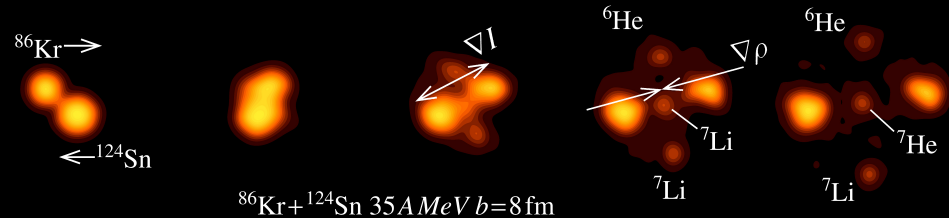


Dissipative regimes in heavy-ion transport models: *in a nutshell, in practice, in progress*

P. Napolitani *IJCLab, Orsay*



• main approximations & schemes • hints & tools • under construction

NUSDAF, Catania 22 – 10 – 2025

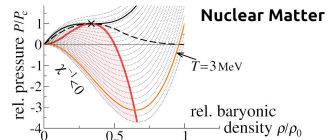


HIC for the nuclear EOS and stars

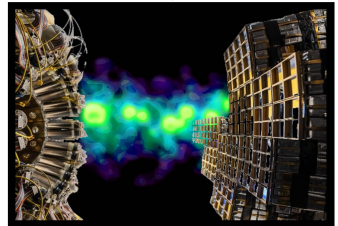
HIC probe the *nuclear interaction*, the *nuclear EoS* and give inputs to the *phenomenology of compact stars* (neutron stars, supernovae, mergers).

However, the link *NM-lab-stars* is indirect :

- even violent astrophysical scenarios are slow compared to the nucl.interaction relaxation time, while **HIC timescales reflect out-of-equilibrium processes** → *large-amplitude ρ fluctuations, bulk instabilities.*
- ⇒ nucl.interaction leads to **clusterisation below $\rho_{\text{saturation}}$ and in unstable conditions** (negative incompressibility).
- HIC are open systems → **surface and related instabilities** to treat in addition.

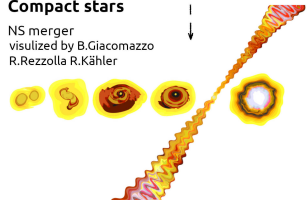


Lab



Compact stars

NS merger
visualized by B.Giacomazzo
R.Rezzolla R.Kähler



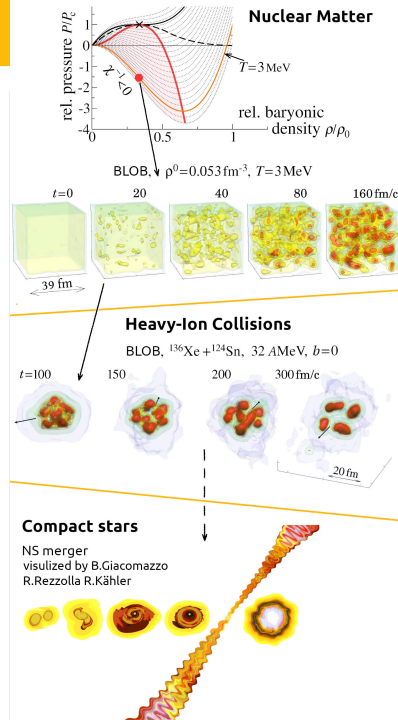
need of microscopic approaches

Nuclear dynamics, rather than focusing on specific ending states, tracks the **onset of resonances, threshold phenomena**, the **fragment/cluster-formation process** as a function of the properties of the medium, i.e. pressure, baryonic density, energy density, neutron excess.

→ suitable to explore *NM-lab-stars* link!

In practice, use **HIC transport models** :

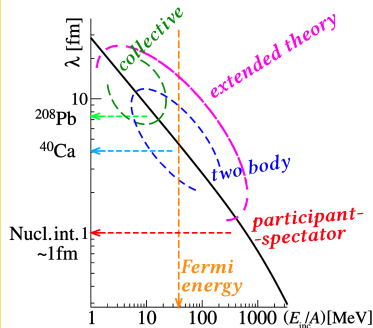
- **no equilibrium assumption**, still they can describe equilibration processes
- **able to associate n, p drifts to distinct EOS-related mechanisms**, (e.g. symmetry energy versus isospin transport coefficients)



transport approaches for dissipative HIC with rare beams

FRIB energy-range challenge for transport models [SORENSEN PPNP 2024] :

- even the most catastrophic processes, i.e. fragments and clusters formation, must be linked to the EOS properties $\Rightarrow$ thorough description of instabilities ;
- describe mean-field behaviour together with (beyond-mean-field) 'measurable' correlation observables, i.e. chronology, femtoscopy correl.func., particle-particle velocity correl. $\Rightarrow$ understand and possibly dismiss common approximations related to the quantum n-body representation



20 years ago (Texas A&M, WCI 2005), many 'transporters' in one shot

An early transport-model-comparison review → [ONO RANDRUP EPJA30 2006](#)

More recently (TMEP project) → [WOLTER COLONNA ET AL. PROG.PART.NUC.PHY. 2022](#)



Typical viewpoint : **mean-field vs molecular-dynamics fight!**

Instead, *two alternative viewpoints to proceed* :

- (1) **applying successive approximations** to a full n-body theory, or
- (2) **applying successive extensions** (correlations) beyond one-body ...

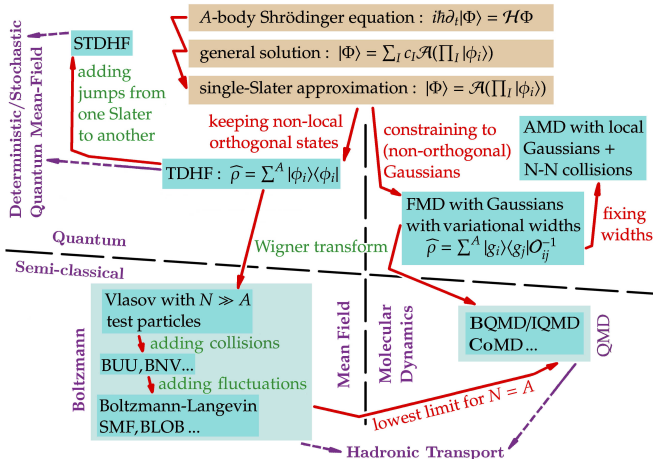
Viewpoint 1. Scheme of successive approximations, simple version

BUU Type

BLOB	P. Napolitani, M. Colonna
BUU-VM	S. Mallik
DJBUU	Y. Kim, S. Jeon,
	M. Kim, C.-H. Lee, K. Kim
GiBUU	J. Weil, T. Gaitanos,
	K. Gallmeister, U. Mosel
IBL	W.J. Xie, F.S. Zhang
IBUU	J. Xu, L.W. Chen, B.A. Li
LBUU(LHV)	R. Wang, Z. Zhang, L.-W. Chen
pBUU	P. Danielewicz
PHSD	E. Bratkovskaya, W. Cassing
RBUU	T. Gaitanos
RVUU	Z. Zhang, C.M. Ko, T. Song
SMASH	D. Oliinychenko, H. Elfner, A. Sorensen
SMF	M. Colonna, P. Napolitani
χ BUU	Z. Zhang, C.M. Ko

QMD Type

AMD	A. Ono
AMD+JAM	N. Ikeno, A. Ono
BQMD/IQMD	A. Le Fèvre, J. Aichelin,
	C. Hartnack, R. Kumar
CoMD	M. Papa
ImQMD	Y.X. Zhang, N. Wang, Z.X. Li
IQMD-BNU	J. Su, F.S. Zhang
IQMD-SINAP	G.Q. Zhang
JAM	A. Ono, N. Ikeno,
	Y. Nara, A. Ohnishi
JQMD 2.0	T. Ogawa, K. Niita,
	S. Hashimoto, T. Sato
LQMD(IQMD-IMP)	Z.Q. Feng, H.G. Cheng
TuQMD/dcQMD	D. Cozma
UrQMD	Y. J. Wang, Q. F. Li, Y. X. Zhang



Viewpoint 2. successive extensions beyond one-body

- **Splitting of a nuclear complex** into fragments requires *large-amplitude dynamics*, **beyond pure MF** equations

BBGKY (Born Bogoliubov Green Kirkwood Yvon) hierarchy

for a 2-body interaction V_{ij} :

$$i\hbar \frac{\partial \rho_1}{\partial t} = [k_1, \rho_1] + \text{Tr}_2[V_{12}, \rho_{12}]$$

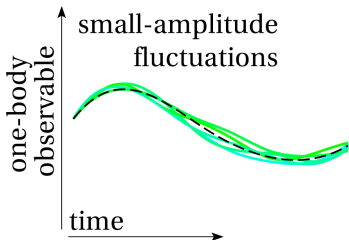
$$i\hbar \frac{\partial \rho_{12}}{\partial t} = [k_1 + k_2 + V_{12}, \rho_{12}] + \text{Tr}_3[V_{13} + V_{23}, \rho_{123}]$$

kinetic equations

$$i\hbar \frac{\partial \rho_{1\dots k}}{\partial t} = \sum_{i=1}^k [k_i + \sum_{j<i}^k V_{ij}, \rho_{1\dots k}] + \sum_{i=1}^{k+1} \text{Tr}_{k+1}[V_{ik+1}, \rho_{1\dots k+1}]$$

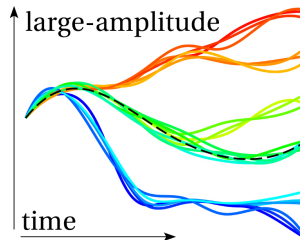
*high coupling
regimes*

($\rho_{1\dots k}$ =many-body dens. matrix, k_i =Kin.E operator, Tr_{ij} =partial trace)



beyond 2nd order

bifurcations,
non-linear regimes,
chaos,
instabilities



From (S)TDHF to BLOB

- Correlations beyond kinetic eq. approximately recovered by handling several mean fields
 → generated from the action of a **fluctuating seed** on the one-body density
 For one MF trajectory n in τ_{BL} :

Stochastic-TDHF scheme

$$i\hbar \frac{\partial \rho_1^{(n)}}{\partial t} \approx [k_1^{(n)} + V_1^{(n)}, \rho_1^{(n)}] + \overbrace{\bar{I}_{\text{coll}}^{(n)}}^{\text{average coll. term}} + \underbrace{\delta I_{\text{coll}}^{(n)}}_{\text{fluctuating coll. term}}$$

after τ_{BL} , it yields $\rho_1^{(n)} \rightarrow \{\rho_1^{(n_\lambda)}; \lambda = 1, \dots, \text{sub}_\lambda\}$

[REINHARD, SURAUD ANNPHYS 216 (1992); ANNPHYS 355 (2015)]

LACOMBE, REINHARD, SURAUD, DINH ANNPHYS 373 (2016)]

Boltzmann-Langevin

$f^{(n)}$: distribution functions
 → Fermi stat. at equilibrium

$$\frac{\partial f^{(n)}}{\partial t} = \{h^{(n)}, f^{(n)}\} + I_{UU}^{(n)} + \underbrace{\delta I_{UU}^{(n)}}_{\text{Markovian contrib. :}}$$

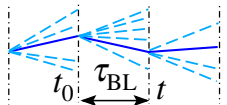
$$\langle \delta I_{UU}^{(n)}(\mathbf{r}, \mathbf{p}, t) \delta I_{UU}^{(n)}(\mathbf{r}', \mathbf{p}', t') \rangle =$$

$$= \text{gain} + \text{loss} = 2\mathcal{D}(\mathbf{r}, \mathbf{p}; \mathbf{r}', \mathbf{p}', t') \delta(t - t')$$

[AYIK, GRÉGOIRE PLB212(1988); NPA513(1990)]

COLONNA, CHOMAZ, RANDRUP NPA567(1994)]

- Diffusion coeff. $\mathcal{D}$ from *Langevin term* →
 → intermittent **fluctuation revival** and **bifurcations**
 in the spirit of the *Brownian motion*



BLOB collision term

Boltzmann-Langevin One Body [NAPOLITANI, COLONNA PLB726 2013; PRC96 2017]

$$\frac{\partial f^{(n)}}{\partial t} - \{h^{(n)}, f^{(n)}\} = I_{UU}^{(n)} + \delta I_{UU}^{(n)} = g \int \frac{d\mathbf{p}_b}{h^3} \int W(AB \leftrightarrow CD) F(AB \rightarrow CD) d\Omega$$

transition rate occupancy

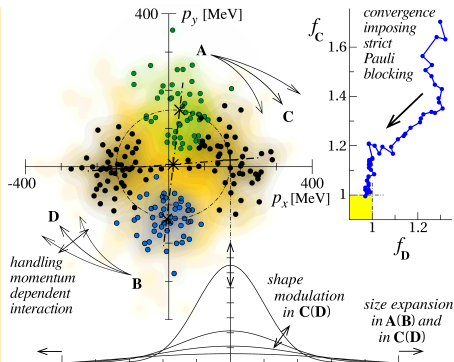
$$W(AB \leftrightarrow CD) = |v_A - v_B| \frac{d\sigma}{d\Omega} ; \quad F(AB \rightarrow CD) = [(1-f_A)(1-f_B)f_C f_D - f_A f_B (1-f_C)(1-f_D)]$$

A, B, C, D : extended equal-isospin phase-space portions of size=nucleon imposed by the variance $f(1-f)$ in h^3 cells at equilibrium

At each Δt :

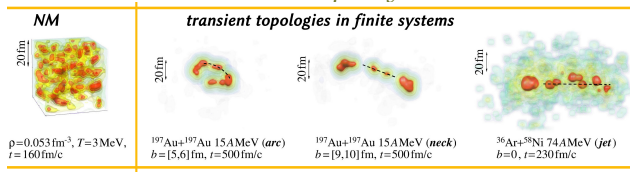
- one-body phase-space scanned
- redefinition of packets A_i, B_i
- N_{test} -scaled probability $W \times F$ accounts for initial/final states
⇒ stochastic kick
- modulation in shape of packets
⇒ strict Pauli

→ correct Fermi statistics



Clusters and fragments emerging in open systems, instabilities

- most probable spinodal fragment size corresponds to the **leading k** in NM, i.e. O to Ne, while heavier elements arise from **recombination** due to mean-field **resilience** in *isotropic systems*.

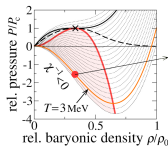
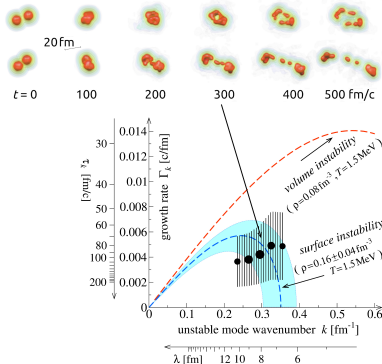


- Open systems introduce a surface, ruled by the same interaction term imposing UV cutoff for k , deformations and instabilities of Plateau-Rayleigh type, depending on surface tension, local density and temperature
- Clusters** in BLOB are treated like the formation of any other heavier fragment : they emerge naturally **from potential ripples** and are not related to cluster-production cross sections.

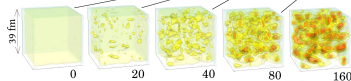
Example : BL description of surface and volume instabilities

split in four fragments $A \geq 20$ in $^{197}\text{Au}+^{197}\text{Au}$ 15 A MeV

Boltzmann-Langevin PRC 100 (2019) 054614

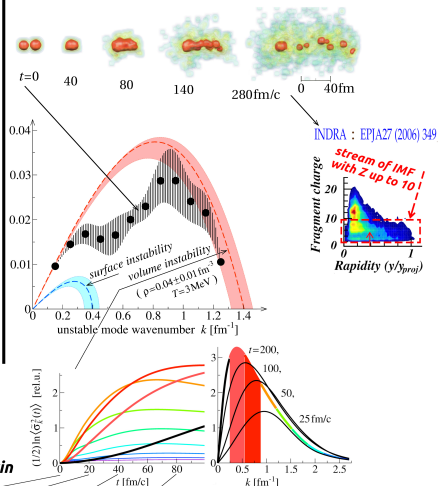


nuclear matter
Boltzmann-Langevin

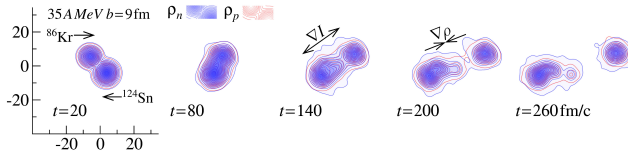


stream of clusters (jet) in $^{36}\text{Ar}+^{58}\text{Ni}$ 74 A MeV

Boltzmann-Langevin PLB 797 (2019) 134833



Example : BL description of combined drifts and sources of isospin-signals



- Stretched/expanded parts of the system experience **density gradients**

⇒ they are favored sites for **isospin transport**

⇒ isovector signals (equilibration, neck...)

⇒ study the form of S [MONTROYA ET AL. PRL73 (1994),

BARAN ET AL. PREP410 (2005), LIONTI ET AL. PLB625 (2005), DiTORO ET AL. EPJA30 (2006),

B.A.LI ET AL. PREP464 (2008), BARAN ET AL. PRC85 (2012), HUDAN ET AL. PRC86 (2012),

DE FILIPPO ET AL. PRC86 (2012), BROWN ET AL. PRC87 (2013), JEDELE ET AL. PRL118 (2017),

RODRIGUEZ MANSO ET AL. PRC95 (2017), A.B. McINTOSH, S.J. YENNELLO PPNP108 (2019), A. PAGANO ET AL., EPJA56 (2020)...

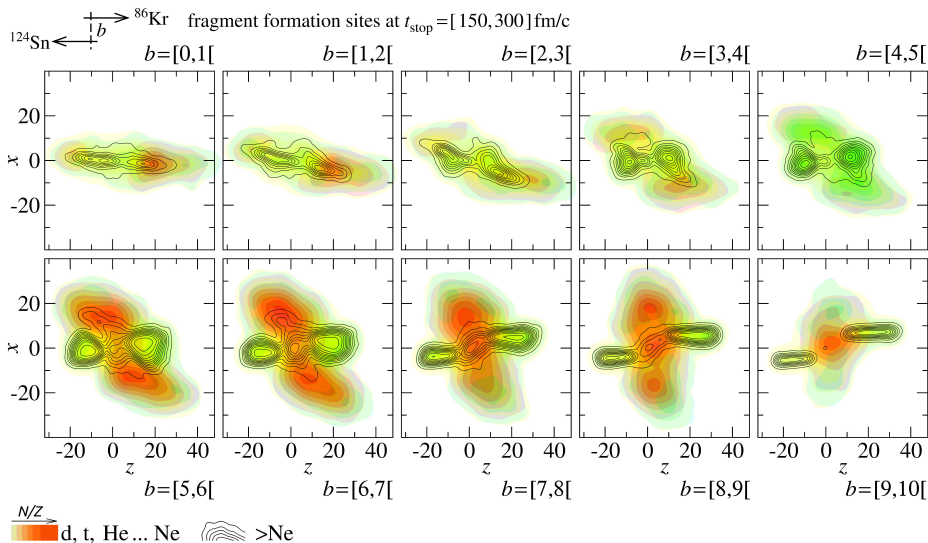
$$j_n - j_p \propto \overbrace{\frac{\partial S(\rho)}{\partial \rho} IV \rho}^{\text{migration}} + \overbrace{S(\rho) \nabla I}^{\text{diffusion}}$$

- If the ultimate fate is the disassembly of the system in a **phase-transition-like process**, the isospin content of the fragments is additionally affected by the phase separation (distillation).

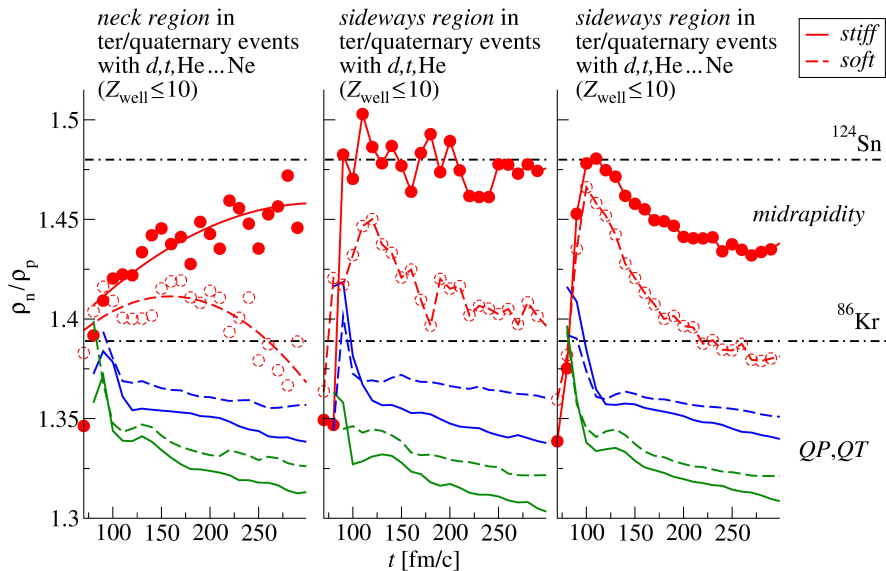
CHOMAZ, COLONNA, RANDRUP PHYS.REP.389 (2004) ; MÜLLER, SEROT PRC52 (1995)

Example : BL description of sources of isospin-signals, densities

- we can associate both neck and sideways areas to iv-transport hotspots



Microscopic tracking of cluster-/fragment-precursor properties



BLOB ready-to-use modes

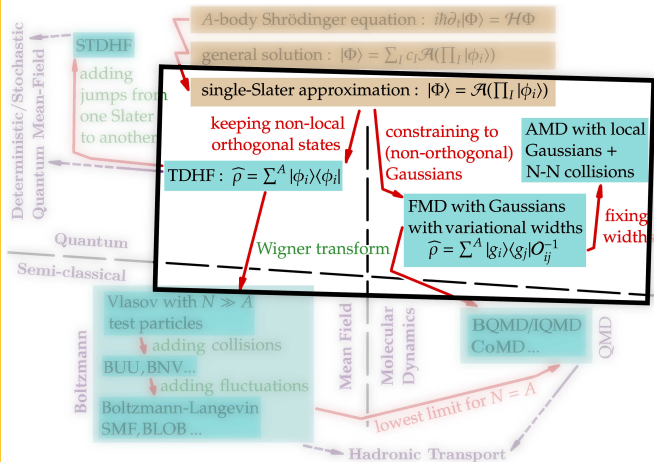
- dissipative heavy-ion collisions with fragment production, 10 to 400 AMeV [P.NAPOLITANI,M.COLONNA PLB726 \(2013\) 382](#)
- proton-nucleus (spallation) up to 1AGeV [P.NAPOLITANI,M.COLONNA PRC92 \(2015\) 034607](#)
- nuclear matter and very-large systems [P.NAPOLITANI,M.COLONNA PRC96 \(2017\)](#)
- properties of cluster- and fragment- precursors (before separation)
- light-charged particles and clusters from density fluctuations and related density observables [NAPOLITANI COLONNA PLB797 134833 \(2019\)](#)
- improved handling for decay (no freeze-out approximation)
[NAPOLITANI SAINTE-MARIE COLONNA PRC100 054614 \(2019\)](#)
- tracking of isospin currents [COLONNA BARAN NAPOLITANI](#)
- allows for several Skyrme interactions, including SAMI interactions (including MDI)

A new framework to dismiss some approximations, work in progress

This is the sector to focus on :

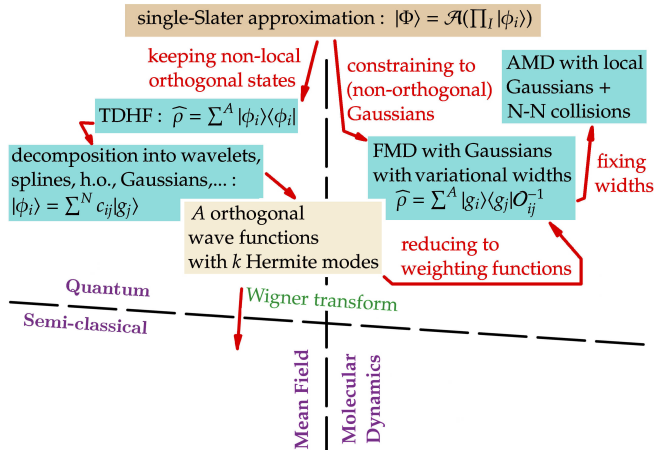
A new framework should be able to :

- Track wave-function evolution (width, shape)
- Preserve nucleonic correlations (avoiding splitting of a WF among two nuclei)
- Describe a large range of energy regimes, starting from low energy
- Improve stability



A new scheme to handle orthogonal wave-function dynamics

- fully orthogonal scheme : one Gaussian per nucleon used as a weighting function to build a hierarchy of Hermite modes
- obtain AMD / FMD (with variational widths) by restricting to the weighting function



Test. Collisional dynamics in $\text{Ca}^{40}+\text{Ca}^{40}$

Knowing that,

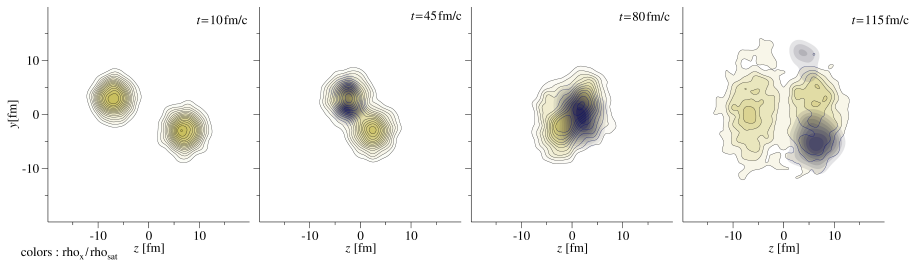
- One-body density is not sufficient to build up N-N corr.
- No decoherence approximation was done

1) *Mean-free path* from collapsing at random two wave functions on their one-body density distribution $\rightarrow$ collision probability

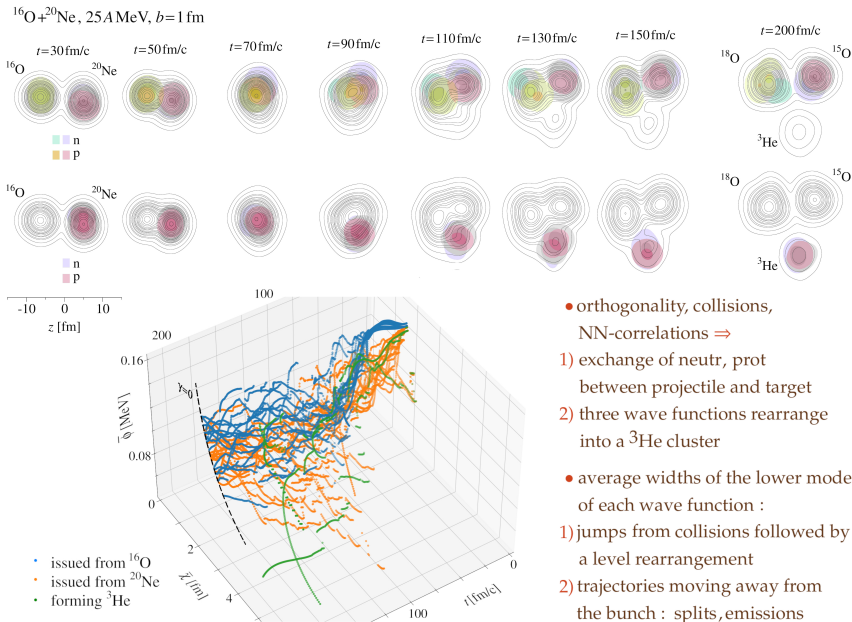
2) centroids boosted like in a semiclassical approach, rotated, translated

3) *Pauli* : not from phase space occupancy but **from the probability of finding an orthogonal solution for the scattered states $\Rightarrow$ new variances and new level scheme**

$^{40}\text{Ca}+^{40}\text{Ca}$, 35 A MeV, $b=6$ fm



Testing transport and cluster formation in OWFD



FRIB energy-range challenge requires :

- thorough description of instabilities, fragment formation, clustering, and their link to the EOS,
- to dismiss common approximations of the quantum n-body representation.

Among many other models...

BLOB is available and validated for several applications,
while OWFD is in progress.

Hermite parameterization

$$\varphi = g_n(\vec{x}) \cdot \sum_I \frac{C_I}{\text{norm}} H_I \left(\frac{x-x_n}{\sqrt{2\chi_{n,x}}}, \frac{y-y_n}{\sqrt{2\chi_{n,y}}}, \frac{z-z_n}{\sqrt{2\chi_{n,z}}} \right)$$

- I is a level superindex, e.g. $N_{\text{max}} = 6$

→ number of levels : $\sum_k^{N_{\text{max}}} \frac{(k+1)(k+2)}{2}$

→ $|I| = 84$

- built on the weighting function

$$g_n(\vec{x}) = g_n^x(x) g_n^y(y) g_n^z(z)$$

$$g_n^x(x) = \left(\frac{1}{2\pi\chi_n} \right)^{1/4} e^{-\xi_n \frac{(x-x_i)^2}{2} + i k_n (x-x_i)}$$

with $\xi_n = \frac{1}{2\chi_n} - 2i\gamma_n$

- $\gamma_n = \frac{\sigma_n}{2\chi_n}$ links the variational widths

- this set is orthogonal

- A collision term would act on the level weights and numbers

very gentle situation
(without collisions)

