RIBDCE – Few-body systems (this talk: $^3\mathrm{H}(p, n)^3\mathrm{He}$ reaction . . .)

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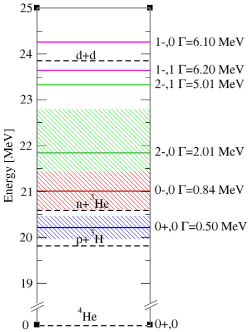
INFN
Istituto Nazionale di Fisica Nucleare

Outline

Collaborators

- A. Kievsky, L.E. Marcucci, A. Grassi (Post-doc) & M. Sagina (PhD student) *INFN-Pisa & Pisa University, Pisa (Italy)*
- L. Girlanda & Y. Capitani (Post-doc) *University of Salento & INFN-Lecce, Lecce (Italy)*
- E. Filandri (Post-doc) *ECT*, Trento (Italy)*
- A. Gnech *ODU, Norfolk & JeffersonLab, Newport News (USA)*
- V. Barlucchi (PhD student) *Mainz U., Mainz (Germany) & Pisa University, Pisa (Italy)*

The ^{4}He spectrum



[Tilley, Weller, & Hale, 1992]

No true excited states – only resonances
1st and 2nd excited states "narrow" resonances

Interactions

- NN potentials
  - N3LO500, N3LO600 [Entem & Machleidt, 2003, 2011]
  - LO500-N4LO500 [Nosyk, Entem & Machleidt, 2017]
  - NVIa & NVIb [Piarulli *et al.*, 2018]
- + accompanying 3N potential at N2LO [Epelbaum *et al.*, 2002]

Numerical techniques for $A = 4$ for scattering

- Faddeev-Yakubovsky methods [Lazauskas & Carbonell, 2004], [Deltuva & Fonseca, 2007]
- Expansion on a basis: NCSM [Quaglioni, Navratil & Roth, 2010], Gaussians [Aoyama *et al.*, 2011], R-matrix [Descouvemont & Baye, 2010], HH [Kievsky, Marcucci, MV, *et al.*, 2008], . . .

HH method for continuum states

$AB \to AB + CD + \cdots$ process

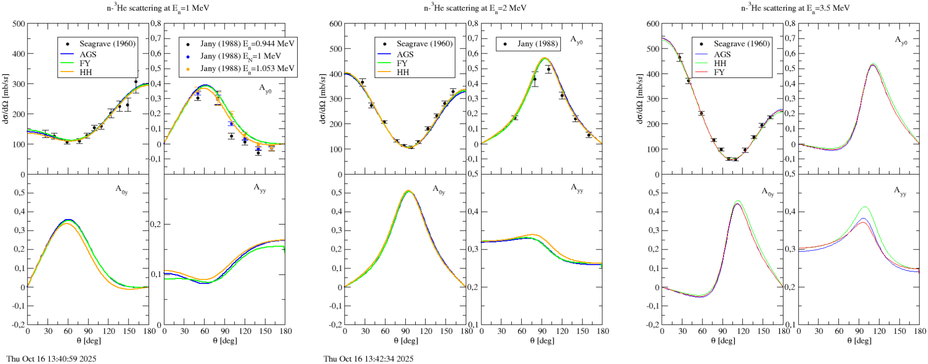
$$\Omega^{\pm}_{AB,LS}(q_{AB}) = \sqrt{\frac{1}{N}} \sum_{perm.=1}^{N} \Big[Y_L(\hat{\mathbf{y}}_p) \otimes [\phi_A \otimes \phi_B]_S \Big]_{JJ_z} \left(f_L(y_p) \frac{G_L(\eta, q_{AB} y_p)}{q_{AB} y_p} \pm \mathrm{i} \frac{F_L(\eta, q_{AB} y_p)}{q_{AB} y_p} \right)$$

$$|\Psi^{JJ_z}_{AB,LS}\rangle = \sum_{n,[K]} a_{AB,LS,[K]} |n,[K]\rangle + |\Omega^{-}_{AB,LS}(q_{AB})\rangle - \sum_{L'S'} \mathcal{S}^{(AB,AB),J}_{LS,L'S'} |\Omega^{+}_{AB,L'S'}(q_{AB})\rangle - \sum_{L'S'} \mathcal{S}^{(AB,CD),J}_{LS,L'S'} |\Omega^{+}_{CD,L'S'}(q_{CD})\rangle - \cdots$$

- $|n,[K]\rangle$ HH states – essentially, homegeneous polynomials of degree K
- Asymptotically $\Omega^{\pm}_{AB,LS}(q) \sim e^{\pm iqy}$
- $\mathcal{S}^{(AB,CD),J}_{LS,L'S'}$ = S-matrix ($T = (\mathcal{S} - I)/2i$)
- $a_{AB,LS,[K]}$, $\mathcal{S}^{(AB,AB),J}_{LS,L'S'}$, $\mathcal{S}^{(AB,CD),J}_{LS,L'S'}$, ... computed using the Kohn variational principle
- For more details, see [MV *et al.* PRC **35**, 063101 (2020)]

Benchmark test of 4N scattering calculations - $n - {}^3\mathrm{He}$ scattering

N3LO500 potential – ^{3}He$(n, n)^3$He elastic scattering

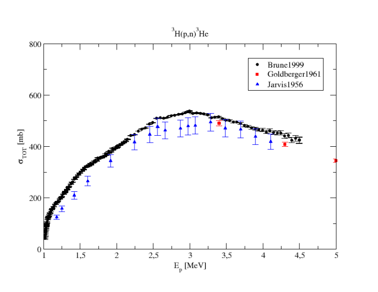


AGS= Deltuva & Fonseca – FY= Lazauskas & Carbonell – HH= present work

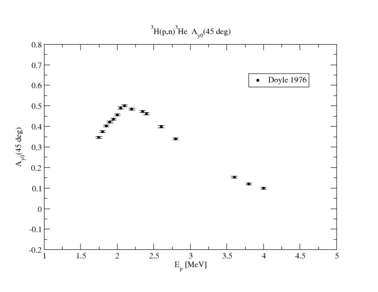
The $^3\mathrm{H}(p,n)^3\mathrm{He}$ reaction

Previous experimental studies
$^3\mathrm{H}(p,n)^3\mathrm{He}$ most data taken before 1980; $p(^3\mathrm{H},{}^3\mathrm{He})n$ a few data (1978-1985)
$^3\mathrm{He}(n,p)^3\mathrm{H}$ most for low energy 2020 LNS + 2024 ORNL (cold neutrons)
In this work, we are interested in the analysis of these data:

Total cross section



Analyzing power at $\theta_{CM} = 45$ deg



Detailed study in the past [MV *et al.*, 2020]: Interactions: N3LO500+3N & N3LO600+3N (2003)
Recently, we started new calculations using LO500-N4LO500 interactions [Nosyk, Entem & Machleidt, 2017] and NVIa & NVIb [Piarulli *et al.*, 2018] still in progress
Other calculations [Deltuva, 2007] no 3NF

T-matrix elements

$$T^{(AB,CD),J}_{LS,L'S'} = \left(\mathcal{S}^{(AB,CD),J}_{LS,L'S'} - I\right)/2i$$ We are interested in $T^{(pt,nh),J}_{LS,L'S'}$

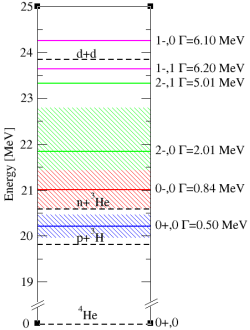
J^π	$^{2S+1}L_J$
0^+	$^1S_0$
0^-	$^3P_0$
1^+	$^3S_1, ^3D_1$
1^-	$^1P_1, ^3P_1$
2^+	$^1D_2, ^3D_2$
2^-	$^3P_2, ^3F_2$
3^+	$^3D_3, ^3G_3$
3^-	$^1F_3, ^3F_3$

Up to the $d-d$ threshold ($E_p \approx 5.4$ MeV)

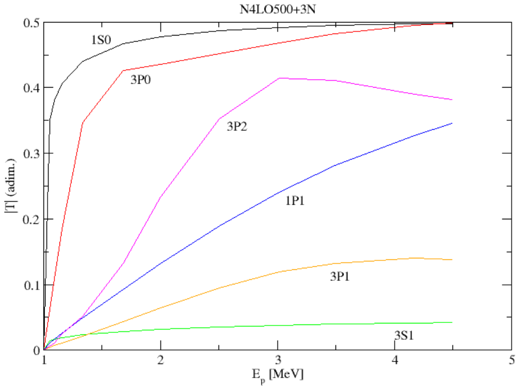
$$\mathcal{S} = \begin{pmatrix} \mathcal{S}^{pt,pt} & \mathcal{S}^{pt,nh} \\ \mathcal{S}^{nh,pt} & \mathcal{S}^{nh,nh} \end{pmatrix}$$

In the talk, we focus on the reaction $^3\mathrm{H}(p,n)^3\mathrm{He}$
threshold in the CM: $E_p + E_{^3\mathrm{H}} > \Delta B \approx 0.75$ MeV
threshold in the LAB: $E_p > 4/3\Delta B \approx 1$ MeV

T-matrix elements (2)

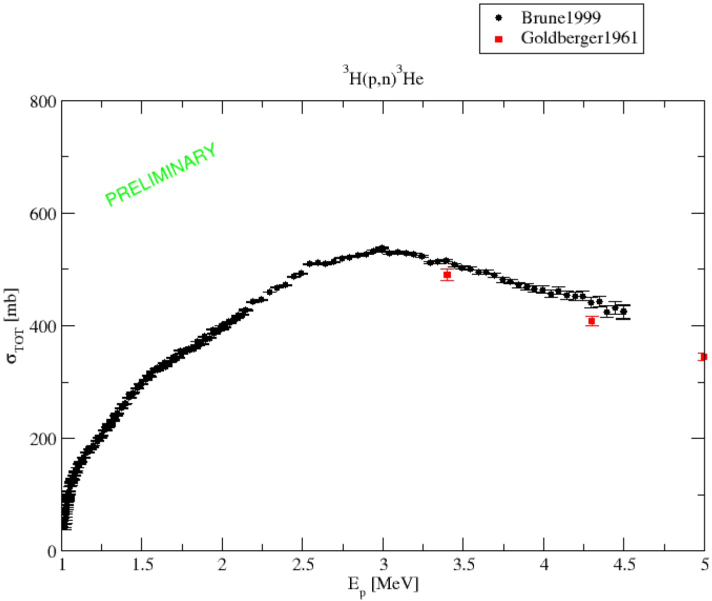


[Tilley, Weller, & Hale, 1992]



Note: if $|T^{(pt,nh),J}_{LS,L'S'}| = 1/2 \Rightarrow |\mathcal{S}^{(pt,nh),J}_{LS,L'S'}| = 1$

Total cross section



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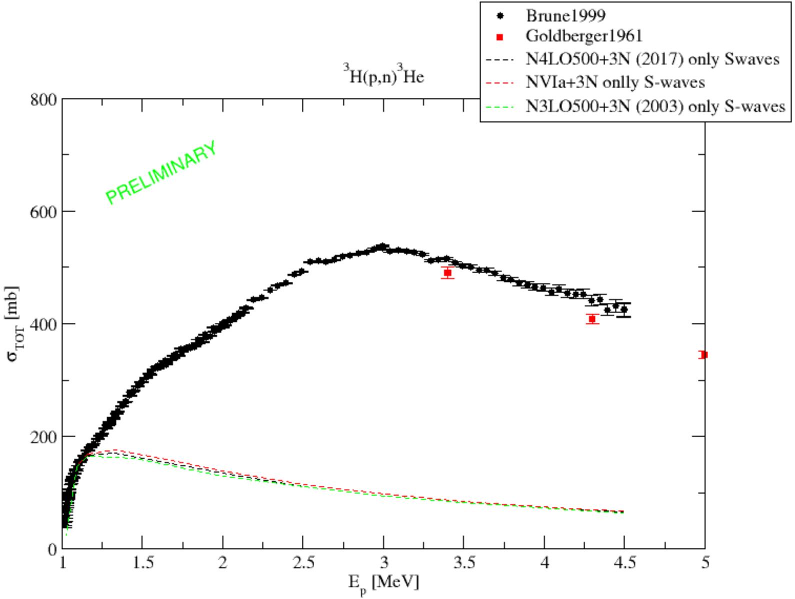

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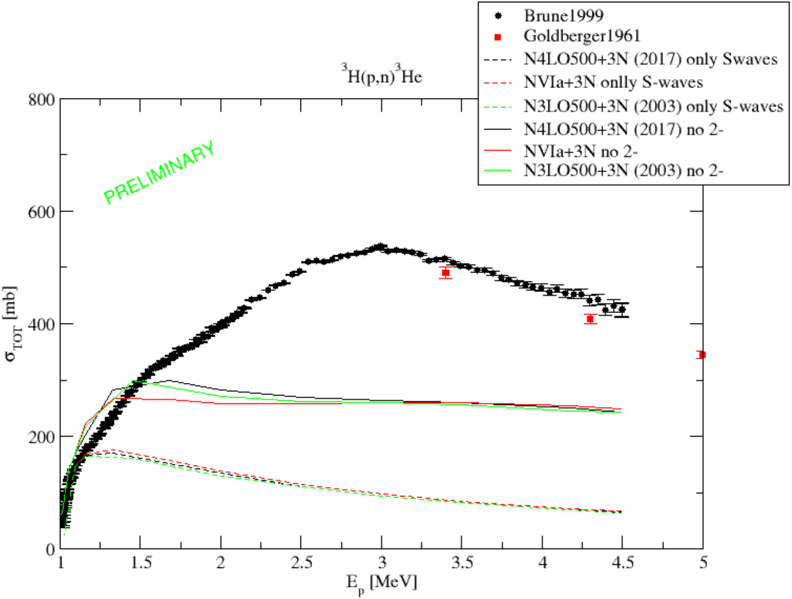

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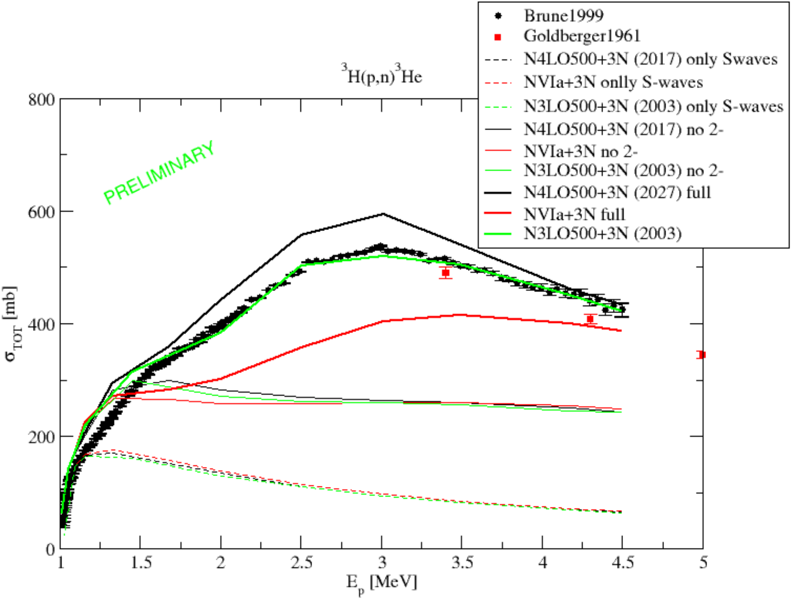
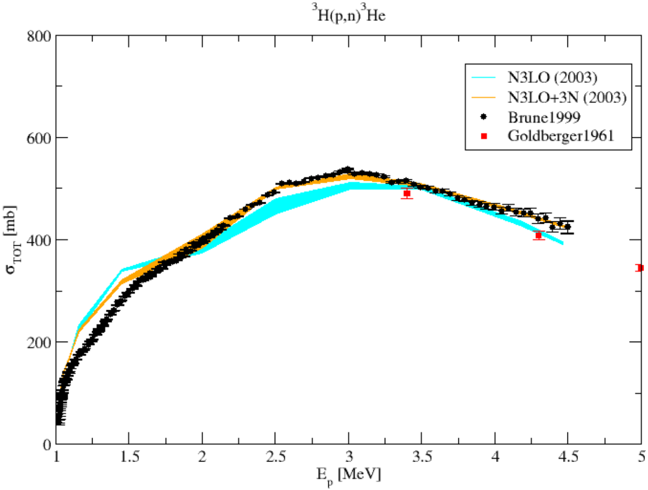


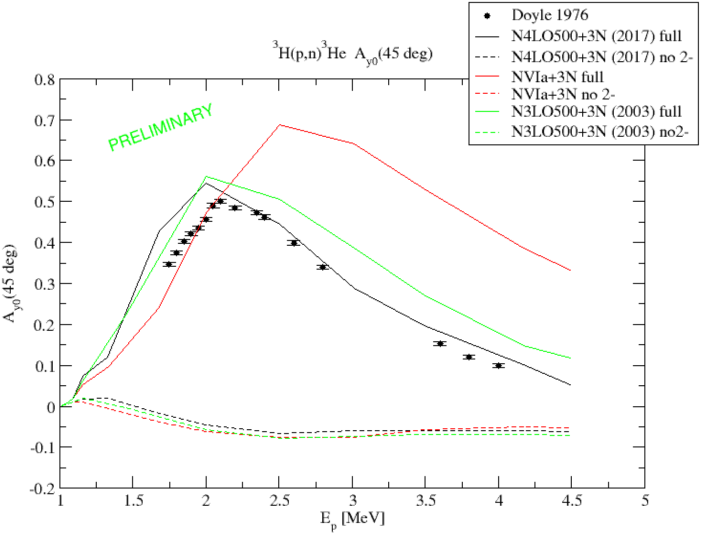
Total cross section – effect of 3NF



Cyan band:
N3LO500 &
N3LO600

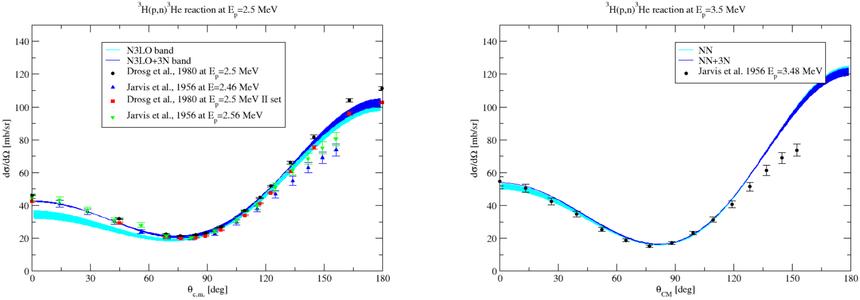
Orange band
N3LO500+3N &
N3LO600+3N

Analyzing power at 45 deg



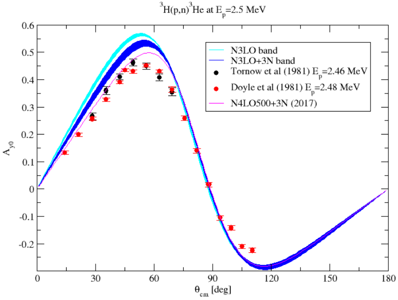
With only S-waves $A_{y0}(\theta) = 0$

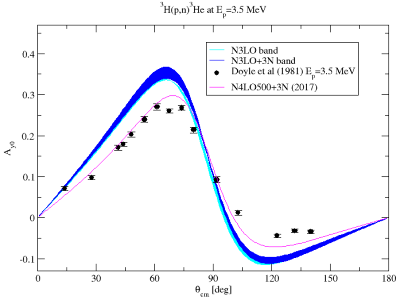
Differential unpolarized cross section



Cyan band: N3LO500 & N3LO600 – Blue band N3LO500+3N & N3LO600+3N
Most of the data at variance from each other

Analyzing power





Cyan band: N3LO500 & N3LO600 – Blue band N3LO500+3N & N3LO600+3N

With only S-waves $A_{y0}(\theta) = 0$

Conclusions

Summary of this study of the $^3\mathrm{H}(p, n)^3\mathrm{He}$ reaction

- Accurate data for σ_{TOT} [Brune *et al.*, 1999] (older data at variance with these data)
- Effect of 0^- and 2^- $^4\mathrm{He}$ resonances clearly visible
- A lot of data for $d\sigma/d\Omega$
  - at variance between each other
  - at variance with Brune's data
- Good data for A_{y0}
- Great sensitivity of the interaction models – optimal to excite resonances!

Future work

- Complete the calculations with the LO500→N4LO500 chiral interactions . . .
- . . . and analyze also low energy data (BBN)
- Improved models of 3N force [Girlanda *et al.*, 2023]
- $^3\mathrm{He}(n, p)^3\mathrm{H}$ at BBN energies [Pizzone *et al.*, 2020]

→ extension to double-exchange reactions! $^3\mathrm{He}(^3\mathrm{H}, 3p)3n$

The $^3\mathrm{He}(^3\mathrm{H}, 3p)3n$ reaction

- $^3\mathrm{H}$, $^3\mathrm{He}$ bound states, no problem
- $3n$ system studied in "Nonresonant density of states enhancement at low energies for three or four neutrons" [Higgins *et al.*, 2020]
- $3p$ system studied in "Convergence of the *ppp* correlation function within the hyperspherical adiabatic basis" [Garrido *et al.*, 2025]
- (3α system, see Elena Filandri's talk)
- Difficulty: six-body problem!!! We will try . . .

Thank you for your attention!

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