# Clustering and Equation of State within extended density functional approaches

$1^{st}$ Collaboration Meeting on
NUclear Structure, Dynamics and Astrophysics at FRIB
(NUSDAF 2025)

Physics and Astronomy Department - University of Catania (Italy)

$22^{th}$ - $24^{th}$ October 2025

MOdeling
Nuclear
STructure and
REactions

**Authors**: S. Burrello, M. Colonna, R. Wang

INFN - Laboratori Nazionali del Sud, Catania

# Outline of the presentation

1. **Many-body (MB) correlations and clustering phenomena in nuclear systems**
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)

2. **Extended EDF-based models: recent developments and results**
   - ✏ Unified (thermodynamic) description of few-body correlations and clusters
     - Embedding short-range correlations within relativistic mean-field approaches
     - Global mass-shift parameterization for a multi-purposes EOS
   - ✏ Dynamical approach with light clusters as degrees of freedom (DOF)
     - Phase-space excluded-volume approach in dilute nuclear medium
     - Quasi-analytical characterization of spinodal instability and growth rates

3. **Further developments and outlooks**
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent description of fragment formation mechanisms in heavy-ion collisions

4. **Summary**

# Outline of the presentation

1. **Many-body (MB) correlations and clustering phenomena in nuclear systems**
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)

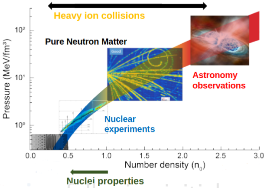
2. **Extended EDF-based models: recent developments and results**
   - Unified (thermodynamic) description of few-body correlations and clusters
   - Dynamical approach with light clusters as degrees of freedom (DOF)

3. **Further developments and outlooks**
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent description of fragment formation mechanisms in heavy-ion collisions

4. **Summary**

# Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\rm beam} \approx (30 - 300)\,A$MeV $\Rightarrow$ EOS



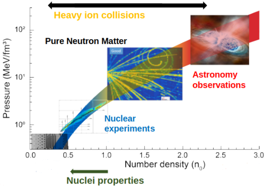

- Expansion following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - **Spinodal** instabilities $\rightarrow$ fragments
  - **Few-body** correlations $\rightarrow$ light clusters
- **Phenomenological** EDF with **clusters** DOF
  - NM $\rightarrow$ (nucleons+nuclei)

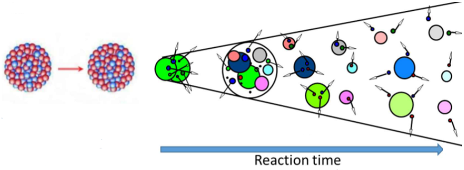
**Theoretical challenge**

Consistent dynamical approach for light clusters and heavier **fragments**

# Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\mathrm{beam}} \approx (30 - 300)\, A\mathrm{MeV} \Rightarrow \mathrm{EOS}$



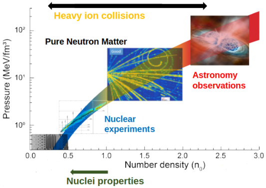

- **Expansion** following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - **Spinodal** instabilities $\rightarrow$ fragment
  - **Few-body** correlations $\rightarrow$ light clusters
- **Phenomenological** EDF with **clusters** DOF
  - Dilute NM $\rightarrow$ mixture (nucleons+nuclei)

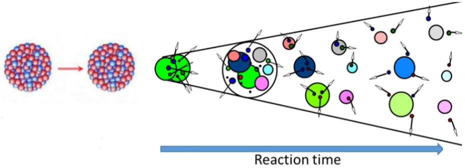
**Theoretical challenge**

Consistent dynamical approach for light clusters and heavier fragments

# Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\text{beam}} \approx (30 - 300)\,A\text{MeV} \Rightarrow$ EOS

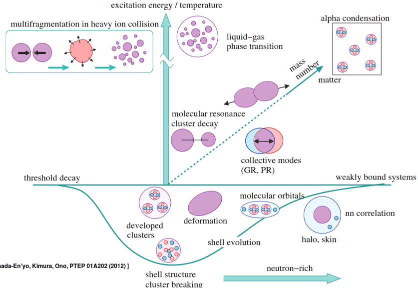



- **Expansion** following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - **Spinodal** instabilities $\rightarrow$ fragment
  - **Few-body** correlations $\rightarrow$ light clusters
- Phenomenological EDF with clusters DOF
  - Dilute NM $\rightarrow$ mixture (nucleons+nuclei)

**Theoretical challenge**

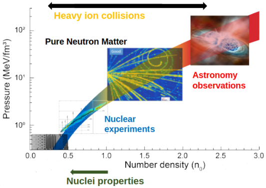
Consistent dynamical approach for light clusters and heavier fragments

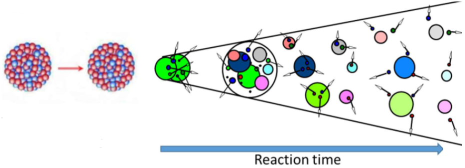



[Kanada-En'yo, Kimura, Ono, PTEP 01A202 (2012)]

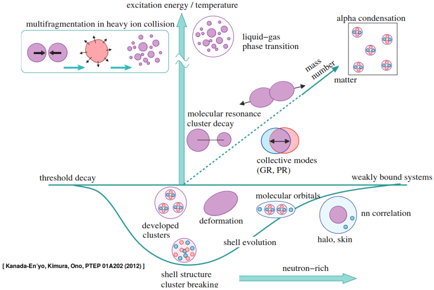
# Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\rm beam} \approx (30 - 300)\,A$MeV $\Rightarrow$ EOS




- **Expansion** following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - **Spinodal** instabilities $\rightarrow$ **fragment**
  - **Few-body** correlations $\rightarrow$ light **clusters**
- **Phenomenological** EDF with **clusters** DOF
  - **Dilute** NM $\rightarrow$ **mixture** (nucleons+nuclei)

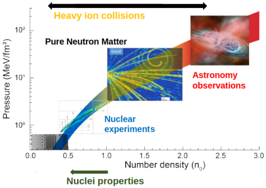


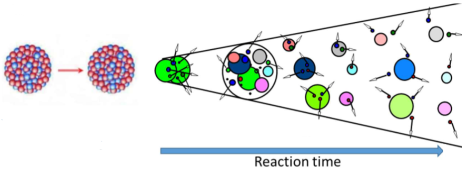

**Theoretical challenge**

Consistent dynamical approach for light clusters and heavier fragments

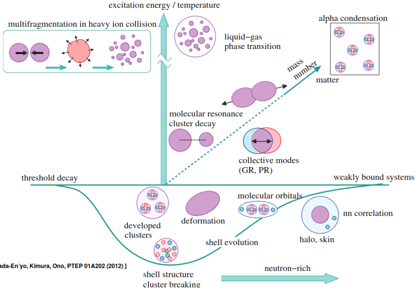
# Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\mathrm{beam}} \approx (30 - 300)\,A\mathrm{MeV} \Rightarrow$ EOS




- **Expansion** following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - **Spinodal** instabilities $\rightarrow$ **fragment**
  - **Few-body** correlations $\rightarrow$ light **clusters**
- **Phenomenological** EDF with **clusters** DOF
  - **Dilute** NM $\rightarrow$ **mixture** (nucleons+nuclei)




[Kanada-En'yo, Kimura, Ono, PTEP 01A202 (2012)]

**Theoretical challenge**

Consistent dynamical approach for **light clusters** and heavier **fragments**

# In-medium (Mott) effects and cluster dissolution

- Pauli-blocking (Mott) effect $\Rightarrow$ cluster **dissolution**
  - **Microscopic** in-medium effects
  - (Effective) **binding energy** $\rightarrow B^{\text{eff}} = B - \Delta m$
- $\Delta m^{(\text{low})}$ from in-medium Schrödinger equation (**SE**)

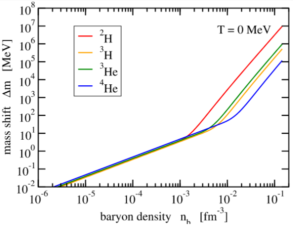
  [G. Röpke, NPA 867 (2011) 66–80]
- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\text{c.m.}}) \Rightarrow$ **heuristic** $\Delta m^{(\text{high})}$ beyond **Mott density**
  - **Bound** clusters survive only if $|\mathbf{P}_{\text{c.m.}}| > P_{\text{Mott}}$ (Mott momentum)
  - Few-body short-range correlations (SRCs) in the

  [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

# In-medium (Mott) effects and cluster dissolution

- Pauli-blocking (Mott) effect ⇒ cluster **dissolution**
  - **Microscopic** in-medium effects ⇒ **Mass-shift** ($\Delta m$)
  - (Effective) **binding energy** → $B^{\rm eff} = B - \Delta m$

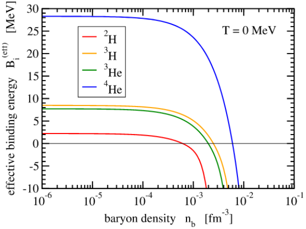
  [S. Typel et al., PRC 81, 015803 (2010)]




- $\Delta m^{(\rm low)}$ from in-medium Schrödinger equation (**SE**)

  [G. Röpke, NPA 867 (2011) 66–80]

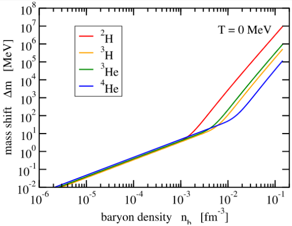
- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\rm c.m.})$ ⇒ **heuristic** $\Delta m^{(\rm high)}$ beyond **Mott density**



# In-medium (Mott) effects and cluster dissolution

- Pauli-blocking **(Mott) effect** $\Rightarrow$ cluster **dissolution**
  - **Microscopic** in-medium effects $\Rightarrow$ **Mass-shift** ($\Delta m$)
  - (Effective) **binding energy** $\rightarrow B^{\text{eff}} = B - \Delta m$

    [S. Typel et al., PRC 81, 015803 (2010)]

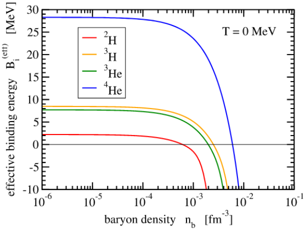



- $\Delta m^{(\text{low})}$ from in-medium Schrödinger equation (**SE**)

  [G. Röpke, NPA 867 (2011) 66–80]

- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\text{c.m.}}) \Rightarrow$ **heuristic** $\Delta m^{(\text{high})}$ beyond **Mott density**
  - Bound clusters survive only if $|\mathbf{P}_{\text{c.m.}}| > \mathbf{P}_{\text{Mott}}$ (Mott momentum)
  - Few-body short-range correlations (SRCs) in the continuum

    [S. Burrello, S. Typel, EPJA 58, 120 (2022)]



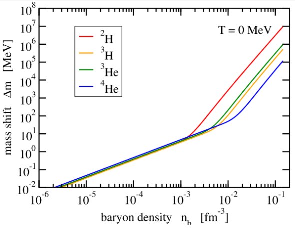
# In-medium (Mott) effects and cluster dissolution

- Pauli-blocking **(Mott) effect** $\Rightarrow$ cluster **dissolution**
  - **Microscopic** in-medium effects $\Rightarrow$ **Mass-shift** ($\Delta m$)
  - (Effective) **binding energy** $\rightarrow B^{\text{eff}} = B - \Delta m$

  [S. Typel et al., PRC 81, 015803 (2010)]

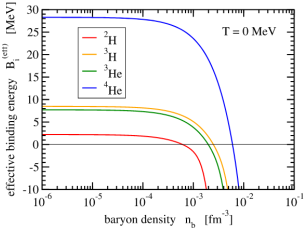
- $\Delta m^{(\text{low})}$ from in-medium Schrödinger equation (**SE**)

  [G. Röpke, NPA 867 (2011) 66–80]




- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\text{c.m.}}) \Rightarrow$ **heuristic** $\Delta m^{(\text{high})}$ beyond **Mott density**
  - **Bound** clusters survive only if $|\mathbf{P}_{\text{c.m.}}| > \mathbf{P}_{\text{Mott}}$ (**Mott momentum**)
  - Few-body short-range correlations (SRCs) in the **continuum**

  [S. Burrello, S. Typel, EPJA 58, 120 (2022)]



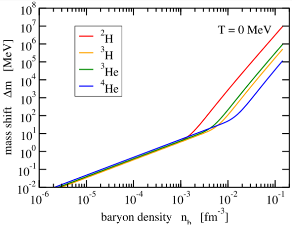
# In-medium (Mott) effects and cluster dissolution

- Pauli-blocking **(Mott) effect** $\Rightarrow$ cluster **dissolution**
  - **Microscopic** in-medium effects $\Rightarrow$ **Mass-shift** $(\Delta m)$
  - (Effective) **binding energy** $\rightarrow B^{\mathrm{eff}} = B - \Delta m$

  [S. Typel et al., PRC 81, 015803 (2010)]

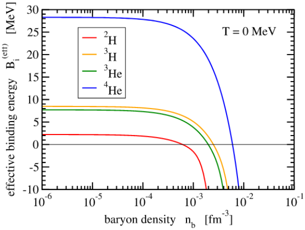
- $\Delta m^{(\mathrm{low})}$ from in-medium Schrödinger equation (**SE**)

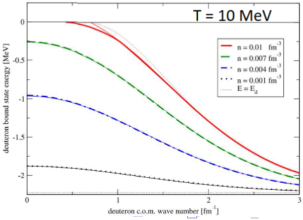
  [G. Röpke, NPA 867 (2011) 66–80]




- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\mathrm{c.m.}}) \Rightarrow$ **heuristic** $\Delta m^{(\mathrm{high})}$ beyond **Mott density**
  - **Bound** clusters survive only if $|\mathbf{P}_{\mathrm{c.m.}}| > \mathbf{P}_{\mathrm{Mott}}$ (**Mott momentum**)
  - Few-body short-range correlations (SRCs) in the **continuum**

  [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

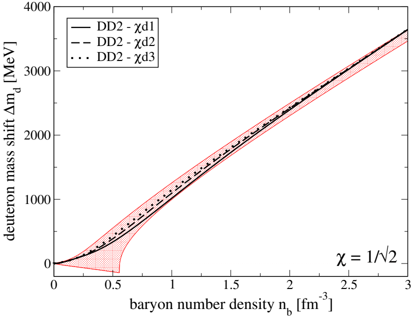


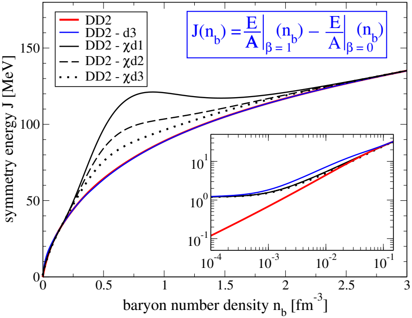



# Clusters as surrogate for SRCs in extended EDFs

- Unified mass-shift **parameterization** for bound $d$ / $np$ SRCs ($\rho_B \lesssim \rho_{\mathrm{Mott}}$)

$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1-\tanh(x)\right] + fx\tanh(gx), \qquad x = \frac{\rho_b}{\rho_0}$$




Eur. Phys. J. A (2022) 58:120
https://doi.org/10.1140/epja/s10050-022-00765-z
THE EUROPEAN PHYSICAL JOURNAL A
Check for updates
Regular Article - Theoretical Physics

**Embedding short-range correlations in relativistic density functionals through quasi-deuterons**

S. Burrello[1,a], S. Typel[1,2,b]

# Outline of the presentation

1. Many-body (MB) correlations and clustering phenomena in nuclear systems
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)

2. **Extended EDF-based models: recent developments and results**
   - ✏ Unified (thermodynamic) description of few-body correlations and clusters
     - Embedding short-range correlations within relativistic mean-field approaches
     - Global mass-shift parameterization for a multi-purposes EOS
   - ✏ **Dynamical approach with light clusters as degrees of freedom (DOF)**
     - Phase-space excluded-volume approach in dilute nuclear medium
     - Quasi-analytical characterization of spinodal instability and growth rates

3. Further developments and outlooks
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent description of fragment formation mechanisms in heavy-ion collisions

4. Summary

# Effective phase-space excluded-volume approach

- Modeling dynamics of HIC at intermediate energies ⇒ **Transport** theories
  - Boltzmann–Uehling–Uhlenbeck (**BUU**) equation for the **distribution function** $f_\tau$

$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \ldots], \qquad \tau = n, p, d, t, h, \alpha$$

- **Extended** kinetic approaches ⇒ Light clusters & in-medium effects
  - Solving **in-medium SE** $\forall$ $(\mathbf{r}, t)$ point (**computationally** very demanding)
  - Effective phase-space excluded-volume approach
    [P. Danielewicz, G. F. Bertsch, NPA 533, 712 (1991)]
    [C. Kuhrts et al., PRC 63, 034605 (2001)]

# Effective phase-space excluded-volume approach

- Modeling **dynamics** of HIC at intermediate energies ⇒ **Transport** theories
  - Boltzmann–Uehling–Uhlenbeck (**BUU**) equation for the **distribution function** $f_\tau$

$$(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}})\, f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \ldots], \qquad \tau = n, p, d, t, h, \alpha$$

- **Extended kinetic** approaches ⇒ Light **clusters** & **in-medium** effects
  - Solving **in-medium SE** ∀ $(\mathbf{r}, t)$ point (**computationally** very demanding)
  - Effective **phase-space excluded-volume approach**
    [P. Danielewicz, G. F. Bertsch, NPA 533, 712 (1991)]
    [C. Kuhrts et al., PRC 63, 034605 (2001)]

$$\langle f_\tau \rangle_\nu(\mathbf{P}) = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3}\, f_\tau^{\mathrm{tot}}(\mathbf{p})\, |\tilde{\phi}_{\nu,\mathbf{P}}(\mathbf{p})|^2 < F_A^{\mathrm{cut}}$$

$\tilde{\phi}_{\nu,\mathbf{P}}$ = free-space 1-body probability (Gaussian) distribution
$f_\tau^{\mathrm{tot}}$ = total phase-space occupation of the medium
(including contributions from light clusters)

# Effective phase-space excluded-volume approach

- Modeling **dynamics** of HIC at intermediate energies ⇒ **Transport** theories
  - Boltzmann–Uehling–Uhlenbeck (**BUU**) equation for the **distribution function** $f_\tau$

$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \ldots], \qquad \tau = n, p, d, t, h, \alpha$$

- **Extended kinetic** approaches ⇒ Light **clusters** & **in-medium** effects
  - Solving **in-medium SE** ∀ (**r**, $t$) point (**computationally** very demanding)
  - Effective **phase-space excluded-volume approach**

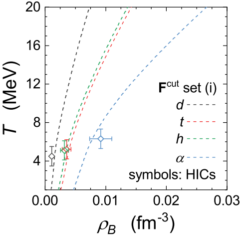
[P. Danielewicz, G. F. Bertsch, NPA 533, 712 (1991)]

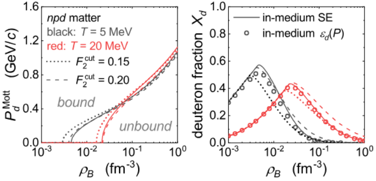
[C. Kuhrts et al., PRC 63, 034605 (2001)]

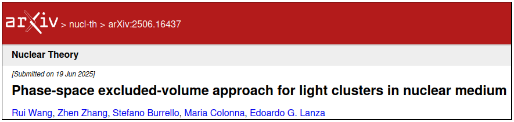
$$\langle f_\tau \rangle_\nu(\mathbf{P}) \equiv \int \frac{\mathrm{d}\mathbf{p}}{(2\pi\hbar)^3} f_\tau^{\mathrm{tot}}(\mathbf{p}) |\tilde{\phi}_{\nu,\mathbf{P}}(\mathbf{p})|^2 < F_A^{\mathrm{cut}}$$

$\tilde{\phi}_{\nu,\mathbf{P}} \equiv$ **free-space** 1-body probability (**Gaussian**) distribution

$f_\tau^{\mathrm{tot}} \equiv$ **total** phase-space **occupation** of the medium (including contributions from light **clusters**)





arXiv > nucl-th > arXiv:2506.16437

Nuclear Theory

[Submitted on 19 Jun 2025]

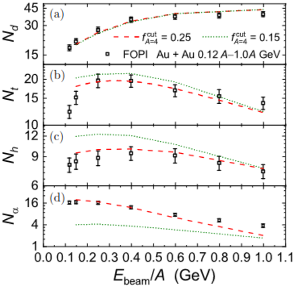
**Phase-space excluded-volume approach for light clusters in nuclear medium**

Rui Wang, Zhen Zhang, Stefano Burrello, Maria Colonna, Edoardo G. Lanza

# Kinetic approach for HIC with light-clusters DOF

- Integrating **phase-space excluded-volume** in **transport** models ⇒ **cut-off** in $I_\tau^{\mathrm{coll}}$
  ⇒ **Description** of cluster **yields** from **FOPI** collaboration for $A \leq 4$

[R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, Phys. Rev. C 108, L031601 (2023)]




- Kinetic approach for HIC at intermediate energies
  - No consistent light & heavier fragments production

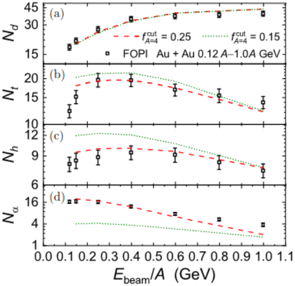
- Spinodal instability ⇒ Mean-field (Vlasov) dynamics

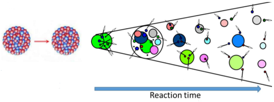
$$(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}})\, f_\tau = 0$$

# Kinetic approach for HIC with light-clusters DOF

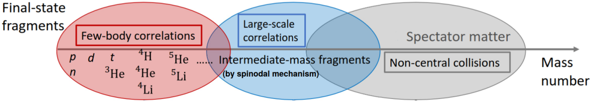
- Integrating **phase-space excluded-volume** in **transport** models ⇒ **cut-off** in $I_\tau^{\mathrm{coll}}$
  ⇒ **Description** of cluster **yields** from **FOPI** collaboration for $A \leq 4$

[R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, Phys. Rev. C 108, L031601 (2023)]



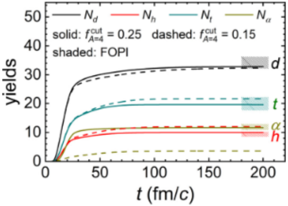




- **Kinetic** approach for HIC at intermediate energies
  - No **consistent light** & heavier **fragments** production




- **Spinodal** instability ⇒ **Mean-field** (**Vlasov**) dynamics

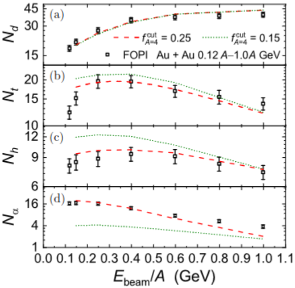
$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = 0$$

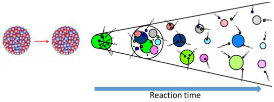


# Kinetic approach for HIC with light-clusters DOF

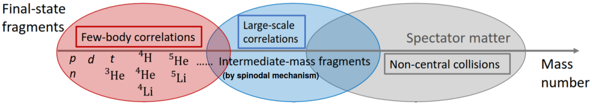
- Integrating **phase-space excluded-volume** in **transport** models $\Rightarrow$ **cut-off** in $I_\tau^{\rm coll}$
  $\Rightarrow$ **Description** of cluster **yields** from **FOPI** collaboration for $A \leq 4$

[R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, Phys. Rev. C 108, L031601 (2023)]



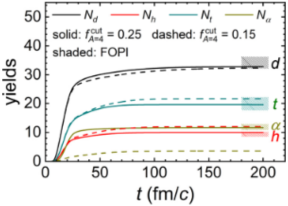




- **Kinetic** approach for HIC at intermediate energies
  - No **consistent light** & heavier **fragments** production




- **Spinodal** instability $\Rightarrow$ **Mean-field** (**Vlasov**) dynamics

$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = 0$$




**Our goal**

Assess if light **clusters** (from **compression** phase) affect **spinodal** instability (**expansion** stage)

# Density-dependent (Mott) momentum cut-off

PHYSICAL REVIEW C **110**, L031601 (2024)

Letter | Editors' Suggestion

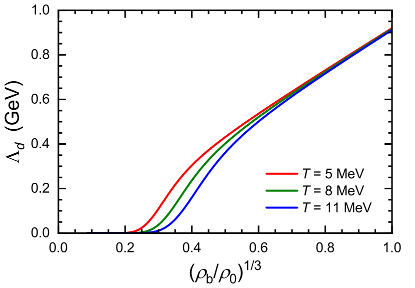
**Dynamics of dilute nuclear matter with light clusters and in-medium effects**

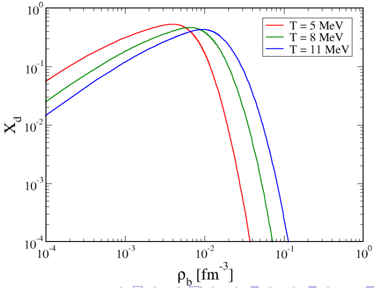
Rui Wang[1,*] Stefano Burrello[2,†] Maria Colonna[2,‡] and Francesco Matera[3,§]

- **Pauli-blocking** $\Rightarrow$ Cut-off (Mott) momentum $P_j^{\mathrm{Mott}} \equiv \Lambda_j(\rho_b, T)$ **parameterization**

$$\rho_j = g_j \int_{|\mathbf{p}|>\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3} f_j \qquad j = n, p, d \qquad (\Lambda_q = 0, \text{ for } q = n, p)$$

- Chemical equilibrium $\Rightarrow X_d = \dfrac{A_d \rho_d}{\rho_0}$ consistent with **microscopic** calculations



# Linearized Vlasov equations for NM+deuterons

- **Linear** response to collision-less Boltzmann ⇒ linearized **Vlasov** equations for NMd

$$\partial_t\left(\delta f_j\right)+\nabla_{\mathbf{r}}(\delta f_j)\cdot\nabla_{\mathbf{p}}\varepsilon_j-\nabla_{\mathbf{p}} f_j\cdot\nabla_{\mathbf{r}}(\delta\varepsilon_j)=0 \quad \Rightarrow \quad \delta\rho_j=-\chi_j\sum_l\left(F_0^{jl}+\tilde{F}_\lambda^{jl}\right)\delta\rho_l-\delta_{jd}\sum_l\Phi_\lambda^{dl}\delta\rho_l$$

- **Single-particle** energy $\varepsilon_j\equiv\dfrac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (from EDF $\mathcal{E}=\mathcal{K}+\mathcal{U}$)

$$\varepsilon_j=\frac{p^2}{2m_j}+U_j+\tilde{\varepsilon}_j^\lambda \qquad \left(\tilde{\varepsilon}_j^\lambda\propto\Phi_\lambda^{dj}\sim\frac{\partial\Lambda_d}{\partial\rho_j}\right)$$

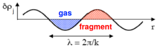
- Momentum-independent **Skyrme**-like interaction (= for bound and free nucleons)

$$\mathcal{U}=\frac{A}{2}\frac{\rho_b^2}{\rho_0}+\frac{B}{\alpha+2}\frac{\rho_b^{\alpha+2}}{\rho_0^{\alpha+1}}+\frac{C(\rho)}{2}\frac{\rho_3^2}{\rho_0}+\frac{D}{2}(\nabla_{\mathbf{r}}\rho_b)^2-\frac{D_3}{2}(\nabla_{\mathbf{r}}\rho_3)^2$$

- **Density-dependent** (Mott) momentum **cut-off** ⇒ extra-terms in both $\delta\rho_j$ and $\varepsilon_j$

$$\rho_j=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_j \quad j=n,p,d \quad \rightarrow \quad \delta\rho_j(\mathbf{r},t)=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j-\delta_{jd}\sum_{l=n,p,d}\Phi_\lambda^{dl}\delta\rho_l$$

- $\Phi_\lambda^{dl}\neq 0 \Rightarrow$ adding **in-medium** effects for cluster appearance/dissolution in dynamics
- **Landau** procedure $\left(F_0^{jl}\sim\dfrac{\partial U_j}{\partial\rho_l},\tilde{F}_\lambda^{jl}\sim\dfrac{\partial\tilde{\varepsilon}_j^\lambda}{\partial\rho_l}\right)$ for $\delta f_j\sim\sum_{\mathbf{k}}\delta f_j^{\mathbf{k}}\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

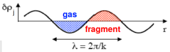


# Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations $\Rightarrow$ **dispersion relation** $\omega = \omega(k)$

$$\delta\rho_j = -\chi_j \sum_l \left( F_0^{jl} + \tilde{F}_\lambda^{jl} \right) \delta\rho_l - \delta_{jd} \sum_l \Phi_\lambda^{dl} \delta\rho_l$$

- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ **unstable** mode (**spinodal region**)




[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]

- $\omega = 0$ $(\chi_j = 1) \Rightarrow$ **border**
- $\mathrm{Im}(\omega) \Rightarrow$ **growth rate**

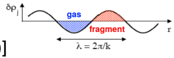
# Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations $\Rightarrow$ **dispersion relation** $\omega = \omega(k)$

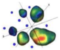
$$\delta\rho_j = -\chi_j \sum_l \left(F_0^{jl} + \tilde{F}_\lambda^{jl}\right) \delta\rho_l - \delta_{jd} \sum_l \Phi_\lambda^{dl} \delta\rho_l$$

- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ **unstable** mode (**spinodal region**)

[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]




$\longrightarrow$



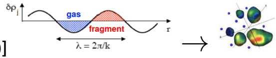
- $\omega = 0$ $(\chi_j = 1) \Rightarrow$ border
- $\mathrm{Im}(\omega) \Rightarrow$ growth rate

# Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations ⇒ **dispersion relation** $\omega = \omega(k)$

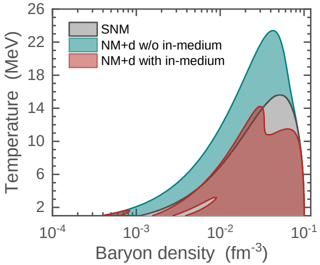
$$\delta\rho_j = -\chi_j \sum_l \left( F_0^{jl} + \tilde{F}_\lambda^{jl} \right) \delta\rho_l - \delta_{jd} \sum_l \Phi_\lambda^{dl} \delta\rho_l$$

- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ **unstable** mode (**spinodal region**)




[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]

- $\omega = 0$ $(\chi_j = 1)$ ⇒ **border**
- $\mathrm{Im}(\omega)$ ⇒ growth rate



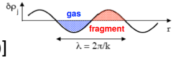

**Legend**
- full
- $\Phi_\lambda^{dl} = \tilde{F}_\lambda^{jl} = 0$

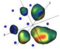
# Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations ⇒ **dispersion relation** $\omega = \omega(k)$

$$\delta\rho_j = -\chi_j \sum_l \left(F_0^{jl} + \tilde{F}_\lambda^{jl}\right)\delta\rho_l - \delta_{jd}\sum_l \Phi_\lambda^{dl}\delta\rho_l$$

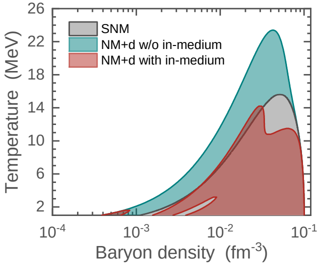
- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ **unstable** mode (**spinodal region**)

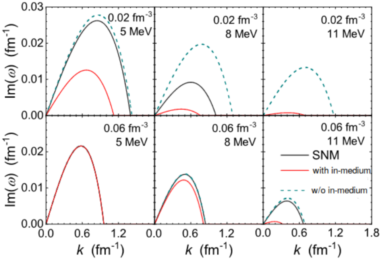


→


[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]

- $\omega = 0$ $(\chi_j = 1) \Rightarrow$ **border**
- $\mathrm{Im}(\omega) \Rightarrow$ **growth rate**



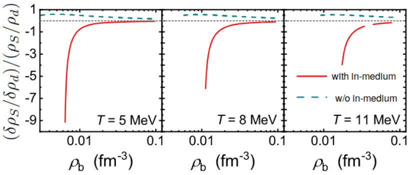

**Legend**
- full
- $\Phi_\lambda^{dl} = \tilde{F}_\lambda^{jl} = 0$

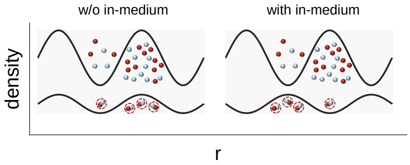
**In-medium effects in dynamics**

- **Slowdown** of instability rate & different **fragmentation** modes

# Instability direction: "distillation" mechanism

- **Direction of instability** in space of density fluctuations: $\frac{\delta\rho_S}{\delta\rho_d}$ $(\rho_S = \rho_n + \rho_p)$
  - $\frac{\delta\rho_S}{\delta\rho_d} \gtrless 0 \Rightarrow$ **Nucleons** and **deuterons** fluctuations move in (out) of phase




[R. Wang, S. Burrello, M. Colonna, F. Matera, PRC 110, L031601 (2024)]

- **NMd with no in-medium effects:**
  - Favored growth of instabilities
  - **Cooperation** to form fragments
- **NMd with in-medium effects:**
  - Deuterons move to **low densities**
  - They might be separately emitted
    $\Rightarrow$ **"distillation" mechanism**

# Outline of the presentation

1. Many-body (MB) correlations and clustering phenomena in nuclear systems
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)
2. Extended EDF-based models: recent developments and results
   - Unified (thermodynamic) description of few-body correlations and clusters
   - Dynamical approach with light clusters as degrees of freedom (DOF)
3. **Further developments and outlooks**
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent description of fragment formation mechanisms in heavy-ion collisions
4. Summary

# Further developments and outlooks

- **Hydrodynamics** vs **linearized Vlasov** approach

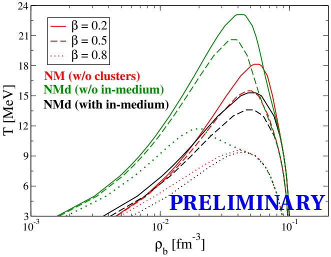
  [S. Burrello et al., in preparation]

  - Interplay ($d$ + $\alpha$) & different **cut-off**

    [Carmelo Piazza's Master's Thesis work]

  - **Microscopic** description from **in-medium SE**

    [Pablo Nieto Gallego's Master's Thesis work]



# Further developments and outlooks

- **Hydrodynamics** vs **linearized Vlasov** approach

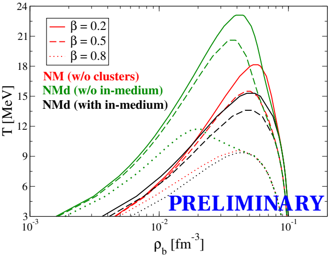
  [S. Burrello et al., in preparation]

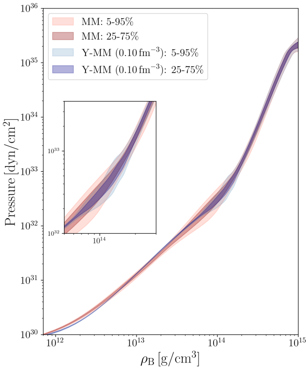
  - Interplay ($d$ + $\alpha$) & different **cut-off**

  [Carmelo Piazza's Master's Thesis work]

  - **Microscopic** description from **in-medium SE**

  [Pablo Nieto Gallego's Master's Thesis work]




## Work in progress

- **Implementation** within **transport** simulations
- **Connection** with **Bayesian** methods

PHYSICAL REVIEW C **112**, 035802 (2025)

**Bayesian inference of neutron star crust properties using an *ab-initio*–benchmarked metamodel**

S. Burrello[1,*] F. Gulminelli[2] M. Antonelli[2] M. Colonna[1] and A. F. Fantina[3]
[1]*INFN, Laboratori Nazionali del Sud, I-95123 Catania, Italy*
[2]*Université de Caen Normandie, ENSICAEN, CNRS/IN2P3, LPC Caen UMR6534, 14000 Caen, France*
[3]*Grand Accélérateur National d'Ions Lourds (GANIL), CEA/DRF - CNRS/IN2P3, Boulevard Henri Becquerel, 14076 Caen, France*

(Received 6 June 2025; revised 23 July 2025; accepted 8 August 2025; published 5 September 2025)

# Outline of the presentation

1. Many-body (MB) correlations and clustering phenomena in nuclear systems
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)
2. Extended EDF-based models: recent developments and results
   - Unified (thermodynamic) description of few-body correlations and clusters
   - Dynamical approach with light clusters as degrees of freedom (DOF)
3. Further developments and outlooks
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent description of fragment formation mechanisms in heavy-ion collisions
4. **Summary**

# Final remarks and conclusions

## Main topic

- Description of **correlations** & clustering with phenomenological **EDF** models
- **Dynamics** of dilute NM with light **clusters** DOF and local **in-medium** effects

## Main results

- Unified **mass-shift** parametrization for deuterons & **SRCs** and impact on EOS
- **Validation** of **phase-space excluded-volume** approach against in-medium SE
- Role of clusters on SNM **spinodal** instability and fragmentation dynamics

## Further developments and outlooks

- Screening effects for **bound** nucleons and connection with **hydrodynamics**
- Extension to ANM with other light clusters and **cut-off** parameterizations
- Numerical calculations & **consistent** description of **HIC** fragment formation

# Final remarks and conclusions

## Main topic

- Description of **correlations** & clustering with phenomenological **EDF** models
- **Dynamics** of dilute NM with light **clusters** DOF and local **in-medium** effects

## Main results

- Unified **mass-shift** parametrization for deuterons & **SRCs** and impact on EOS
- **Validation** of **phase-space excluded-volume** approach against in-medium SE
- Role of clusters on SNM **spinodal** instability and fragmentation dynamics

## Further developments and outlooks

- Screening effects for **bound** nucleons and connection with **hydrodynamics**
- Extension to ANM with other light clusters and **cut-off** parameterizations
- Numerical calculations & **consistent** description of **HIC** fragment formation

THANK YOU FOR YOUR ATTENTION!