Generalized Symmetries
and
Particle Physics

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KAIST

COST "COSMIC WISPers" Colloquium

Opening Remarks

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Symmetry: most essential and powerful concept in the pursuit of fundamental physics

Philip W. Anderson (Nobel Prize in Physics in 1977)

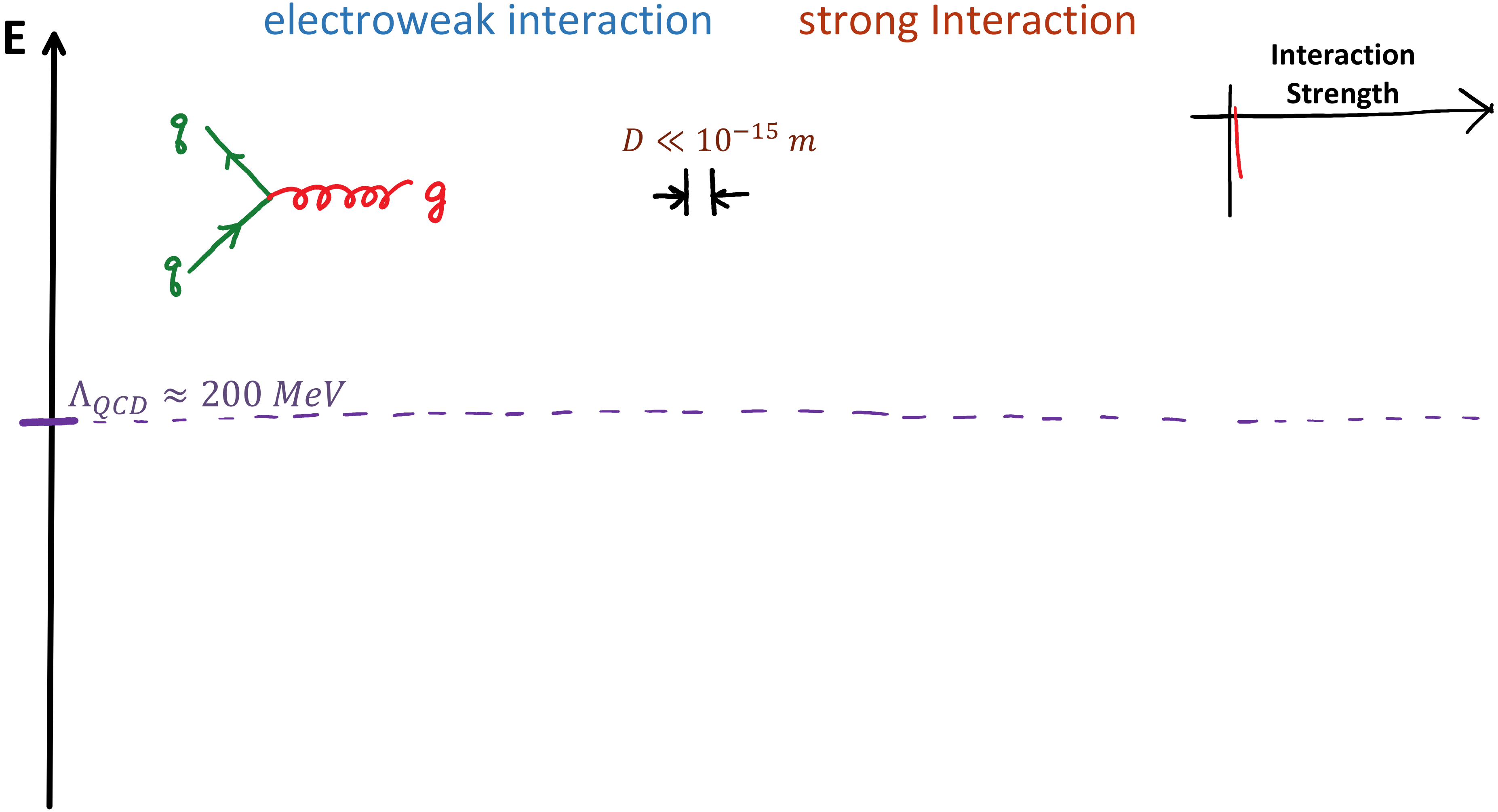
In 1972 article [More is Different]:

"It is only slightly overstating the case to say that physics is the study of symmetry"

Power of **Symmetries**

- In Standard Model (SM) $\supset$ strong interaction

$$\underbrace{\gamma,\quad W^{\pm},\quad Z,}_{\text{electroweak interaction}} \qquad \underbrace{gluon(g^{a=1,\dots,8})}_{\text{strong Interaction}}$$



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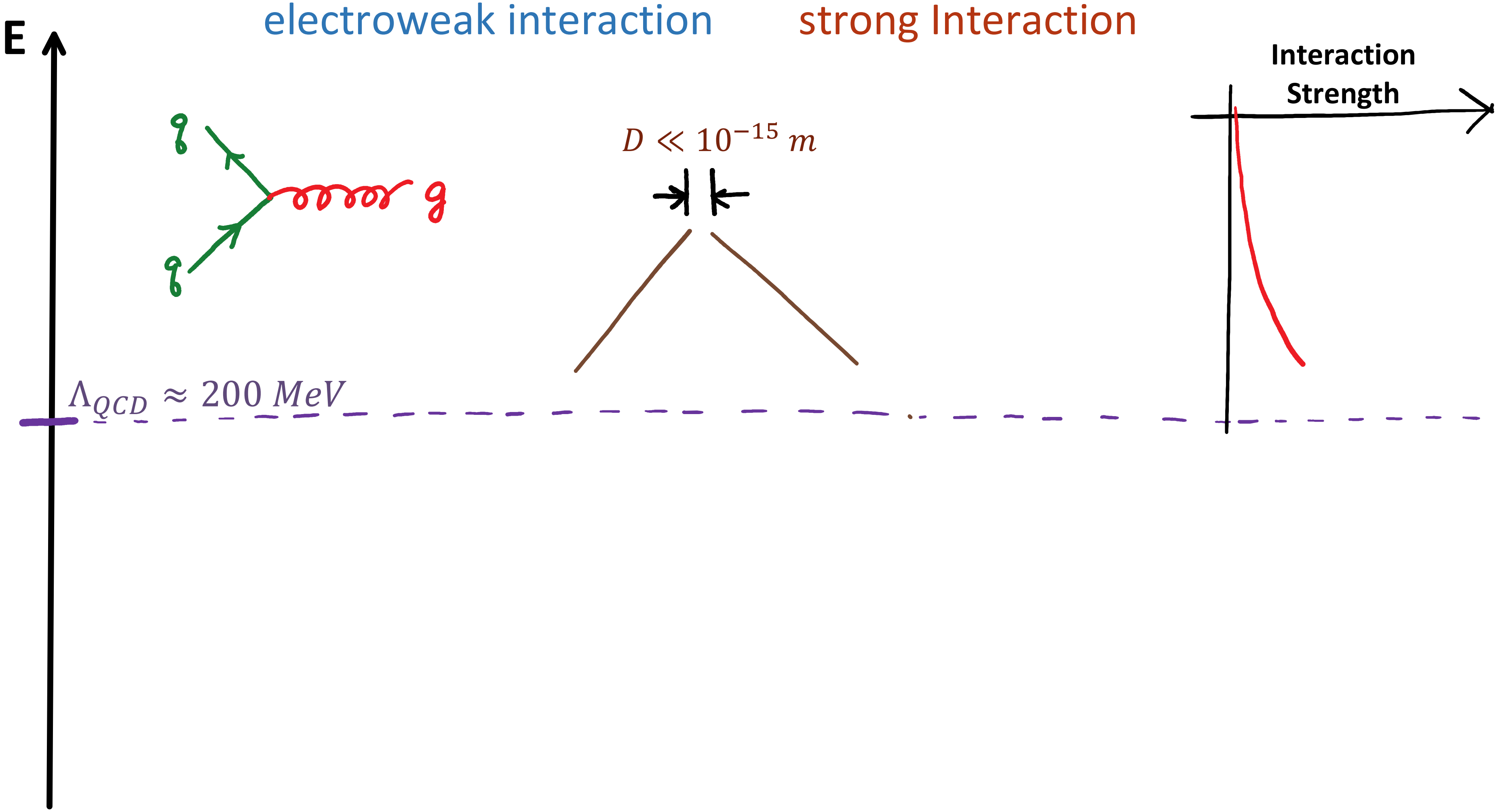

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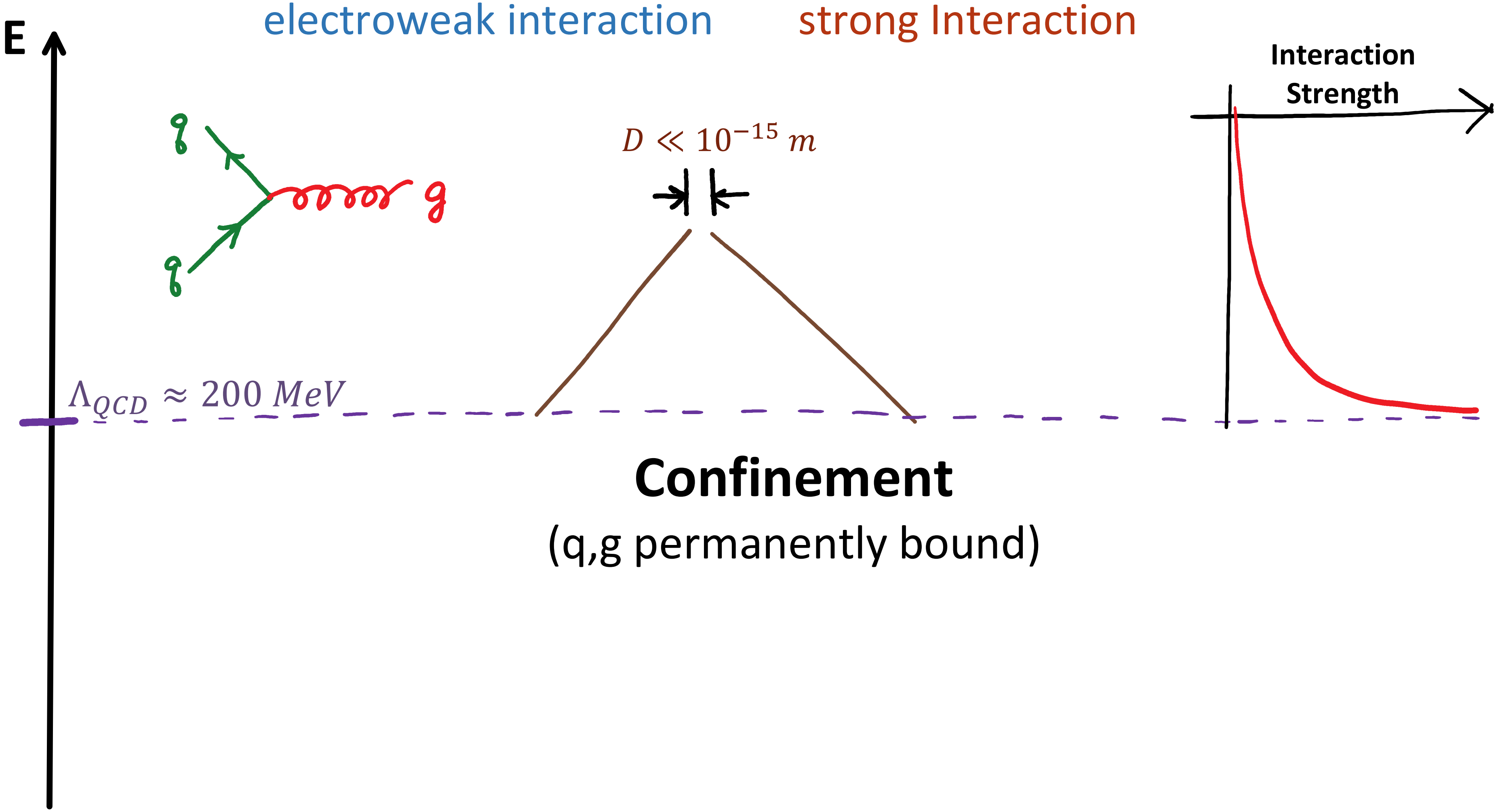

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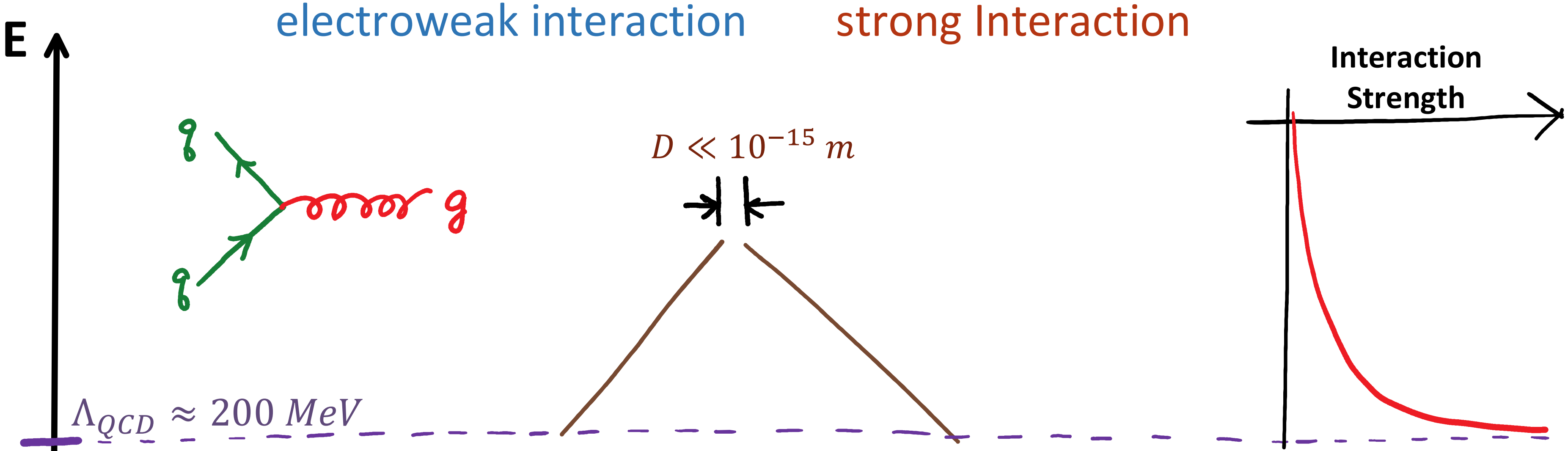

Confinement

(q,g permanently bound)

Composite Particles

(Mesons, Hadrons)

- In Standa

CMI

ABOUT PROGRAMS PEOPLE MILLENNIUM PROBLEMS PUBLICATIONS EVENTS NEWS

Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

E

g

g

$\Lambda_{QCD} \approx$

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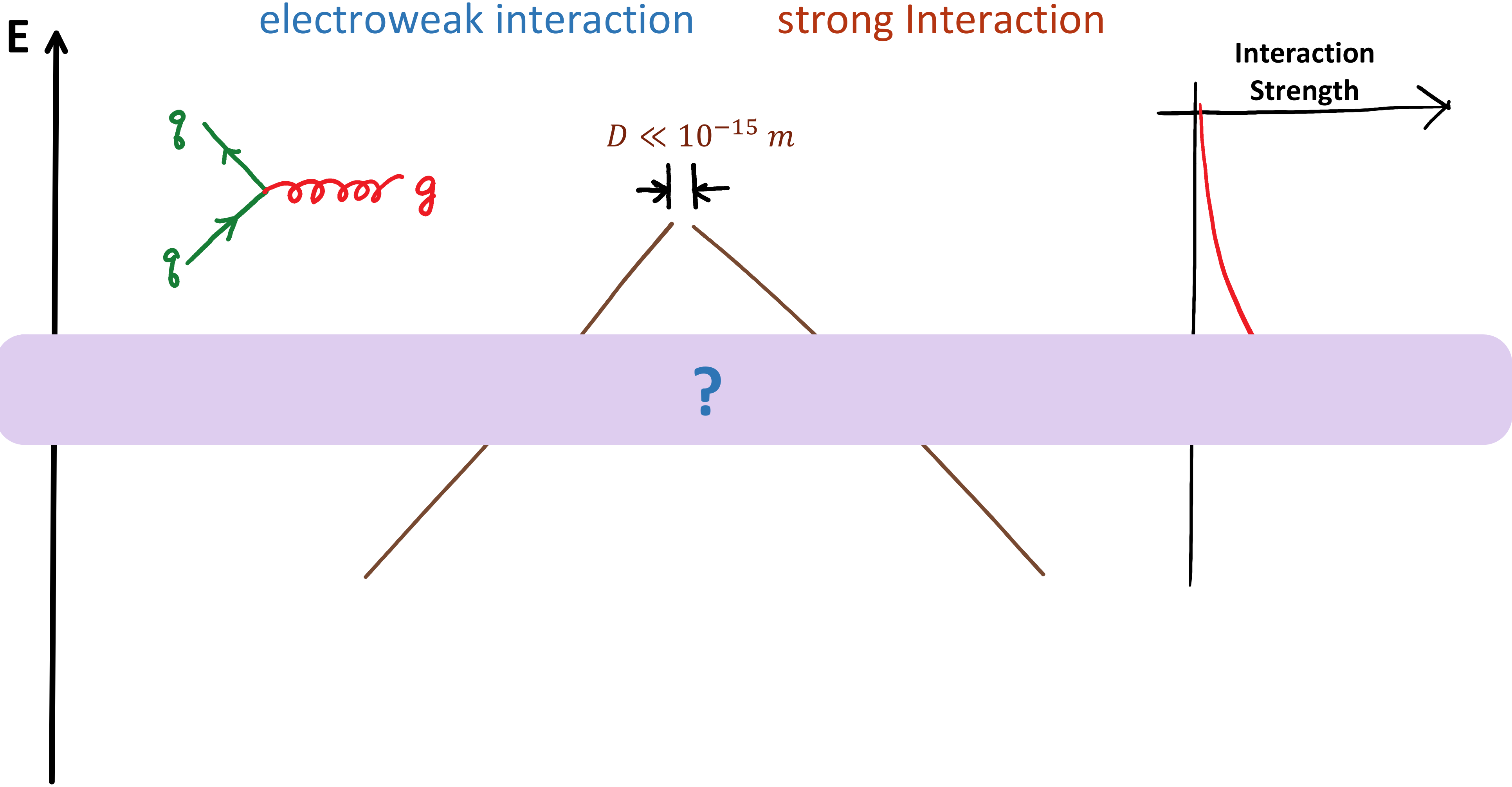

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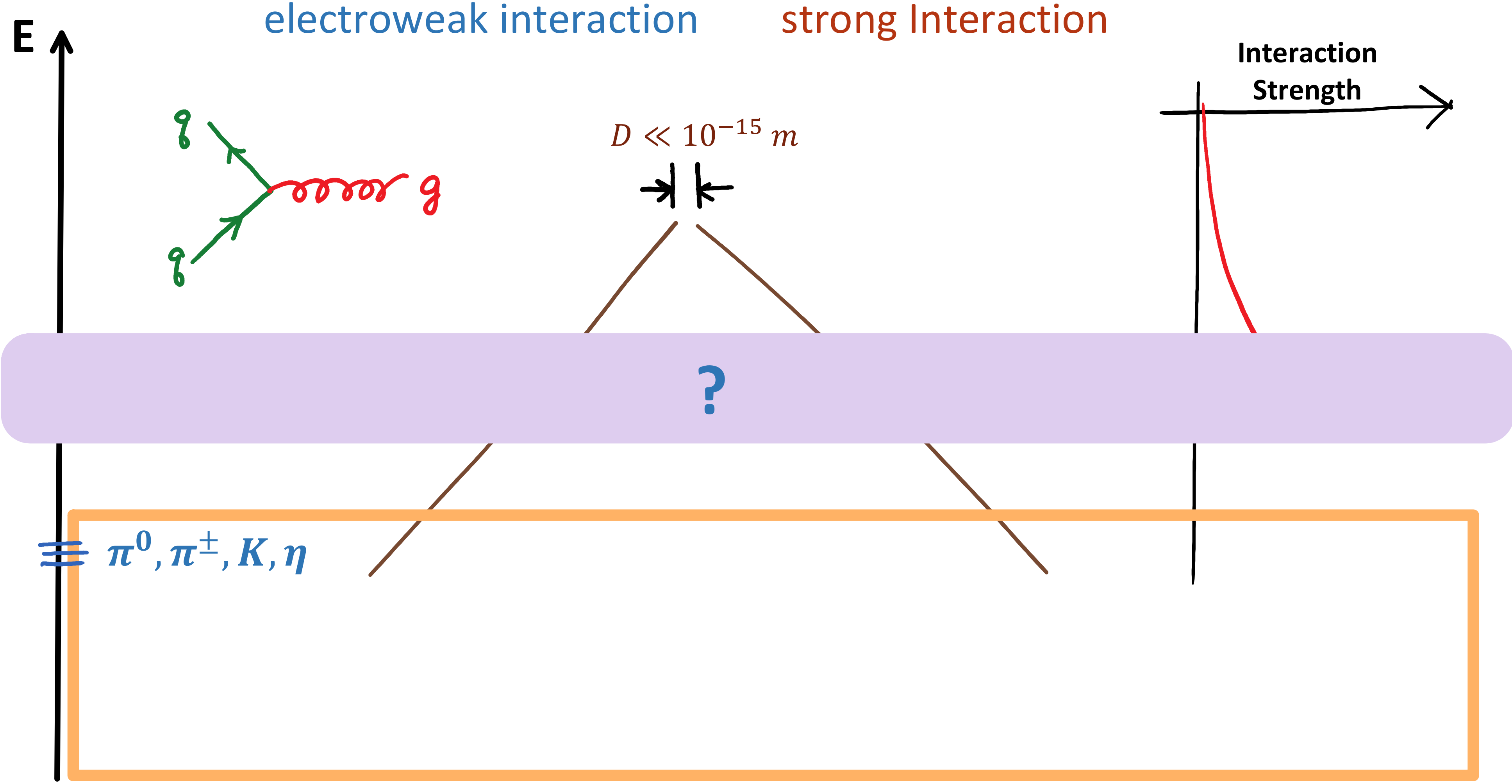

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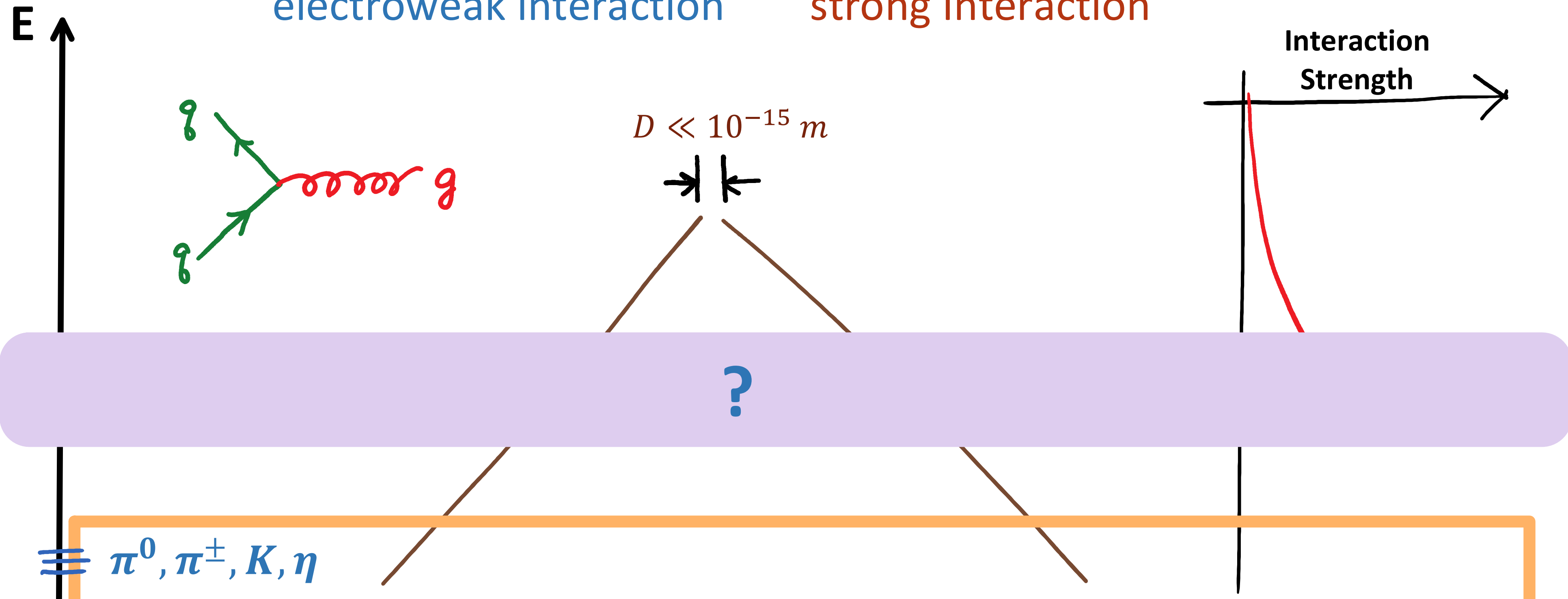

Naively, it is impossible to understand meson physics
without first solving "confinement" problem

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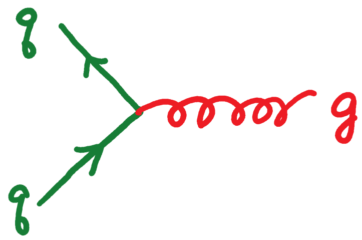

[Global symmetry: $\boldsymbol{G = SU(3)_L \times SU(3)_R}$]

$$\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \to \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \to \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

?

$\boldsymbol{\pi^0, \pi^{\pm}, K, \eta}$

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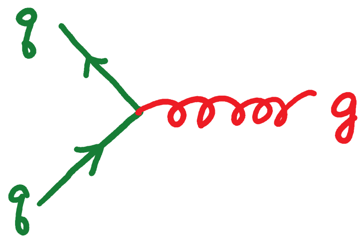

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$\equiv \boldsymbol{\pi^0, \pi^{\pm}, K, \eta}$

[Experimental observations]

physics of mesons (π, K, η) respects only

$$\boldsymbol{H = SU(3)_V}$$

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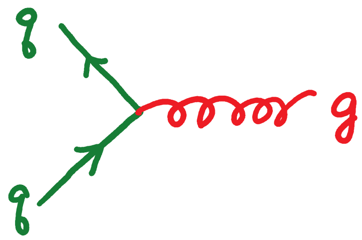

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QCD ground state

$\boldsymbol{\pi^0, \pi^{\pm}, K, \eta}$

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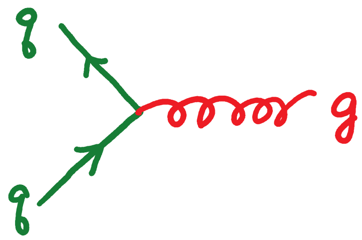

[Global symmetry: $\boldsymbol{G = SU(3)_L \times SU(3)_R}$]

$\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \to \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$ and $\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \to \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$

SSB

$\equiv$ $\boldsymbol{\pi^0, \pi^{\pm}, K, \eta}$

[Experimental observations]

$$\mathcal{L} \sim \frac{f^2}{4} Tr\left(\partial_\mu U^\dagger \partial^\mu U\right) + \cdots, U = e^{i\lambda \cdot \frac{\Pi}{f}}$$

$$\Pi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

physics of mesons (π, K, η) respects only

$\boldsymbol{H = SU(3)_V}$

CCWZ (Callan, Coleman, Wess, Zumino 1969)
→ Weinberg 1979

Power of Symmetries

Missing Properties of Symmetry?

Despite spectacular successes of Chiral Perturbation Theory....

(i) Repeat the exercise with $U(1)_L \times U(1)_R \to U(1)_B$

$\Rightarrow$ expect another light meson (η'), but not found?

(ii) meansured $\Gamma(\pi^0 \to \gamma\gamma) \gg$ theoretical estimation?

(iii) CP violation: $KK \to \pi\pi\pi$ observed but it can not happen in ChPT?

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Anomalous Symmetry

Adler-Bell-Jackiw (ABJ) anomaly, 't Hooft anomaly
RG-invariance of anomaly (anomaly matching)

Opening Remarks

Naturalness Problems and Global Symmetries

1. Electroweak Hierarch Problem

$$\left(\frac{\text{Gravity}}{\text{weak}}\right) \sim \left(\frac{v}{M_{pl}}\right)^2 \sim \left(\frac{100\ GeV}{10^{19}\ GeV}\right)^2 \sim 10^{-34} \ll 1$$

A source of challenge: **no apparent symmetry** acting on (generic) **scalar $\boldsymbol{\Phi}$**

Exception-1) Shift symmetry: Higgs = PNGB $\Rightarrow$ Composite Higgs / Little Higgs

Exception-2) Chiral symmetry (scalar $\leftrightarrow$ fermion): SUSY $\Rightarrow$ (N)MSSM

In these cases, hierarchy problem becomes **Technical Naturalness Problem.**

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Naturalness Problems and Global Symmetries

2. Strong CP Problem

$$\tilde{J} = \mathrm{Im}\,\det[y_u^\dagger y_u, y_d^\dagger y_d\,] \propto \sin\delta_{CKM} \sim O(1) \quad \text{vs} \quad \bar{\theta} \equiv \arg e^{-i\theta}\det(y_u y_d) \ll 1$$

"Jarlskog invariant"

source of challenge 1: **no clean symmetry structure**

CP (=T), Anomalous $U(1)_{PQ}$, flavor symmetry, …
renormalization of $\bar{\theta}$ from other CPV sources

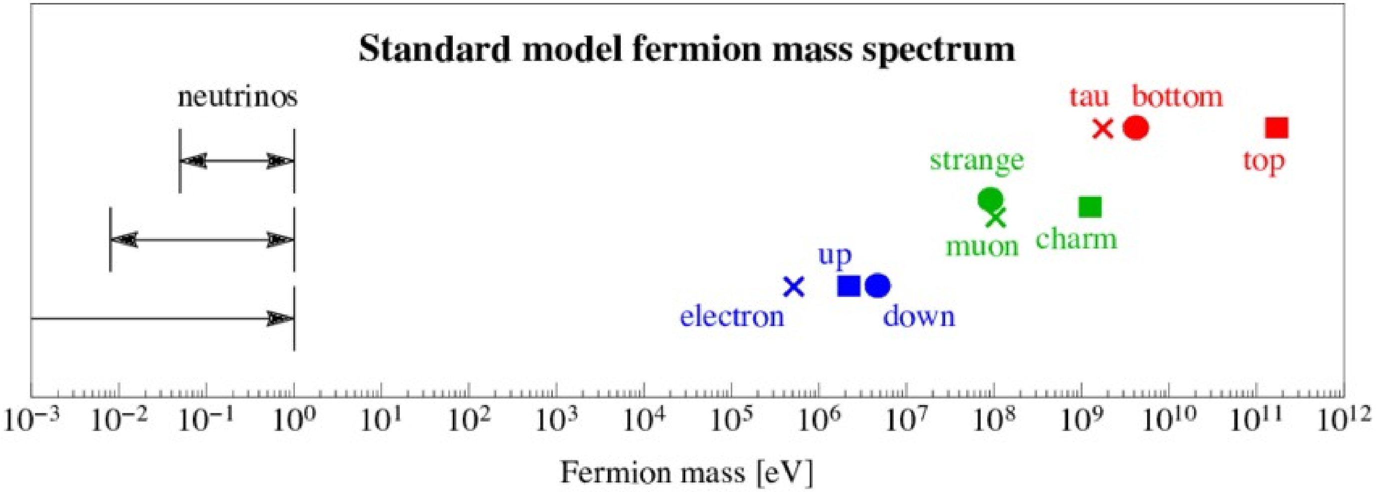
source of challenge 2: the limit $\bar{\theta} \to 0$ does not enhance the symmetry of QFT

Strong CP problem = Dirac Naturalness Problem

Opening Remarks

Naturalness Problems and Global Symmetries

3. Flavor Problem [e.g. m_ν]



https://www.researchgate.net/figure/Mass-spectrum-of-standard-model-fermions-Charged-leptons-up-type-quarks-and-downtype_fig1_361578459

$$M_\nu \sim 10^{-2}\ eV$$

Requires Dynamical Explanation!

Several attractive theories exist.

(1) Seesaw models based on $U(1)_L$
(2) Extradimension, clockwork: localization
(3) Radiatively generated m_ν

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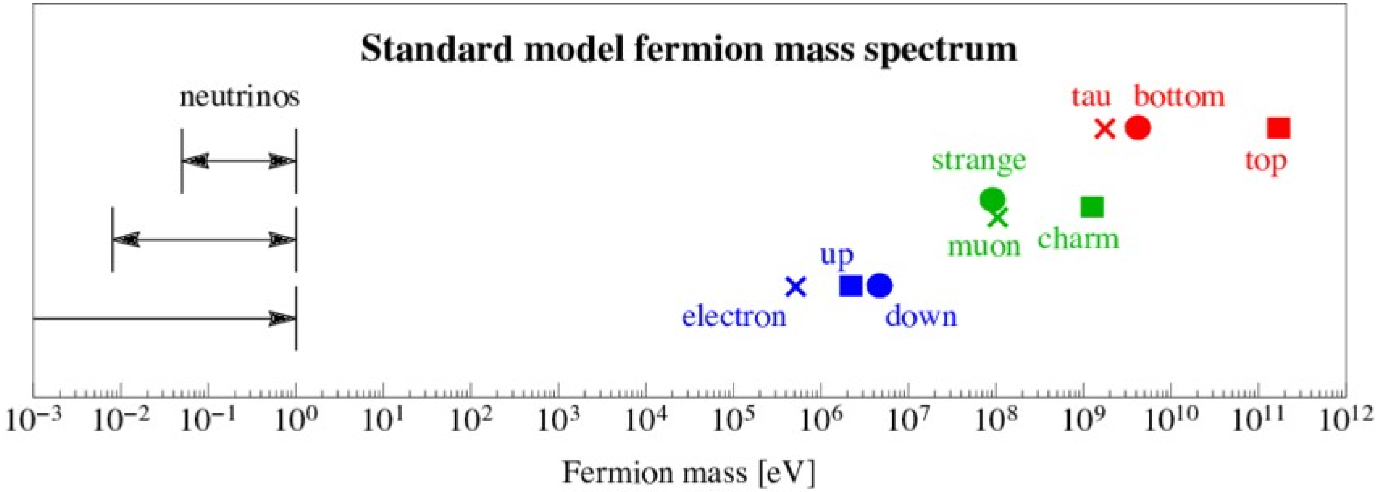

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A source of challenge: ultimate mechanism still to be confirmed.

=> more feasible, testable, and motivating **theoretical ideas** should be laid out.

Opening Remarks

Global Structure of G_{SM} and Global vs Gauge Symmetries (?)

All existing observables of SM probe and are consistent with Lie algebra

$$g_{SM} = su(3) \times su(2) \times u(1)$$

(1) ∃ Ambiguity in the global structure of gauge group

$$G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}, \qquad \Gamma = \mathbb{Z}_6, \mathbb{Z}_3, \mathbb{Z}_2, \mathbb{I}$$

(2) As I will discuss soon, at deeper level this is a question about the global symmetry.

(3) This is not at all an "academic interest", but probably answers to this question may provide the best test and probe of short-distance (UV) fate of our universe (SM).

Opening Remarks

Topological Defects and Global Symmetries

Topological defects are quite ubiquitous in theories in particle physics.

(1) Topological defects from SSB (either global or gauge): $G \to H \subset G$

i. Domain Wall: $\Pi_0(G/H) \neq 0$
ii. Cosmic String (Vortex): $\Pi_1(G/H) \neq 0$
iii. Monopole: $\Pi_2(G/H) \neq 0$
iv. Various hybrid defects: $G \to H_1 \to H_2 \to \cdots$

(2) These can be produced in the early universe, and possibly at colliders.

(3) Topological defects provide additional probes of (B)SM theories.

(4) I will show later that they are essential/only probes of Generalized Symmetries.

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Particle Physics and Global Symmetries so far ...

Philip W. Anderson (Nobel Prize in Physics in 1977)

In 1972 article [More is Different]:

"It is only slightly overstating the case to say that physics is the study of symmetry"

(1) Naturalness

(2) Global Structure ambiguity of G_{SM}

(3) Topological defects

(4) Strong dynamics and QCD Confinement

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Somewhat disparate types and contexts of symmetries.

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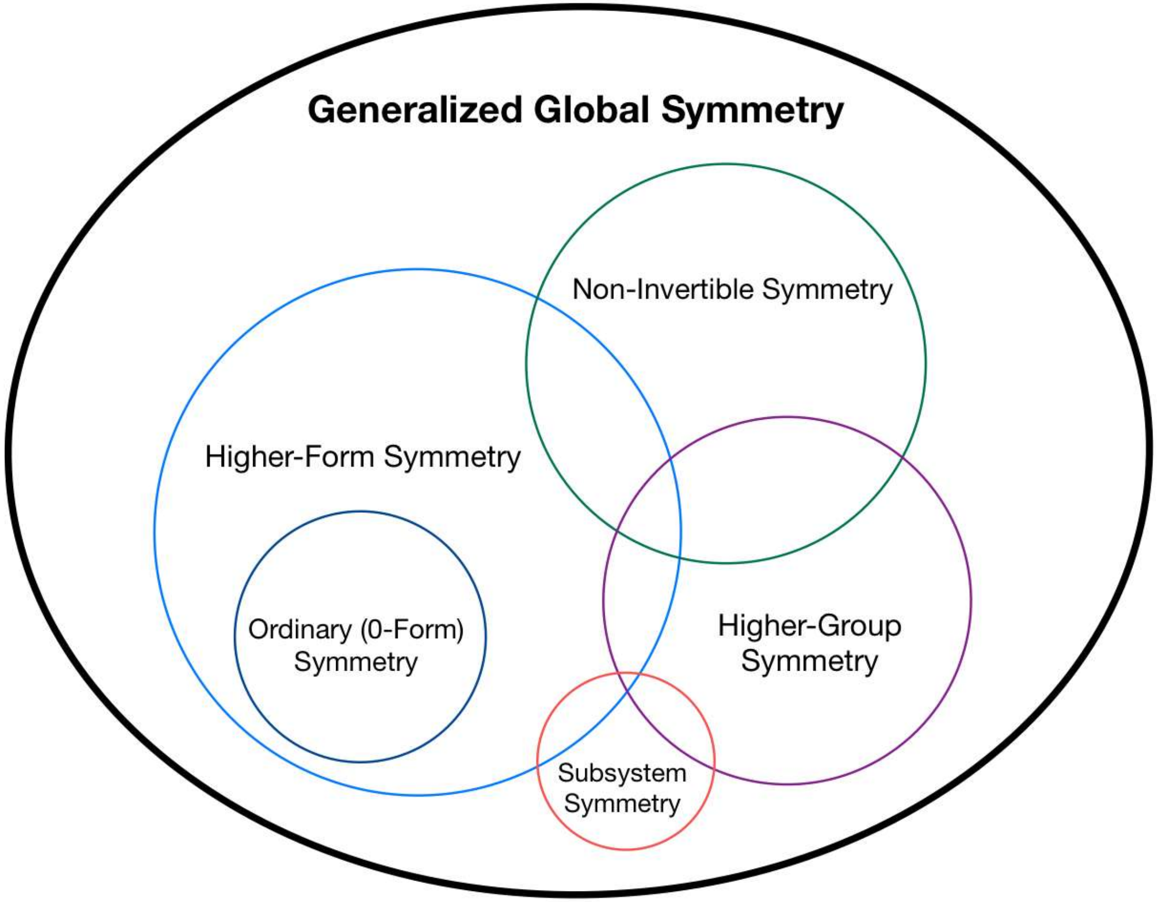
Generalized Global Symmetries



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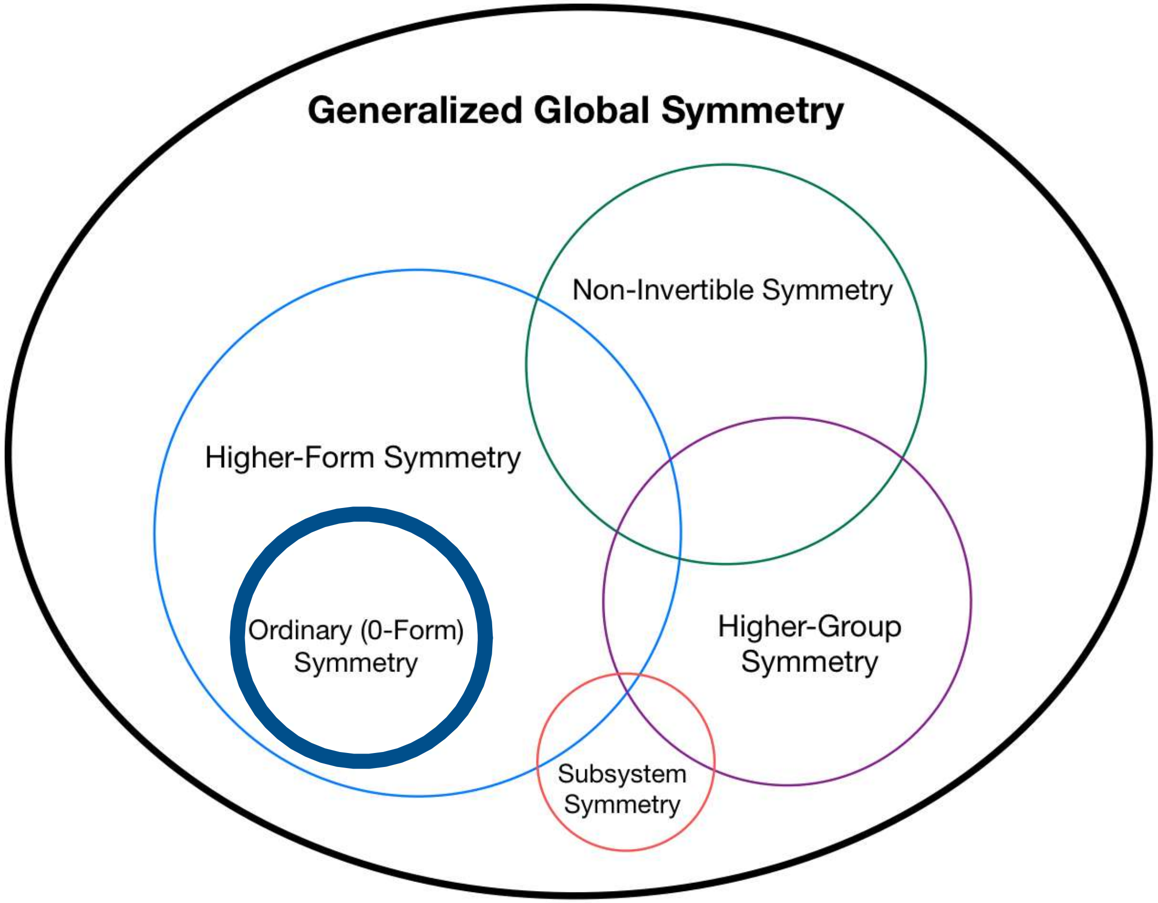

Opening Remarks

Generalized Global Symmetries in Particle Physics

1. Well-motivated and timely to think about new ideas and breakthroughs that Generalized Global Symmetry can provide.
2. Potentially, it can provide a unified framework in which the followings can be organized or appear simultaneously.
    i. Naturalness
    ii. Global Structure ambiguity of G_{SM}
    iii. Topological defects
    iv. Strong dynamics and QCD Confinement
3. Generalized Symmetry may provide new rules for effective field theory, e.g. new spurion analysis, stronger hints to UV from IR

Opening Remarks

Generalized Global Symmetries in Particle Physics

0. Noninvertible Chiral Symmetry and Exponential Hierarchies '22 (C. Cordova, K. Ohmori)
   Noninvertible Global Symmetries in the Standard Model '22 (Y. Choi, H.T. Lam, S.-H Shao)

1. Neutrino Masses from Generalized Symmetry Breaking '22 (C. Cordova, **SH**, S. Koren, K. Ohmori)

2. Higher Flavor Symmetries in the Standard Model '22 (C. Cordova, S. Koren)

3. Coupling a Cosmic String to a TQFT '23 (T.D. Brennan, **SH**, LT Wang)
   Quantization of Axion-Gauge Couplings and Non-Invertible Higher Symmetries '23 (Y. Choi, M. Forslund, H. T. Lam, S-H. Shao)
   Axion-Gauge Coupling Quantization with a Twist '23 (M. Reece)
   Axion Domain Walls, Small Instantons, and Non-Invertible Symmetry Breaking '23 (C. Cordova, **SH**, L. Wang)
   Axion Couplings in Heterotic String Theory '24 (P. Agrawal, M. Nee, M Reig)

4. Non-invertible Peccei-Quinn Symmetry and the Massless Quark Solution to Strong CP Problem '24 (C. Cordova, **SH**, S. Koren)
   Spontaneously Broken (-1)-Form U(1) Symmetry '24 (D. Aloni, E. Garcia-Valdecasas, M. Reece, M. Suzuki)
   High-Quality Axions from Higher-Form Symmetries in Extra Dimensions '24 (N. Craig, M. Kongsore)

5. Nonperturbative effects in the Standard Model with gauged 1-form symmetry '21 (M. Anber, E. Popptiz)
   Fractional-charge hadrons and leptons to tell the Standard Model group apart '24 (R. Alonso, D. Dimakou, M. West)
   The Standard Model Gauge Group, SMEFT, and Generalized Symmetries '24 (H-L. Li, L-X. Xu)

6. A New Solution to the Callan-Rubakov Effect '23 (T. D. Brennan)
   Monopoles, Scattering, and Generalized Symmetries '23 (M. Beest, P. B. Smith, D. Delmastro, Z. Komargodski, D. Tong)
   Fermion-Monopole Scattering in the Standard Model '23 (M. Beest, P. B. Smith, D. Delmastro, R. Mouland, D. Tong)

Outline

I. Briefly on Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Non-invertible Naturalness

II-1. Non-invertible leptonic symmetry and m_ν
II-2. Non-invertible PQ symmetry and strong CP problem
II-3. Other Naturalness Problems

III. UV fate of the universe: Global Structure ambiguity

III-1. Global Structure and global symmetries
III-2. Global Structure from axion non-invertible symmetries
III-3. Global Structure from fractionally charged particle search

IV. Outlook

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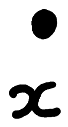
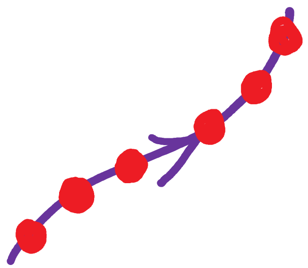
Higher-form symmetries

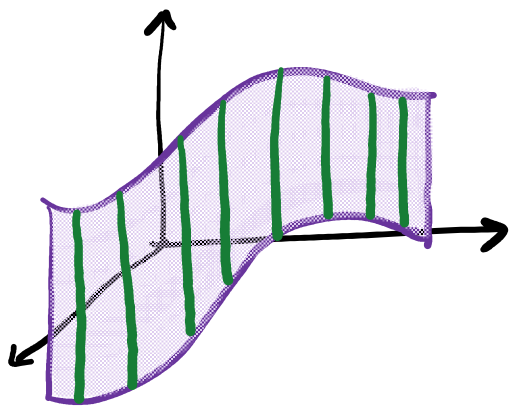
Various **extended objects** appear in broad class of theories.



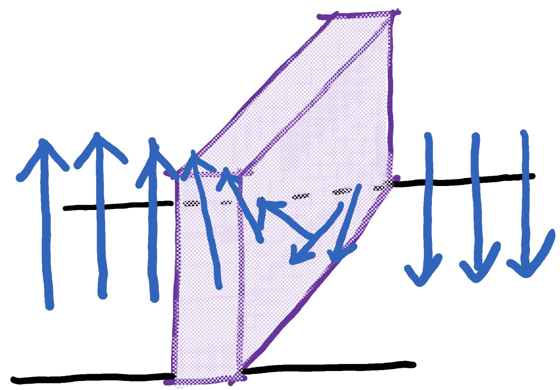
Local operator
e.g. particle
0-form symmetry



Line operator
e.g. Wilson line
't Hooft line
1-form symmetry



Surface operator
e.g. Cosmic string
2-form symmetry



Volume operator
e.g. Domain Wall
3-form symmetry

Higher-form symmetries

1. 0-form symmetry

Consider 4d two Weyl fermions Ψ_+, Ψ_- : $U(1)_+ \times U(1)_-$

$U(1)_V$: $\Psi_+ \to e^{i\alpha}\, \Psi_+$, $\Psi_- \to e^{-i\alpha}\, \Psi_-$ (can be gauged)

$U(1)_A$: $\Psi_+ \to e^{i\alpha}\, \Psi_+$, $\Psi_- \to e^{i\alpha}\, \Psi_-$ $\Rightarrow$ $d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$

Higher-form symmetries

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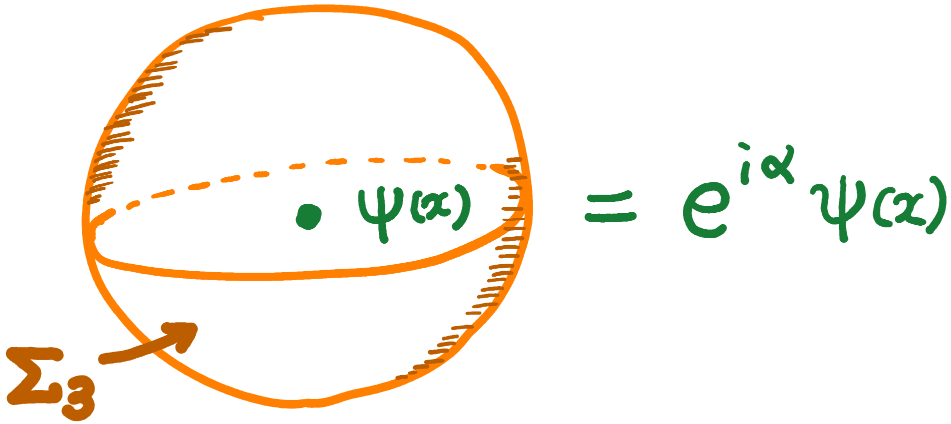
"Symmetry Defect Operator"

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$\langle U(\alpha, \Sigma_3)\psi(x)\rangle \sim e^{i\alpha}\psi(x)$$



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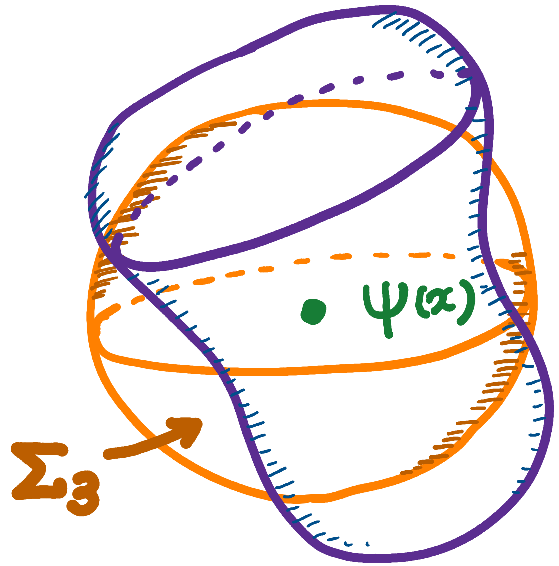


$= e^{i\alpha}\psi(x)$

$$\Delta Q = \int_{\hat{\Sigma}_4} d * J_1 = 0$$

$U(\alpha, \Sigma_3) =$ topological

Higher-form symmetries

2. p-form symmetry

0-form $\leftrightarrow$ local op (particle)

0-form $\leftrightarrow$ j_1 (j_μ)

0-form $\leftrightarrow$ A_1 (A_μ)

$$S \supset \int d^4x\, A_\mu j^\mu = \int A_1 \wedge * j_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha \int * j_1}$$

Higher-form symmetries

2. p-form symmetry

0-form $\leftrightarrow$ local op (particle)

0-form $\leftrightarrow$ $j_1 \;\; (j_\mu)$

0-form $\leftrightarrow$ $A_1 \;\; (A_\mu)$

$$S \supset \int d^4x\, A_\mu j^\mu = \int A_1 \wedge * j_1$$

$$U(\alpha, \Sigma_3) = e^{i\alpha \int * j_1}$$

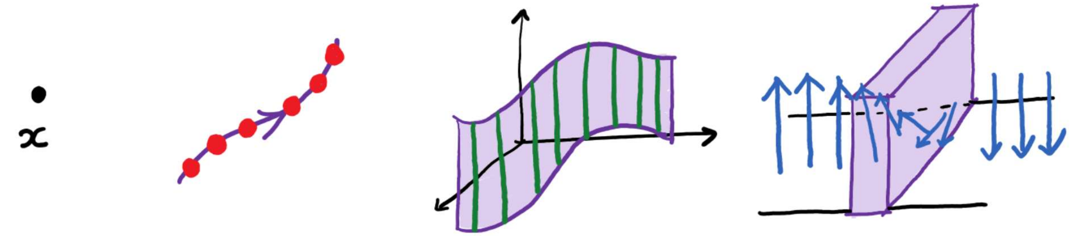
p-form $\leftrightarrow$ p-dim op

p-form $\leftrightarrow$ j_{p+1}

p-form $\leftrightarrow$ A_{p+1}

$$S \supset \int A_{p+1} \wedge * j_{p+1}$$

$$U(\alpha, \Sigma_{d-p-1}) = e^{i\alpha \int * j_{p+1}}$$



Local operator e.g. particle **0-form symmetry**

Line operator e.g. Wilson line 't Hooft line **1-form symmetry**

Surface operator e.g. Cosmic string **2-form symmetry**

Volume operator e.g. Domain Wall **3-form symmetry**

Higher-form symmetries

2. p-form symmetry

0-form $\leftrightarrow$ local op (particle)
0-form $\leftrightarrow$ j_1 (j_μ)
0-form $\leftrightarrow$ A_1 (A_μ)

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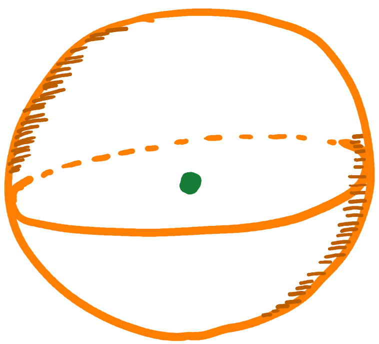
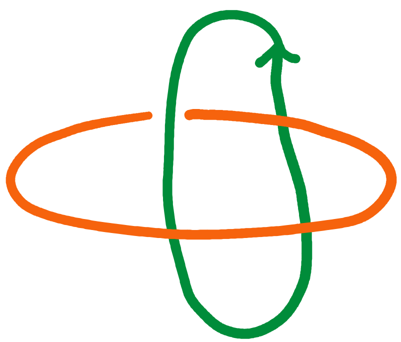
$$U\left(\alpha, \Sigma_{d-p-1}\right) = e^{i\alpha \int * j_{p+1}}$$

E.g.) 0- and 1-form symmetry in 3d





Higher-form symmetries

2. p-form symmetry

2-1. $U(1)_{EM}$ with Ψ_+, Ψ_-

EoM: $d * F_2 = j_\Psi \quad \left(d * F_2 = 0 \;\Rightarrow\; U(1)^{(1)}(e)\right)$

charged op: Wilson $W_1 = e^{i\oint A_1}$, SDO $U(\Sigma_2) = e^{i\oint *F_2}$

Bianchi id: $dF_2 = 0 \;\Rightarrow\; U(1)^{(1)}(m)$

charged op: 't Hooft $T_1 = e^{i\oint \tilde{A}_1}$, SDO $U(\Sigma_2) = e^{i\oint F_2}$

$U(1)^{(0)}{}_A$: $\Psi_+ \to e^{i\alpha}\Psi_+$, $\quad \Psi_- \to e^{i\alpha}\Psi_- \;\Rightarrow\; d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$

Higher-form symmetries

2. p-form symmetry

2-2. $SU(N)$ YM (either pure YM or with only adj matter)

$\exists\ Z_N^{(1)}(e)$: under 0-form center $\Psi \to e^{\frac{2\pi i}{N} * N}\, \Psi$

$\to$ Wilson line with charge $= 0,1,\cdots,(N-1)$ not screened

$\nexists$ mag 1-form : $\Pi_1\left(SU(N)\right) = \emptyset$

Higher-form symmetries

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$\to$ Wilson line with charge $= 0,1,\cdots,(N-1)$ not screened

$\nexists$ mag 1-form : $\Pi_1\left(SU(N)\right) = \emptyset$

2-3. $PSU(N) = \frac{SU(N)}{Z_N}$: $Z_N^{(1)}(e)$ is gauged (electric states projected out)

$\nexists$ electric 1-form

$\exists\ Z_N^{(1)}(m) : \Pi_1\left(PSU(N)\right) = Z_N$ or $"N * \frac{1}{N} = 1"$

$\Rightarrow\ \oint G_2 = 2\pi/N\,,\quad \int \mathrm{tr}(G_2 \wedge G_2) \supset \frac{N-1}{2N} \int w_2 \wedge w_2$

Fractional Instanton

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Consider again $U(1)_{EM}$ with Ψ_+, Ψ_-

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$\Rightarrow \quad \Pi_3\big(U(1)\big) = \emptyset$ and $U(1)_A$ turns into non-invertible symm

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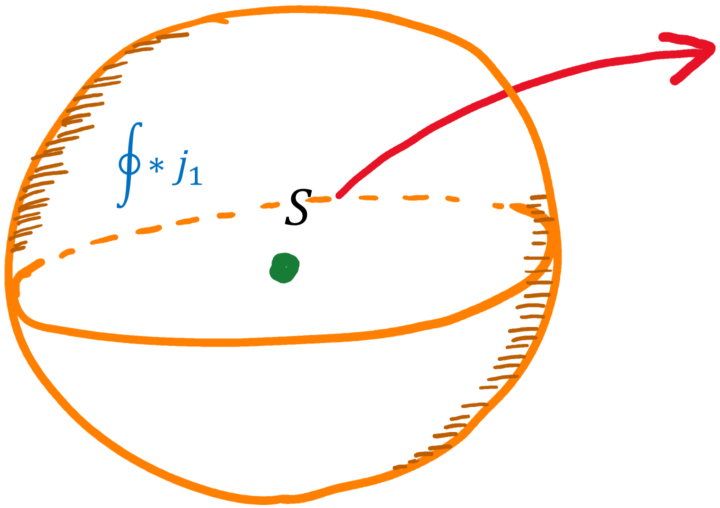
Simplified picture:

$$U(1)^{(0)}{}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \;, \quad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$$



$$S \;\to\; S + \frac{2\pi i K}{z} \int \frac{F_2 \wedge F_2}{8\pi^2}$$

$$\underbrace{\exp\left(\frac{2\pi i}{z} \oint * j_1\right)}_{U\left(\frac{2\pi}{z}, \Sigma_3\right)}$$

Non-Invertible Symmetry

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Simplified picture:

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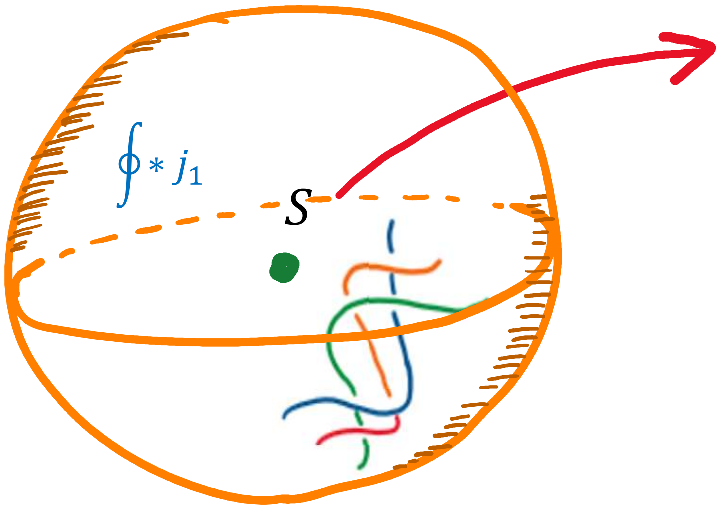

$$S \;\to\; S + \frac{2\pi i K}{z}\int \frac{F_2 \wedge F_2}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F_2 \wedge F_2}{8\pi^2} \;\to\; S$$

$$\underbrace{\exp\left(\frac{2\pi i}{z}\oint * j_1\right)}_{U\left(\frac{2\pi}{z},\,\Sigma_3\right)} \times \mathcal{A}^{\mathrm{N,p}}(\mathrm{F}_2/\mathrm{N})$$

Non-Invertible Symmetry

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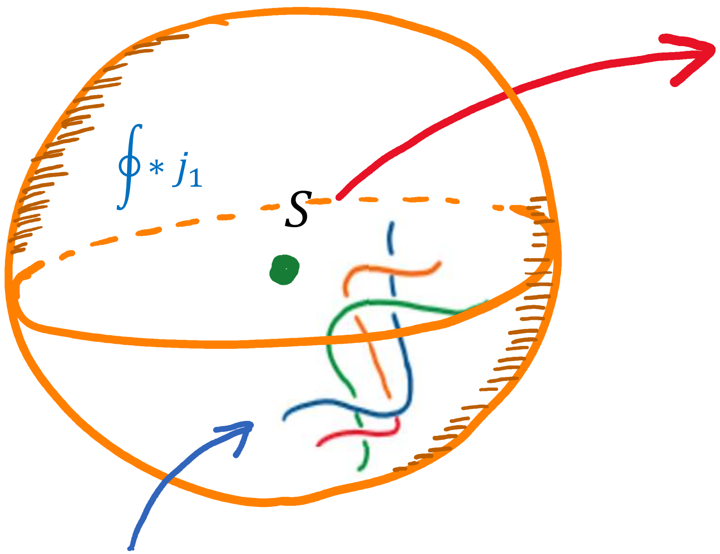

$$S \;\to\; S + \frac{2\pi i K}{z}\int \frac{F_2 \wedge F_2}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F_2 \wedge F_2}{8\pi^2} \;\to\; S$$

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$$S_{3d} = \frac{iN}{4\pi}\int_{\Sigma_3} a_1 \wedge da_2 + \frac{i}{2\pi}\int_{\Sigma_3} a_1 \wedge F_2$$

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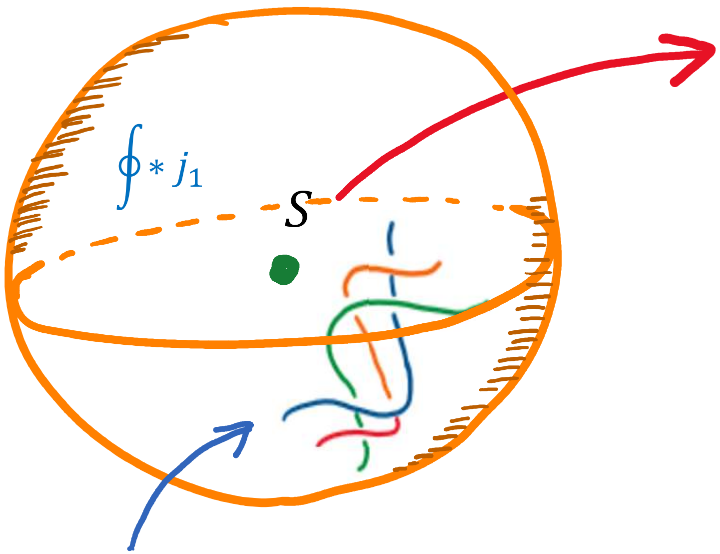

$$S \;\to\; S + \frac{2\pi i K}{z}\int \frac{F_2 \wedge F_2}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F_2 \wedge F_2}{8\pi^2} \;\to\; S$$

$$\boldsymbol{\mathcal{D}}_{\frac{2\pi}{z}}(\boldsymbol{\Sigma_3}) = \underbrace{\exp\left(\frac{2\pi i}{z}\oint * j_1\right)}_{U\left(\frac{2\pi}{z},\,\Sigma_3\right)} \times \mathcal{A}^{\mathrm{N,p}}(\mathrm{F_2/N})$$

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$$\mathcal{D}_k(\Sigma_3) \times \overline{\mathcal{D}}_k(\Sigma_3) \sim \sum_S \xi(S)\,\exp\left(\frac{i}{2\pi N}\int_S F_2\right) \neq 1$$

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z} \underbrace{\boxed{\int_{M_4} \frac{G \wedge G}{8\pi^2}}}_{\in Z} + \frac{2\pi K i}{z} \underbrace{\boxed{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}}_{\in Z_L}$

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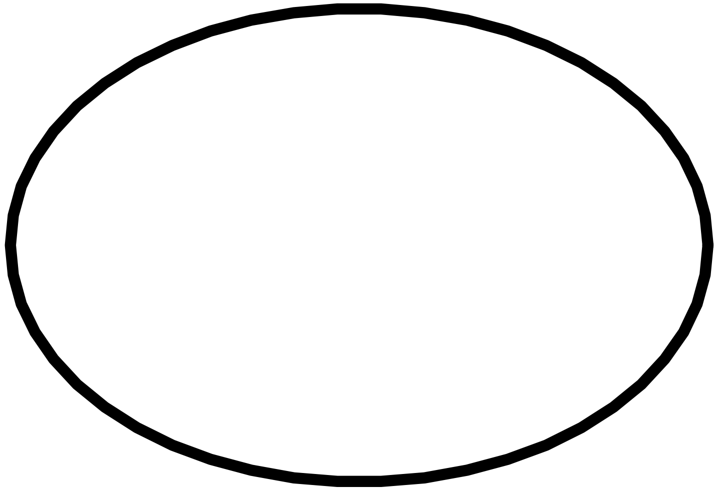
Global $U(1)_A$



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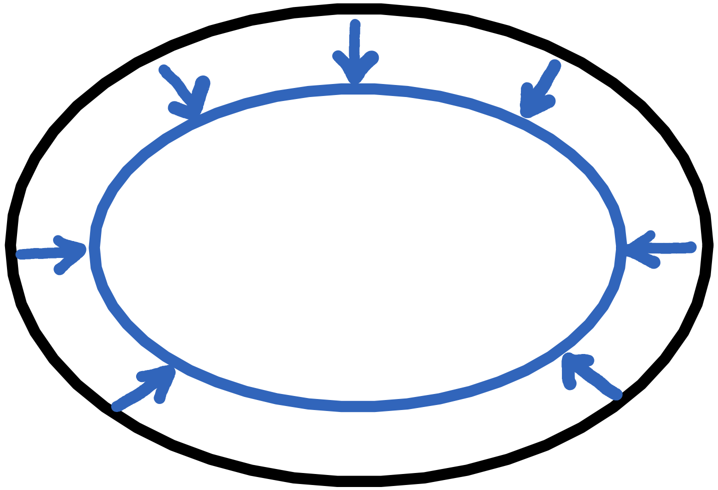

$\to$ Z **Instanton**

Non-Invertible Symmetry

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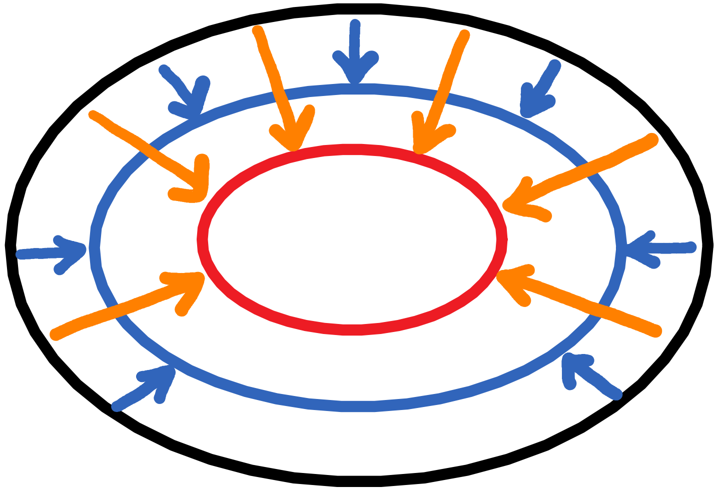
Global $U(1)_A$



$\to$ Z **Instanton**

$\to$ Z_L **(fractional) Instanton**

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

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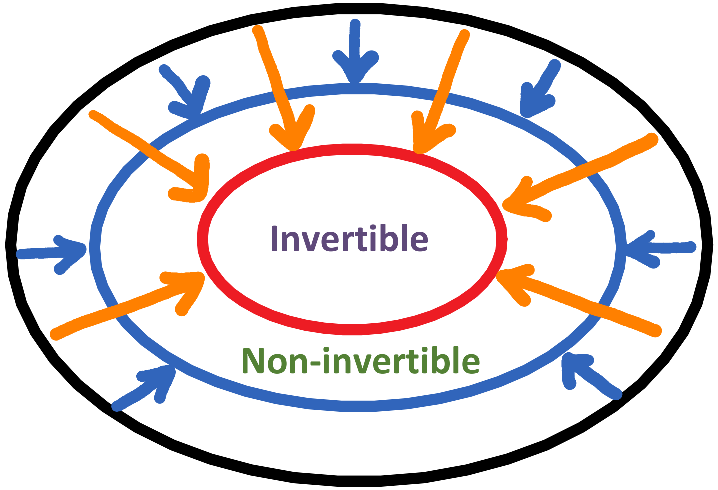
magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z}\underbrace{\boxed{\int_{M_4}\frac{G\wedge G}{8\pi^2}}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\boxed{\left(\frac{L-1}{L}\right)\int_{M_4}\frac{w_2\wedge w_2}{2}}}_{\in Z_L}$

Global $U(1)_A$



$\rightarrow$ Z Instanton

$\rightarrow$ Z_L (fractional) Instanton

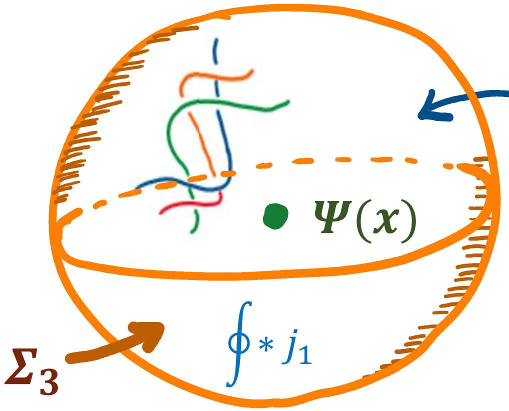
Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$$U(1)_A \text{ with } \alpha = \frac{2\pi}{z}, \qquad S \to S + \frac{2\pi K i}{z}\underbrace{\boxed{\int_{M_4}\frac{G\wedge G}{8\pi^2}}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\boxed{\left(\frac{L-1}{L}\right)\int_{M_4}\frac{w_2\wedge w_2}{2}}}_{\in Z_L}$$



$$S_{3d} = \frac{iN}{4\pi}\int_{\Sigma_3} a_1 \wedge da_2 + \frac{i}{2\pi}\int_{\Sigma_3} a_1 \wedge w_2$$

$$\boxed{U\left(\frac{2\pi}{z},\Sigma_3\right) \to D_z = U\left(\frac{2\pi}{z},\Sigma_3\right) \times \mathcal{A}^{N,p}(w_2) \text{ with } \frac{p}{N} = \frac{K}{z}}$$

Outline

I. Briefly on Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Non-invertible Naturalness

II-1. Non-invertible leptonic symmetry and m_ν
II-2. Non-invertible PQ symmetry and strong CP problem
II-3. Other Naturalness Problems

III. UV fate of the universe: Global Structure ambiguity

III-1. Global Structure and global symmetries
III-2. Global Structure from axion non-invertible symmetries
III-3. Global Structure from fractionally charged particle search

IV. Outlook

Non-Invertible Naturalness

Basic Idea:

1. ∃ Very small (or zero) observable quantities (local operators) in particle physics: $\mathcal{L} \ni c\,\mathcal{O}(x)$, $c \ll 1$

   e.g. $\lambda_{ij}(HL_i)(HL_j) \sim M_\nu^{ij}\,\nu_i\nu_j$, $y_d HQd^c\ (\to \bar{\theta} < 10^{-10})$

Non-Invertible Naturalness

Basic Idea:

1. $\exists$ Very small (or zero) observable quantities (local operators) in particle physics: $\mathcal{L} \ni c\,\mathcal{O}(x)$, $c \ll 1$

   e.g. $\lambda_{ij}(HL_i)(HL_j) \sim M_\nu^{ij}\,\nu_i\nu_j$, $\quad y_d HQd^c\ (\to \bar{\theta} < 10^{-10})$

2. if $\mathcal{O}(x)$ charged under 0-form non-invertible symmetry
   $\Rightarrow$ enforces such Wilson coefficient c to 0

   : local operator of interests (e.g. neutrino mass or quark mass) are primarily protected by non-invertible symmetry

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3. ∃ unique and universal way that non-inv symmetry is broken by dynamical monopole / small instantons

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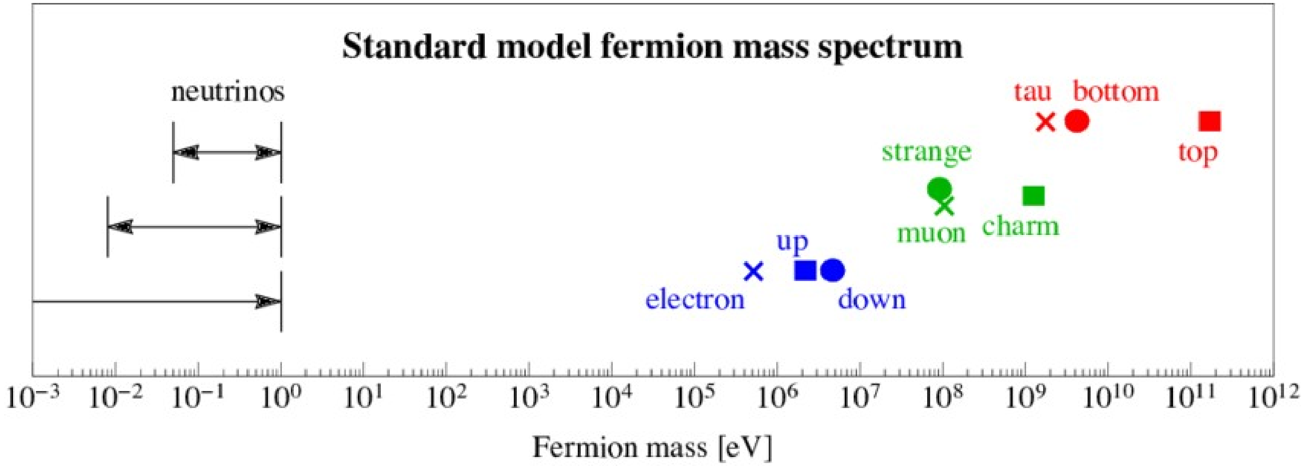
3. ∃ unique and universal way that non-inv symmetry is broken by dynamical monopole / small instantons

4. Exponentially small c obtained by UV embedding via breaking of non-inv symmetry coming from UV monopole/instanton effects

Non-Invertible Naturalness

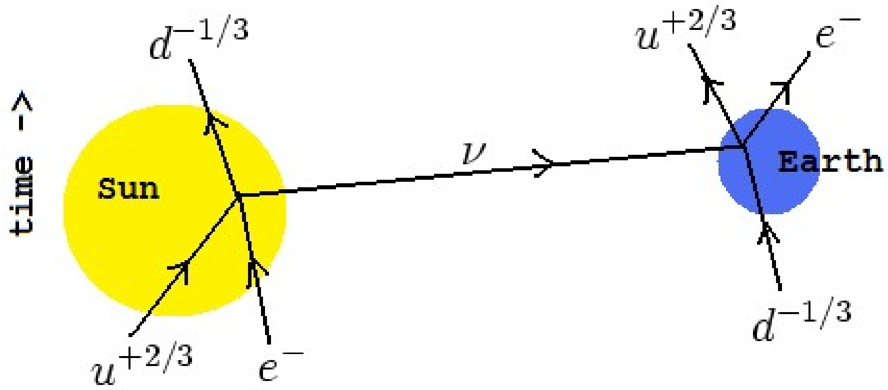
1. non-inv leptonic symmetry and m_ν: ('22 C. Cordova, **SH**, S. Koren, K. Ohmori)



https://www.researchgate.net/figure/Mass-spectrum-of-standard-model-fermions-Charged-leptons-up-type-quarks-and-downtype_fig1_361578459

$$M_\nu \sim 10^{-2}\, eV$$

Requires Dynamical Explanation!



https://carlbrannen.wordpress.com/2008/06/21/neutrino-oscillation-or-interference/

Neutrino Oscillation

$$P_{e\to\mu} \propto \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Non-Invertible Naturalness

1. non-inv leptonic symmetry and m_ν: ('22 C. Cordova, **SH**, S. Koren, K. Ohmori)

(1) Quantum Invertible Symmetry of SM :

$$\mathrm{U}(1)_{L_e-L_\mu} \times U(1)_{L_\mu-L_\tau} \times \frac{U(1)_{B-N_cL}}{Z_{N_c}}$$

(2) Symmetry of $G_{SM} \times U(1)_{L_\mu-L_\tau}$:

- Invertible: $U(1)_{B-N_gN_cL_e}/Z_{N_c}$
- Non-invertible: $U(1)_{L_e-L_\mu} \supset Z^L_{N_g} \ (\subset U(1)_L)$

(3) Forbidding m_ν by non-invertible symmetry

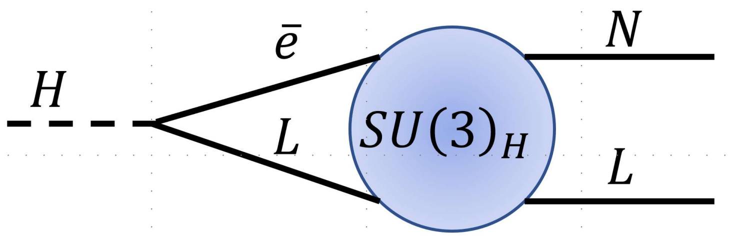
$$\mathcal{L} \sim \lambda_{ij}(HL_i)(HL_j) \sim m_\nu^{ij}\, \nu_i\nu_j$$

Non-Invertible Naturalness

1. non-inv leptonic symmetry and m_ν: ('22 C. Cordova, **SH**, S. Koren, K. Ohmori)

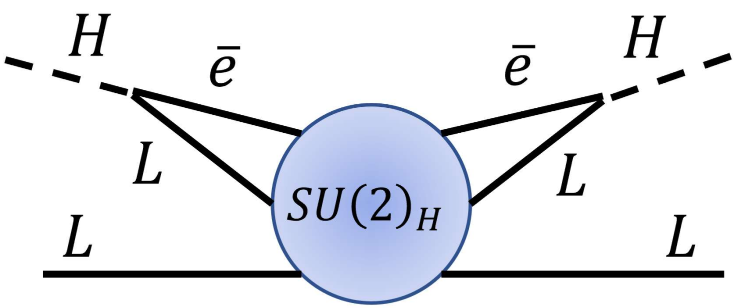
(4) Small m_ν from UV small instantons

i. Dirac m_ν from $U(1)_{L_\mu - L_\tau} \subset SU(3)_H$ instanton



$$\mathcal{L} \sim y_\tau e^{-\frac{2\pi}{\alpha_H}} \tilde{H} L N$$

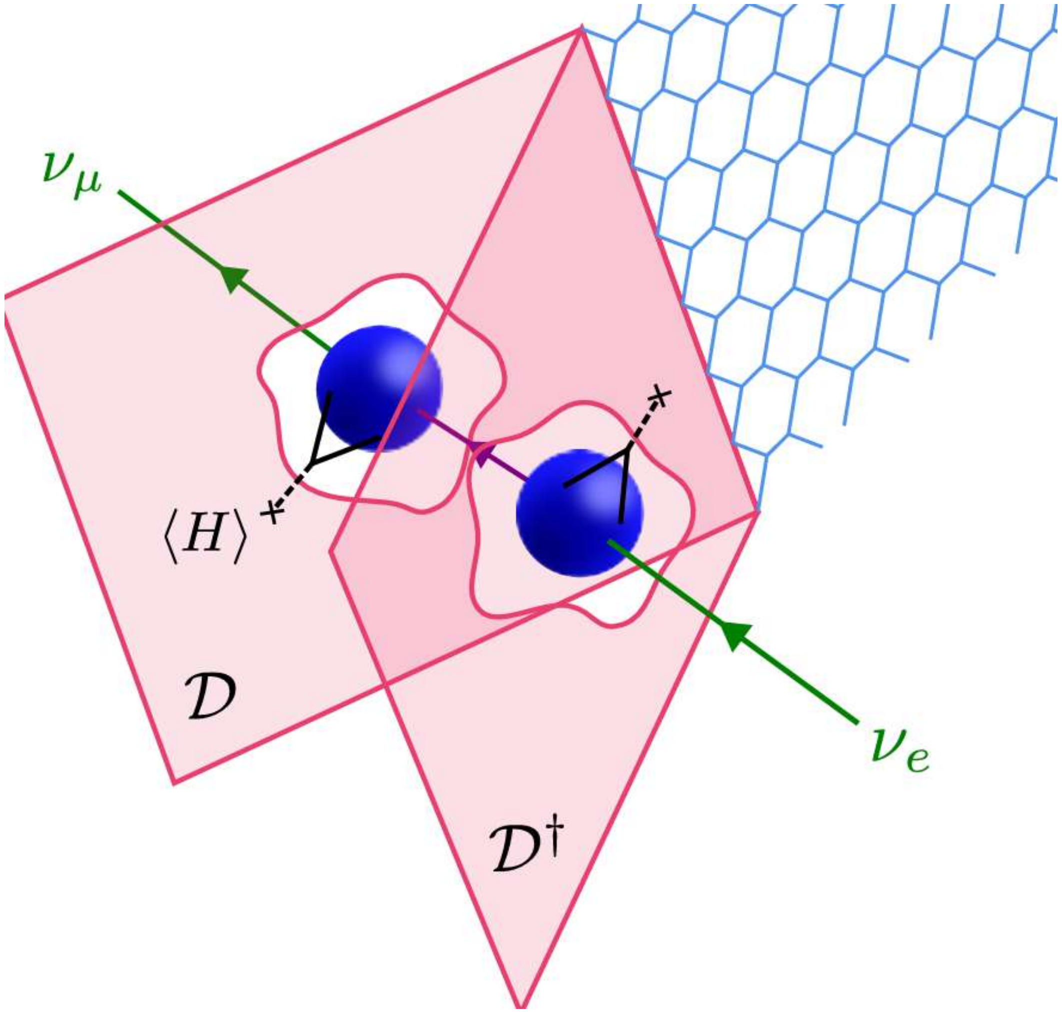
ii. Majorana m_ν from $U(1)_{L_\mu - L_\tau} \subset SU(2)_H \times U(1)_Z$ instanton



$$\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi} \, e^{-\frac{2\pi}{\alpha_H}} \, (HL)^2$$

Non-Invertible Naturalness

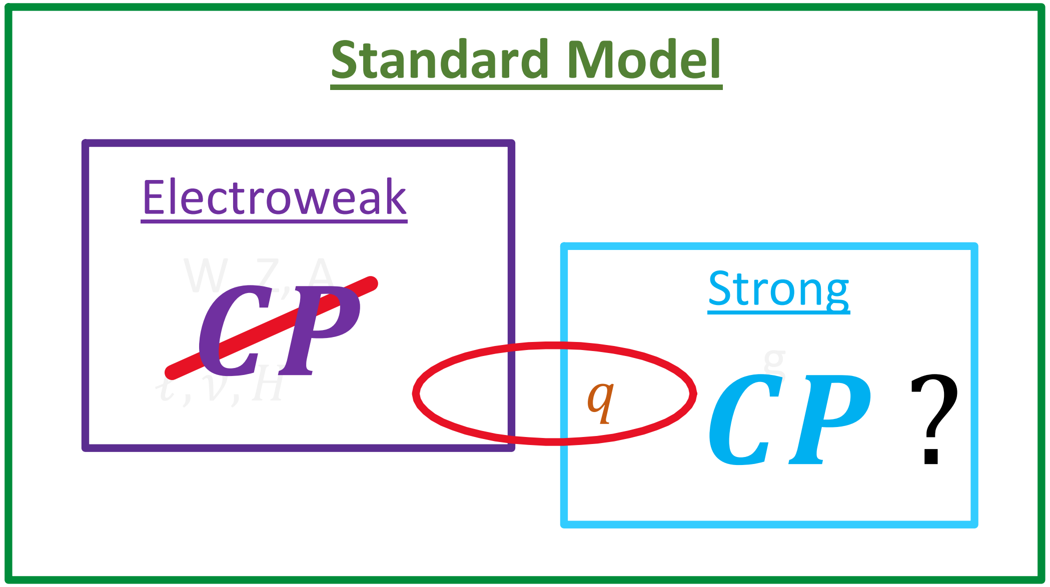
1. non-inv leptonic symmetry and m_ν: ('22 C. Cordova, **SH**, S. Koren, K. Ohmori)



Non-Invertible Naturalness

2. non-inv PQ symmetry and strong CP: ('24 C. Cordova, **SH**, S. Koren)

Strong CP Problem



Expectation based on general rules of effective field theory

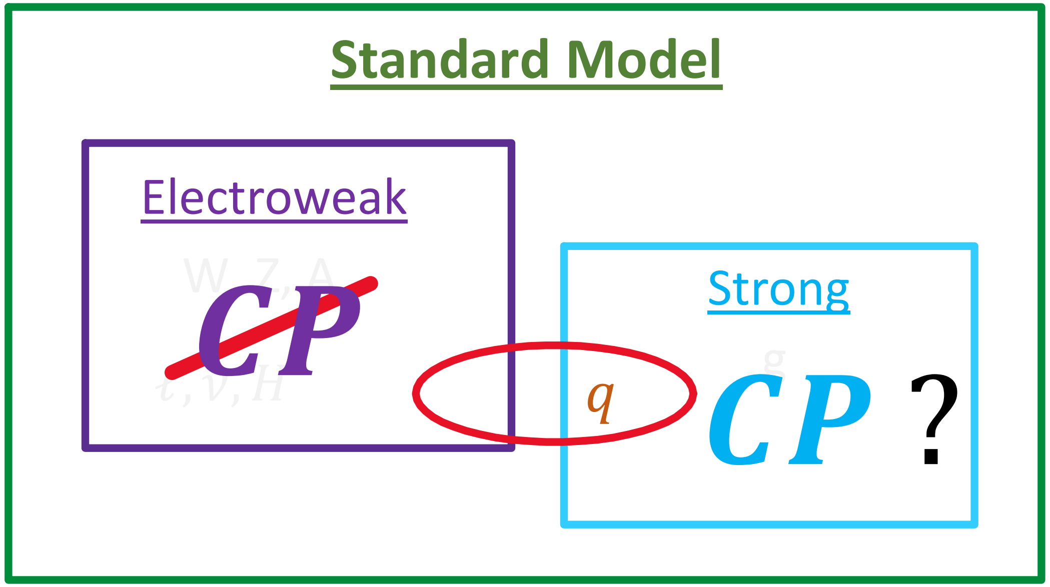
$$\tilde{J} = \mathrm{Im}\,\det[y_u^\dagger y_u, y_d^\dagger y_d\,] \propto \sin\delta_{CKM}$$

"Jarlskog invariant"

Non-Invertible Naturalness

2. non-inv PQ symmetry and strong CP: ('24 C. Cordova, **SH**, S. Koren)

Strong CP Problem



Expectation based on general rules of effective field theory

$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

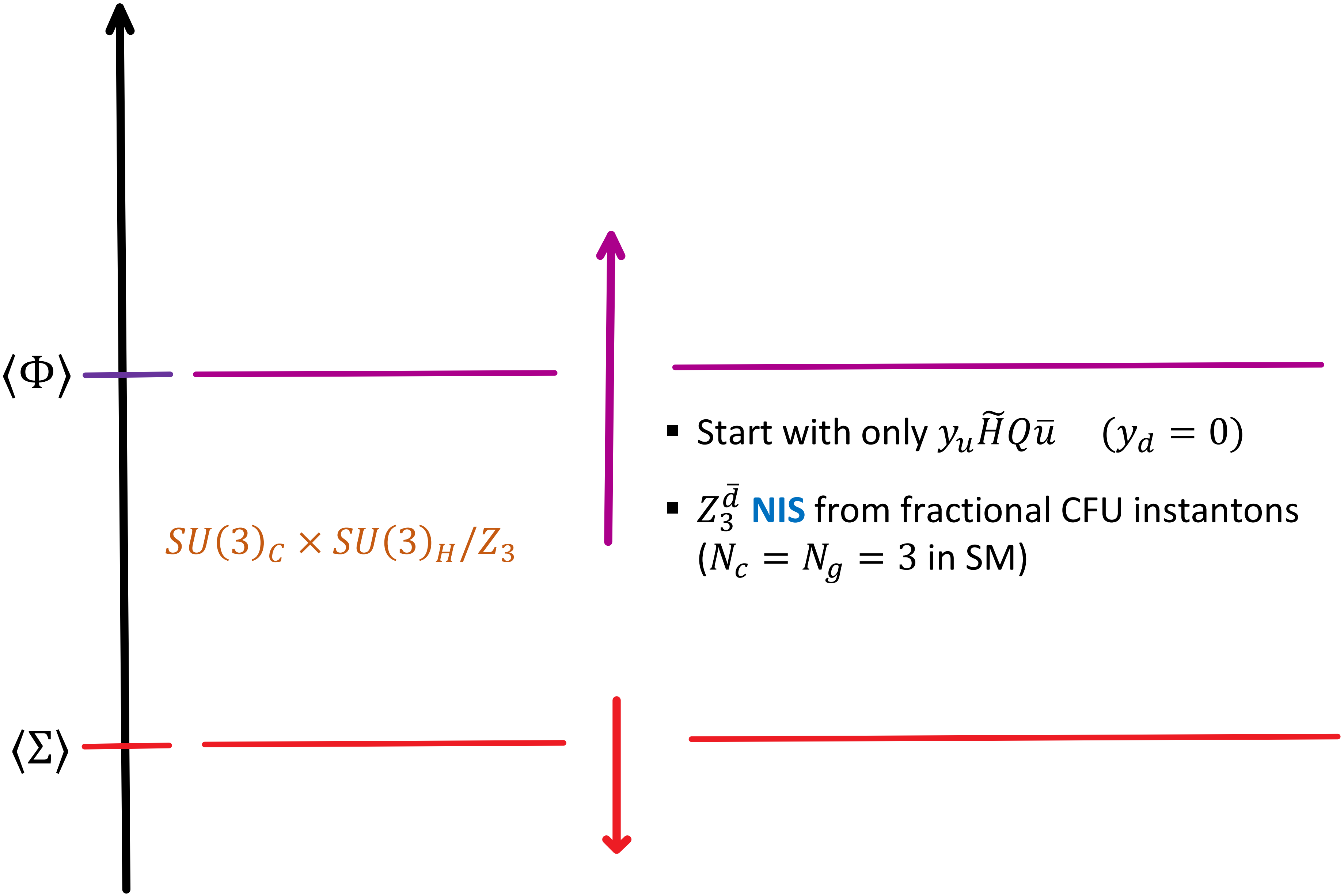
Neutron Electric Dipole Moment

$$d_n \sim 3 \times 10^{16}\,\bar{\theta} \;\rightarrow\; \bar{\theta} < 10^{-10}$$

$\tilde{J} = \mathrm{Im}\,\det[y_u^\dagger y_u, y_d^\dagger y_d\,] \propto \sin\delta_{CKM}$ vs $\bar{\theta} \equiv \arg e^{-i\theta}\det(y_u y_d)$

"Jarlskog invariant"

Solving Strong CP with Non-Invertible Symmetry



- Start with only $y_u \widetilde{H} Q \bar{u}$ $\quad (y_d = 0)$
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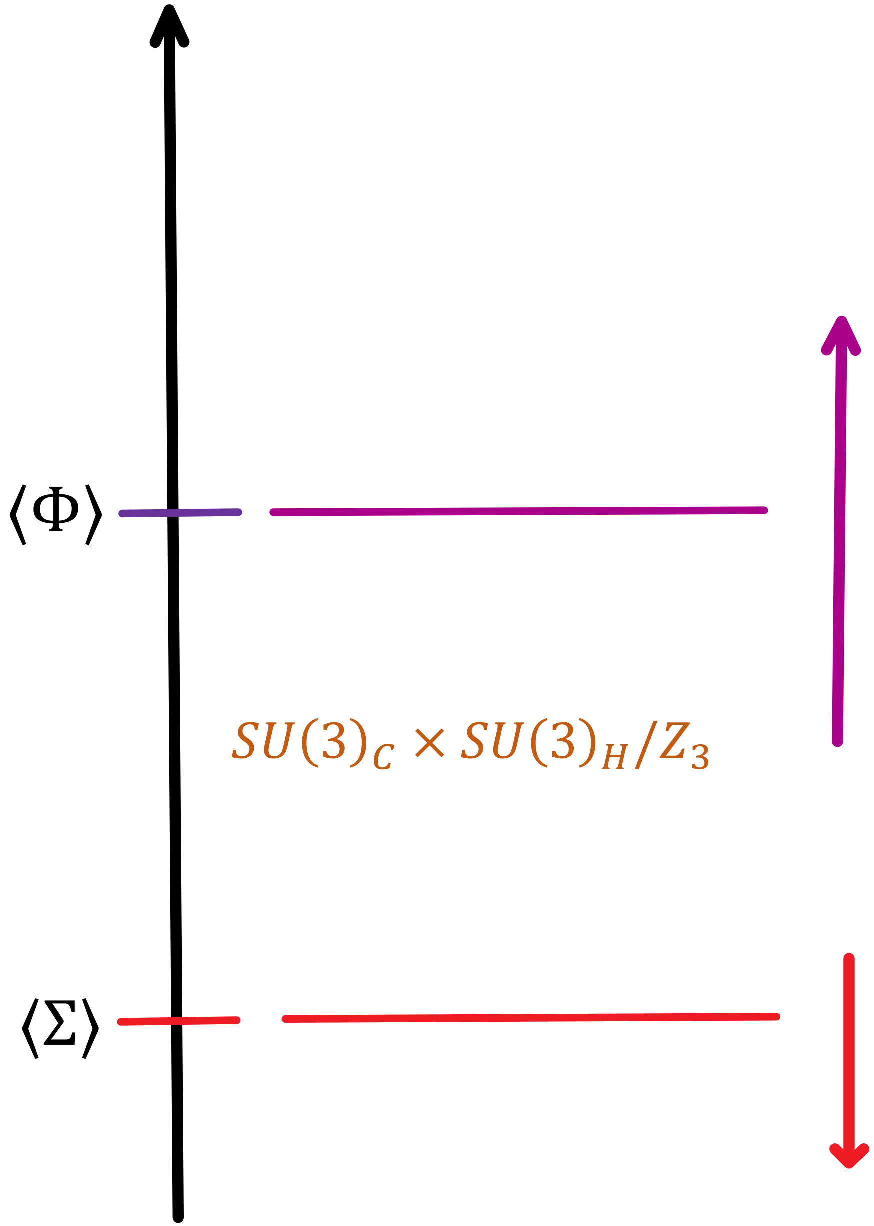

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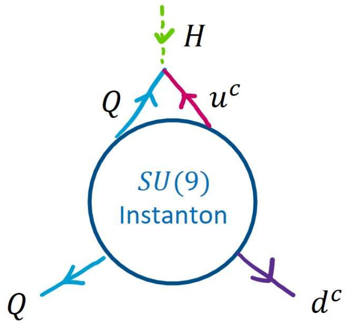

$SU(9)$

$\langle \Phi \rangle$

$$\mathcal{L}_{UV} \supset y_u \widetilde{H} Q \bar{u} + y_u^* e^{-\frac{2\pi}{\alpha_9}} H Q \bar{d} + \cdots$$

with $\bar{\theta} = 0$

$SU(3)_C \times SU(3)_H / Z_3$

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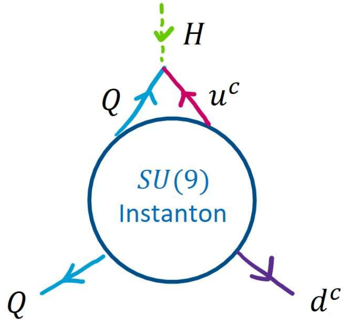

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$SU(3)_C \times SU(3)_H / Z_3$

$\langle \Sigma \rangle$

SM

Yukawa texture and CKM CPV (+possible $\bar{\theta}$)

with $\bar{\theta} = 0$ or $\bar{\theta} \ll 1$

Non-Invertible Naturalness

3. Other Naturalness Problems

(1) Natural 2HDM Alignment from non-inv PQ symmetry

('24 C. Delgado, S. Koren)

$$V(\Phi_1, \Phi_2) \supset -m_{12}^2 \Phi_1^+ \Phi_2 + [\lambda_6(\Phi_1^+ \Phi_1) + \lambda_7(\Phi_2^+ \Phi_2)](\Phi_1^+ \Phi_2)$$

i. with two scalars, non-inv symm acting on scalars

Ii. Small m_{12}^2 (and $\lambda_{6,7}$) generated by UV small instanton effects

(2) Non-inv Axion-shift symmetry and axion-DW problem

('22 C. Cordova, K. Ohmori) ('23 Y. Choi, M. Forslund, H. T. Lam, S-H. Shao) ('23 M. Reece)
('23 C. Cordova, **SH**, L. Wang) [see also ('22 P. Agrawal, M. Nee, M Reig) ('24 P. Agrawal, M. Nee, M Reig)]

i. $\exists$ non-inv discrete axion-shift symm (depending on Γ)

ii. Axion potential $V(a)$ breaks this $\rightarrow$ (non-inv) axion domain walls

iii. Solving domain wall problem $\rightarrow$ small bias term $\delta V(a)$

Non-Invertible Naturalness

3. Other Naturalness Problems

(3) Electroweak Hierarchy Problem ?

i. Any non-inv symm acting the Higgs (scalar) ?
ii. Any other generalized symmetry protecting $H^{+}H$?

(4) Cosmological Constant Problem ?

Outline

I. Briefly on Generalized Global Symmetries

I-1. Higher-form symmetry
I-2. Non-invertible symmetry

II. Non-invertible Naturalness

II-1. Non-invertible leptonic symmetry and m_ν
II-2. Non-invertible PQ symmetry and strong CP problem
II-3. Other Naturalness Problems

III. UV fate of the universe: Global Structure ambiguity

III-1. Global Structure and global symmetries
III-2. Global Structure from axion non-invertible symmetries
III-3. Global Structure from fractionally charged particle search

IV. Outlook

UV fate of the universe: Global Structure ambiguity

1. Global Structure and Global Symmetries

$$G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}, \quad \Gamma = 1, Z_2, Z_3, Z_6$$

i. The entire SM matter fields are neutral under $\mathbb{Z}_6$ transformation generated by

$$e^{\frac{2\pi i}{3}\lambda_8} = e^{\frac{2\pi i}{3}} I_3 \in SU(3)_C, \quad e^{\frac{2\pi i}{2}T_3} = -I_2 \in SU(2)_L, \quad e^{\frac{2\pi i}{6}Q_Y} \in U(1)_Y$$

ii. For $\Gamma = \mathbb{I}$,

$\mathbb{Z}_6$ Wilson lines are not screened $\Rightarrow$ $\mathbb{Z}_6^{(1)}$ electric 1-form center symmetry

iii. Gauging $\Gamma = \mathbb{Z}_p^{(1)} \subset \mathbb{Z}_6^{(1)} \Rightarrow \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\mathbb{Z}_{6/p}^{(1)}(e) \times \mathbb{Z}_p^{(1)}(m)$

UV fate of the universe: Global Structure ambiguity

2. Global Structure from axion non-inv symmetries

('23 Y. Choi, M. Forslund, H. T. Lam, S-H. Shao) ('23 C. Cordova, **SH**, L. Wang) [see also ('23 M. Reece)]

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B \wedge B$$

1. $G, W, B =$ field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = 1$: $\ell_{1,2,3} \in Z$
$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$
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e.g. Experimental observation: $\ell_1 = 0$, $\ell_2 = 2$, $\ell_3 = 2$

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$\Gamma = Z_6$: $SU(5)$, $SO(10)$, E_6
$\Gamma = Z_3$: $SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]
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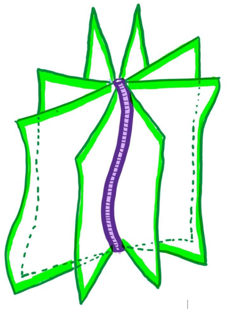
4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

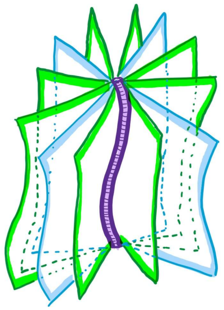
$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3}$ where $K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Axion-Standard Model

III. Axion-SM



VS



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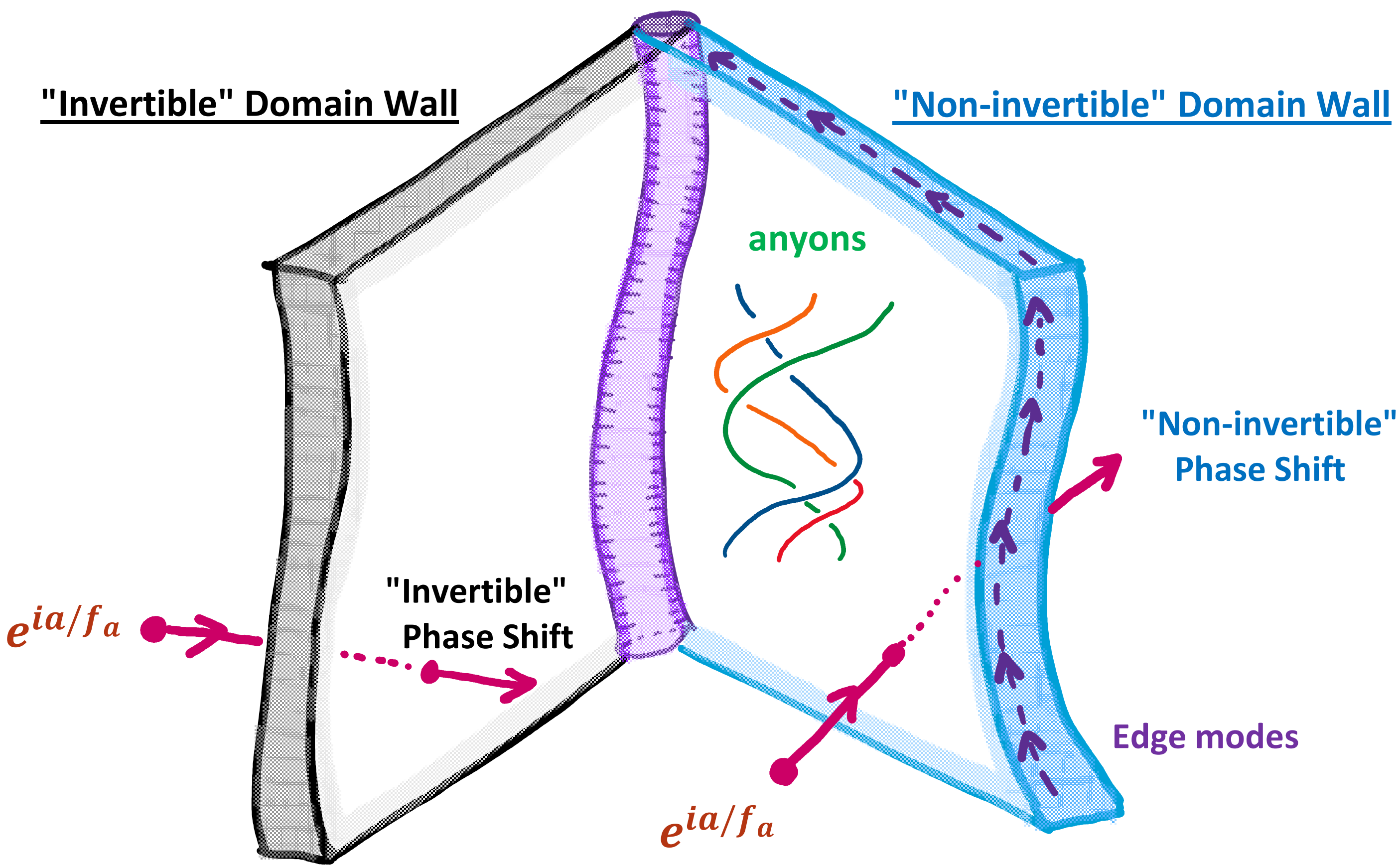
4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3 \;\; (\ell_1 = 18, \ell_2 = 0, \ell_3 = 3)$

$Z^{I}_{K=1} \subset Z^{NI}_{\ell_3=3}$ where $K = \gcd\left(\frac{\ell_1}{3}, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW



UV fate of the universe: Global Structure ambiguity

3. Global Structure from fractional charged particle search

('24 S. Kore, A. Martin) [see also ('24 R. Alonso, D. Dimakou, M. West) ('24 H.-L. Li, L.-X. Xu)]

Recall:
$$\Gamma = \mathrm{Z}_6 : \quad SU(5)\,, \qquad SO(10)\,, \qquad E_6$$
$$\Gamma = Z_3 : \quad SU(4)_C \times SU(2)_L \times SU(2)_R \quad \text{[Pati-Salam]}$$
$$\Gamma = Z_2 : \quad SU(3)_C \times SU(3)_L \times SU(3)_R \quad \text{[Trinification]}$$

i. (heavy) fractionally charged particle can exists if Γ is non-trivial

ii. e.g. Pati-Salam: $e/2$ states is allowed
  Trinification: $e/3$ states are allowed
  $\Gamma = \mathbb{I}$: $e/6$ states are allowed

iii. Discovery of (heavy) fractionally charged particle at colliders can probe/rule-out certain deep UV unified theories.

Outline

Outlook

I. Generalized symmetry = vastly more symmetries

→ no need refined argument to justify why we should think and work to see what these new symmetries can do for us.

II. In formal theory side, very active research is underway both high-energy and condensed matter.

III. Only very recently, we began to see concrete realizations and applications of generalized symmetries in particle physics problems.

IV. There are many questions, many directions to pursue. It is matter of time and efforts to uncover big surprises.

THANK YOU

FOR

YOUR ATTENTION!