Bosons on the edge: many-body physics in uncommon geometries

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Florence Theory Group Day - March 19, 2025

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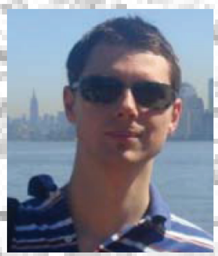

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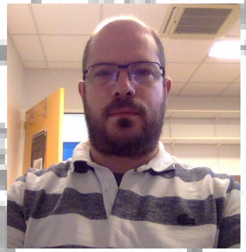

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Giuseppe Pellicane
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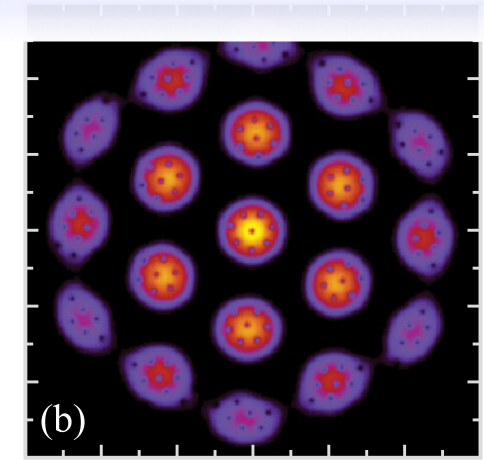
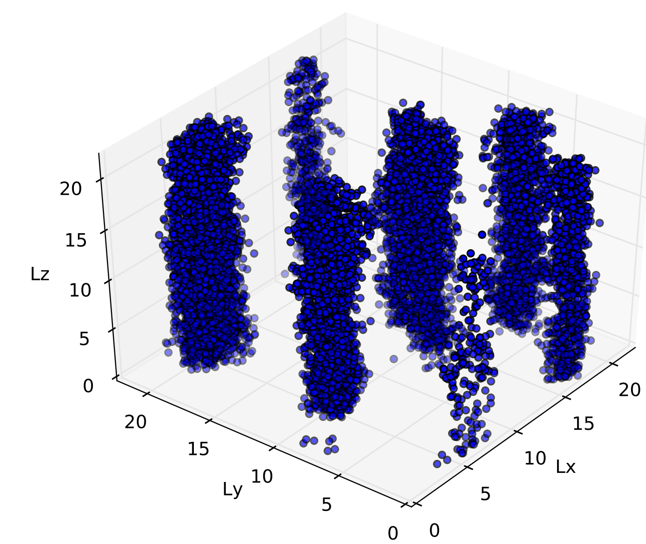
Clustering…

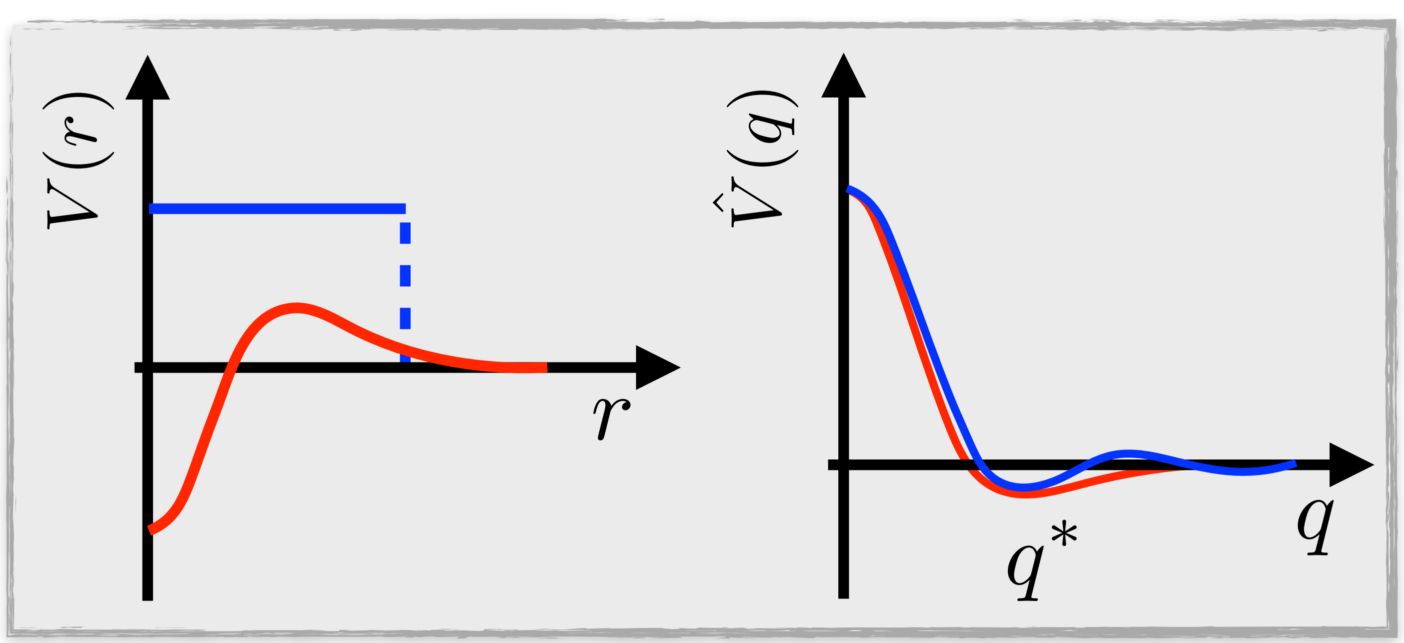
- Broad class of physical potentials: $V(r)$ whose FT have a negative region exhibit an instability leading to clustering at a density modulation $\lambda = 2\pi/q^*$
- Soft matter: colloidal particles, polymer chains, etc…
- Cold-atoms: Rydberg-blockade regime / Soft-core bosons / Dipoles
- Electron bubble crystals in highly degenerate Landau levels (Goebig et al. PRB 2004).

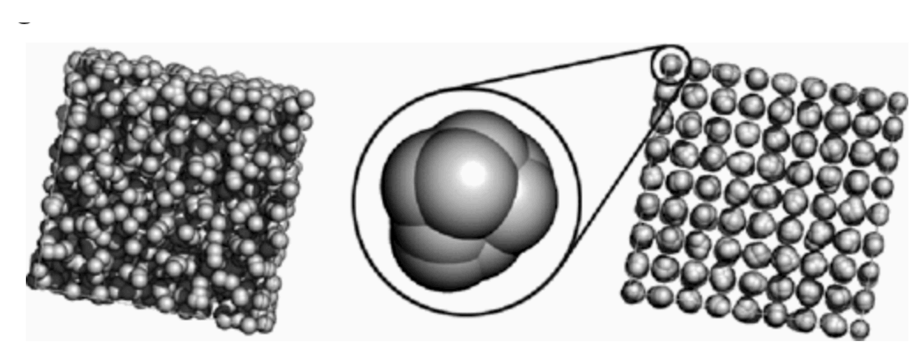


Cinti et al. Nat. Comm. 2014



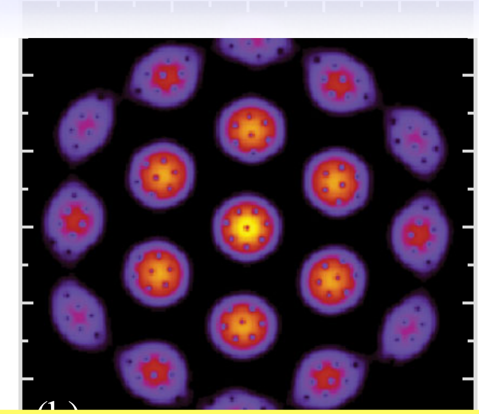
Cinti et al. PRL 2017



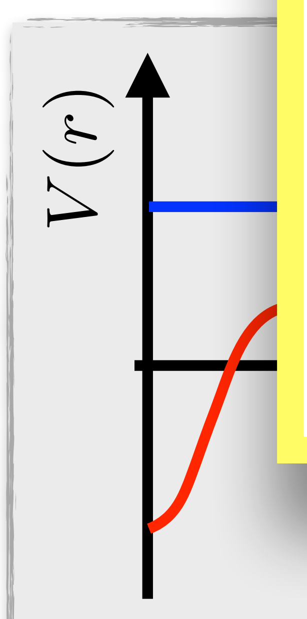


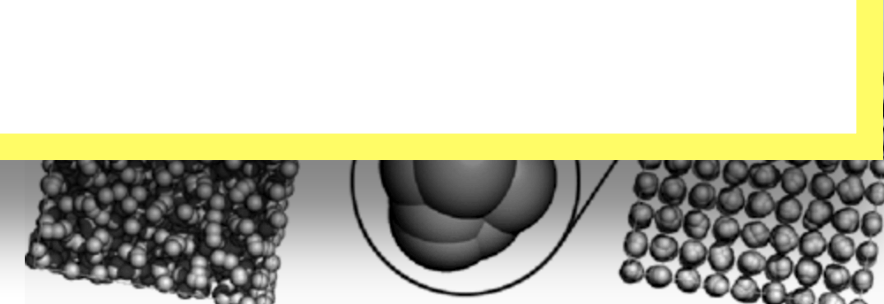
Likos Phys. Rep. 2001

Clustering…



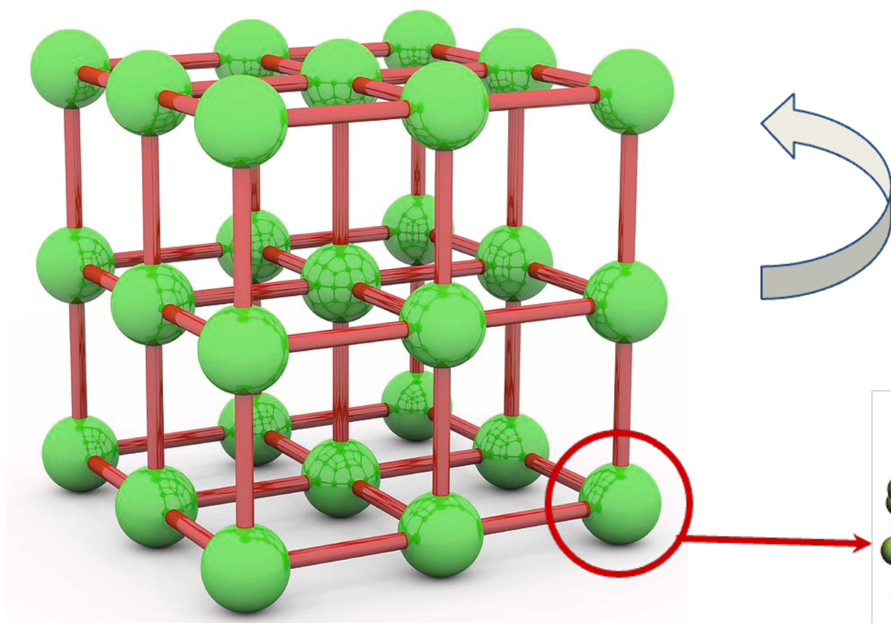
- Broad class of physical potentials: $V(r)$ whose FT have a negative region exhibit an instability leading to clustering at a density modulation $\lambda = 2\pi/q^*$
- Soft matter: c
- Cold-atoms: F
- Electron bubb et al. PRB 200



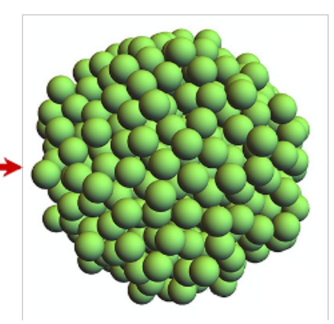


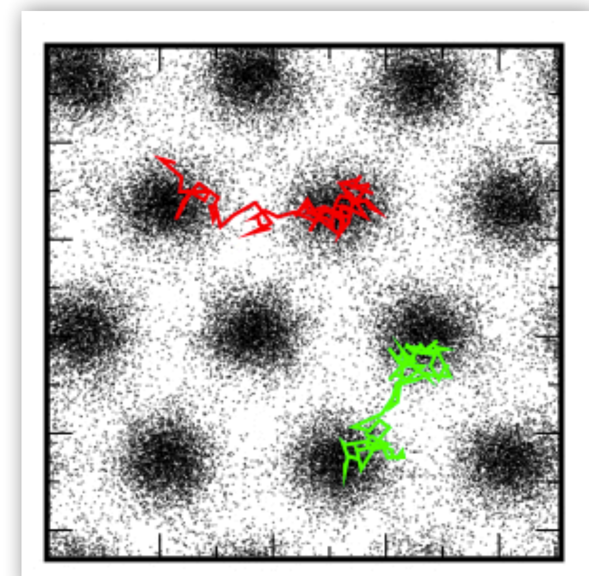
Likos Phys. Rep. 2001

<u>Bosons clusters are supersolid!</u>



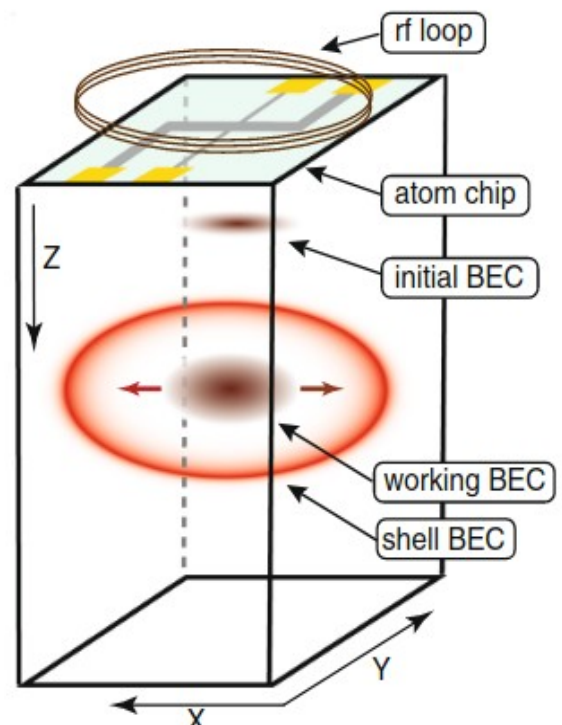
$I < I_{classical}$

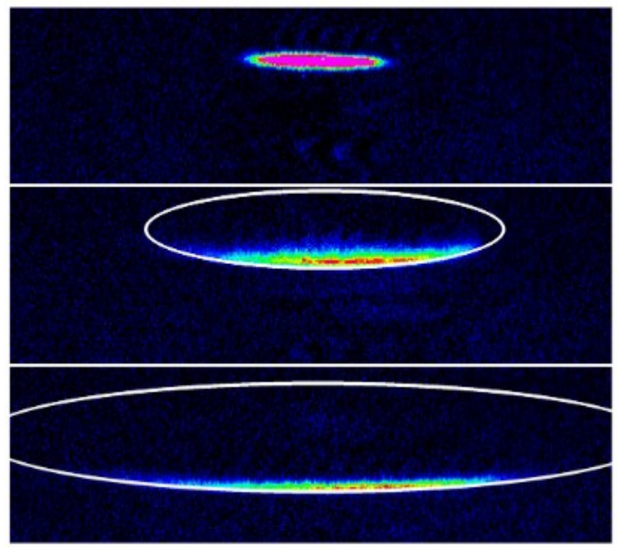




Bosons on a spherical surface

Ultracold atoms experiments

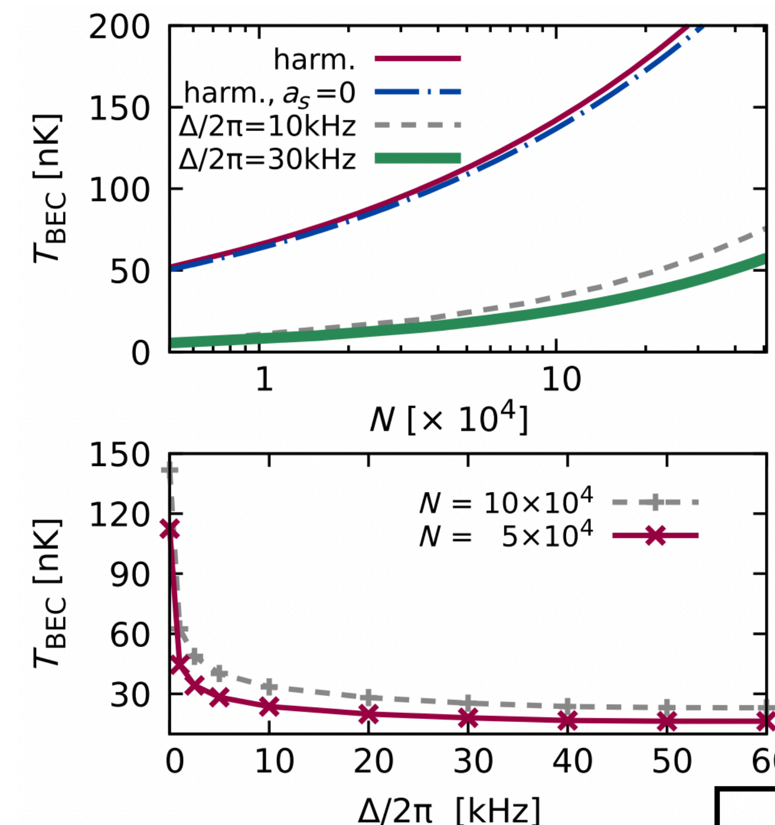


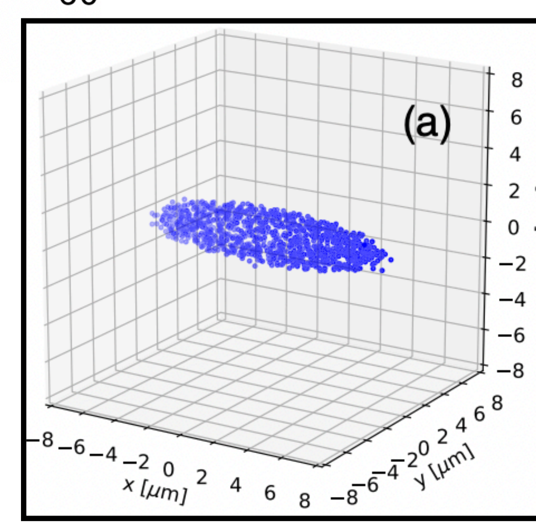


quadrupolar field dressed by radio frequency field via adiabatic deformation

Colombe et al. Eur Phys Lett 2004
Garraway et al. J. Phys B 2016
Lundblad et al. Micrograviry 2019

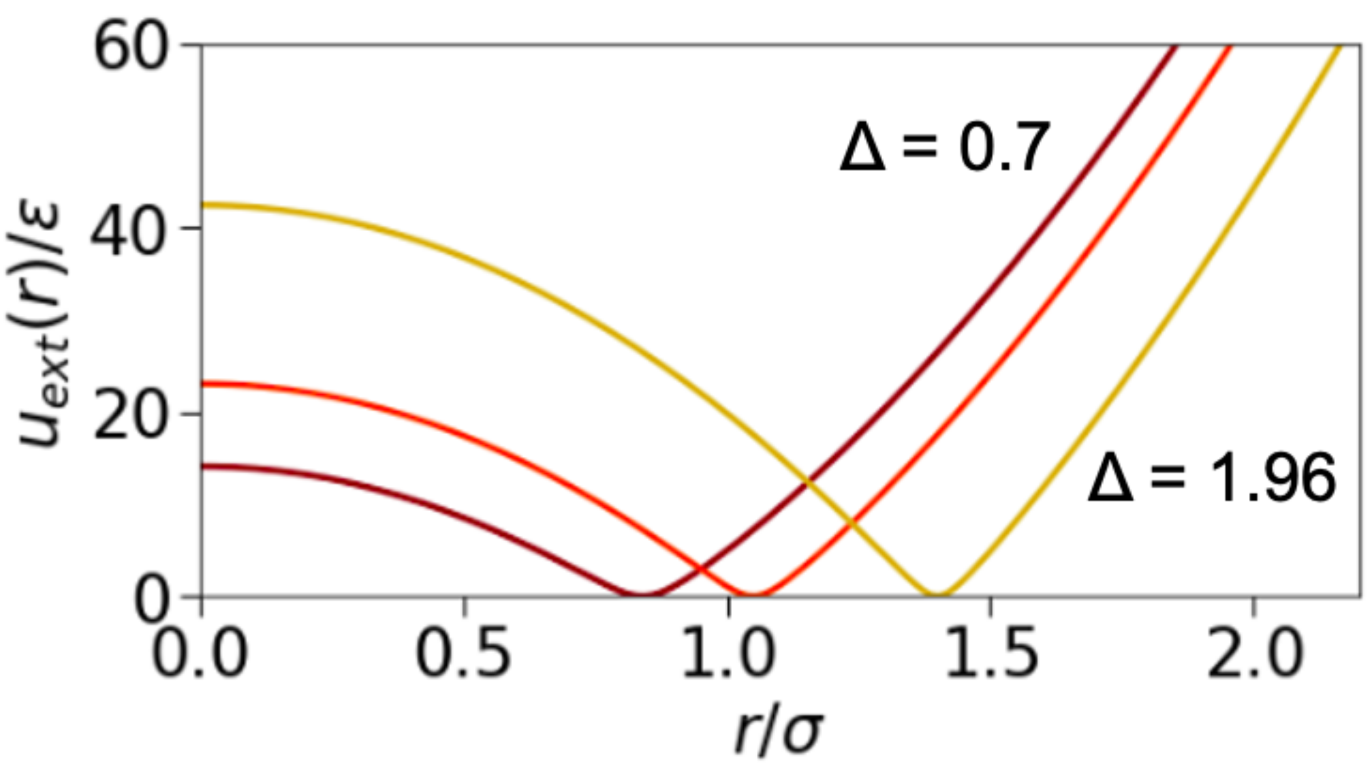
BEC and Superfluidity
So far weak hard-core interactions…

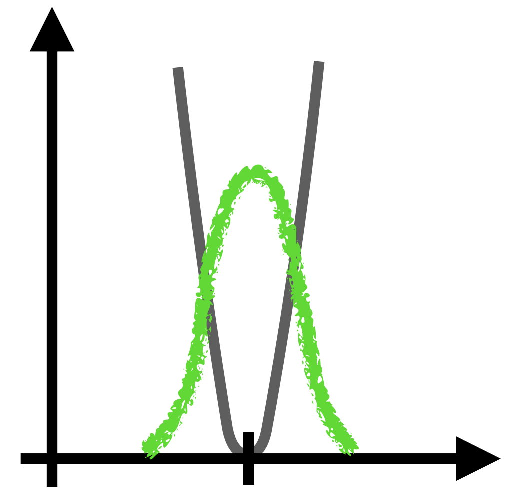




Tononi, FC, Salasnich PRL 2020
Tononi et al. Nat Phys Rev 2013

Bubble traps





$$r = \sqrt{\Delta} \;\Rightarrow\; u_{ext}(\sqrt{\Delta}) = 0$$

Small oscillation limit

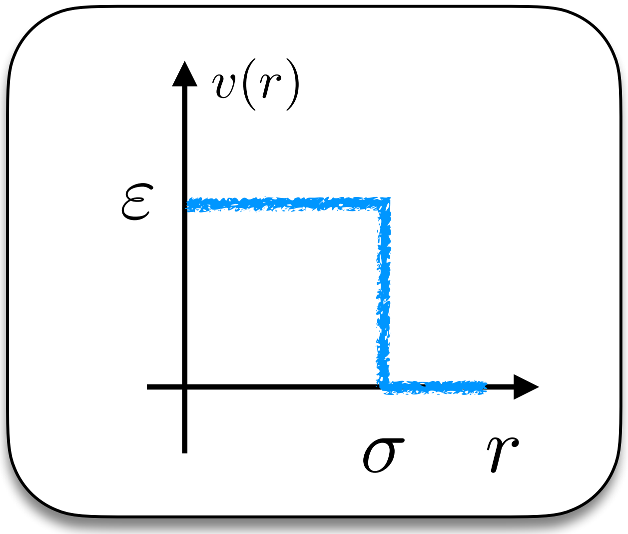
$$r = x + \sqrt{\Delta}$$

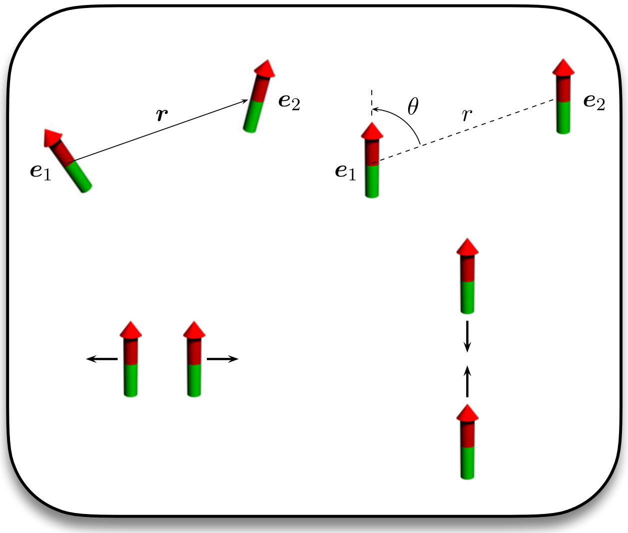
$$u_{ext}(x) \approx \frac{u_0 \Delta}{2\Omega^2} x^2$$

$$u_{ext}(r) = u_0 \left(\sqrt{\frac{(r^2 - \Delta)^2}{4\Omega^2} + 1} - 1 \right)$$

Sun et al. PRA 2018

Physical model and interactions





$$v_{int}(r) = \varepsilon\theta(r-\sigma)$$

$$v_{int}(\mathbf{r}) = v_{HS}(r) + \frac{\mu_0 d_m^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

Soft-core interactions - Rydberg atoms
e.g. see Srakaew NatPhys 2023

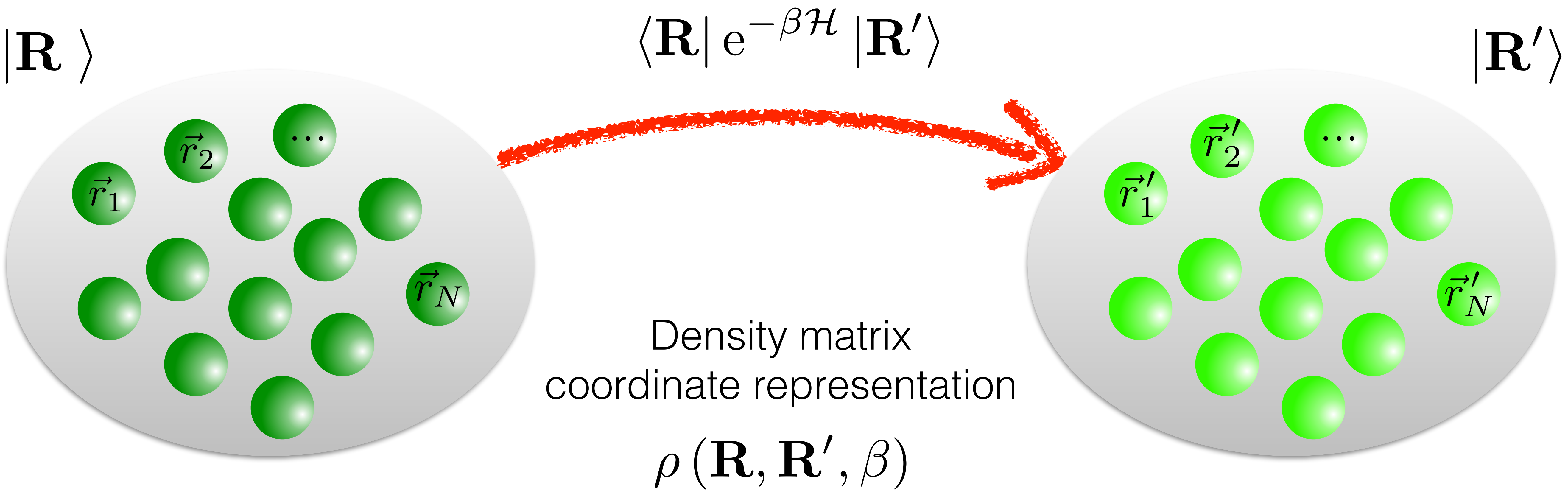
Dipole-dipole interactions - lanthanides
e.g. see Biagioni Nature 2024

$$H = \sum_{i=1}^{N}\left(-\lambda\nabla^2_{\mathbf{r}_i} + u_{ext}(|\mathbf{r}_i|)\right) + \sum_{i<j} v_{int}(\mathbf{r}_i - \mathbf{r}_j)$$

Further info: • spineless bosons • canonical ensemble • finite temperature • $\lambda = \hbar^2/2m$

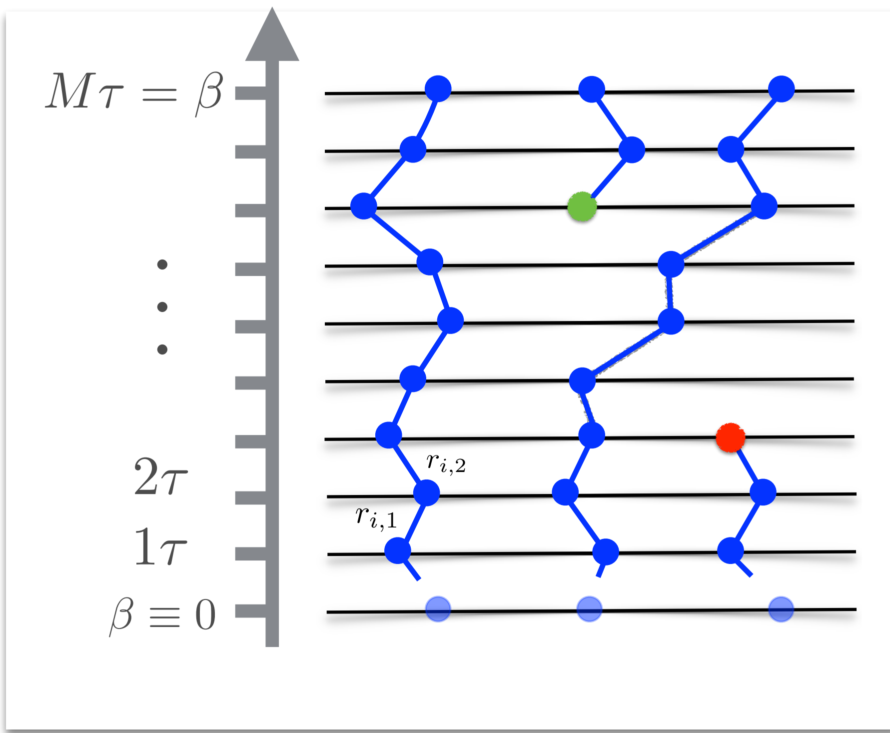
Path Integral Monte Carlo

- ☑ Accurate: no adjustable parameter (microscopic Hamiltonian only input)
- ☑ Unbiased: no a priori assumption needed (e.g., trial wave function)
- ☑ Numerically exact for Bose systems
- ☑ Finite temperature and *T* —> 0
- ☑ Access to quantum thermodynamic properties

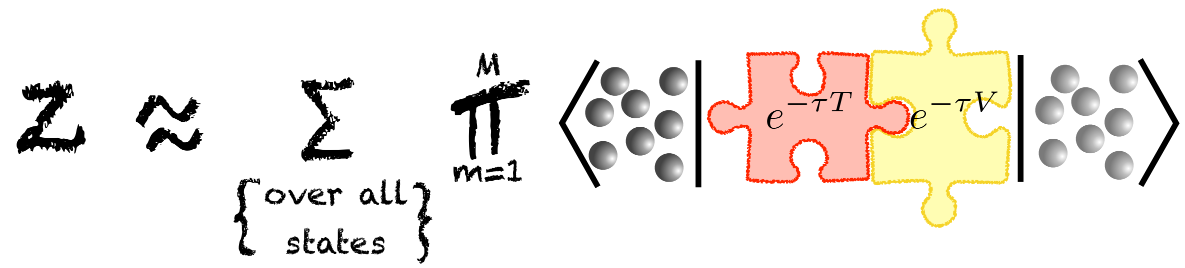


Ceperley RMP 1995

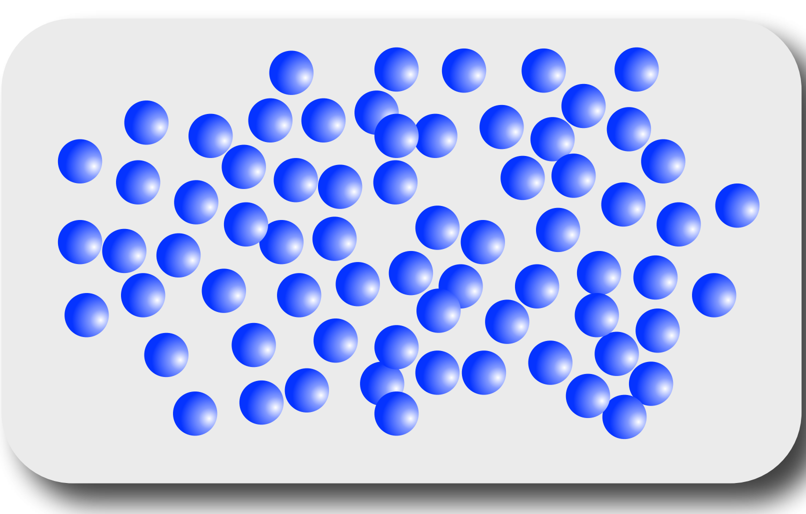
PIMC & Worm algorithm

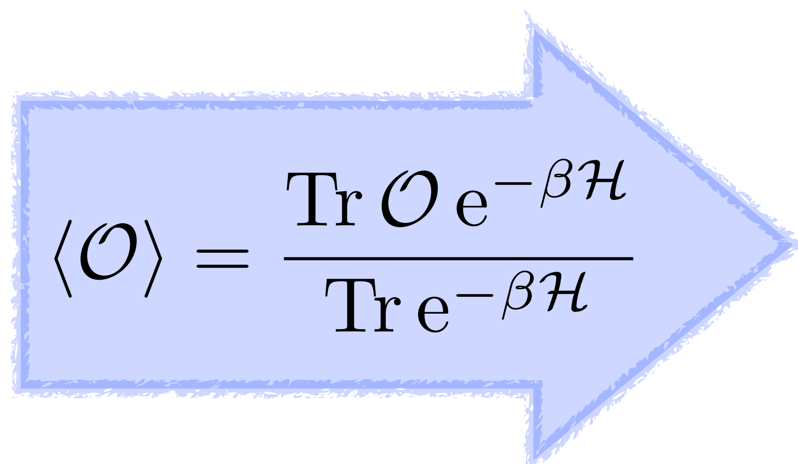


The configuration space is generalised from the partition function to the Matsubara-Green function



Important: approximation of the many-body density matrix! There exist many recipes on the market…





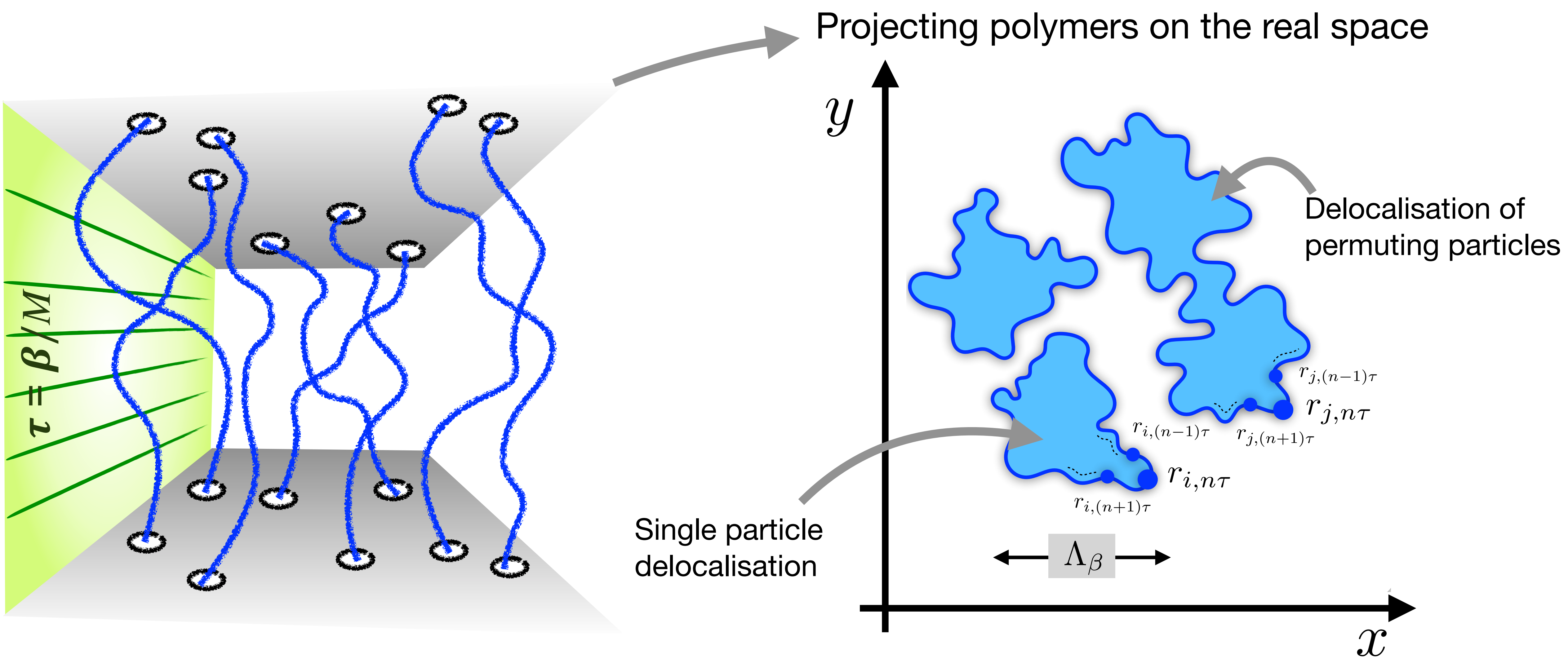
$$\langle \mathcal{O} \rangle = \frac{\mathrm{Tr}\, \mathcal{O}\, \mathrm{e}^{-\beta \mathcal{H}}}{\mathrm{Tr}\, \mathrm{e}^{-\beta \mathcal{H}}}$$

- Energy
- Superfluidity
- Condensation
- Correlations & dynamical properties
- Structural properties
- …

Ceperley RMP 1995

Boninsegni et al. PRL 2006

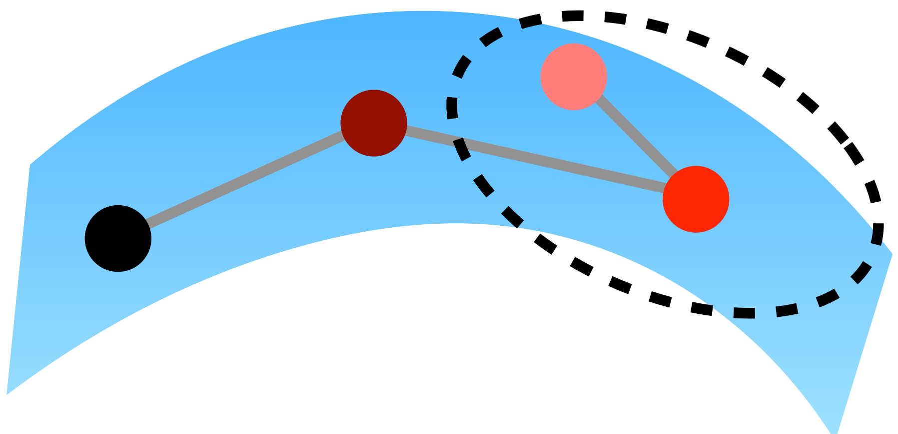
Path Integral Monte Carlo



- Partition function becomes a classical system of polymers. Every polymer is a necklace of beads connected by springs (i.e. the kinetic energy)
- The mean square displacement of the polymer's beads is of the order of the de Broglie thermal wave length

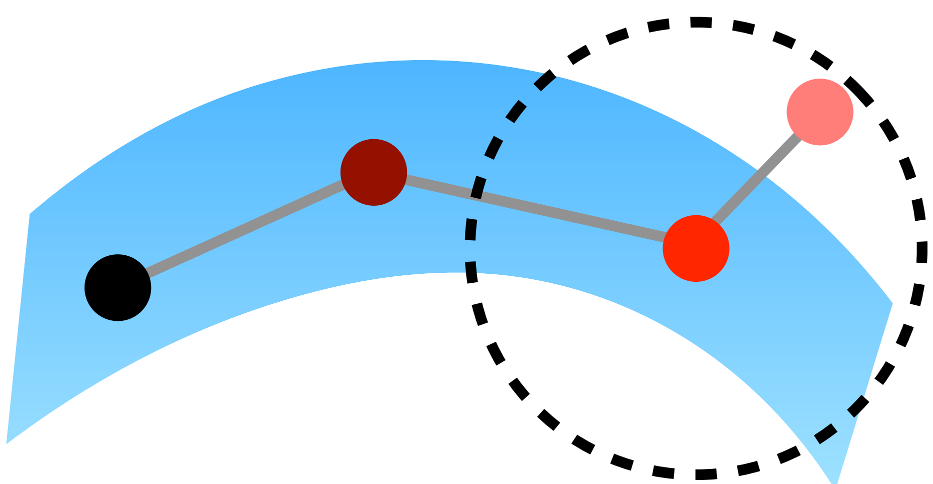
Monte Carlo sampling

$\sigma_r, \, \sigma_\theta$



Biased PIMC

$$\sigma = \sqrt{2\lambda\tau}$$

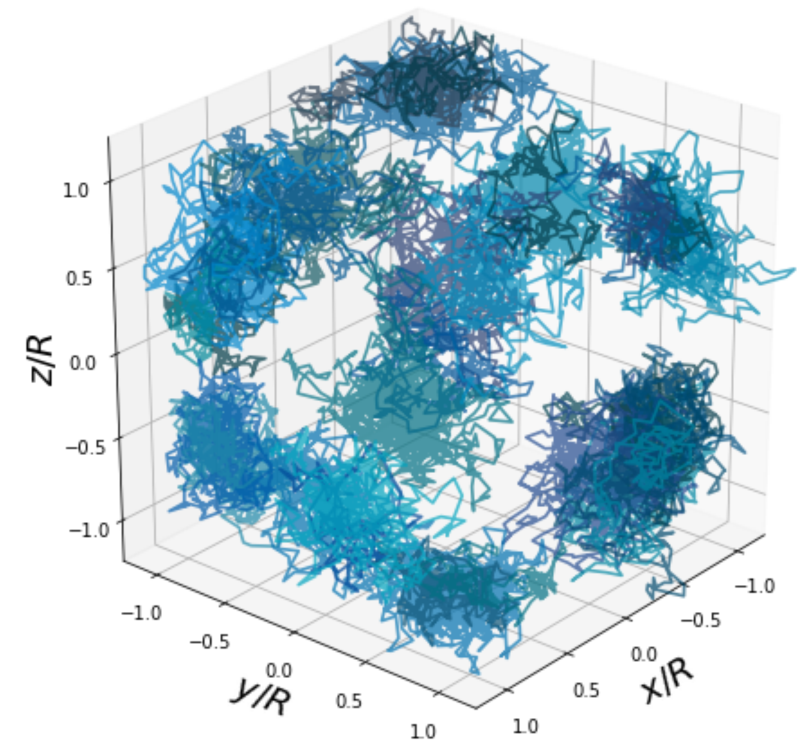


Standard PIMC

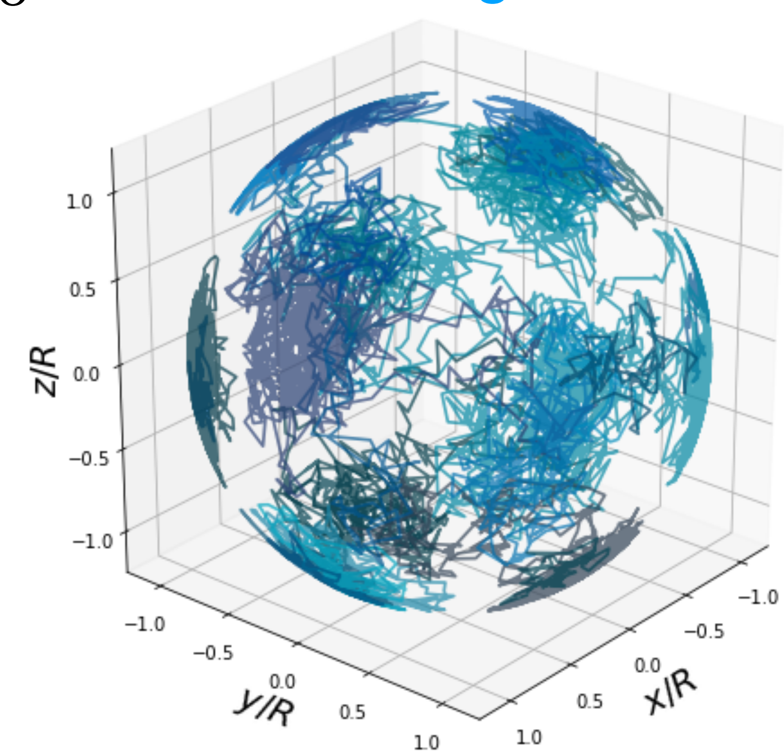
$$\rho_{free}(\mathbf{r},\mathbf{r}';\tau) = \frac{1}{(4\pi\lambda\tau)^{d/2}} \exp\left\{-\frac{(\mathbf{r}-\mathbf{r}')^2}{4\lambda\tau}\right\}$$

$$\rho(\mathbf{r},\mathbf{r}';\tau) = \rho_{free}(\mathbf{r},\mathbf{r}';\tau)\, e^{-\tau V(\mathbf{r})}$$

$u_0 = 2$ Weak confinement

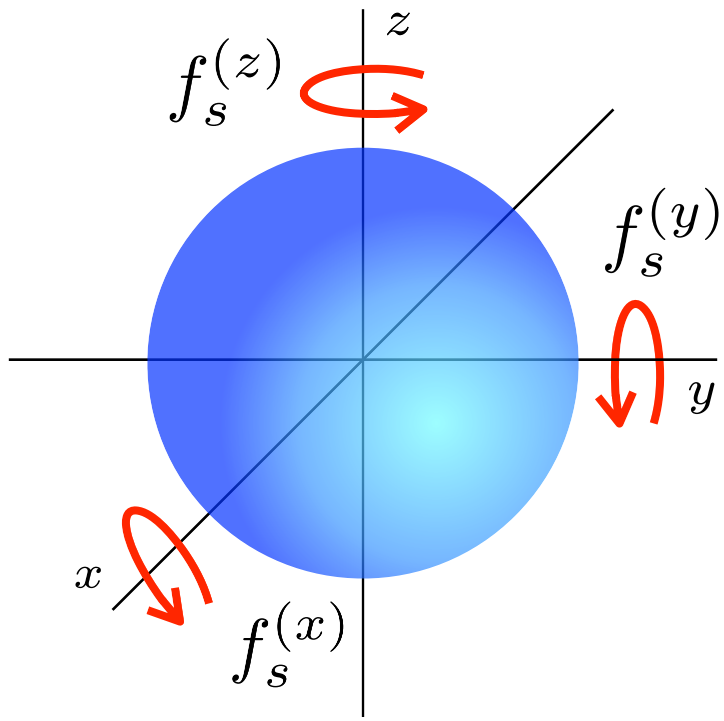


$u_0 \to \infty$ Strong confinement



Monte Carlo sampling

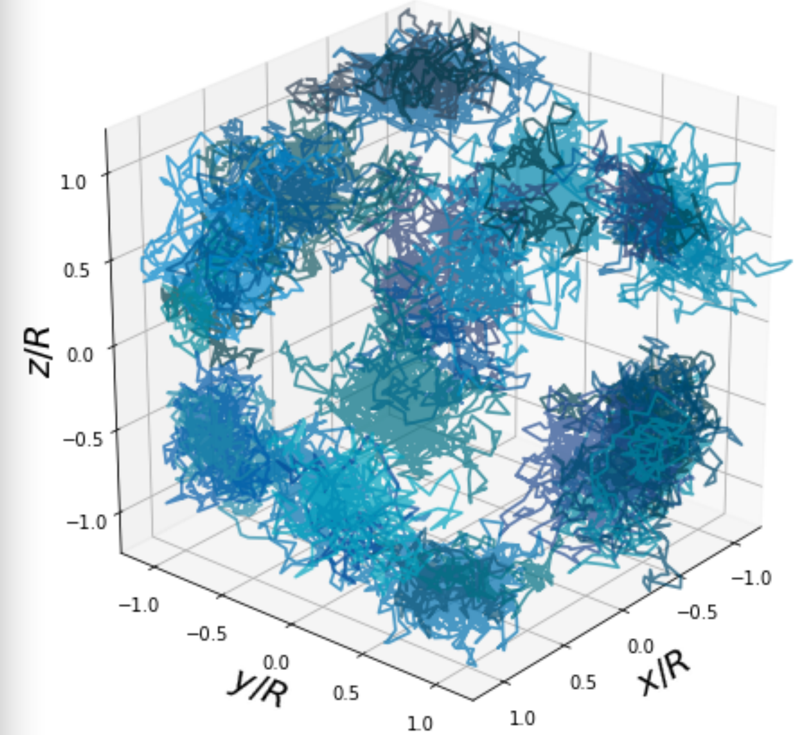
☞ Sampling superfluid fraction ☜



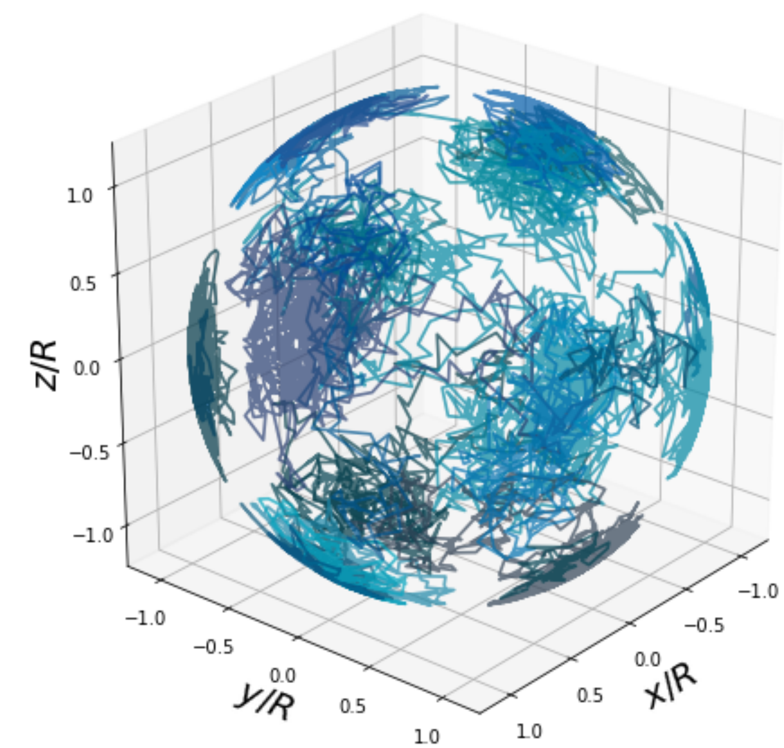
$$f_s^{(k)} = \frac{4m^2}{\hbar^2 \beta} \frac{\langle A_k^2 \rangle - \langle A_k \rangle^2}{I_{cl}^{(k)}}$$

$$A_k = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=0}^{M-1} \left(\mathbf{r}_i^j \times \mathbf{r}_i^{j+1} \right)_k$$

$u_0 = 2$

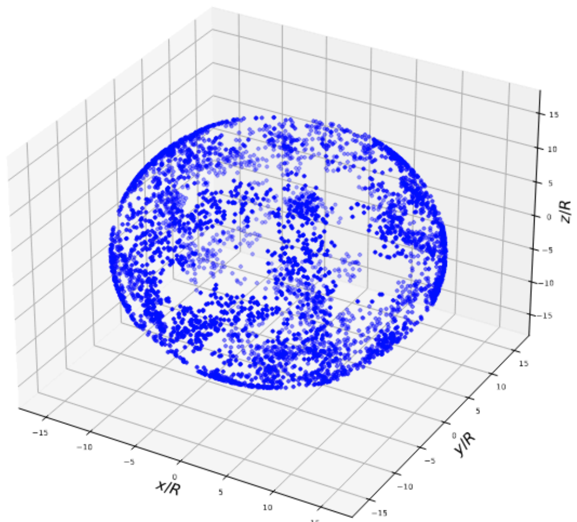


$u_0 \to \infty$

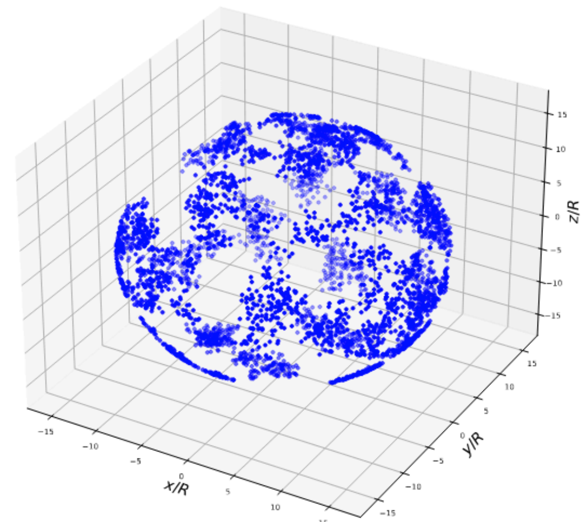


BKT transition on spheres

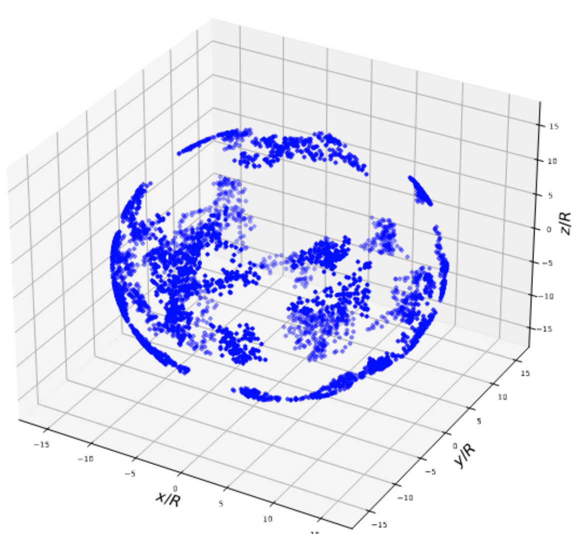
T/T_0=0.06



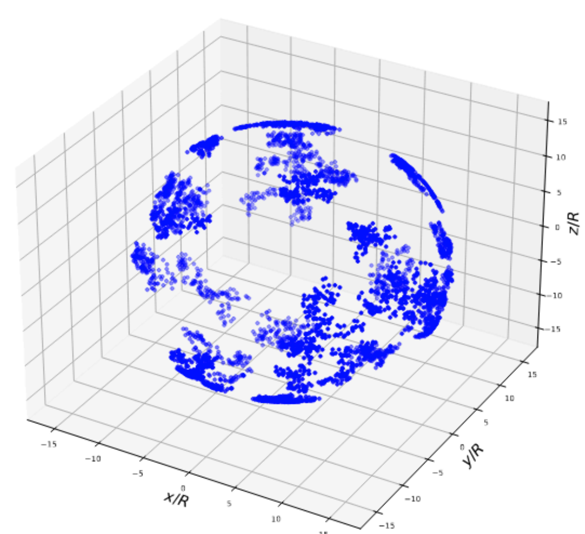
T/T_0=0.08



T/T_0=0.18



T/T_0=0.30



Adimensional superfluid density

$$K = \beta J = \frac{n_s \hbar^2}{m k_B T}$$

Fugacity

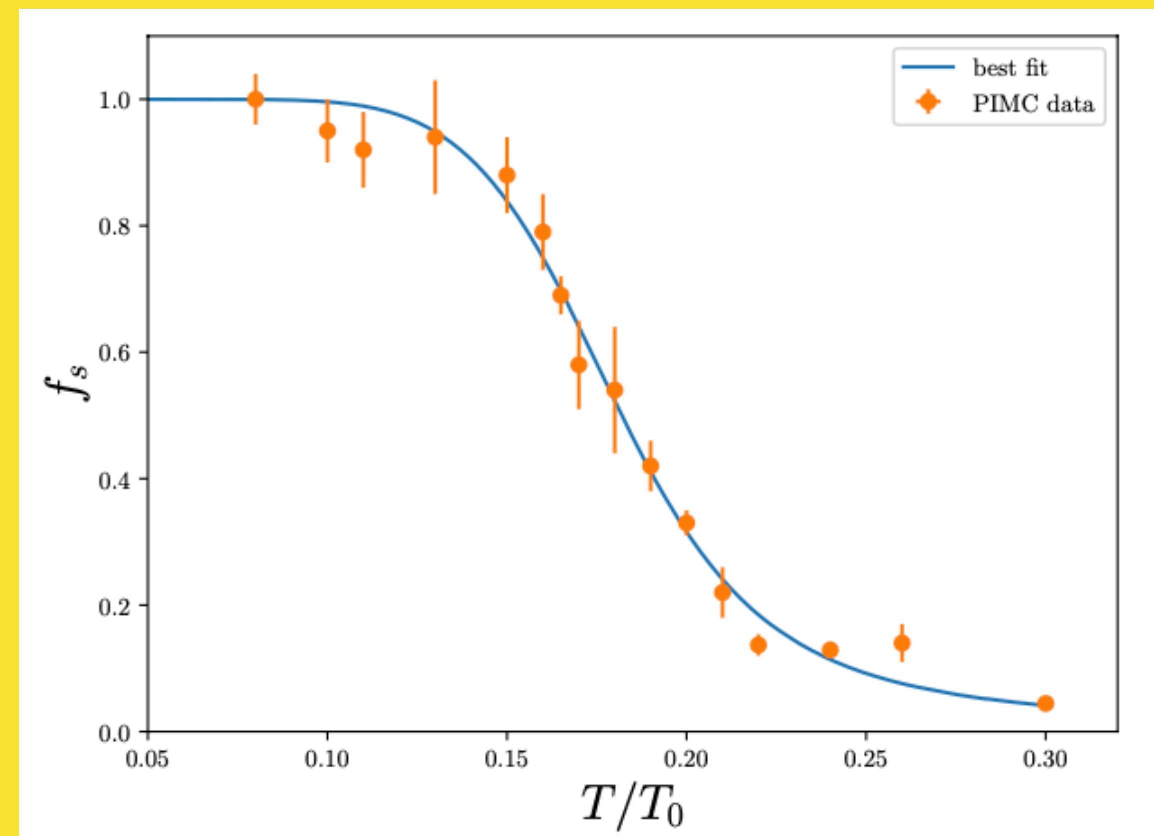
$$y = e^{-\beta E_c}$$

Spherical Case

$$\begin{cases} \dfrac{d\,K^{-1}(\theta)}{dl(\theta)} = 4\pi^3 y(\theta)^2 \\ \dfrac{dy}{dl(\theta)} = (2 - \pi K)\, y(\theta)^2 \end{cases}$$

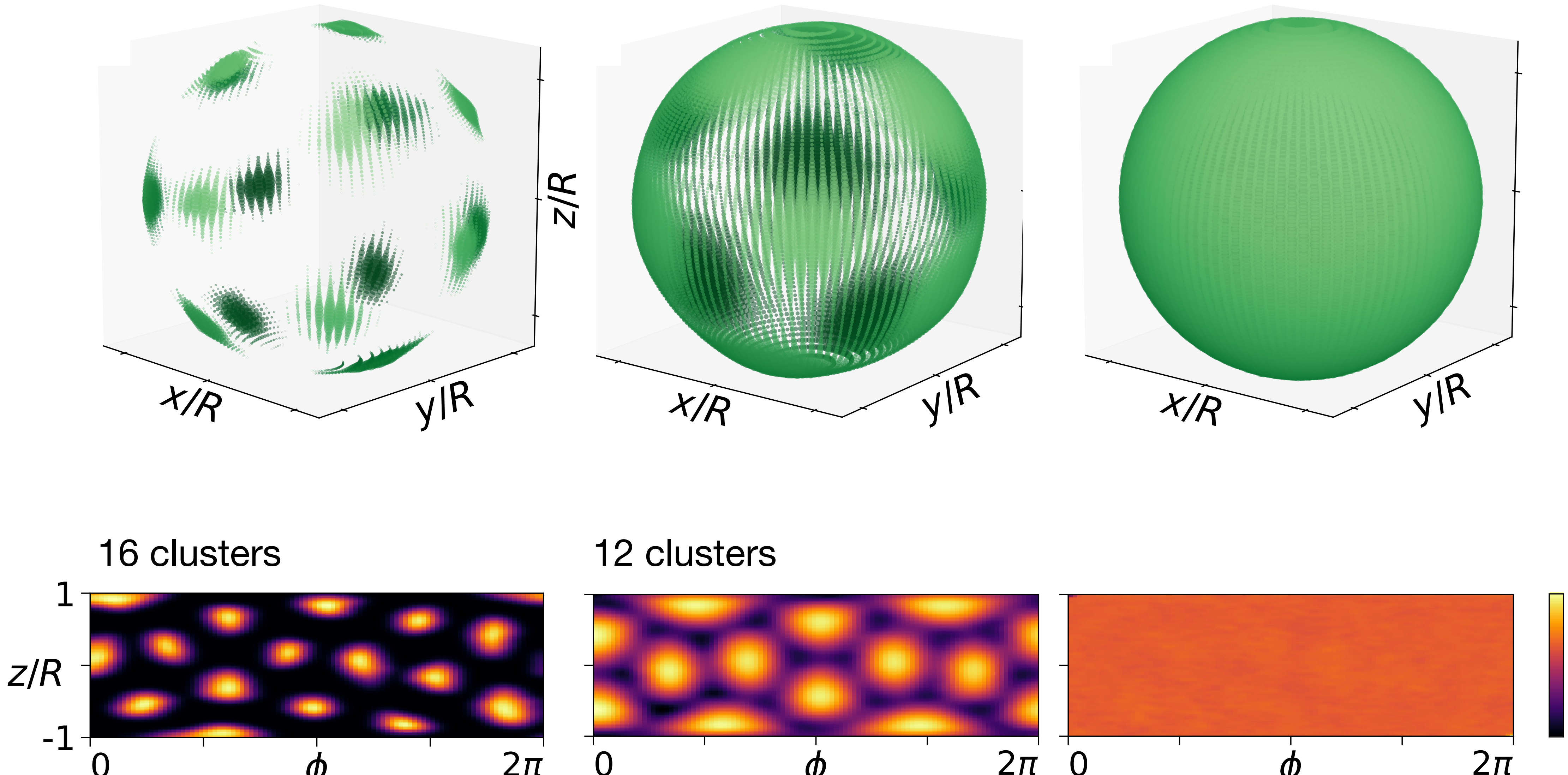
↓

Studying vortex interaction energy on a sphere



Bettarini, Ciardi, FC in preparation…

Soft-core interactions

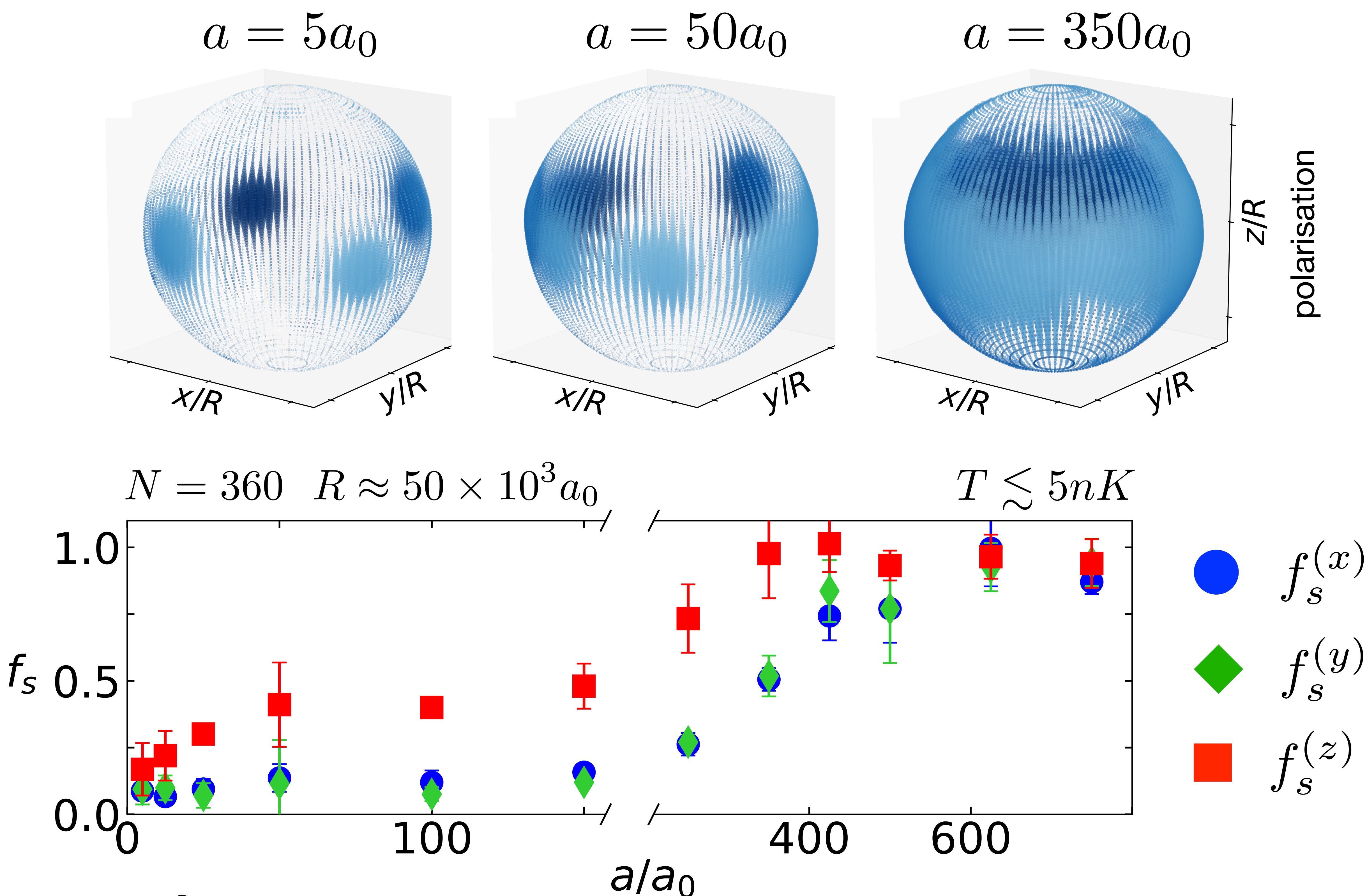


Fixing density and temperature

λ

Ciardi, FC, Pellicane, Prestipino PRL (2024)

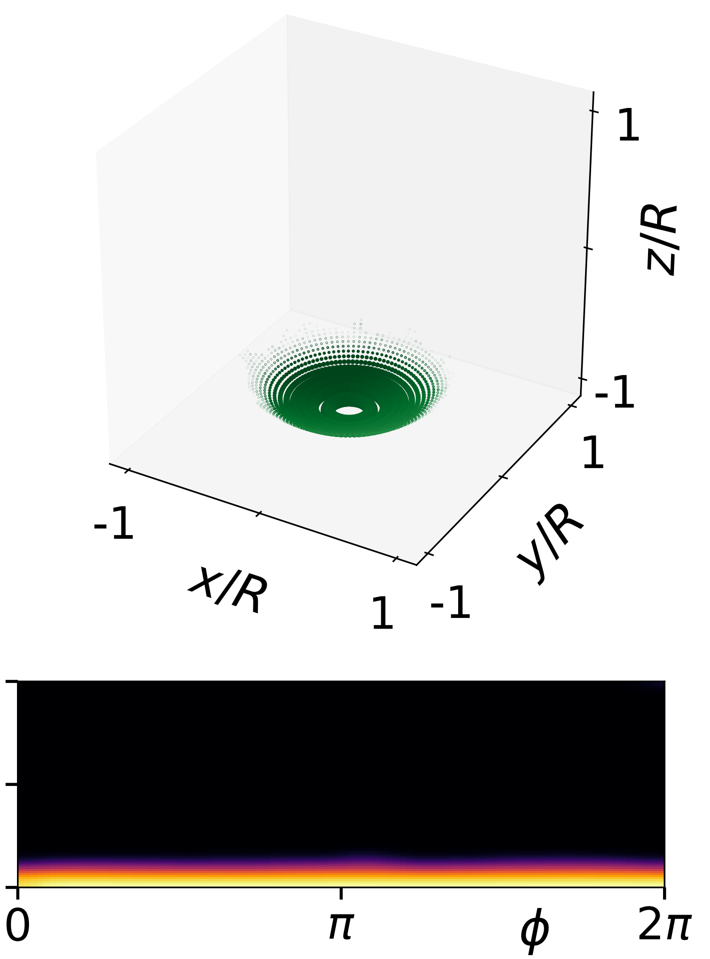
Dipolar bosons, thin-shell limit for Dy

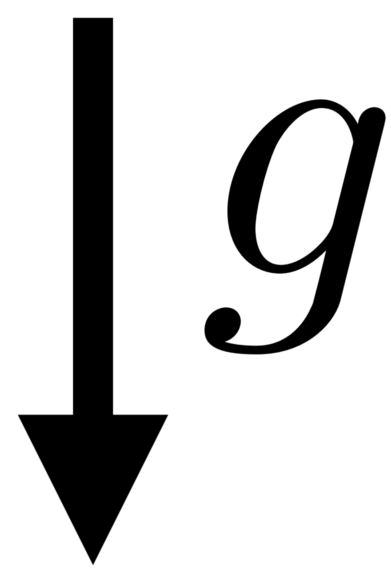


$\rho = 4\mu m^{-2}$ Using Lundblad et al. Microgravity 2019

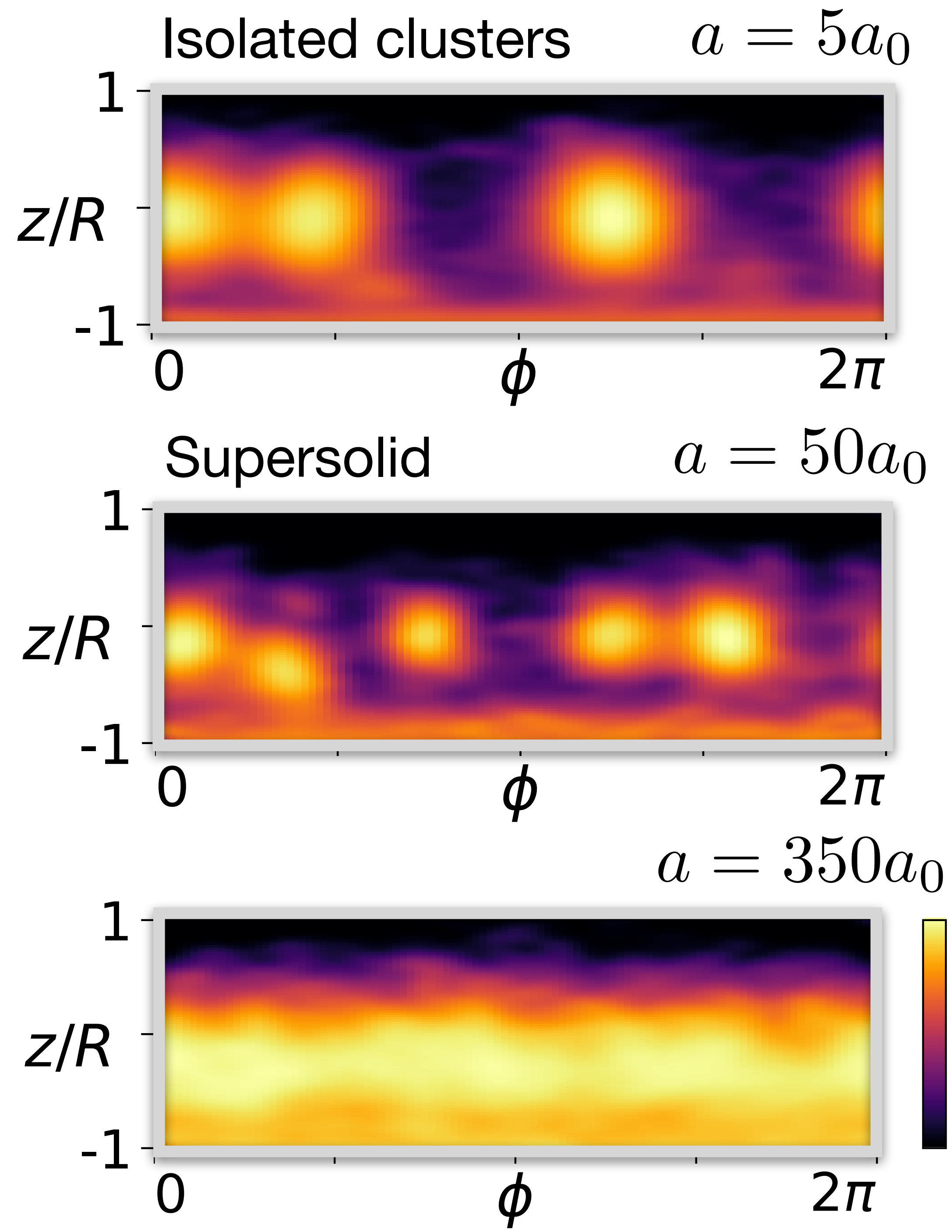
Gravity…

Hard core $a = 50a_0$



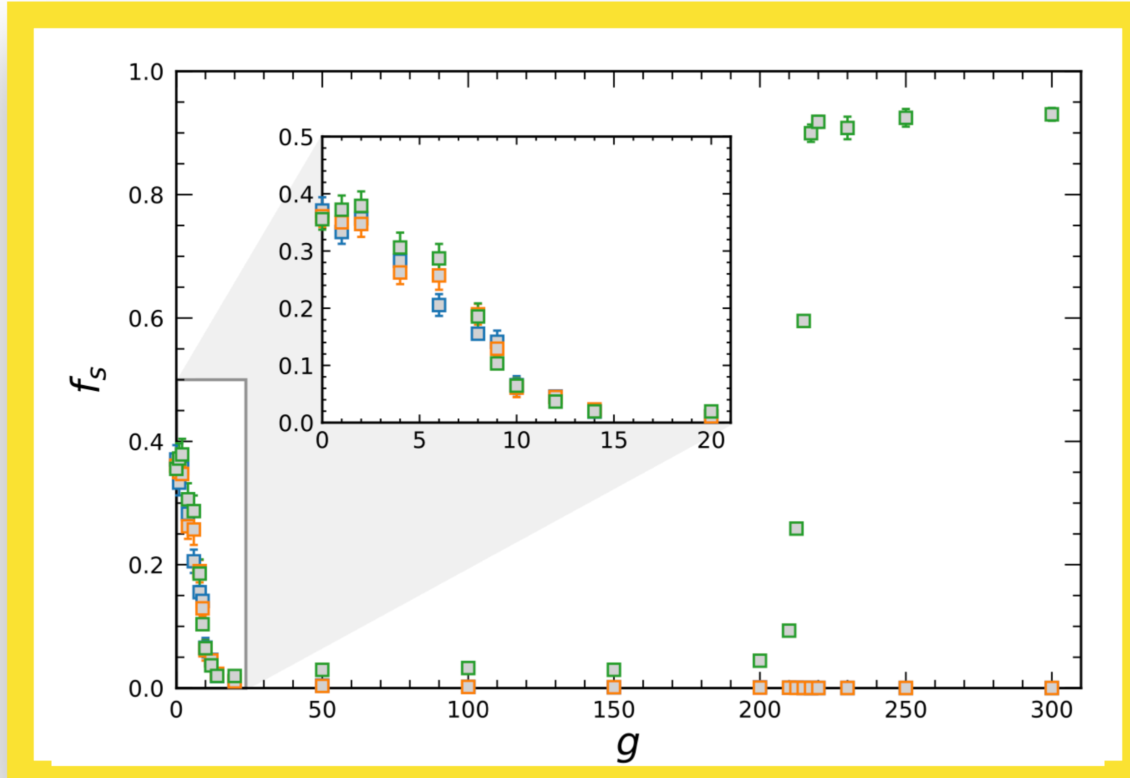


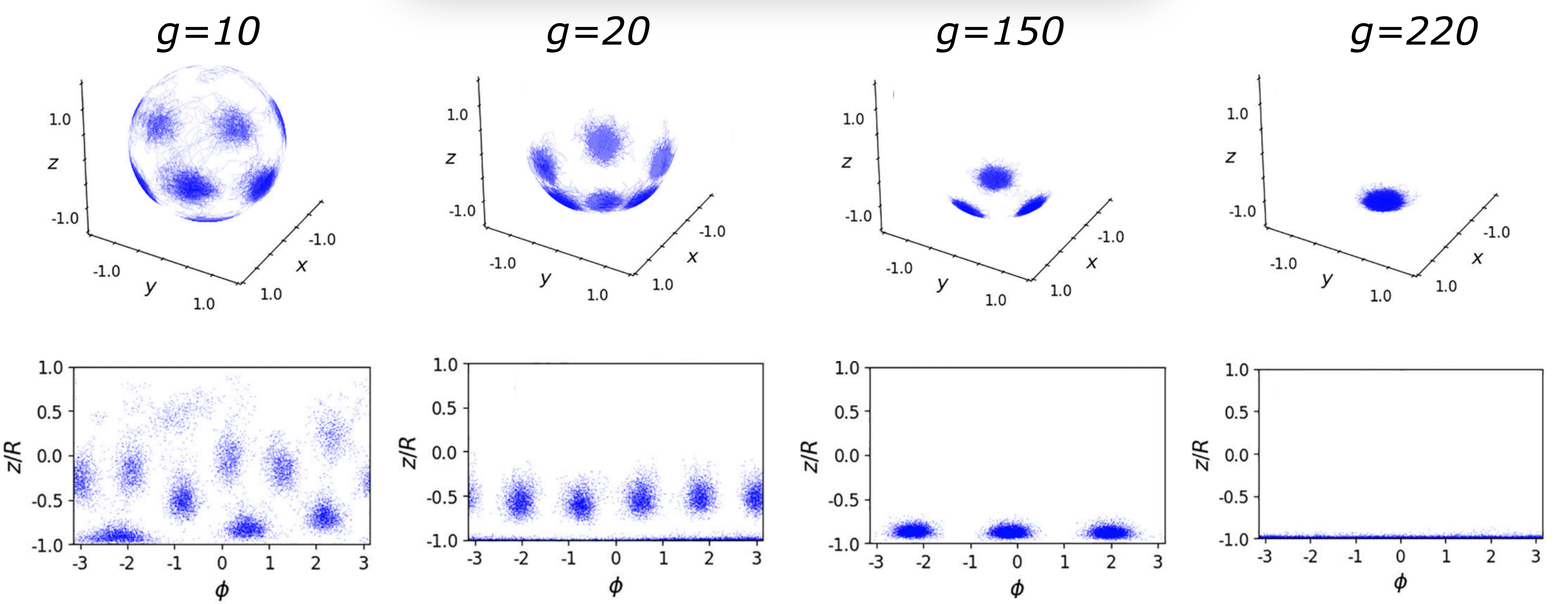
Dipoles



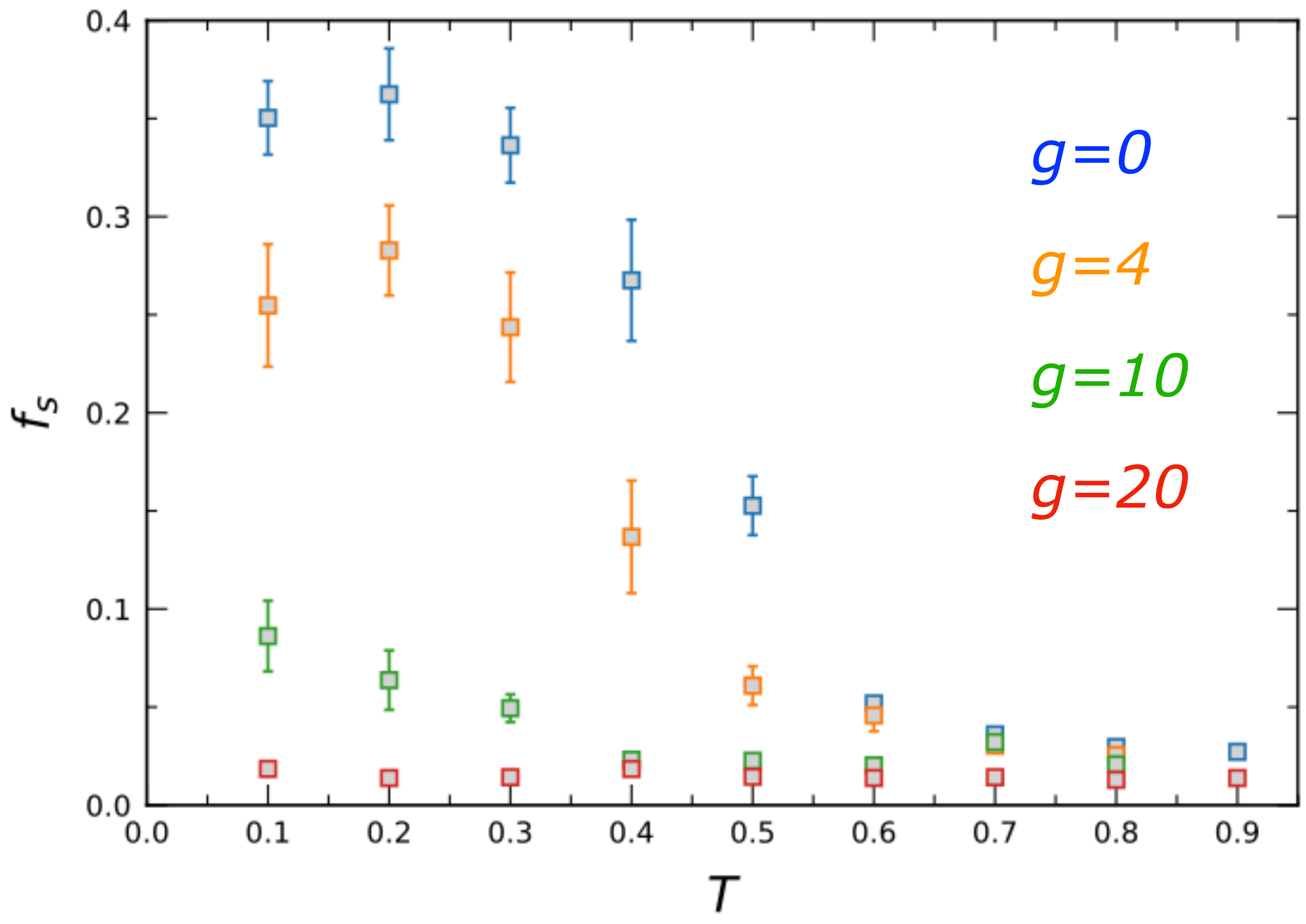
Phases are only slightly disturbed by gravity

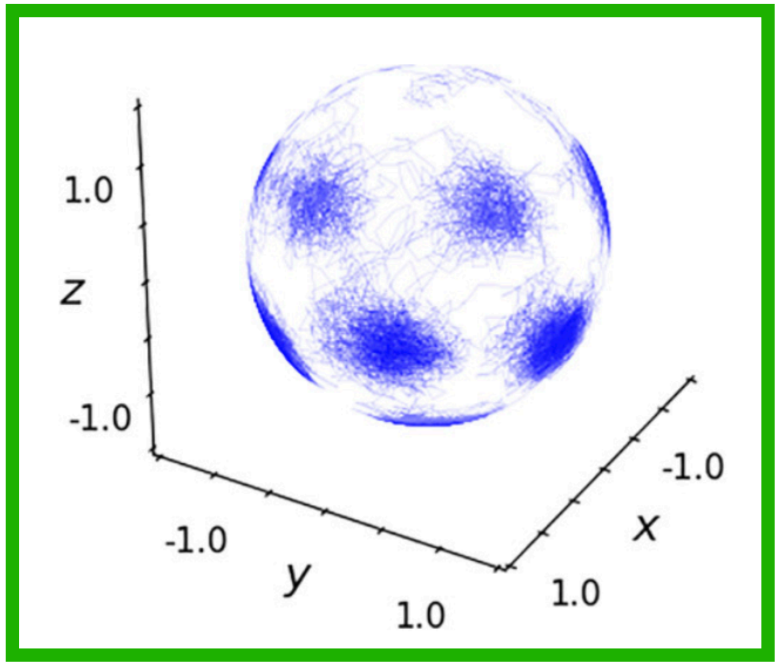
Ground state limit: pick the planet you like…





Temperature and feasible parameters





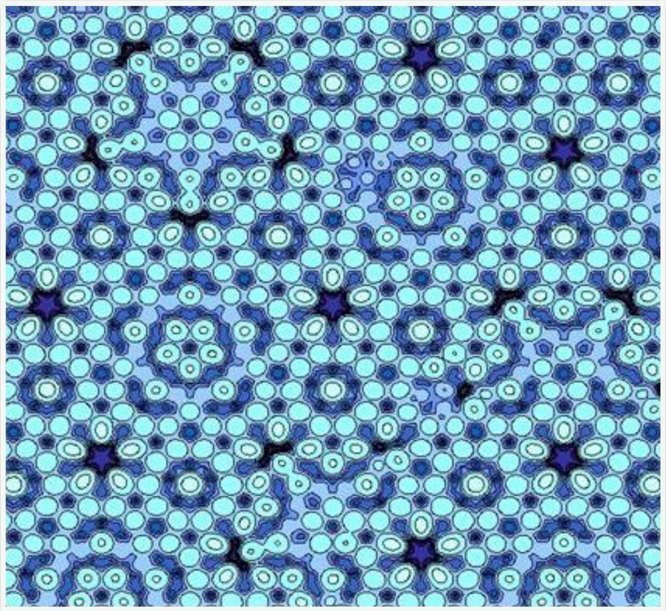
What is g in here?…

$$V(z) = mgz$$

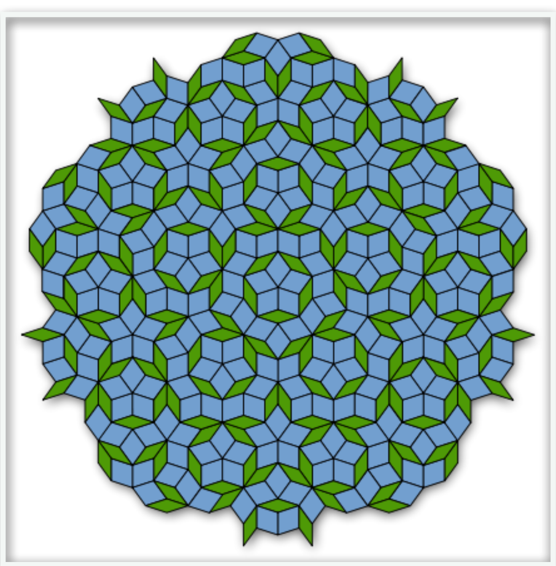
Considering Rb atoms and g_{earth}= 9.81, corresponds to g ~ 8 in these simulations

Ciardi, FC, Pellicane, Prestipino PRB (2025)

Quasicrystals



Silver depositing on Al-Pd-Mn quasicrystal surface



Math: Penrose tiling

- Symmetries in crystallography: 1-, 2-, 3-, 4- and 6-fold (from 2π to $2\pi/6$) symmetries are allowed
- But 5-, 7-fold and higher fold are forbidden
- Everything changed when Shechtman et al. (1984) observed an icosahedral single grain with a 5-fold symmetries…

VOLUME 53, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1984

Metallic Phase with Long-Range Orientational Order and No Translational Symmetry

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and

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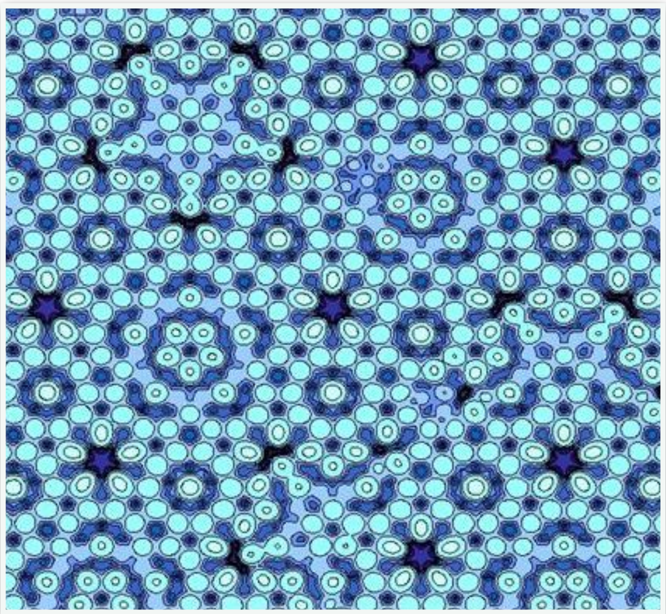
and

J. W. Cahn
Center for Materials Science, National Bureau of Standards, Gaithersburg, Maryland 20760
(Received 9 October 1984)

We have observed a metallic solid (Al–14-at.%-Mn) with long-range orientational order, but with icosahedral point group symmetry, which is inconsistent with lattice translations. Its diffraction spots are as sharp as those of crystals but cannot be indexed to any Bravais lattice. The solid is metastable and forms from the melt by a first-order transition.

Quasicrystals



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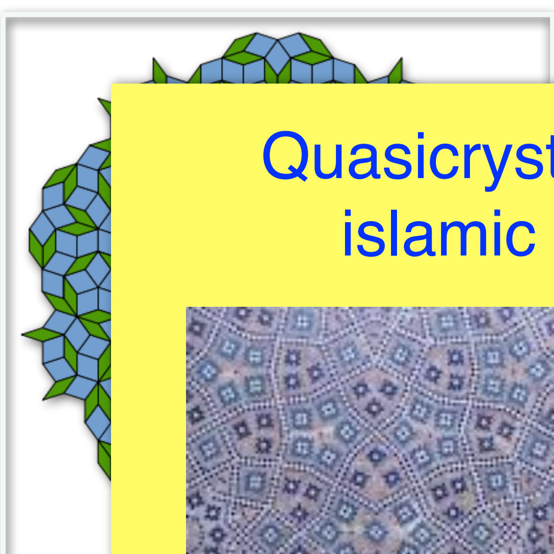
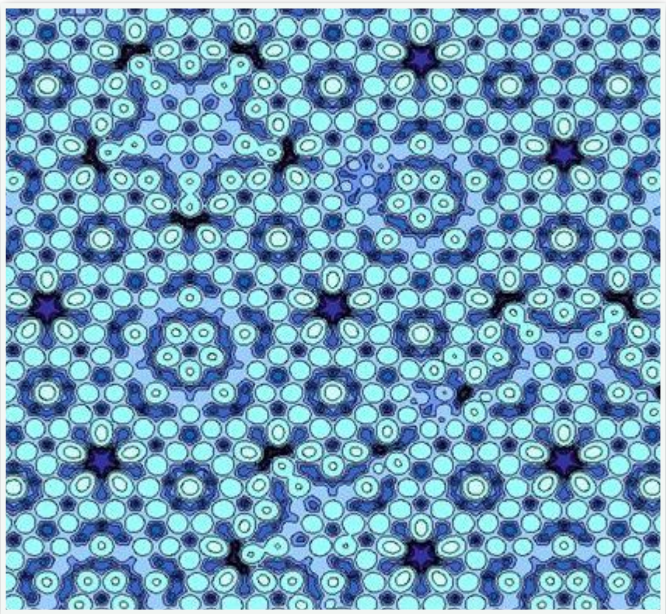


Math…

Quasicrystals in <u>medieval</u> islamic architecture!

Quasicrystals



Silver depositing on Al-Pd-Mn quasicrystal surface

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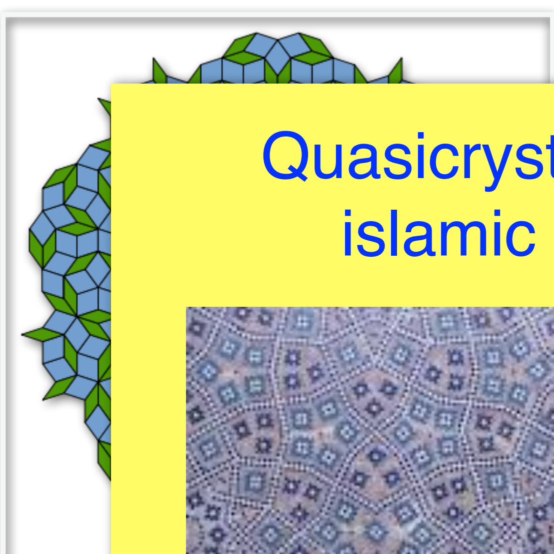
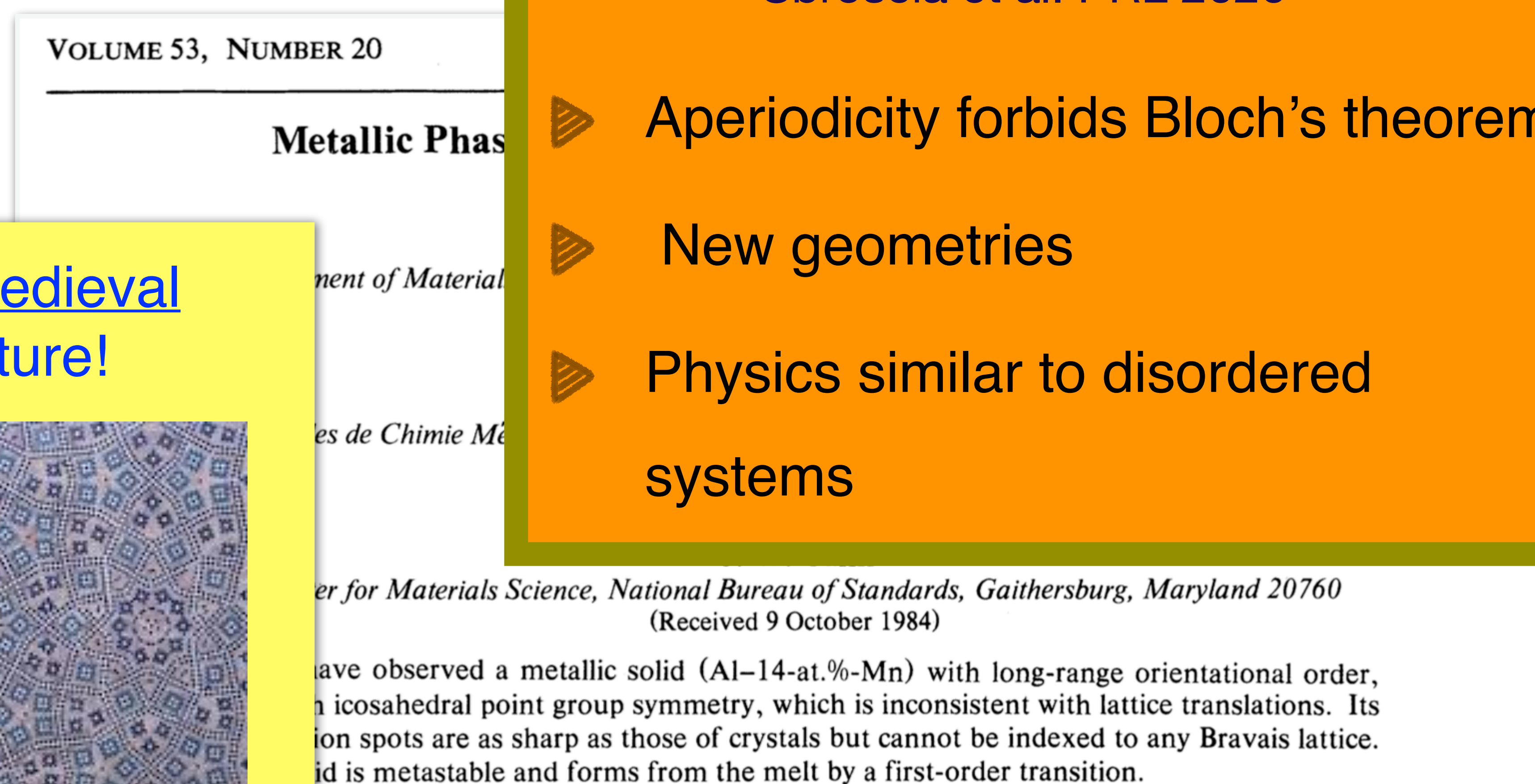
  observed an icos
  symmetries…



Math



VOLUME 53, NUMBER 20

Metallic Phas

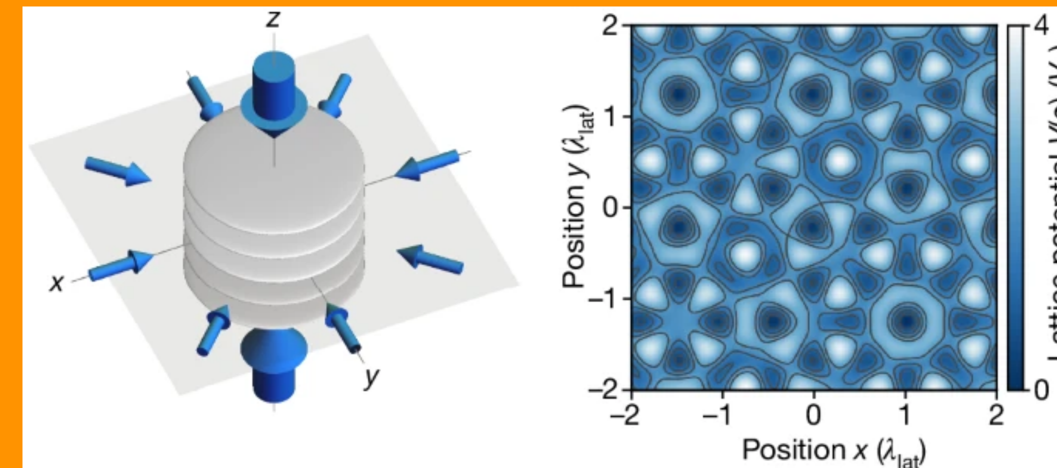
nent of Material

es de Chimie Mé

er for Materials Science, National Bureau of Standards, Gaithersburg, Maryland 20760
(Received 9 October 1984)

ave observed a metallic solid (Al–14-at.%-Mn) with long-range orientational order,
n icosahedral point group symmetry, which is inconsistent with lattice translations. Its
ion spots are as sharp as those of crystals but cannot be indexed to any Bravais lattice.
id is metastable and forms from the melt by a first-order transition.

Quasicrystals in medieval islamic architecture!

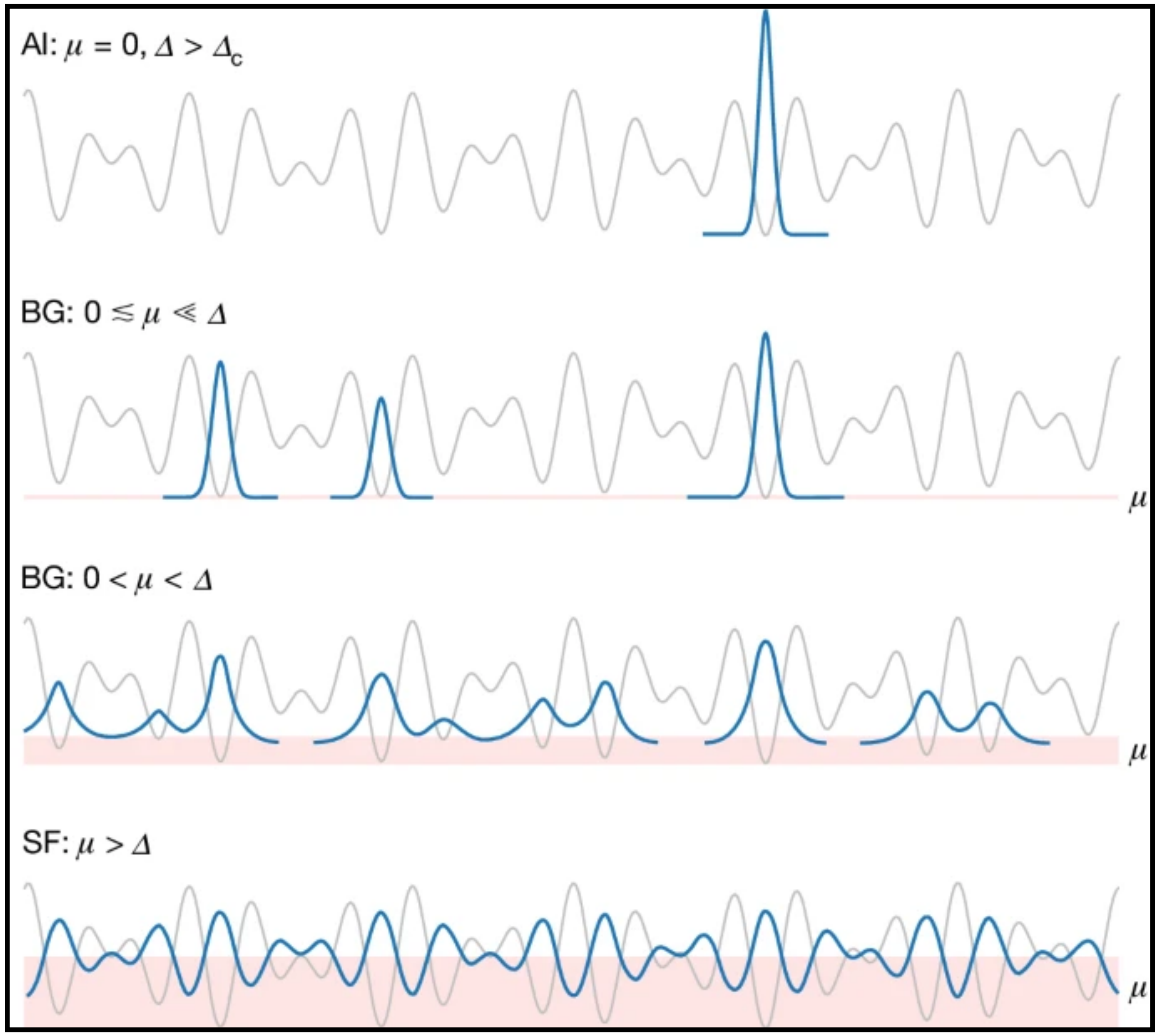


$$V_{qc}(\mathbf{r}) = V_0 \sum_{i=1}^{4} \cos^2(\mathbf{k}_i \cdot \mathbf{r})$$

Sbroscia et al. PRL 2020

- Aperiodicity forbids Bloch's theorem
- New geometries
- Physics similar to disordered systems

Optical quasicrystal lattice (QCL)



Nature **633**, 328 (2024)

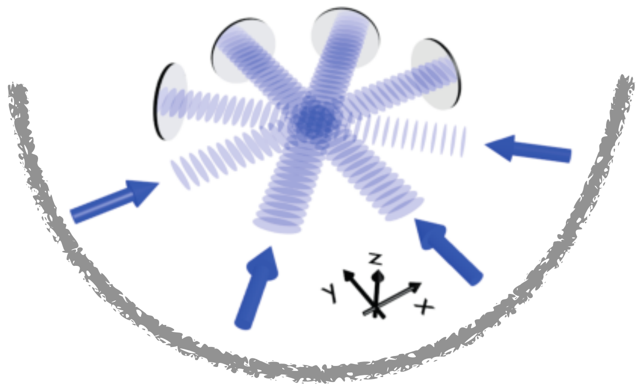
- No interactions $(\mu = 0) \Rightarrow$ Anderson Localization (AL)
- Increasing interaction (density) $\Rightarrow$ Bose Glass (BG)
- Eventually, the system turns superfluid (SF)
- **What is the effect of long-range interactions?**

μ: Chemical potential

Δ: Disorder intensity

Δ_c: Critical Δ for AL at $\mu = 0$

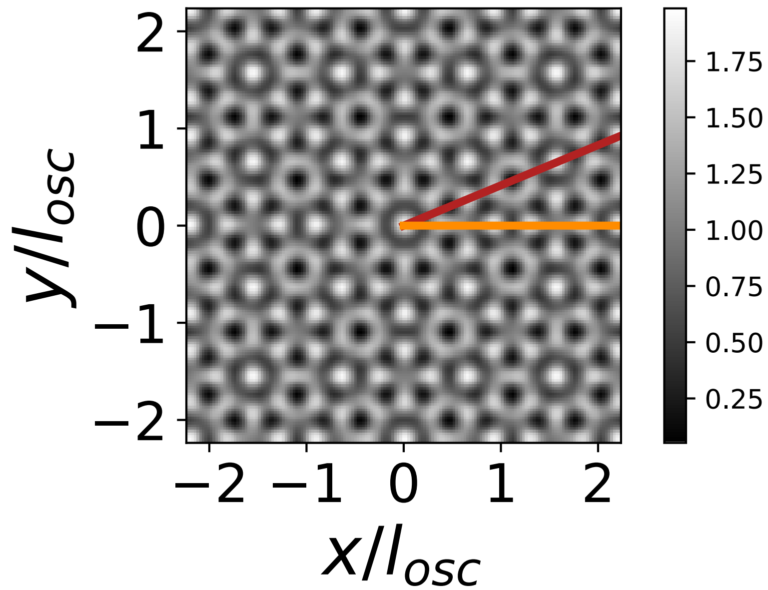
Bosons in quasi-periodic lattices

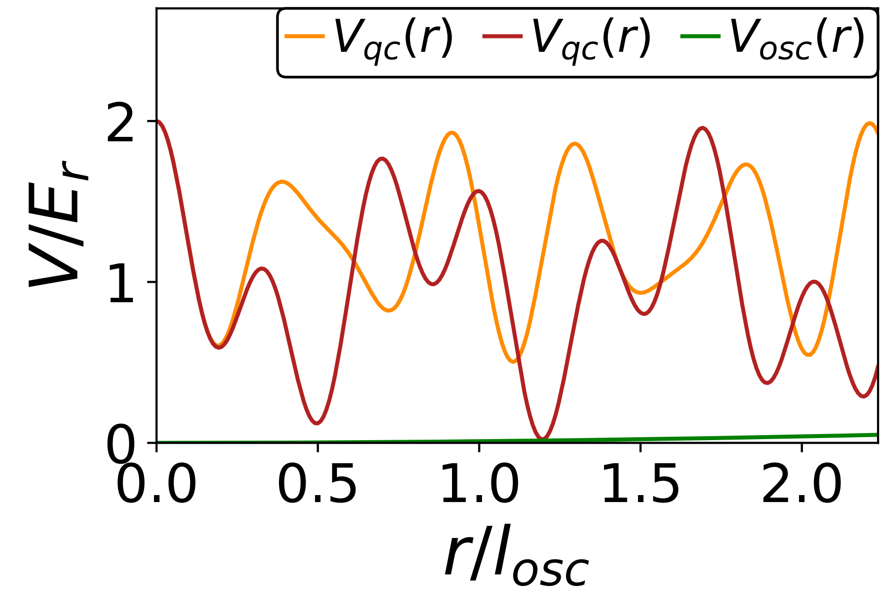


$$V_{qc}(\mathbf{r}) = V_0 \sum_{i=1}^{4} \cos^2(\mathbf{k}_i \cdot \mathbf{r})$$

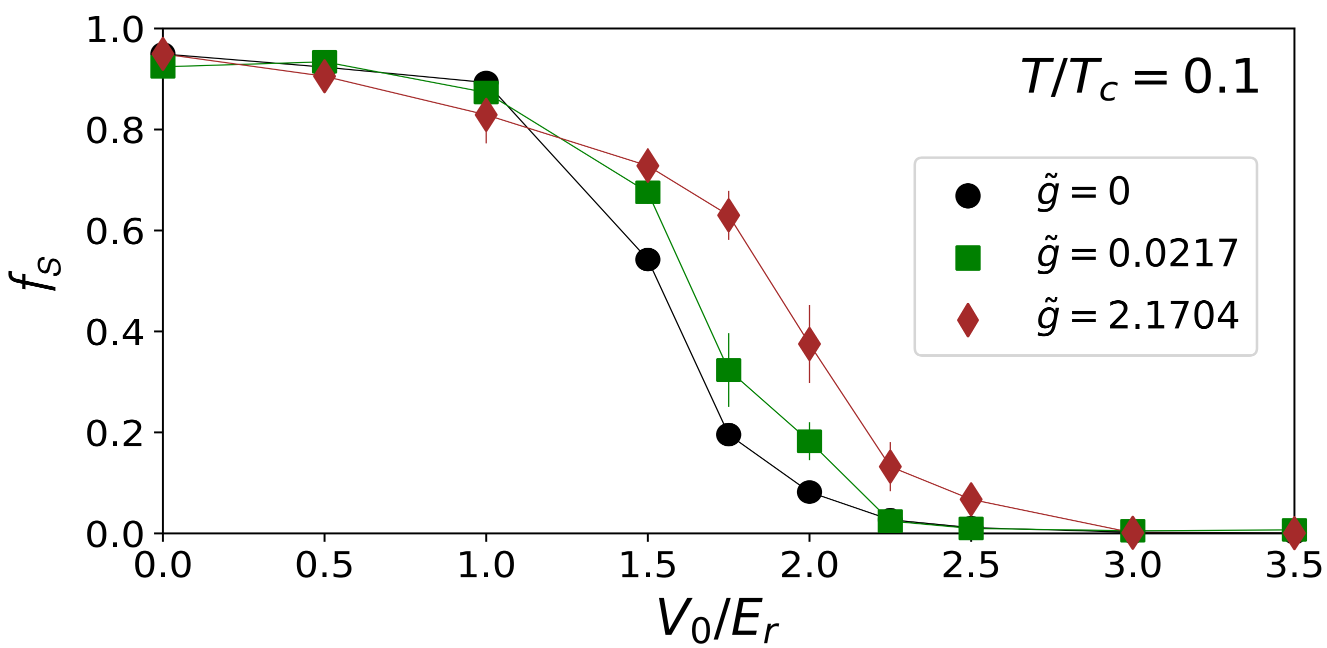
Sbroscia et al. PRL 2020

Viebahn et al. PRL 2019





Global superfluity T —> 0



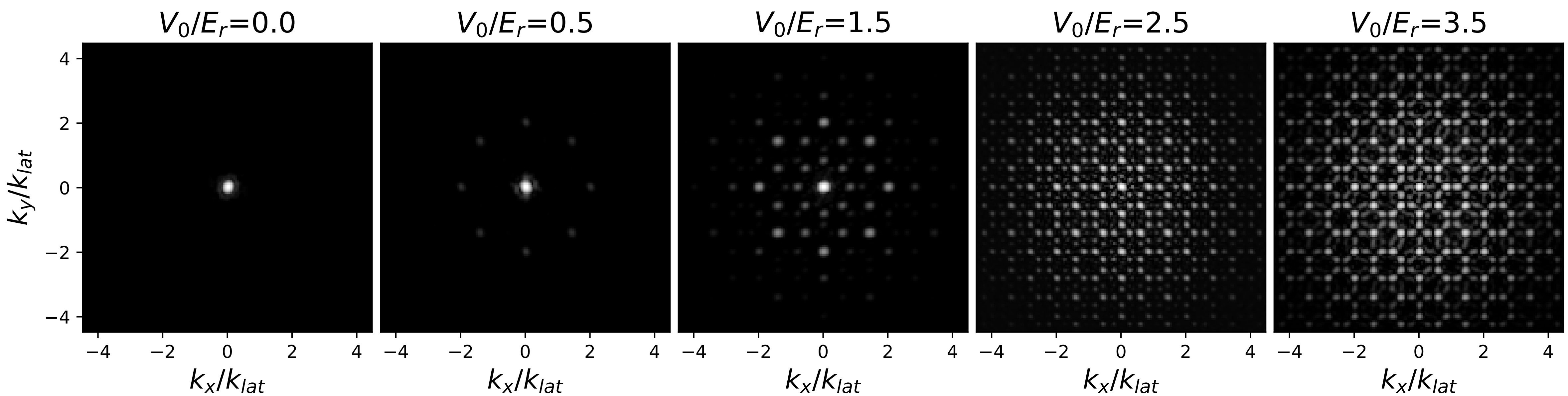
- Superfluid phase: $f_s = 1$
- Bose Glass: κ finite but f_s *local only*
- Insulator: f_s and κ both zero

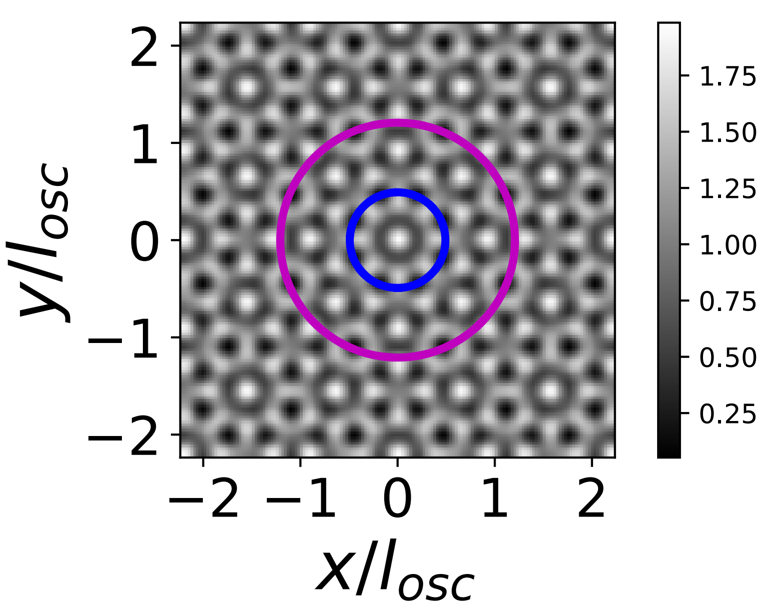
Ciardi, Macri, FC PRA 2022

Ciardi, Angelone, Mezzacapo, FC PRL 2023

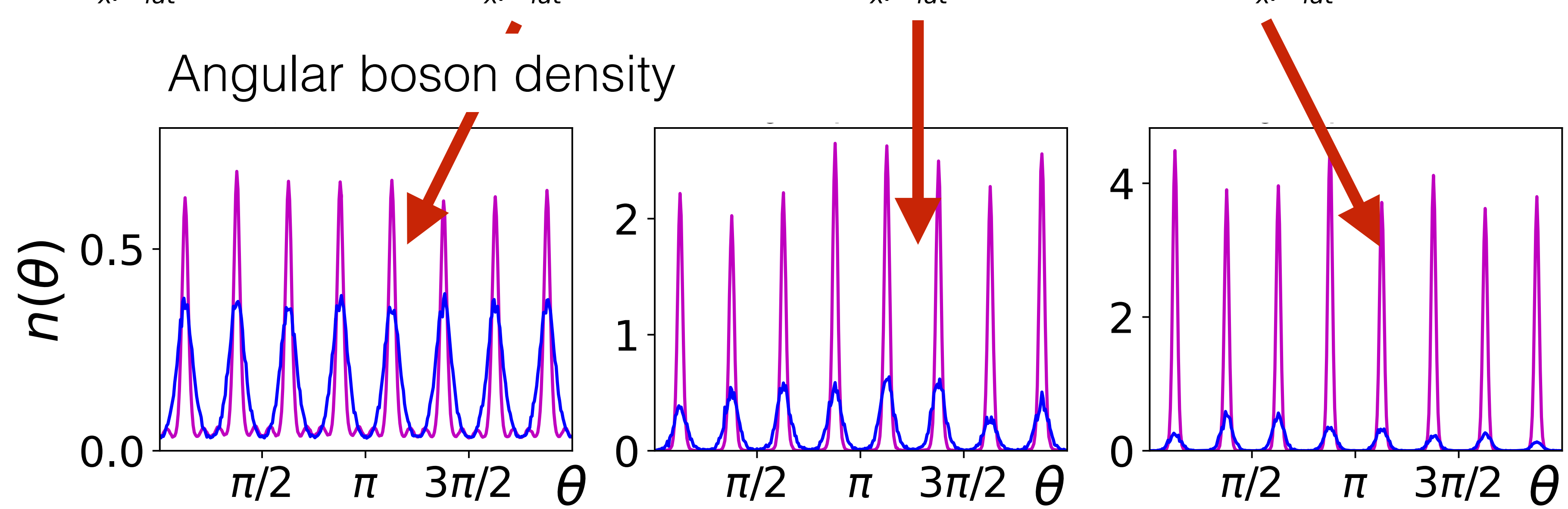
Bosons in quasi-periodic lattices

Diffraction patterns increasing V_0 at finite temperature $T< T_c$





Angular boson density



Superfluid phase in its GS

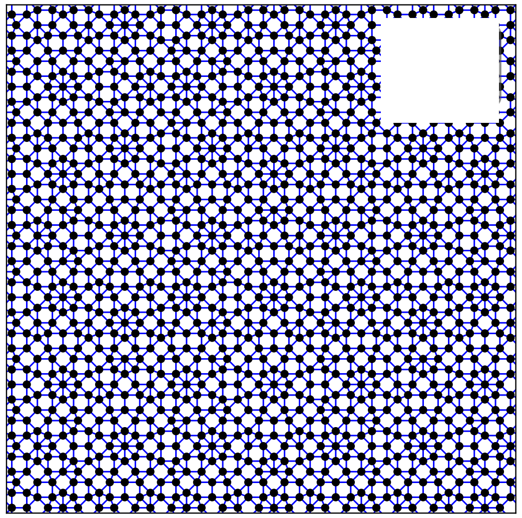
Superfluid puddles Bose Glass!

Insulating phase

$$l_{osc} = \sqrt{\hbar/m\omega}$$

$$E_r = \hbar^2 k_{lat}^2/2m$$

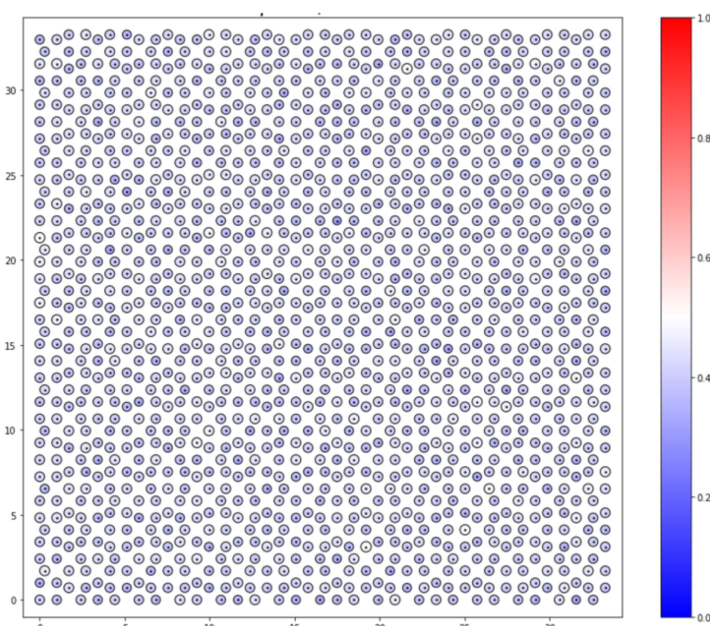
Removing the disorder…



$$H = -\sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + \mathrm{h.c.} \right) - \mu \sum_{i=1}^{N} n_i + V \sum_{\langle i,j \rangle} n_i n_j$$

Quasicrystalline substrate by taking as lattice sites the vertices of approximants of the Ammann-Beenker tiling

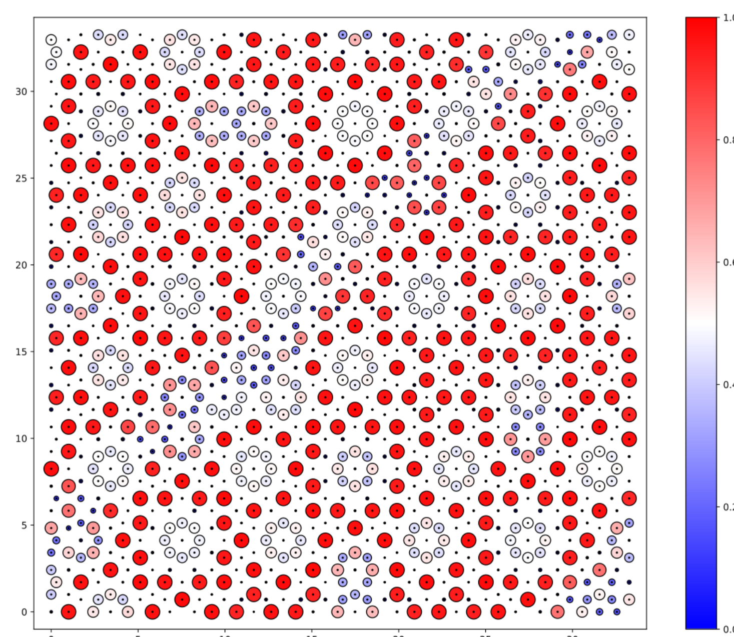
$\mu = 1 \; V = 4$



Condensate

delocalisation

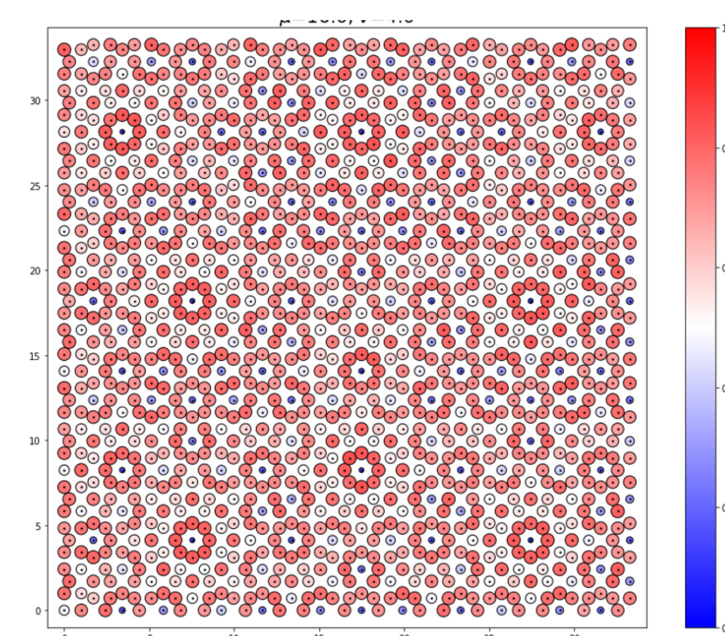
$\mu = 4 \; V = 8$



Bose Glass

Wheels host delocalised particles

$\mu = 16 \; V = 4$



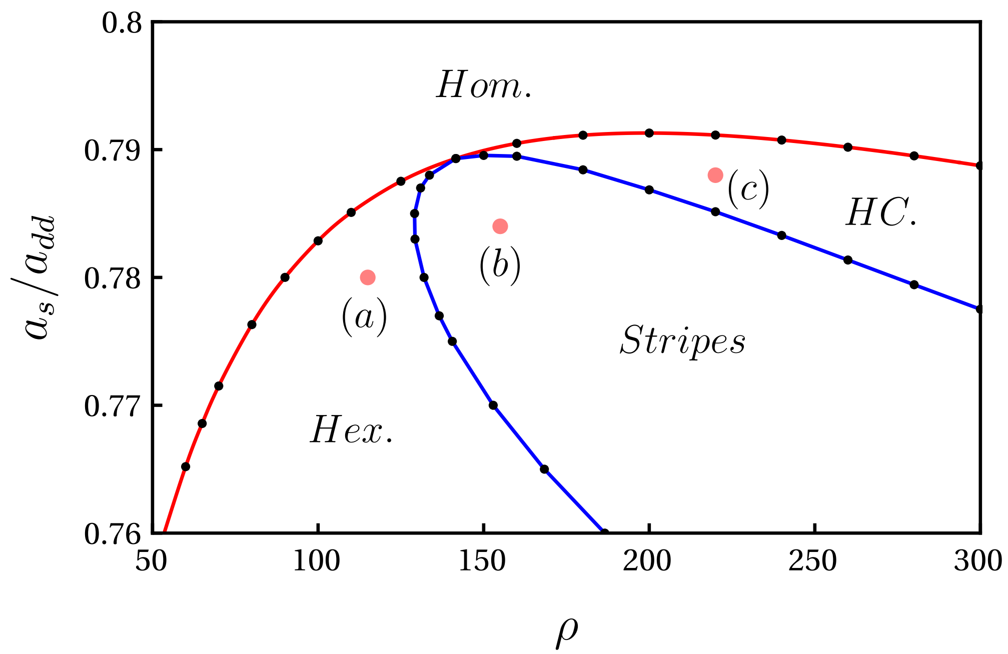
Modulated phase

no condensate

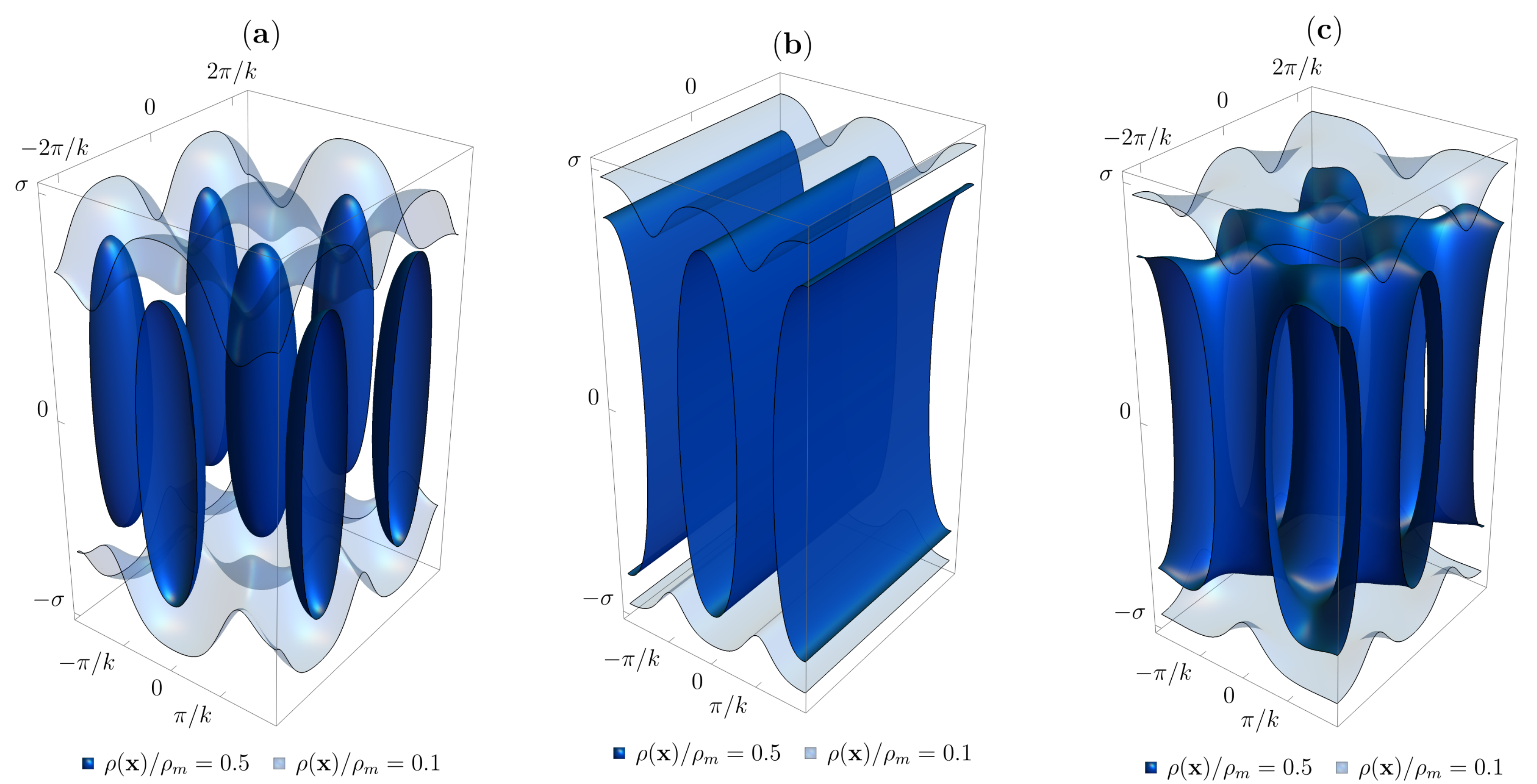
Quasi periodicity and dipolar interactions…

Parameters space:

- contact scattering length (a_s/a_{dd})
- 2D particles density ($\rho_\perp$)
- transversal trap frequency (ω_z)



Typical 3D supersolid solutions after functional minimization on $\psi_\perp(x,y)$:



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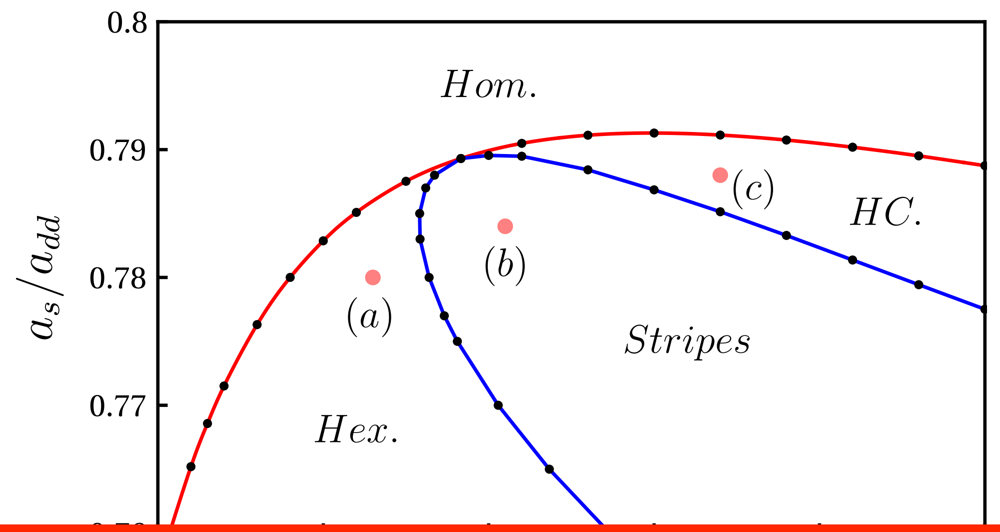


Mean field energy per particle functional of the system at $T = 0$:

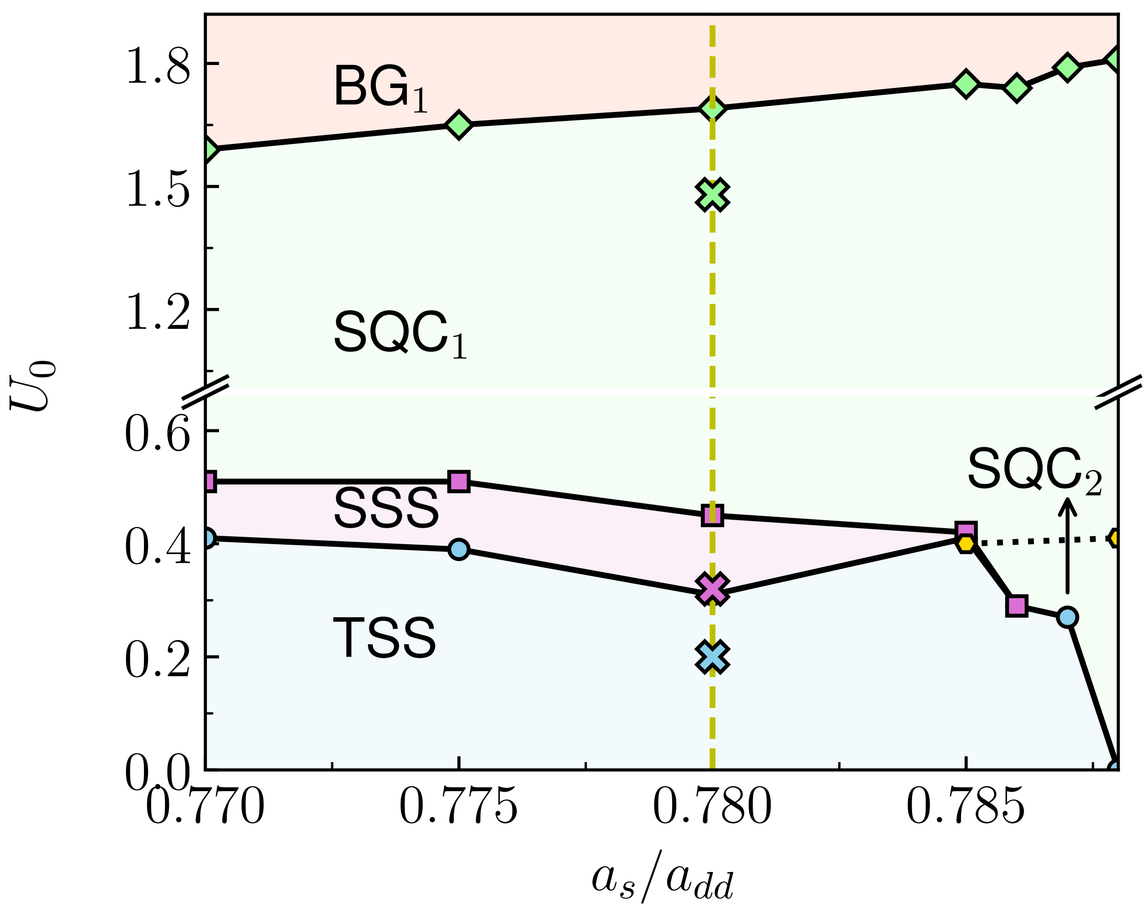
$$\frac{E[\psi]}{N} = \int d\mathbf{r} \Big\{ \frac{1}{2}|\nabla\psi(\mathbf{r})|^2 + U(z)|\psi(\mathbf{r})|^2 + \frac{2}{5}\gamma N^{3/2}|\psi(\mathbf{r})|^5 + N\frac{a_s}{6a_{dd}}|\psi(\mathbf{r})|^4 + \frac{N}{8\pi}\int d\mathbf{r}' V_d(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r})|^2|\psi(\mathbf{r}')|^2 \Big\}.$$

$V_d(\mathbf{r}) = 1/r^3 \times (1 - 3z^2/r^2)$, $U(z) = \frac{1}{2}\omega_z^2 z^2$, $\delta(E_{QF}/N)/(\delta A) = \frac{2}{5}\gamma N^{3/2}|\psi(\mathbf{r})|^5$

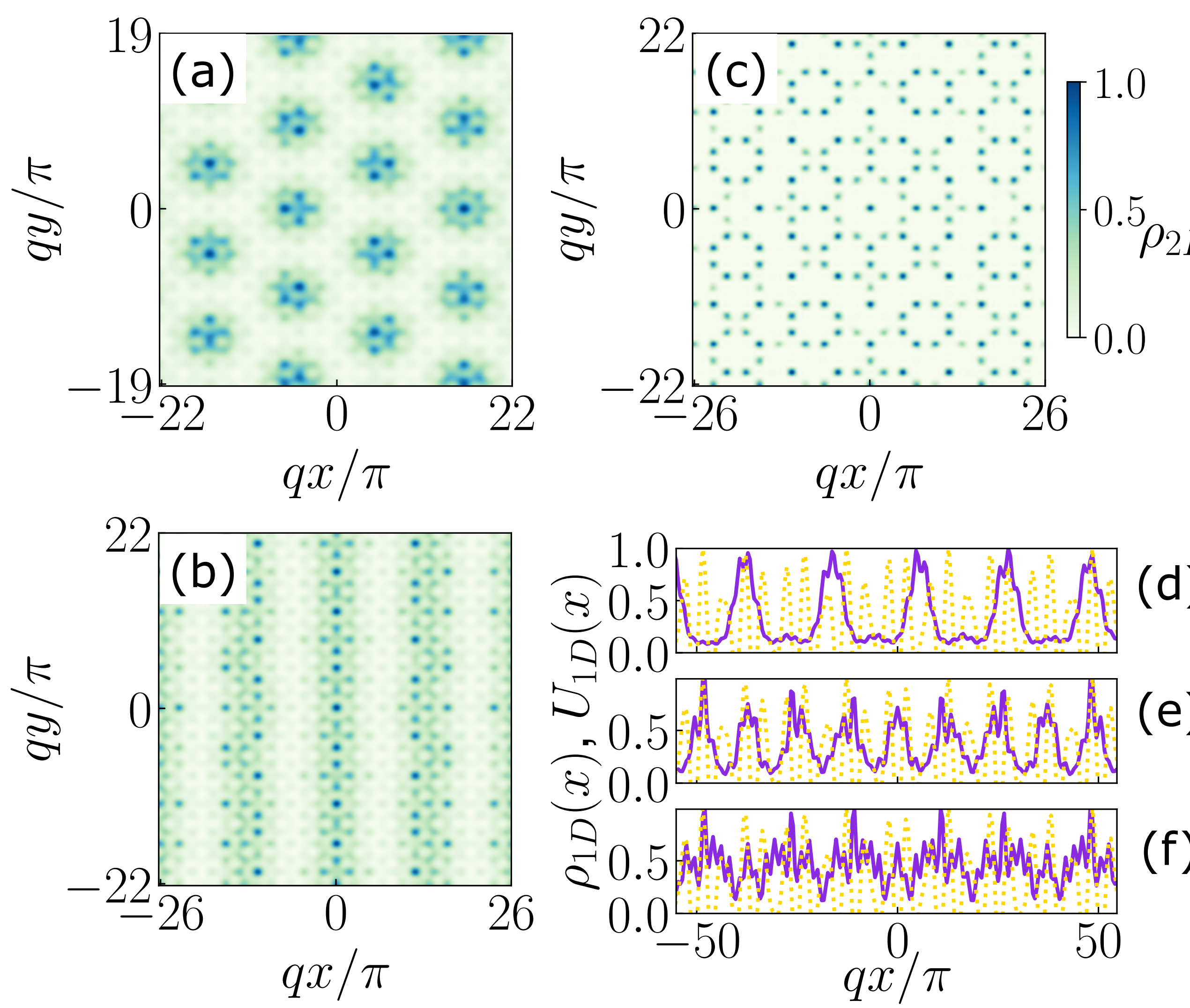
Anzats for the wave function: $\psi(\mathbf{r}) = \sqrt{4(1 - z^2/\sigma^2)/(3\sigma)} \times \psi_\perp(x, y)$ [1, 2]

$\rho(\mathbf{x})/\rho_m = 0.5$ $\rho(\mathbf{x})/\rho_m = 0.1$ $\rho(\mathbf{x})/\rho_m = 0.5$ $\rho(\mathbf{x})/\rho_m = 0.1$ $\rho(\mathbf{x})/\rho_m = 0.5$ $\rho(\mathbf{x})/\rho_m = 0.1$

Phase diagram from the GP equation



Parameters: $\rho_\perp = 120,\ \omega_z = 0.08,\ q = 1.8$
$(q \sim 4k_0)$



Phases identified:

TSS: Triangular supersolid

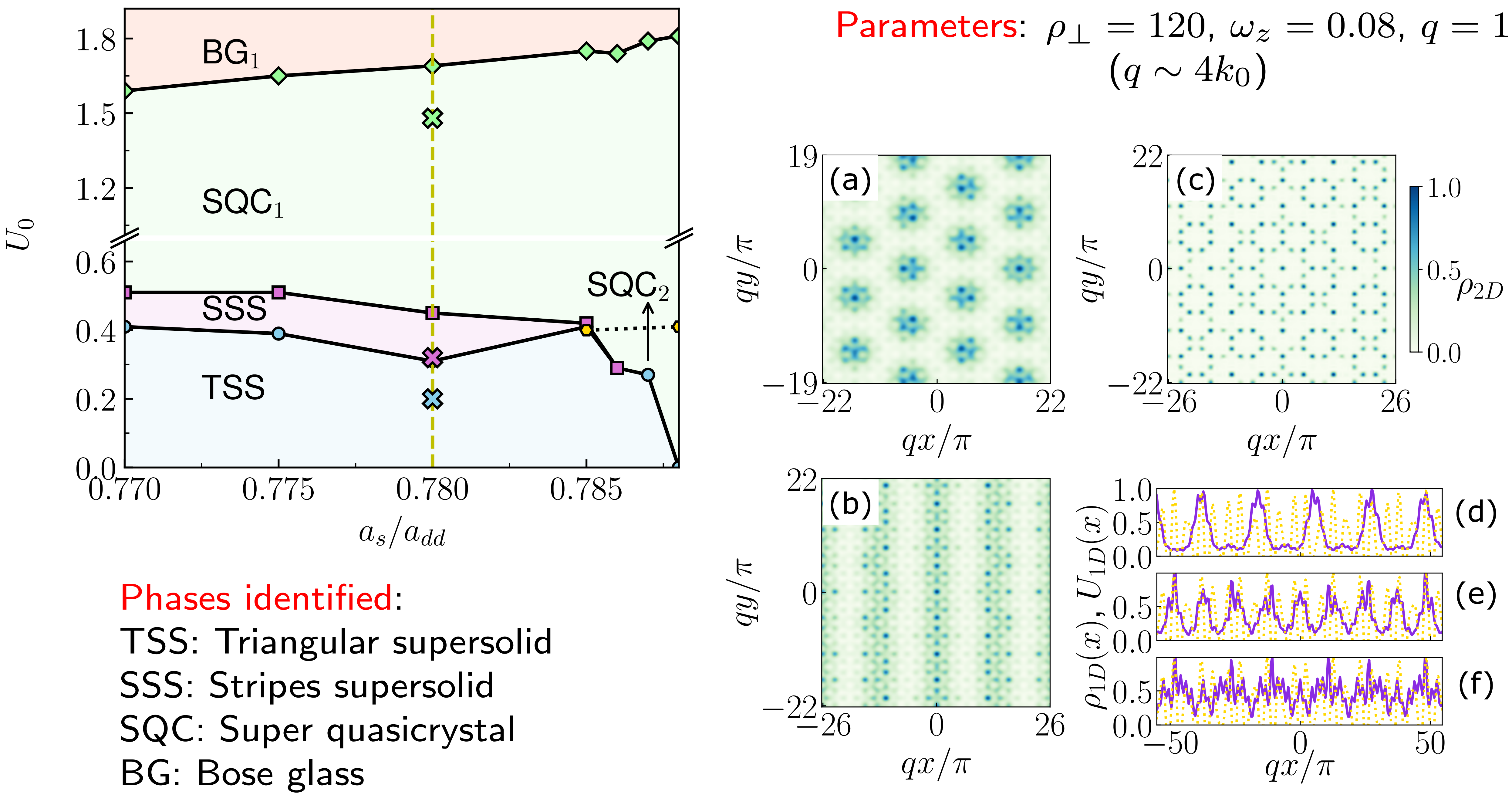
SSS: Stripes supersolid

SQC: Super quasicrystal

BG: Bose glass

Zampronio, Mendoza-Coto, Macrì, and FC PRL **133** (2024)

Phase diagram from the GP equation



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Thank you for listening!

Zampronio, Mendoza-Coto, Macrì, and FC PRL **133** (2024)