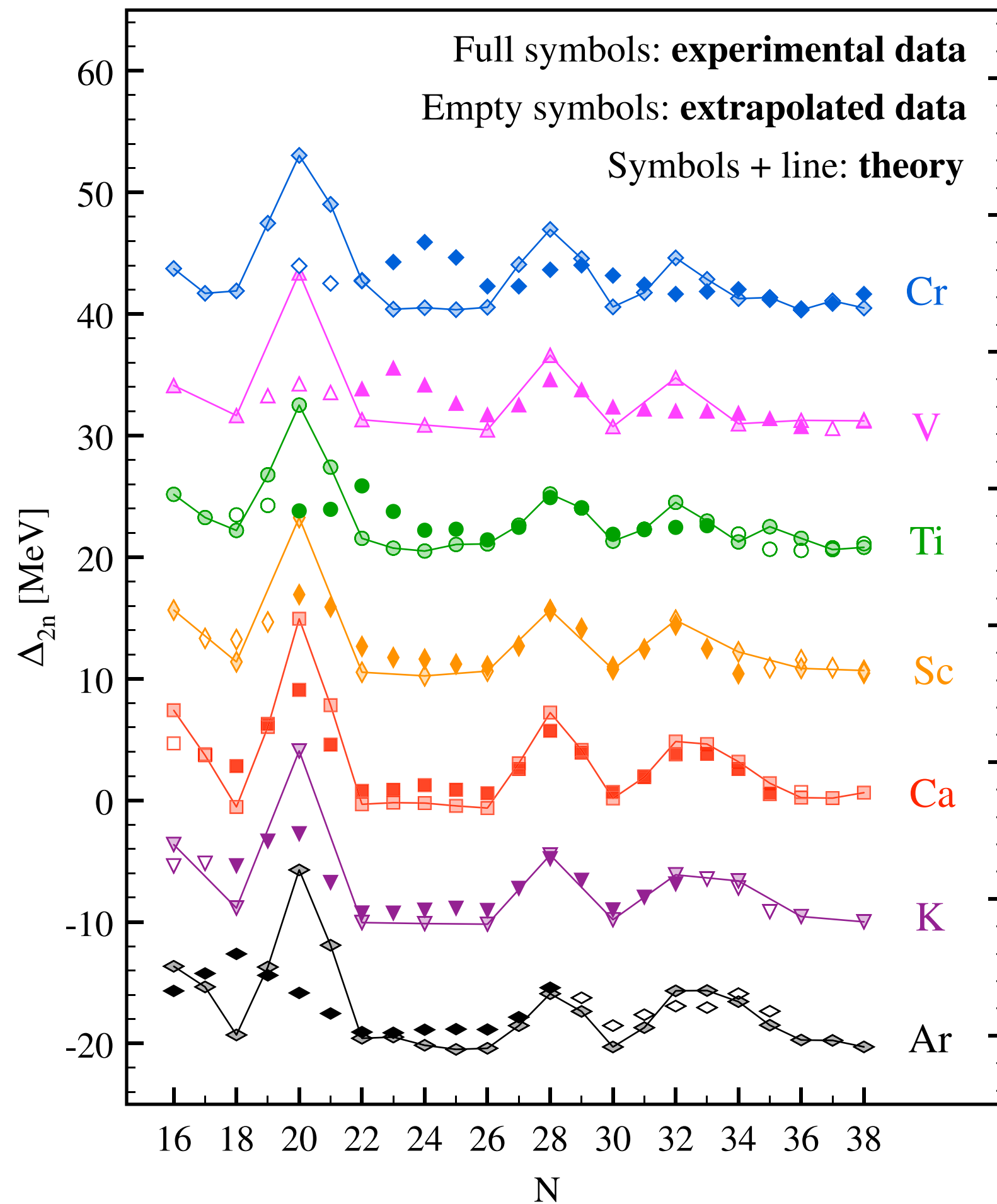
→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

Magic numbers

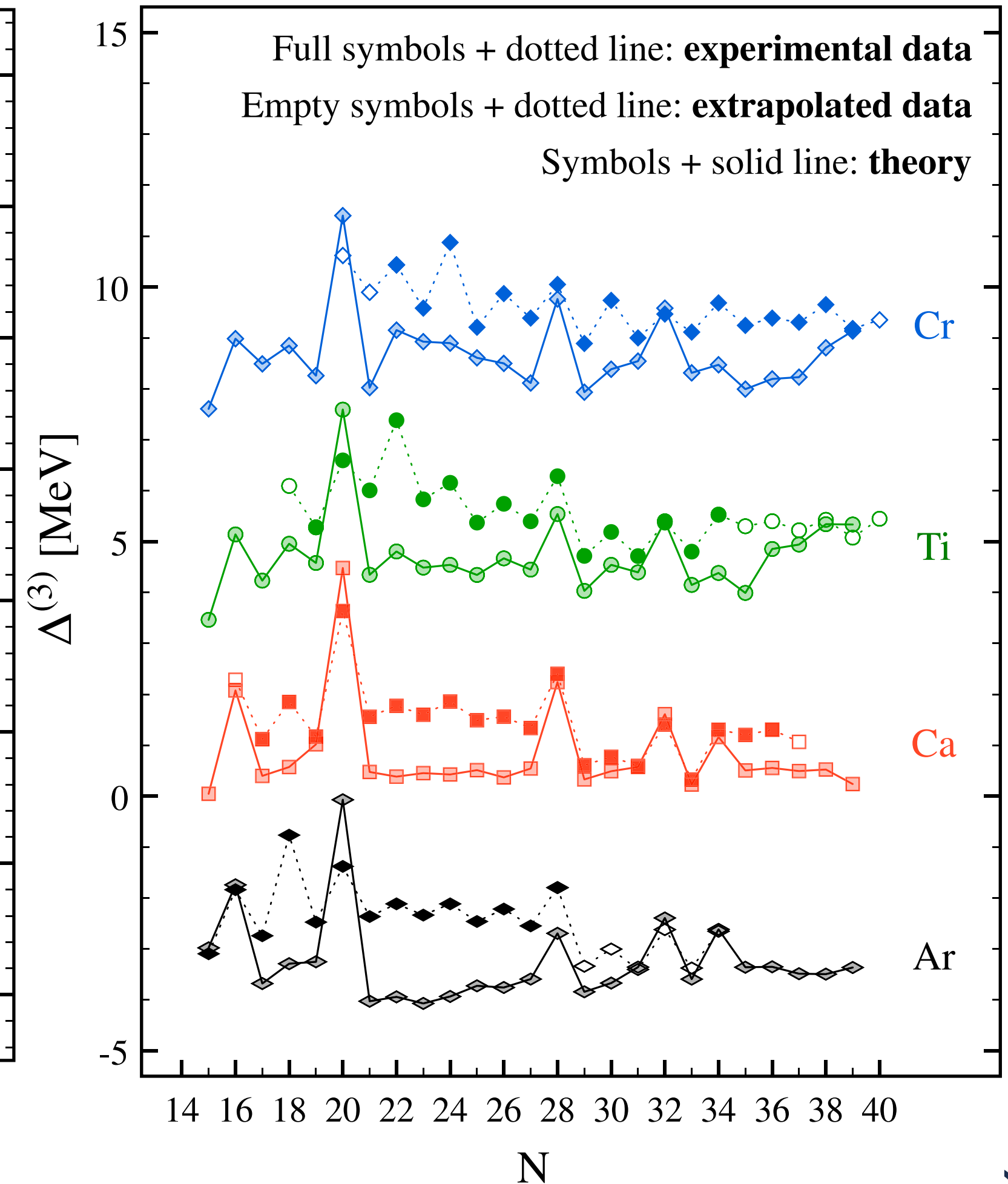
- Magic numbers: extra-stable combinations of N & Z



$$\Delta_{2n}(N, Z) \equiv S_{2n}(N, Z) - S_{2n}(N + 2, Z)$$

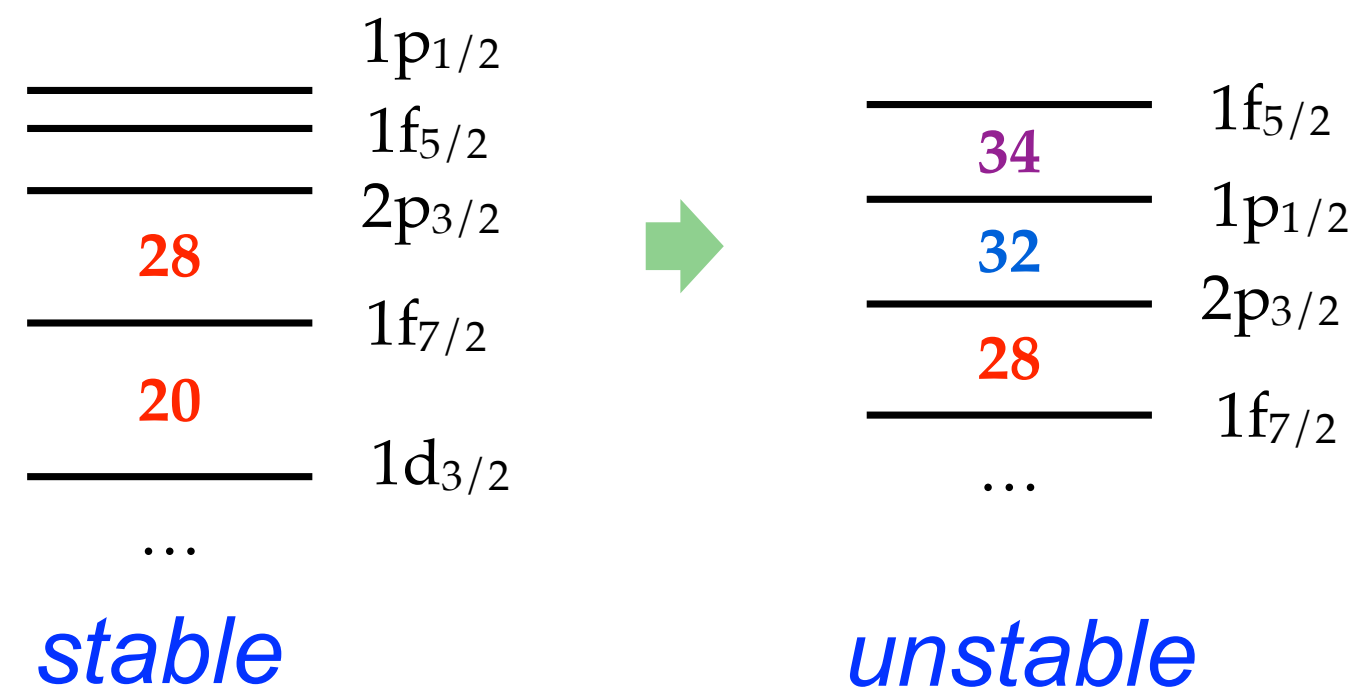


$$\Delta^{(3)}(N, Z) \equiv \frac{(-1)^N}{2} [E(N-1, Z) - 2E(N, Z) + E(N+1, Z)]$$

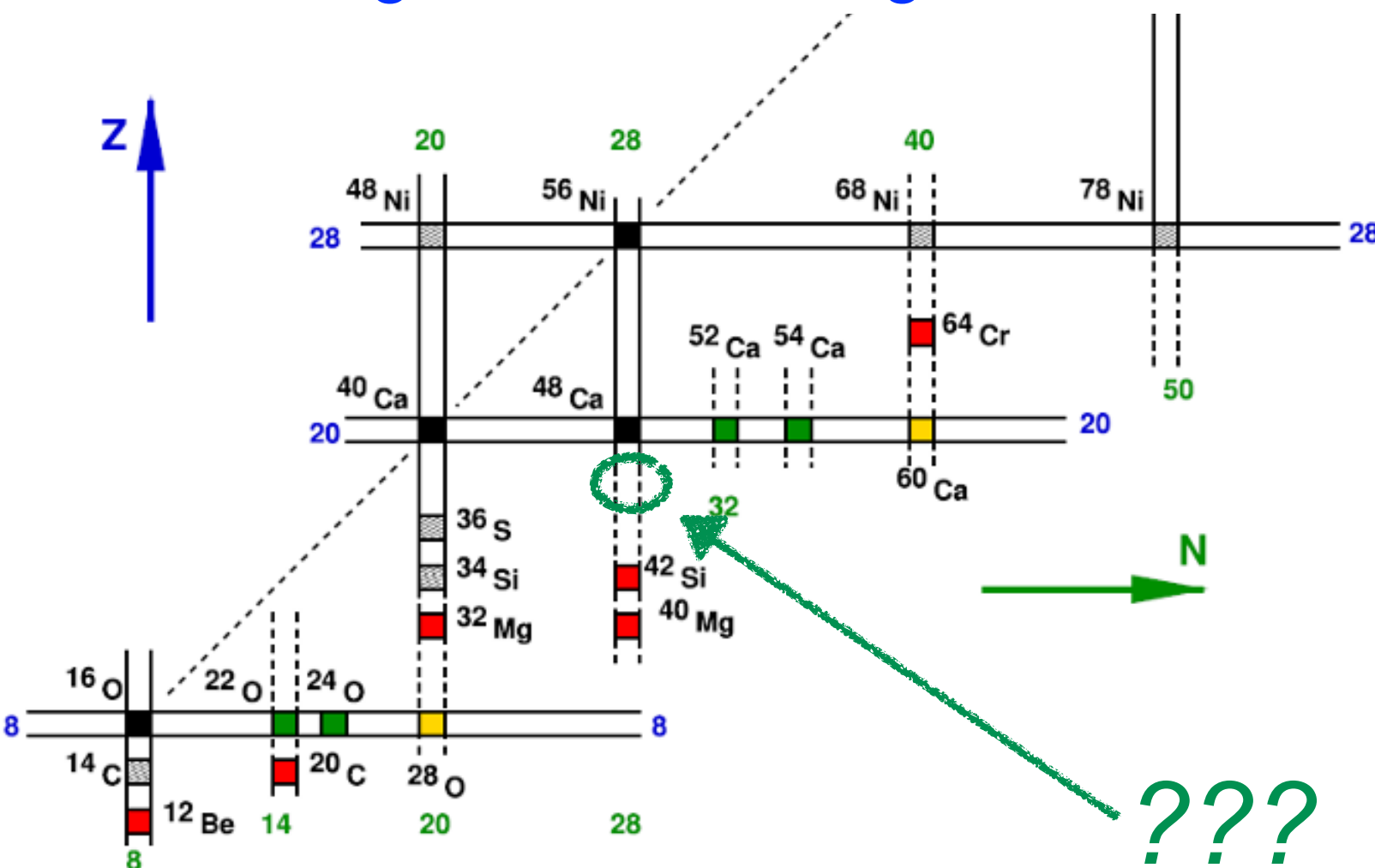


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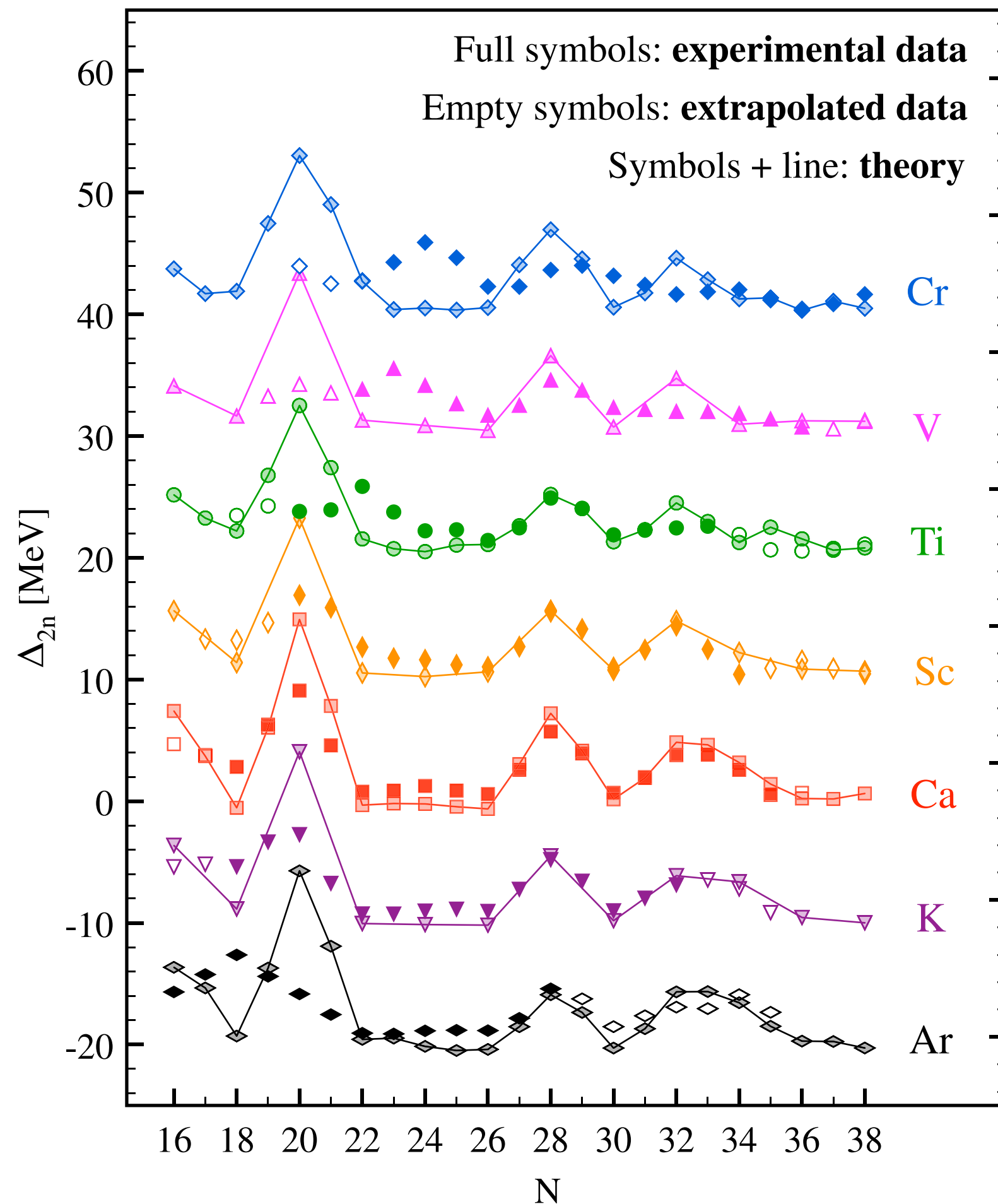
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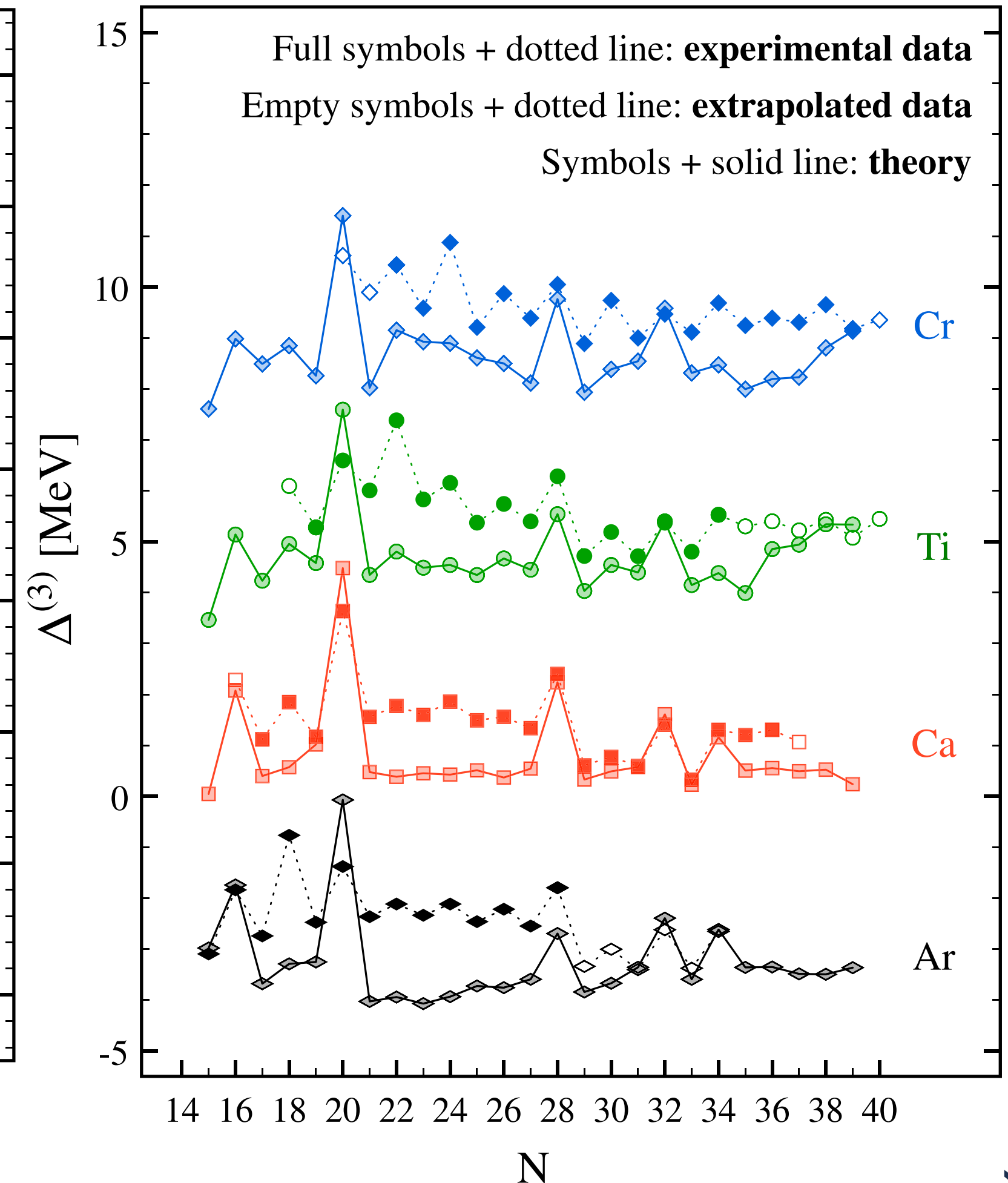
Known magic and semimagic nuclei:



$$\Delta_{2n}(N, Z) \equiv S_{2n}(N, Z) - S_{2n}(N + 2, Z)$$



$$\Delta^{(3)}(N, Z) \equiv \frac{(-1)^N}{2} [E(N-1, Z) - 2E(N, Z) + E(N+1, Z)]$$



[Somà, CB et al. Eur. Phys. J. A **57**, 135 (2021)]

[Nowacki, Obertelli, Poves, PPNP 120, 103866 (2021)]



DIPARTIMENTO DI FISICA



The physics contained in the **one-body** Green's function

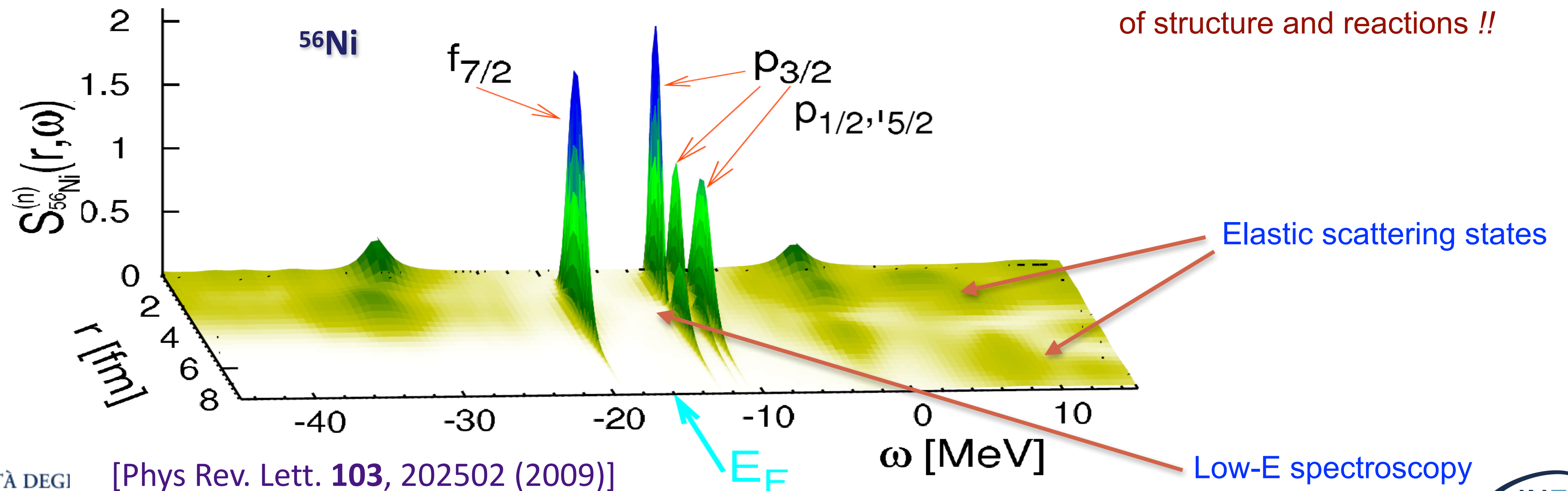
One-body propagator: $g_{\alpha\beta}(t - t') = -i/\hbar \langle \Psi_0^A | T[c_\alpha(t) c_\beta^\dagger(t')] | \Psi_0^A \rangle$

Dyson Equation: $g_{\alpha\beta}(\omega) = g_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} g_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^*(\omega) g_{\delta\beta}(\omega)$

(One-body) spectral function: $S(\alpha, \omega) := \pm \frac{1}{2\pi} \Im m \{ g_{\alpha\alpha}(\omega) \}$

The self-energy $\Sigma_{\alpha\beta}^*(\omega)$ 'decides' both ground state observables and elastic scattering.

==> Consistent computation of structure and reactions !!



Ab initio optical potentials from propagator theory

Relation to Feshbach theory:

Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)

Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

Previous SCGF work:

CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005)

S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011)

A. Idini, CB, P. Navrátil, Phys. Rv. Lett. **123**, 092501 (2019)

M. Vorabbi, CB, et al., in preparation

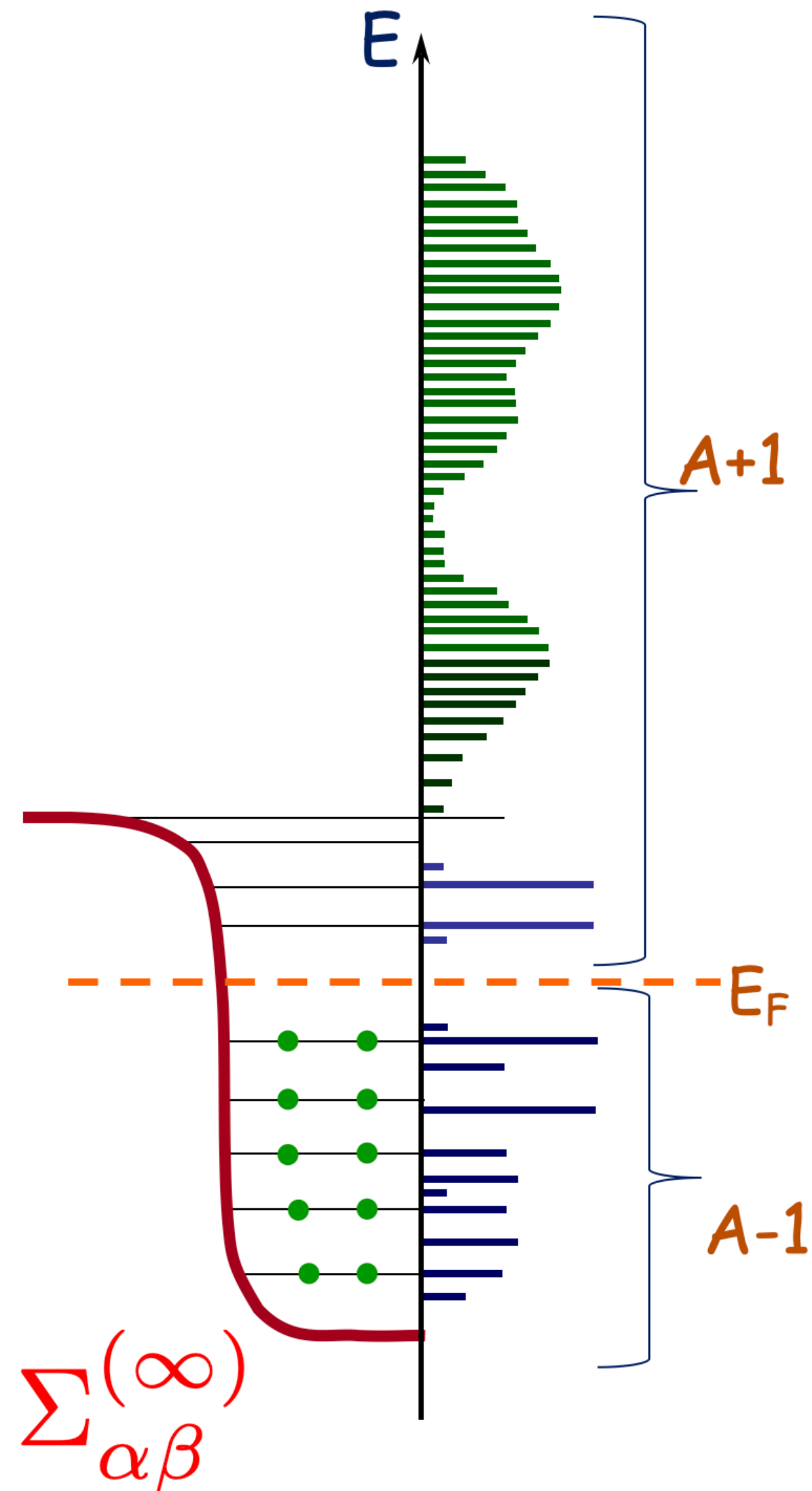
State-of-the-art of the field:

C. Hebborn, et al., CB, “*Optical potentials for the rare-isotope beam era*”, arXiv:2210.07293 (WP)

Jour. Phys. G (2023), in press.



Microscopic optical potential

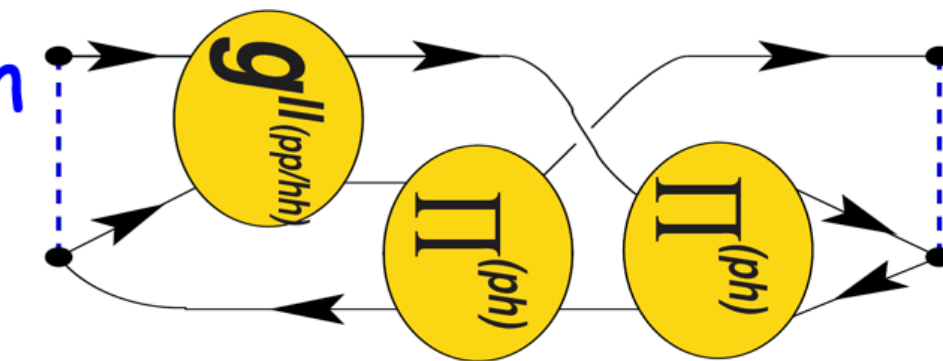


Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

- contains **both** **particle** and **hole** props.
- it is proven to be a **Feshbach opt. pot.**, in general it is **non-local** !

$$\Sigma_{\alpha\beta}^*(\omega) = \underbrace{\Sigma_{\alpha\beta}^{(\infty)} + \sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{\text{Particle-vibration couplings:}}$$

Particle-vibration couplings:

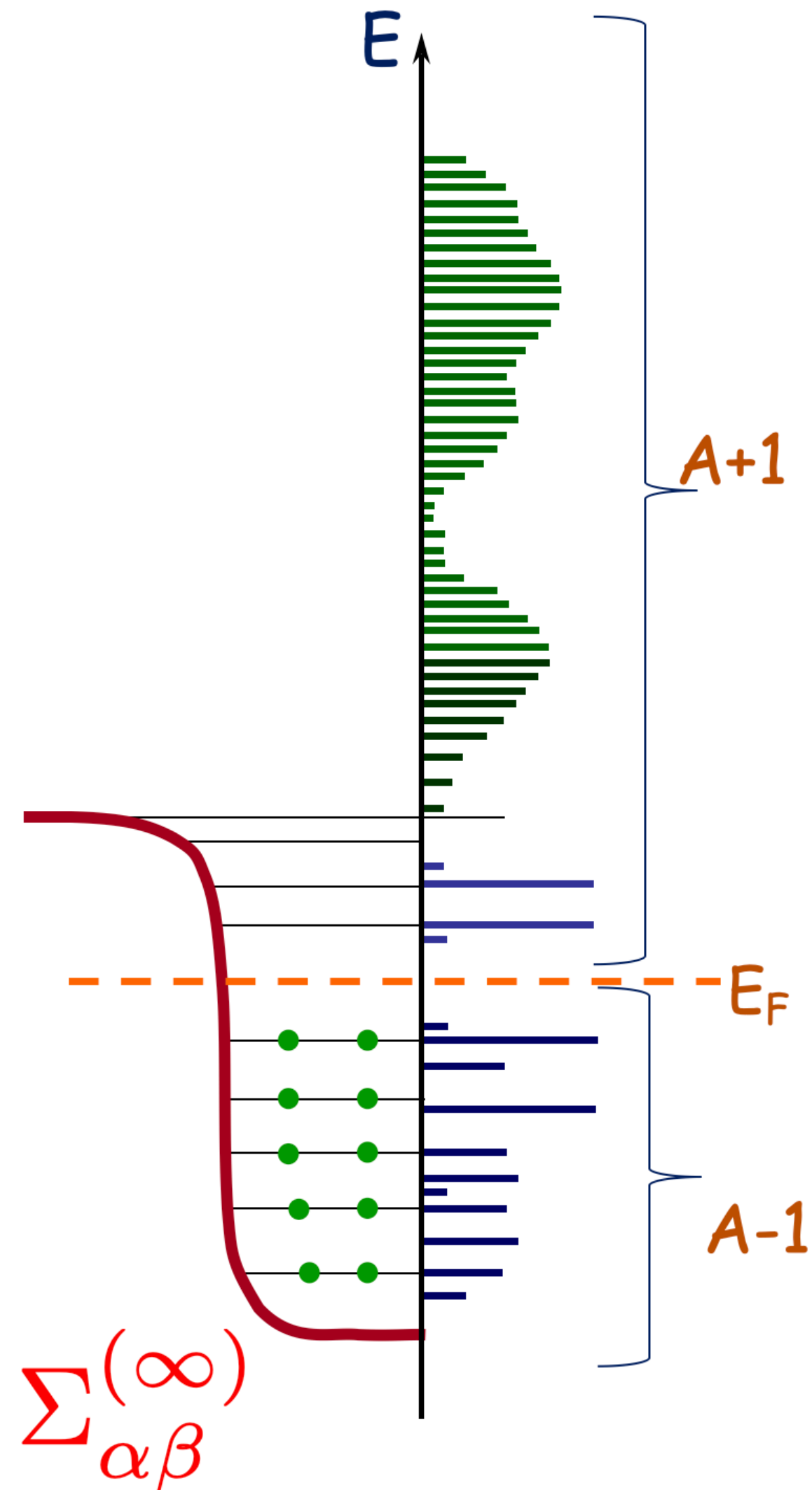


Solve scattering and overlap functions directly in momentum space:

$$\Sigma^{*l,j}(k, k'; E) = \sum_{n, n'} R_{nl}(k) \Sigma_{n, n'}^{*l,j} R_{nl}(k')$$

$$\frac{k^2}{2\mu} \psi_{l,j}(k) + \int dk' k'^2 \Sigma^{*l,j}(k, k'; E_{c.m.}) \psi_{l,j}(k') = E_{c.m.} \psi_{l,j}(k)$$

Microscopic optical potential

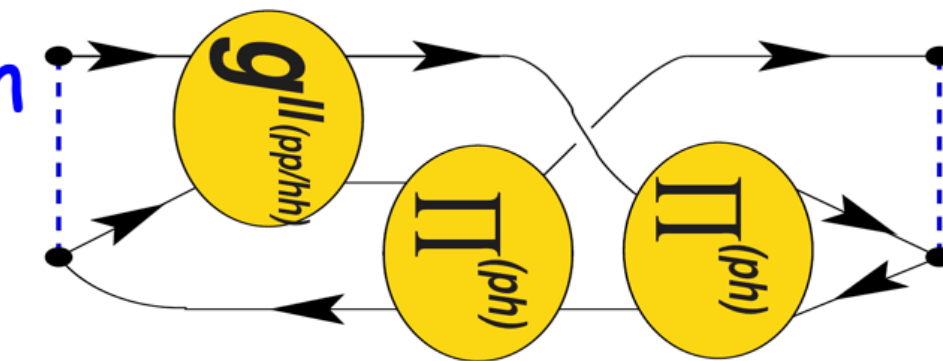


Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

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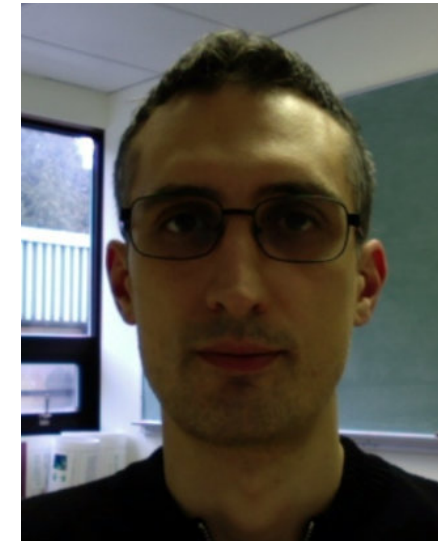
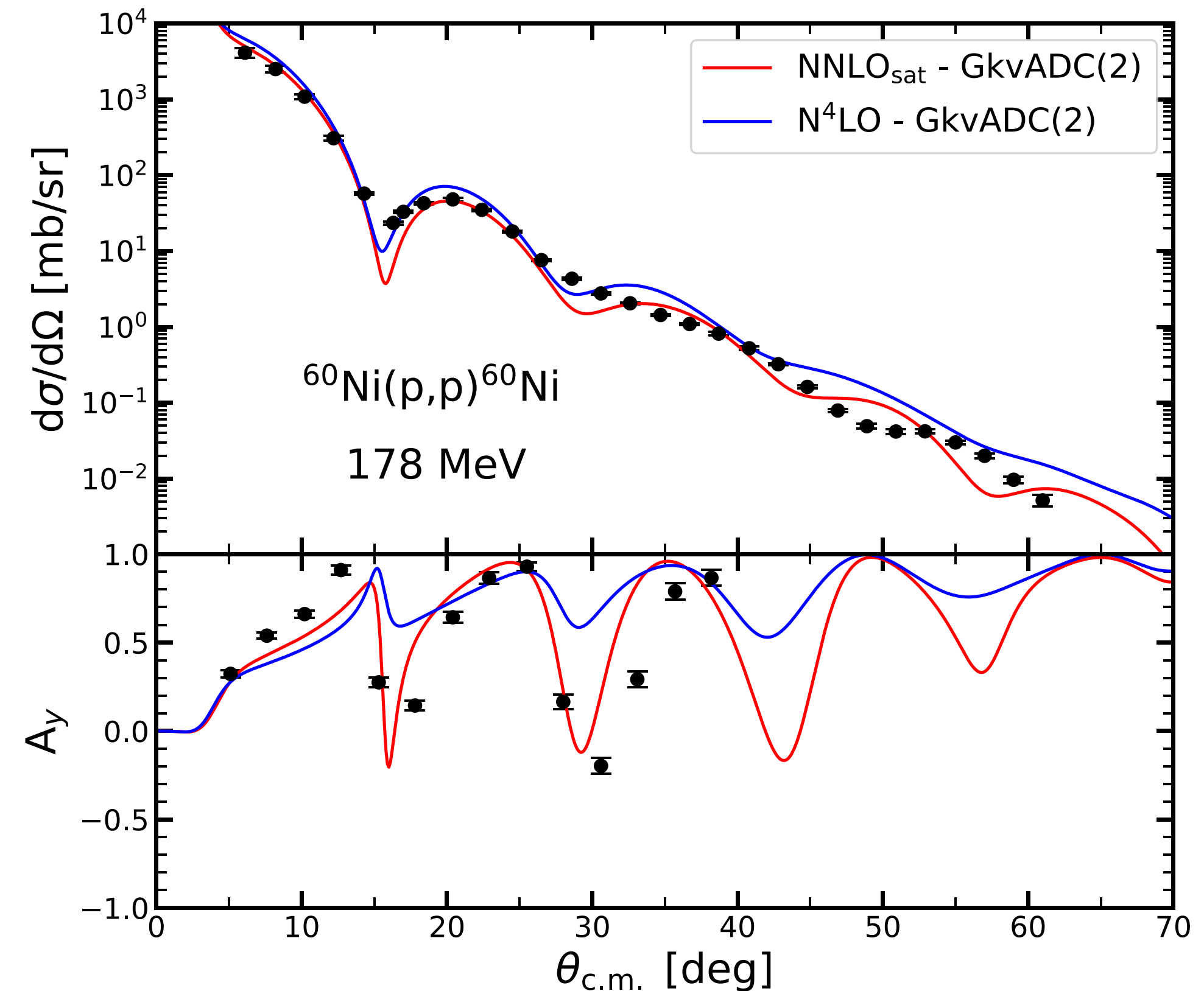
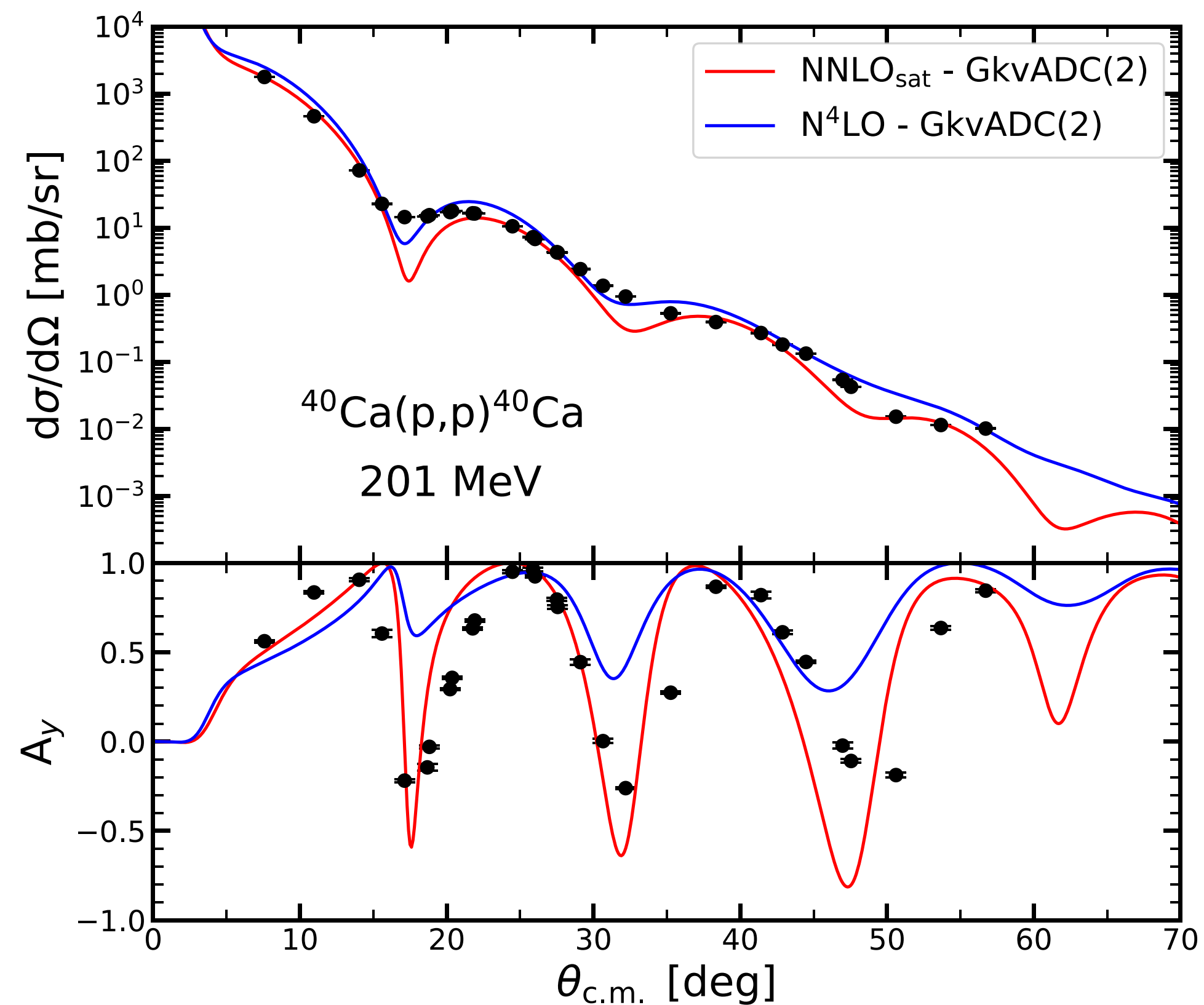
$$\Sigma_{\alpha\beta}^*(\omega) = \underbrace{\Sigma_{\alpha\beta}^{(\infty)}}_{\text{mean-field}} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{\text{Particle-vibration couplings}}$$

Particle-vibration couplings:



$$\Sigma_{\alpha\beta}^{(\infty)} = \text{diagram of a particle line with a self-energy loop}$$

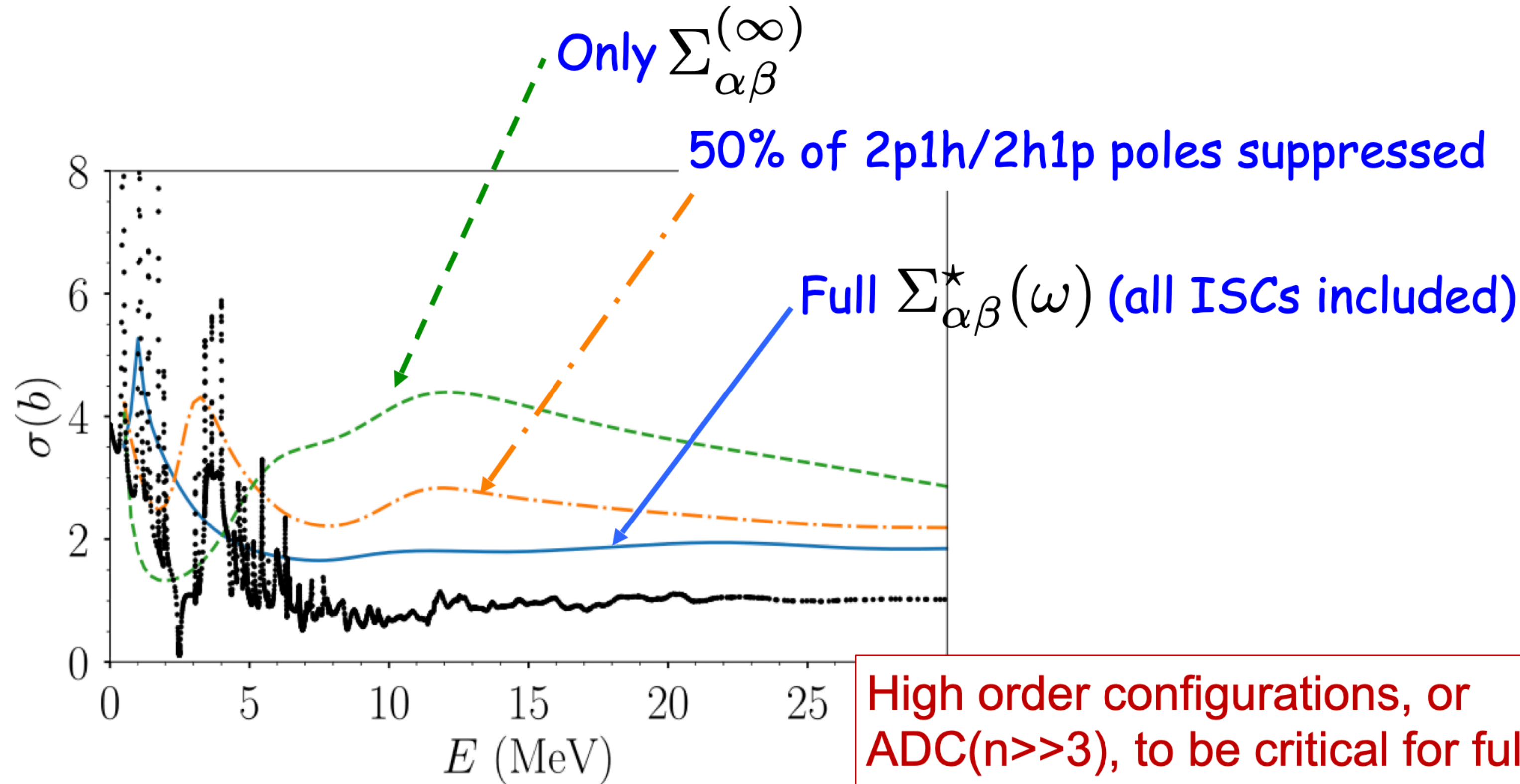
Elastic nucleon nucleus scattering



M. Vorabbi
U. of surrey

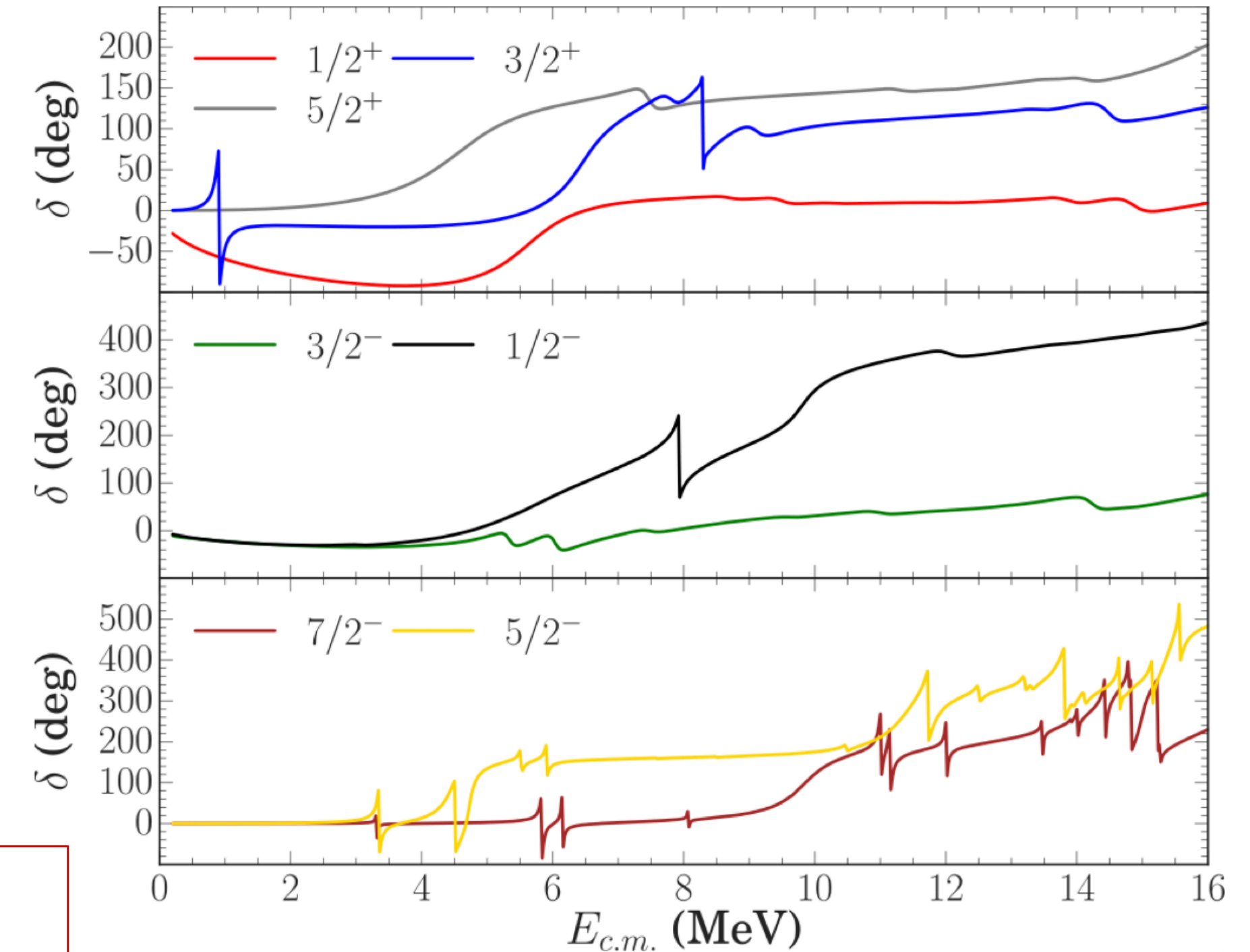
Role of intermediate state configurations (ISCs)

n - ^{16}O , total elastic cross section

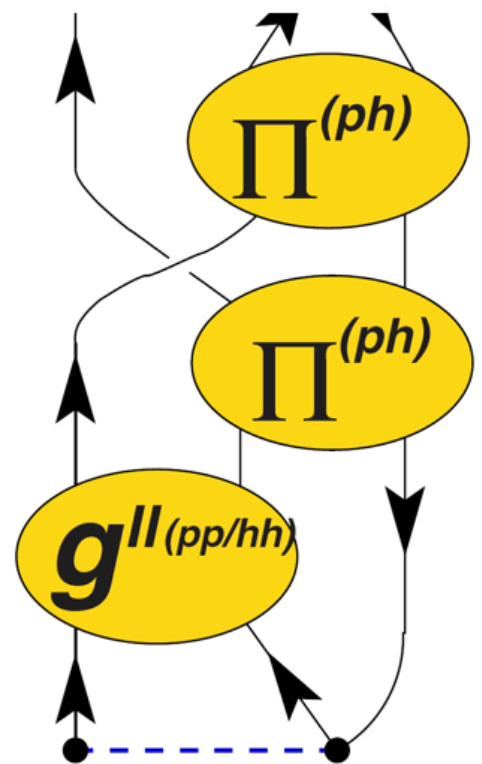


High order configurations, or ADC($n \gg 3$), to be critical for fully ab initio optical potentials

[A. Idini, CB, Navrátil, Phys. Rev. Lett. **123**, 092501 (2019)]



$$\Sigma_{\alpha\beta}^*(\omega) = \Sigma_{\alpha\beta}^{(\infty)} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left(\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right)_{i,j} \mathbf{M}_{j,\beta}}_{2p1h} + \underbrace{\sum_{r,s} \mathbf{N}_{\alpha,r} \left(\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right)_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{2h1p}$$



Green's function theory beyond ADC(3)? **DiagMC !!**

The Green's function is found as the exact solution of the Dyson equation:

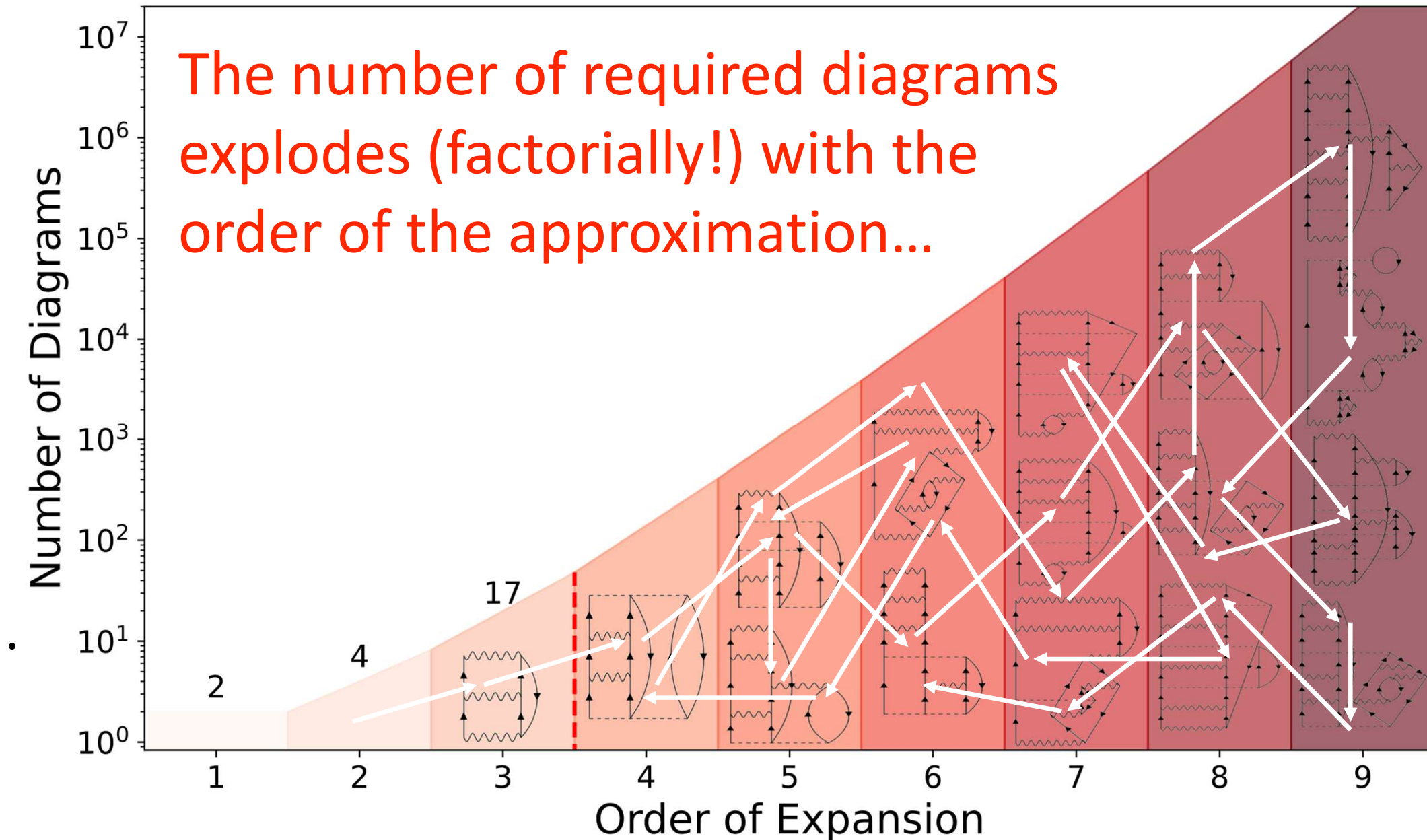
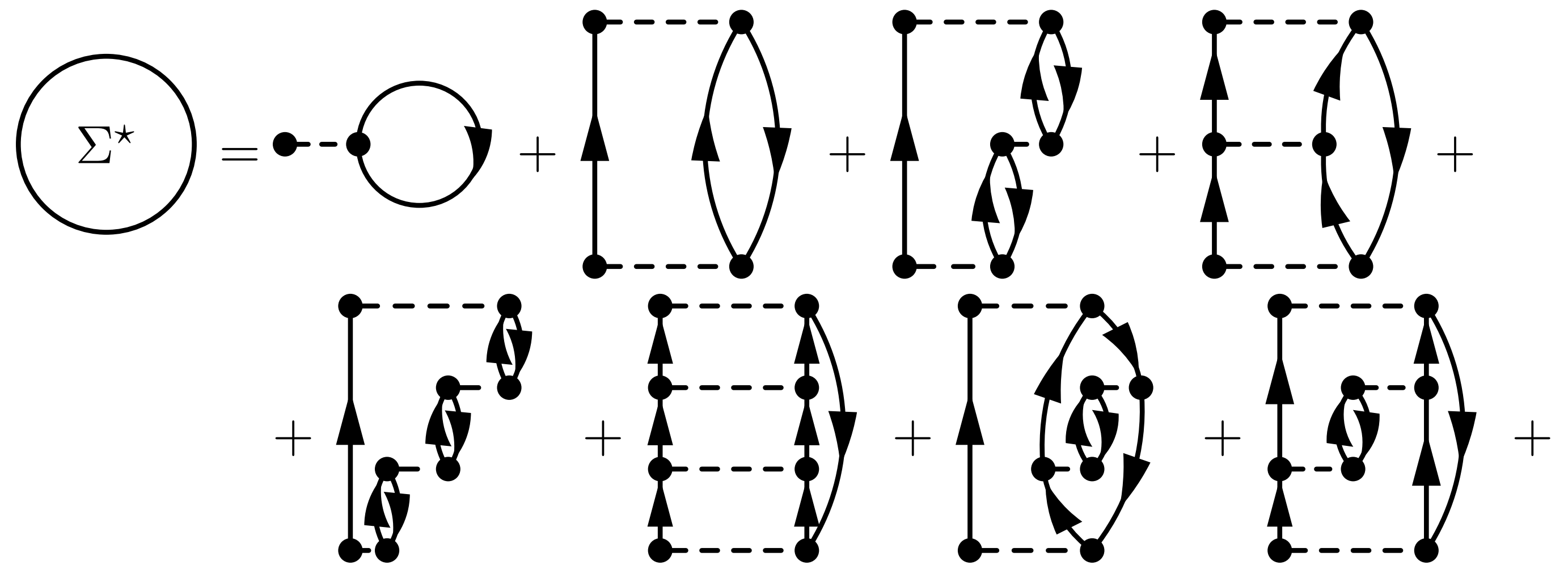
$$G_{\alpha\beta}(\omega) = G_{\alpha\beta}^{(0)}(\omega) + \sum_{\gamma\delta} G_{\alpha\gamma}^{(0)}(\omega) \Sigma_{\gamma\delta}^*(\omega) G_{\delta\beta}(\omega)$$

S. Brolli

(PhD candidate,
Milan)



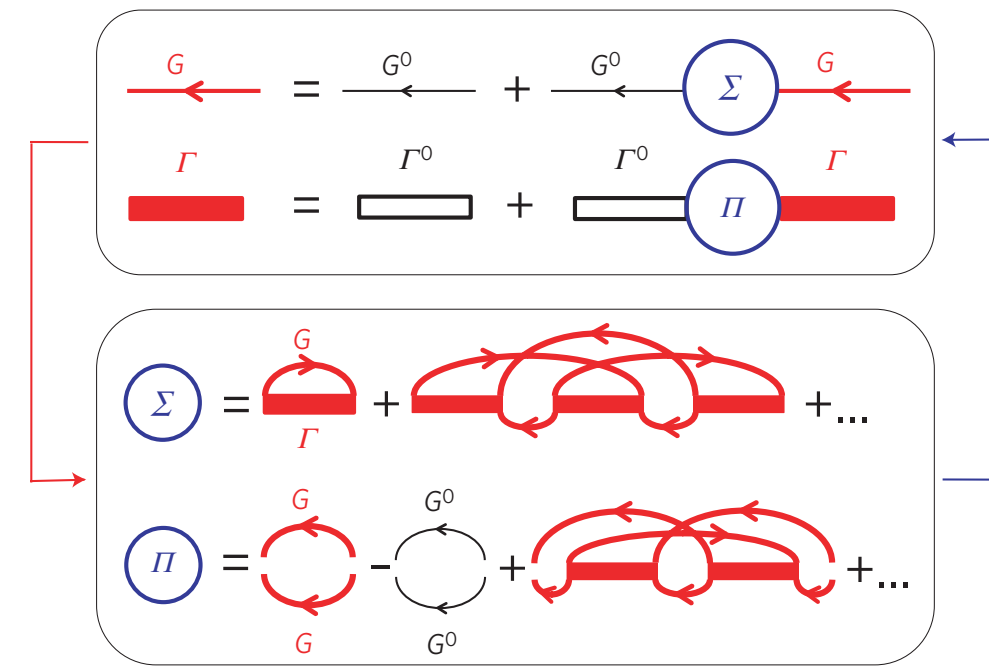
It requires knowing the self-energy which is the sum of an *infinite series* of Feynman diagrams:



Diagrammatic Monte Carlo (DiagMC) *samples diagrams in their topological space* using a Markov chain.

[Brolli, CB, Vigezzi, Phys. Rev. Lett. **134**, 182502 (2025)]

Diagrammatic MC in solid state physics...



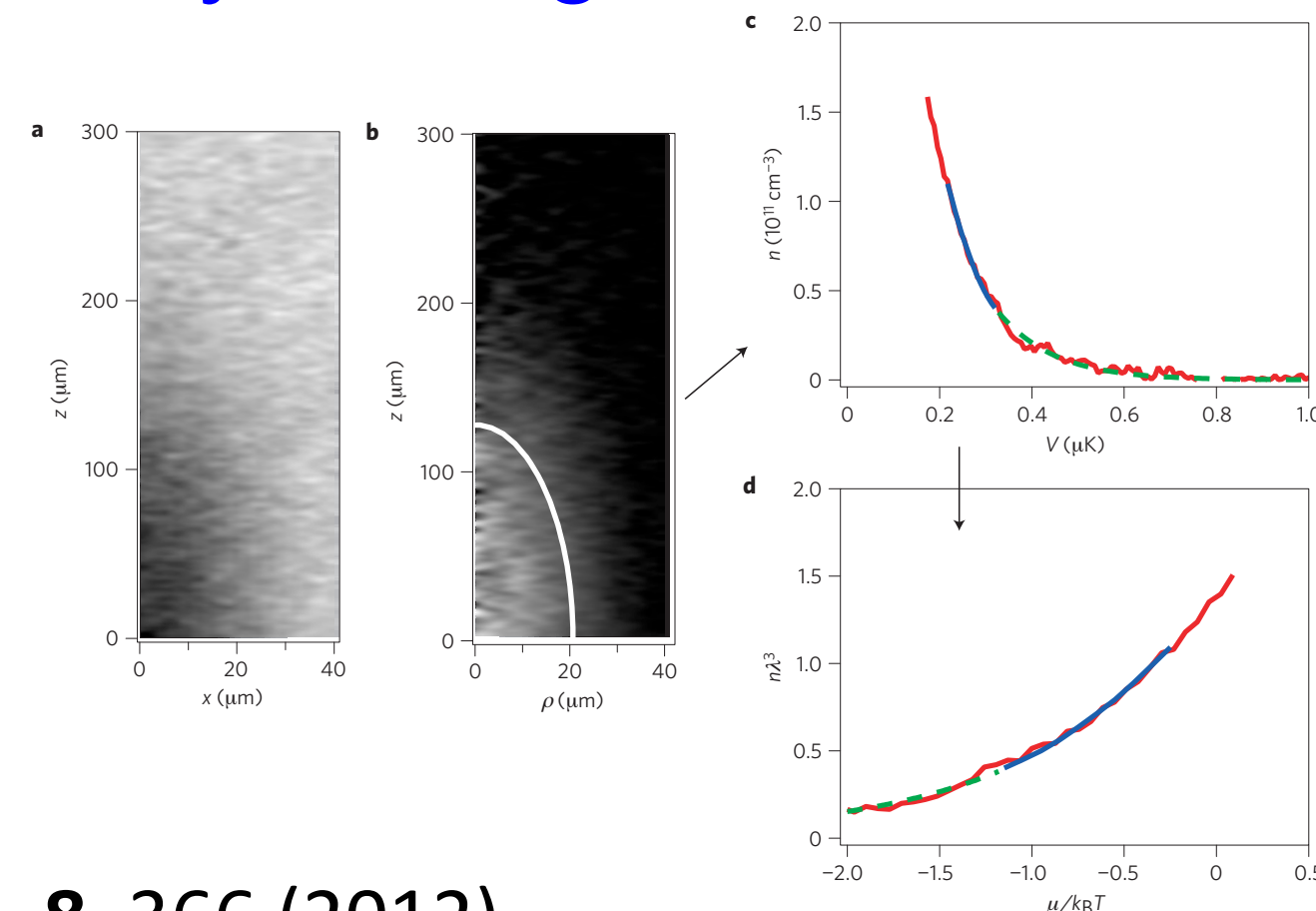
DiagMC:
 Phys. Rev. Lett. 81, 2514 (1998)
 Phys. Rev. B 99, 035140 (2019)
 ...

Mostly infinite matter at $T \neq 0$...

...what about finite nuclei??

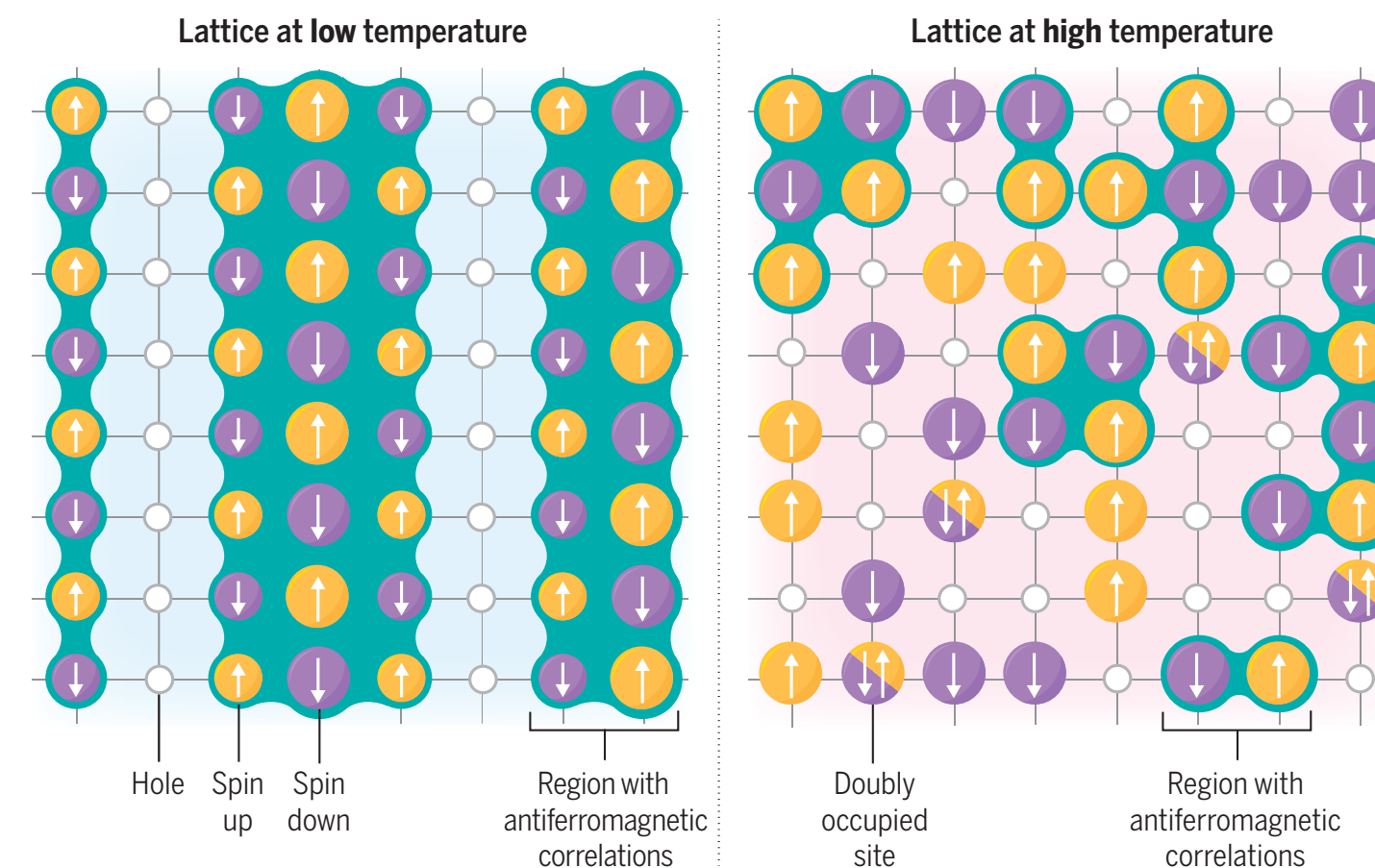
Figure 1 | Bold diagrammatic Monte Carlo The skeleton diagrammatic series for the self-energy Σ and the pair self-energy Π is evaluated

EOS unitary fermi gas...



Nature Phys. 8, 366 (2012)

Spins in the Hubbard model...



Science 385, eade9194 (2024)

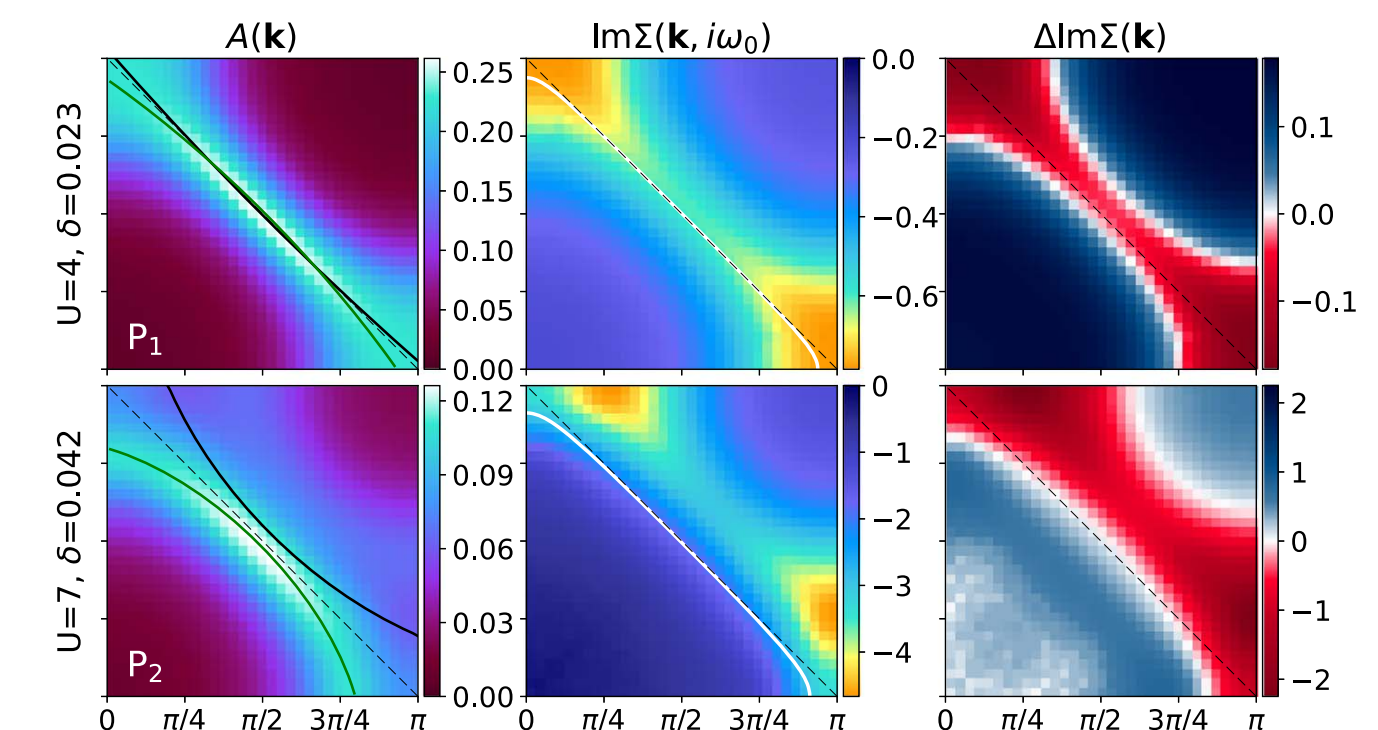


Fig. 2. Fingerprints of the different regimes. The momentum-resolved spectral function, $A(\mathbf{k})$ (left); the imaginary part of the self-energy, $\text{Im}\Sigma(\mathbf{k}, i\omega_0)$ (middle); and the difference between the imaginary part of the self-energy at the two lowest Matsubara frequencies, $\Delta \text{Im}\Sigma(\mathbf{k}) = \text{Im}\Sigma(\mathbf{k}, i\omega_0) - \text{Im}\Sigma(\mathbf{k}, i\omega_1)$ (right) are shown for

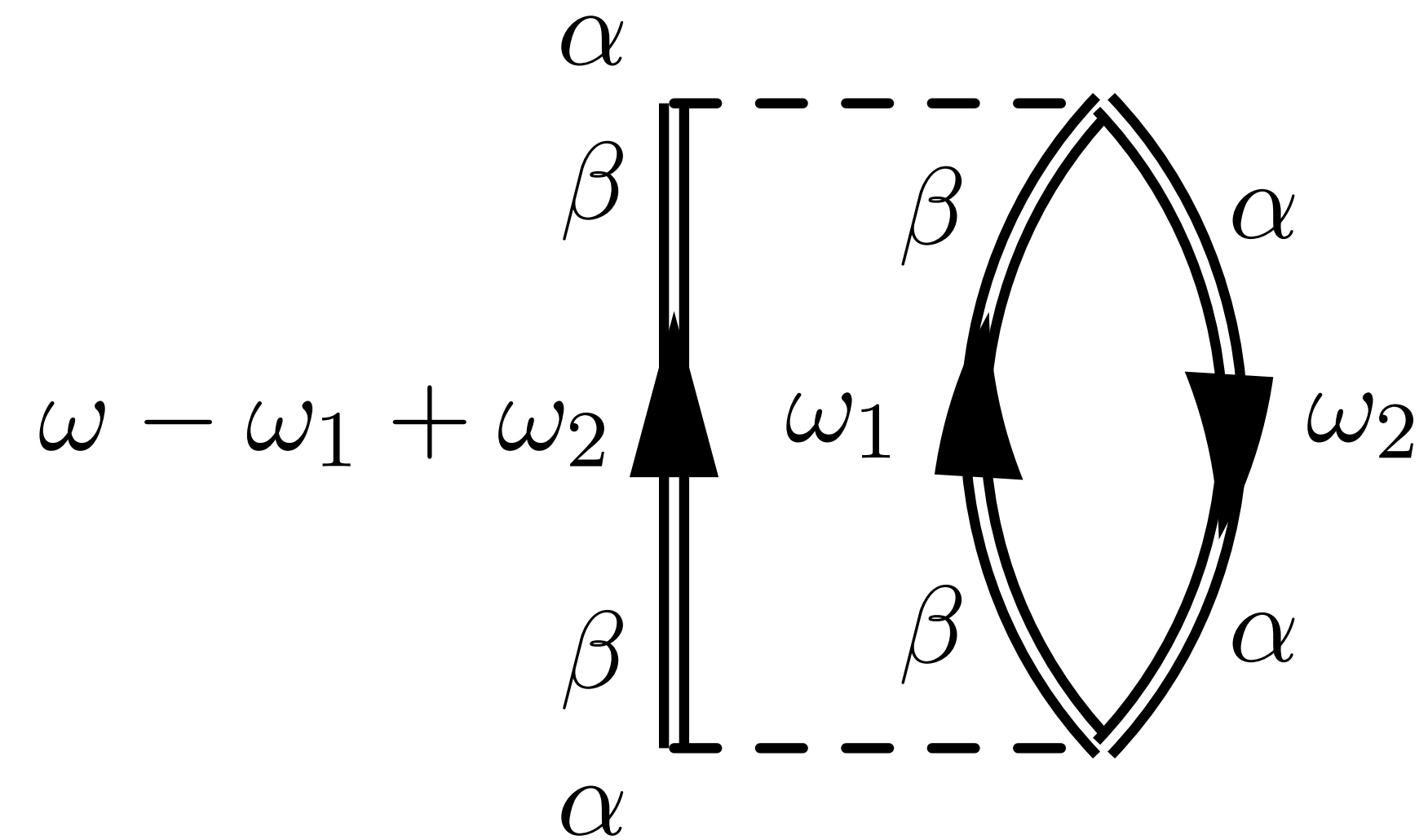
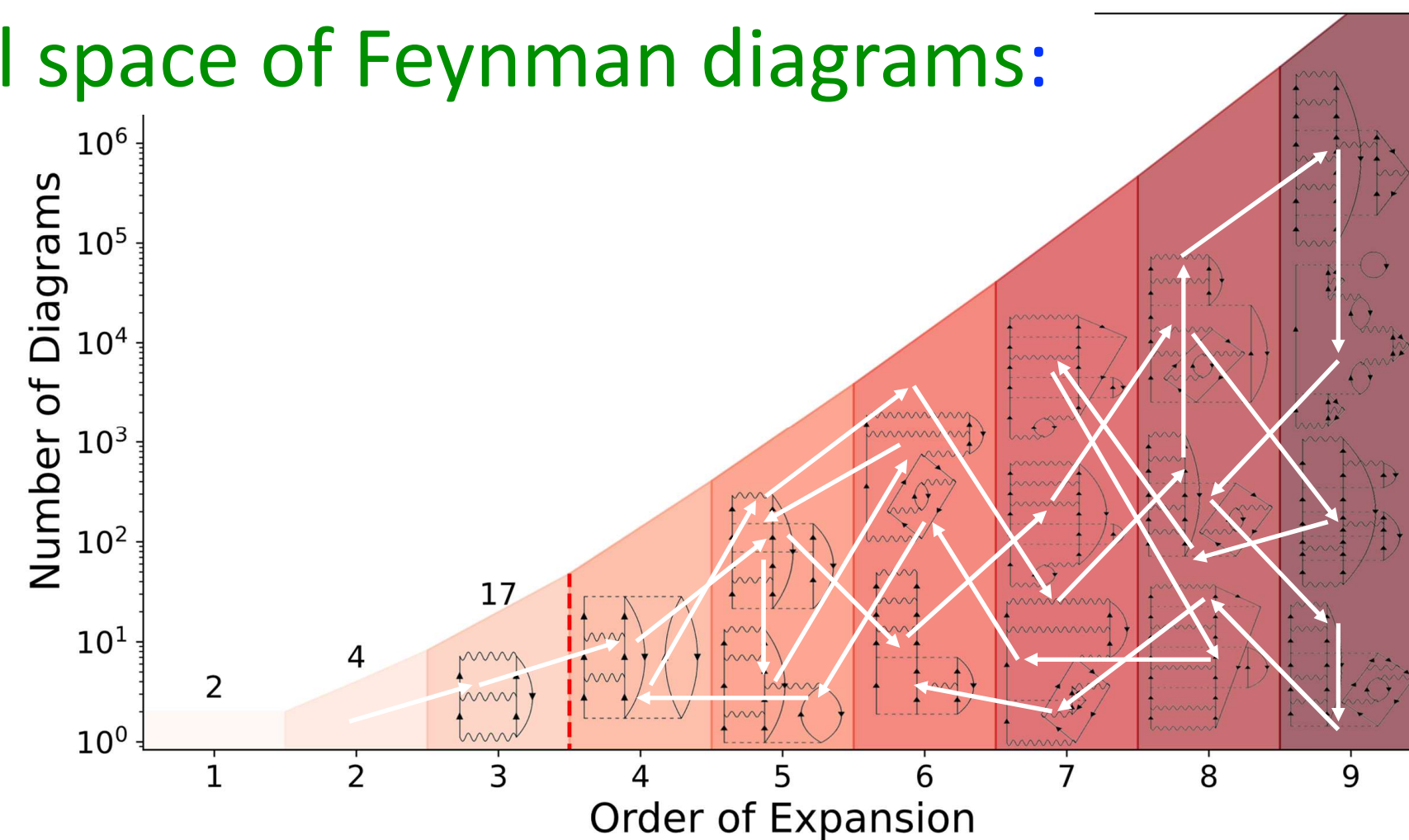
Diagrammatic Monte Carlo: overview

$$\Sigma_{\alpha\beta}^*(\omega) = \sum_{\mathcal{T}} \sum_{\gamma_1 \dots \gamma_n} \int d\omega_1 \dots d\omega_m \mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{T}; \gamma_1 \dots \gamma_n; \omega_1 \dots \omega_m) 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^*}}$$

Infinte Feynman
series of diagrams

Monte Carlo integration - "101"

Monte Carlo integration in the
topological space of Feynman diagrams:



Diagrammatic Monte Carlo: overview

$$\Sigma_{\alpha\beta}^*(\omega) = \sum_{\mathcal{T}} \sum_{\gamma_1 \dots \gamma_n} \int d\omega_1 \dots d\omega_m \mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{T}; \gamma_1 \dots \gamma_n; \omega_1 \dots \omega_m) 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^*}}$$

We define $\mathcal{C} := (\mathcal{T}; \gamma_1 \dots \gamma_n; \omega_1 \dots \omega_m)$

$$\Sigma_{\alpha\beta}^*(\omega) = \int d\mathcal{C} |\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})| e^{i \arg[\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})]} 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^*}}$$

$$\Sigma_{\alpha\beta}^*(\omega) = \mathcal{Z}_{\alpha\beta}^{\omega} \int d\mathcal{C} \frac{|\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})| W_o(N)}{\mathcal{Z}_{\alpha\beta}^{\omega}} \frac{e^{i \arg[\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})]}}{W_o(N)} 1_{\mathcal{T} \in \mathcal{S}_{\Sigma^*}}$$

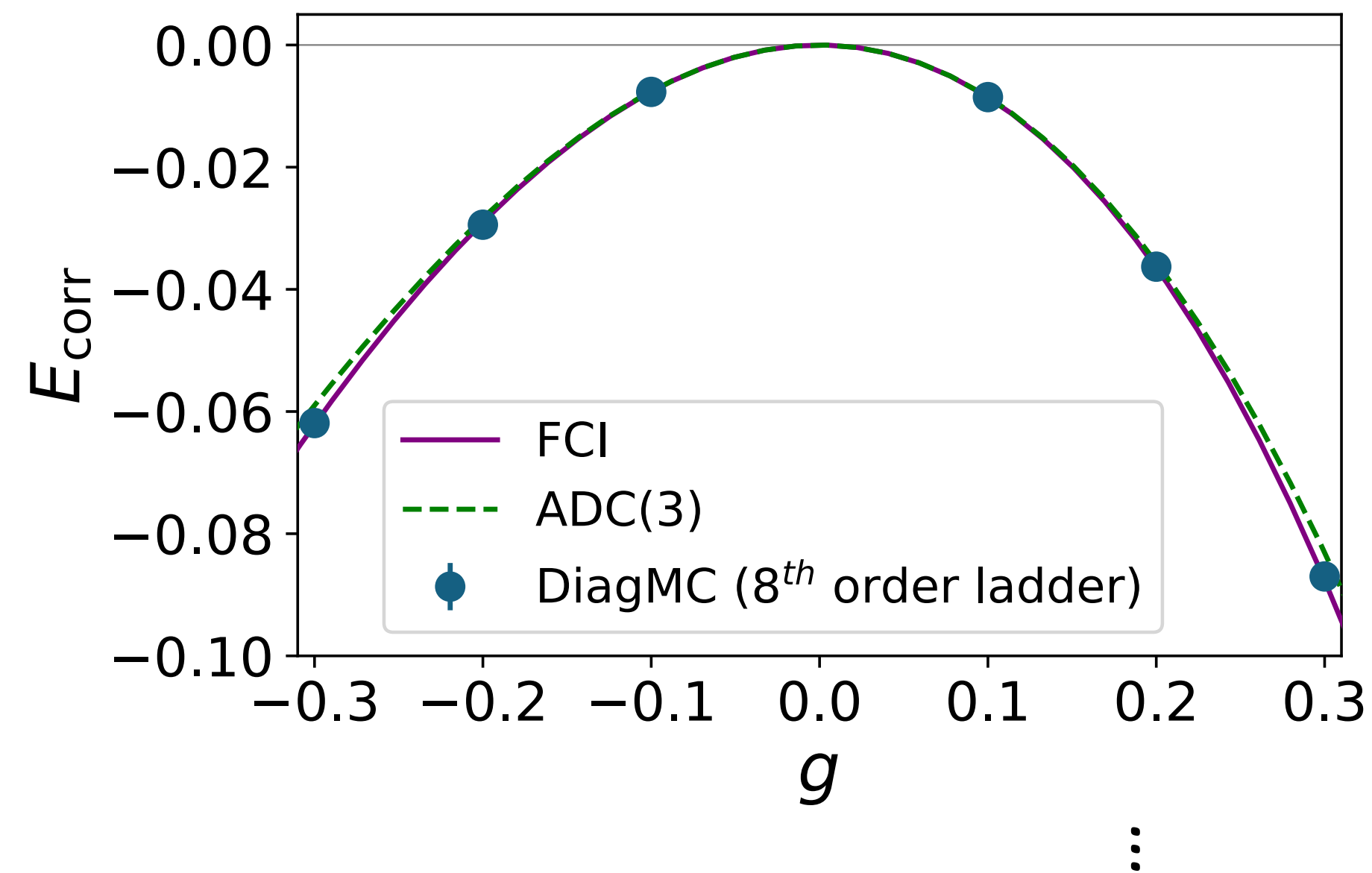
- $W_o(N)$ is an order dependent reweighting factor
- $\mathcal{Z}_{\alpha\beta}^{\omega} = \int d\mathcal{C} |\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})| W_o(N)$ is a normalization factor
- $w_{\alpha\beta}^{\omega}(\mathcal{C}) := \frac{|\mathcal{D}_{\alpha\beta}^{\omega}(\mathcal{C})| W_o(N)}{\mathcal{Z}_{\alpha\beta}^{\omega}}$ is a probability distribution function

Results of the simulation for $D=2-10$ levels

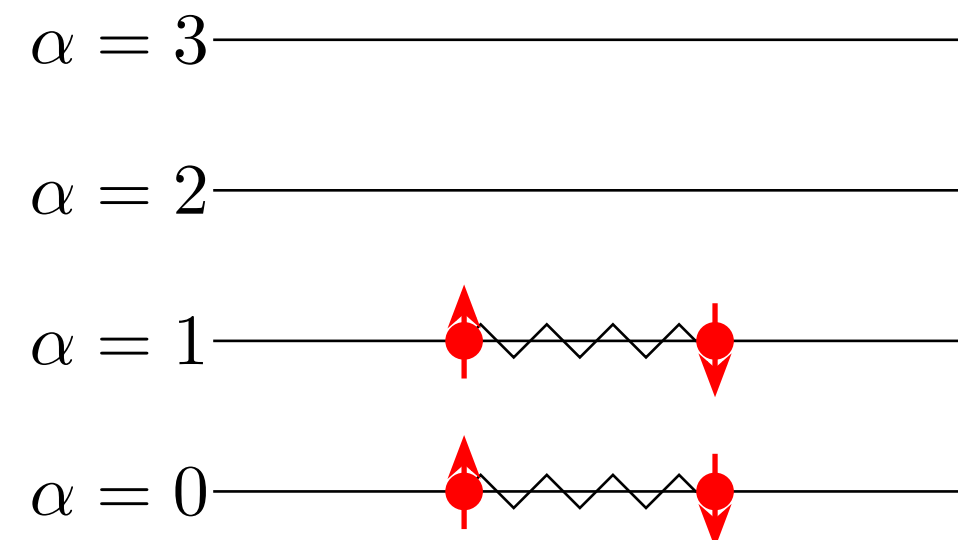
S. Brolli
(PhD Milan)



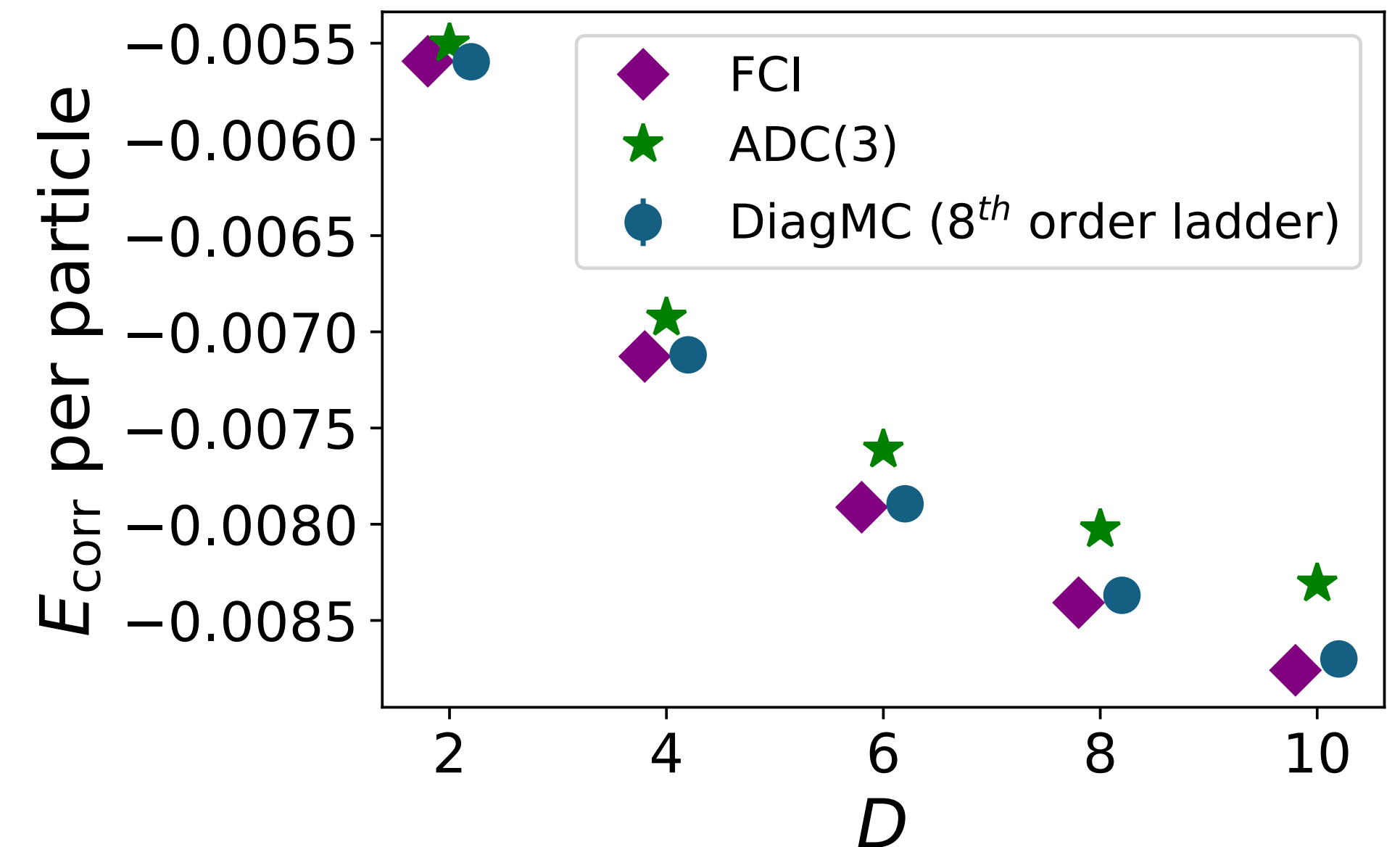
Correlation energy $\Delta E = E - E_{HF}$ as a function of interaction strength (g):



Spectroscopic function
for $D=10$ levels :



Accuracy for different model spaces model spaces ($D=2-10$ levels):



$$H = \xi \sum_{\alpha=0}^{D-1} \sum_{\sigma=+,-} \alpha c_{\alpha\sigma}^{\dagger} c_{\alpha\sigma} - \frac{g}{2} \sum_{\alpha,\beta=0}^{D-1} c_{\alpha+}^{\dagger} c_{\alpha-}^{\dagger} c_{\beta-} c_{\beta+}$$

Applying DiagMC to nuclei...

S. Brolli
(PhD Milan)

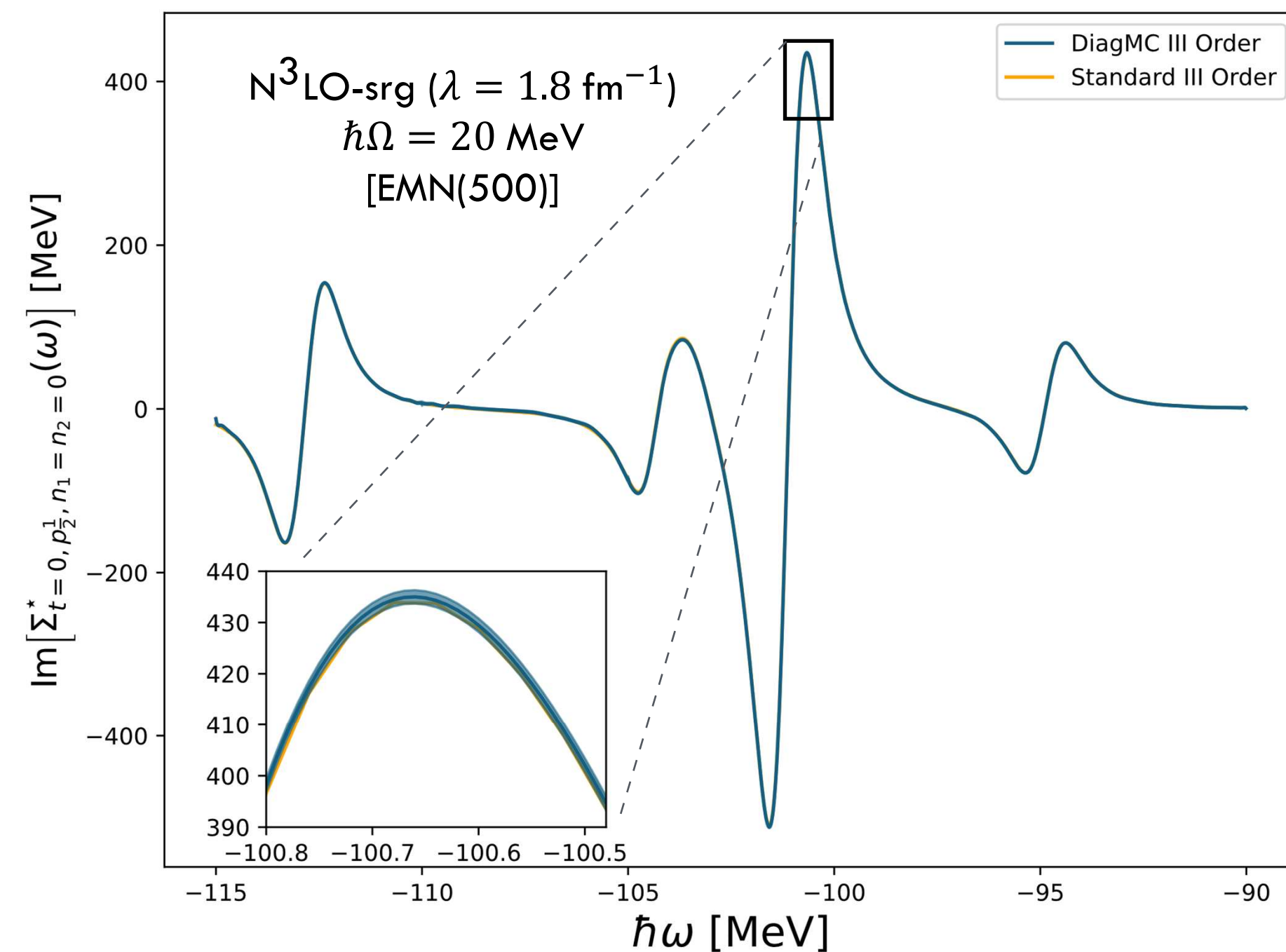


DiagMC is being extended to treat realistic microscopic nuclear Hamiltonians

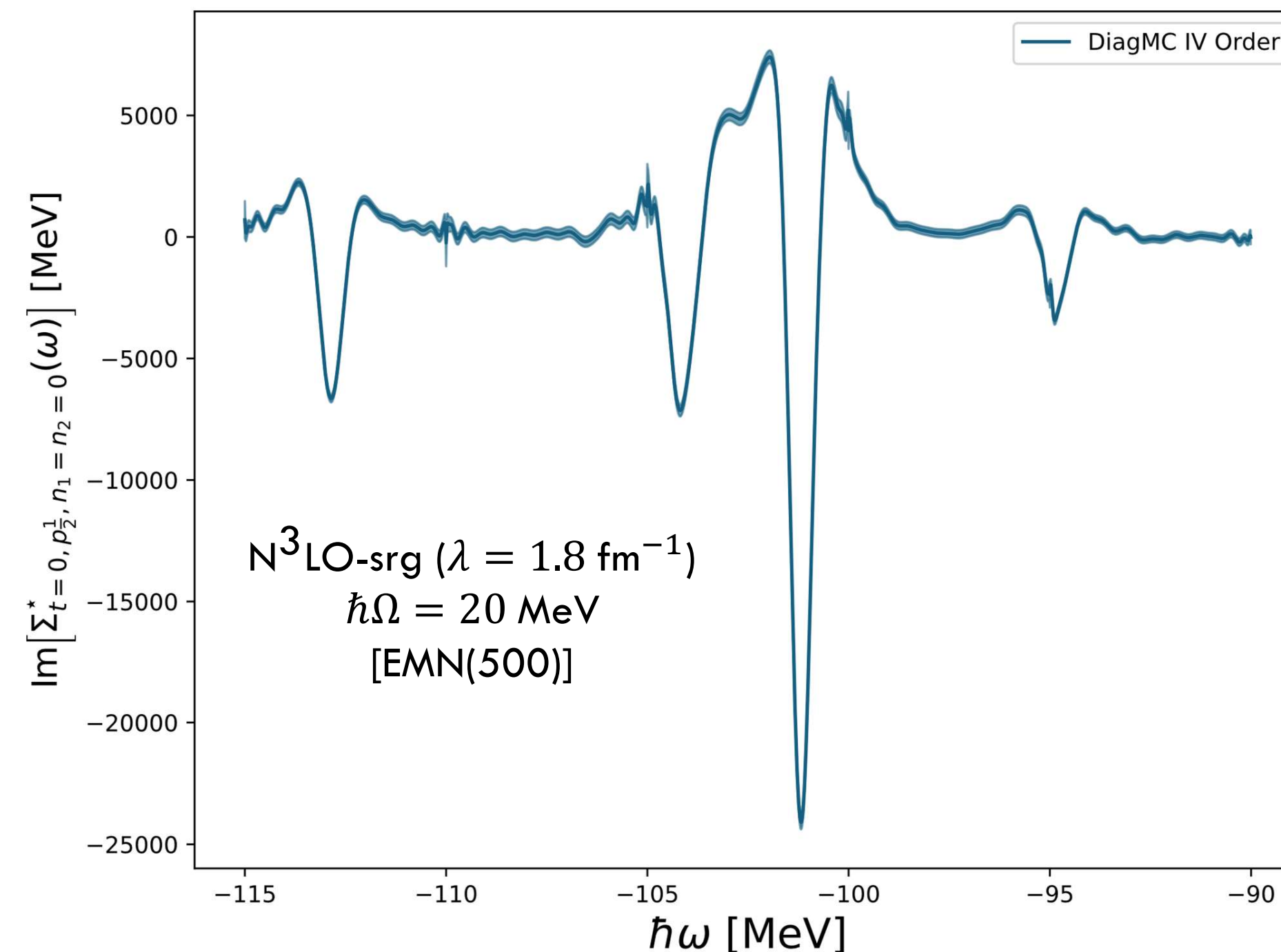
^{16}O in harmonic oscillator space ($N_{\text{max}} = 2$, for now)

Example of neutron $p_{1/2}$ self-energy up to 3rd order

THIRD ORDER RESULTS



FOURTH ORDER RESULTS



Applying DiagMC to nuclei...

S. Brolli
(PhD Milan)



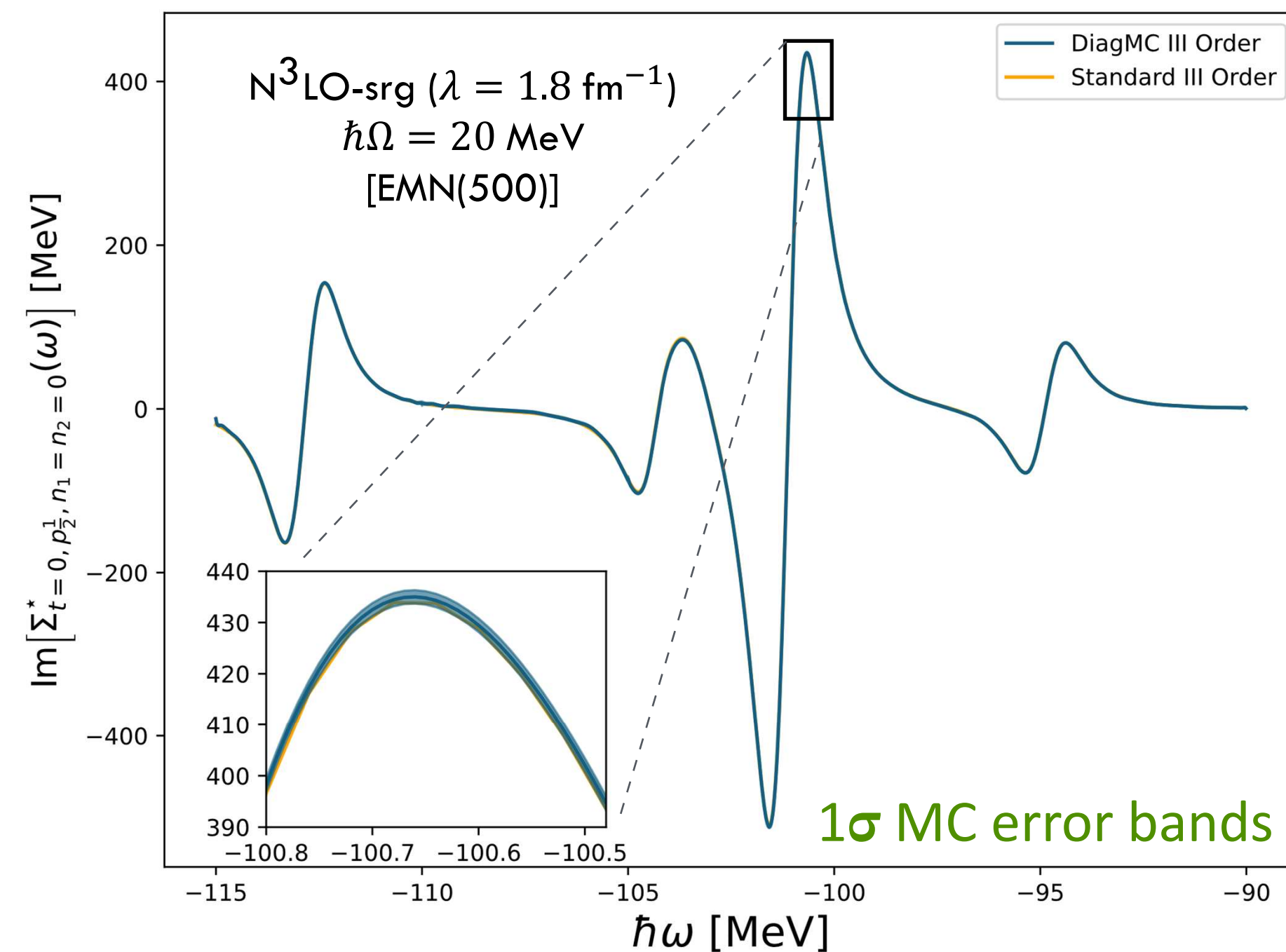
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^{16}O in harmonic oscillator space ($N_{\text{max}} = 2$, for now)

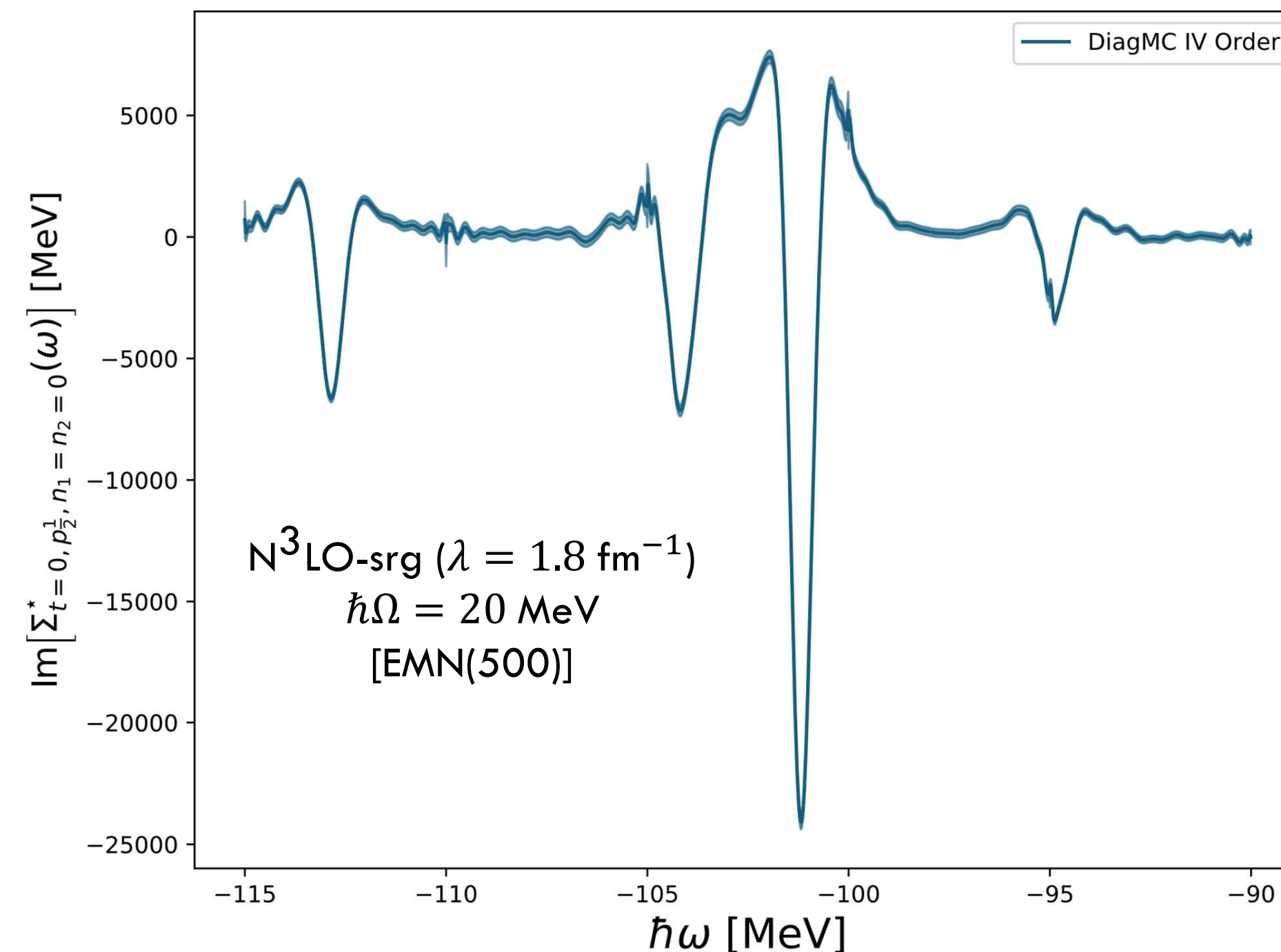
Example of neutron $p_{1/2}$ self-energy up to 3rd order

S. Brolli & CB — preliminary !!!

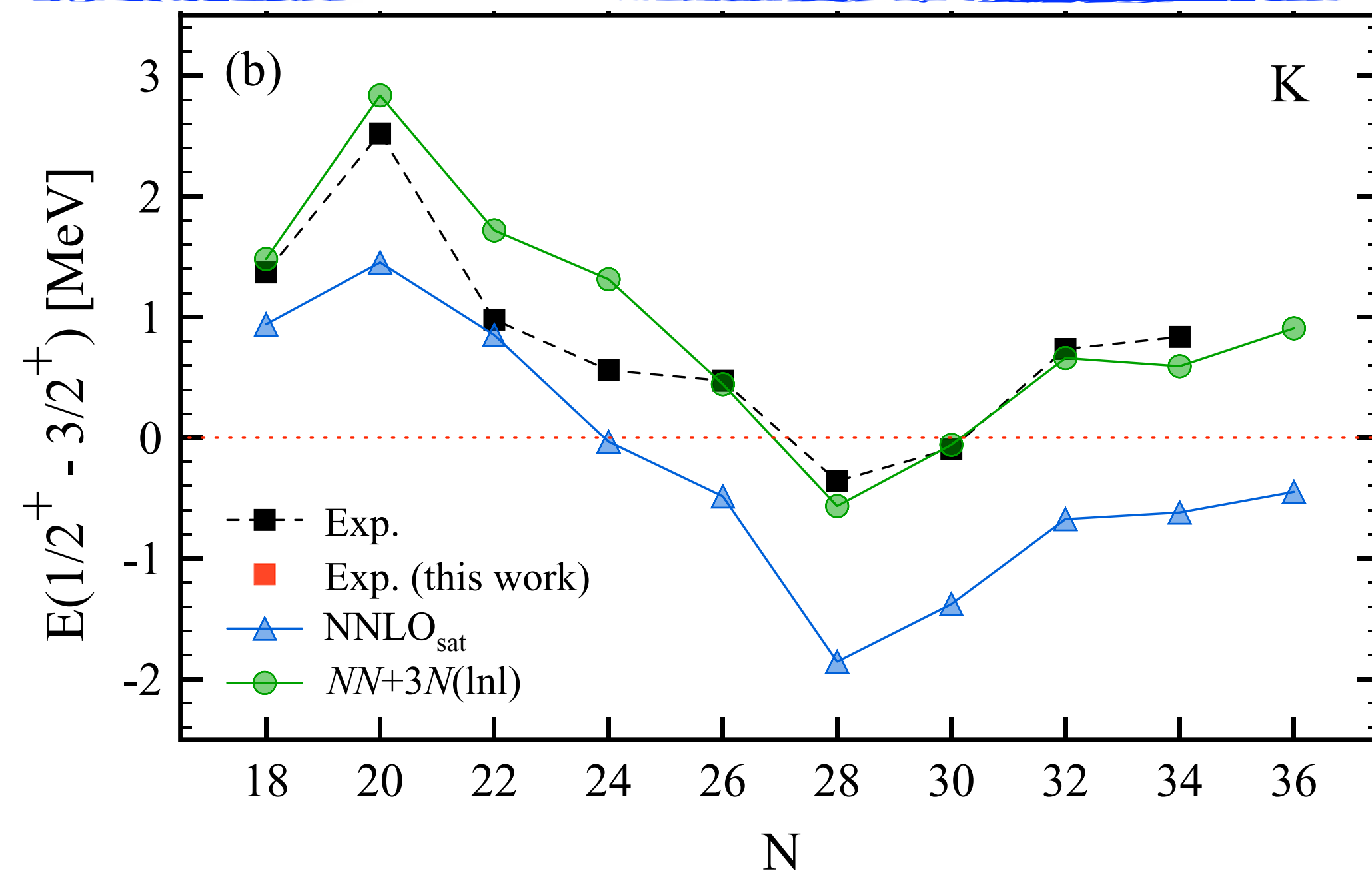
THIRD ORDER RESULTS



FOURTH ORDER RESULTS

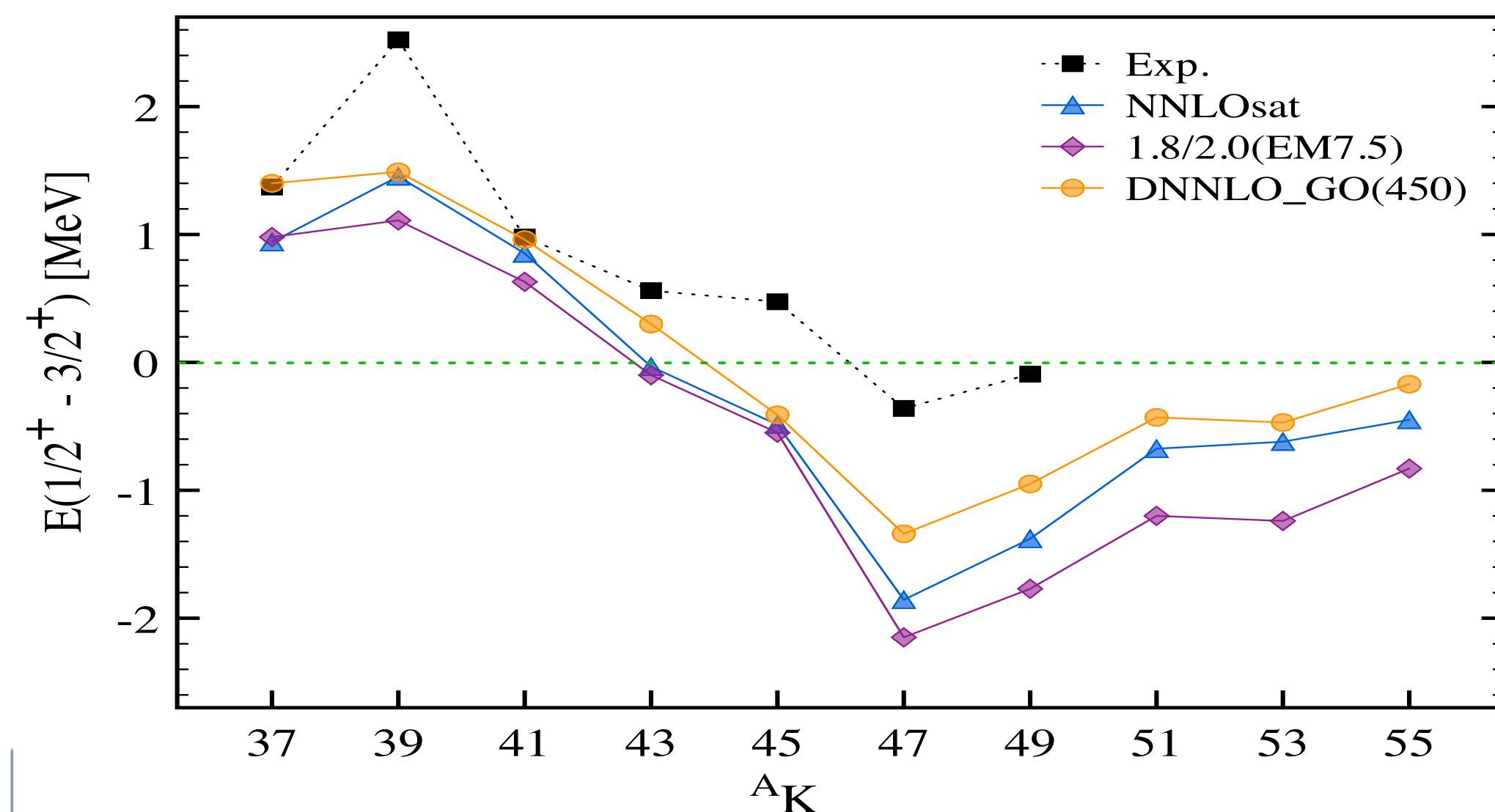


$d_{3/2} - s_{1/2}$ inversion of protons at N=28



V. Somà, P. Navrátil, F. Raimondi, CB,
T. Duguet, PRC**104**, 024315 (2021)

Both states populated by proton removal from ^ACa

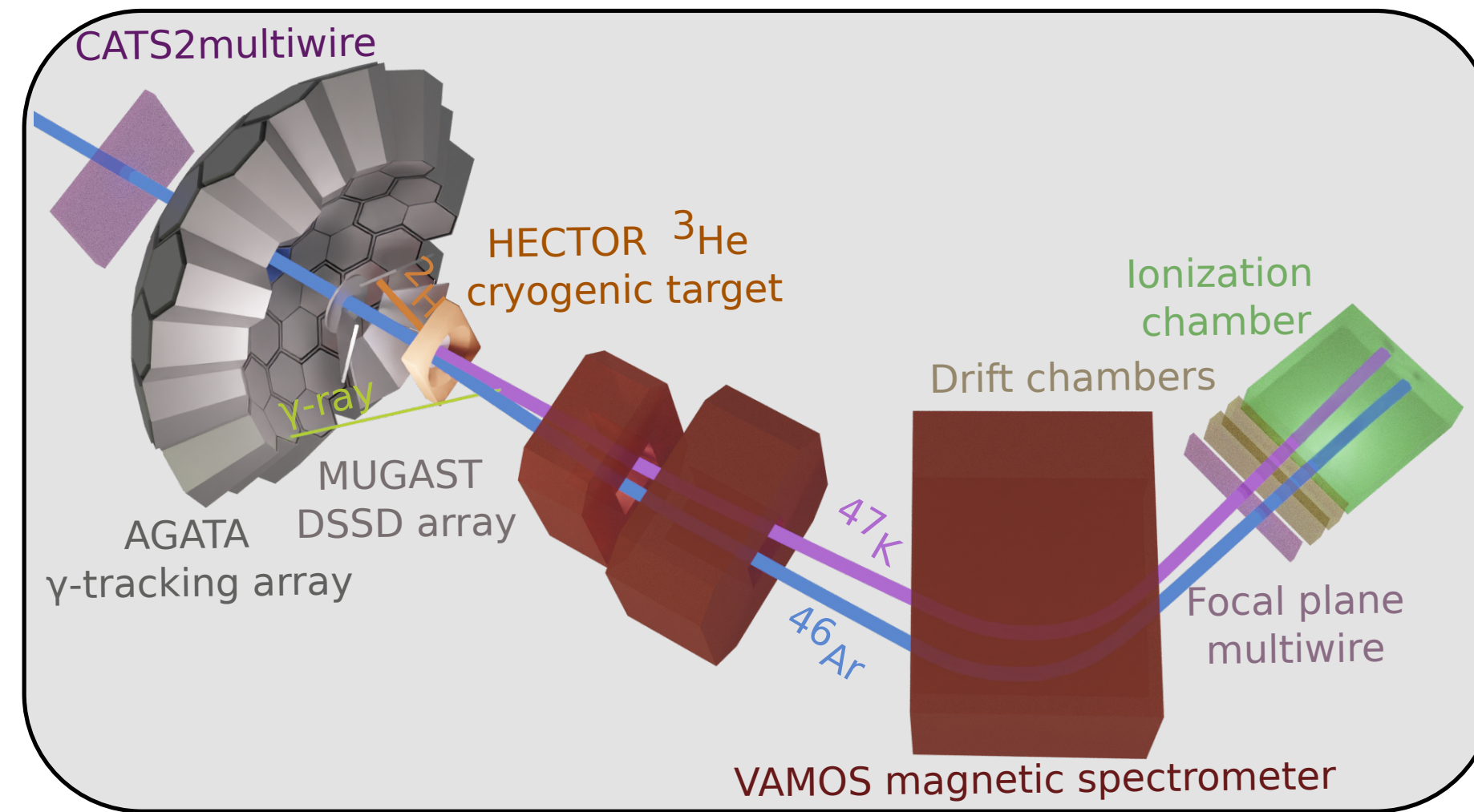
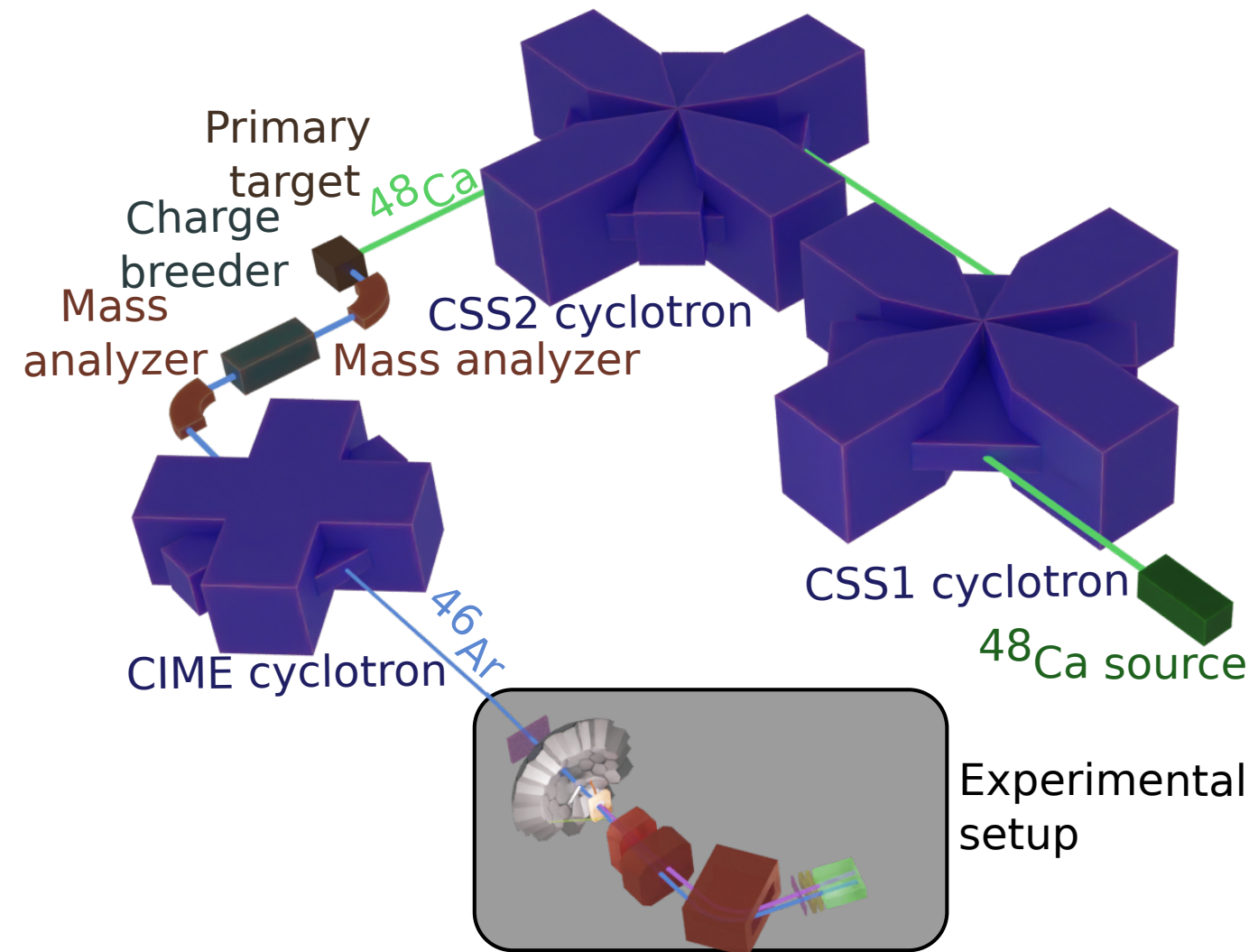


Papuga et al., PRL**110**, 172503 (2013); PRC**90**, 034321 (2014)

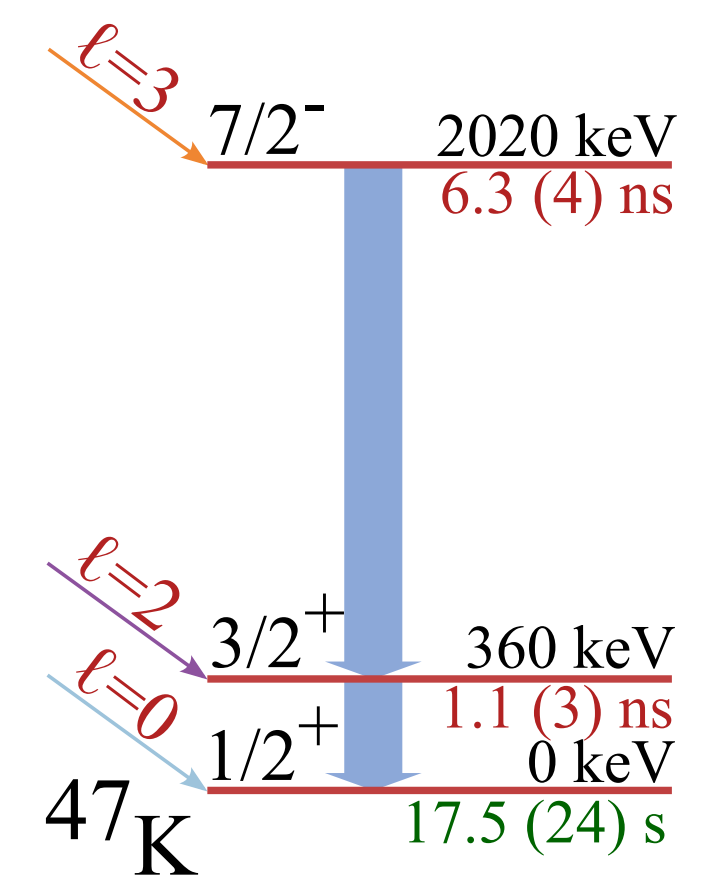
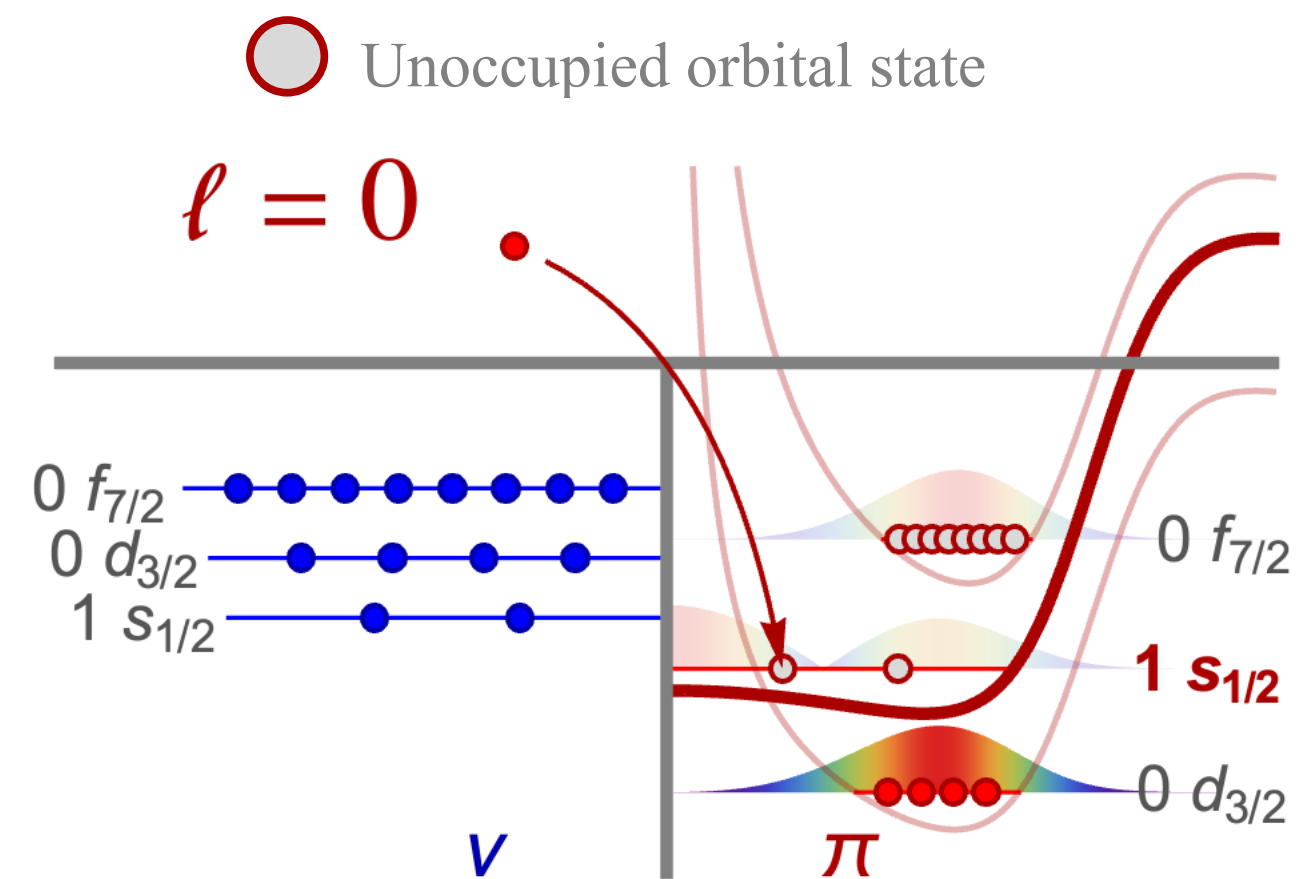
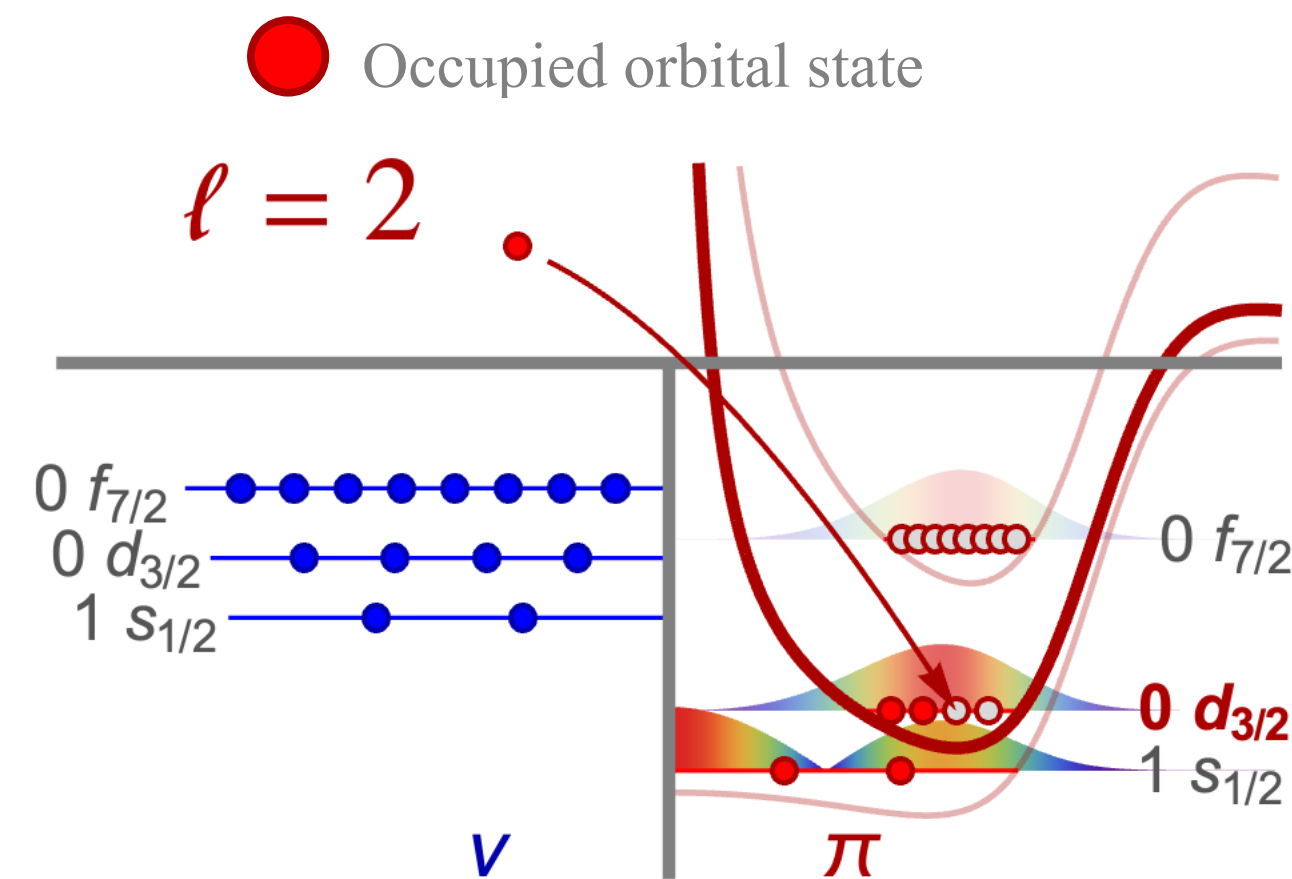
RIKEN, SEASTAR coll., Phys. Lett. **B802** 135215 (2020)

Linh, Gillibert, et al., PRC**104**, 044331 (2021)

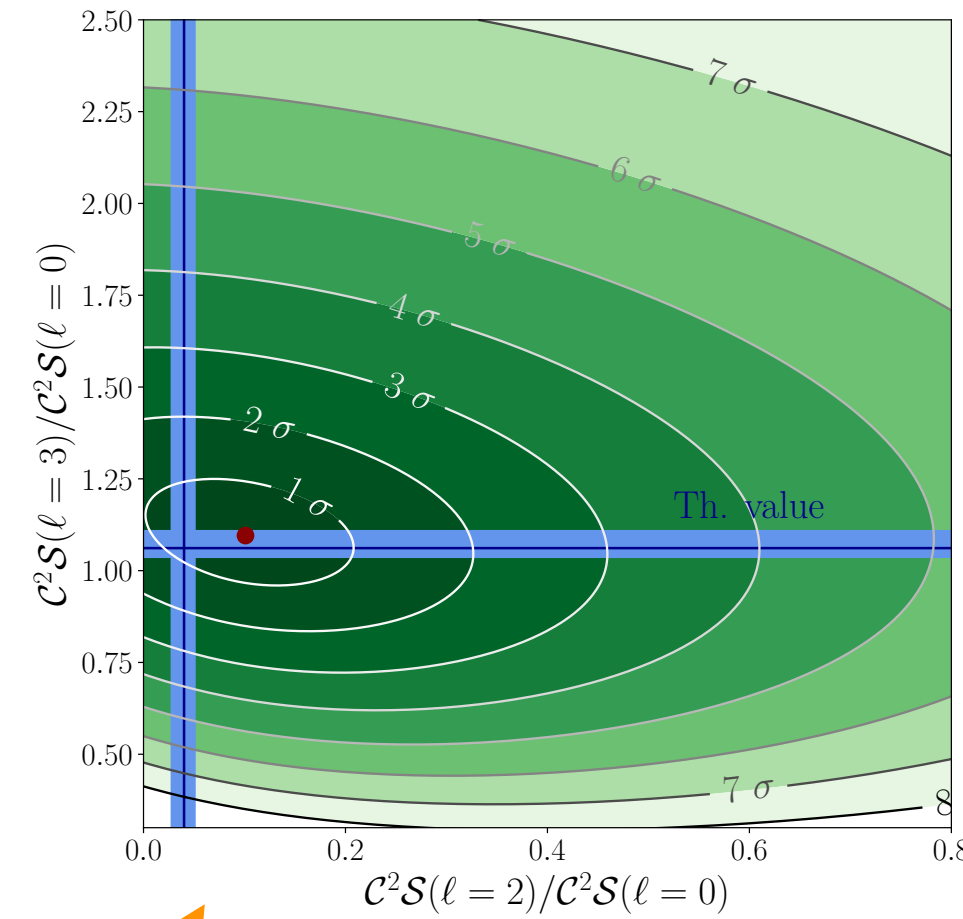
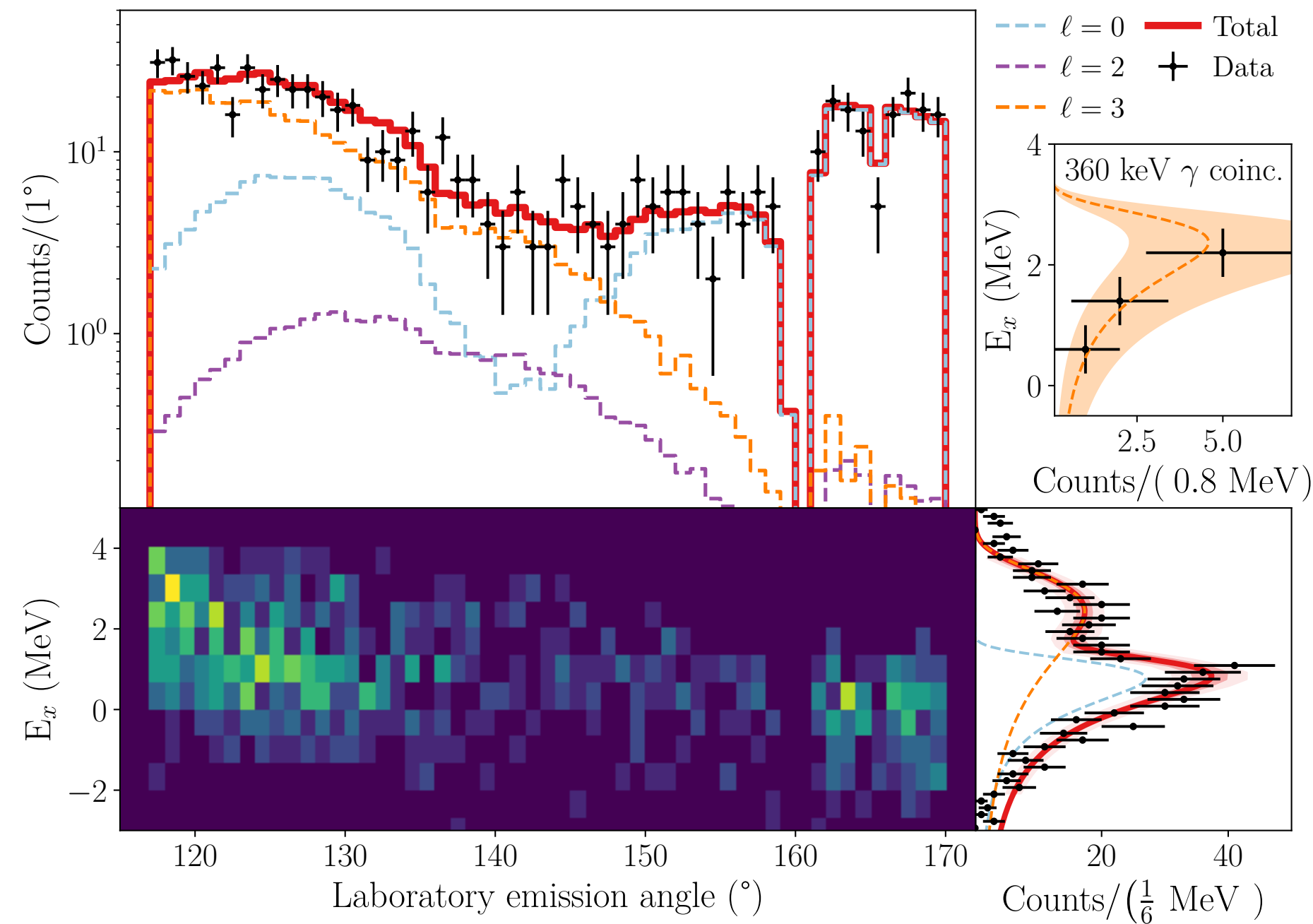
$^{46}\text{Ar}(^3\text{He},d)^{47}\text{K}$ at GANIL



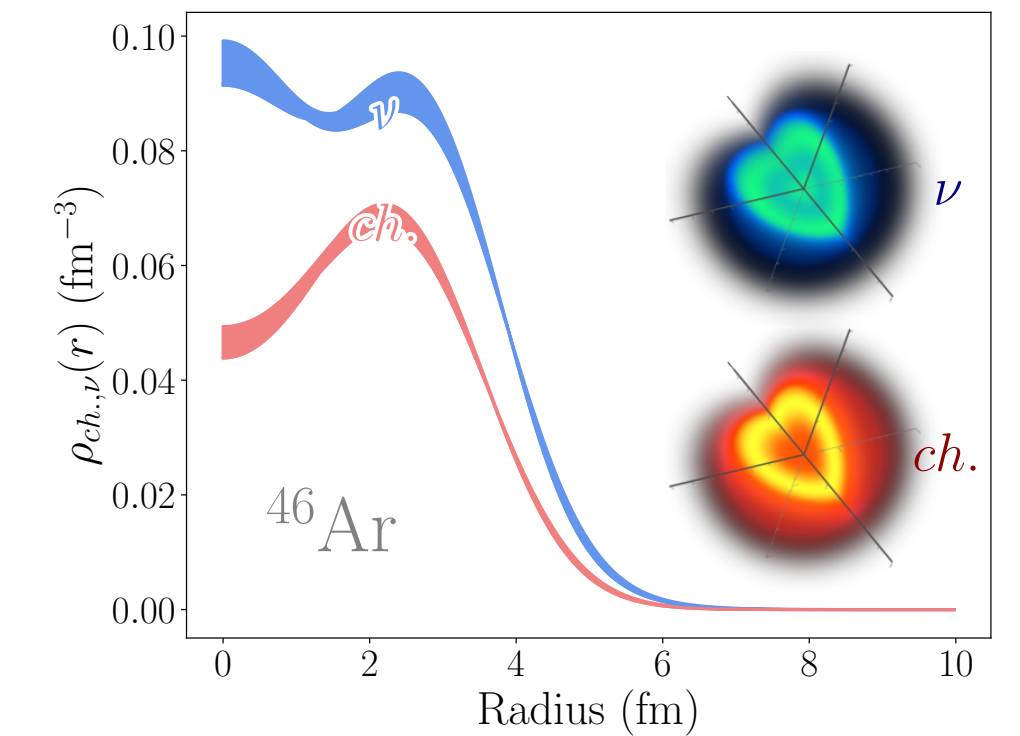
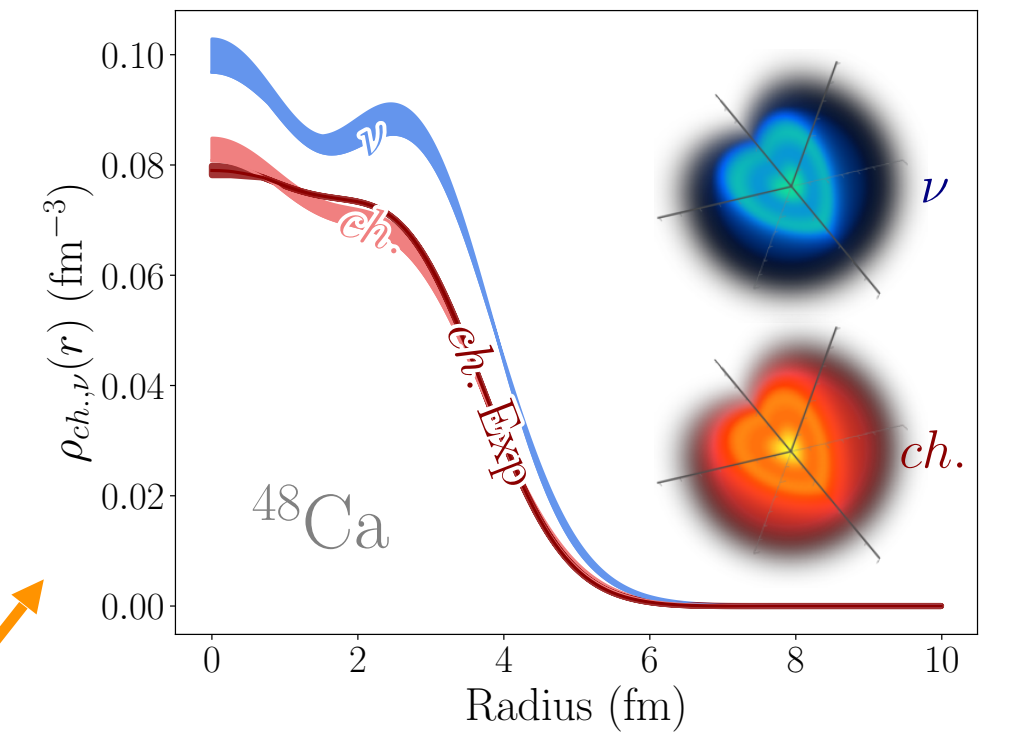
d3/2 - s1/2 inversion
revisited from adding
protons to ^{46}Ar



$^{46}\text{Ar}(^3\text{He},d)^{47}\text{K}$ at GANIL : New charge bubble in ^{46}Ar



$$\frac{d\sigma}{d\Omega} = \sum_k g_k C^2S_k \frac{d\sigma_k^{SP}}{d\Omega}$$



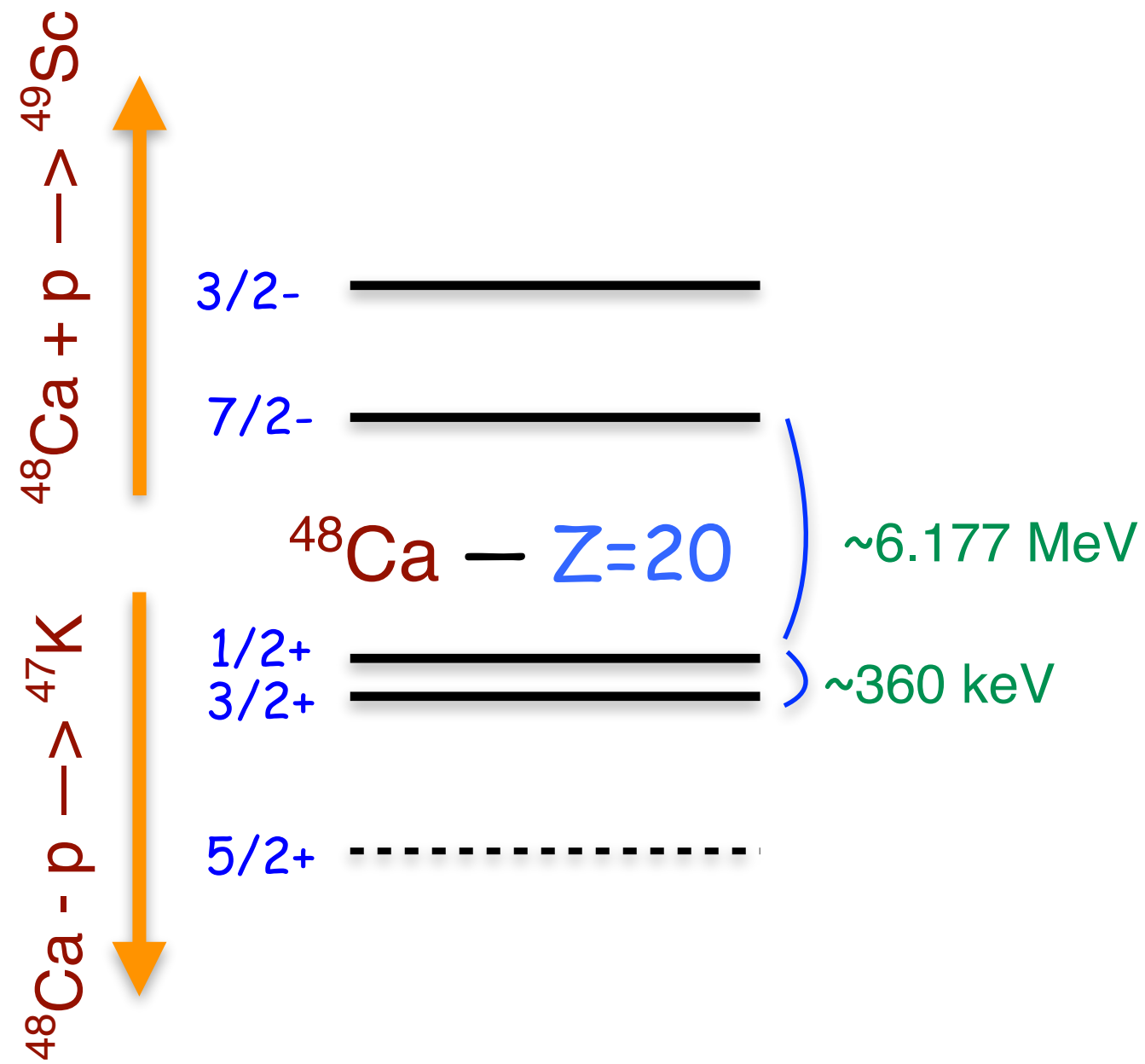
d3/2 - s1/2 inversion revisited
from adding protons to ^{46}Ar

Theory & experiment for relative
SFs agree within 1 sigma and
confirms charge depletion in ^{46}Ar

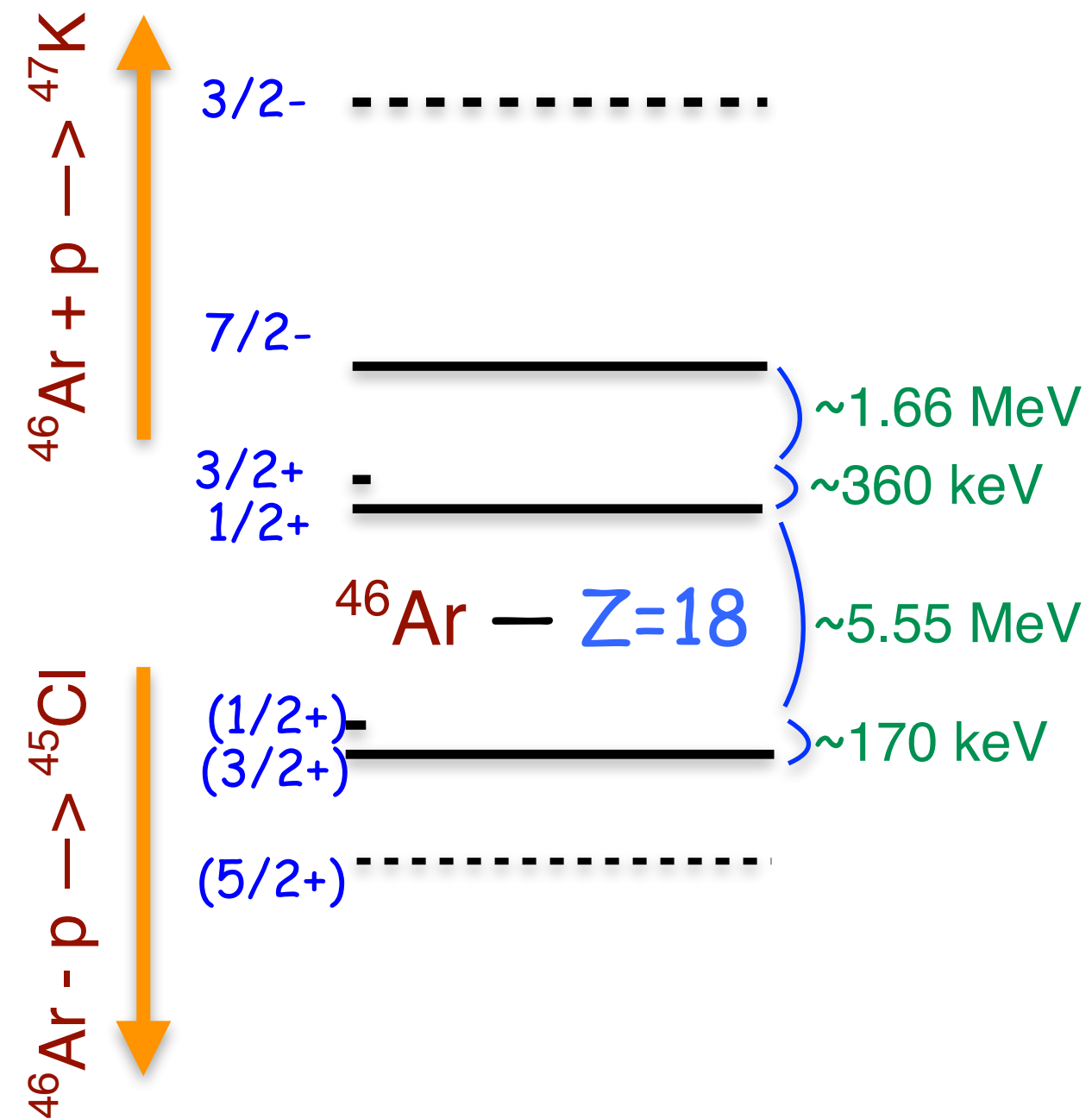
Theory bands are a combination of NNLOsat, Δ -full DN²LO(394) and DN²LO(450), as well as Darmstadt's saturating "magic force" 1.8/2.0 (7.5) —all of these have constrained LECs using the ^{16}O radius and few other mid-mass nuclei data.



$^{46}\text{Ar}(^3\text{He},d)^{47}\text{K}$ at GANIL : New charge bubble in ^{46}Ar



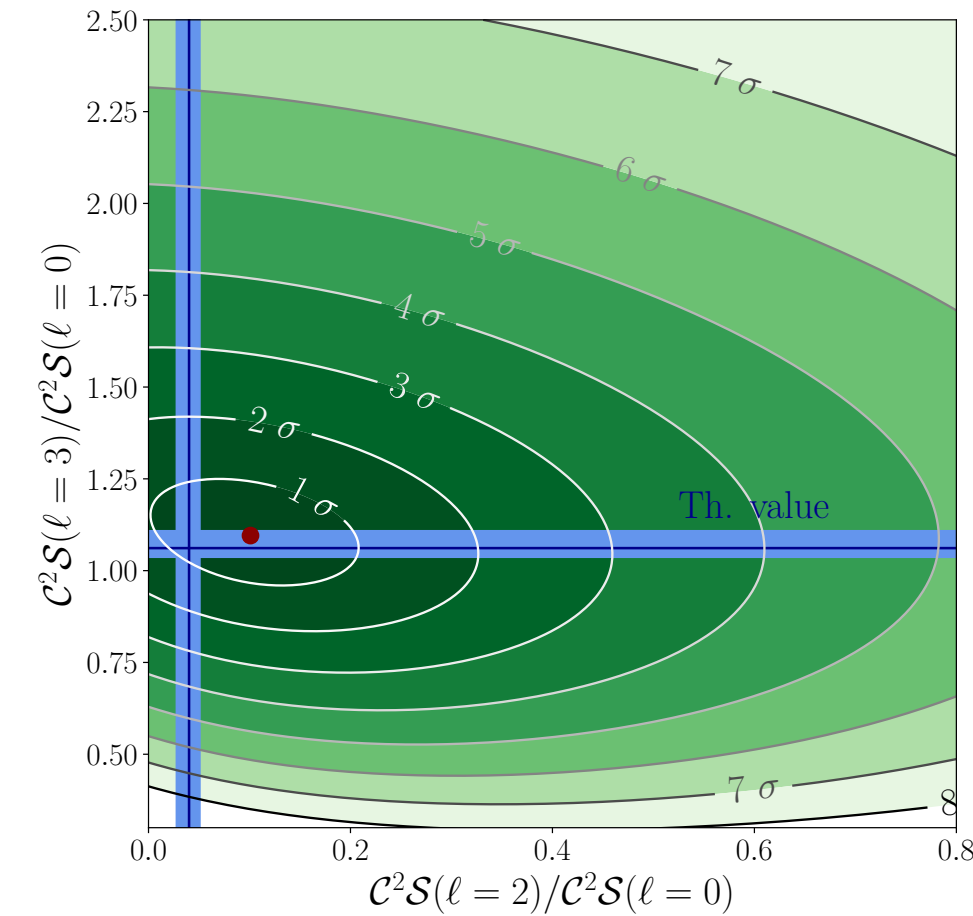
$$\Delta E_{1/2+ - 3/2+} (\text{exp}) = 360 \text{ keV}$$



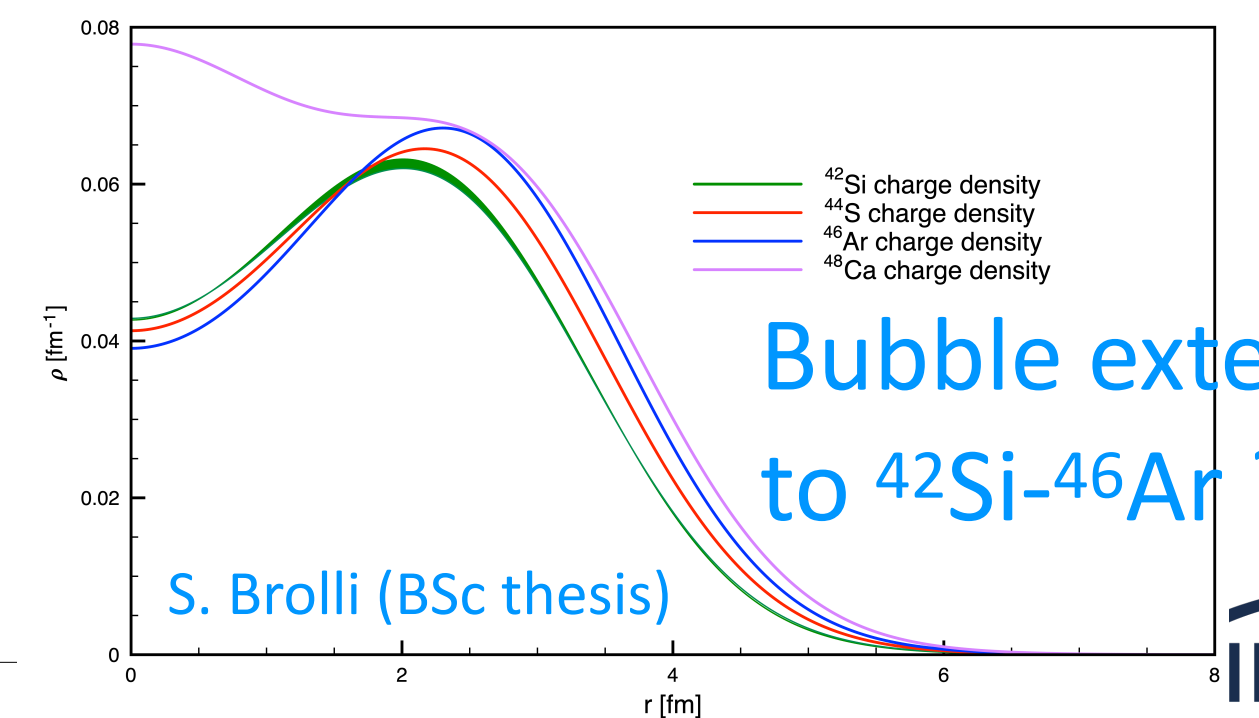
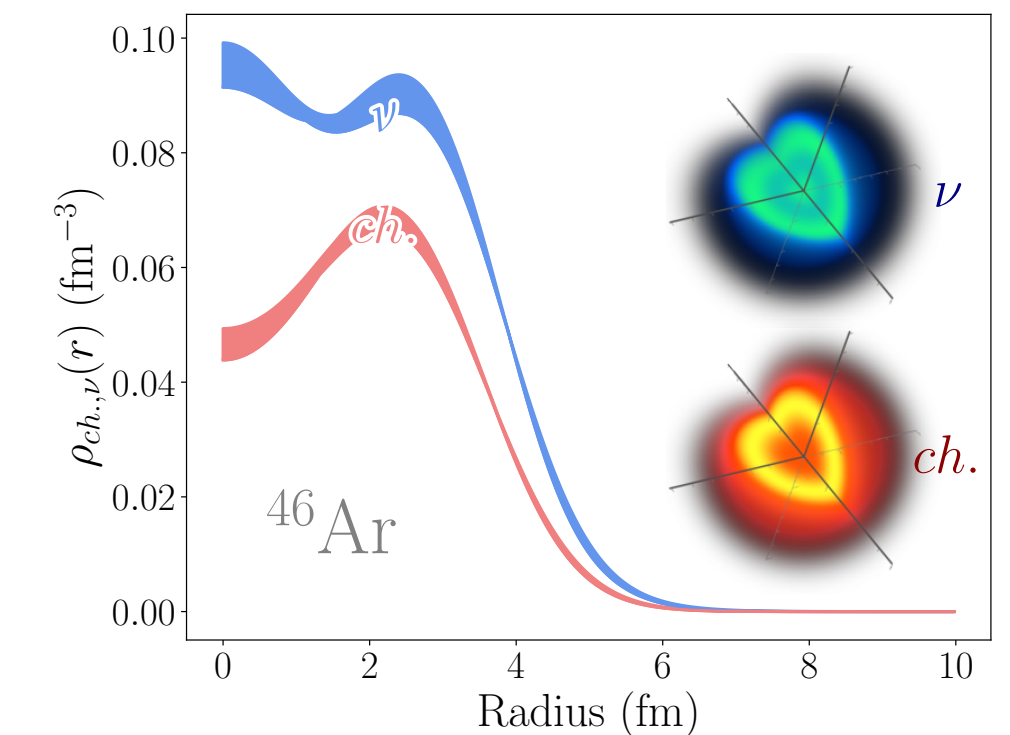
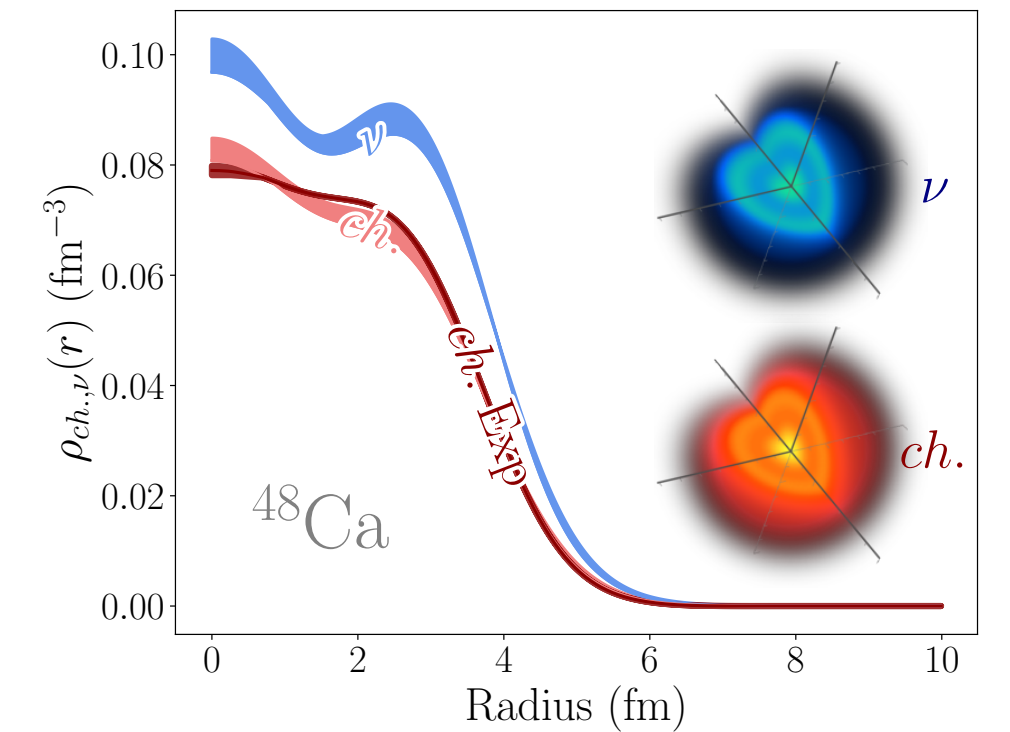
$$\Delta E_{1/2+ - 3/2+} (\text{exp}) = 5,55 \text{ MeV}$$

$$(\text{theo}) = 2,59 \text{ MeV}$$

The $d_{3/2} - s_{1/2}$ gap opening and new shell closure at $Z=18$!!



$$\frac{d\sigma}{d\Omega} = \sum_k g_k C^2 S_k \frac{d\sigma_k^{SP}}{d\Omega}$$

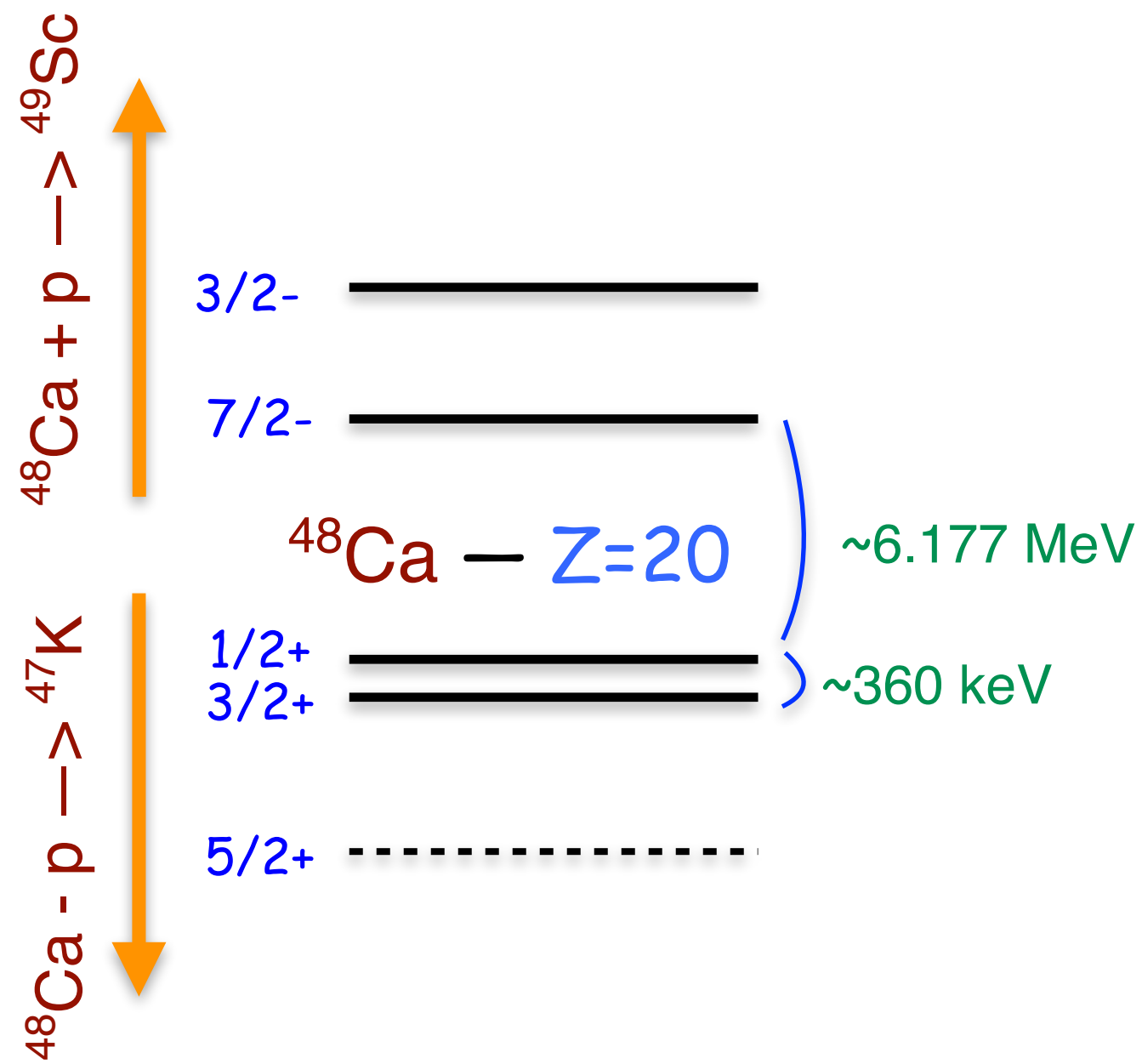


Bubble extends to ^{42}Si - ^{46}Ar ??

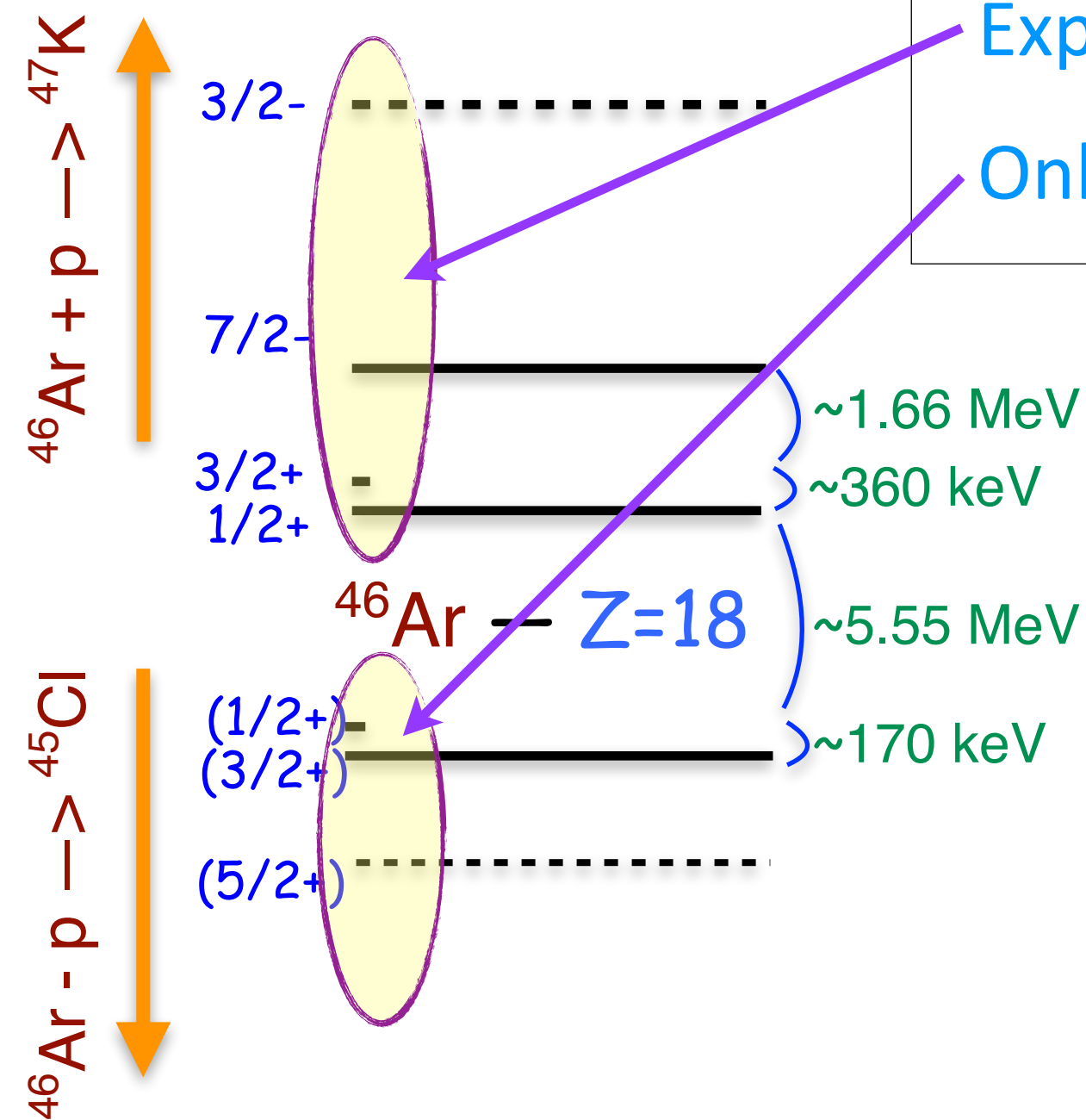
S. Brolli (BSc thesis)



$^{46}\text{Ar}(^3\text{He},d)^{47}\text{K}$ at GANIL : New charge bubble in ^{46}Ar



$$\Delta E_{1/2+ - 3/2+} (\text{exp}) = 360 \text{ keV}$$



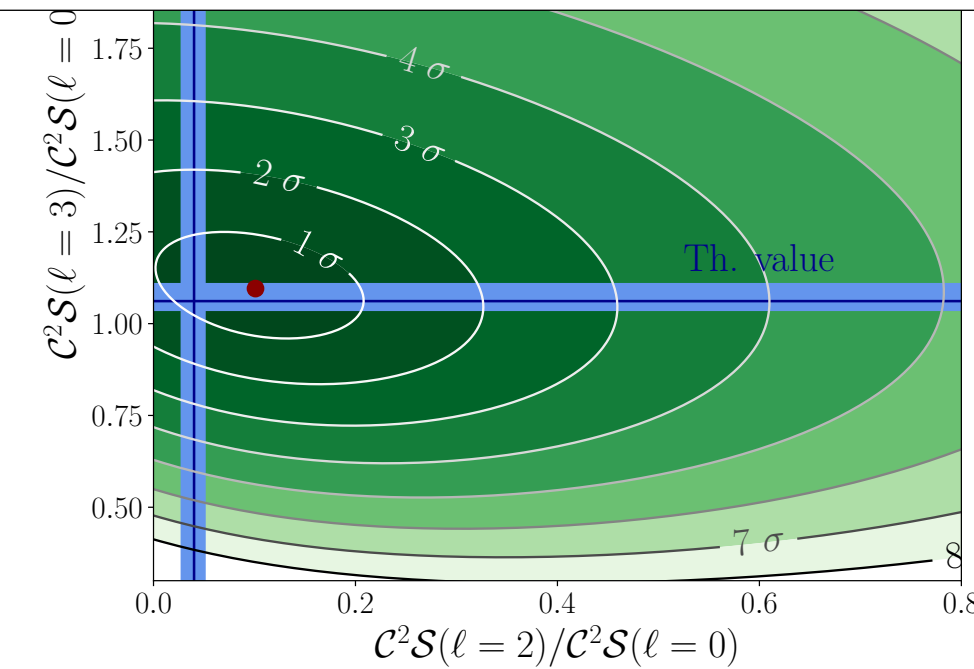
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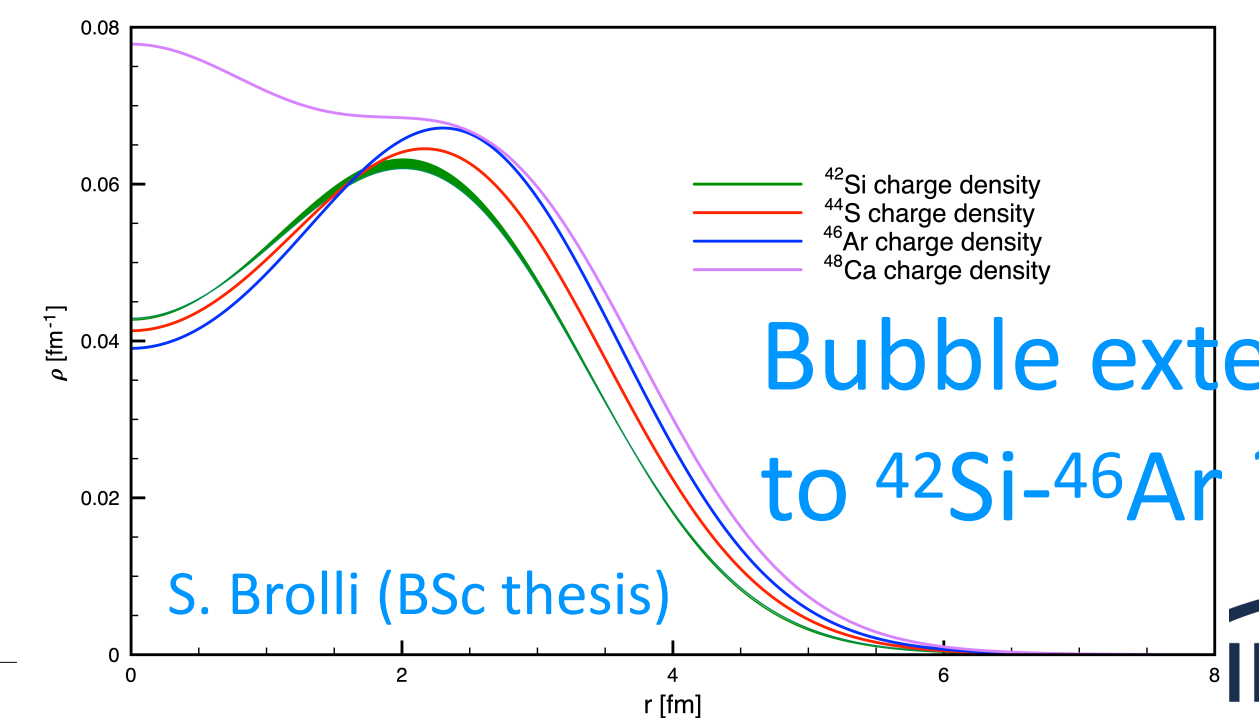
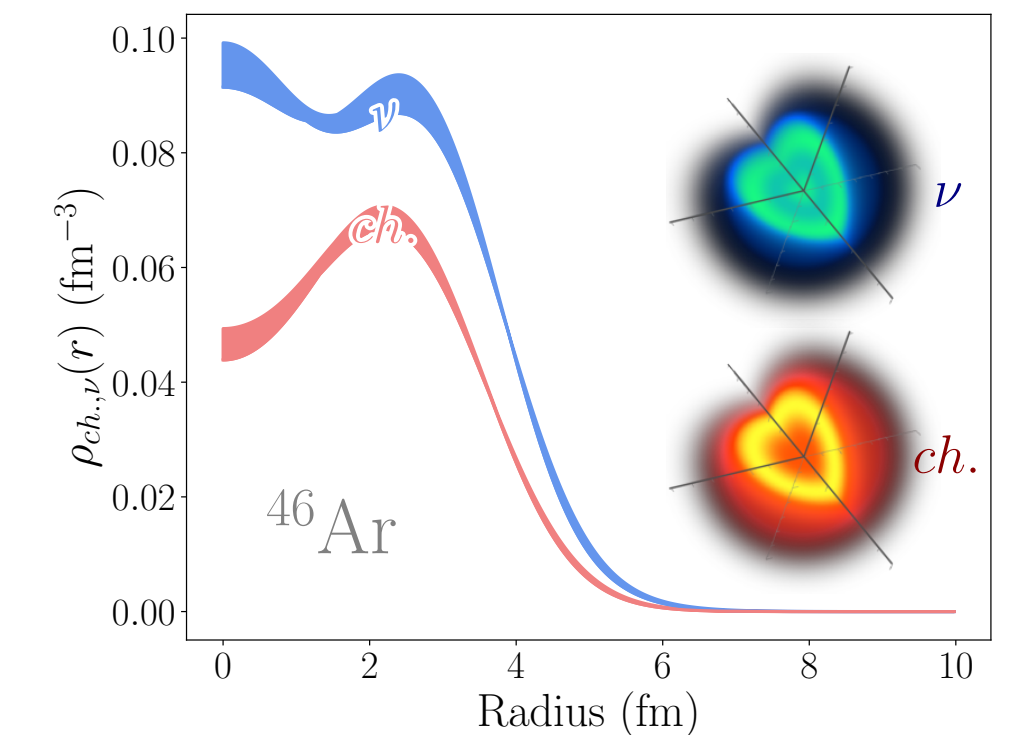
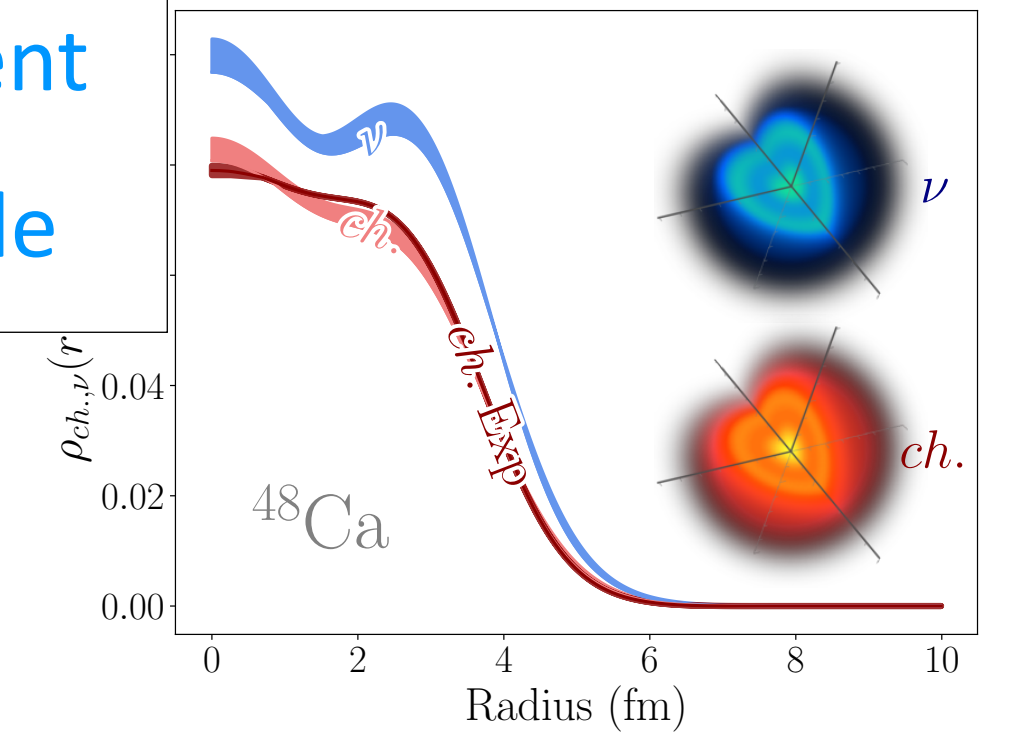
The $d3/2 - s1/2$ gap opening and new shell closure at $Z=18$!!

Experiment & theory in agreement

Only theory prediction is available



$$\frac{d\sigma}{d\Omega} = \sum_k g_k C^2S_k \frac{d\sigma_k^{SP}}{d\Omega}$$



Bubble extends to ^{42}Si - ^{46}Ar ??

S. Brolli (BSc thesis)



Nuclear Density Functional from Ab Initio Theory

See talk by
G. Colò this morning

PHYSICAL REVIEW C **104**, 024315 (2021)

Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino^{1,2,*} C. Barbieri^{1,2} A. Carbone³ G. Colò^{1,2} A. Lovato^{4,5} F. Pederiva^{6,5} X. Roca-Maza^{1,2}
and E. Viguzzi²

¹Dipartimento di Fisica “Aldo Pontremoli,” Università degli Studi di Milano, 20133 Milano, Italy

²Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy

³Istituto Nazionale di Fisica Nucleare—CNAF Viale Carlo Rort Pichat 6/2 40127 Bologna Italy

F. Marino
(EPS PhD thesis
prize, 2025)

DFT is in principle exact – but the energy
density functional (EDF) is not known

For nuclear physics this is even more
demanding: need to link the EDF to
theories rooted in QCD!

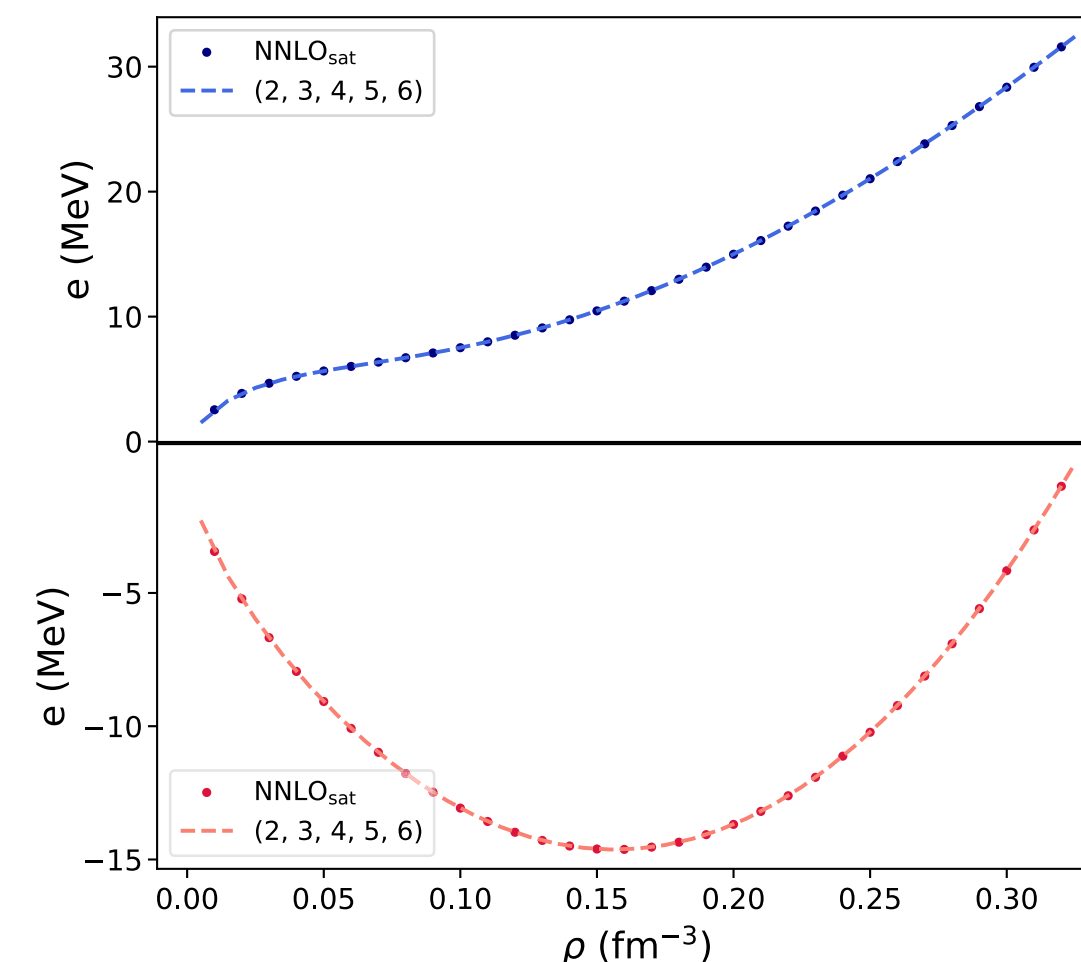
Machine-learn DFT functional
on the nuclear equation of state

Jacob's ladder



+ approximate GA

Benchmark in finite systems



$$E = \int d\mathbf{r} \mathcal{E}(\mathbf{r}) = E_{\text{kin}} + E_{\text{pot}} + E_{\text{Coul}}$$

$$E_{\text{GA}} = E_{\text{LDA}} + E_{\text{surf}}$$

$$E_{\text{surf}} = \int d\mathbf{r} \left[\sum_{t=0,1} C_t^\Delta \rho_t \Delta \rho_t - \frac{W_0}{2} \left(\rho \nabla \cdot \mathbf{J} + \sum_q \rho_q \nabla \cdot \mathbf{J}_q \right) \right]$$

Algebraic diagrammatic construction [ADC(3)] for infinite matter

Finite size box (of length L) with periodic Boundary conditions:

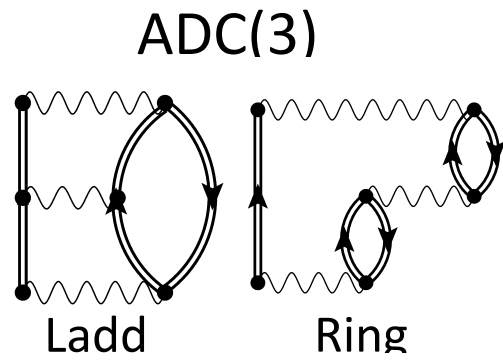
$$\rho = \frac{A}{L} \qquad p_F = \sqrt[3]{\frac{6\pi^2\rho}{v_d}} \qquad \mathbf{k} = \frac{1}{L} \left(2\pi \mathbf{n} + \boldsymbol{\theta} \right), \qquad |\mathbf{k}|^2 = k_x^2 + k_y^2 + k_z^2 \leq \frac{4\pi^2}{L^2} N_{max}$$

$$\Sigma^{11}(\omega) = \Sigma^{11(\infty)} + \tilde{\Sigma}(\omega)$$



ADC(3) self energy:

$$\Sigma_{\alpha\beta}^{(\star)}(\omega) = -U_{\alpha\beta} + \Sigma_{\alpha\beta}^{(\infty)} + M_{\alpha,r}^\dagger \left[\frac{1}{\omega - [E^> + C]_{r,r'} + i\eta} \right]_{r,r'} M_{r',\beta} + N_{\alpha,s} \left[\frac{1}{\omega - (E^< + D) - i\eta} \right]_{s,s'} N_{s',\beta}^\dagger$$



Gorkov superfluid formulation (at 1st order)

$$g(\omega) = g^{(0)}(\omega) + g^{(0)}(\omega) \Sigma^\star(\omega) g(\omega)$$

Normal self-energy Pairing field

$$\Sigma^\star(\omega) = \begin{pmatrix} \Sigma^{11}(\omega) & \Delta \\ \Delta^* & \Sigma^{22}(\omega) \end{pmatrix}$$

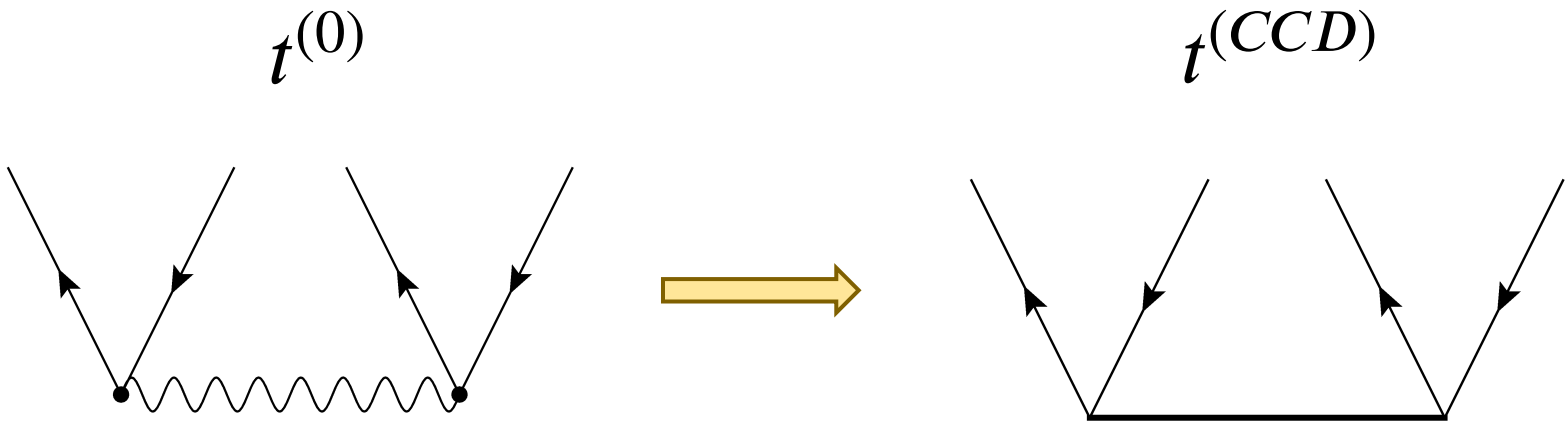
Extension of ADC(3)



ADC(3)-D

In $\tilde{\Sigma}(\omega)$, replace $t^{(0)}$ with converged T_2 amplitudes

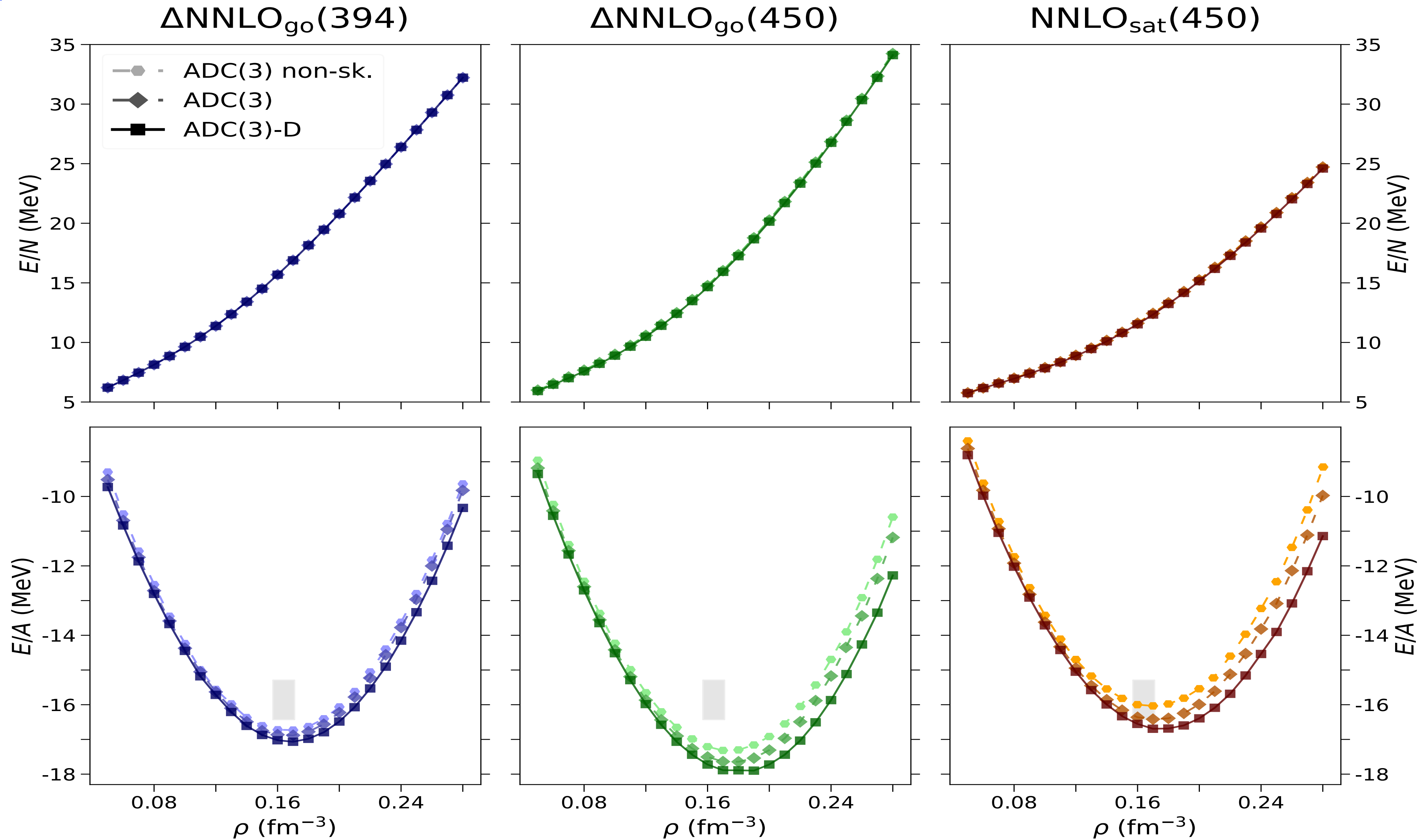
$$(t^{(0)})_{ij}^{ab} = \frac{\langle ab | v | ij \rangle_A}{\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b}$$



Somà et al., Phys. Rev. C **84**, 064317 (2011)
 Raimondi et al., Phys Rev C **97**, 054308 (2018)
 Barbieri et al., Lect. Notes Phys. **936**, 571 (2017) - Ch 11
 Barbieri et al., Phys. Rev. C **105**, 044330 (2022)

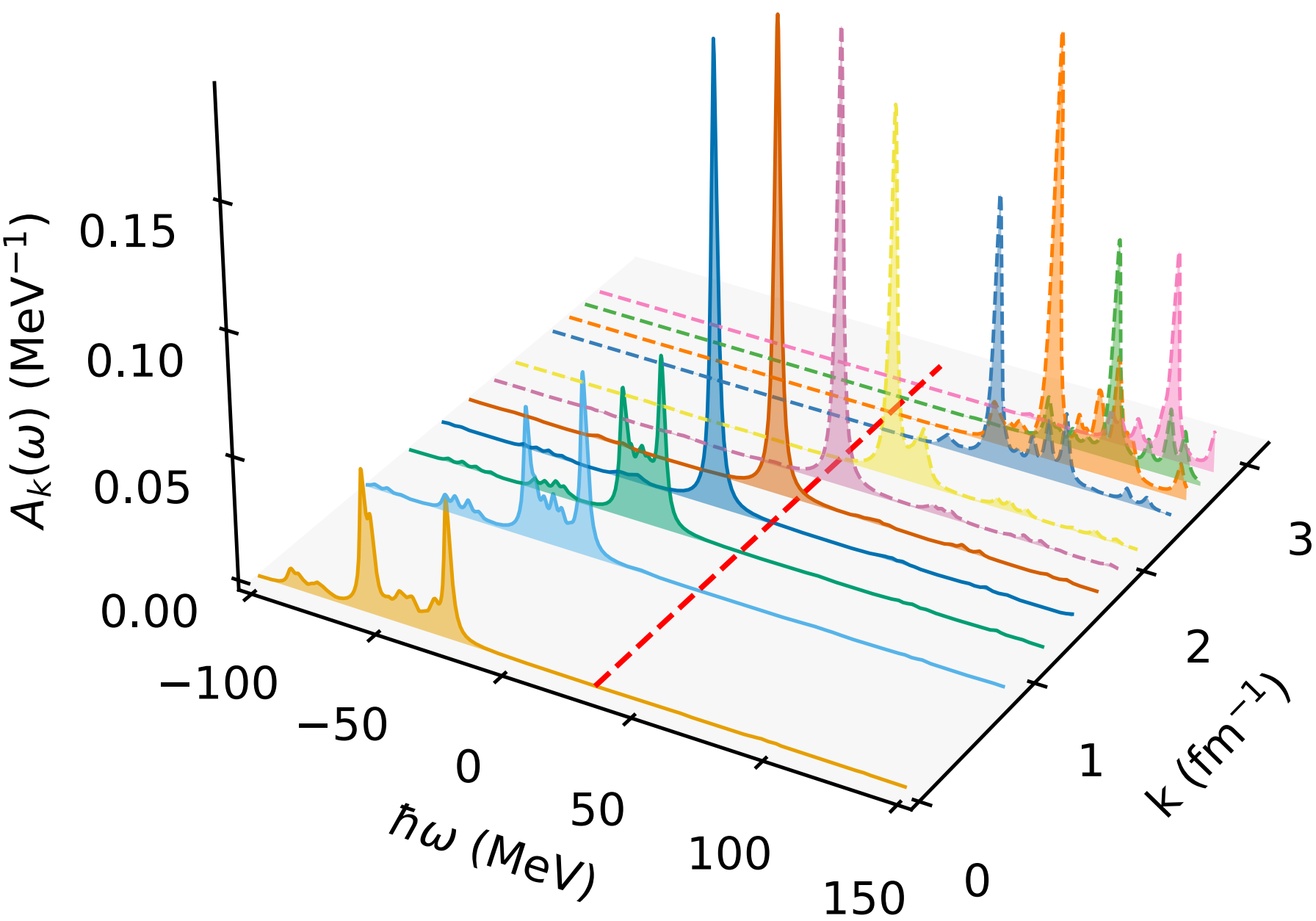


Equations of state ($T=0$)



Removing finite size effects - Twisted Angle BC

Ordinary PBC

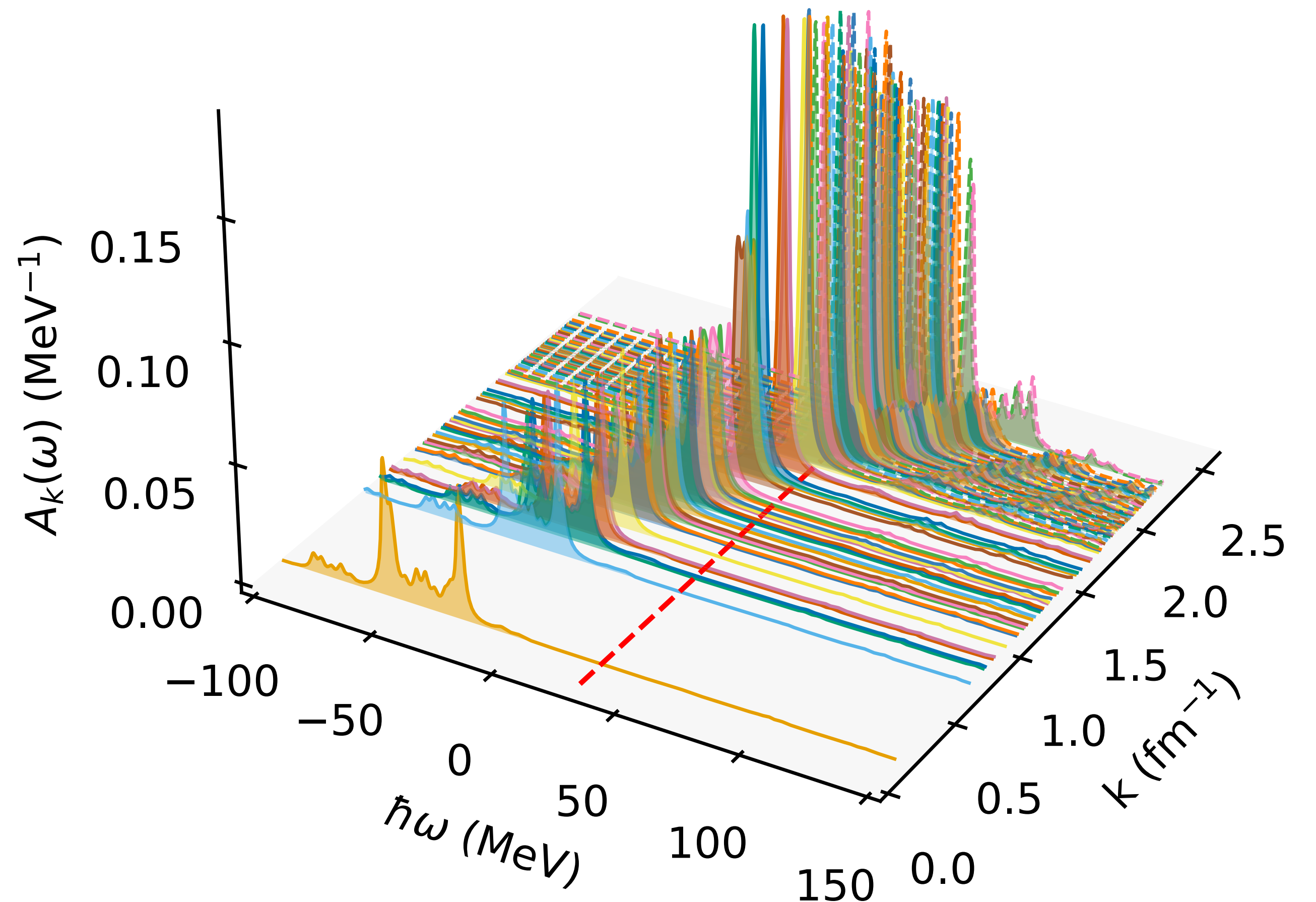


$$S^p(\mathbf{p}, \omega) = \sum_n |\langle \Psi_n^{A+1} | \psi_{\mathbf{p}}^\dagger | \Psi_0^A \rangle|^2 \delta(\hbar\omega - (E_n^{A+1} - E_0^A))$$

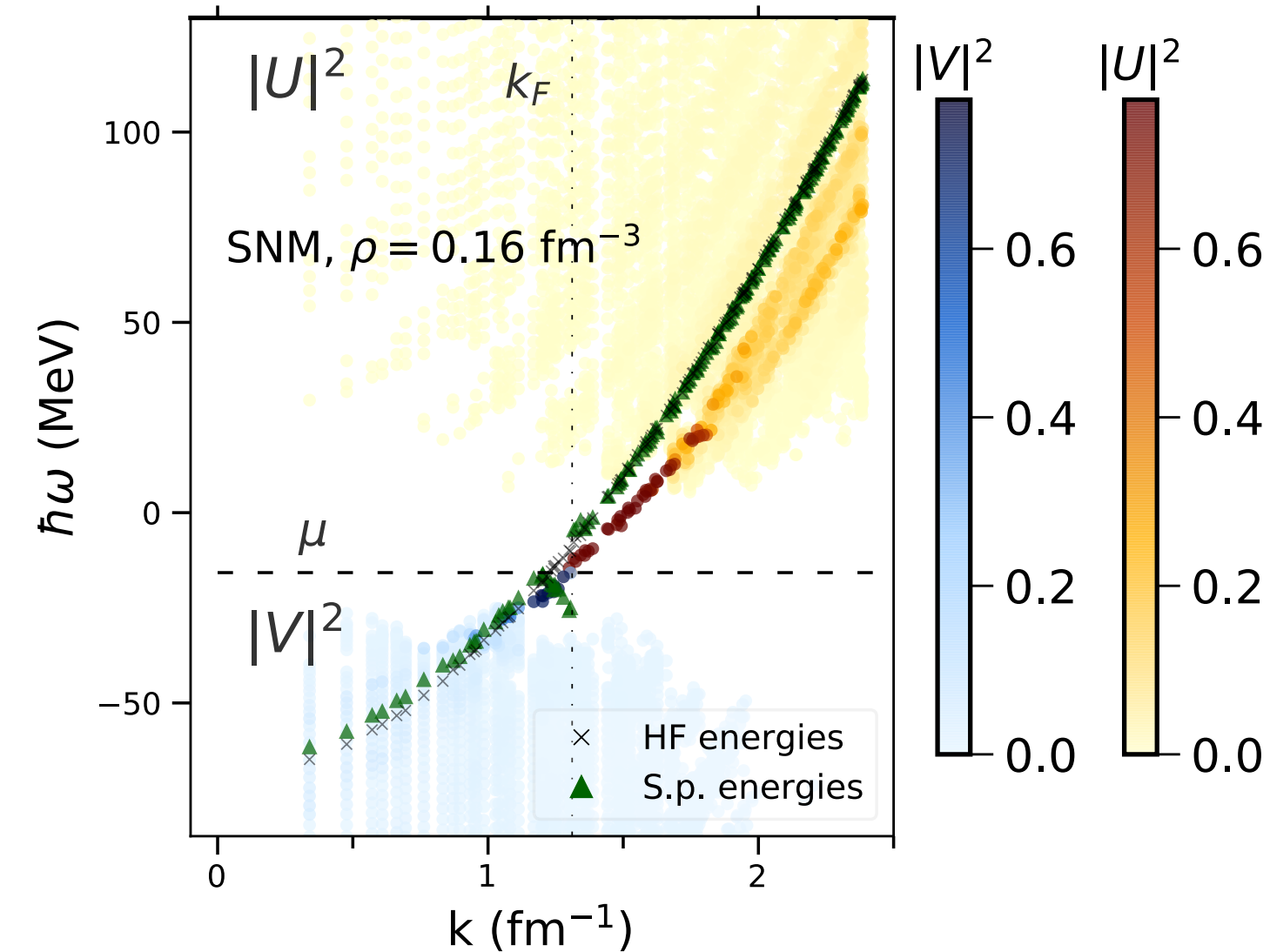
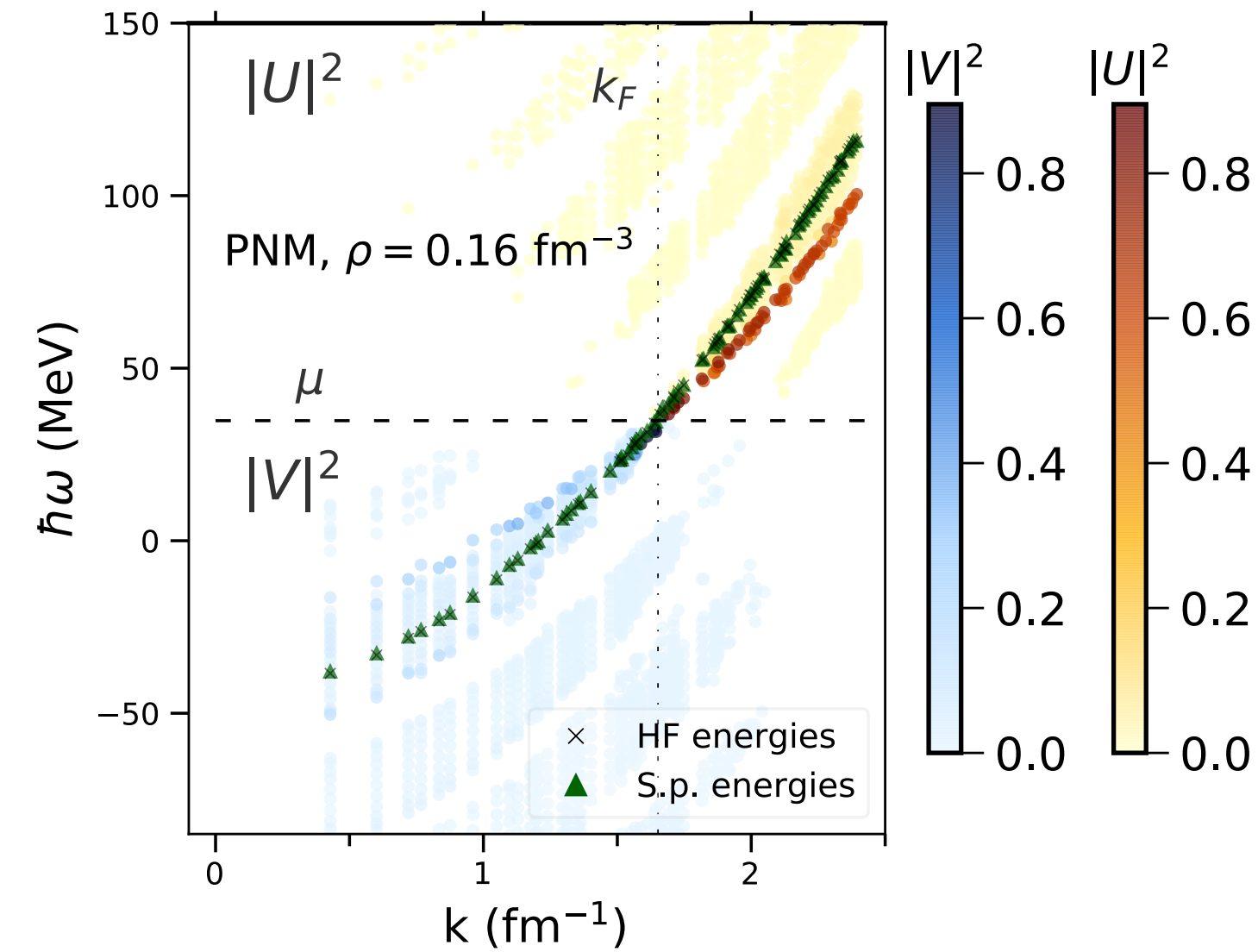
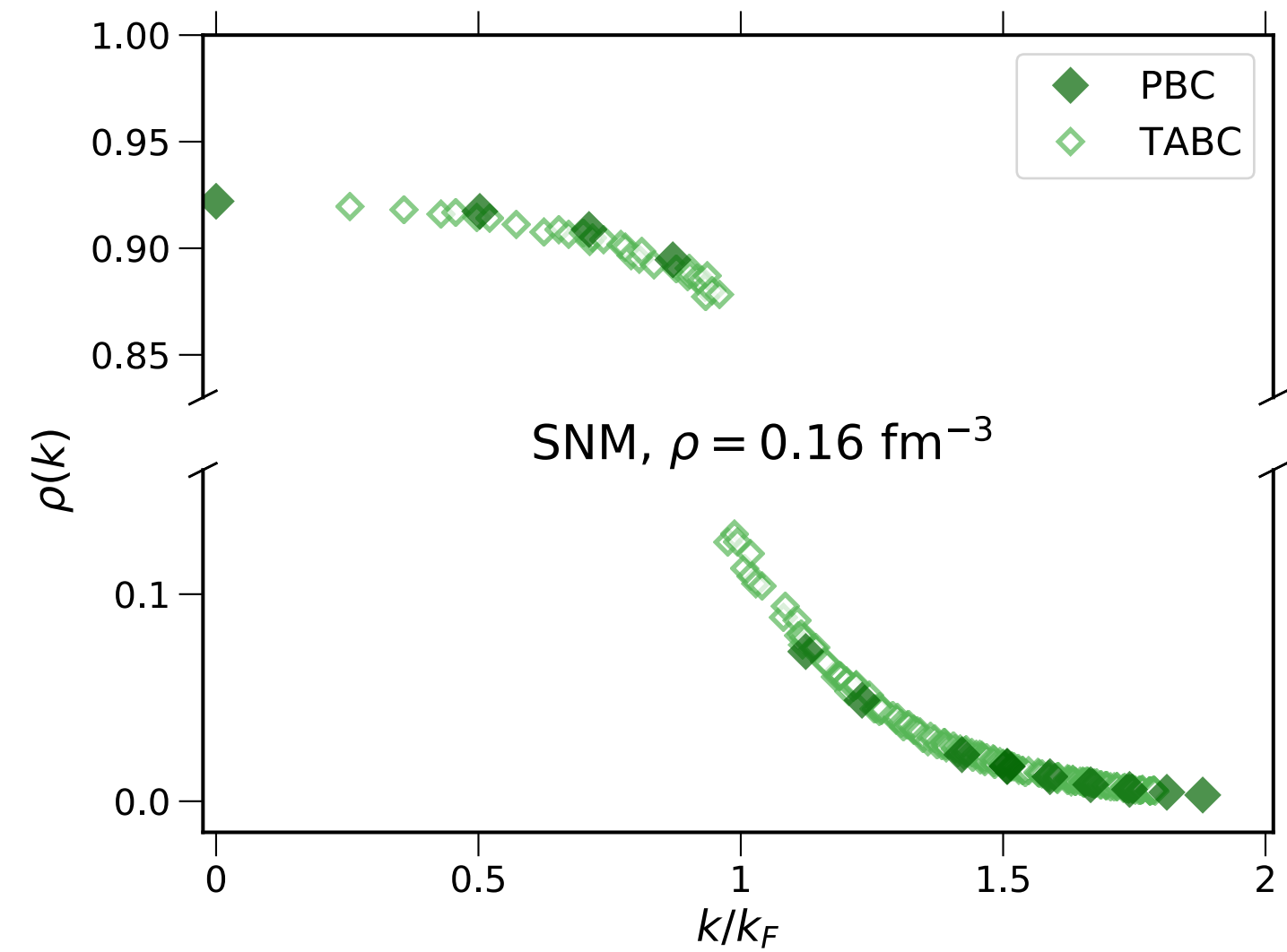
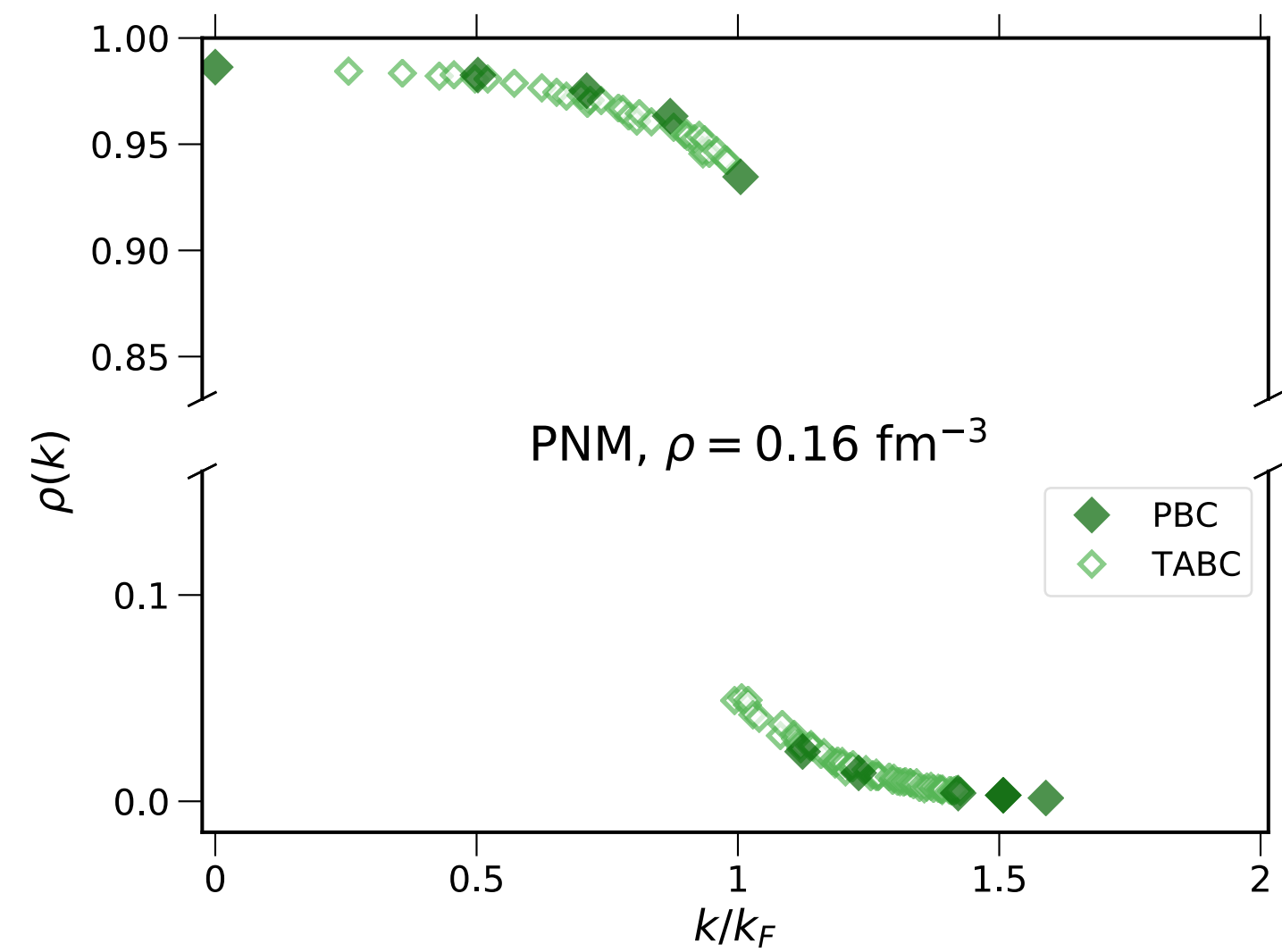
$$S^h(\mathbf{p}, \omega) = \sum_k |\langle \Psi_k^{A-1} | \psi_{\mathbf{p}} | \Psi_0^A \rangle|^2 \delta(\hbar\omega - (E_0^A - E_k^{A-1}))$$

sp-TABC: chose just one set of twisted angles

- Finer resolution in momenta
- Better description of the thermodynamic Limit

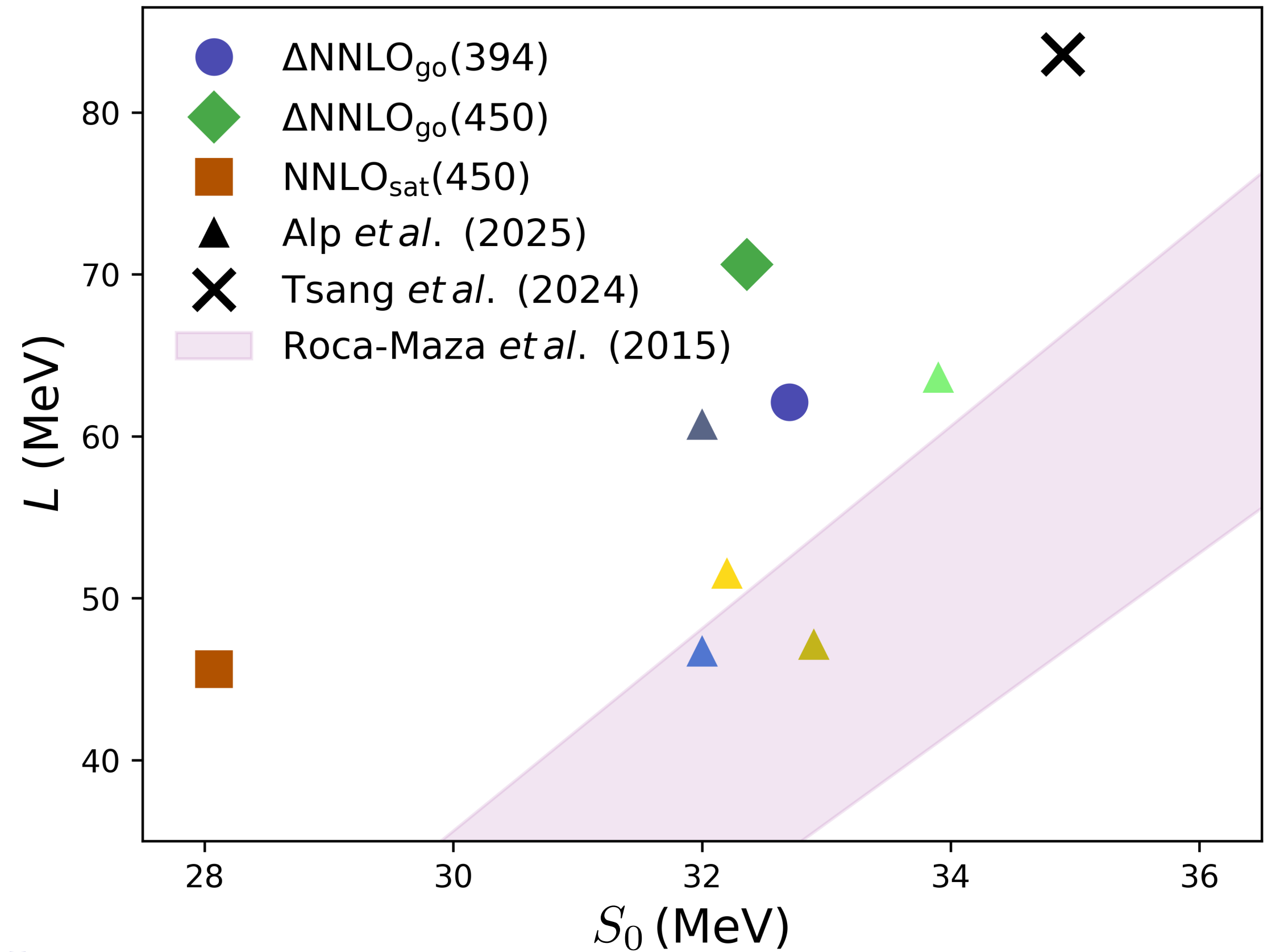
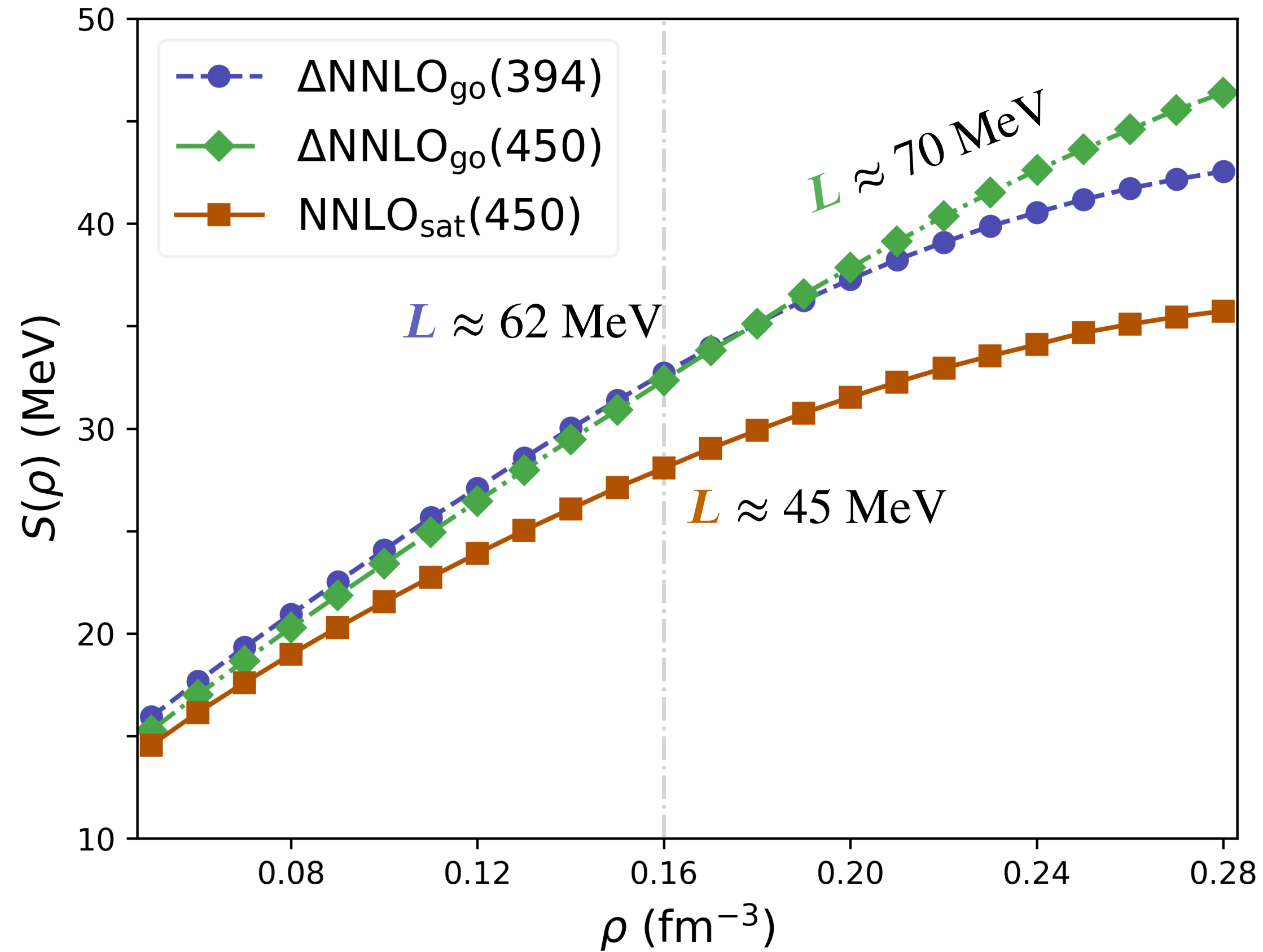


Removing finite size effects - Twisted Angle BC



$\Delta\text{NNLO}_{\text{go}}(450)$

Symmetry energy



$$S(\rho) = S(\rho_0) + \frac{L}{3\rho_0}(\rho - \rho_0) + \dots$$

F. Marino, CB, and G. Colò, arXiv:2510.xxx, to be submitted.

Alp *et al.*, arXiv:2504.18259
 Burgio *et al.*, Front. Astron. Space Sci. **11**, 1505560 (2024)
 Lynch and Tsang, Phys. Lett. B **830**, 137098 (2022)
 Roca-Maza *et al.*, Phys. Rev. C **92**, 064304 (2015)





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14-18 Sept 2026 (main event)

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Thank you for your attention !!!

Thank you for your attention !!!

